Supply Chain Analytics 11 (2025) 100151

Contents lists available at ScienceDirect

Supply Chain Analytics

journal homepage: www.sciencedirect.com/journal/supply-chain-analytics

A machine learning approach to dynamic pricing in multi-channel transportation supply chains

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ARTICLE INFO

Keywords:
Machine learning
Transportation analytics
Multi-channel supply chain
Dynamic pricing
Revenue optimization
Supply chain resiliency

ABSTRACT

Effective pricing strategies are critical for optimizing revenue and maintaining competitiveness in transportation supply chains, particularly in multi-channel environments. This paper presents a machine learning-driven dynamic pricing approach designed to optimize ticket prices across multiple transportation modes, service classes, and sales channels. In particular, the proposed approach integrates predictive analytics and optimization techniques to estimate customer demand, price sensitivity, and revenue potential while accounting for operational constraints such as capacity limits and market share requirements. Machine learning enables the optimization model to dynamically adjust pricing strategies based on historical demand patterns and real-time market fluctuations. To demonstrate its applicability, the approach is applied in a case study in the transportation sector, illustrating its role in optimizing pricing decisions. Additionally, sensitivity analysis highlights the model's robustness against capacity changes, demand fluctuations, and pricing constraints. The findings emphasize the role of supply chain analytics in enhancing pricing strategies, making them more adaptive, data-driven, and resilient to market dynamics.

1. Introduction

The transportation industry faces growing challenges in developing effective pricing strategies, particularly in response to dynamic market conditions and evolving consumer behavior.[1] Pricing plays a fundamental role in shaping demand, influencing customer choices, and ensuring the financial sustainability of transportation services within supply chains. Studies have shown that ticket prices can fluctuate significantly, even for adjacent seats, as transportation providers continuously adjust fares based on real-time demand and supply conditions [18,24]. The rise of digital platforms and multiple sales channels has further increased the complexity of pricing decisions, pushing transportation companies to adopt advanced revenue optimization models. These models must account for factors such as seasonal variations, travel schedules, and service classes, while ensuring profitability and efficiency within transportation supply chains [26–28]. One of the most effective pricing strategies in this environment enables firms to continuously adjust fares in response to shifting demand patterns, helping them balance supply and demand while maximizing revenue. This adaptability plays a crucial role in strengthening the responsiveness and resilience of transportation supply chains [14].

Pricing adjustments in response to real-time market fluctuations play a crucial role in transportation supply chains. One widely used approach is dynamic pricing, which involves continuously updating fares based on changes in demand, supply levels, and external market conditions [17]. In multi-channel pricing, transportation providers adjust fares across different sales platforms to account for variations in consumer behavior and competitive pressures. For example, airlines may offer lower fares on third-party websites to attract budget-conscious customers while maintaining premium rates on their own platforms for brand-loyal

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[1] The transportation industry faces a wide range of challenges beyond pricing, including operational complexity, uncertain demand, capacity constraints, and the need for resilience in the face of disruptions (see, e.g., [11,13,22]).

https://doi.org/10.1016/j.sca.2025.100151
Received 27 March 2025; Received in revised form 29 July 2025; Accepted 29 July 2025
Available online 30 July 2025

passengers [31,32]. Similarly, multi-class pricing categorizes fares based on service tiers (e.g., economy, business), aligning prices with perceived value and willingness to pay [10,25]. These pricing strategies enable firms to manage supply-demand imbalances, optimize revenue, and remain competitive in volatile markets. By incorporating data-driven decision-making, firms can adjust prices dynamically to maximize revenue during peak periods, stimulate demand during off-peak times, and allocate resources more efficiently, ensuring both profitability and operational stability [9].

Beyond pricing structures, an essential aspect of real-time pricing adjustments is the customer utility function, which models how consumers perceive value based on price, service quality, and other factors [15]. Transportation firms use utility-based decision-making to align pricing strategies with customer expectations and market dynamics, improving profitability and competitiveness [12]. However, accurately estimating the parameters of these utility functions – such as utility thresholds and potential demand estimates – is crucial for effective pricing decisions. The importance of such estimation has grown, as recent studies show that real-time demand patterns and consumer behavior insights can significantly improve pricing strategies [20,33]. Machine learning has emerged as a powerful tool in this regard, enabling firms to refine pricing models by capturing price sensitivity, forecasting demand fluctuations, and optimizing revenue across multiple sales channels [5,29].[2] By integrating machine learning into dynamic pricing frameworks, transportation companies can develop more adaptive pricing strategies that address multi-channel complexities and evolving customer preferences, leading to improved efficiency and profitability [8,14].[3]

Several studies have explored the role of machine learning and optimization techniques in pricing and revenue management within transportation and retail sectors. Research in this area has focused on integrating predictive analytics, dynamic pricing, and optimization models to improve pricing strategies and demand forecasting. While prior studies have contributed to pricing optimization in various contexts, much of the existing research remains limited to single-channel or single-mode settings, often overlooking the complexities of multi-channel pricing with heterogeneous customers. Furthermore, the dynamic nature of customer utility functions and price sensitivities across different sales channels remains insufficiently addressed, highlighting the need for more integrated, adaptive pricing frameworks that accommodate multi-channel interactions and evolving consumer behavior.

Against this background, the contributions of this paper are as follows:

- This study presents a pricing optimization approach that accounts for multiple transportation modes (air, rail, bus), two service classes (economy, business), and three sales channels (company website, ticketing apps, travel agencies). Unlike previous studies that focus on isolated pricing settings, our approach captures the interactions between different channels and customer groups, leading to more effective pricing strategies.
- By incorporating predictive analytics, our approach links machine learning-based demand estimates directly to pricing decisions. This integration enhances the accuracy of estimating customer preferences and price sensitivity, enabling more precise demand forecasting and revenue optimization.
- Unlike conventional approaches that treat often customer preferences as fixed, our approach accounts for variations in price sensitivity and purchasing behavior across sales channels and over time. By analyzing historical transaction data and real-time demand fluctuations, the proposed approach dynamically adjusts pricing strategies to align with changing market conditions.
- The proposed approach not only aims to maximize revenue but also ensures feasibility by incorporating capacity constraints and minimum market share requirements, ensuring that pricing decisions remain practical for real-world transportation systems.

To this end, Section 2 reviews the related literature. Section 3 presents the proposed approach. Section 4 illustrates its applicability through a case study and provides a sensitivity analysis to evaluate robustness under varying market conditions. Section 5 concludes the paper with key insights and directions for future research.

2. Literature review

The literature on transportation pricing has evolved considerably in response to increasing complexities in customer behavior, technological innovation, and distribution channel diversification. This section provides an overview of relevant academic contributions across key domains that shape modern pricing strategies in the transportation industry.

2.1. Theoretical foundations of transportation pricing

The evolution of transportation pricing theory has progressed from cost-based fare structures to sophisticated revenue management systems that integrate advanced analytics and behavioral insights. Foundational research established key economic principles linking pricing strategies to market efficiency, consumer welfare, and operator profitability. These theoretical underpinnings form the conceptual basis for understanding how pricing decisions affect transportation system performance and broader market dynamics [5]. Initial models focused on cost recovery and basic demand-supply relationships, assuming relatively homogeneous and static customer preferences. However, empirical studies soon revealed significant heterogeneity in consumer behavior, price sensitivity, and service expectations across market segments and time periods. This recognition led to more refined pricing theories that incorporated customer segmentation, utility maximization, and strategic interaction among transportation providers. Advances in microeconomic theory – particularly in consumer choice modeling and competition dynamics – have significantly shaped the development of transportation pricing. Game-theoretic approaches have enhanced the understanding of competitive pricing, while behavioral economics has provided insight into consumer decision-making under varying market conditions and information constraints [30].

2.2. Pricing strategies and mechanisms

Dynamic pricing has emerged as one of the most transformative innovations in transportation revenue management, enabling real-time fare adjustments in response to shifting market conditions. Elmaghraby and Keskinocak [14] laid the theoretical groundwork for dynamic pricing, illustrating how real-time price adjustments can enhance supply chain responsiveness and operational efficiency. Building on this, Gönsch et al. [17] developed models for continuous fare optimization that account for demand variability, capacity constraints, and external market forces. Their empirical studies demonstrated significant revenue gains through systematic price adjustments, especially during periods of demand volatility. Den Boer [9] further contributed by incorporating data analytics and machine learning into dynamic pricing frameworks.

[2] Machine learning is increasingly used to support business decision-making processes in areas such as supply chain management, customer behavior prediction, and supplier evaluation. Examples include its application in manufacturing supply chains [16], customer experience modeling in online services [3], and strategic supplier selection [2].

[3] For an introduction to machine learning models, see, e.g., Burger [7] and Kubat [19]. For an overview of data science methods and their relevance to machine learning in business administration, see Afsharian [1] and Broere et al. [6].

These data-driven methods allow providers to optimize pricing strategies across peak and off-peak periods while maintaining service quality and customer satisfaction. More recent studies, including Narangajavana et al. [24] and Kamandanipour et al. [18], provide detailed analyses of fare fluctuations, revealing that dynamic pricing systems are now widely adopted and play a pivotal role across various transportation modes.

