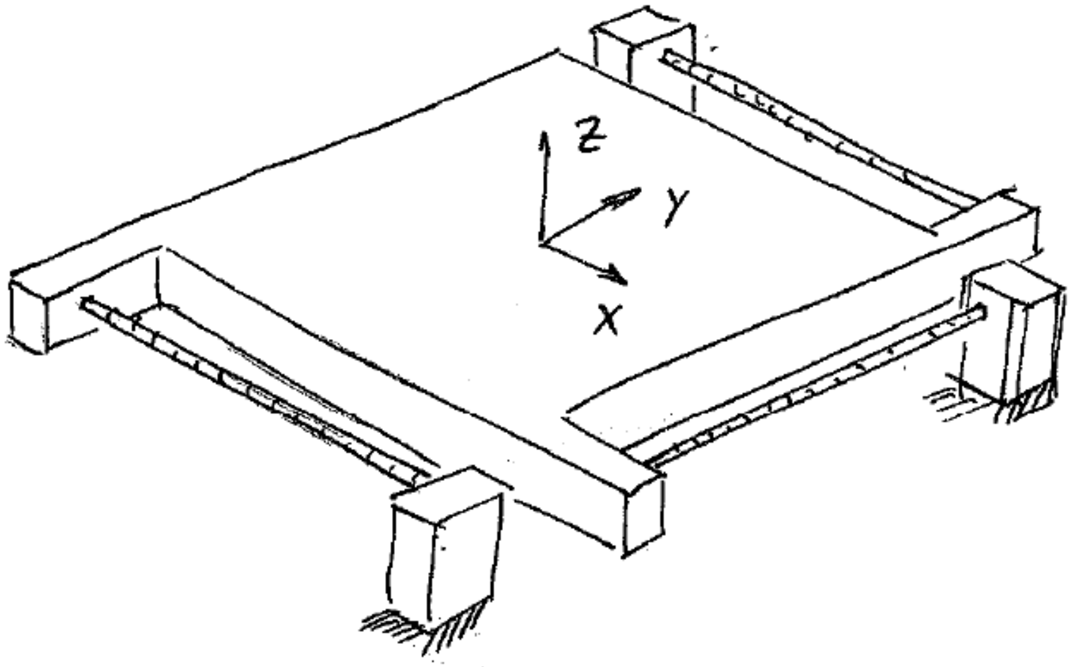
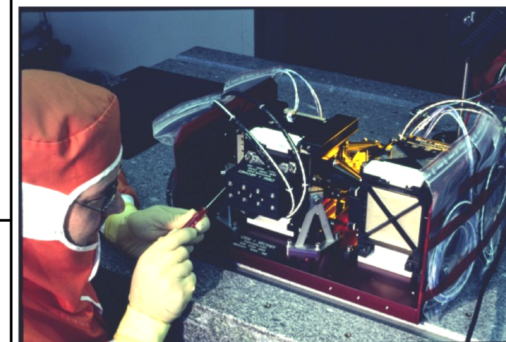
KINEMATICS OF FLEXURE-MECHANISMS



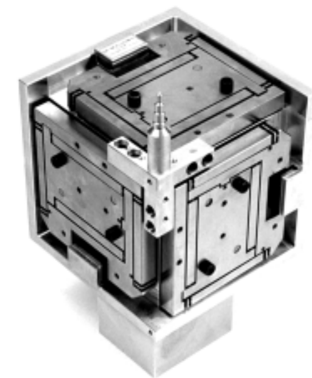
Prof. Simon Henein, Dr. Etienne Thalmann

Complexity scale

Flexure **mechanism**
(robot, machine…)



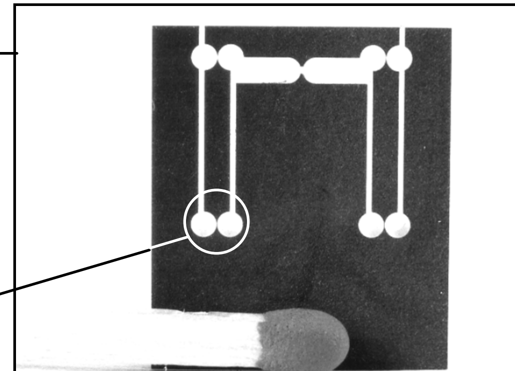
Flexure **structure**



Flexure **joint**

“Basics”

Basic flexure

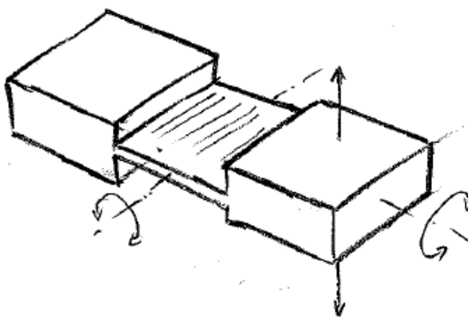
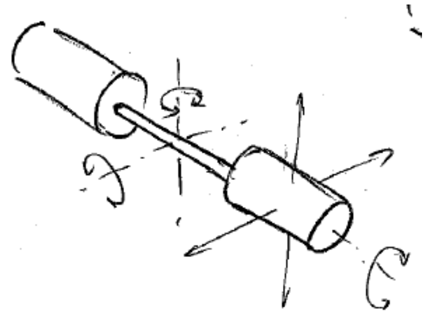
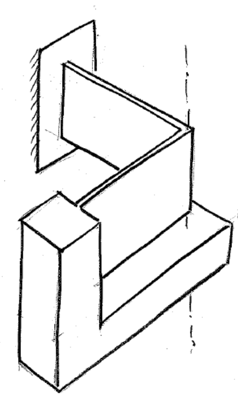
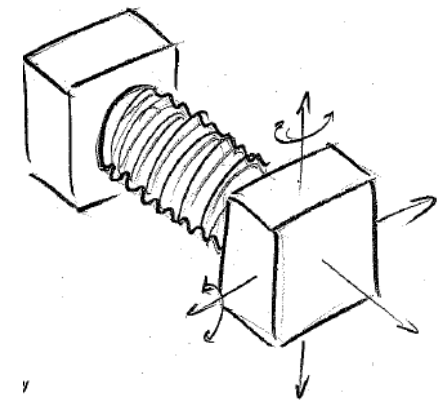


DOF of basic flexures

Simple examples of flexures as kinematical joints

Freedom

Ground

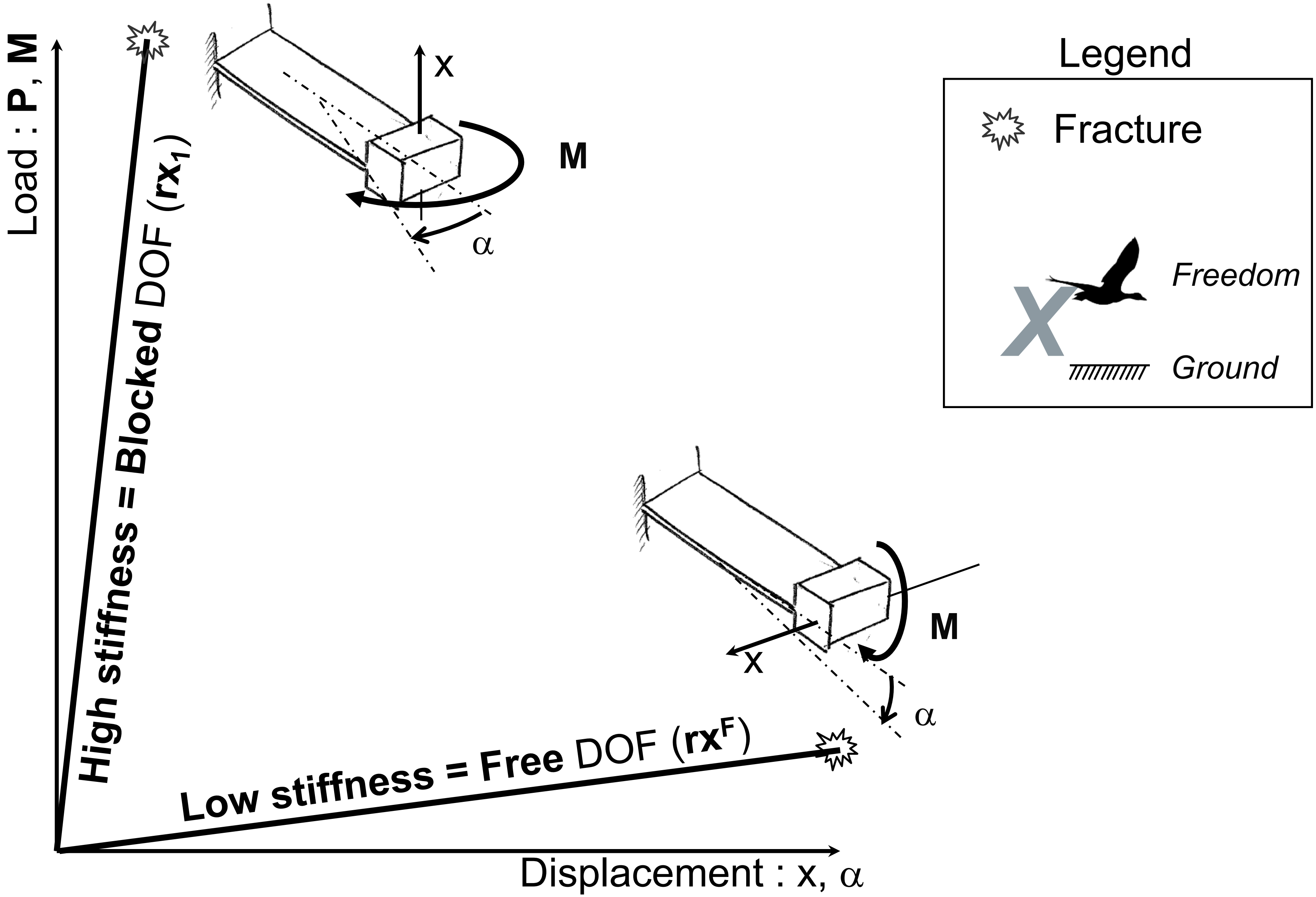





Name :	**Leaf spring**	**Rod**	**Corner-blade**	**Bellow**
Degrees of freedom:	**DOF = 3**	**DOF = 5**	**DOF = 5**	**DOF = 5**
Notation :	$\boldsymbol{L}_3^3$	$\boldsymbol{R}_1^5$	$\boldsymbol{C}_1^5$	$\boldsymbol{B}_1^5$

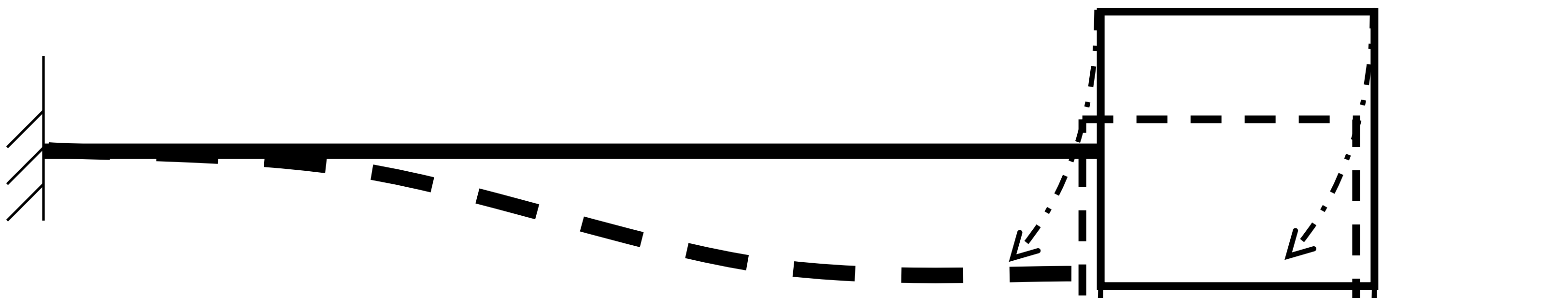
Remarks : - the sum of the *Freedom* and *Ground* indices is always equal to 6

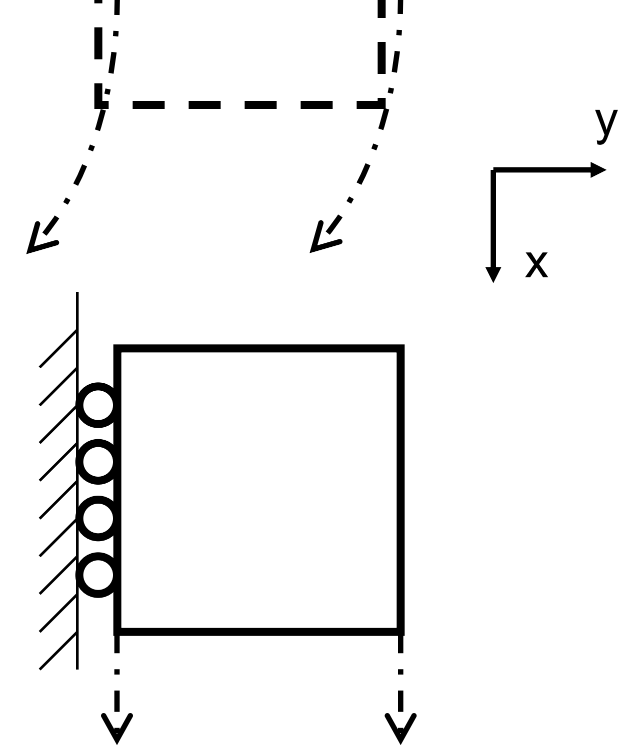
- we assume that all loads are smaller than the critical loads (i.e. buckling is excluded)
- we assume that Hooke's law and classical mechanics of material beam theory apply

DOFs of flexure-joints and respective stiffness



…^{Fp} : Parasitic motion of a leaf spring or a rod used in translation (no rotation of the extremity)





Curvilinear translation

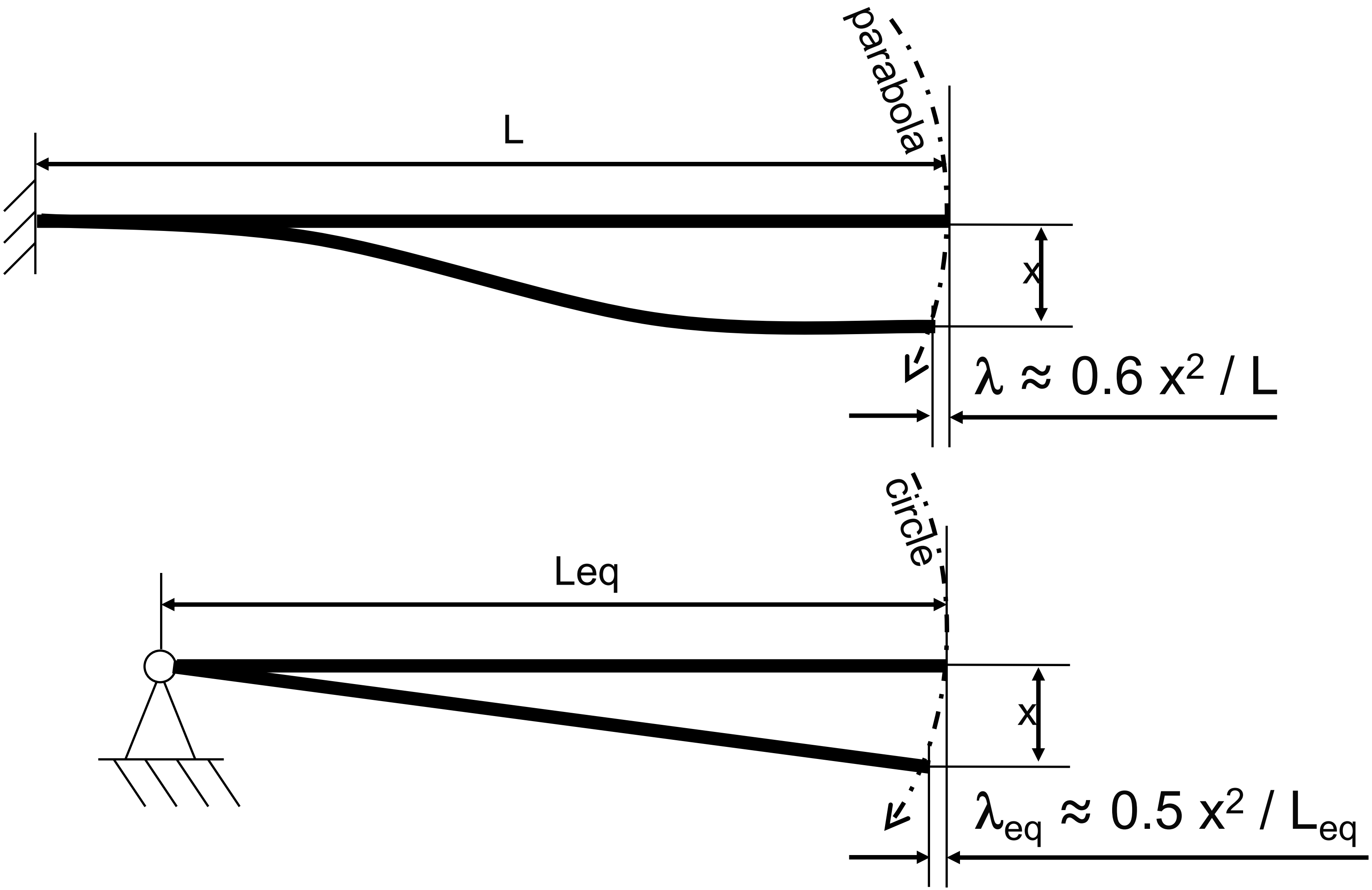
=

Rectilinear translation along x,

accompanied by a parasitic shift along y

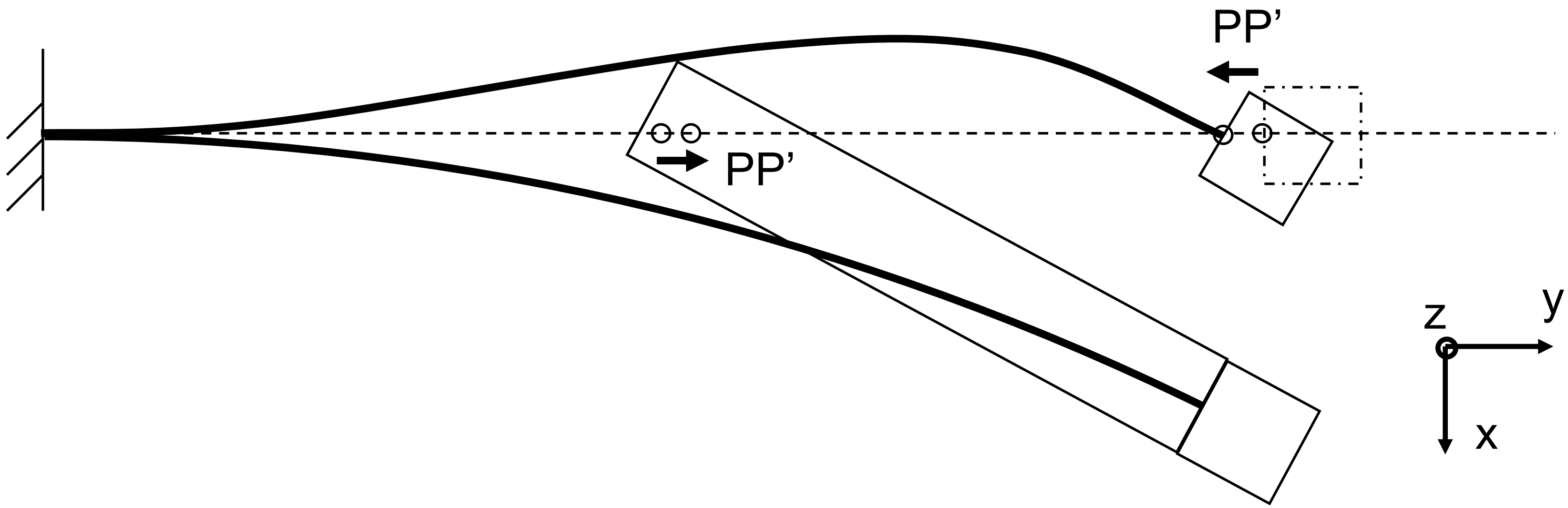
$$x^{Fp}$$

…[Fp] : Parasitic motion of a leaf spring or a rod used in translation (no rotation of the extremity)



