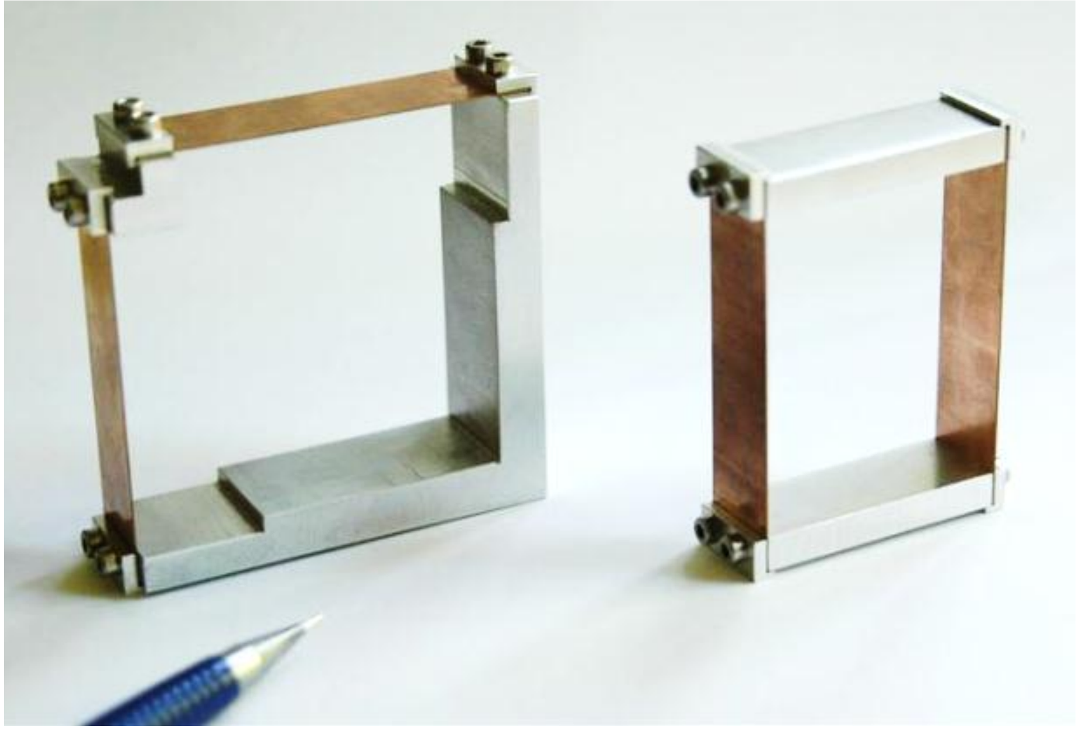
Dimensioning of flexure joints



Prof. Simon Henein, Dr. Etienne Thalmann

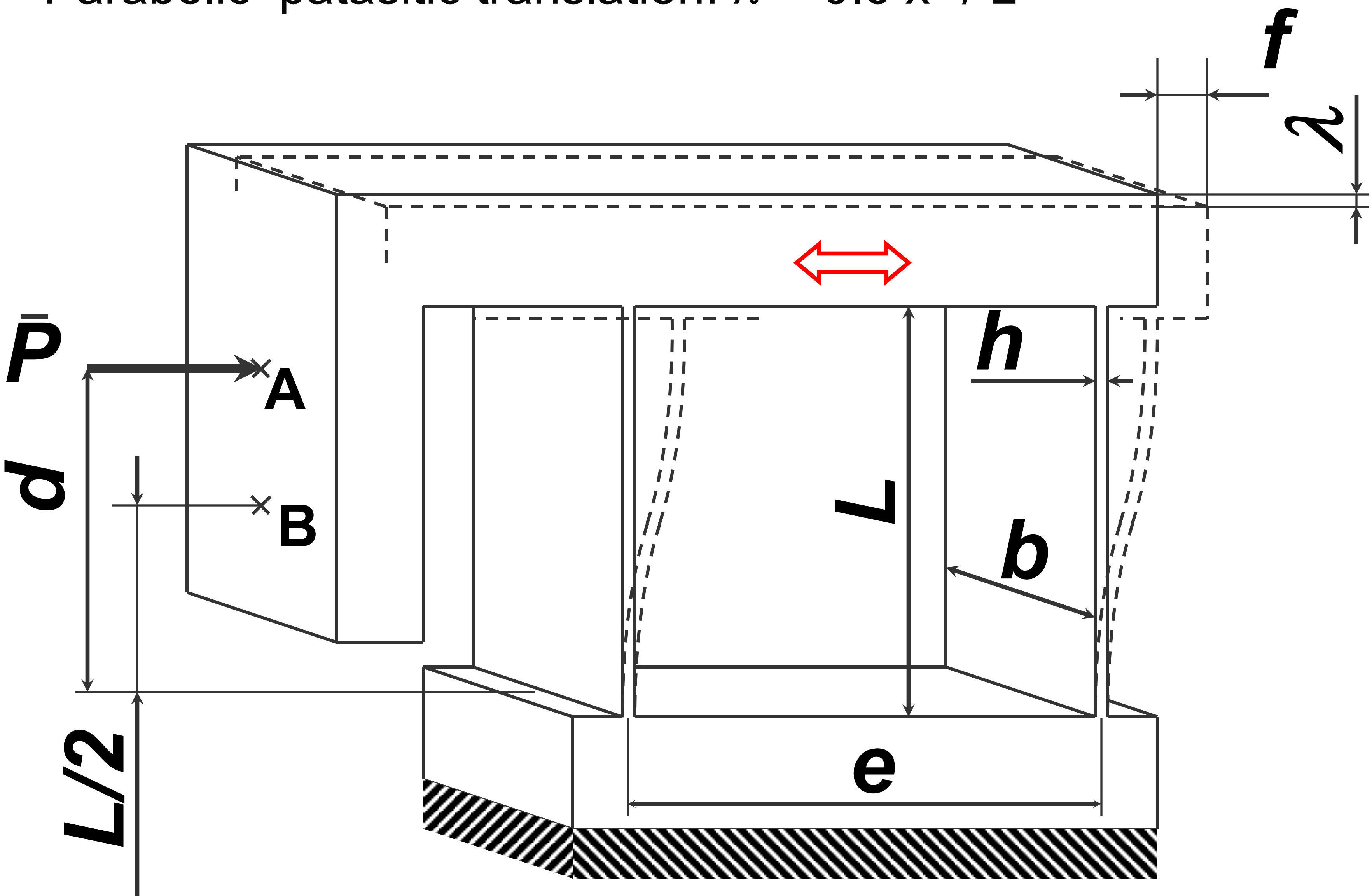
Main sizing considerations

- Parasitic shift
- Stiffness
- Admissible stroke
- Manufacturability
- Critical load (buckling)