2.3. Customer utility modeling and behavioral analysis

Understanding customer decision-making processes represents a critical component of effective transportation pricing strategies. Customer utility theory provides the conceptual foundation for modeling how consumers evaluate transportation options based on price, service quality, convenience, and other relevant factors. Friedman [15] established fundamental principles for utility-based pricing models in transportation contexts, demonstrating how utility functions can capture customer preferences and guide pricing decisions. This foundational work provided theoretical frameworks for understanding customer choice behavior and predicting demand responses to pricing changes. Dutta and Mitra [12] advanced utility-based pricing research by developing practical methodologies for estimating utility function parameters and incorporating these estimates into revenue optimization models. Their work emphasized the critical importance of accurate utility parameter estimation for effective pricing decisions and demonstrated how utility-based approaches can improve both profitability and customer satisfaction. Recent developments in behavioral analysis have revealed the dynamic nature of customer preferences and utility functions. Liu et al. [21] and Zhang et al. [33] have shown that customer behavior patterns, price sensitivity levels, and utility function parameters vary significantly over time and across different market conditions. This research challenges traditional assumptions about static customer preferences and highlights the need for adaptive pricing strategies capable of responding to evolving consumer behavior.

2.4. Machine learning applications in revenue management

The integration of machine learning techniques into transportation pricing and revenue management has emerged as a transformative development, enabling providers to analyze vast datasets and identify complex patterns that traditional analytical approaches cannot capture effectively. Machine learning applications in this domain focus on demand forecasting, price optimization, customer segmentation, and competitive analysis. Bertsimas and Perakis [5] pioneered the application of advanced machine learning techniques in pricing optimization, demonstrating how these approaches can significantly improve demand forecasting accuracy and revenue performance. Their research established methodological foundations for integrating predictive analytics into pricing decision-making processes.

Chen and Hu [8] have advanced machine learning applications by developing sophisticated models for multi-channel revenue optimization. Their work demonstrates how machine learning enables transportation providers to capture complex interactions between customer segments, pricing strategies, and market conditions across different distribution channels simultaneously. Yuan et al. [29] have contributed to this literature by developing integrated frameworks combining machine learning-based demand forecasting with real-time pricing optimization. Their research shows how these integrated approaches can achieve substantial improvements in revenue performance while maintaining operational feasibility and customer satisfaction. The predictive capabilities of machine learning have proven particularly valuable for capturing temporal variations in customer behavior, price sensitivity patterns, and demand fluctuations across different market segments and distribution channels. These capabilities enable more sophisticated approaches to revenue optimization that account for the heterogeneous and dynamic nature of transportation markets.

2.5. Multi-modal transportation pricing

Multi-modal transportation pricing represents an emerging area of research addressing the complexities of pricing optimization across different transportation modes serving similar markets. This research stream recognizes that customers often consider multiple transportation options when making travel decisions, creating interdependencies between pricing strategies across different modes. Ding et al. [10] and Otero & Akhavan-Tabatabaei [25] conducted foundational research on multi-class pricing strategies, examining how service differentiation across economy and business classes influences customer choice behavior and revenue optimization opportunities. Their work demonstrated the importance of aligning pricing strategies with perceived value propositions and customer willingness to pay across different service tiers.

Despite substantial progress in transportation pricing research, several critical gaps remain that limit the practical applicability and effectiveness of existing approaches. Current literature predominantly focuses on single-mode or single-channel pricing optimization, failing to capture the complex interdependencies characterizing modern transportation markets. The integration of multiple transportation modes, service classes, and distribution channels within unified pricing frameworks remains largely unexplored, despite the practical importance of these interactions for revenue optimization. Existing research typically treats these elements in isolation, missing opportunities for comprehensive optimization across entire transportation ecosystems.

Furthermore, the dynamic nature of customer utility functions and behavioral patterns across different channels and time periods requires more sophisticated analytical approaches than currently available in the literature. Most existing research treats customer preferences as static parameters, failing to capture the adaptive and context-dependent nature of consumer behavior in dynamic market environments. The incorporation of practical operational constraints, such as capacity limitations and market share requirements, into pricing optimization models also requires further development. Many existing approaches focus primarily on theoretical optimization without adequate consideration of real-world implementation challenges and constraints.

These research gaps highlight significant opportunities for developing more comprehensive, integrated approaches to transportation pricing optimization that can address the full complexity of contemporary transportation markets while maintaining practical and operational effectiveness.

3. The proposed approach

3.1. Overview

The primary objective of this approach is to determine optimal ticket prices and sales volumes across multiple dimensions, including modes of transportation (airplane, train, bus), service classes (economy and business), and sales channels (company websites, ticketing applications, travel agencies) within a 30-day decision period. It accounts for variable customer demand, capacity constraints, and market share requirements, while ensuring compliance with price limits and daily fluctuation thresholds.

To achieve this, the approach combines historical sales data with real-time market indicators to continuously assess key demand drivers. Machine learning techniques estimate channel-specific customer preferences and price sensitivity parameters, enabling dynamic pricing adjustments based on seasonality, holidays, and event-driven demand variations. These insights ensure that pricing decisions remain both consumer-responsive and operationally feasible, incorporating fixed transportation capacities and regulatory pricing constraints such as minimum and maximum fare limits.

Beyond revenue maximization, the approach integrates critical business requirements by enforcing minimum market share thresholds across transportation modes and service categories to maintain competitive positioning. It assumes that ticket demand is a deterministic function of price and time, treated independently across modes, classes, and channels. Additionally, it considers customer purchasing decisions as influenced by a threshold utility function, which varies based on pricing and sales channel attributes. However, competitive pricing dynamics, cancellations, and refunds are not included, as the focus remains on the interaction between price, demand, and operational constraints.

3.2. Mathematical assumptions

Building upon the overview provided in Section 3.1, the following assumptions define the operational boundaries and mathematical structure of the dynamic pricing optimization model in Section 3.3.

1. Transportation modes and service classes: The model considers three transportation modes (airplane, train, and bus) and two service classes (economy and business), each with distinct price elasticities and utility functions to reflect differentiated customer segments.
2. Time horizon: A discrete 30-day decision period before departure is implemented, with daily pricing decisions to capture temporal variations in customer purchasing behavior.
3. Multi-channel distribution: The model incorporates three sales channels (company website, ticketing applications, and travel agencies), each with channel-specific utility parameters aligned with the data-driven insights outlined in Section 3.1.
4. Demand characteristics: Demand is modeled as a deterministic linear function of price, with time-dependent parameters. Price elasticities differ between economy and business classes, reflecting the market segmentation outlined in Section 3.1. Demand is also assumed to be independent across transportation modes, service classes, and sales channels.
5. Capacity constraints: Each transportation mode has a fixed capacity that cannot be exceeded across all time periods and service classes, ensuring operational feasibility.
6. Price Dynamics and Boundaries: Daily price fluctuations are restricted to a maximum $\pm 10\%$ change to maintain pricing stability. Predefined minimum and maximum price limits can be applied to each transportation mode and service class.
7. Market share requirements: Each transportation mode and service class must maintain minimum market share thresholds, aligning with the competitive positioning objectives stated in Section 3.1.
8. Customer decision process: Purchasing decisions follow a utility-based model, where customers buy tickets only when their perceived utility exceeds a defined threshold, influenced by price and channel-specific attributes.
9. Simplifying exclusions: The model does not consider competitor-driven price adjustments, ticket cancellations, modifications, refunds, or cross-elasticities between transportation modes and service classes.

3.3. Mathematical notations

To formulate the optimization model given in Section 3.4, we first introduce the following notations, which define its key variables and parameters.

Indices:	
v	Index for transportation modes (airplane, train, bus), where $v \in \{1, 2, \ldots, V\}$
c	Index for service classes (economy, business), where $c \in \{1, \ldots, C\}$
s	Index for sales channels (company website, ticketing applications, travel agencies), where $s \in \{1, 2, \ldots, S\}$
t	Index for days remaining until departure, where $t \in \{1, 2, 3, \ldots, T\}$

Parameters:	
K_v	Capacity of transportation mode v.
$P^{\min}_{v,c,t}$	Minimum price threshold for transportation mode v, service class c, on day t.
$P^{\max}_{v,c,t}$	Maximum price threshold for transportation mode v, service class c, on day t.
$Q^{\min}_{v,c,s,t}$	Minimum quantity bound for mode v, class c, channel s, day t.
$Q^{\max}_{v,c,s,t}$	Maximum quantity bound for mode v, class c, channel s, day t.
$\alpha_{s,t}$, $\gamma_{s,t}$	Base utility parameters for sales channel s and on day t.
β_{c_1} , δ_{c_2}	Price sensitivity coefficients for economy and business classes, respectively.
$\epsilon_{c_1,s,t}$, $\epsilon_{c_2,s,t}$	Utility threshold required for a purchase in service class c, through sales channel s, on day t.
$D_{v,c,s,t}$	Potential demand for mode v, class c, channel s, on day t.
$M_{v,c}$	Minimum market share requirement for mode v and service class c.
$	Unit of measurement for all monetary values in the model.

Decision variables:	
$p_{v,c,t}$	Ticket price for transportation mode v and service class c on day t.
$q_{v,c,s,t}$	Quantity of tickets sold for mode v, class c, through channel s, on day t.

