PHYSICS 334 - ADVANCED LABORATORY I

UNIVERSAL GRAVITATIONAL CONSTANT

Spring 2001

Purposes: Determine the value of the universal gravitation constant *G*.

Background: Classical mechanics topics-moments of inertia, central forces, torques, gravitation, and damped harmonic motion.
MIT and Tel-Atomic laboratory write-ups

Skills: Statistical analysis.

Protocol: ***Moving the apparatus is to be avoided. The tungsten wire that suspends the boom is very fragile and difficult to replace. Should you need to move the Cavendish apparatus, request assistance and do so very carefully after lowering the boom support so that tension is off the wire. Lowering the boom will necessitate a time-consuming re-centering process when the boom is raised again. When removing the test masses, remove both simultaneously to avoid tipping the apparatus.***

The forces being measured are **extremely** ***small. Changes in the environment around the apparatus can produce effects that swamp the effects sought. Be sure to electrically ground the test masses and the boom support before each measurement.***

The device to be used is the Tel-Atomic TEL-2000 Cavendish Balance system, illustrated schematically in Fig. 1. This system uses a symmetric differential capacitance transducer that, to first order, eliminates measurement errors associated with the 'pendulum' vibrations of the suspended beam. For this laboratory, you will first calibrate this capacitance transducer to determine the correspondence between voltage output from the transducer and angular displacement.

Using this calibration, you will then measure the angular displacement as a function of time for different configurations of the test masses. Fitting the angular displacement functions will enable you to determine equilibrium positions. These equilibrium positions are related to the combined effects of torsional forces from the suspending tungsten wire and gravitational forces between the test masses. By knowing the positions and magnitudes of the masses, *G* can be determined.

Remember: the forces involved are very small. Injudicious movements or changes in the local environment while the data runs are in progress can compromise your measurements.

Part A: Angular displacement calibration

1. In preparation for data taking below, start the Cavendish data acquisition software application on the PC connected to the apparatus, but do not start a data-taking run at this point. Adjust the differential capacitance control unit (DCCU) to its highest gain setting. Move the test masses to the neutral position.

2. You now need to ensure the suspended boom is near the null position by verifying that the output voltage from the DCCU is zero. You can also verify this by placing a laser beam along a perpendicular line from the Cavendish apparatus centerline so that it strikes the mirror on the central axis support and reflects a spot back to the laser exit. If the suspension beam is not near the null position, contact either the instructor or the teaching assistant.

3. Use the 'optical lever' method to determine the angular displacement, similar to that noted in the Tel-Atomic write-up. Arrange the laser so that the beam strikes the mirror mounted on the tungsten wire lower support, with the beam as close to horizontal as possible.

4. Position a measuring screen so the reflected laser beam spot strikes the screen.

5. Move the test masses to the near position. The masses on the suspended boom will be attracted towards the lead balls, and the reflected laser spot will move across the measuring screen. The period of the boom motion is about 200 seconds.

6. Adjust the orientation and location of the measuring screen so the full motion of the reflected laser spot remains on the screen. Once this orientation is fixed, carefully determine the distances needed to accurately determine the angular displacements as a function of the place where the spot strikes the measuring screen.

7. Begin a data-taking run covering about 10 full cycles of motion. Record in your notebook the positions for the maximum excursions of the laser spot on the measuring screen. The data acquisition system will record the voltage output of the DCCU as a function of time. Your measurements of the maximum excursions can be correlated with the maxima of the voltage swings from the DCCU. By determining the angular changes (*in radians!*) corresponding to these maxima, determine the calibration of the DCCU voltage. Note that a shift of the mirror by $\Delta\Theta$ about the central axis produced an angle shift of 2 $\Delta\Theta$.

8. After finishing a data run of about 10 full swings, fit the data obtained to the form

$$V(t) = A\sin(\omega t + \varphi)\exp(-\beta t)$$

   The factor β is the damping constant for the system. You should compare this value with the damping constants measured in subsequent analyses below.

Part B: Measurement of G

1. Overview

In the near position, the test masses produce torques on the suspended boom due to the gravitational attraction of the suspended masses and the boom. The torque produced by the test masses on the nearest corresponding suspended mass is

$$\tau_n = \frac{2GMmd}{R^2}$$

where the factor of 2 arises due to the presence of two sets of masses, *M* is the mass of one of the test masses, *d* is the distance of each of the suspended masses from the central axis of the boom, and *R* is the separation of the test and suspended masses. This is the primary source of torque in the system. Clearly, all of the terms in this expression must be determined with the greatest accuracy possible in order to arrive at an accurate value of *G*.