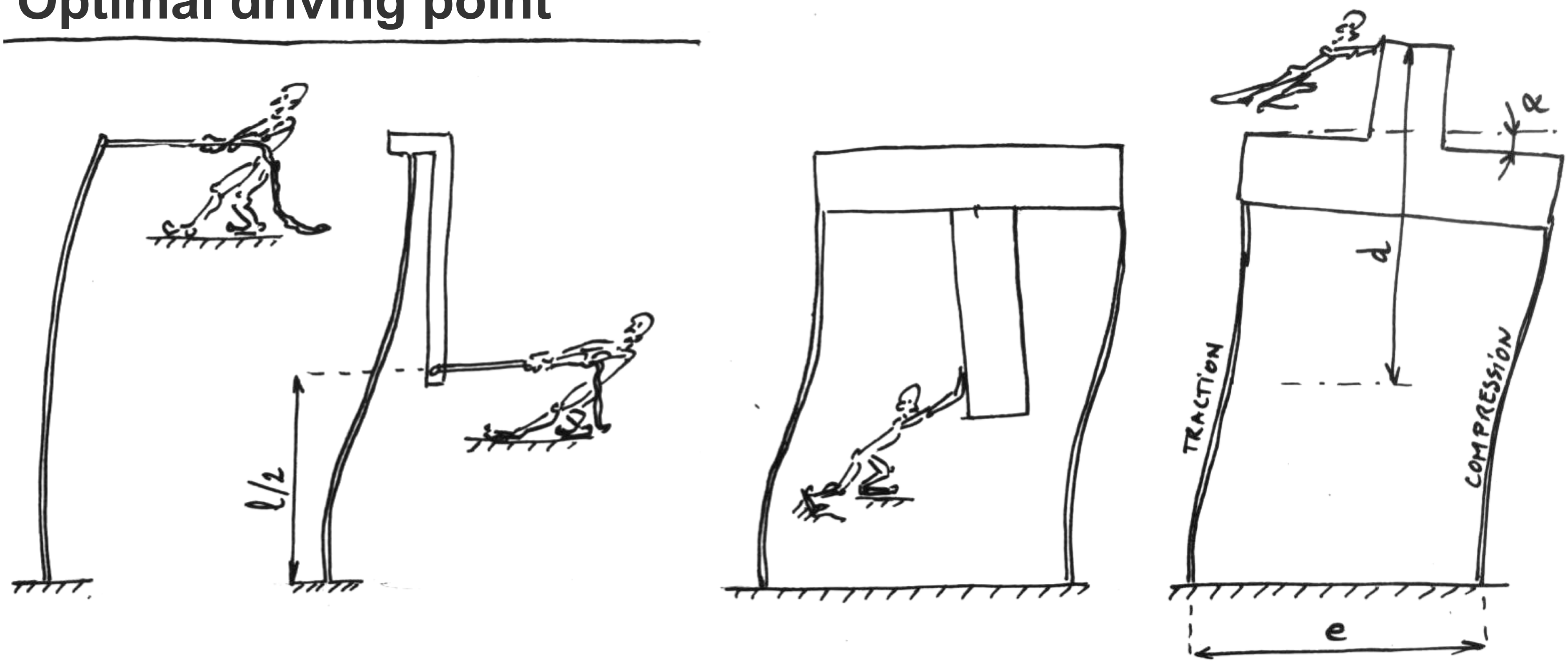
Parallel flexure stage

Parallel spring stage

Parabolic patasitic translation: $\lambda \approx 0.6\ x^2 / L$



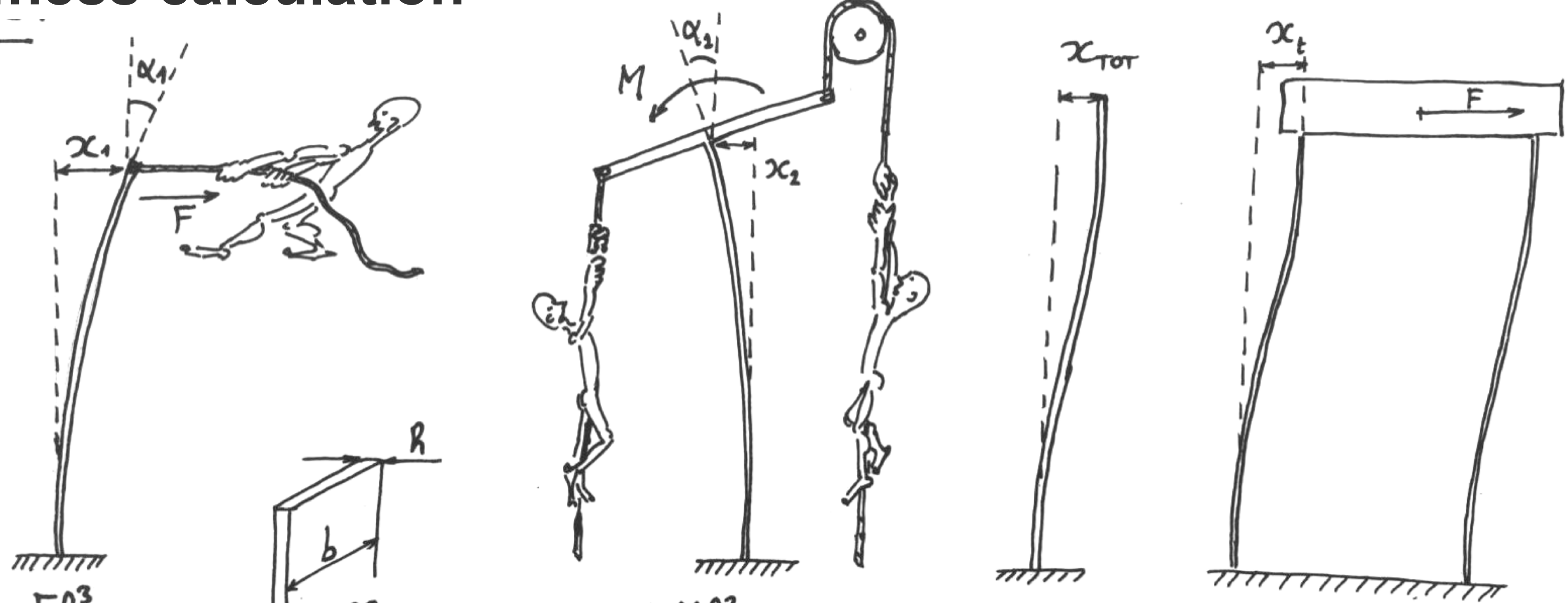
Optimal driving point



To minimize the parasitic rotation caused by the cantilever effect :