3.4. The base optimization model

With the problem outlined in Section 3.1, the assumptions defined in Section 3.2, and the notation introduced in Section 3.3, this section presents the mathematical model, beginning with the objective function and followed by the corresponding constraints.

$$\max\ R = \sum_{v}\sum_{c}\sum_{s}\sum_{t} p_{v,c,t} \cdot q_{v,c,s,t} \tag{1}$$

The objective function aims to maximize total revenue from ticket sales across all transportation modes, service classes, sales channels, and time periods.

The optimization model is subject to several constraints that ensure feasibility, regulatory compliance, and alignment with operational and market requirements. These constraints include capacity limitations, demand restrictions, pricing boundaries, market share requirements, and consumer purchasing behavior:

- Capacity constraint

$$\sum_{c} q_{v,c,s,t} \le K_v, \forall v, t, s \tag{2}$$

ensures that the total number of tickets sold does not exceed the available capacity for each transportation mode, preventing overbooking and ensuring that sales remain within the operational limits.

- Demand constraint

$$q_{v,c,s,t} \le D_{v,c,s,t}, \forall v, c, s, t \tag{3}$$

guarantees that ticket sales do not surpass potential demand for each combination of transportation mode, service class, sales channel, and time period, ensuring that sales remain aligned with market projections.

- Pricing boundaries

$$P^{\min}_{v,c,t} \le p_{v,c,t} \le P^{\max}_{v,c,t}, \forall v, c, t \tag{4}$$

maintains ticket prices within predefined minimum and maximum limits, reflecting market conditions, customers' willingness to pay, and legal or regulatory constraints.

- Market share requirement

$$\sum_s \sum_t q_{v,c,s,t} \geq M_{v,c}, \forall v, c \tag{5}$$

requires that each transportation mode and service class meets its minimum market share threshold, supporting competitive positioning and long-term stability.

- Customer utility functions

$$U_{c_1,s,t}(p) = \alpha_{s,t} - \beta_{c_1}.p, \forall c_1, s, t \tag{6}$$

$$U_{c_2,s,t}(p) = \gamma_{s,t} - \delta_{c_2}.p, \forall c_2, s, t \tag{7}$$

model customer utility as a function of ticket price and sales channel attributes, capturing the relationship between price, customer satisfaction, and the likelihood of purchase for different service classes.

- Projected demand restrictions

$$D_{v,c_1,s,t}(p) = \max\left(0, U_{c_1,s,t}(p) - \epsilon_{c_1,s,t}\right), \forall v, c_1, s, t \tag{8}$$

$$D_{v,c_2,s,t}(p) = \max\left(0, U_{c_2,s,t}(p) - \epsilon_{c_2,s,t}\right), \forall v, c_2, s, t \tag{9}$$

approximates potential ticket demand using utility functions, ensuring that price fluctuations are accurately reflected in projected demand estimates.

- Non-negativity constraint

$$q_{v,c,s,t} \geq 0 \quad \forall v, c, s, t \tag{10}$$

ensures that ticket sales values remain non-negative.

3.5. The mixed-integer linear program

The objective function (1) is bilinear because it involves the product of two decision variables, $p_{v,c,t}$ (ticket price) and $q_{v,c,s,t}$ (quantity of ticket sold). This nonlinearity complicates the optimization process. To handle this, we apply a separable quadratic transformation technique, following the approach of Asghari et al. [4].[4] More precisely, the product term is expressed as the difference of squares of two auxiliary variables y_1 and y_2, defined as the half-sum and half-difference of the original variables.

The resulting quadratic terms y_1^2 and y_2^2 are then approximated using piecewise linear functions, enabling the reformulation of the model as a mixed-integer linear program (MILP) that can be solved efficiently by commercial solvers. This transformation relies on known bounds of the original variables to define feasible domains for the auxiliary variables and ensure approximation accuracy. When applicable, if one of the original variables appears only within these products and is nonnegative, simpler bounding strategies can also be employed.

It is important to note that this linearization yields an approximate representation of the original bilinear model. The quality of the approximation depends on the number and spacing of the piecewise segments and the tightness of the variable bounds.

Now we define the auxiliary variables used in the transformation:

$$y_{1,v,c,s,t} = \frac{1}{2}\left(p_{v,c,t} + q_{v,c,s,t}\right), \quad \forall v, c, s, t \tag{11}$$

$$y_{2,v,c,s,t} = \frac{1}{2}(p_{v,c,t} - q_{v,c,s,t}). \quad \forall v, c, s, t \tag{12}$$

Using these definitions, the original bilinear product $p_{v,c,t}.q_{v,c,s,t}$ can be reformulated as:

$$y_{1,v,c,s,t}^2 - y_{2,v,c,s,t}^2 := p_{v,c,t}.q_{v,c,s,t}. \tag{13}$$

This transformation converts the bilinear term into a difference of two univariate quadratic functions, each depending on a single auxiliary variable. This structure is essential for applying piecewise linear approximations to each quadratic term separately, as detailed below.

To ensure approximation accuracy, we define bounds for the auxiliary variables based on the known lower and upper bounds of the original decision variables. These bounds keep the approximated values within feasible and interpretable ranges during optimization:

$$\frac{1}{2}\left(p_{v,c,t}^{\min} + q_{v,c,s,t}^{\min}\right) \leq y_{1,v,c,s,t} \leq \frac{1}{2}\left(p_{v,c,t}^{\max} + q_{v,c,s,t}^{\max}\right), \tag{14}$$

$$\frac{1}{2}(p_{v,c,t}^{\min} - q_{v,c,s,t}^{\max}) \leq y_{2,v,c,s,t} \leq \frac{1}{2}(p_{v,c,t}^{\max} - q_{v,c,s,t}^{\min}). \tag{15}$$

These bounds ensure that the auxiliary variables stay within valid domains, which is crucial for both the accuracy of the approximation and the feasibility of the overall optimization.

For the quadratic terms $y_{1,v,c,s,t}^2$ and $y_{2,v,c,s,t}^2$, we employ piecewise linear approximation: Let N be the number of breakpoints for approximation. For each auxiliary variable y_k (where k represents either y_1 or y_2), the domain is discretized into N uniformly spaced intervals:

- Breakpoints: $b_0, b_1, \ldots, b_N$ where b_0 = lower bound, b_N = upper bound
- λ_i: weight associated with breakpoint i, $\forall i \in \{0, 1 \ldots, N\}$

To ensure the piecewise linear approximation maintains its structural integrity, we impose Special Ordered Set of Type 2 constraints. These constraints enforce that the approximation adheres to the structure of the underlying quadratic function:

$$\sum_{i=0}^{N} \lambda_i = 1 \tag{16}$$

$$y_k = \sum_{i=0}^{N} \lambda_i.b_i \tag{17}$$

$$y_k{}^2 = \sum_{i=0}^{N} \lambda_i.b_i{}^2 \tag{18}$$

At most two adjacent λ_i can be nonzero. (19)

This adjacency constraint is crucial for maintaining the linear interpolation between breakpoints. It ensures that the quadratic terms are accurately approximated through convex combinations, while preserving the overall linear structure of the optimization model.

Constraints (8) and (9) include a nonlinear expression involving a maximum operator, which introduces nonlinearity. To linearize these constraints, we introduce an auxiliary continuous variable $\phi_{v,c,s,t}$ a binary variable $y_{v,c,s,t} \in \{0, 1\}$ and the following constraints for all indices v, c, s, t, to replace the utility-based demand functions:

$$\phi_{v,c_1,s,t} = U_{c_1,s,t}(p) - \epsilon_{c_1,s,t} \quad \forall v, c_1, s, t \tag{20}$$

$$\phi_{v,c_2,s,t} = U_{c_2,s,t}(p) - \epsilon_{c_2,s,t} \quad \forall v, c_2, s, t \tag{21}$$

We then introduce the following linear constraints to represent the max operator:

$$\phi_{v,c,s,t} \geq U_{c,s,t}(p) - \epsilon_{c,s,t} \quad \forall v, c, s, t \tag{22}$$

$$\phi_{v,c,s,t} \geq 0 \quad \forall v, c, s, t \tag{23}$$

$$\phi_{v,c,s,t} \leq U_{c,s,t}(p) - \epsilon_{c,s,t} + M(1 - y_{v,c,s,t}) \quad \forall v, c, s, t \tag{24}$$

[4] See also, Marandi et al. [23].

$$\phi_{v,c,s,t} \leq My_{v,c,s,t} \quad \forall v,c,s,t \tag{25}$$

$$y_{v,c,s,t} \in \{0,1\} \quad \forall v,c,s,t \tag{26}$$

Here, M is a sufficiently large constant that bounds the utility difference term. These constraints ensure that $\phi_{v,c,s,t}$ equals either the positive part of the utility gap or zero, correctly replicating the max operator in the linearized form.

