In [66]:
```
import numpy as np
import math as m
import matplotlib.pyplot as plt
%matplotlib inline
from mpl_toolkits.mplot3d import Axes3D
```

In [67]:
```
def plotter(x,y,z):
    fig = plt.figure()
    ax = fig.add_subplot(111, projection='3d')

    ax.scatter(x,y,z,s=5)

    ax.set_xlabel('AU x-axis')
    ax.set_ylabel('AU y-axis')
    ax.set_zlabel('AU z-axis')

    plt.tight_layout()
```

In [68]:
```
def gravforcex(planet1, planet2):
    x1 = planet1.x
    y1 = planet1.y
    z1 = planet1.z
    x2 = planet2.x
    y2 = planet2.y
    z2 = planet2.z
    rsep = ((x1-x2)**2+(y1-y2)**2+(z1-z2)**2)**(0.5)
    return -4*m.pi**2*(x1-x2)*planet2.mass/(rsep)**3

def gravforcey(planet1, planet2):
    x1 = planet1.x
    y1 = planet1.y
    z1 = planet1.z
    x2 = planet2.x
    y2 = planet2.y
    z2 = planet2.z
    rsep = ((x1-x2)**2+(y1-y2)**2+(z1-z2)**2)**(0.5)
    return -4*m.pi**2*(y1-y2)*planet2.mass/(rsep)**3

def gravforcez(planet1, planet2):
    x1 = planet1.x
    y1 = planet1.y
    z1 = planet1.z
    x2 = planet2.x
    y2 = planet2.y
    z2 = planet2.z
    rsep = ((x1-x2)**2+(z1-z2)**2+(z1-z2)**2)**(0.5)
    return -4*m.pi**2*(x1-x2)*planet2.mass/(rsep)**3
```

In [69]:
```
class makeplanet:

    def __init__(self, name,mass,x,y,z,vx,vy,vz):
        self.name = name
        self.x = x
        self.y = y
        self.z = z
        self.vx = 365*vx
        self.vy = 365*vy
        self.vz = 365*vz
        self.mass = mass

    def accx(self,x,y,z):
        r = ((x)**2+(y)**2+(z)**2)**(0.5)
        accx = -4*m.pi**2*x/(r**3)
        return accx

    def accy(self,x,y,z):
        r = ((x)**2+(y)**2+(z)**2)**(0.5)
        accy = -4*m.pi**2*y/(r**3)
        return accy

    def accz(self,x,y,z):
        r = ((x)**2+(y)**2+(z)**2)**(0.5)
        accz = -4*m.pi**2*z/(r**3)
        return accz

    def movex(self,xnew):
        self.x=xnew
    def movey(self,ynew):
        self.y=ynew
    def movez(self,znew):
        self.z=znew

    def movevx(self,vxnew):
        self.vx=vxnew
    def movevy(self,vynew):
        self.vy=vynew
    def movevz(self,vznew):
        self.vz=vznew
```

In [96]:
```
test = makeplanet("Test", 1, 1,1,1,1,1,1)
mercury = makeplanet("Mercury",1.65E-07,2.80E-01,1.73E-01,-1.18E-02,-2.0
1E-02,2.53E-02,3.91E-03)
venus = makeplanet("Venus", 2.45E-06, -7.02E-01, 1.36E-01, 4.24E-02, -3.
81E-03, -1.99E-02, -5.40E-05)
earth = makeplanet("Earth", 3.0E-06, -9.88E-01, 8.50E-02, -1.52E-04, -1.
68E-03, -1.71E-02, 4.35E-07)
mars = makeplanet("Mars", 3.3E-07, 7.78E-01, 1.28, 7.56E-03, -1.14E-02,
8.47E-03, 4.58E-04)
jupiter = makeplanet("Jupiter", 0.00095, -5.23, -1.53, 1.23E-01,2.02E-3,
 -6.88E-03, -1.67E-05)
saturn = makeplanet("Saturn", 0.000275, -1.48,-9.93, 2.32E-01, 5.212E-
03, -8.39E-04, -1.93E-04)
uranus = makeplanet("Uranus", 0.000044, 1.82E01, 8.08, -2.06E-01, -1.62E
-03, 3.41E-03, 3.38E-05)
neptune = makeplanet("Neptune", 0.0000515, 2.84E01, -9.47, -4.60E-01,9.7
11E-04, 2.997E-03, -8.38E-05)
pluto = makeplanet("Pluto", 6.55E-09, 9.89, -3.18E01, 5.396E-01, 3.06E-0
3, 2.906E-04, -9.09E-04)
```