For $L_{eq} = 5/6\ L$ we get $\lambda_{eq} \approx \lambda$

…^{Fp} : Parasitic motion of a leaf spring or a rod used as revolute joint

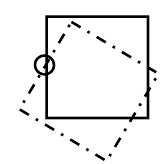


Rotation with centre shift

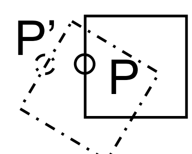
=

Pure rotation about an axis parallel to z, accompanied by a parasitic shift in the xy plane

$$rz^{Fp}$$

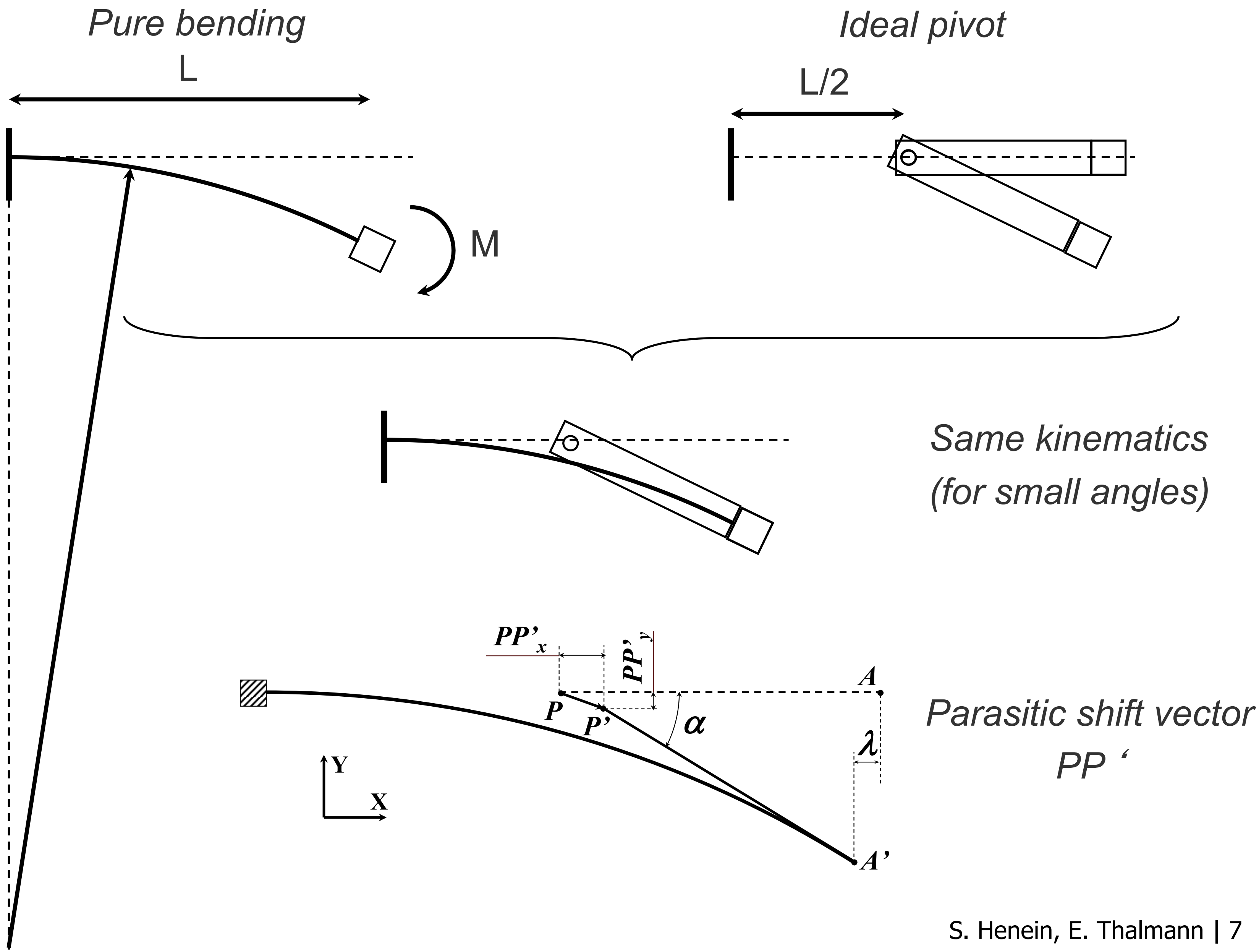


pure rotation

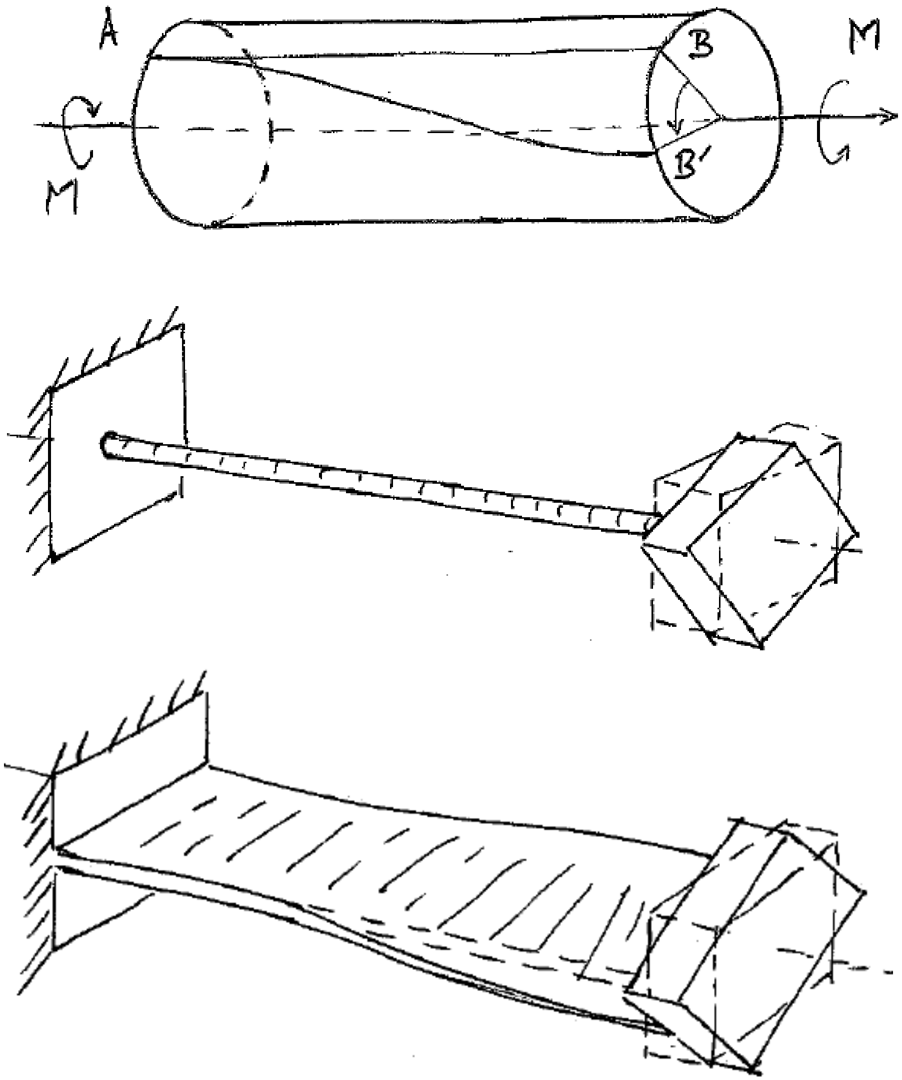


pure rotation + shift PP'

…[Fp] : Parasitic motion of a leaf spring or a rod used as revolute joint (pure bending)

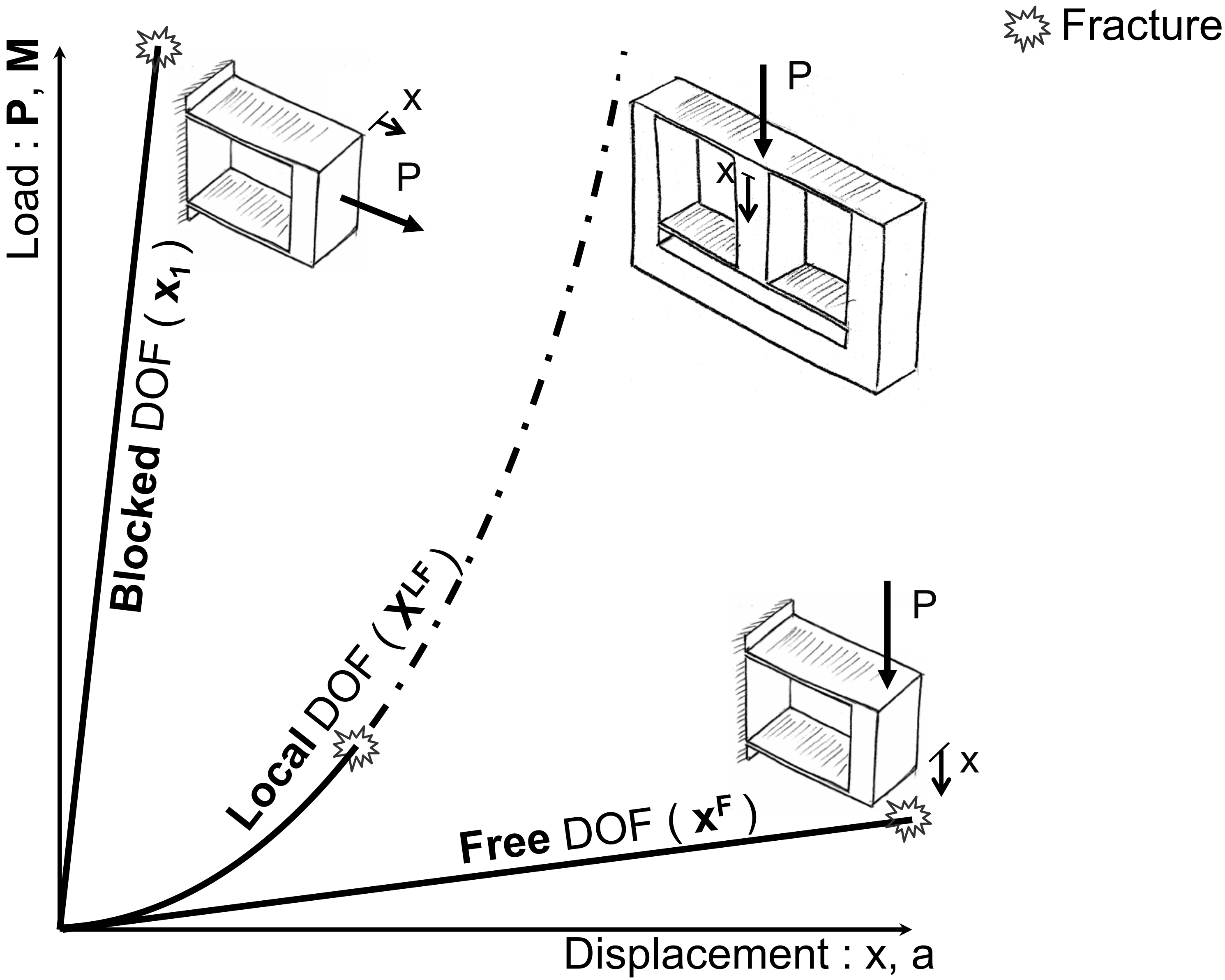


…F : Parasitic motion in torsion ≈ zero

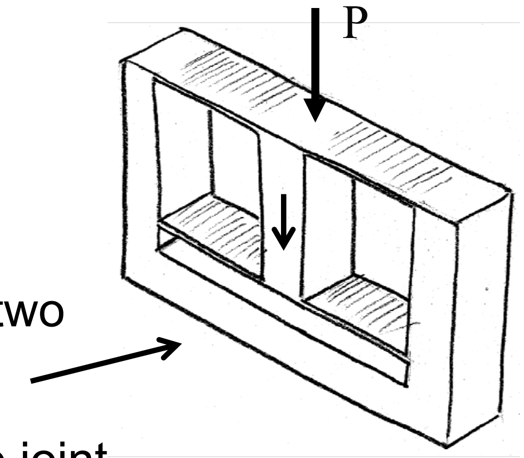


For rotations about the natural torsion axis of the flexures, we neglect the shortening effects and thus assume that the parasitic shift is zero.

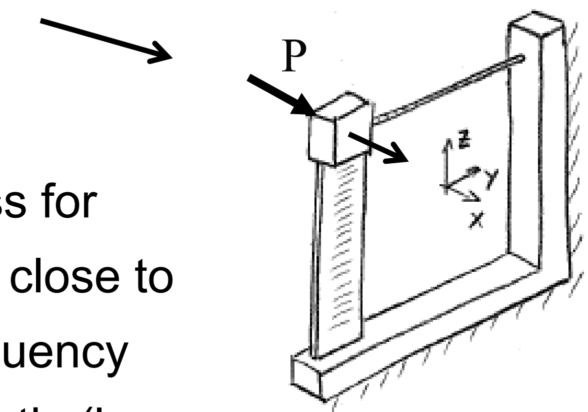
DOFs of flexure-bearings and respective stiffness



…^{LF}: Local Degrees of Freedom

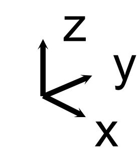


- Causes
  - Conflict between the parasitic motion of at least two flexure-joint
  - Conflict between the parasitic motion of a flexure joint and the blocked DOF of another flexure-joint



- Consequences
  - Corresponding DOF not blocked : low stiffness for microscopic displacement (slope at the origin close to free DOFs), i.e. free to vibrate, low eigen-frequency
  - Progressive force vs displacement characteristic (large forces required to produce macroscopic displacements)
  - High stresses induced in the flexure-joints and the rigid segments (distortions)
  - Short admissible strokes compared to free DOFs

Notation



- x, y, z : translations parallel to the respective axes
- rx, ry ,rz : rotations about axes parallel to the respective axes
- …$^{\mathbf{F}}$: Free DOF
  - example : x^{F} = rectilinear translation along x
  - example: rx^{F} = pure rotation about an axis parallel to x
- …$^{\mathbf{Fp}}$: Free DOF, accompanied with a parasitic motion
- …$^{\mathbf{LF}}$: Local Degree of Freedom i.e. “free to vibrate”
- …$^{\mathbf{LFp}}$: Local Degree of Freedom, with parasitic motion
- …$_{1}$: DOF blocked once
- …$_{\mathbf{2}}$: DOF blocked twice, i.e. DOH = 1 (Degree of Hyperstaticity)
- …$_{\mathbf{3}}$: DOF blocked thrice, i.e. DOH = 2

