Central Limit Theorem (CLT)

Theorem

If $X_1, \ldots, X_n$ are i.i.d. with $\mathbb{E}[f(X_1)^2] < \infty$, then,

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left(f(X_i) - \mu_f\right) \xrightarrow{(d)} \mathcal{N}(0, \sigma_f^2) \quad \text{with} \quad \mu_f = \mathbb{E}[f(X_1)],\ \sigma_f^2 = \mathrm{Var}(f(X_1)).$$

where $\xrightarrow{(d)}$ is the *convergence in distribution.*

In particular,

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \xrightarrow{(d)} \mathcal{N}(0, 1) \quad \text{with} \quad \mu = \mathbb{E}[X_1],\ \sigma^2 = \mathrm{Var}(X_1).$$