In [71]:
```
print(test.x)
print(test.accx(test.x,test.y, test.z))
```

```
1
-7.5976250103520755
```

In [72]:
```
#Functionality is working for changing the x coordinate
test.movex(2)
test.x
```

Out[72]: 2

In [73]:
```
test.accx(test.x,test.y, test.z)
```

Out[73]: -5.372332165732466

In [74]:
```
test.accy(test.x,test.y, test.z)
```

Out[74]: -2.686166082866233

In [75]:
```
test.accz(test.x,test.y, test.z)
```

Out[75]: -2.686166082866233

In [76]:
```
test.movez(3)
test.accz(test.x,test.y, test.z)
```

Out[76]: -2.260939482359547

In [77]:
```
def buildaccx(planet, list_of_planets):
    accel_x = planet.accx(planet.x, planet.y, planet.z)
    for i in list_of_planets:
        if i.name!=planet.name:
            #print(i.name)
            accel_x += gravforcex(planet, i)
    return accel_x

def buildaccy(planet, list_of_planets):
    accel_y = planet.accy(planet.x, planet.y, planet.z)
    for i in list_of_planets:
        if i.name!=planet.name:
            accel_y += gravforcey(planet, i)
    return accel_y

def buildaccz(planet, list_of_planets):
    accel_z = planet.accz(planet.x, planet.y, planet.z)
    for i in list_of_planets:
        if i.name!=planet.name:
            accel_z += gravforcez(planet, i)
    return accel_z
```

In [78]:
```
earth.x
```

Out[78]: -0.988

In [79]:
```
earth.accx(earth.x, earth.y, earth.z)
```

Out[79]: 39.99833339111919

In [80]:
```
planets = [mercury, venus, earth, mars, jupiter, saturn, uranus,
neptune, pluto]
```

In [81]:
```
#Just testing a function to make sure that the x acceleration makes sens
e
buildaccx(earth, planets)
```

Out[81]: 39.99773374665994

In [82]:
```
buildaccy(earth,planets)
```

Out[82]: -3.4417092456222957

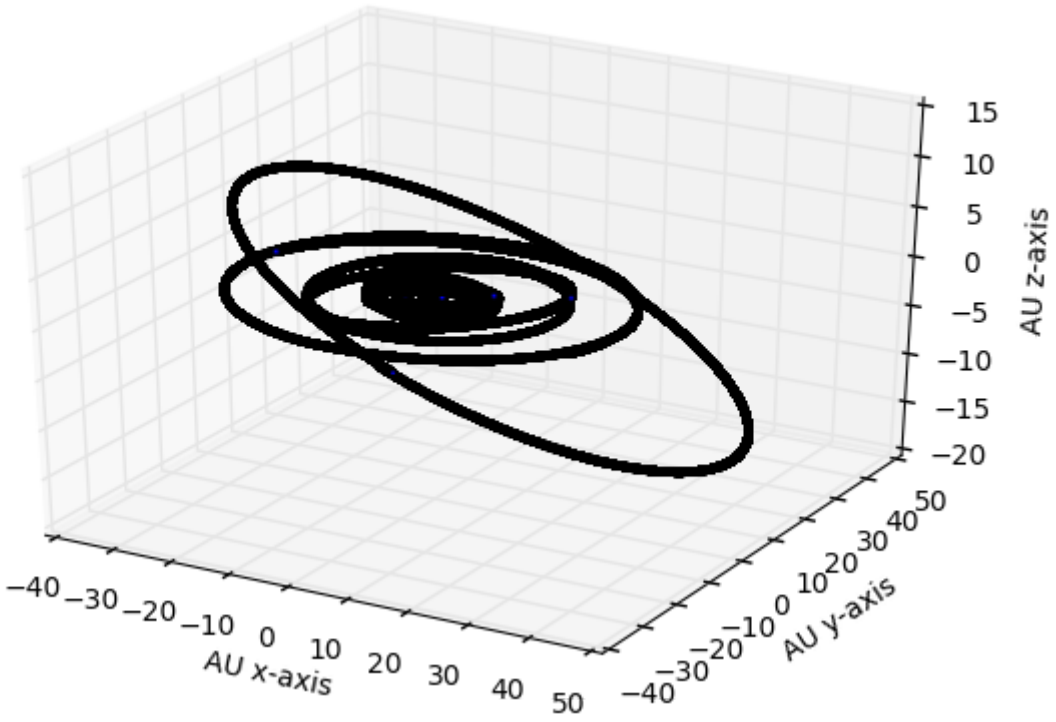
In [83]:
```
def coorx(xi, h,vxi, axi):
    return xi + h*vxi+h**2*axi/2

def velx(vxi, h, ax_i_1,ax_i):
    return vxi + (h/2)*(ax_i_1+ax_i)

def coory(yi, h,vyi, ayi):
    return yi + h*vyi+h**2*ayi/2

def vely(vyi, h, ay_i_1,ay_i):
    return vyi + (h/2)*(ay_i_1+ay_i)

def coorz(zi, h,vzi, azi):
    return zi + h*vzi+h**2*azi/2

def velz(vzi, h, az_i_1,az_i):
    return vzi + (h/2)*(az_i_1+az_i)
```

