EPFL - Autumn 2021	Dr. Pablo Antolin
Analysis III SV MT	**Exercises**
Serie 13	December, 23

Note: several exercises are directly taken from [B.Dacorogna and C.Tanteri, *Analyse avancée pour ingénieurs* (2018)]. Their corrections can be found there.

Exercise 1 (Ex 17.7 page 271)**.**
Find the function $y : \mathbb{R} \to \mathbb{R}$ solution of:

$$\begin{aligned} \forall x \in \mathbb{R} \quad &: \; y''(x) + 5y'(x - \pi) - y(x) = \cos x - 3\sin(2x) + 2, \\ &: \; y(x + 2\pi) = y(x). \end{aligned}$$

Exercise 2.
Compute:

$$\int_0^{+\infty} \frac{t}{(t^2+4)^2} \sin\left(\frac{t}{2}\right) dt.$$

Hint: The Fourier transform of the function f defined by $f(x) = xe^{-\omega|x|}$ with $\omega > 0$ is given by:

$$\mathfrak{F}(f)(\alpha) = \frac{4\omega}{i\sqrt{2\pi}} \frac{\alpha}{(\alpha^2+\omega^2)^2}.$$

Exercise 3 (Exemple 17.7 page 269)**.**
Find a solution y of the following equation by using the properties of the Fourier transform (Convolution):

$$y(x) + 3\int_{-\infty}^{+\infty} e^{-|t|} y(x-t)dt = e^{-|x|}, \quad \forall x \in \mathbb{R}.$$

Hint: The Fourier transform of the function f defined by $f(x) = \dfrac{e^{-\omega|x|}}{\omega}$ with $\omega > 0$ is given by:

$$\mathfrak{F}(f)(\alpha) = \sqrt{\frac{2}{\pi}} \frac{1}{\alpha^2+\omega^2} \; .$$

Exercise 4 (Ex 17.11 page 272)**.**
Find a solution y of the following equation by using the properties of the Fourier transform (Convolution and Differentiation):

$$3y(x) + \int_{-\infty}^{+\infty} [y''(t) - y(t)]\, f(x-t)dt = g(x), \quad \forall x \in \mathbb{R}.$$

where $f(x) = e^{-|x|}$, and $g(x) = xe^{-x^2}$.

Hint: The Fourier transforms of functions f and g are respectively given by:

$$\mathfrak{F}(f)(\alpha) = \sqrt{\frac{2}{\pi}}\frac{1}{\alpha^2+1} \quad \text{and} \quad \mathfrak{F}(g)(\alpha) = \frac{-i\alpha}{2\sqrt{2}}e^{-\frac{\alpha^2}{4}}.$$