However, two additional torques exist within this apparatus that must be taken into account in order to find *G*. One is an additional torque produced by the interaction of each test mass with the suspended mass at the distant end of the boom. This produces a torque τ_d which acts in the opposite direction of the torque τ_n. By inspection, this torque is seen to be

$$\tau_d = -\frac{2GMm}{r^2}\cdot d\sin\theta = -\frac{2GMm}{R^2+(2d)^2}\cdot d\cdot\frac{R}{\sqrt{R^2+(2d)^2}}$$

$$= -\frac{2GMmd}{R^2}\cdot\frac{R^3}{(R^2+(2d)^2)^{3/2}}$$

$$= -\tau_n f_d \quad where \quad f_d = \frac{R^3}{(R^2+(2d)^2)^{3/2}}$$

An additional torque arises from the attraction of the suspension beam to the test mass. By following the analysis detailed in the Tel-Atomic write-up, this additional torque is given by

$$\tau_b = \frac{2GMmd}{R^2}\cdot f_b'$$

$$= \tau_n \cdot f_b' \quad where \quad f_b' = \frac{(m_b f_b - m_h)}{m}$$

with the various terms defined in the discussion above equation 26 of the Tel-Atomic write-up. The combined effects of these last two items amounts to about 3-4% of τ_n, and the total torque due to these gravitational effects is

$$\tau_{total}^{grav} = \tau \ \ (1 + f_b' + f_d)$$

Opposing the sum of these torques is the restoring torque $K(\Delta\theta)$ supplied by the tungsten wire. This wire twists and reaches an equilibrium angle given by

$$\tau_{wire} = \tau_{total}^{grav}$$

which means

$$K(\Delta\theta) = \tau_{total}^{grav} = \frac{2GMmd}{R^2}(1 + f_d + f_b')$$

$$\rightarrow G = \frac{K(\Delta\theta)\, R^2}{2\, Mmd\, (1 + f_d + f_b')}$$

Thus, the measurement of K and $\Delta\theta$, along with the masses and dimensions of the apparatus, will suffice to permit a determination of G.

2. Measurement protocol

In practice, determining the equilibrium angular shift $\Delta\theta$ is difficult and time-consuming because the deflection is small and the time needed to reach equilibrium can be long. For that reason, we will determine the equilibrium angle $\Delta\theta$ by fitting the angular deflection as a function of time when the masses are in the near position to the expression

$$\theta(t)_{near} = \Delta\theta + \theta_1 \sin(\omega t + \theta)\exp(-\beta t)$$

Note that the damping coefficient β determined in Part A enters here as well. You should nonetheless vary β as a parameter in your fits here and compare the results to those in Part A. This expression approaches the equilibrium angle $\Delta\theta$ as t becomes large. When the suspended beam finally comes to rest, the DCCU should indicate this angle; you should note whether this final resting angle indeed corresponds to the results of your fit.

You should repeat this process for the test masses moved to the alternate near position. Then repeat the measurement at both near positions a second time, for a total of four measurements.

3. The torsion constant K

The torsion constant K can be determined two ways and you should try both methods:

a) The torsion constant can be estimated from the properties of the tungsten wire directly using

$$K = \frac{\pi\mu\phi^4}{32\ell}$$

where ϕ is the diameter of the wire (25 μm), Young's modulus μ for tungsten is 1.57 x 10^{11} N/m^2 and the length of the wire is l.

b) From your fit to the oscillations of the suspended beam, you will have determined the oscillation frequency ω. This frequency of oscillation corresponds to

$$K = \omega^2 + \beta^2 I$$

where I is the moment of inertia of the suspended system of beam and masses. Since this is a composite system, you will have to determine the total moment of inertia by summing the individual moments of inertia and using the parallel axis theorem.

4. Determining R

For all gravitational torques, R enters in terms as second or third power. Thus, determining R accurately is important. To measure R, use the following method, estimating uncertainties at each step.

a) Measure the width between the glass plates *W*.

b) Determine the diameters of each of the lead test masses and find their average *D*.

c) Determine the size of the gaps between the test masses and the glass plates when the test masses are in the near position. Find the average of this gap *g*.

d) Determine the approximate reduction in R due to the rotation of the boom ($d\,\Delta\theta$).

e) Using these values *R* is given by

$$R=\frac{W}{2}+\frac{D}{2}+\frac{g}{2}-d(\Delta\theta)$$

5. Recording data

Since you will be doing the same calculation of *G* for all four trials, you should transfer your data to a spreadsheet to automate your calculations.

6. Uncertainties

You should accept the values indicated in Figure 1 with their stated uncertainties. Do not disassemble the apparatus to make any measurement beyond removing the test masses for weighing.