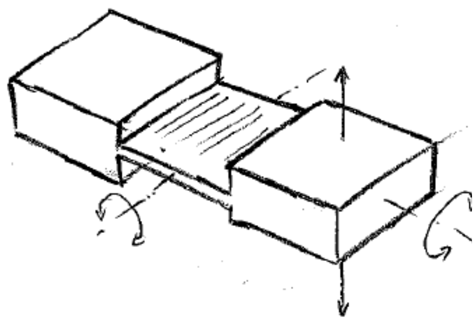
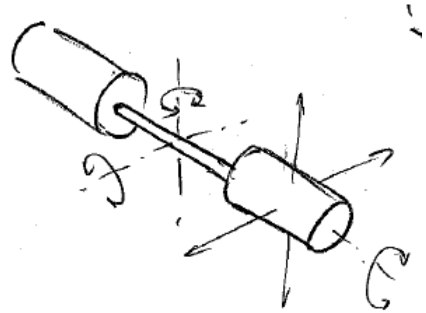
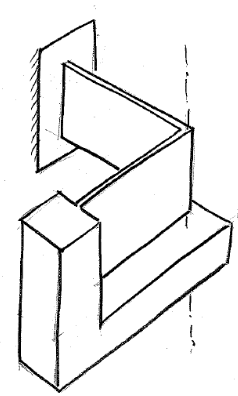
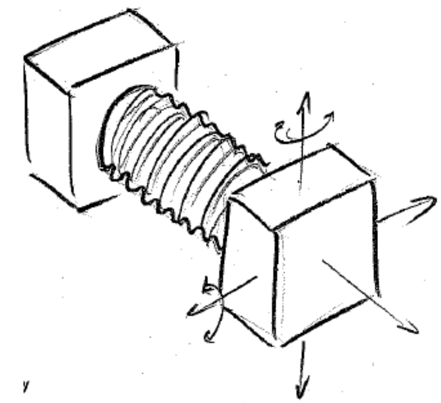
Freedom ⟶ *_ , F, Fp, LF, LFp*

Ground ⟶ *_ , 1, **2**, **3**, **4**, **5**,…*

Basic flexures

Freedom

Ground

Name :	**Leaf spring**	**Rod**	**Corner-blade**	**Bellow**
Degrees of freedom:	**DOF = 3**	**DOF = 5**	**DOF = 5**	**DOF = 5**
Notation :	$\boldsymbol{L}_3{}^3$	$\boldsymbol{R}_1{}^5$	$\boldsymbol{C}_1{}^5$	$\boldsymbol{B}_1{}^5$

Remarks : - the sum of the *Freedom* and *Ground* indices is always equal to 6
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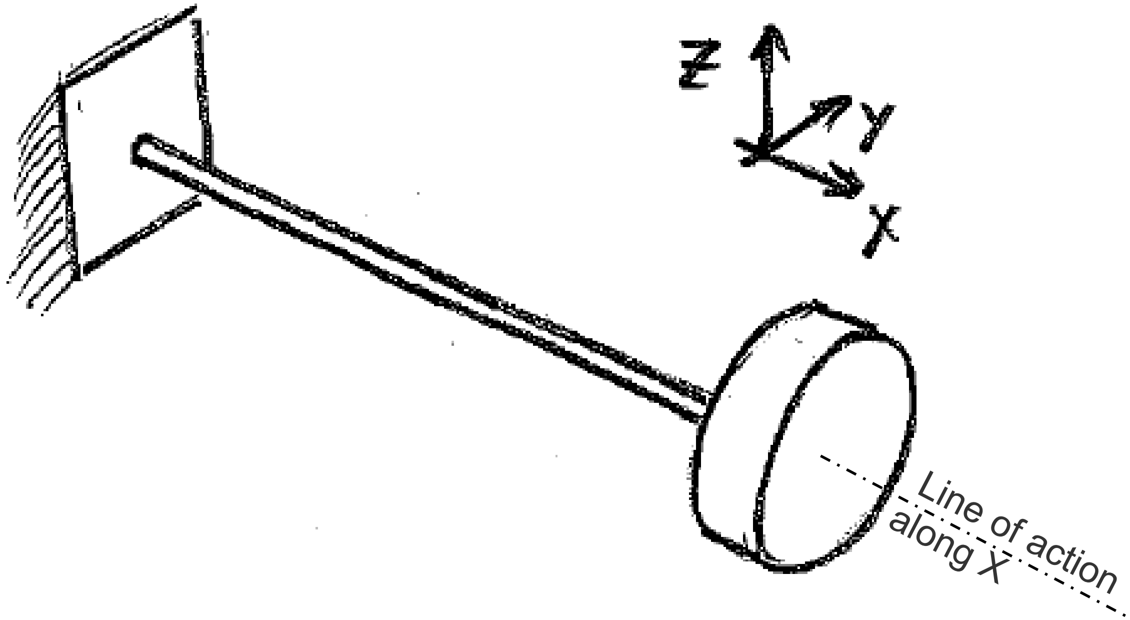
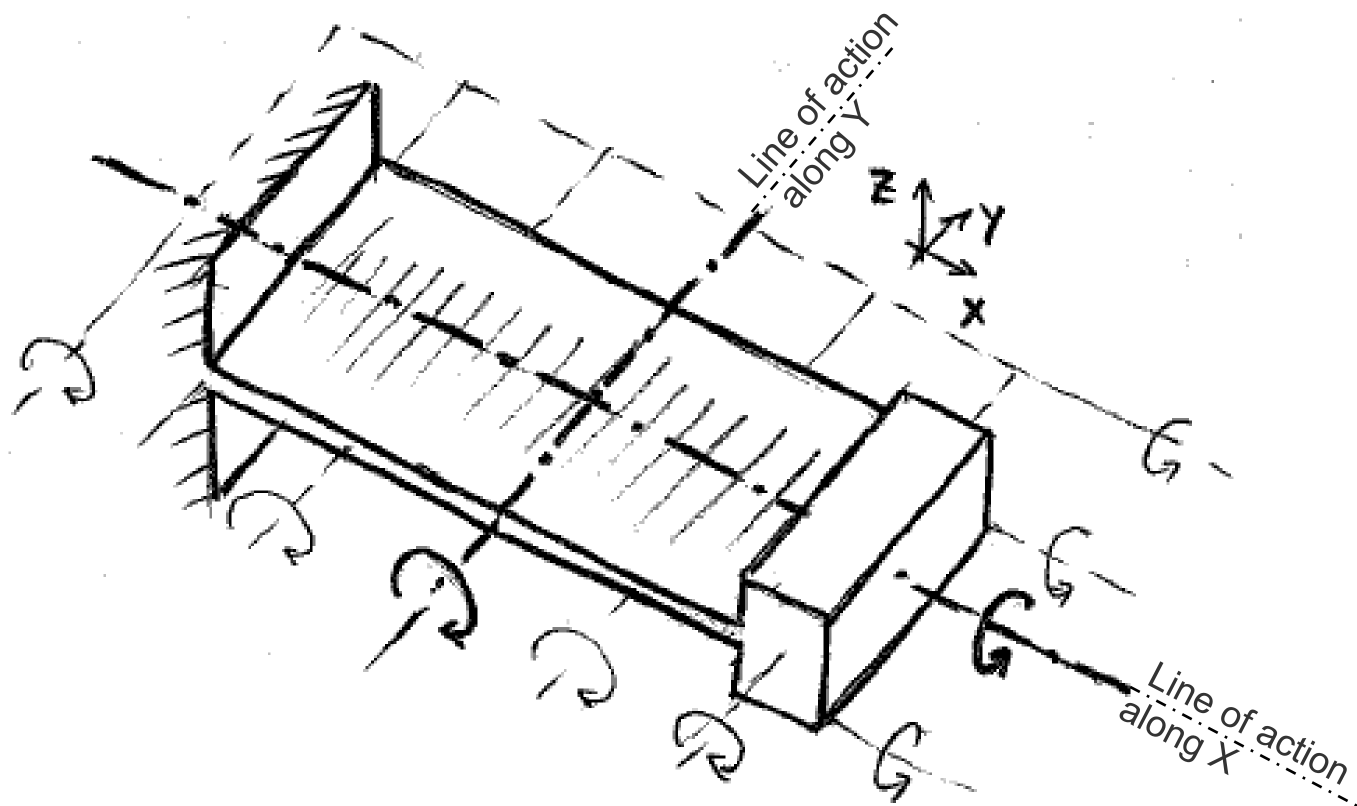
Rod flexure R_1^5



x_1, $\mathbf{y^{Fp}}$, $\mathbf{z^{Fp}}$, $\mathbf{rx^{F}}$, $\mathbf{ry^{Fp}}$, $\mathbf{rz^{Fp}}$

DOF = 5; M = 5; DOH = 0

Leaf spring flexure L_3^3

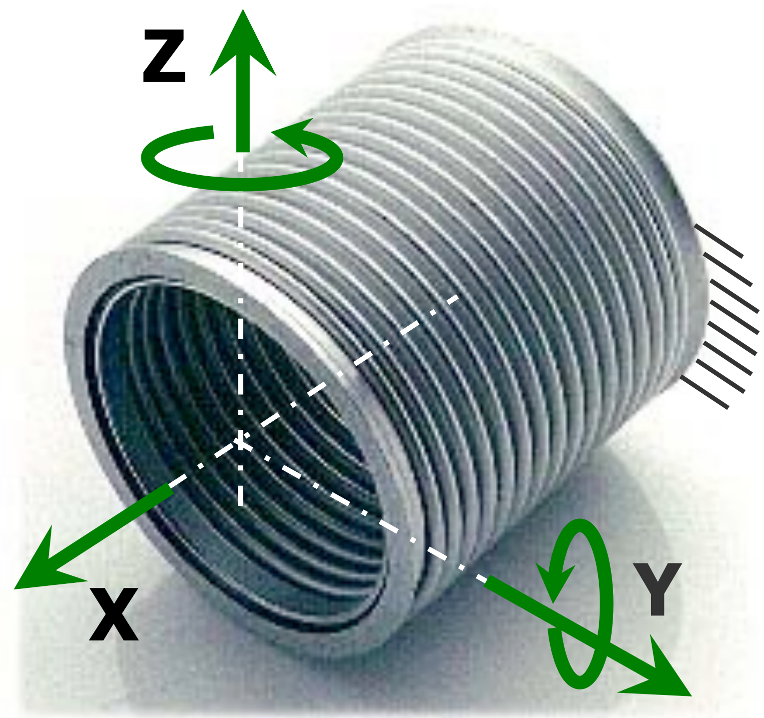


- Translation **z** (bending in S shape)
- Rotations **ry** (bending)
- Rotations **rx** (torsion)

x_1, y_1, $\mathbf{z^{Fp}}$, $\mathbf{rx^{F}}$, $\mathbf{ry^{Fp}}$, rz_1

DOF = 3; M = 3; DOH = 0

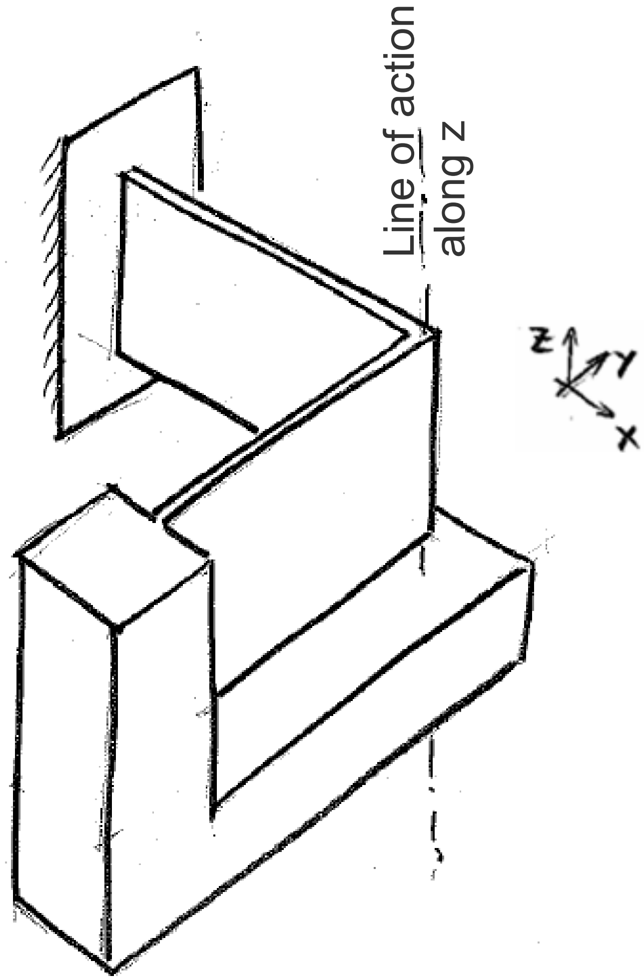
Bellow flexure B_1^5



$\mathbf{x^F}$, $\mathbf{y^F}$, $\mathbf{z^F}$, rx_1, $\mathbf{ry^F}$, $\mathbf{rz^F}$

DOF = 5; M = 5; DOH = 0

Corner-blade flexure C_1^5



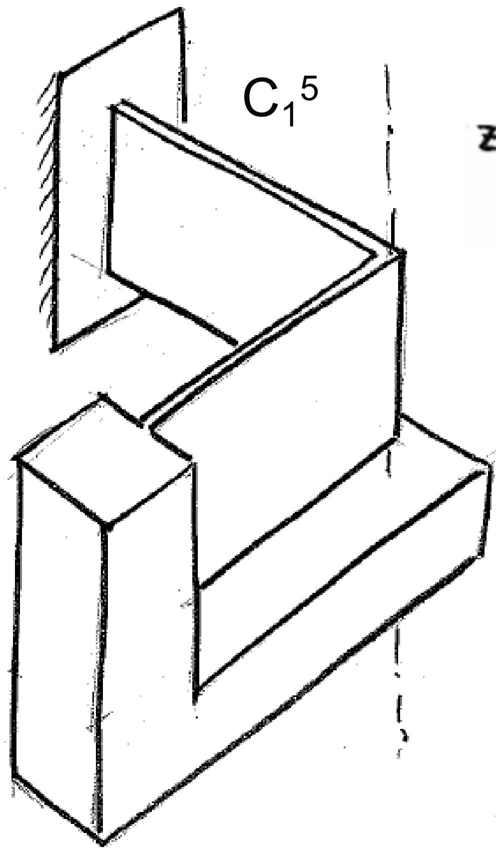
x^F, y^F, z_1, rx^F, ry^F, rz^F

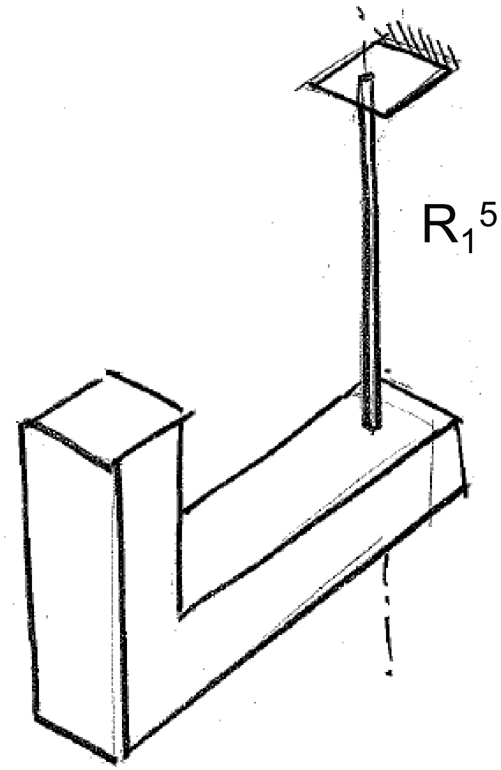
DOF = 5; M = 5; DOH = 0

Corner blade and equivalent rod

$x^F, y^F, z_1, rx^F, ry^F, rz^F$

DOF = 5; M = 5; DOH = 0

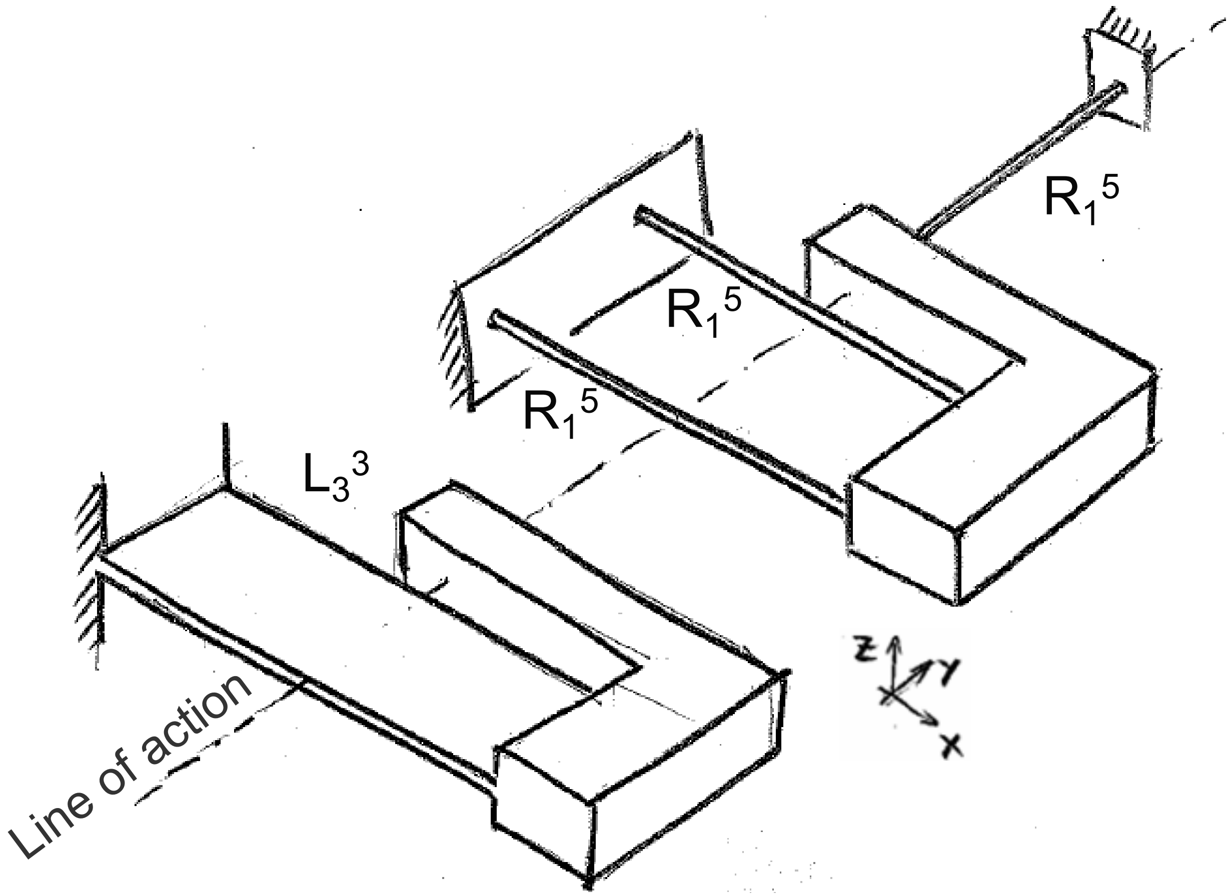




$x^{Fp}, y^{Fp}, z_1, rx^{Fp}, ry^{Fp}, rz^F$

DOF = 5; M = 5; DOH = 0

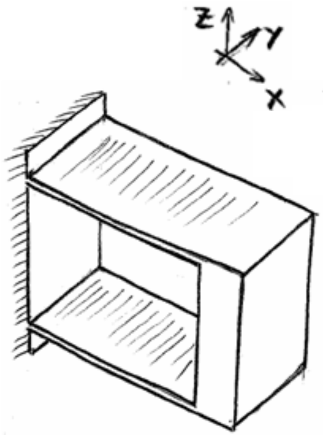
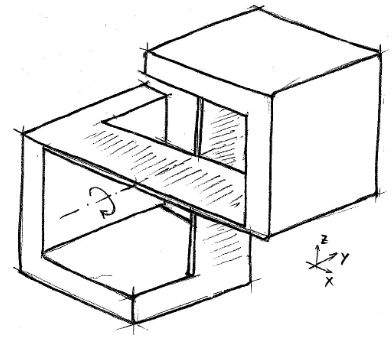
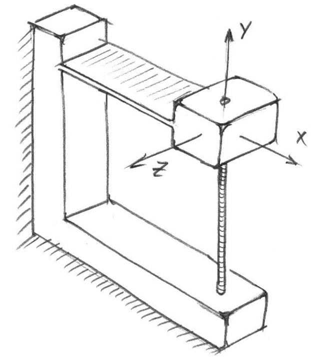
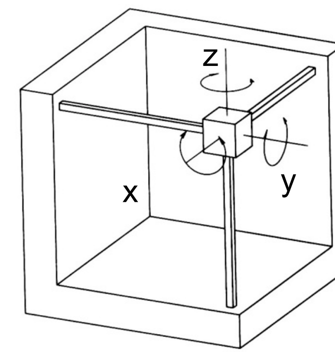
Leaf spring and equivalent rod assembly