- Minimize *d*
- Maximize the distance between blades *e*

Stiffness calculation



$$x_1 = \frac{Fl^3}{3EI} \qquad I = \frac{bh^3}{12} \qquad x_2 = \frac{-Ml^2}{2EI} \qquad x_{TOT} = x_1 + x_2$$

$$\alpha_1 = \frac{Fl^2}{2EI} \qquad \alpha_2 = \frac{-Ml}{EI} \qquad \alpha_{TOT} = \alpha_1 + \alpha_2 = 0$$

$$\alpha_{TOT} = \frac{Fl^2}{2EI} - \frac{Ml}{EI} = 0$$

$$\boxed{M = \frac{Fl}{2}}$$

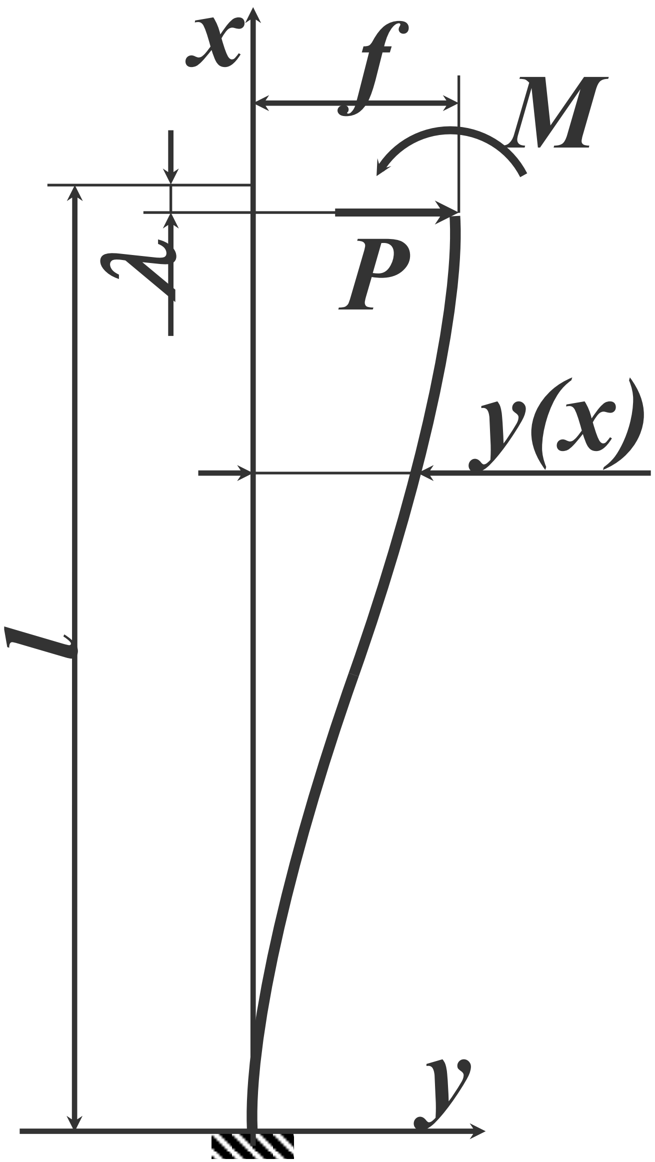
$$x_{TOT} = \frac{Fl^3}{3EI} - \frac{Fl^3}{4EI}$$

$$x_{TOT} = \frac{Fl^3}{12EI}$$

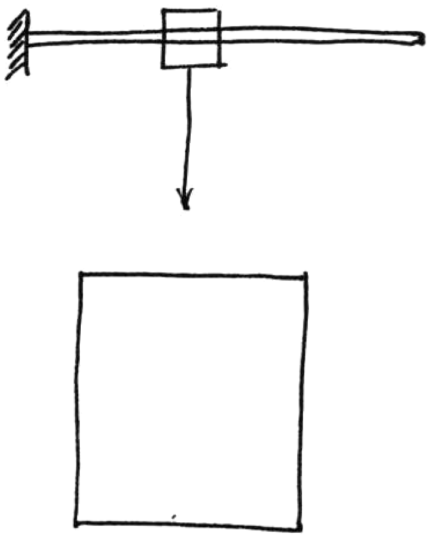
$$K = \frac{F}{x_{TOT}} = \frac{12EI}{l^3} = \frac{Ebh^3}{l^3}$$

