Elec-H-401: Modulation and Coding

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Contents

1 Optimal communication over the ideal channel

1.1 Introduction

The goal of this first part is to implement a minimal working simulation of a QAM communication. In order to achieve that, the up- and downsampling blocks, the RRC filtering blocks and the baseband equivalent model of the channel, as represented in figure 1 must be implemented. The modulator

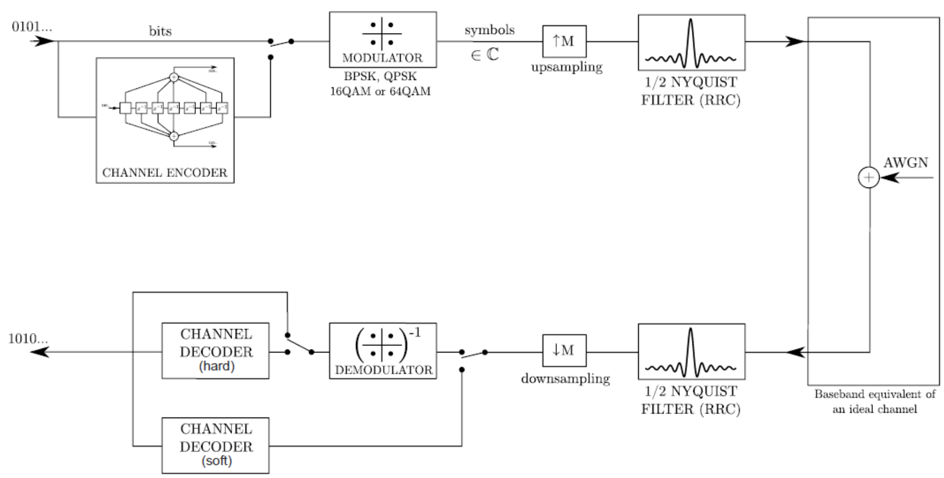

Figure 1: Block diagram of the communication system [source: Assignment introduction].

and demodulator are supplied with the assignment statement.

1.2 Halfroot Nyquist filtering

After its modulation, the message is upsampled and filtered with a root raised cosine filter to limit its bandwidth occupation. The effect on the PSD of the signal is shown in figure 2.

In order to maximize the SNR at the output, the low pass filtering is split between the transmitter and the receiver. The halfroot nyquist filter $g(t)$ is such that the resulting operation $h(t) = g(t) * g(t)$ forms a nyquist filter which does not introduce inter symbol interference, as shown in figure 3.

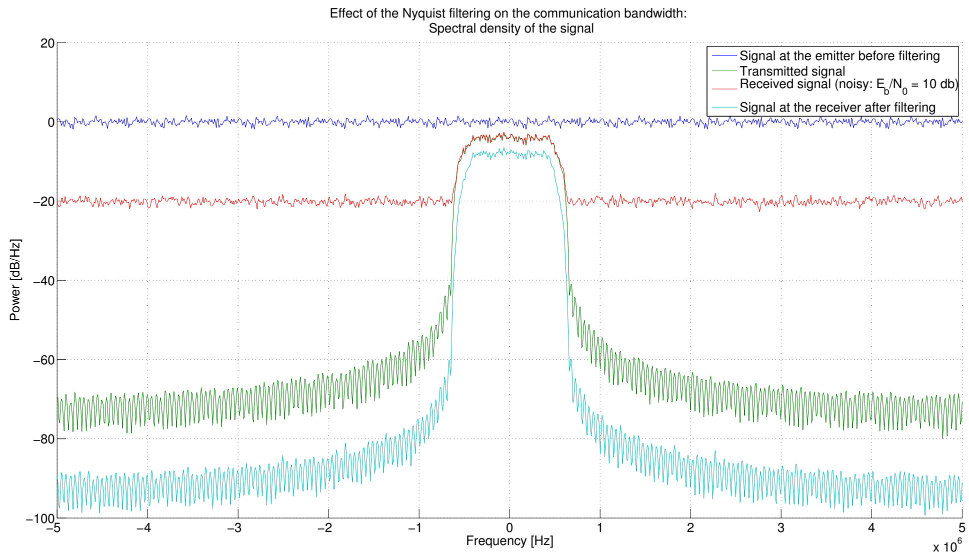

Figure 2: Nyquist filtering limits the communication bandwidth. $\beta = 0.3$, $n_{taps} = 20$ $f_m = 1\,\mathrm{MHz}$.