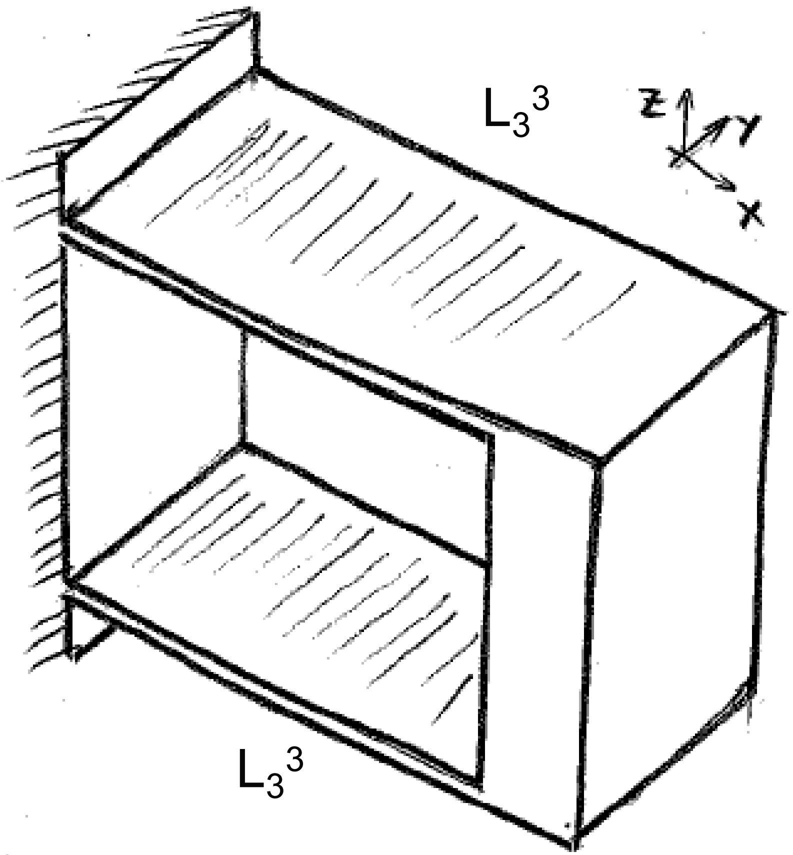
x_1, y_1, $\mathbf{z^{Fp}}$, $\mathbf{rx^F}$, $\mathbf{ry^{Fp}}$, rz_1

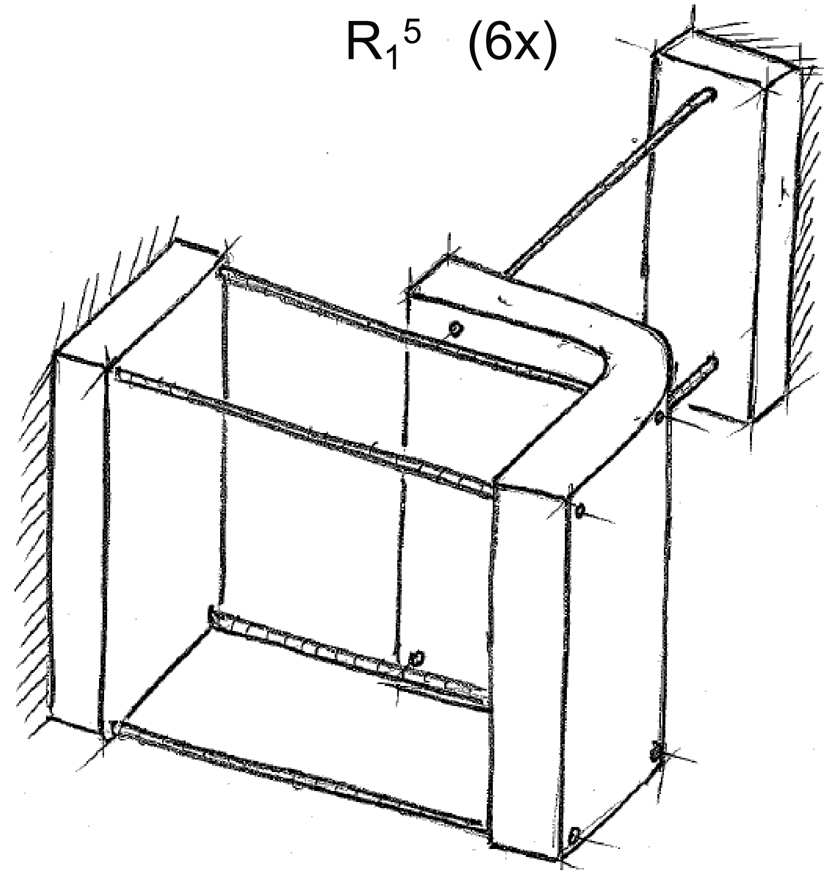
DOF = 3; M = 3; DOH = 0

Flexure joint examples

				
Name :	**Parallel flexure**	**Cross-spring pivot**	**Flexure Cardan**	**Flexure rotula**
Degrees of freedom:	$x_1, y_1, \mathbf{z^{Fp}}, rx_1, ry_1, rz_\mathbf{2}$	$x_1, y_\mathbf{2}, z_1, rx_1, \mathbf{ry^{Fp}}, rz_1$	$x_1, y_1, z_1, \mathbf{rx^{Fp}}, ry_1, \mathbf{rz^{Fp}}$	$x_1, y_1, z_1, \mathbf{rx^{Fp}}, \mathbf{ry^{Fp}}, \mathbf{rz^{Fp}}$
Equivalent ideal joint	**Slider**	**Pivot**	**Cardan (universal joint)**	**Ball joint**

Parallel spring stage and equivalent rod assembly



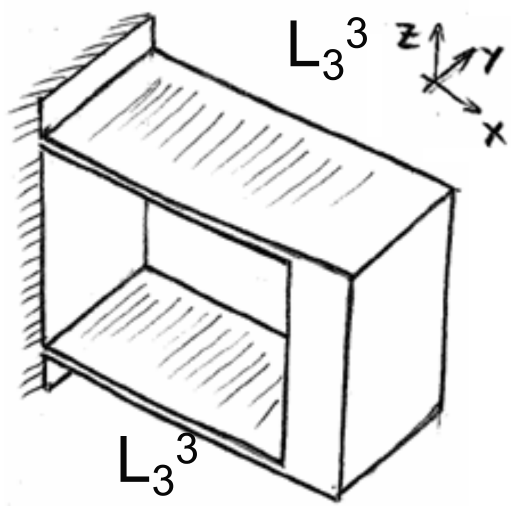


x_1, y_1, $\mathbf{z^{Fp}}$, rx_1, ry_1, $rz_{\mathbf{2}}$

DOF = 1; M = 0; DOH = 1

Kinematic analysis of basic flexure-bearings

Example : Parallel spring stage



Intuitive method

$$x_1 \quad y_1 \quad z^{Fp} \quad rx_1 \quad ry_1 \quad rz_{\color{red}2}$$

Blocked once

Free, but accompanied by parasitic motion

Blocked twice, i.e. 1 degree of hyperstaticity

DOF = number of "F" = 1

DOH = ∑ (ground indices - 1) = 1

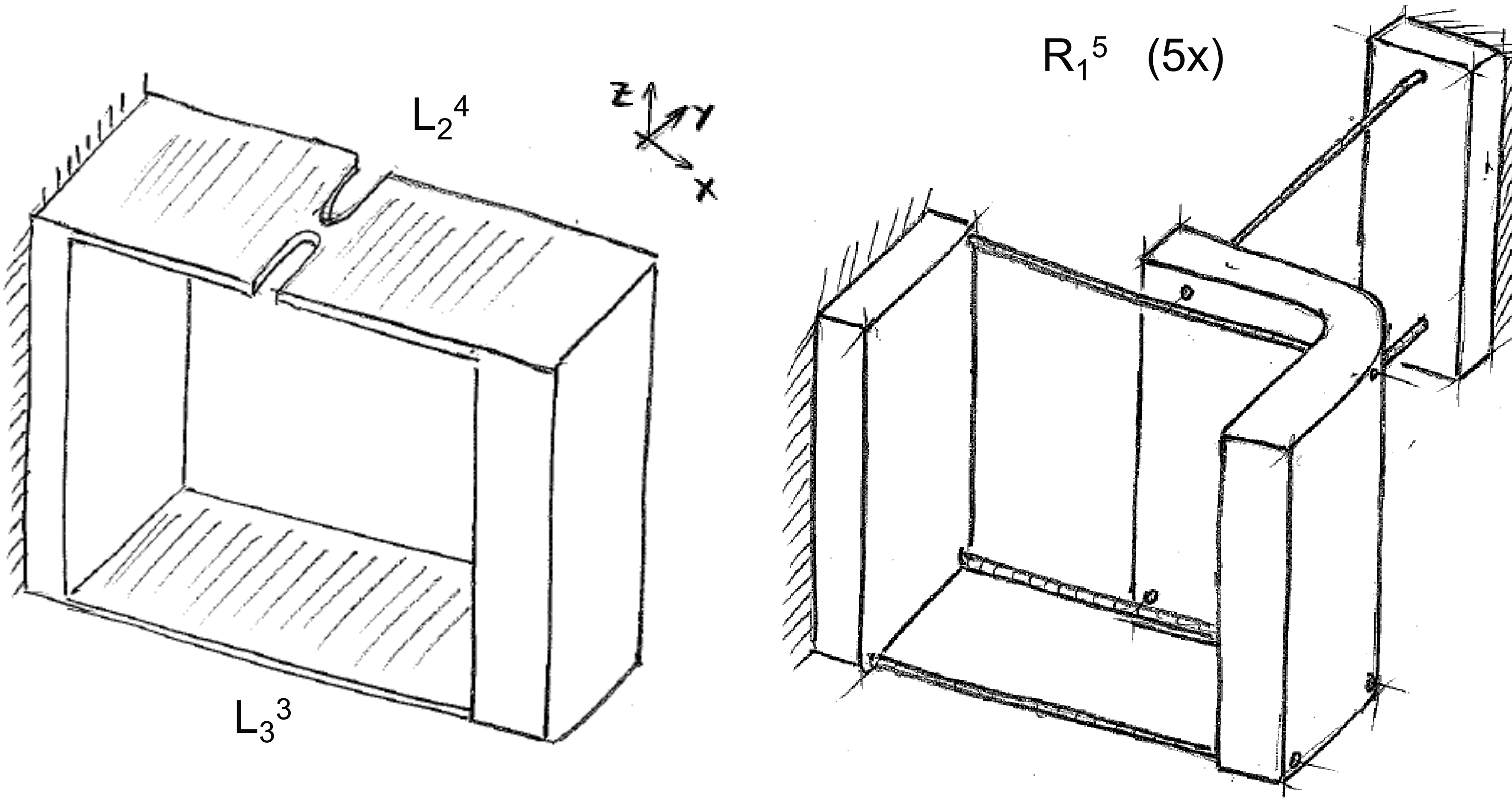
Check 1

The sum of the DOFs blocked by all the flexure-joints should be equal to the sum of all ground indices : $L_3^3 + L_3^3 = x_1 + y_1 + z^{Fp}\ rx_1 + ry_1 + rz_2$ → ***OK***

Check 2

DOH = DOF – M : presenty we have M = 2x3–1x6 = 0 → DOH = 1 - 0 = 1 → ***OK***

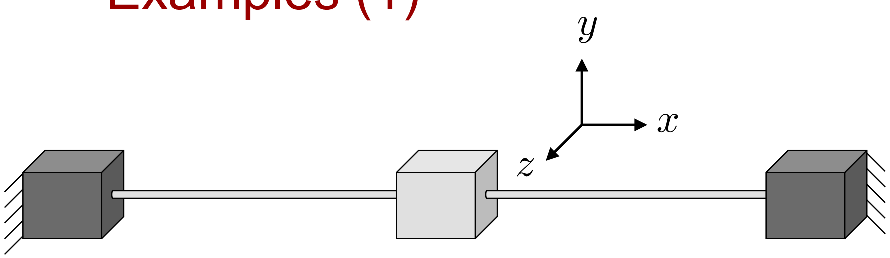
Parallel spring stage with notch and equivalent rod assembly



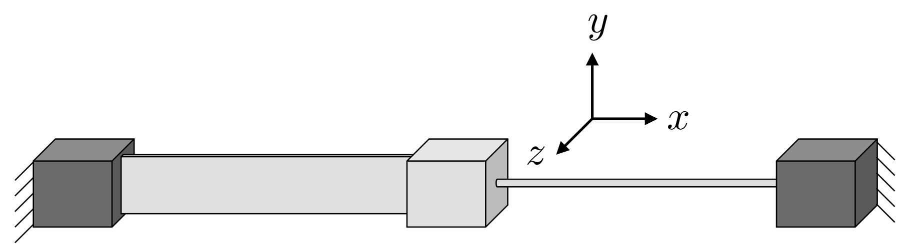
x_1, y_1, $\mathbf{z^{Fp}}$, rx_1, ry_1, rz_1

DOF = 1; M = 1; DOH = 0

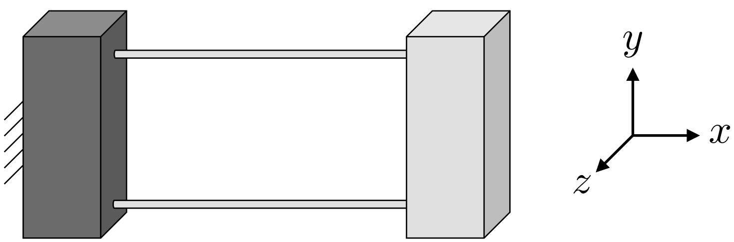
Examples (1)



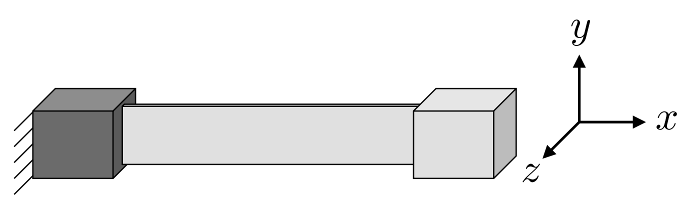
(a) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...



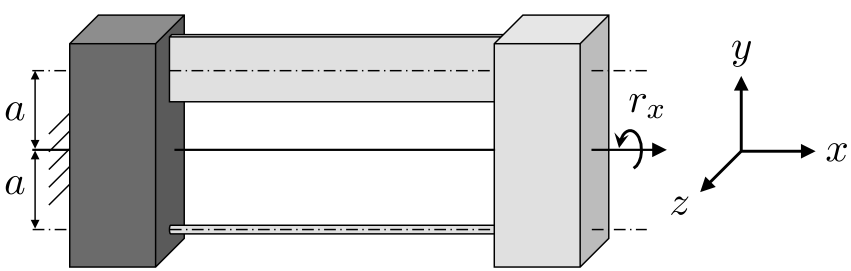
(b) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...



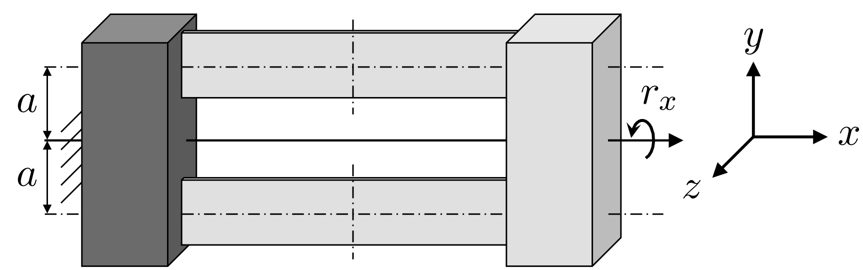
(c) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...