Now, having linearized all nonlinear components, the MILP formulation is as follows:

$$\max\ R = \sum_v \sum_c \sum_s \sum_t y^2_{1,v,c,s,t} - y^2_{2,v,c,s,t} \tag{27}$$

$$\text{s.t.} \sum_c q_{v,c,s,t} \leq K_v \quad \forall v,t,s \tag{28}$$

$$q_{v,c,s,t} \leq D_{v,c,s,t} \quad \forall v,c,s,t \tag{29}$$

$$P^{\min}_{v,c,t} \leq p_{v,c,t} \leq P^{\max}_{v,c,t} \quad \forall v,c,t \tag{30}$$

$$\sum_s \sum_t q_{v,c,s,t} \geq M_{v,c} \quad \forall v,c \tag{31}$$

$$U_{c_1,s,t}(p) = \alpha_{s,t} - \beta_{c_1}.p \quad \forall c_1,s,t \tag{32}$$

$$U_{c_2,s,t}(p) = \gamma_{s,t} - \delta_{c_2}.p \quad \forall c_2,s,t \tag{33}$$

$$y_{1,v,c,s,t} = \frac{1}{2}(p_{v,c,t} + q_{v,c,s,t}) \quad \forall v,c,s,t \tag{34}$$

$$y_{2,v,c,s,t} = \frac{1}{2}(p_{v,c,t} - q_{v,c,s,t}) \quad \forall v,c,s,t \tag{35}$$

$$\frac{1}{2}\left(p^{\min}_{v,c,t} + q^{\min}_{v,c,s,t}\right) \leq y_{1,v,c,s,t} \leq \frac{1}{2}\left(p^{\max}_{v,c,t} + q^{\max}_{v,c,s,t}\right), \quad \forall v,c,s,t \tag{36}$$

$$\frac{1}{2}(p^{\min}_{v,c,t} - q^{\max}_{v,c,s,t}) \leq y_{2,v,c,s,t} \leq \frac{1}{2}(p^{\max}_{v,c,t} - q^{\min}_{v,c,s,t}). \quad \forall v,c,s,t \tag{37}$$

$$\sum_{i=0}^{N} \lambda_i = 1 \tag{38}$$

$$y_k = \sum_{i=0}^{N} \lambda_i.b_i \tag{39}$$

$$y_k{}^2 = \sum_{i=0}^{N} \lambda_i.b_i{}^2 \tag{40}$$

$$\text{At most two adjacent } \lambda_j \text{ can be nonzero} \tag{41}$$

$$\phi_{v,c_1,s,t} = U_{c_1,s,t}(p) - \epsilon_{c_1,s,t} \quad \forall v,c_1,s,t \tag{42}$$

$$\phi_{v,c_2,s,t} = U_{c_2,s,t}(p) - \epsilon_{c_2,s,t} \quad \forall v,c_2,s,t \tag{43}$$

$$\phi_{v,c,s,t} \geq U_{c,s,t}(p) - \epsilon_{c,s,t} \quad \forall v,c,s,t \tag{44}$$

$$\phi_{v,c,s,t} \geq 0 \quad \forall v,c,s,t \tag{45}$$

$$\phi_{v,c,s,t} \leq U_{c,s,t}(p) - \epsilon_{c,s,t} + M(1 - y_{v,c,s,t}) \quad \forall v,c,s,t \tag{46}$$

$$\phi_{v,c,s,t} \leq My_{v,c,s,t} \quad \forall v,c,s,t \tag{47}$$

$$y_{v,c,s,t} \in \{0,1\} \quad \forall v,c,s,t \tag{48}$$

$$q_{v,c,s,t} \geq 0 \quad \forall v,c,s,t \tag{49}$$

$$\lambda_i \geq 0 \quad \forall i \in \{1,2\} \tag{50}$$

3.6. Parameter estimation

The dynamic pricing model introduced in the previous sections requires accurate estimation of several critical parameters to effectively capture market dynamics and customer behavior. This integration creates a closed-loop system where historical data drives parameter estimation, which in turn guides optimization decisions, generating new transaction data that further refines parameter estimates in subsequent iterations. To ensure consistency with the optimization framework and preserve economic interpretability, regression is employed for parameter estimation. The estimated parameters directly inform both the objective function and the constraints of the model in Section 3.5:[5]

1. Utility functions are defined with the estimated base utility parameters ($\alpha_{s,t}$, $\gamma_{s,t}$) and price sensitivity coefficients (β_{c_1}, δ_{c_2}).
2. Utility thresholds ($\epsilon_{c_1,s,t}$, $\epsilon_{c_2,s,t}$) to determine the minimum utility required for ticket purchases.
3. Potential demand estimates ($D_{v,c,s,t}$), regulating ticket availability based on projected demand.

At its core, our parameter estimation approach utilizes regression models to systematically analyze historical transaction data, temporal patterns, and market conditions. This regression-based framework quantifies the relationship between observed market responses and pricing variables while preserving the economic interpretability required for practical implementation. The methodology also accounts for variations in customer preferences across sales channels (company websites, ticketing applications, travel agencies) and service classes (economy, business).

The parameter estimation process employs distinct regression models for each combination of service class and sales channel, allowing for targeted capture of segment-specific behavioral patterns. The complete regression models are specified as follows:

For economy class (c_1):

$U_{c_1,s,t}(p) = \beta_{0,s} + \beta_t.t + \beta_{t^2}.t^2 + \beta_p.p + \beta_w.S_w + \beta_h.S_h + \sum_{j=1}^{6} \beta_{W_j}.W_j + \beta_B.B_L.I(s=1) + \beta_v.B_v.I(s=1) + \beta_A.A_r.I(s=2) + \beta_x.A_x.I(s=2) + \beta_m.F_m.I(s=3) + \beta_o.F_o.I(s=3) + \varepsilon_{c_1,s,t}$

For business class (c_2):

$U_{c_2,s,t}(p) = \theta_{0,s} + \theta_t.t + \theta_{t^2}.t^2 + \theta_p.p + \theta_w.S_w + \theta_h.S_h + \sum_{j=1}^{6} \theta_{W_j}.W_j + \theta_B.B_L.I(s=1) + \theta_v.B_v.I(s=1) + \theta_A.A_r.I(s=2) + \theta_x.A_x.I(s=2) + \theta_m.F_m.I(s=3) + \theta_o.F_o.I(s=3) + \varepsilon_{c_2,s,t}$

where:

- $U_{c,s,t}(p)$ represents the observed utility for service class c, through sales channel s, on day t
- p is the ticket price variable
- t is a continuous variable representing days until departure, ranging from 1 to 30 (days)
- S_w is a binary variable (0,1) indicating high season periods
- S_h is a binary variable (0,1) indicating holiday periods
- W_j are variables for each day of the week ($j = 1,2,\ldots,7$)
- $I(s = k)$ is an indicator function equal to 1 if $s = k$ and 0 otherwise.
- B_L is the average page load time on the company website (seconds)
- B_v is the website conversion rate (percentage)
- A_r is the average app store rating (1–5 scale)
- A_x is the crash rate percentage for mobile applications (percentage)

[5] A regression-based approach is applied in this study to construct the estimation framework, offering a transparent and interpretable method that aligns with the structure of the linear optimization model. Alternative methods such as regression trees or random forests may also be applied, provided that the nature of the input data and the requirements of the pricing context support their use (for an introduction to machine learning models, see, e.g., Burger [7]; Kubat [19].

- F_m is the commission percentage for travel agencies (percentage)
- F_o is the historical volume indicator for travel agencies (percentage of total sales)
- $\beta_{0,s}$ and $\theta_{0,s}$ are the channel-specific intercept terms for economy class and business class, respectively
- β_t and θ_t are the coefficients for days until departure in economy class and business class, respectively (utility units per day)
- β_{t^2} and θ_{t^2} are the coefficients for squared days until departure in economy class and business class, respectively (utility units per day^2)
- β_p and θ_p are the price sensitivity coefficients for economy class and business class, respectively (utility units per \$), equivalent to β_{c_1} and δ_{c_2} in the simplified notation
- β_w and θ_w are the coefficients for high season indicator in economy class and business class, respectively
- β_h and θ_h are the coefficients for holiday indicator in economy class and business class, respectively
- $\beta_{W,j}$ and $\theta_{W,j}$ are coefficients for day of week indicators in economy class and business class, respectively
- β_B, β_v and θ_B, θ_v are coefficients for website variables in economy class and business class, respectively
- β_A, β_x and θ_A, θ_x are coefficients for mobile app variables in economy class and business class, respectively
- β_m, β_o and θ_m, θ_o are coefficients for travel agency variables in economy class and business class, respectively
- $\varepsilon_{c_1,s,t}$ and $\varepsilon_{c_2,s,t}$ represent the error terms in the respective regression models.