In [97]:
```
planets = [mercury, venus, earth, mars, jupiter, saturn, uranus,
neptune, pluto]
def solar_system(time):
    #The number of years we want to loop over, 10 years for this test
    h = 1/365 #The step size, defined as one day
    n = int(time/h) #The total numbers of iterations

    all_x =[]
    all_y = []
    all_z = []


    for obj in planets:
        coordinatesx = np.zeros(n+1)
        coordinatesy = np.zeros(n+1)
        coordinatesz = np.zeros(n+1)
        velocitiesx = np.zeros(n+1)
        velocitiesy = np.zeros(n+1)
        velocitiesz = np.zeros(n+1)

        coordinatesx[0] = obj.x
        coordinatesy[0] = obj.y
        coordinatesz[0] = obj.z

        velocitiesx[0] = obj.vx
        velocitiesy[0] = obj.vy
        velocitiesz[0] = obj.vz

        for j in range(n):
            x_j = coordinatesx[j]
            vx_j = velocitiesx[j]
            y_j = coordinatesy[j]
            vy_j = velocitiesy[j]
            z_j = coordinatesz[j]
            vz_j = velocitiesz[j]

            ax_j = buildaccx(obj, planets)
            ay_j = buildaccy(obj, planets)
```

```
            az_j = buildaccz(obj, planets)

            x_j_1 = coorx(x_j, h,vx_j,ax_j)
            y_j_1 = coory(y_j, h,vy_j,ay_j)
            z_j_1 = coorz(z_j, h,vz_j,az_j)

            coordinatesx[j+1] = x_j_1
            coordinatesy[j+1] = y_j_1
            coordinatesz[j+1] = z_j_1

            obj.movex(x_j_1)
            obj.movey(y_j_1)
            obj.movez(z_j_1)

            ax_j_1 = buildaccx(obj, planets)
            ay_j_1 = buildaccy(obj, planets)
            az_j_1 = buildaccz(obj, planets)

            vx_j_1 = velx(vx_j,h,ax_j_1,ax_j)
            vy_j_1 = vely(vy_j,h,ay_j_1,ay_j)
            vz_j_1 = velz(vz_j,h,az_j_1,az_j)

            velocitiesy[j+1] = vy_j_1
            velocitiesx[j+1] = vx_j_1
            velocitiesz[j+1] = vz_j_1

            obj.movevx(vx_j_1)
            obj.movevy(vy_j_1)
            obj.movevz(vz_j_1)

        all_x.append(coordinatesx)
        all_y.append(coordinatesy)
        all_z.append(coordinatesz)

        print(obj.name, "is done!")
    plotter(all_x, all_y, all_z)
```