(d) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...

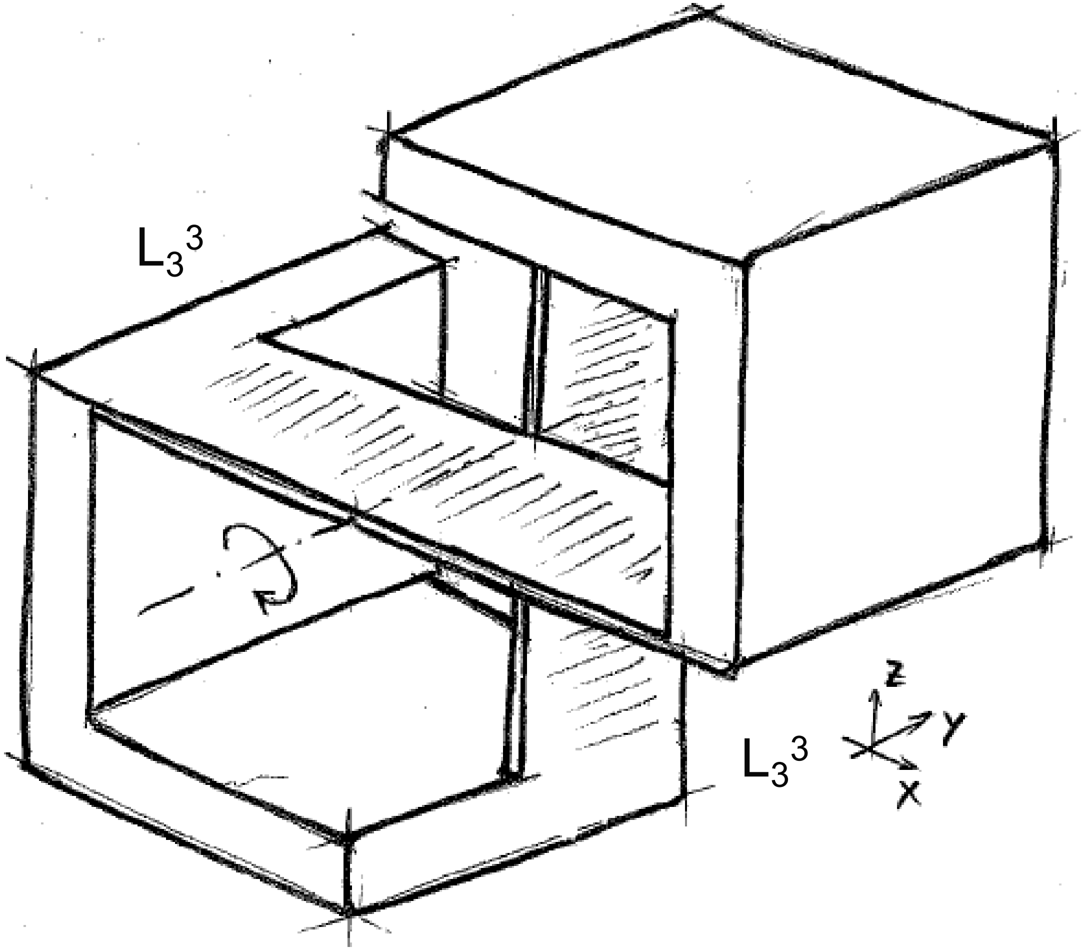


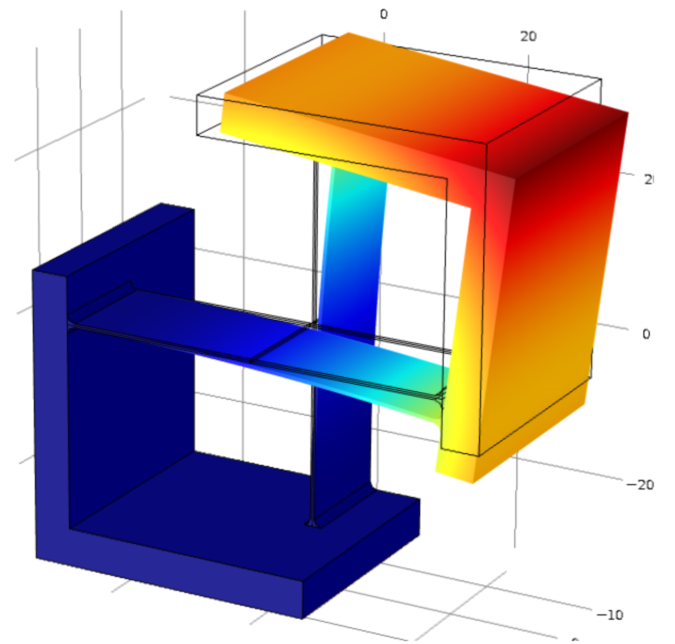
(e) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...



(f) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...

Cross-spring flexure pivot

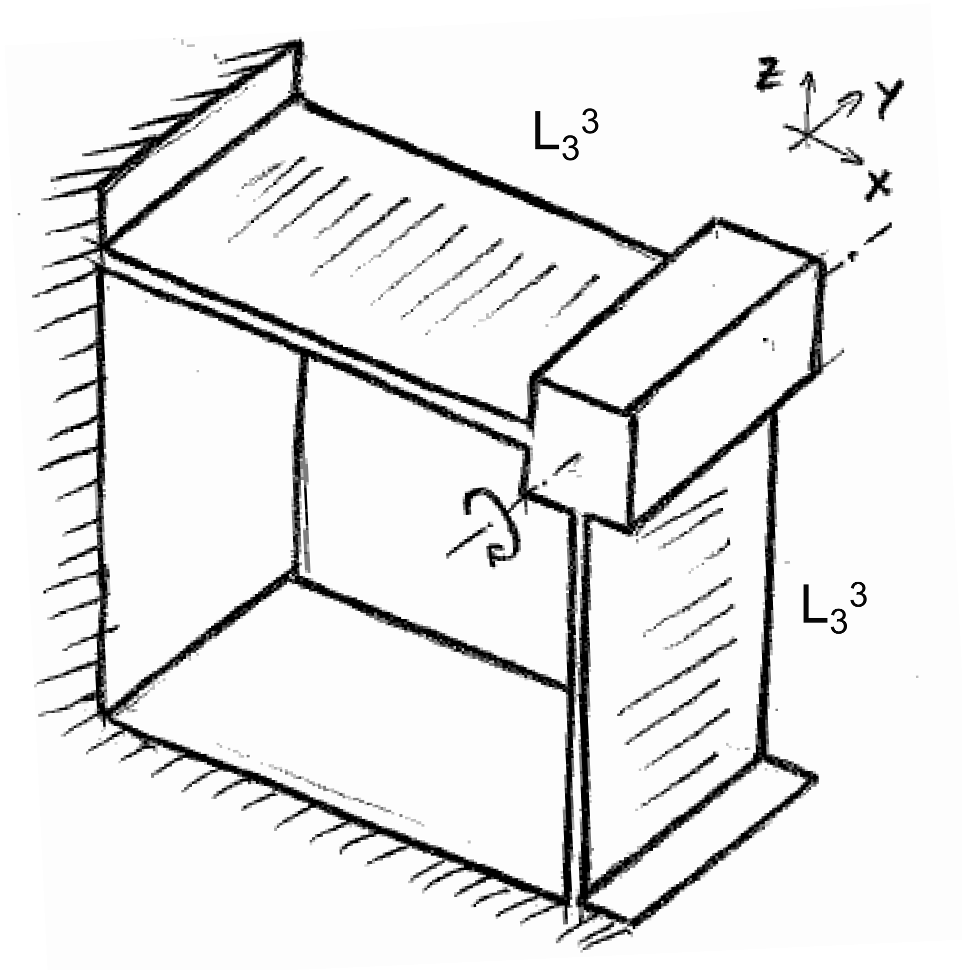


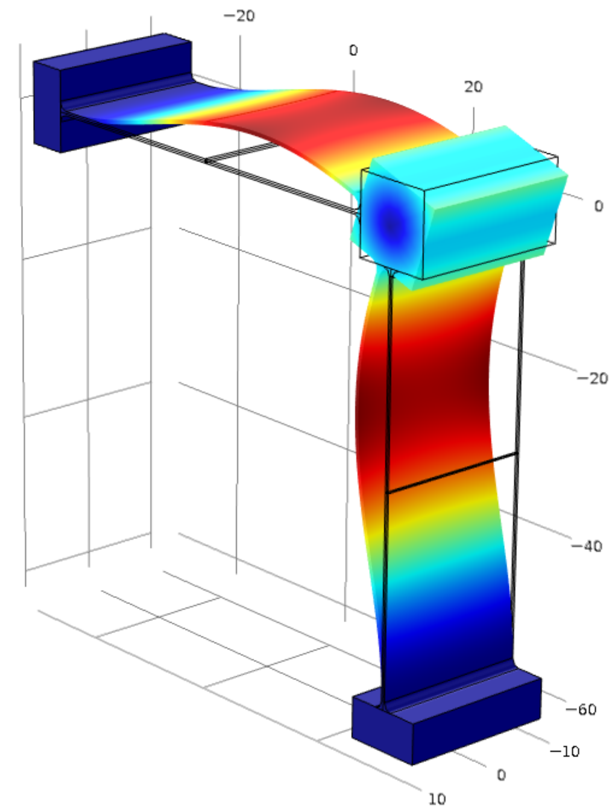


x_1, $y_{\color{red}2}$, z_1, rx_1, $\mathbf{ry^{Fp}}$, rz_1

DOF = 1; M = 0 ; DOH = 1

Remote Center Compliance flexure pivot

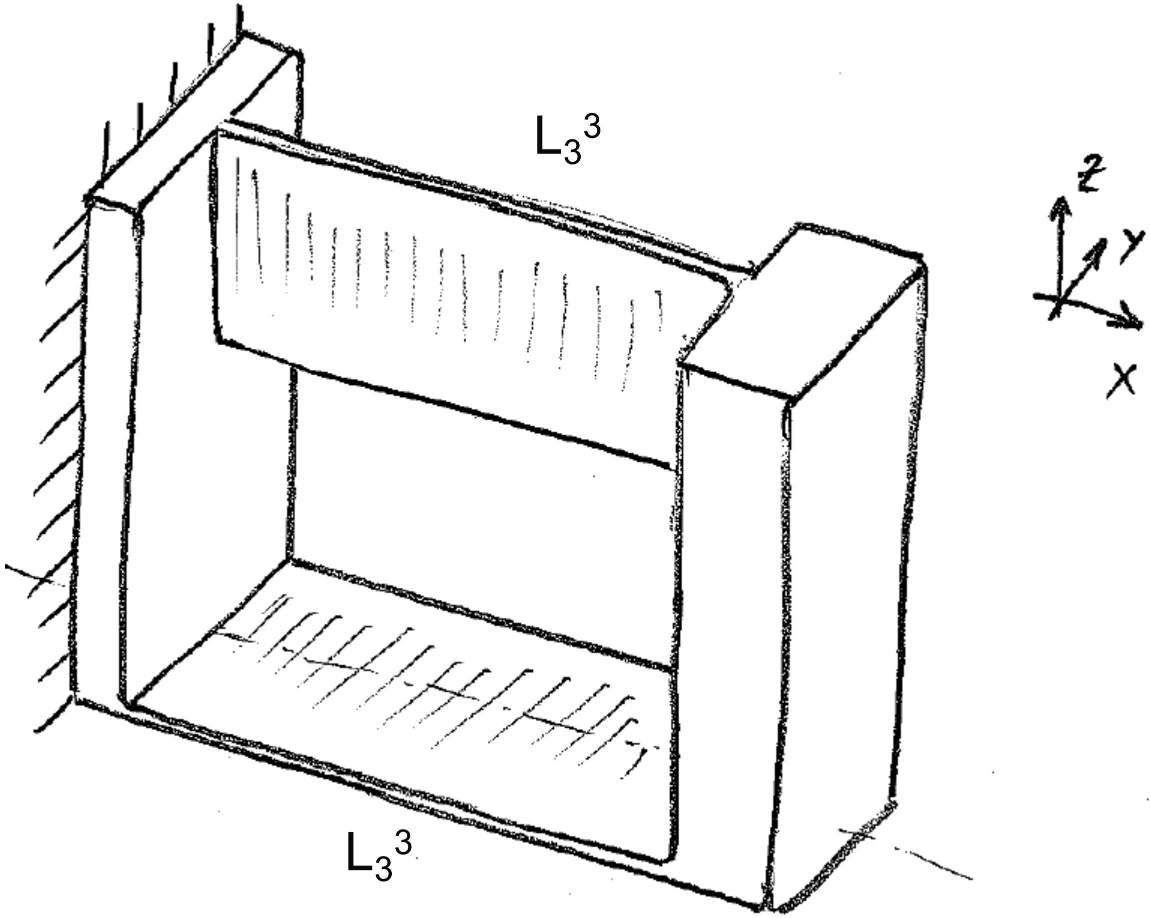




x_1, $y_{\color{red}{2}}$, z_1, rx_1, $\mathbf{ry^{Fp}}$, rz_1

DOF = 1 ; M = 0; DOH = 1

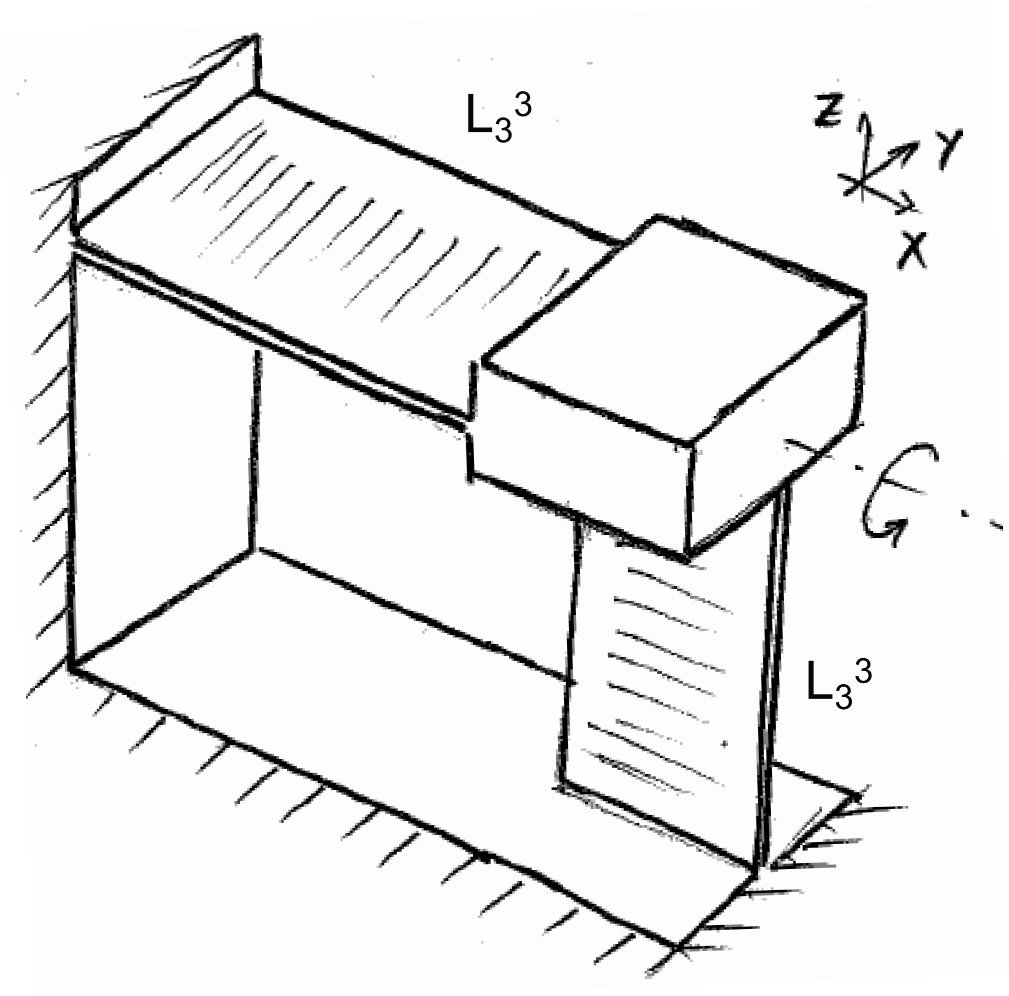
Torsion flexure pivot (squirrel cage)