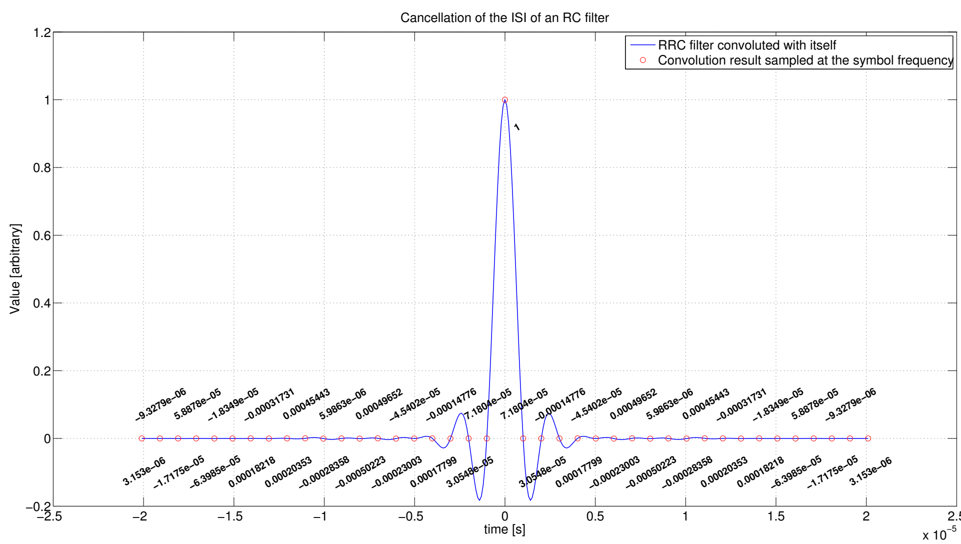

Figure 3: Cancellation of the inter symbol interference of a raised cosine filter. $\beta = 0.3$, $n_{taps} = 20$ $f_m = 1\,\mathrm{MHz}$.

1.3 Impact of the noisy channel

Theory shows that a channel affected by AWGN can be modelled in the baseband by AWGN of corresponding power. This allows to easily simulate the BER of the noisy channel. The results of the simulations are summarized by the BER curves of figure 4.

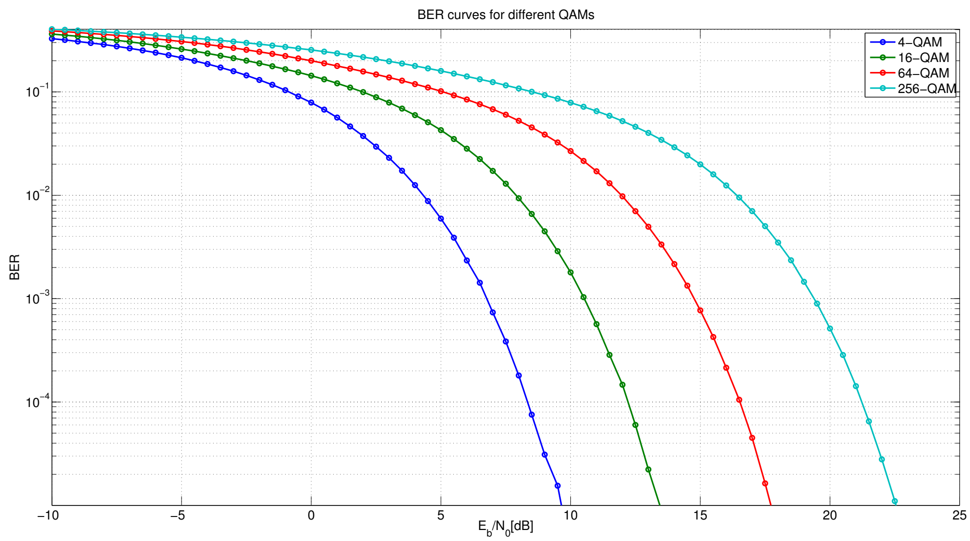

Figure 4: BER in function of $\frac{E_b}{N_0}$ for different QAM modulations.

1.4 Questions

1.4.1 Simulation

It is proposed to use the baseband equivalent model of the AWGN channel. Would it be possible to live with a bandpass implementation of the system? Simulating in the baseband has the advantage of reducing the sampling frequency needed (at least roughly twice the carrier frequency, which is unrealistic for modern GHz links) and allowing to implement and simulate modulation and demodulation techniques regardless of this carrier frequency. This is why baseband equivalent is always preferred in modelling wireless communication channels.

How do you choose the sample rate in Matlab? To be able to observe the effects of Nyquist filtering, the sampling rate must be at least twice as high as the symbol frequency. To be able to simulate sample time shift, even higher sampling rates should be used.