For implementation in the optimization model, the base utility parameters $\alpha_{s,t}$ and $\gamma_{s,t}$ from the simplified utility functions are defined as:

$$\alpha_{s,t} = \beta_{0,s} + \beta_t.t + \beta_{t^2}.t^2 + \beta_p.p + \beta_w.S_w + \beta_h.S_h + \sum_{j=1}^{7} \beta_{W_j}.W_j + \beta_B.B_L.I(s = 1) + \beta_v.B_v.I(s = 1) + \beta_A.A_r.I(s = 2) + \beta_x.A_x.I(s = 2) + \beta_m.F_m.I(s = 3) + \beta_o.F_o.I(s = 3)$$

$$\gamma_{s,t} = \theta_{0,s} + \theta_t.t + \theta_{t^2}.t^2 + \theta_p.p + \theta_w.S_w + \theta_h.S_h + \sum_{j=1}^{7} \theta_{W_j}.W_j + \theta_B.B_L.I(s = 1) + \theta_v.B_v.I(s = 1) + \theta_A.A_r.I(s = 2) + \theta_x.A_x.I(s = 2) + \theta_m.F_m.I(s = 3) + \theta_o.F_o.I(s = 3)$$

And the price sensitivity coefficients are:

$\beta_{c_1} = -\beta_p$ and $\delta_{c_2} = -\theta_p$

The negative signs are applied to align with the utility functions, where price has a negative impact on utility. These models are estimated using ordinary least squares (OLS) with heteroskedasticity-robust standard errors to account for potential variance instability across different price points.

In the following, we detail the dependent and independent variables used in our regression models.

3.6.1. Dependent variables

The utility functions $U_{c_1,s,t}(p)$ and $U_{c_2,s,t}(p)$ serve as dependent variables in our regression models. Since utility is not directly observable, we employ a transformation of purchase probability as a proxy for utility:

$$U_{c,s,t}(p) = \ln(N_b/N_{nb})$$

where:

- N_b represents the count of tickets actually purchased at price p for class c, through channel s, on day t
- N_{nb} represents the count of potential customers who viewed but did not purchase tickets at price p for class c, through channel s, on day t.

This approach avoids circular logic by using observed purchasing behavior rather than derived probabilities to estimate utility.

3.6.2. Independent variables

The regression models incorporate several categories of independent variables:

1. Price variable

- The price offered for the ticket, which directly impacts customer utility through price sensitivity effects. This variable is adjusted for inflation and normalized within each service class to enable meaningful cross-temporal comparisons.

2. Temporal variables

- Days until departure (t): A continuous variable representing the number of days remaining before the scheduled departure, ranging from 1 to 30 days. This captures advance purchase behavior patterns.
- Days until departure squared (t^2): Included to capture non-linear effects in booking patterns, particularly the accelerated booking rates observed as departure dates approach.
- Day of week indicators (D_j): Binary variables for each day of the week to capture systematic variations in booking patterns across different days.

3. Seasonal variables

- High season indicator (S_w): A binary variable identifying periods of traditionally high demand based on historical patterns and industry season definitions.
- Holiday indicator (S_h): A binary variable marking official holidays and surrounding travel days that typically experience distinct booking behaviors.

4. Channel-Specific variables

- These variables are activated through the indicator functions ($I(s = k)$) for specific sales channels:

Website channel variables ($s = 1$):

- Page load time (B_L): Average loading time in seconds for the booking pages, capturing the impact of technical performance on conversion rates.
- Website conversion rate (B_v): The general conversion rate (percentage) for the website, serving as a proxy for overall website effectiveness.

Mobile application channel variables ($s = 2$):

- App rating (A_r): Current average rating in app stores on a 1–5 scale, representing customer satisfaction with the application.
- App crash rate (A_x): Percentage of sessions experiencing application crashes, capturing technical reliability impacts on booking behavior.

Travel agency channel variables ($s = 3$):

- Agency commission percentage (F_m): The commission rate offered to travel agencies, which may influence their recommendation behavior.
- Agency volume indicator (F_o): The historical sales volume for each agency as a percentage of total channel sales, capturing agency-specific performance patterns.

3.6.3. Utility threshold determination

The utility thresholds $\epsilon_{c_1,s,t}$ and $\epsilon_{c_2,s,t}$ represent the minimum utility level at which customers are willing to make a purchase. These thresholds are determined through analysis of observed purchase behavior at different utility levels:

$$\epsilon_{c,s,t} = \lambda_0 + \lambda_t.t + \lambda_w.S_w + \lambda_h.S_h + \sum_{j=1}^{7} \lambda_{D,j}.D_j$$

where:

- λ_0 is the baseline threshold value
- λ_t is the coefficient for days until departure
- λ_w is the coefficient for high season indicator
- λ_h is the coefficient for holiday indicator
- $\lambda_{D,j}$ are coefficients for day of week indicators

This approach allows thresholds to vary based on temporal factors, capturing the dynamic nature of customer purchase decisions.

3.6.4. Potential demand calibration

The potential demand parameters $D_{v,c,s,t}$ are derived by combining regression-based utility estimates with empirical market data:

$D_{v,c,s,t} = TMS_{v,c,t} \times CS_{s,t} \times PU_{c,s,t}(p)$

where:

- $TMS_{v,c,t}$ represents the overall market potential for transportation mode v, service class c, on day t, derived from historical industry data, competitor analysis, and market research surveys measured in absolute number of potential customers
- $CS_{s,t}$ captures the proportion of customers using channel s on day t, derived from historical distribution patterns (percentage)
- $PU_{c,s,t}(p)$ is the proportion of customers whose utility exceeds the threshold ($U_{c,s,t}(p) > \epsilon_{c,s,t}$ calculated using the regression parameters (percentage)

The total market size ($TMS_{v,c,t}$) is determined through a combination of:

- Historical passenger volume data for each transportation mode and service class
- Market growth projections based on economic indicators
- Seasonal adjustment factors derived from five-year historical patterns
- Competitive capacity analysis in each transportation mode
- Special event indicators (conferences, sporting events, festivals) that impact specific dates

4. Analytical evaluation and practical insights

4.1. Assumptions and parameters specifications

This section presents a case study to illustrate the application of the dynamic pricing model introduced in Section 3. The study demonstrates how the model can be implemented in the transportation industry to optimize ticket pricing across multiple sales channels using a data-driven approach. The case study is based on a hypothetical transportation company. The company operates three sales channels – its official website, ticketing applications, and travel agencies – and provides passenger services using three primary transportation modes: airplanes, trains, and buses. It also offers two ticket classes: business and economy.

To perform this analysis, synthetic data were generated to simulate historical records, including ticket prices, sales volumes, vehicle capacity, and customer demand patterns across different sales channels and service classes over a specific timeframe. This generated dataset serves as the foundation for evaluating prospective demand for each transportation mode, service class, and sales channel on a daily basis. Additionally, it is used to determine minimum and maximum pricing limits and the base utility parameters for each sales channel over time. The dynamic pricing model was developed based on this structured dataset, incorporating the following key considerations:

- Capacity constraints: Each method of transportation has a predetermined limit on the number of passengers it can accommodate. The overall number of tickets sold, considering all classes and channels, must not exceed this capacity. This limitation guarantees the prevention of overbooking.
- Pricing flexibility: The model permits daily modifications to ticket pricing, with a maximum restriction of a 10 % increase or decrease, in order to avoid sudden and drastic price changes. This limitation is a manifestation of the company's price strategy and the prevailing market conditions.

Table 1
Indices and the specified values.

Indices	Values
v	$v \in \{1,2,3\}$
c	$c \in \{1,2\}$
s	$s \in \{1,2,3\}$
t	$t \in \{1,2,3,\ldots,30\}$

Table 2
Key parameters.

Parameter	Airplane ($v = 1$)	Train ($v = 2$)	Bus ($v = 3$)
K_v - Capacity	200	400	45
$P^{\min}_{v,c,t}$ - Economy Class	1,500,000 $	330,000 $	250,000 $
$P^{\max}_{v,c,t}$- Economy Class	2,700,000 $	330,000 $	430,000 $
$P^{\min}_{v,c,t}$ - Business Class	3,500,000 $	360,000 $	280,000 $
$P^{\max}_{v,c,t}$ - Business Class	6,600,000 $	520,000 $	460,000 $s
$D_{v,c,s,t}$ - Economy Class	Website: 30 App: 90 Agent: 50	Website: 100 App: 200 Agent: 100	Website: 10 App: 20 Agent: 15
$D_{v,c,s,t}$ - Business Class	Website: 6 App: 15 Agent: 9	Website: 50 App: 100 Agent: 50	Website: 8 App: 10 Agent: 7
$M_{v,c}$ - Economy Class	0.3	0.2	0.15
$M_{v,c}$ - Business Class	0.1	0.15	0.1

- Market share requirements: The model sets a minimum market share for each mode of transportation and class to ensure a consistent client base and preserve the company's market presence.
- Customer utility and demand estimation: The model incorporates utility functions that quantify customer satisfaction with ticket sales for the purpose of estimating customer utility and demand. These functions are impacted by pricing and various other factors, assisting in the estimation of demand for each channel and class.
- Pricing constraints: The model guarantees that ticket prices stay within predetermined minimum and maximum thresholds for each mode and class of transportation.

The basic assumptions for the indices are summarized in Table 1. The index v denotes the type of vehicle: 1 for airplanes, 2 for trains, and 3 for buses. The index c refers to service classes, with 1 representing economy and 2 representing business class. The index s captures the sales channel, where 1 corresponds to the company's website, 2 to ticketing applications, and 3 to travel agencies. Finally, the index t indicates the number of days remaining until departure, ranging from 1 to 30.