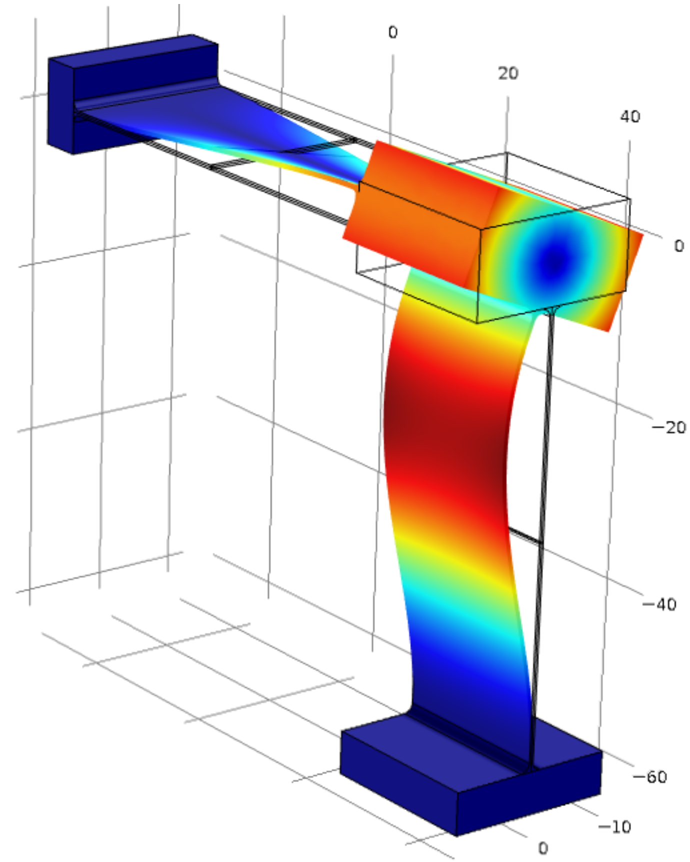


x_1, y_1, z_1, $\mathbf{rx}^{\mathbf{Fp}}$, ry_2, rz_1

DOF = 1; M = 0; DOH = 1

Torsion-Bending flexure pivot

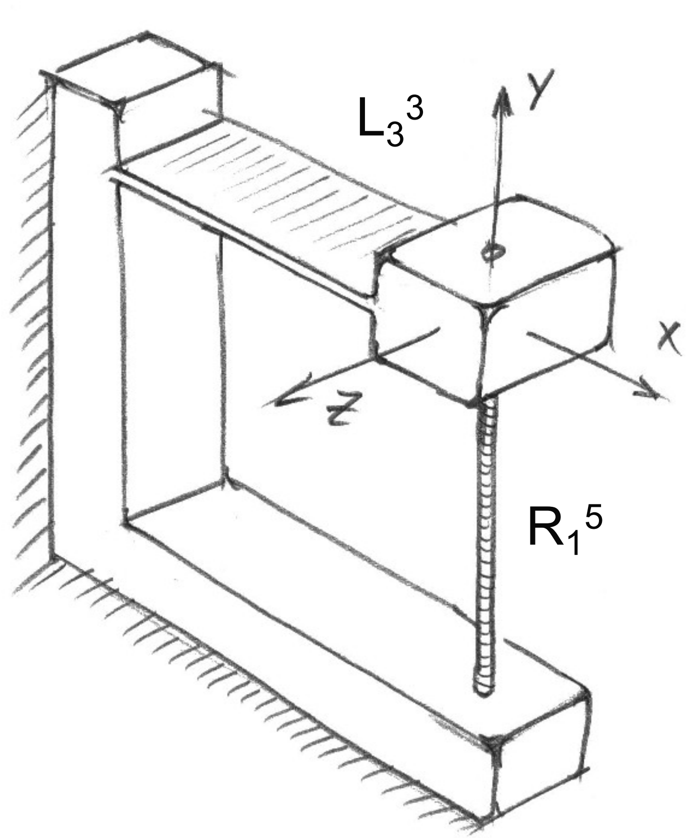




$x_{\color{red}2}$, y_1, z_1, $\mathbf{rx}^{\mathbf{Fp}}$, ry_1, rz_1

DOF = 1; M = 0; DOH = 1

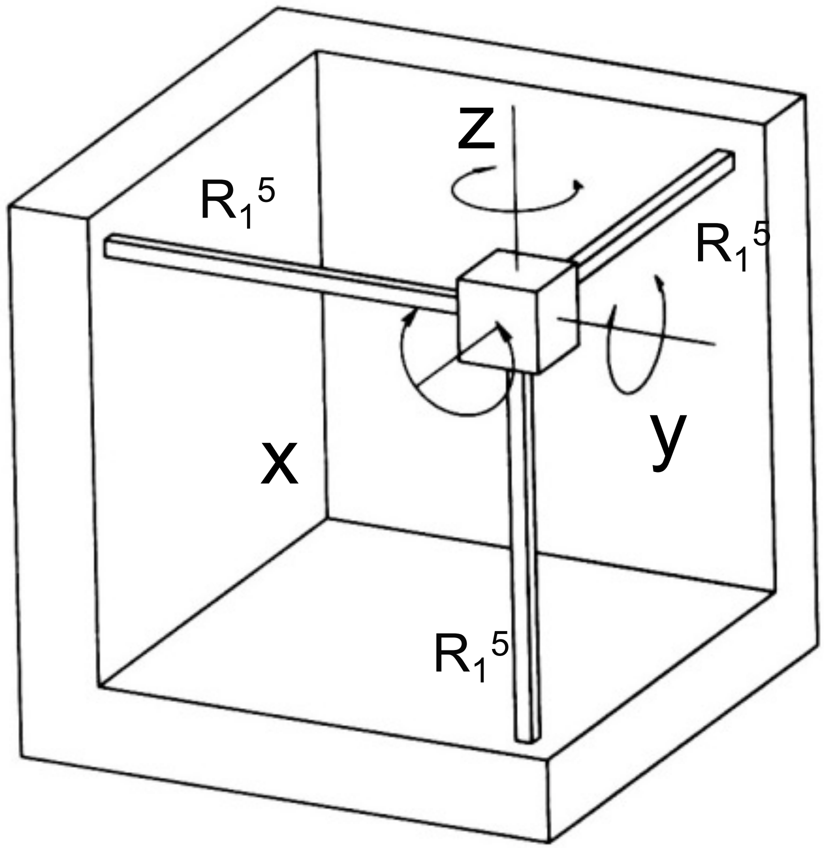
Cardan joint (universal joint)



x_1, y_1, z_1, $\mathbf{rx^{Fp}}$, ry_1, $\mathbf{rz^{Fp}}$

DOF = 2; M = 2; DOH = 0

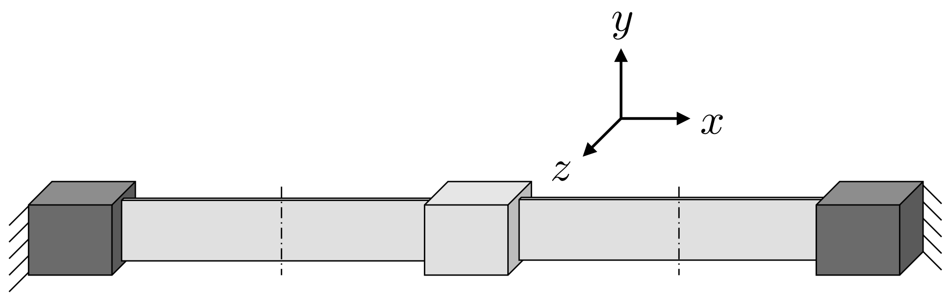
Ball joint



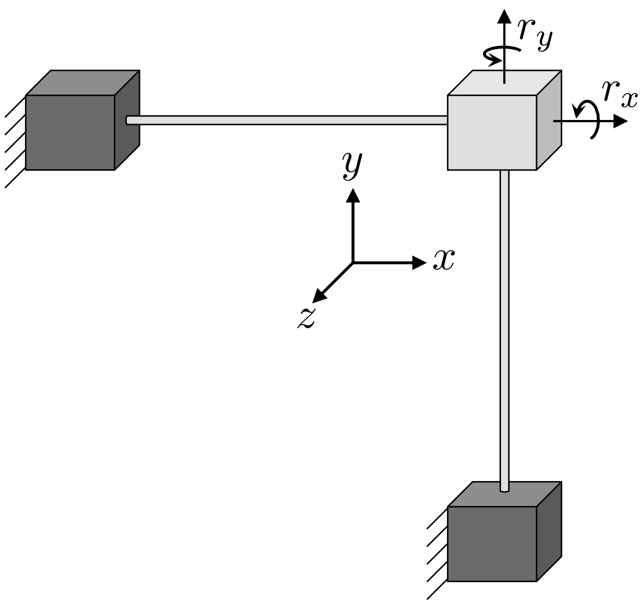
x_1, y_1, z_1, $\mathbf{rx^{Fp}}$, $\mathbf{ry^{Fp}}$, $\mathbf{rz^{Fp}}$

DOF = 3; M = 3; DOH = 0

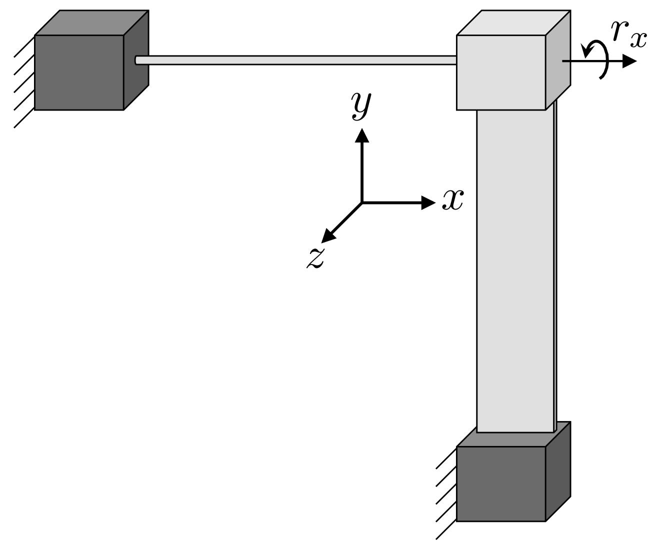
Examples (2)



(g) $x_{\dots}^{\cdots}$ $y_{\dots}^{\cdots}$ $z_{\dots}^{\cdots}$ $rx_{\dots}^{\cdots}$ $ry_{\dots}^{\cdots}$ $rz_{\dots}^{\cdots}$
DOF=... ; M=... ; DOH=...

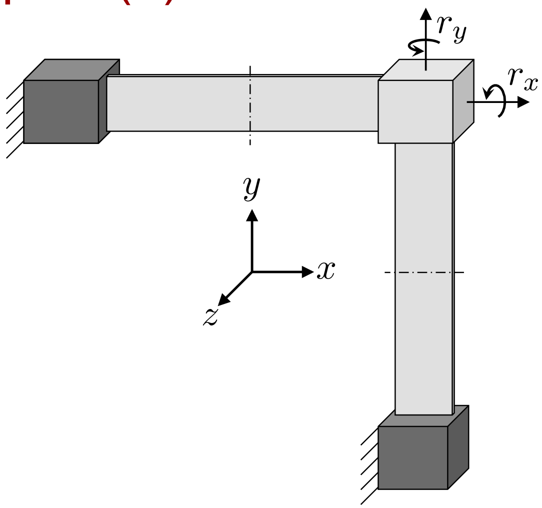


(h) $x_{\dots}^{\cdots}$ $y_{\dots}^{\cdots}$ $z_{\dots}^{\cdots}$ $rx_{\dots}^{\cdots}$ $ry_{\dots}^{\cdots}$ $rz_{\dots}^{\cdots}$
DOF=... ; M=... ; DOH=...

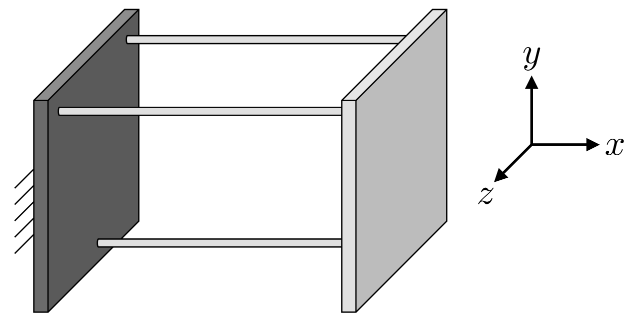


(i) $x_{\dots}^{\cdots}$ $y_{\dots}^{\cdots}$ $z_{\dots}^{\cdots}$ $rx_{\dots}^{\cdots}$ $ry_{\dots}^{\cdots}$ $rz_{\dots}^{\cdots}$
DOF=... ; M=... ; DOH=...

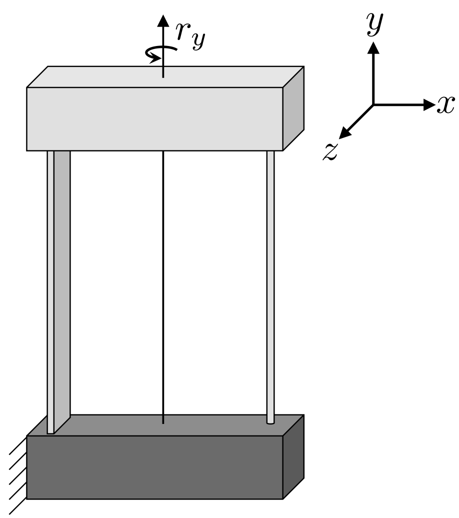
Examples (3)



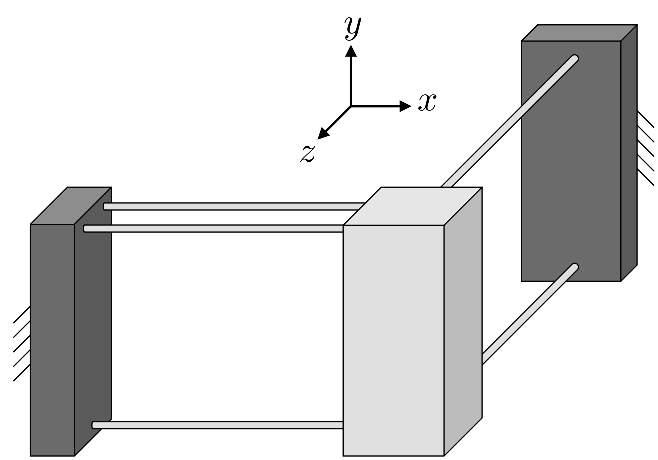
(a) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...



(b) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...

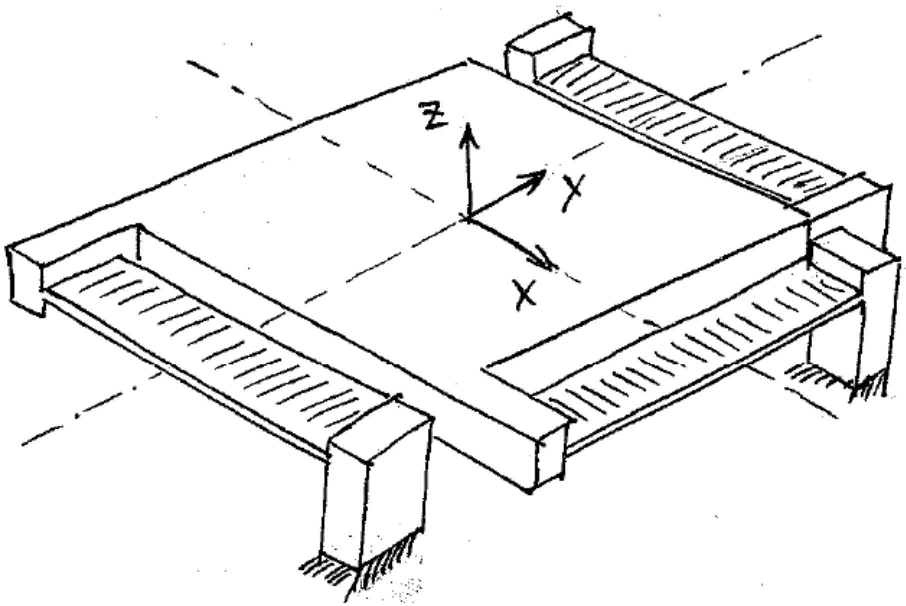


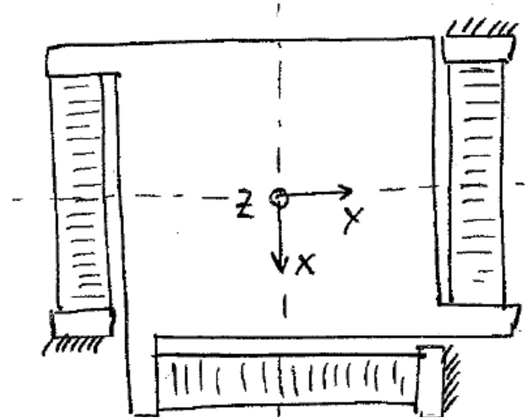
(c) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...

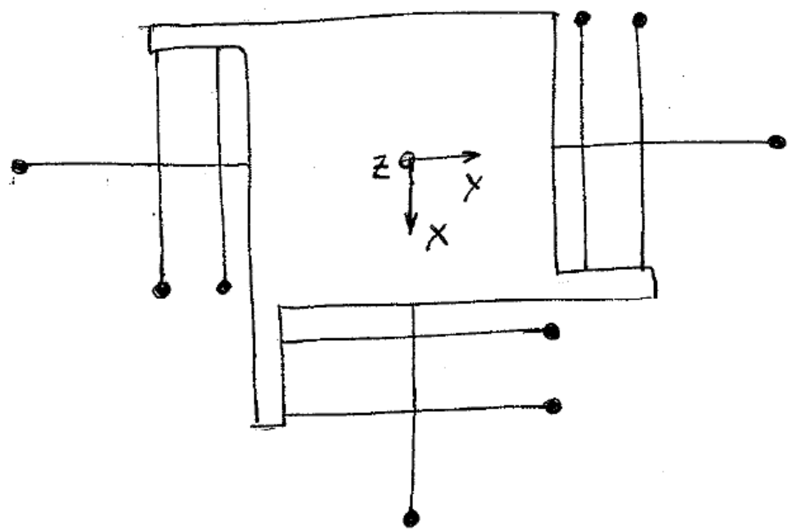


(d) $x_{\ldots}^{\cdots}$ $y_{\ldots}^{\cdots}$ $z_{\ldots}^{\cdots}$ $rx_{\ldots}^{\cdots}$ $ry_{\ldots}^{\cdots}$ $rz_{\ldots}^{\cdots}$
DOF=... ; M=... ; DOH=...