In [98]:
```
solar_system(250)
```

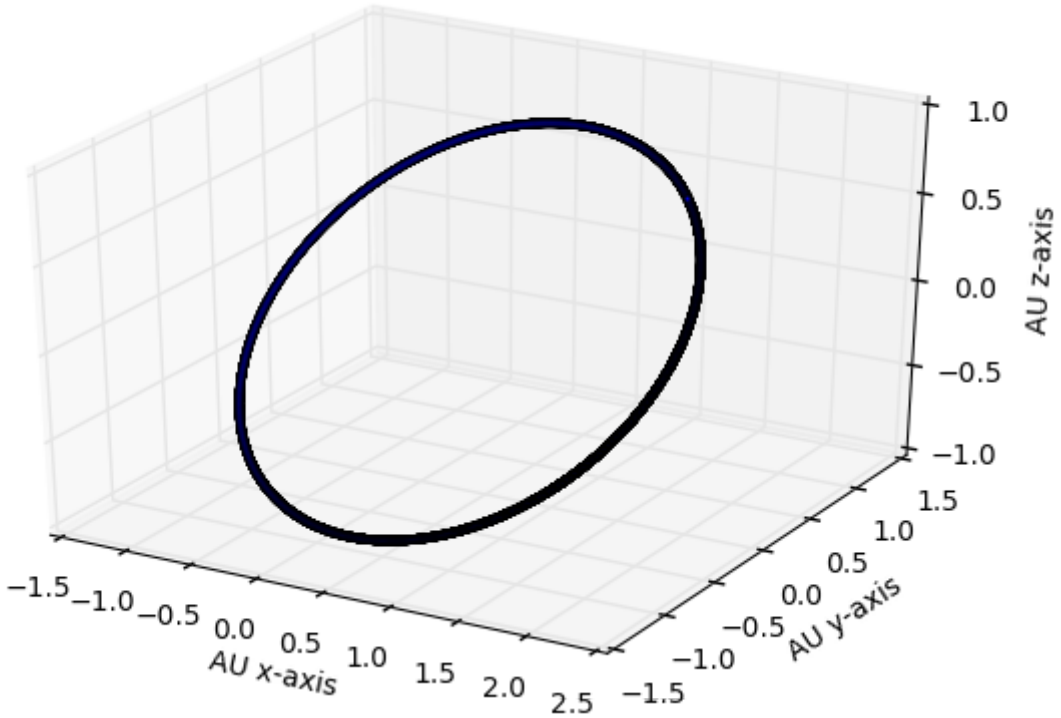
```
Mercury is done!
Venus is done!
Earth is done!
Mars is done!
Jupiter is done!
Saturn is done!
Uranus is done!
Neptune is done!
Pluto is done!
```

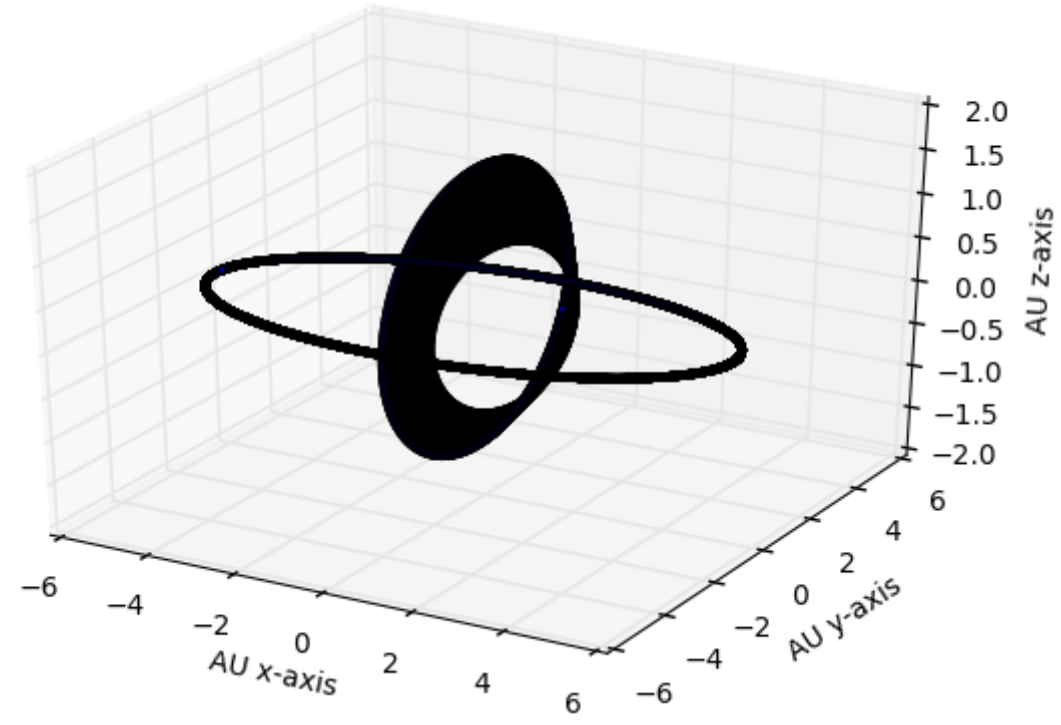


When we start to look at interactions with other planets, then things get a little bit hairy. What we do know is that the alogirthm works, and we know that the effects of Mercury may be altering the orbits of the inner planets.