Subsequently, we proceed to allocate values to the parameters of the mathematical model. The capacity parameter, with three distinct values for each vehicle type. After determining the capacity, we assign specific values for the minimum and maximum price limits for each vehicle type, service class, and the number of days remaining until departure, denoted as t. For instance, the expression $P^{\min}_{1,1,t}$ reflects the minimal price threshold for an economy class flight ticket, which is 1,500,000 $. The expression $P^{\max}_{1,1,t}$ denotes the upper limit of the price for an economy class airplane ticket, which is fixed at 2,700,000 $. Other parameters are defined in a similar manner. The parameter t is held constant to avoid excessive data output and assumes that the minimum price remains the same for all 30 days before departure.

Next, we establish values for a key parameter in the model: potential demand. This example defines potential demand for three vehicle types, two service classes, three sales channels, and the 30 days leading up to departure. For instance, $D_{1,1,1,t}$ reflects the potential demand for economy class airplane tickets sold on the company's website on day t before departure.

The next step is to establish the minimum market share values. Market share represents the proportion of the market controlled by a particular company, product, or service. In this context, it refers to the

Table 3
Ticket prices, sales quantities, and revenue contributions.

Vehicle	Service Class	Sales Channel	Ticket Price ($p_{v,c,t}$) (× 1000 $)	Quantity Sold ($q_{v,c,s,t}$)	Revenue Contribution ($p_{v,c,t} \times q_{v,c,s,t}$) (× 1000 $)
Airplane	Economy	Website	2098	27	56,646
		Ticket App	2098	86	184,298
		Travel Agent	2098	42	88,116
	Business	Website	5012	3	15,036
		Ticket App	5012	11	55,132
		Travel Agent	5012	6	30,072
Train	Economy	Website	406	94	38,164
		Ticket App	406	192	77,952
		Travel Agent	406	95	38,570
	Business	Website	454	42	19,068
		Ticket App	454	96	43,584
		Travel Agent	454	43	19,522
Bus	Economy	Website	347	8	2776
		Ticket App	347	19	6593
		Travel Agent	347	12	4164
	Business	Website	369	6	2214
		Ticket App	369	8	2952
		Travel Agent	369	7	2583
Total Revenue (R) (× 1000 $)	-	-	-	-	687,442

share of overall market demand captured by a specific transportation type and service class. This plays an important role in shaping pricing strategies across different transportation modes, service classes, and sales channels. Market share can be influenced by a variety of factors, including:

1. Pricing strategy: Lower prices may help increase market share by attracting more customers, while higher prices may have the opposite effect.
2. Quality and service: Superior service and product quality can lead to higher market share, even if prices are relatively high.
3. Brand loyalty: Strong brand loyalty can reduce price sensitivity among customers, thereby influencing market share.
4. Competitor actions: The pricing and strategic decisions of competitors directly affect a company's ability to maintain or grow its market share.

For instance, $M_{1,1}$ denotes the minimum market share required for economy class flight tickets in the company under analysis. Table 2 provides a detailed summary of the input parameters used in the dynamic pricing model. The data includes capacity values, pricing thresholds, potential demand estimates, and minimum market share requirements across different vehicle types, service classes, and sales channels.

4.2. Primary results

The optimization model is solved using the specified parameters and indices to determine ticket prices and the number of tickets sold. This process results in the calculation of total revenue. Table 3 presents the aggregated output of the dynamic pricing model, detailing ticket prices, quantities sold, and revenue contributions across different vehicle types, service classes, and sales channels.

For example, the variable $p_{1,1,t}$ represents the price of an economy class airplane ticket on day t before departure. Once ticket prices are determined using the mathematical model, the number of tickets sold—denoted variable $q_{v,c,s,t}$ is also computed. For instance, $q_{1,1,1,t}$ indicates the quantity of economy class airplane tickets sold through the company's website on day t, while $q_{3,2,3,t}$ corresponds to business class bus tickets sold via travel agencies. The total revenue, denoted by R, is calculated as the product of price and quantity sold, considering all relevant dimensions: vehicle type, service class, sales channel, and time until departure.

The ticket price remains consistent across all sales channels for a given vehicle type and service class. However, the quantity of tickets sold differs by channel, as shown in the results, highlighting how sales distribution affects overall revenue. The table provides detailed insights into the model's practical effectiveness in maximizing revenue while balancing sales across multiple channels. After executing the model, the results suggest that the company can significantly improve its pricing strategy to maximize income, while respecting all operational and market constraints. The model generates targeted pricing recommendations for each day, transportation mode, service class, and sales channel, thereby supporting competitive positioning in the market.

4.3. Sensitivity analysis

This section examines how variations in key input parameters affect the outcomes of the proposed model.

Table 4
Sensitivity analysis of the capacity parameter.

Percentage Change in K_1	Value of K_1	Total Revenue R (1000 × $)	Percentage Change in Total Revenue R
−30 %	140	653,069	−5 %
−20 %	160	664,756	−3.3 %
−10 %	180	675,755	−1.7 %
Base Value	200	687,442	0 %
+ 10 %	220	699,128	+ 1.7 %
+ 20 %	240	710,127	+ 3.3 %
+ 30 %	260	721,814	+ 5 %

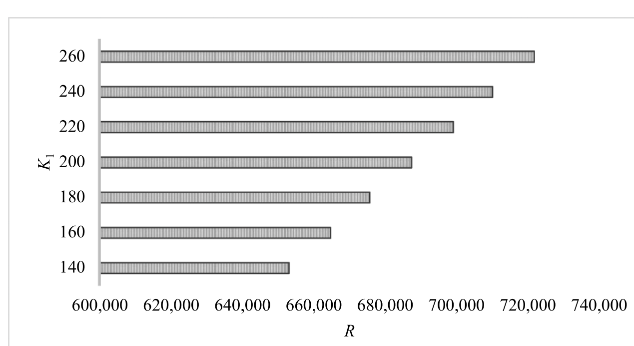

Fig. 1. Sensitivity analysis of airplane capacity parameter.

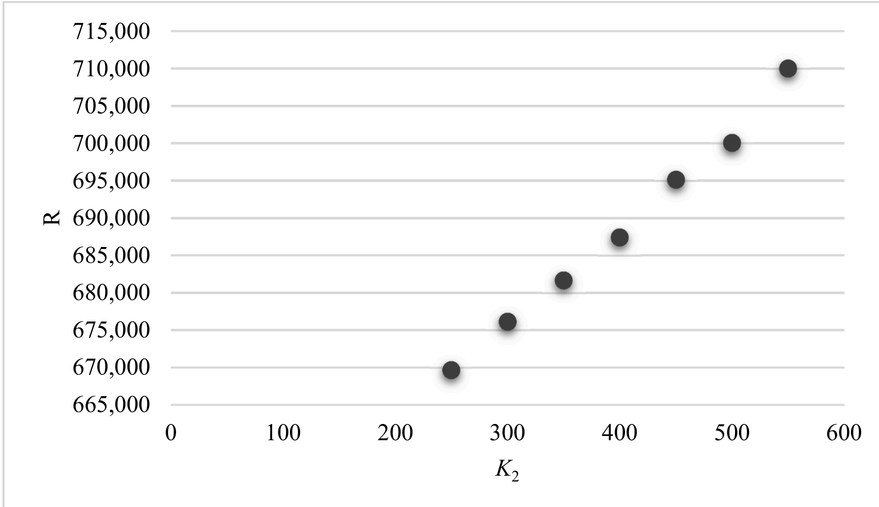

Fig. 2. Sensitivity analysis of train capacity parameter.

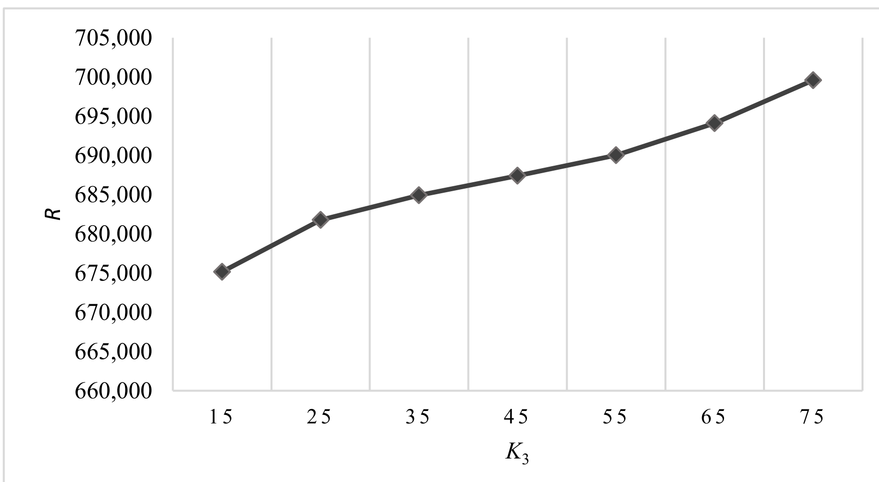

Fig. 3. Sensitivity analysis of bus capacity parameter.

4.3.1. *Changes in the capacity parameter*

Another essential factor in the model is the capacity parameter (K_v). This section specifically examines the sensitivity analysis of the model to variations in airplane capacity (K_1), which represents the primary mode of transportation in this study and is particularly relevant for long-distance travel. The impact of changes in the capacity parameter on the objective function is analyzed and presented in Table 4, assuming that all other parameters remain constant. The model demonstrates a moderate level of sensitivity to variations in airplane capacity. As system or business capacity increases, the total revenue also rises. This indicates that, where feasible and economically justified, expanding airplane capacity can lead to higher income.