Examples (4)



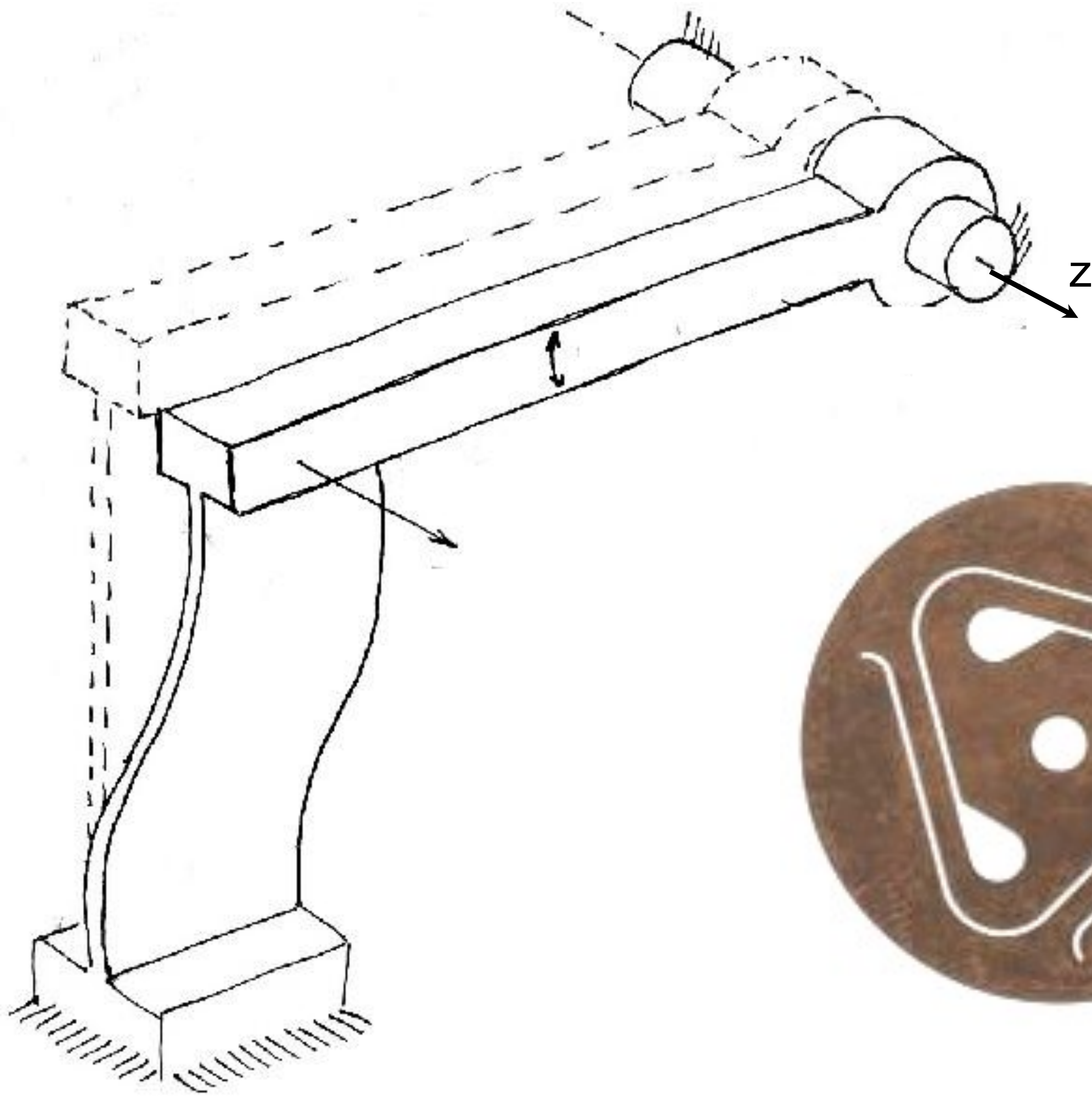




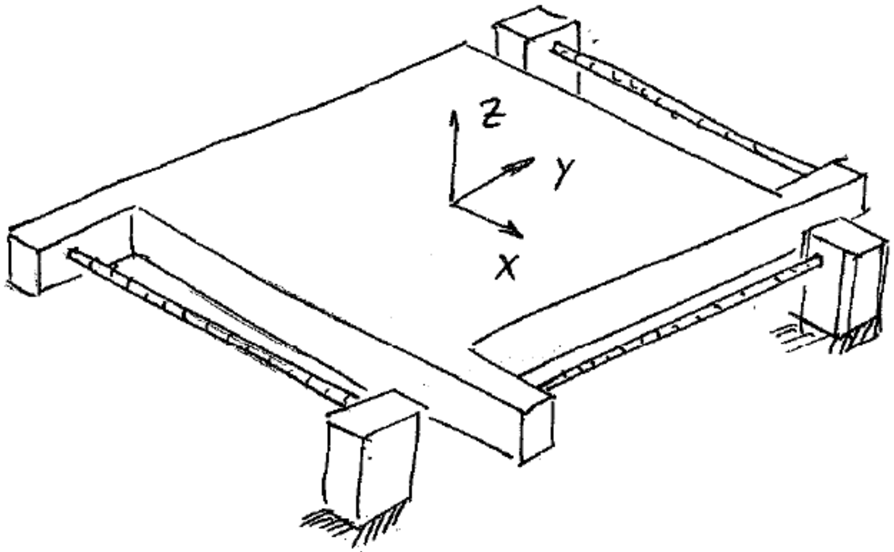
(a) $x_{\dots}^{\cdots}\ y_{\dots}^{\cdots}\ z_{\dots}^{\cdots}\ rx_{\dots}^{\cdots}\ ry_{\dots}^{\cdots}\ rz_{\dots}^{\cdots}$

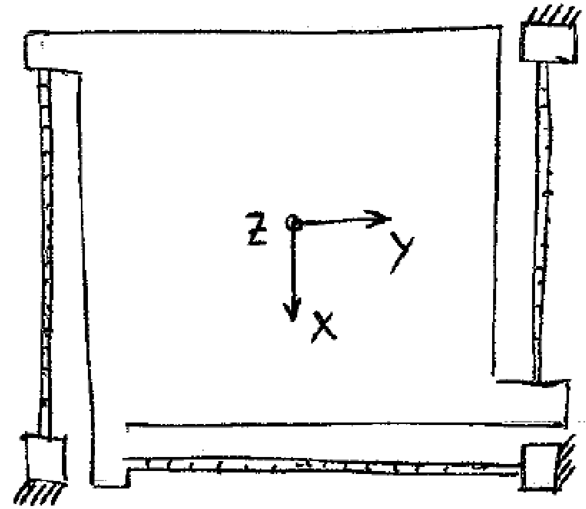
DOF=... ; M=... ; DOH=...

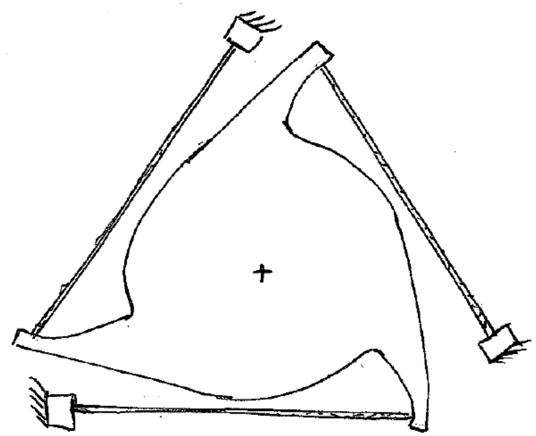
Membrane : Parasitic motion on one blade during axial motion (Z)



Examples (5)

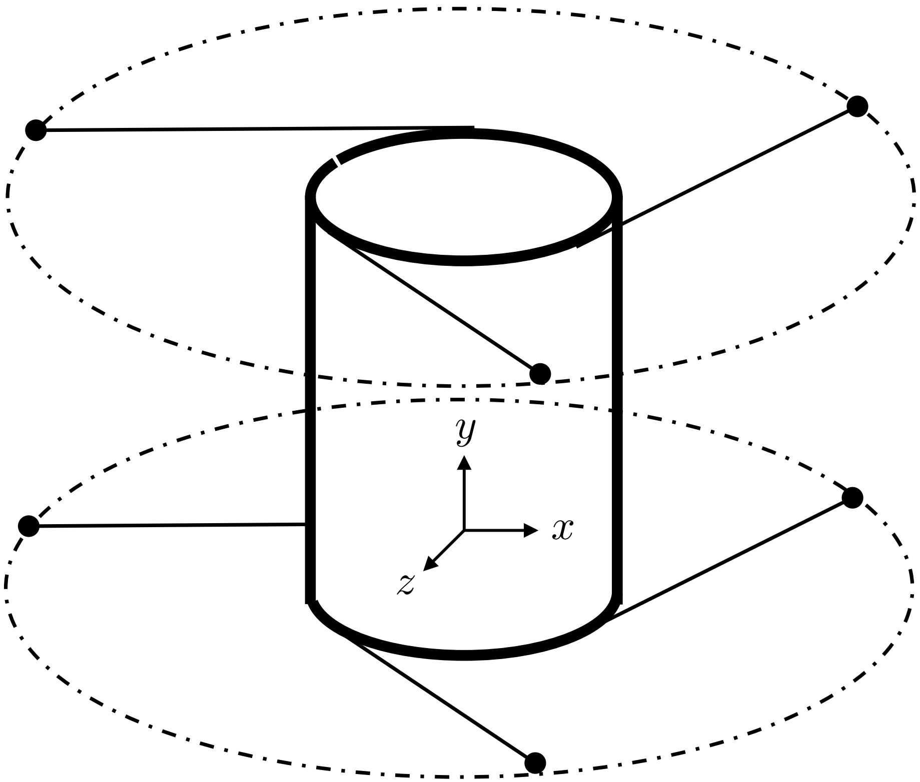




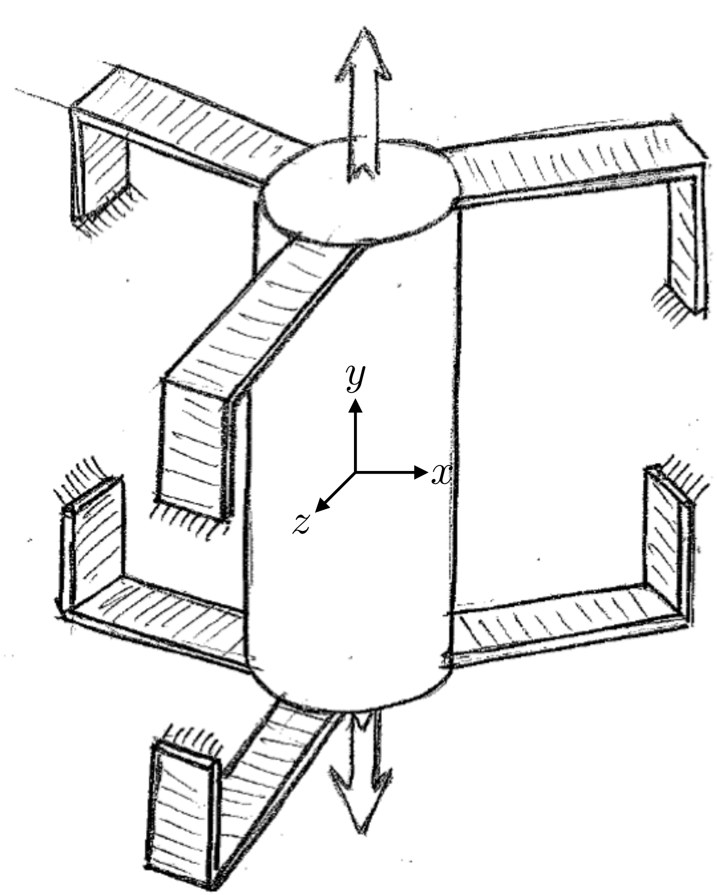


(b) $x_{\dots}^{\cdots}$ $y_{\dots}^{\cdots}$ $z_{\dots}^{\cdots}$ $rx_{\dots}^{\cdots}$ $ry_{\dots}^{\cdots}$ $rz_{\dots}^{\cdots}$
DOF=... ; M=... ; DOH=...

Examples (6)



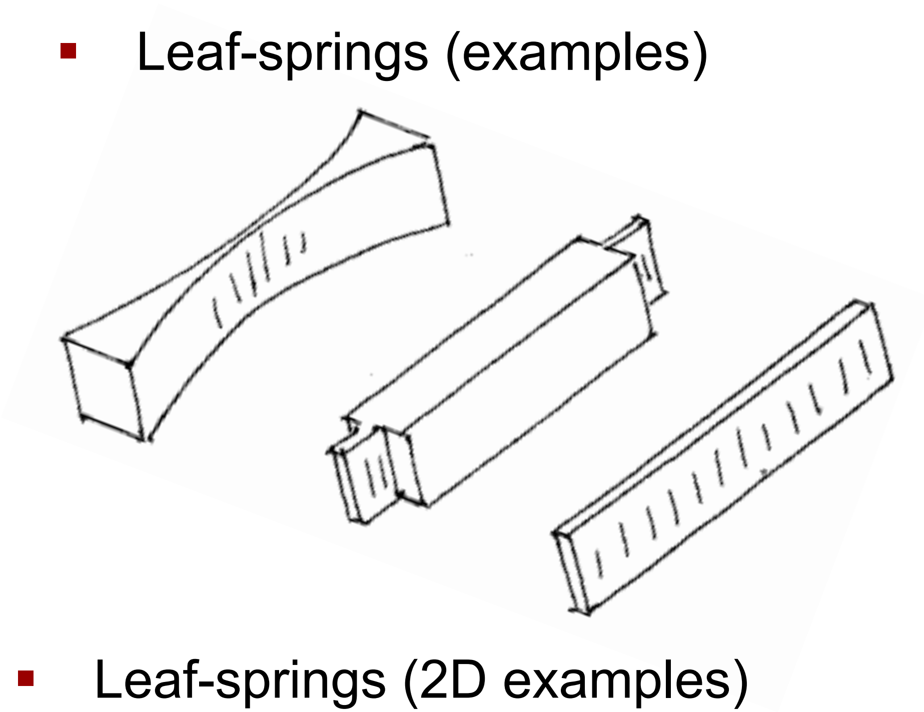
(e) $x_{\dots}^{\dots}$ $y_{\dots}^{\dots}$ $z_{\dots}^{\dots}$ $rx_{\dots}^{\dots}$ $ry_{\dots}^{\dots}$ $rz_{\dots}^{\dots}$
DOF=... ; M=... ; DOH=...



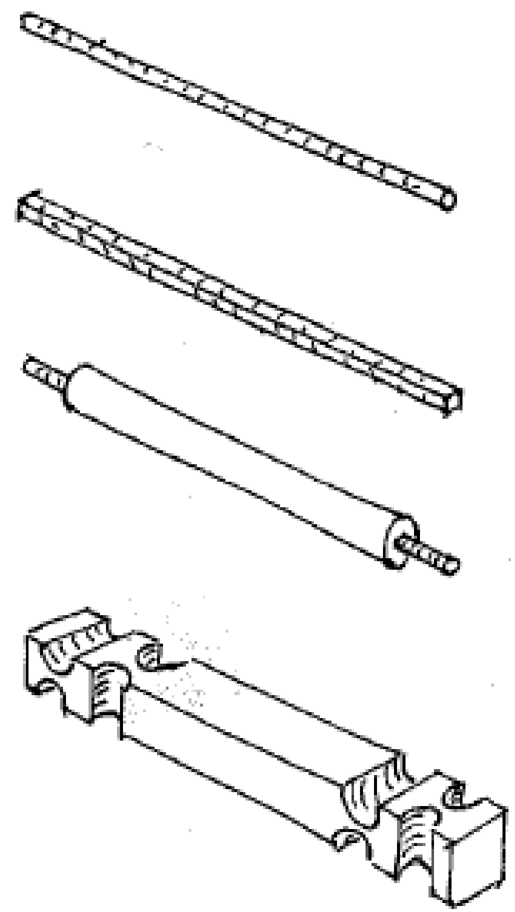
(f) $x_{\dots}^{\dots}$ $y_{\dots}^{\dots}$ $z_{\dots}^{\dots}$ $rx_{\dots}^{\dots}$ $ry_{\dots}^{\dots}$ $rz_{\dots}^{\dots}$
DOF=... ; M=... ; DOH=...