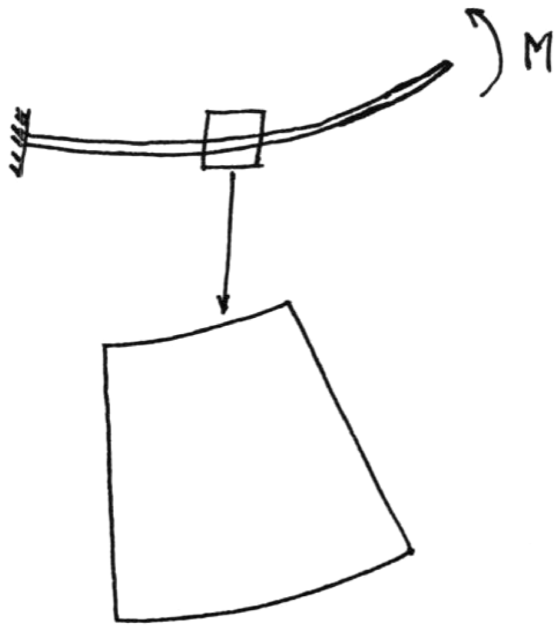
$$K_T = \frac{F}{x_T} = \frac{2Ebh^3}{l^3}$$

Bending moment distribution along the blade



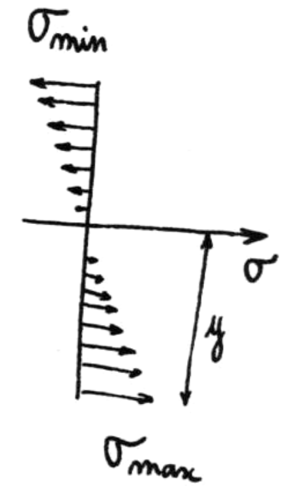
Admissible stroke calculation





$$\sigma_{max} = \frac{M y}{I} \qquad I = \frac{b h^3}{12} \qquad y = \frac{h}{2}$$

$$\boxed{\sigma_{max} = \frac{6M}{b h^2}}$$



$$M_{adm} = \frac{b h^2 \sigma_{adm}}{6}$$

$$F_{adm} = \frac{2 M_{adm}}{l} = \frac{b h^2 \sigma_{adm}}{3 l}$$

$$x_{adm} = \frac{F_{adm}}{K} = \frac{b h^2 \sigma_{adm}}{3 l} \cdot \frac{l^3}{E b h^3}$$