Fig. 1 shows the results of the sensitivity analysis for the airplane capacity parameter. As expected, increasing the airplane's capacity leads to higher revenue, while decreasing the capacity results in lower revenue. However, the relationship is not strictly proportional, indicating diminishing returns as capacity continues to grow.

Additionally, Fig. 2 presents the sensitivity analysis for K_2, which pertain to train capacity.

The bus capacity parameter (K_3) has a significant impact on revenue, especially for medium-distance travel options within the model. This section analyzes the sensitivity of bus capacity while holding other factors constant. Fig. 3 illustrates the relationship between changes in bus capacity and total revenue (R). The results show that increasing capacity leads to higher revenue; however, the rate of increase diminishes as capacity grows. Small increases from a low baseline generate substantial revenue gains, whereas further expansions yield diminishing returns. Conversely, reducing bus capacity leads to a noticeable drop in revenue, emphasizing the importance of maintaining an optimal capacity level. The analysis also shows that revenue tends to stabilize at higher capacity levels, highlighting the need for a balanced capacity management strategy. Thus, while increasing bus capacity can be beneficial, it must be economically justified to avoid inefficiencies

Fig. 4 illustrates the relationship between the capacity parameters K and revenue R for three modes of transportation: airplane (K_1 vs. R_1), train (K_2 vs. R_2), and bus (K_3 vs. R_3). Each line represents a different mode, with data points showing how revenue changes as capacity increases. The blue line with circular markers represents airplane capacity (K_1), revealing a notable rise in revenue as capacity grows, indicating a high sensitivity to capacity changes. This indicates that augmenting airplane capacity can substantially boost income until a specific limit, beyond which increases may begin to decline.

The gray line with square markers represents rail capacity (K_2), showing a moderate increase in revenue with rising capacity: less steep than that of the airplane, indicating measured sensitivity. The line with triangular markers denotes bus capacity (K_3), which exhibits the lowest

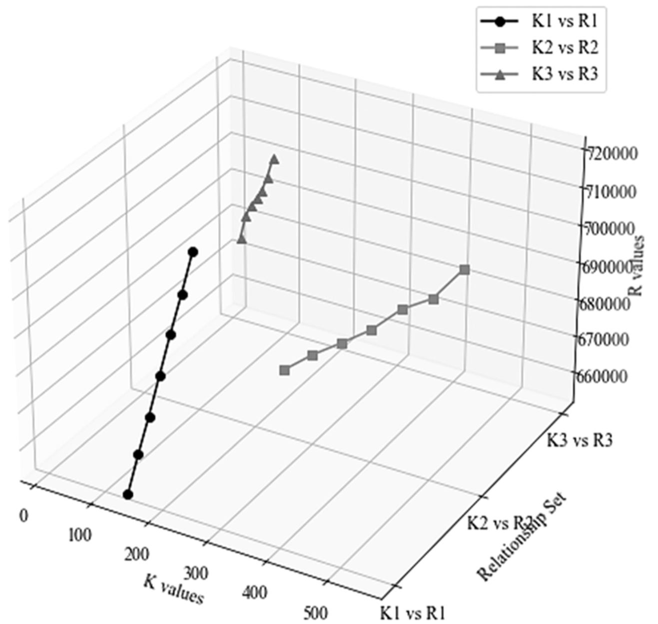

Fig. 4. Sensitivity analysis of capacity parameters for airplane (K_1), train (K_2) and bus (K_3).

Table 5
Sensitivity analysis of minimum market share parameter.

Percentage Change in $M_{1,1}$	Value of $M_{1,1}$	Total Revenue R (1000 × \$)	Percentage Change in Total Revenue R
−30 %	0.21	710,127	+ 3.3 %
−20 %	0.24	702,565	+ 2.2 %
−10 %	0.27	695,003	+ 1.1 %
Base Value	0.30	687,442	0 %
+ 10 %	0.33	679,880	−1.1 %
+ 20 %	0.36	672,318	−2.2 %
+ 30 %	0.39	664,756	−3.3 %

sensitivity, as changes in capacity result in smaller revenue variations. This reduced impact is likely due to shorter travel distances and lower fares typical of bus transport. The figure clearly illustrates that airplane capacity has the greatest effect on revenue, followed by train and then bus. These insights are valuable for designing effective capacity management strategies across transportation modes.

4.3.2. Changes in the minimum market share parameter

This section analyzes the minimum market share parameter, a key component of the model with strategic importance for the company. Specifically, we examine the sensitivity of the model to changes in the minimum market share for airplanes in the economy class ($M_{1,1}$), while keeping all other parameters constant. The variations applied in the analysis are detailed in Table 5. The results show that the model is only moderately sensitive to changes in $M_{1,1}$. As the required market share increases, overall revenue decreases. For instance, a 30 % increase in $M_{1,1}$ leads to a 3.3 % drop in total income.

Fig. 5 presents the sensitivity analysis of the minimum market share parameter for airplane economy class ($M_{1,1}$). The chart shows a downward trend, indicating a negative correlation between $M_{1,1}$ and total revenue. This effect likely stems from the need to offer lower prices to maintain a larger market share, which in turn reduces income. While maintaining market presence is important, setting the required share too high may lead to diminishing returns.

4.3.3. Changes in the maximum price limit parameter

The next parameter examined in this section is the maximum price limit ($P^{\max}_{v,c,t}$), which is another crucial factor in evaluating pricing strategies and overall revenue. Specifically, we examine the sensitivity of variations in the maximum price limit ($P^{\max}_{1,1,t}$) for economy class airplane tickets, which plays a particularly important role compared to other factors. Table 6 displays the percentage variations in this parameter, its numerical values, and its influence on the objective function, while keeping other parameters unchanged. The model exhibits a moderate level of responsiveness to changes in the upper limit of prices for economy class airplane tickets. As the maximum permissible price increases, total revenue also rises.

Table 6
Sensitivity analysis of maximum price limit parameter.

Percentage Change in $P^{\max}_{1,1,t}$	Value of $P^{\max}_{1,1,t}$	Total Revenue R (1000 × \$)	Percentage Change in Total Revenue R
−30 %	1890	653,069	−5 %
−20 %	2160	664,756	−3.3 %
−10 %	2430	675,755	−1.7 %
Base Value	2700	687,442	0 %
+ 10 %	2970	700,012	+ 1.7 %
+ 20 %	3240	710,378	+ 3.3 %
+ 30 %	3510	722,915	+ 5 %

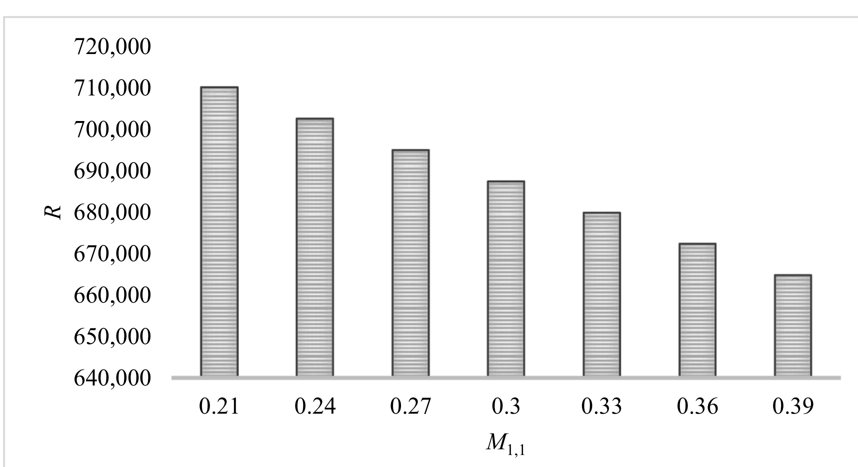

Fig. 5. Sensitivity analysis of the minimum market share parameter for economy class airplane.

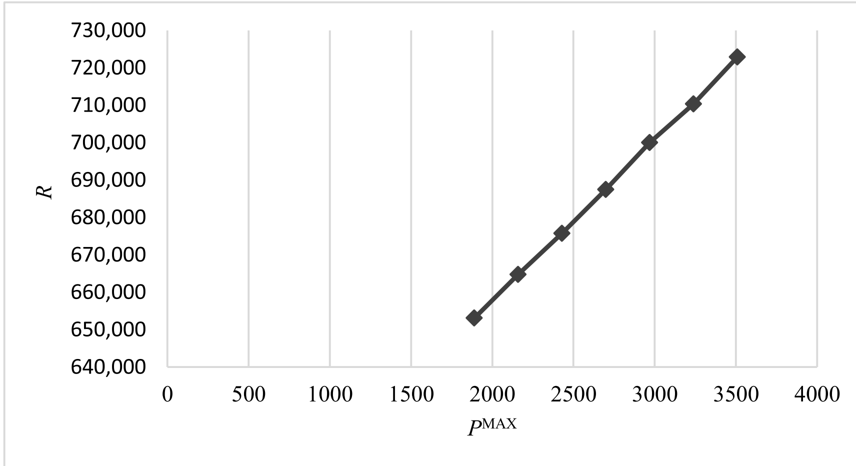

Fig. 6. Sensitivity analysis of the maximum price limit parameter for economy class airplane ($P^{\max}_{1,1,t}$).