In [93]:
```
planets = [earth]
solar_system(100)
planets = [earth, jupiter]
solar_system(100)
```

```
Earth is done!
Earth is done!
Jupiter is done!
```

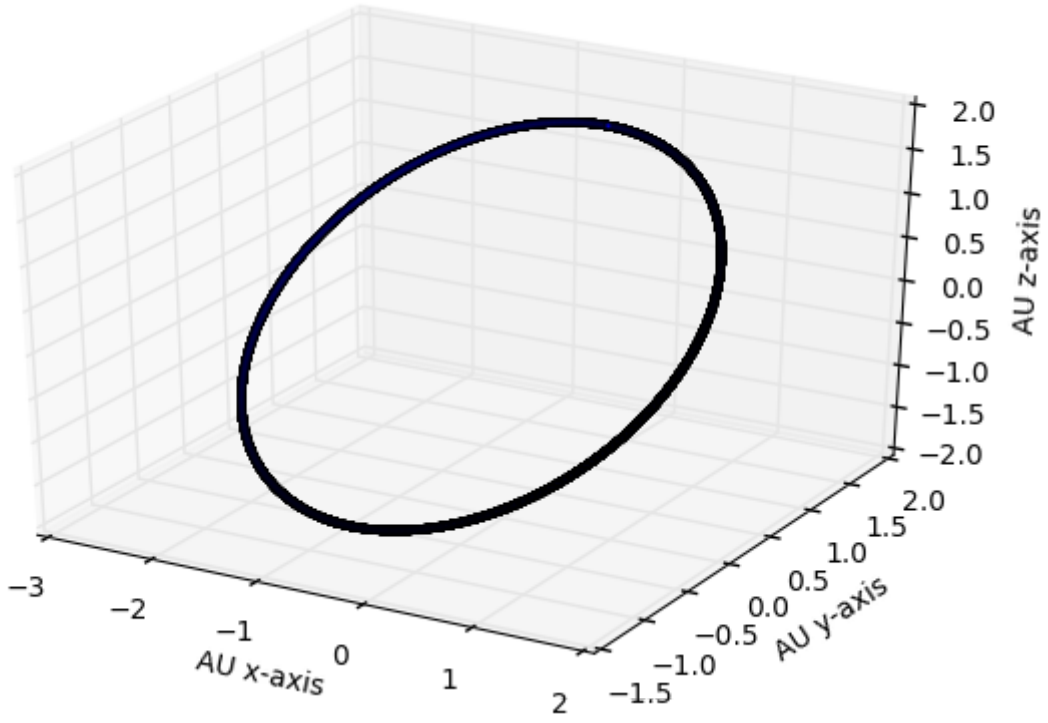


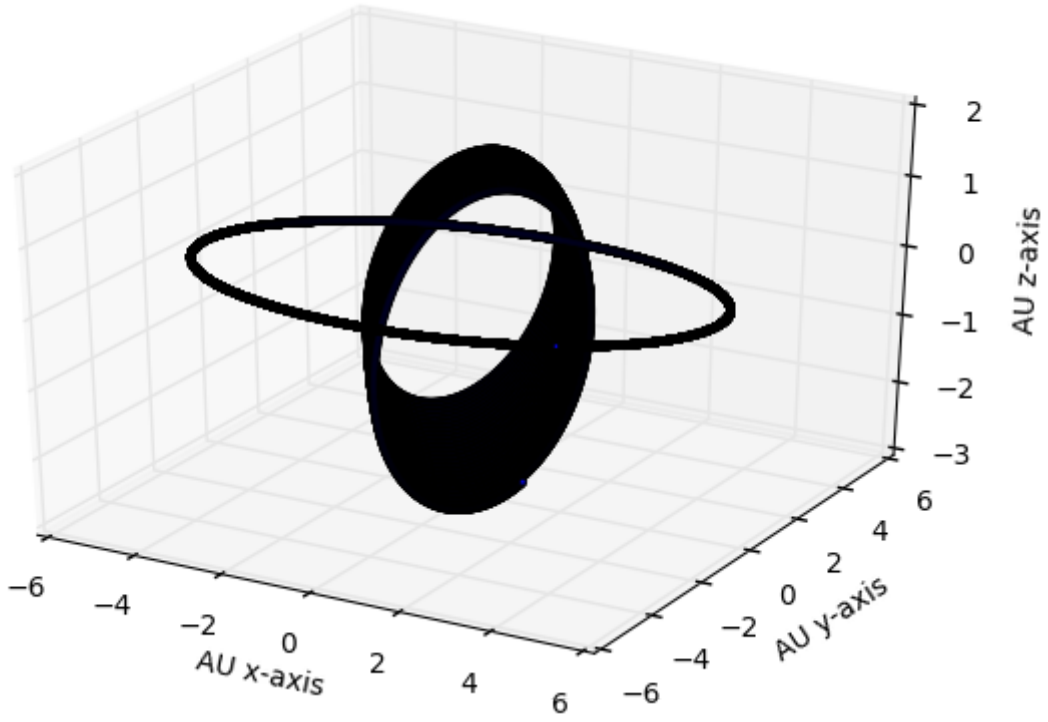


We can see that over the course of a century, there can be some efects on the orbit of Earth. It appears as though Jupiter is pulling the Earth towards it over the course of time. Now I want to rescale the mass of Jupiter.

In [94]:
```
jupiter.mass = 0.0095
planets = [earth]
solar_system(100)
planets = [earth, jupiter]
solar_system(100)
```

```
Earth is done!
Earth is done!
Jupiter is done!
```