Aside from those uncertainties indicated in the figure, you will need to measure quantities and ascertain their associated uncertainties. Since nearly all quantities associated with the determination enter as simple factors or sums, you should use propagation of uncertainties to determine the final uncertainty in your value of *G*. Again, you should use a spreadsheet to automate these calculations.

Table I. Some additional dimensions of the Tel-Atomic apparatus not given in Figure 1.

Item	**Dimension**
Radius of small mass	6.75(1) mm
Thickness of support beam	1.65(1) mm
Width of support beam	1.28(1) cm

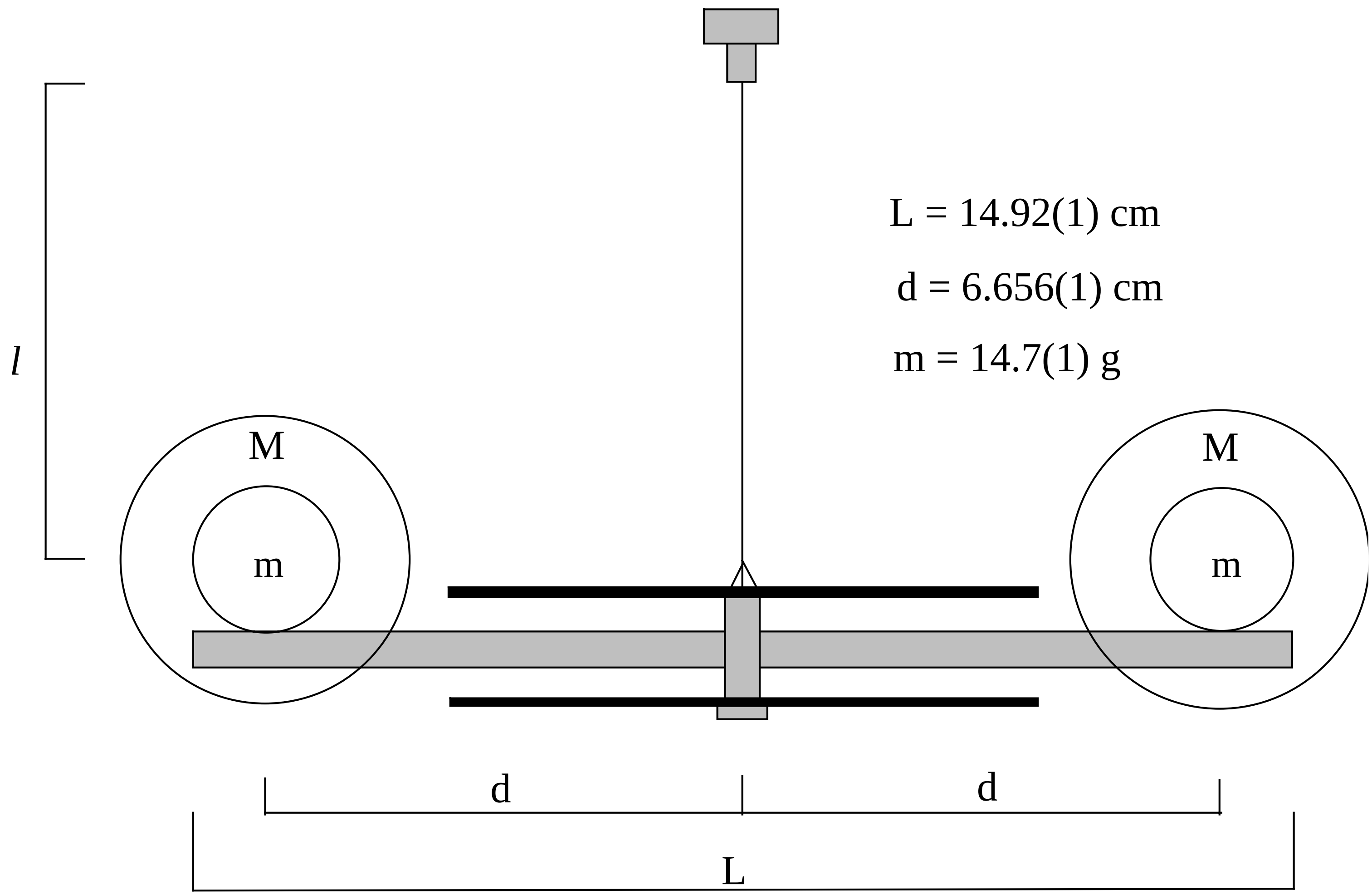

Figure 1. Dimension labels for elements of the Cavendish apparatus for measuring *G*. The enclosing glass case is not shown in this sketch. For additional details, see Figure 1 in the Tel-Atomic write-up.