Série 18

1. Calculer la dérivée des fonctions suivantes :

a) $A(x) = \displaystyle\int_2^{2x} \frac{1}{\arg\cosh t}\, dt\,, \quad x > 1\,;$ b) $B(x) = \displaystyle\int_{\arcsin x}^{x} \frac{\sin t}{t}\, dt\,, \quad 0 < x < 1\,.$

2. Déterminer, si elle existe, la limite suivante : $\displaystyle\lim_{x\to 0} \frac{\displaystyle\int_0^{x^2} \left[e^{(t^2)} - 1\right] dt}{x^6}\,.$

Calculer les primitives des fonctions suivantes :

3. $a(x) = 12\,(x+1)\,(x-2)\,(x-1)\,.$

4. $b(x) = \dfrac{x^3 - \sqrt[3]{x}}{\sqrt{x}}\,.$

5. $c(x) = \sin(2x)\cdot\cos^2(x)\,.$

6. $d(x) = \sin(4x)\cdot\cos(x)\,.$

7. $e(x) = \dfrac{x}{e^{(x^2+1)}}\,.$

8. $f(x) = \tan^2(x)\,.$

9. $g(x) = \dfrac{1}{\sin(x)\cdot\cos(x)}\,.$

10. $h(x) = \dfrac{1}{1-e^x}\,.$

11. $i(x) = \dfrac{1}{\cosh(x)}\,.$

12. $j(x) = \dfrac{\ln x}{x}\,.$

13. $k(x) = \dfrac{1}{x\cdot\ln^2 x}\,.$

14. $l(x) = \dfrac{\cos\left(\sqrt{x}\,\right)}{\sqrt{x}}\,.$

15. $m(x) = \sin^2\left(a\,x\,\right), \quad a \in \mathbb{R}^*.$

Réponses de la série 18

1. a) $A'(x) = \dfrac{2}{\operatorname{arg\,cosh}(2x)}$, b) $B'(x) = \dfrac{\sin x}{x} - \dfrac{x}{\arcsin x} \cdot \dfrac{1}{\sqrt{1-x^2}}$.

2. La limite vaut $\dfrac{1}{3}$.

3. $\displaystyle\int 12\,(x+1)\,(x-2)\,(x-1)\,dx = 3x^4 - 8x^3 - 6x^2 + 24x + C$.

4. $\displaystyle\int \frac{x^3 - \sqrt[3]{x}}{\sqrt{x}}\,dx = \frac{2}{7}\,x^{7/2} - \frac{6}{5}\,x^{5/6} + C$.

5. $\displaystyle\int \sin(2x)\cdot\cos^2(x)\,dx = -\frac{1}{2}\,\cos^4(x) + C$.

6. $\displaystyle\int \sin(4x)\cdot\cos(x)\,dx = -\frac{1}{10}\,\cos(5x) - \frac{1}{6}\,\cos(3x) + C$, ou

$\displaystyle\int \sin(4x)\cdot\cos(x)\,dx = -\frac{8}{5}\,\cos^5(x) + \frac{4}{3}\,\cos^3(x) + C'$.

7. $\displaystyle\int \frac{x}{e^{(x^2+1)}}\,dx = -\frac{1}{2}\,e^{-(x^2+1)} + C$.

8. $\displaystyle\int \tan^2(x)\,dx = \tan(x) - x + C$.

9. $\displaystyle\int \frac{1}{\sin(x)\cdot\cos(x)}\,dx = \ln|\tan(x)| + C$.

10. $\displaystyle\int \frac{1}{1-e^x}\,dx = -\ln|e^{-x} - 1| + C$.

11. $\displaystyle\int \frac{1}{\cosh(x)}\,dx = -2\,\arctan(e^{-x}) + C = 2\,\arctan(e^x) + C' = \arctan\left[\sinh(x)\right] + C''$.

12. $\displaystyle\int \frac{\ln x}{x}\,dx = \frac{1}{2}\,\ln^2(x) + C$

13. $\displaystyle\int \frac{1}{x\cdot\ln^2 x}\,dx = -\frac{1}{\ln x} + C$

14. $\displaystyle\int \frac{\cos\left(\sqrt{x}\,\right)}{\sqrt{x}}\,dx = 2\,\sin\left(\sqrt{x}\,\right) + C$

15. $\displaystyle\int \sin^2\left(a\,x\,\right)\,dx = \frac{x}{2} - \frac{\sin\left(2a\,x\,\right)}{4a} + C$