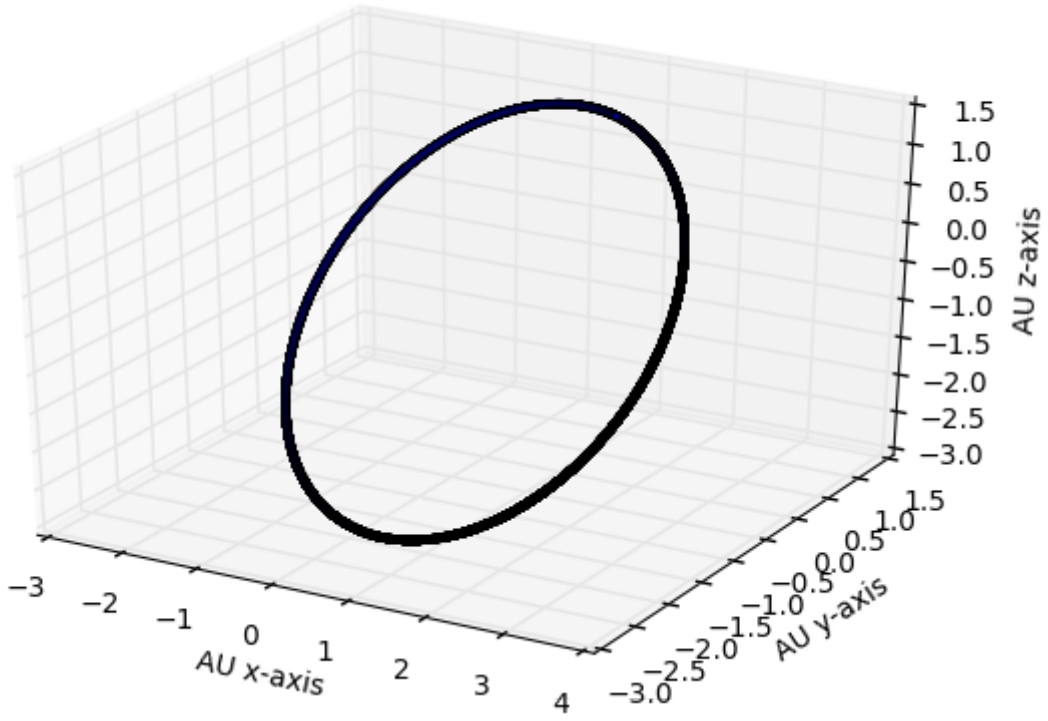


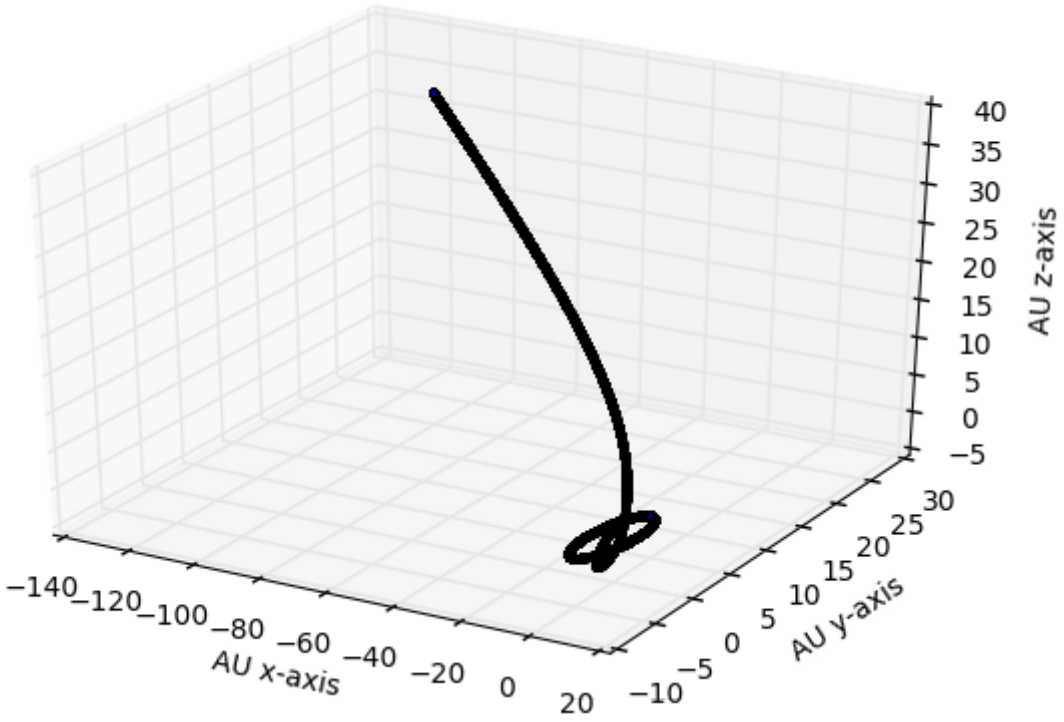


The verlet method is unstable, and we are quickly approaching that threshold. Also, we can see that with just the three body system, there is a significant contribution from Jupiter on Earth's orbit. Since Jupiter is so massive, we hardly notice any difference in its trajectory.

In [95]:
```
jupiter.mass = 0.095
planets = [earth]
solar_system(100)
planets = [earth, jupiter]