How do you make sure you simulate the desired $\frac{E_b}{N_0}$ ratio? Rather than adding a predetermined amount of noise to the signal, we first estimate its power in the useful frequency band, and then choose the noise power in order to obtain the required $\frac{E_b}{N_0}$ ratio.

How do you choose the number of transmitted data packets and their length? In order to reliably observe a BER of 10^{-n}, common practice is to send 10^{n+1} bits of data. We can send those in one simulation because the system is time invariant and the noise is ergodic.

1.4.2 Communication System

Determine the supported (uncoded) bit rate as a function of the physical bandwidth. Obviously, R = log2(M)/T with R = bit rate, M = number of symbols and T = symbol duration. With Nyquist filtering: T = 1/FM which is the bandwidth occupied by the signal (see figure 2).

Explain the trade-off communication capacity/reliability achieved by varying the constellation size. As the constellation size increases for a given SNR, the distance between the "edges" of the constellations spots that are spread out by the noise decreases, which means overlaps and thus errors get more and more frequent.

Why do we choose the halfroot Nyquist filter to shape the complex symbols? Filtering is required at the transmitter in order to limit the bandwidth used by the transmission. However, ideally, filtering is also needed at the receiver in order to maximize the SNR, since the external noise can affect all frequencies. This is why we choose to split the filtering operation between the transmitter and the receiver. Finally, ISI cancellation is required in order to be able to demodulate the signal properly. This is achieved by splitting a raised cosine filter between the transmitter and the receiver.

How do we implement the optimal demodulator? Give the optimisation criterion.

How do we implement the optimal detector? Give the optimisation criterion.

2 Time and frequency synchronisation

2.1 Effect of the CFO-caused ISI on the BER

The degradation of BER due only to the ISI introduced by the CFO is shown in figure 5. Figure 5 shows that for a given value of $\Delta\omega$ and at

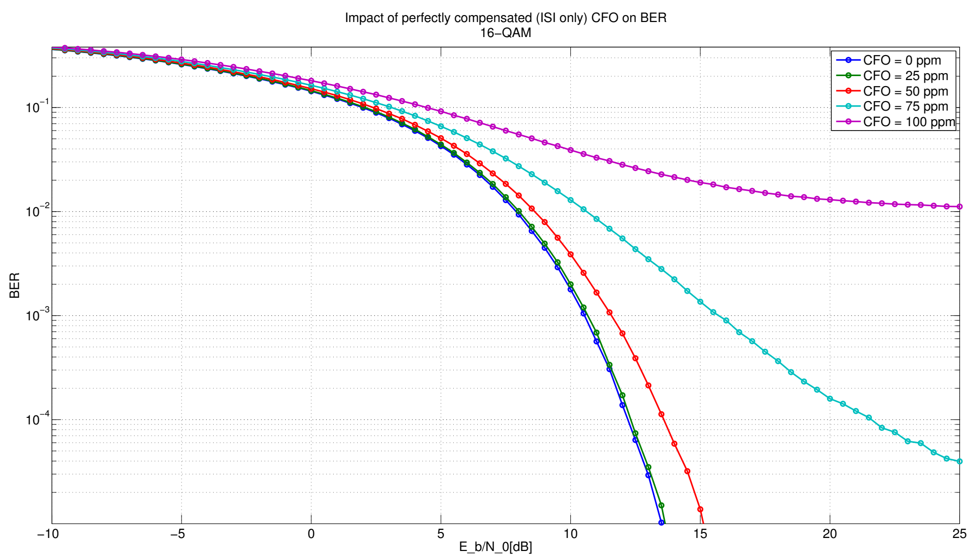

Figure 5: Impact of the ISI introduced by the CFO on the BER.

higher noise levels, the BER curve is worsened but shows similar behaviour as the original curve with no CFO. However, past a certain value of $\frac{E_b}{N_0}$, the BER stops decreasing and only the error introduced by the ISI subsist. Those plateaus in the curves correspond to what we observed when we were testing the simulation with no CFO but with a misdefined filter which did not cancel the ISI. To conclude, figure **??** shows the ISI of two halfroot nyquist filters separated by CFO.

The impact of phase drift over time is immediate past a certain message length, and the BER becomes 0.5.

2.2 Sample time shift errors

Figure 6 shows the impact of the sample time shift on the error rate.

