EPFL - Autumn 2021	Dr. Pablo Antolin
Analysis III SV MT	**Exercises**
Serie 10	December, 02

Note: several exercises are directly taken from [B.Dacorogna and C.Tanteri, *Analyse avancée pour ingénieurs* (2018)]. Their corrections can be found there.

Hint: For the following exercises, we suggest to :

1. *Start by sketching the graph of f and the graph of f', over at least two periods;*
2. *Check that the function f is (piecewise) C^1 ;*
3. *For Exercises 2, 4, and 5, cite the theorem to be used to conclude the value of the sum.*

Exercise 1 (Ex 14.4 page 220).
Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by periodicity of period 2 such that

$$f(x) = x \quad \text{if } x \in [0, 2[.$$

Calculate the complex Fourier series.

Exercise 2 (Ex 14.11 page 221).

1. Using complex notations, calculate the Fourier series of the 2π-periodic and odd function defined on $[0, \pi]$ by

$$f(x) = \begin{cases} x & \text{if} \quad 0 \leq x \leq \frac{\pi}{2}, \\ \pi - x & \text{if} \quad \frac{\pi}{2} < x \leq \pi. \end{cases}$$

2. Then deduce

$$\sum_{k=-\infty}^{\infty} \frac{1}{(2k-1)^2} = \frac{\pi^2}{4}.$$

Exercise 3 (Ex 14.3 page 220).
Calculate the Fourier series of the function $f : \mathbb{R} \to \mathbb{R}$ 2π-periodic defined by

$$f(x) = \begin{cases} \sin(x) & \text{if} \quad 0 \leq x \leq \frac{\pi}{2}, \\ 0 & \text{if} \quad \frac{\pi}{2} < x < \frac{3\pi}{2}, \\ \sin(x) & \text{if} \quad \frac{3\pi}{2} \leq x < 2\pi. \end{cases}$$

Exercise 4 (Ex 14.8 page 221)**.**

1. Calculate the Fourier series of the 2π-periodic function defined by

$$f(x) = |\cos(x)| \quad \text{if} \quad x \in [0, 2\pi[.$$

2. Deduce the sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{4k^2 - 1}.$$

Exercise 5 (Ex 14.10 page 221)**.**

1. For $\alpha \in \mathbb{R} \setminus \mathbb{Z}$, calculate the Fourier series of the 2π-periodic function defined by

$$f(x) = \cos(\alpha x) \quad \text{if} \quad x \in [-\pi, \pi[.$$

2. Deduce the formula

$$\sum_{n=1}^{\infty} \frac{1}{n^2 - \alpha^2} = \frac{1}{2\alpha^2} - \frac{\pi}{2\alpha \tan(\alpha\pi)}.$$