solar_system(100)
```

```
Earth is done!
Earth is done!
Jupiter is done!
```





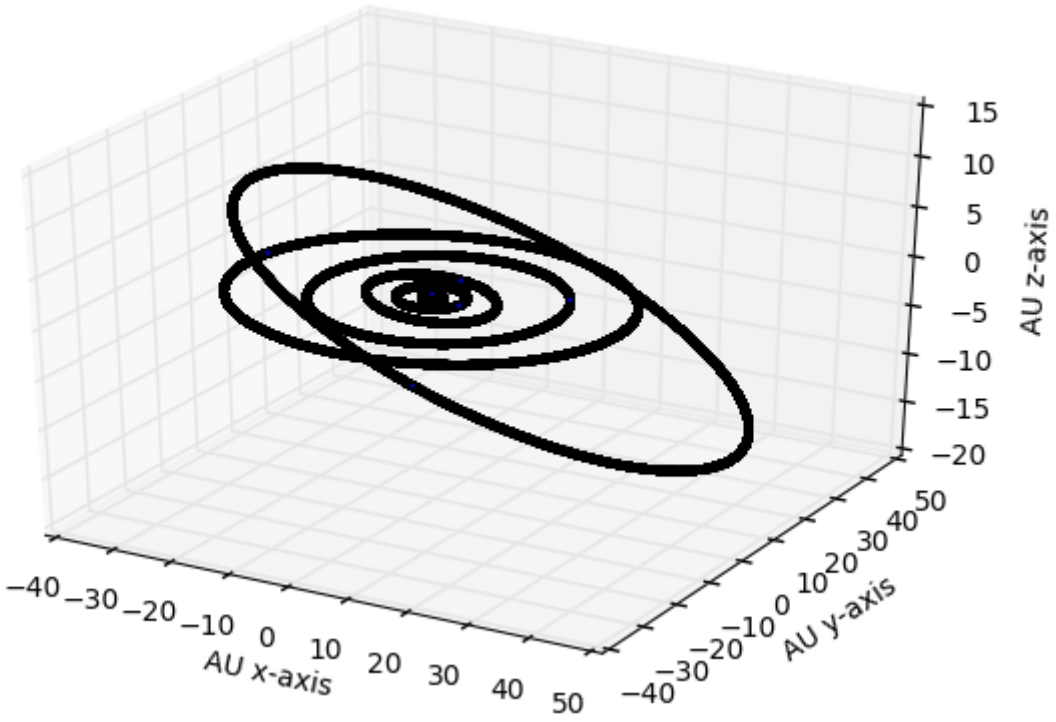
The above plot is the one that we were looking for. If we look at the mass of Jupiter scaled to be 1000 times it's original mass, then the Verlate algorithm breaks down and we lose stability. Here, we see the Earth sprial out of the galaxy. This tells us that there is some limitation to the stability of the verlet solver.