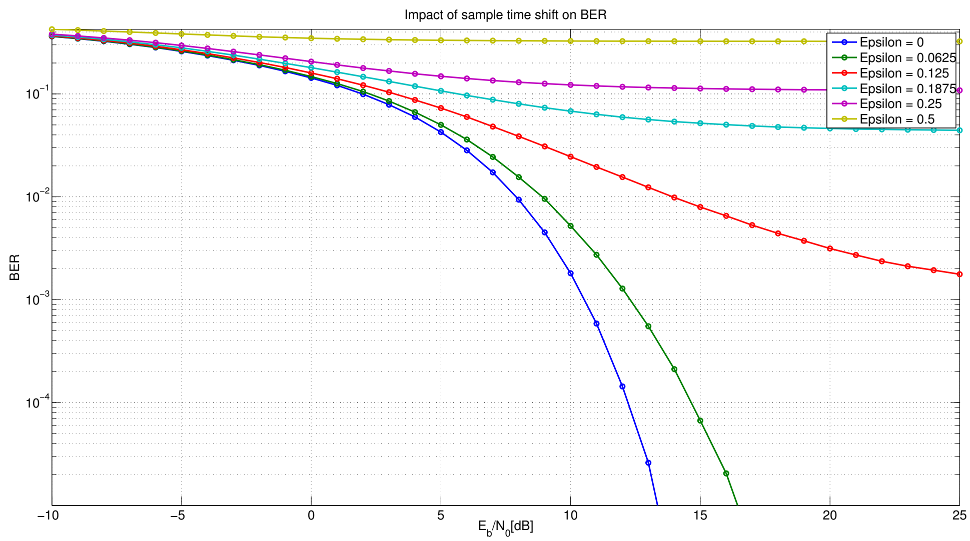

Figure 6: Impact of the sample time shift on the BER.

2.3 Questions

2.3.1 Simulation

Derive analytically the baseband model of the channel including the synchronisation errors. Derivation: pen and paper.

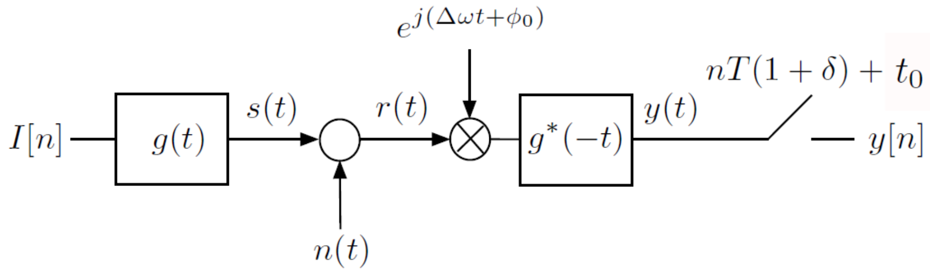

Figure 7: Baseband model with synchronization errors.

How do you separate the impact of the carrier phase drift and ISI due to the CFO in your simulation? We do this by perfectly cancelling the CFO by hand after the filtering operation at the receiver.

How do you simulate the sampling time shift in practice? The modulated message at the symbol frequency is first upsampled before all the other operations. At the receiver, after the filtering operation, is signal is downsampled. During this downsampling operation, a fixed sampling time shift can be introduced by shifting the indexes.

How do you select the simulated E_b/N_o ratio? Typical value is 10 dB. Small enough to get something correct at the end and big enough to get some errors to be able to test our simulated channel with noise.

How do you select the lengths of the pilot and data sequences? The pilot's length should be long enough to get a good estimation of the phase and the length of the data is selected to ensure a correct phase interpolation between two pilot sequences. Furthermore, the pilot sequence length and repetition should be as small as possible in order to maximize the channel throughput.

2.3.2 Communication System

In which order are the synchronisation effects estimated and compensated. Why? First the sampling time shift is estimated and compensated with Gardner's algorithm, because frame and frequency acquisition can only work on a correctly sampled sequence, while Gardner's algorithm is robust to CFO.

Explain intuitively how the error is computed in the Gardner algorithm. Why is the Gardner algorithm robust to CFO? At step n, the time shift error is estimated by looking at the value of the signal between samples n and n-1. If there was a zero crossing, then the middle value should be zero. The estimation of epsilon is then corrected by a term that is proportional to the value of this middle sample.

Explain intuitively why the differential cross-correlator is better suited than the usual cross-correlator? Isn't it interesting to start the summation at $k = 0$ (no time shift)? The differential cross-correlator first estimates the start time, and then the CFO. In doing so, it avoids an exhaustive 2D search which is computationally expensive.

Are the frame and frequency acquisition algorithms optimal? If yes, give the optimisation criterion.