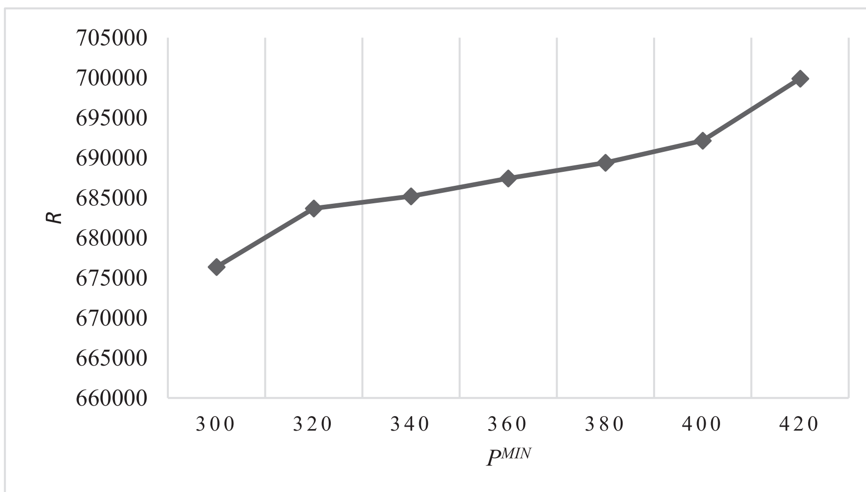

Fig. 7. Sensitivity analysis of the minimum price limit parameter for business class train ($P^{\min}_{2,2,t}$).

Table 7
Sensitivity analysis of the demand parameter for economy class airline tickets via application channel.

Value of $D_{1,1,2,t}$	Total Revenue R (1000 ×$)	Percentage Change in Total Revenue R
40	486,021	−29.3 %
60	558,890	−18.7 %
80	641,383	−6.7 %
90 (base value)	687,442	0 %
100	731,438	+ 6.4 %
120	833,867	+ 21.3 %
140	947,295	+ 37.8 %
160	1,069,659	+ 55.6 %
170	1,071,722	+ 55.9 %
180	1,056,598	+ 53.7 %
190	1,011,914	+ 47.2 %
200	944,545	+ 37.4 %

Fig. 6 also depicts the sensitivity analysis of the maximum price limit parameter for economy class flight tickets. The chart shows an upward-sloping line, indicating a positive correlation between ($P^{\max}_{1,1,t}$) and total revenue. This suggests that adopting a flexible pricing strategy – especially during periods of high demand – can lead to increased revenue. However, it is important to consider that this approach does not account for potential long-term effects of higher prices on customer satisfaction and loyalty.

4.3.4. Changes in the minimum price limit parameter

This section examines the model's sensitivity to fluctuations in the minimum price limit parameter ($P^{\min}_{2,2,t}$), a vital component of the pricing framework that might influence revenue outcomes for particular service categories. The analysis focuses on how changes in the lowest permissible price affect total revenue, while keeping all other parameters constant.

Fig. 7 illustrates the impact of changes in the minimum price limit on total revenue. As the minimum price limit increases, revenue also rises, but at a decreasing rate, indicating diminishing marginal returns. While higher minimum prices can increase revenue, the extra gains become smaller as the prices get higher. This analysis highlights the importance of maintaining a balanced minimum pricing strategy: setting a baseline that supports revenue growth while preserving flexibility to attract price-sensitive customers. The findings highlight the strategic role of the minimum price limit in optimizing pricing policies.

4.3.5. Changes in the potential demand parameter

This section analyzes the model's sensitivity to changes in the demand parameter ($D_{v,c,s,t}$), which represents the projected demand for different vehicle types, service classes, and sales channels across various

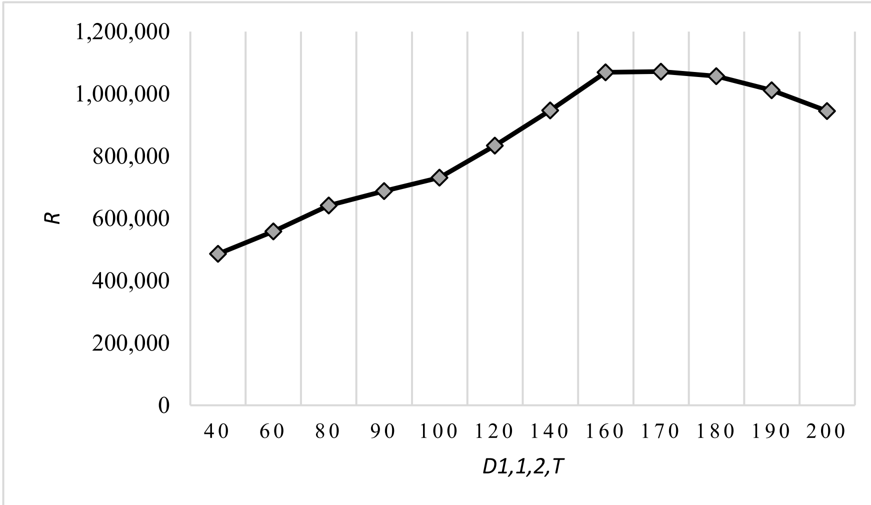

Fig. 8. Sensitivity analysis of potential demand for economy class airplane tickets via the application channel.

time periods. The analysis concentrates on the demand for economy class airplane tickets via the ticketing application channel ($D_{1,1,2,t}$), as this constitutes one of the highest-volume sales channels in the base model. Table 7 presents the percentage variations applied to this parameter.

The analysis uncovers several critical facts regarding the correlation between demand and revenue:

1. Initial growth phase ($D_{1,1,2,t}$= 40–170):

- Revenue demonstrates steady rise in response to rising demand.
- The correlation is positive, though not perfectly linear.
- The system efficiently accommodates growing demand.

2. Optimal point ($D_{1,1,2,t}$= 170):

- Revenue reaches its peak at approximately 466,120,000 $.
- This marks the ideal balance between demand and system capacity.
- Further increases in demand beyond this point become detrimental.

3. Decline phase ($D_{1,1,2,t} > 170$):

- Revenue begins to decline as demand exceeds the optimal level.
- This decrease may be ascribed to multiple factors:
  o Constraints on system capacity
  o Increased operational intricacy
  o Potential deterioration in service quality
  o Price adjustments required to manage excess demand
  o Higher operational costs during peak demand periods

Fig. 8 depicts the non-linear correlation between demand and revenue, illustrating three distinct phases: growth, optimum, and decline.

5. Conclusion and future research

This study has demonstrated the impact of dynamic pricing models and data-driven pricing strategies in the transportation sector, particularly in ticket pricing optimization. The findings highlight the significance of incorporating advanced methodologies, such as machine learning, into pricing systems to enhance revenue management and operational efficiency. By utilizing historical data and applying mathematical modeling, transportation companies can move beyond fixed pricing structures, which often fail to account for real-time market dynamics and consumer behavior. Instead, dynamic pricing models allow for continuous adjustments based on demand patterns, pricing constraints, and competitive conditions, ensuring a more responsive and flexible approach to pricing.

The empirical analysis supports the effectiveness of dynamic pricing models in adapting to market volatility, optimizing revenue, and improving customer satisfaction through competitive and fair pricing strategies. The proposed optimization framework provides a structured and scalable solution that transportation companies can use to refine their pricing strategies and remain competitive in a data-driven industry. The case study results further confirm that the proposed revenue management methodology can determine optimal ticket prices while accounting for operational constraints. Ultimately, the findings demonstrate that dynamic pricing optimization has significant potential to improve revenue outcomes, while continuous data collection and analysis are essential for refining these models and sustaining their long-term effectiveness.

While this study provides insights into dynamic pricing optimization for multi-channel transportation supply chains, several avenues for future research remain open. First, extending the current model to incorporate competitive pricing dynamics and the effects of cancellations, ticket modifications, and refunds could enhance its realism and applicability. Including such market interactions would allow the model to better capture the complexities of real-world transportation environments and improve its predictive accuracy. Second, integrating more advanced machine learning models – such as regression trees, random forests, or deep learning techniques – could improve the estimation of demand and price sensitivity, particularly in capturing nonlinear customer behavior and cross-channel interactions. Future work may focus on developing hybrid frameworks that combine the interpretability of linear models with the predictive power of nonlinear approaches, while maintaining computational efficiency.

Third, incorporating customer segmentation and loyalty factors explicitly into the pricing model could help tailor pricing strategies for different consumer groups and improve long-term revenue sustainability. Examining the impact of external disruptions, such as pandemics or economic shocks, on dynamic pricing strategies would also be a valuable direction for future research. Finally, pilot implementations and real-world case studies across different geographic regions and transportation modes could help validate and refine the model's practical effectiveness, addressing operational constraints and business challenges encountered during deployment. These extensions would further enhance the robustness, adaptability, and strategic value of data-driven dynamic pricing frameworks in increasingly complex and competitive transportation environments.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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