Kinematically equivalent flexure-joints

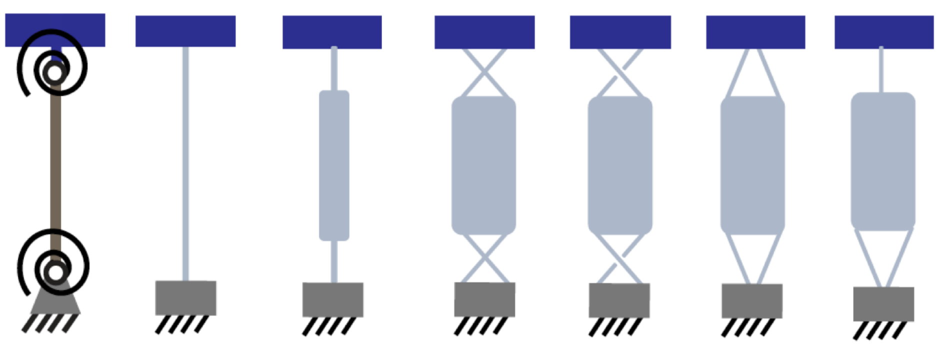
- Leaf-springs (examples)



- Rods (examples)



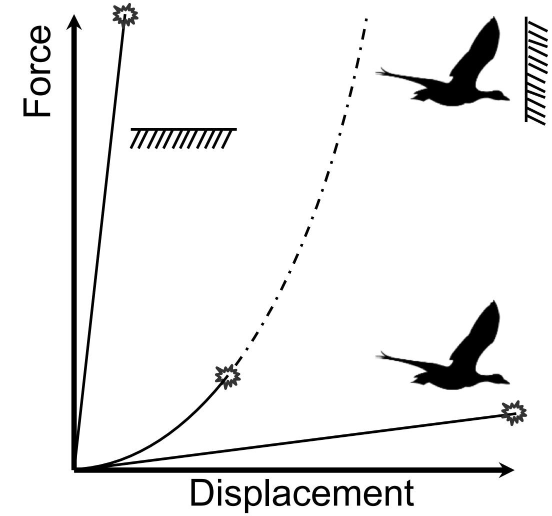
- Leaf-springs (2D examples)



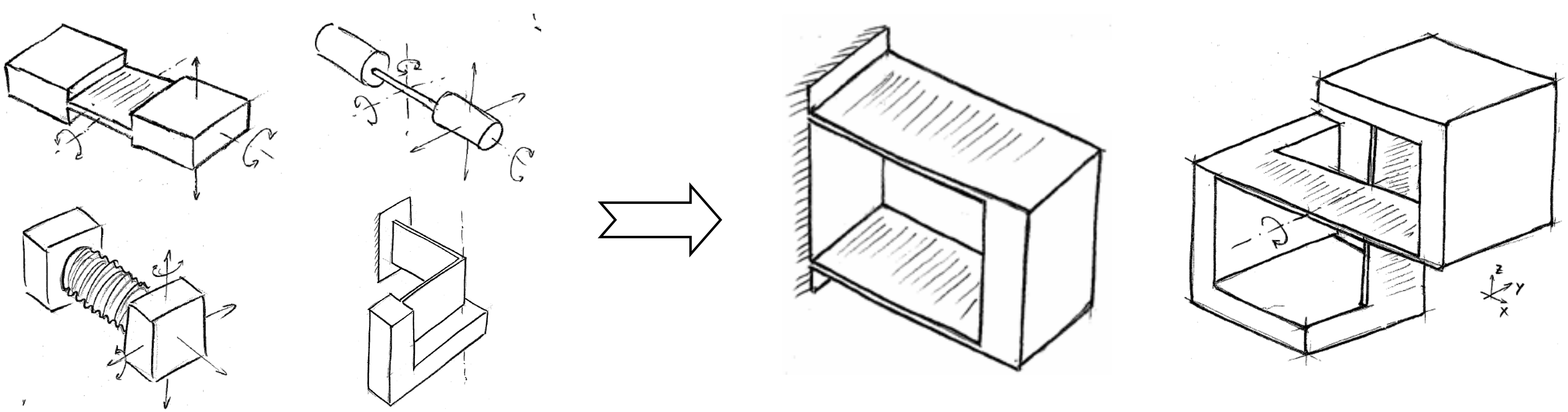
Thalmann, E. (2020). *Flexure Pivot Oscillators for Mechanical Watches* [EPFL Thesis]

Basics: summary

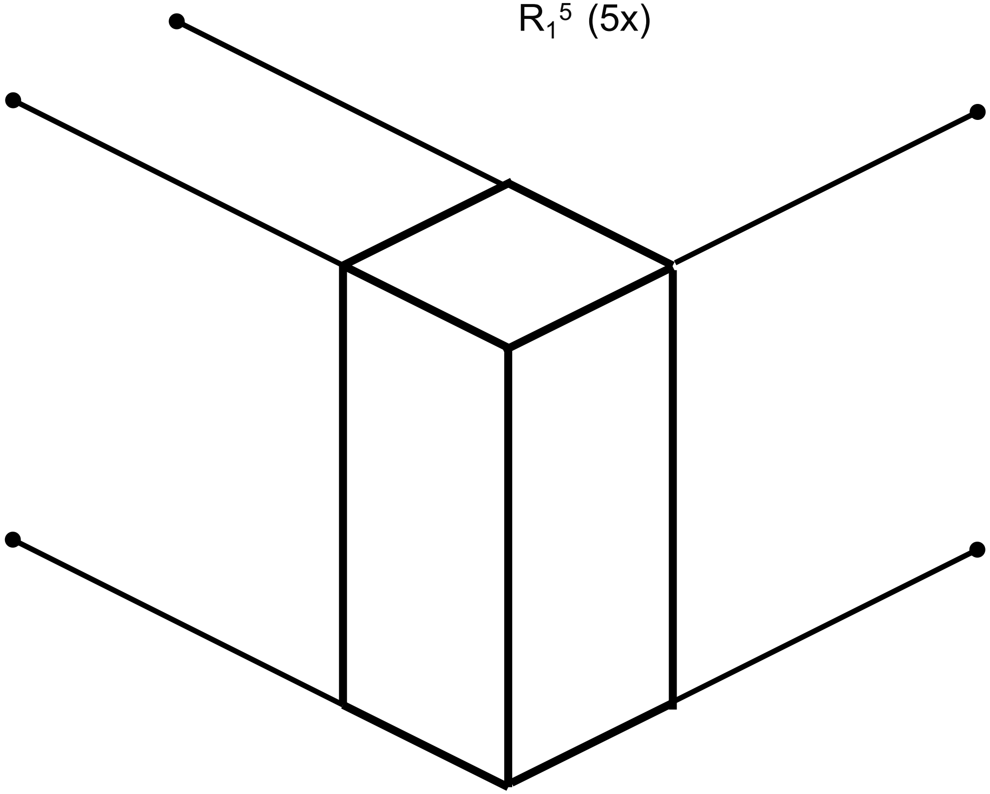
- DOF ≈ stiffness characteristic



- DOF, DOS, M, DOH
- Combination of flex-joints to produce flex-bearings
- Kinematic analysis method with cross-checks



Examples (7)

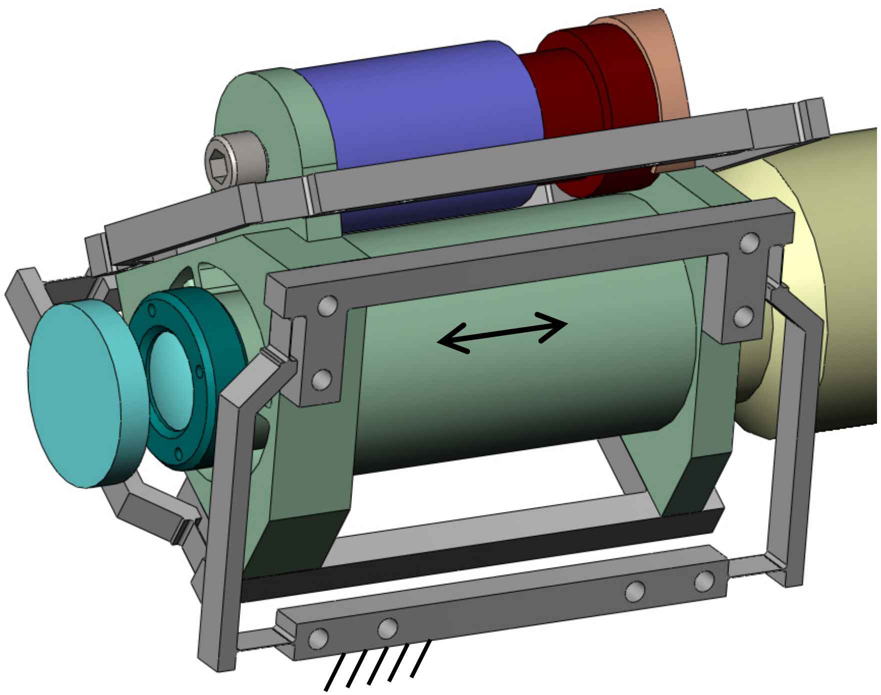


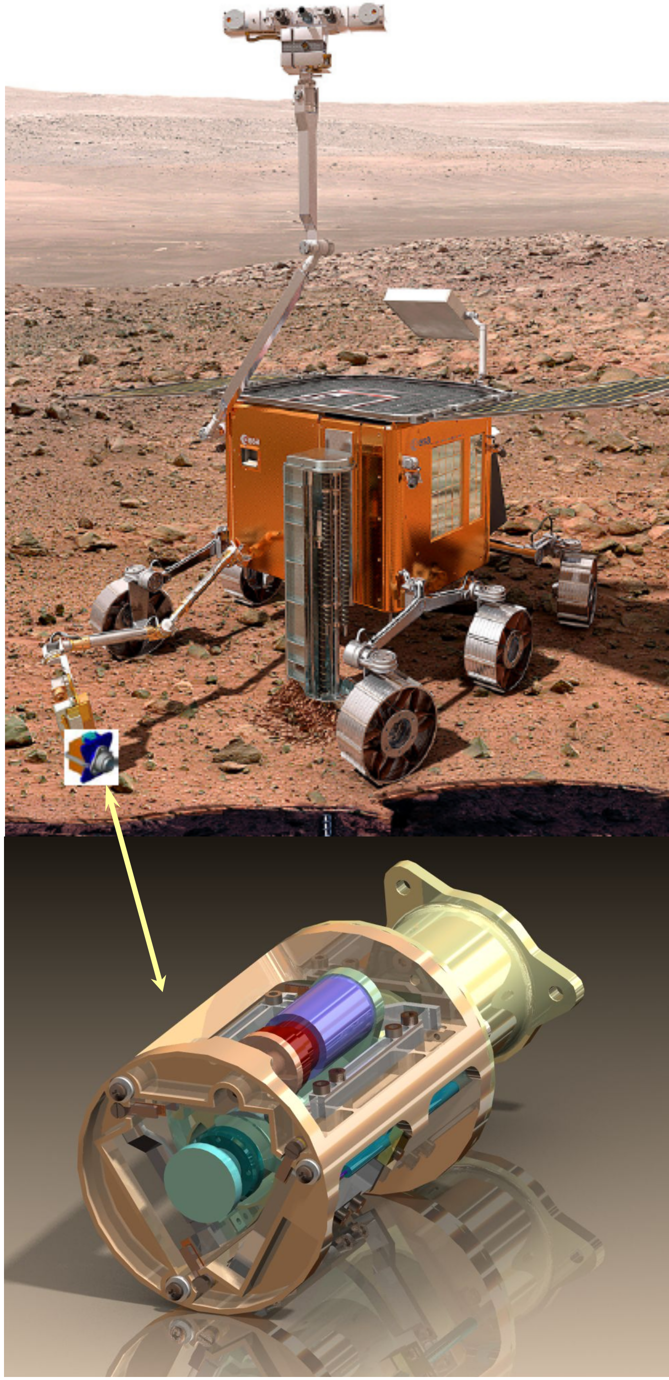
$$x_{\dots}^{\cdots}\ y_{\dots}^{\cdots}\ z_{\dots}^{\cdots}\ rx_{\dots}^{\cdots}\ ry_{\dots}^{\cdots}\ rz_{\dots}^{\cdots}$$

$$\mathrm{DOF}{=}\dots;\ \mathrm{M}{=}\dots;\ \mathrm{DOH}{=}\dots$$

Rectilinear mechanisms

Close-up Imager :
European Mars Mission EXOMARS



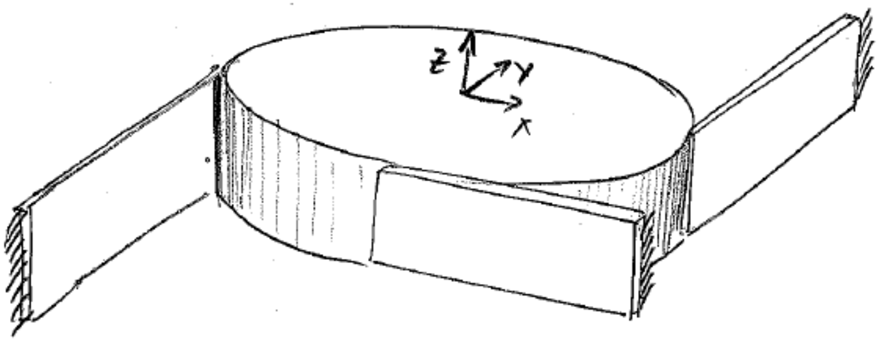


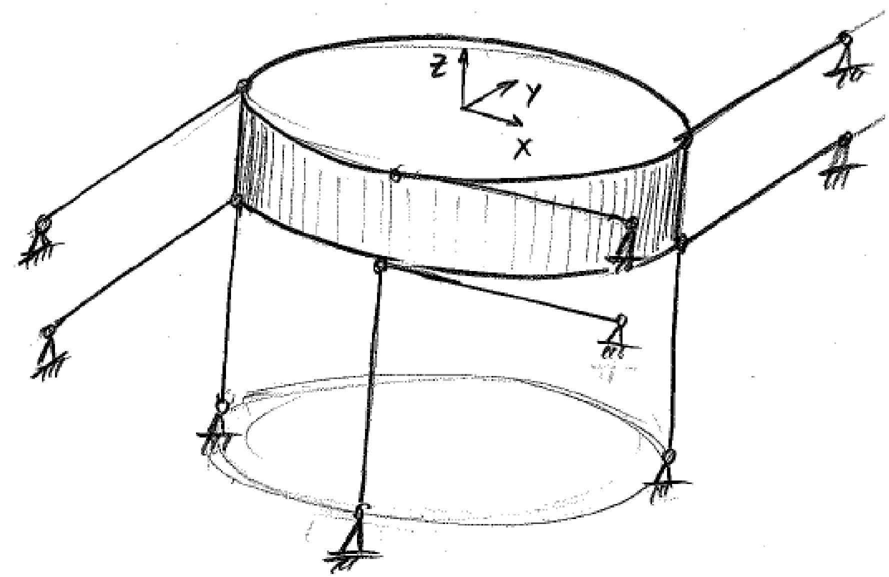
csem

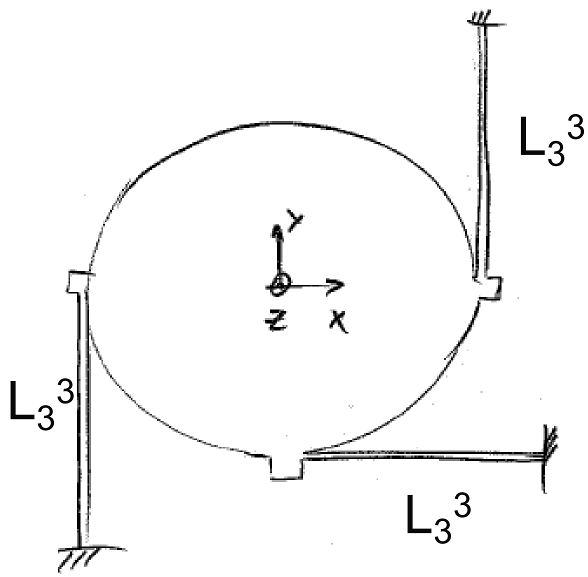
SPACE-X Space Exploration Institute

[*Mars Science and Exploration Conference, ESA/ESTEC, Nov 2007*]

Mirror mounts







2D

x_1, y_1, rz_1

DOF = 0; M = 0; DOH = 0

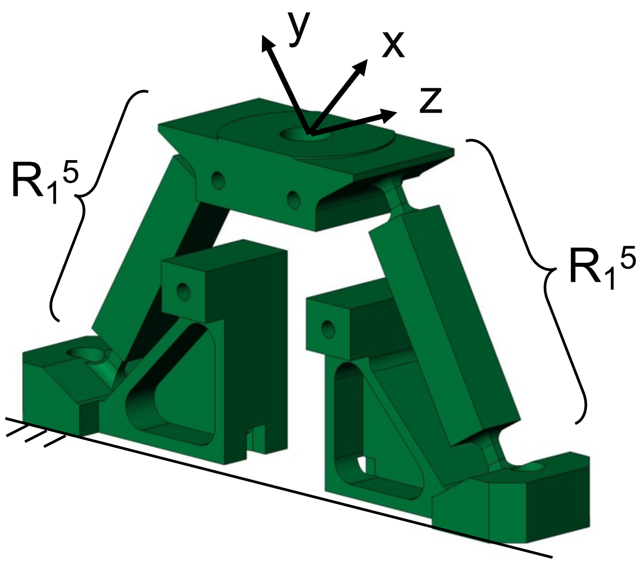
3D

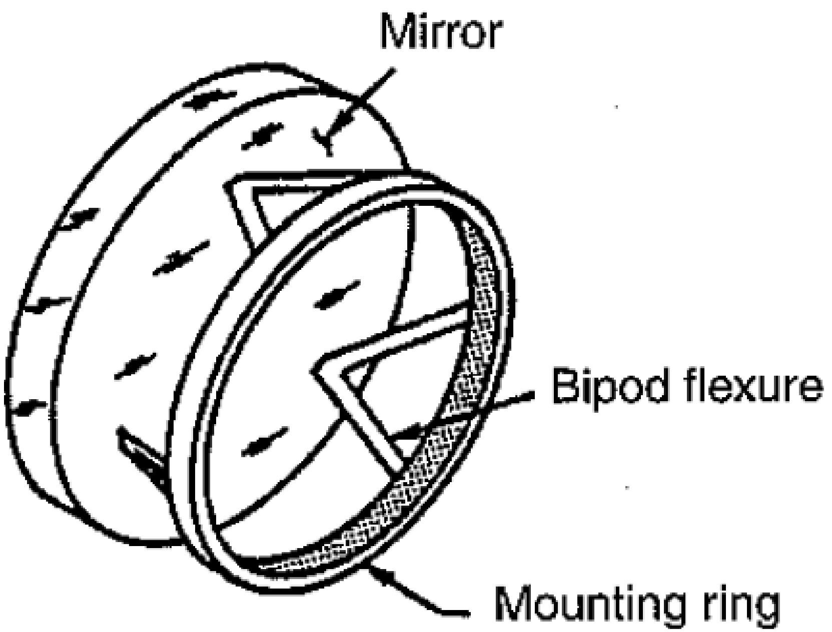
x_1, y_1, z_1, $rx_{\mathbf{3}}$, $ry_{\mathbf{2}}$, rz_1

DOF = 0; M = -3 ; DOH = 3)

Mirror mounts

Bipod flexure





x_1, y_1, $\mathbf{z^{Fp}}$, $\mathbf{rx^{Fp}}$, $\mathbf{ry^{Fp}}$, $\mathbf{rz^{Fp}}$

DOF = 4; M = 4 ; DOH = 0

x_1, y_1, z_1, rx_1, ry_1, rz_1

DOF = 0; M = 0; DOH = 0