In [86]:
```
def solar_system_no_interaction(time):
    #The number of years we want to loop over, 10 years for this test
    h = 1/365 #The step size, defined as one day
    n = int(time/h) #The total numbers of iterations

    all_x =[]
    all_y = []
    all_z = []


    for obj in planets:
        coordinatesx = np.zeros(n+1)
        coordinatesy = np.zeros(n+1)
        coordinatesz = np.zeros(n+1)
        velocitiesx = np.zeros(n+1)
        velocitiesy = np.zeros(n+1)
        velocitiesz = np.zeros(n+1)

        coordinatesx[0] = obj.x
        coordinatesy[0] = obj.y
        coordinatesz[0] = obj.z

        velocitiesx[0] = obj.vx
        velocitiesy[0] = obj.vy
        velocitiesz[0] = obj.vz

        for j in range(n):
            x_j = coordinatesx[j]
            vx_j = velocitiesx[j]
            y_j = coordinatesy[j]
            vy_j = velocitiesy[j]
            z_j = coordinatesz[j]
            vz_j = velocitiesz[j]

            ax_j = obj.accx(x_j,y_j,z_j)
            ay_j = obj.accy(x_j,y_j,z_j)
            az_j = obj.accz(x_j,y_j,z_j)

            x_j_1 = coorx(x_j, h,vx_j,ax_j)
            y_j_1 = coory(y_j, h,vy_j,ay_j)
            z_j_1 = coorz(z_j, h,vz_j,az_j)

            coordinatesx[j+1] = x_j_1
            coordinatesy[j+1] = y_j_1
            coordinatesz[j+1] = z_j_1

            #obj.movex(x_j_1)
            #obj.movey(y_j_1)
            #obj.movez(z_j_1)

            ax_j_1 = obj.accx(x_j_1,y_j_1,z_j_1)
```

```
            ay_j_1 = obj.accy(x_j_1,y_j_1,z_j_1)
            az_j_1 = obj.accz(x_j_1,y_j_1,z_j_1)

            vx_j_1 = velx(vx_j,h,ax_j_1,ax_j)
            vy_j_1 = vely(vy_j,h,ay_j_1,ay_j)
            vz_j_1 = velz(vz_j,h,az_j_1,az_j)

            velocitiesy[j+1] = vy_j_1
            velocitiesx[j+1] = vx_j_1
            velocitiesz[j+1] = vz_j_1

            #obj.movevx(vx_j_1)
            #obj.movevy(vy_j_1)
            #obj.movevz(vz_j_1)

        all_x.append(coordinatesx)
        all_y.append(coordinatesy)
        all_z.append(coordinatesz)

        print(obj.name, "is done!")
    plotter(all_x, all_y, all_z)
```

In [88]:
```
solar_system_no_interaction(250)
```

```
Mercury is done!
Venus is done!
Earth is done!
Mars is done!
Jupiter is done!
Saturn is done!
Uranus is done!
Neptune is done!
Pluto is done!
```



We can see that the program handles both the interacting case and non interacting case wonderfully.

I'm calling it good with this one.

In [ ]:
```
```