Admissible Stress

Admissible Stroke

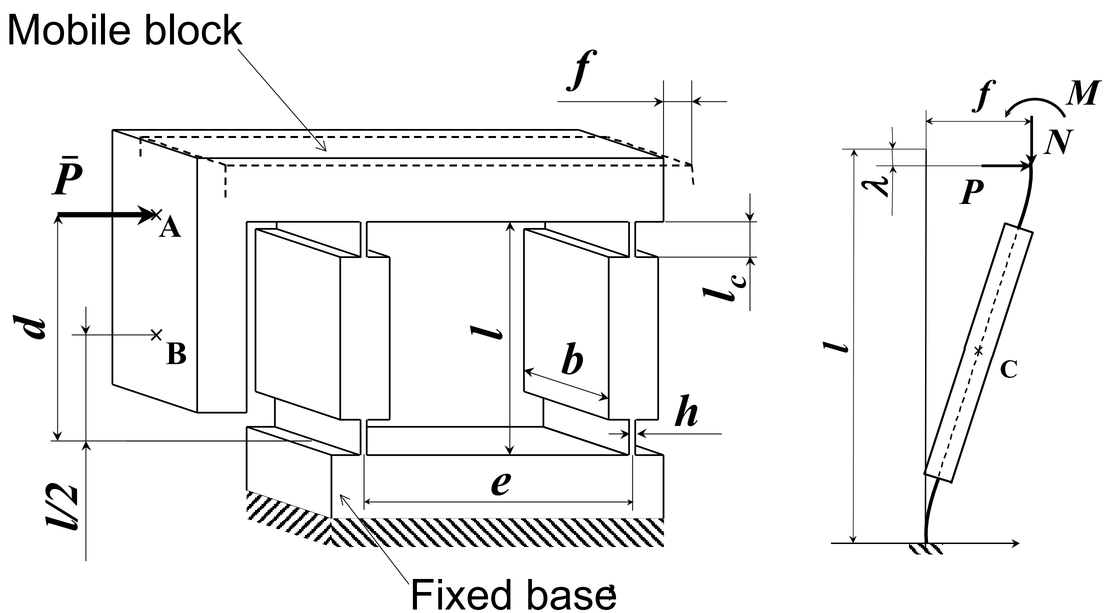
Blade length

$$\boxed{x_{adm} = \frac{\sigma_{adm} \, l^2}{3 E h}}$$

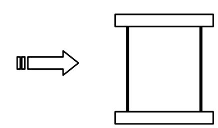
Blade thickness

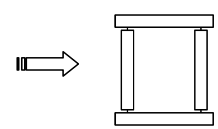
Young's Modulus

Linear stage with necked down flexures



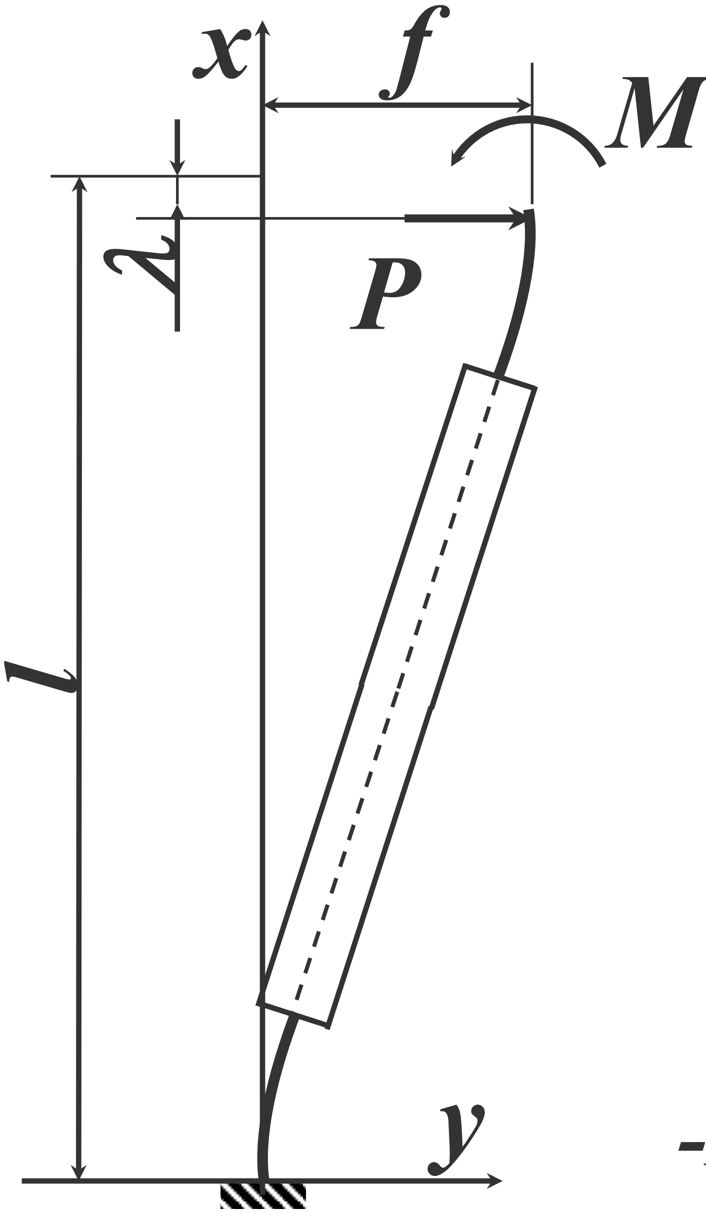
$$\xi = \frac{2l_c}{l} \qquad 0 < \xi \leq 1$$

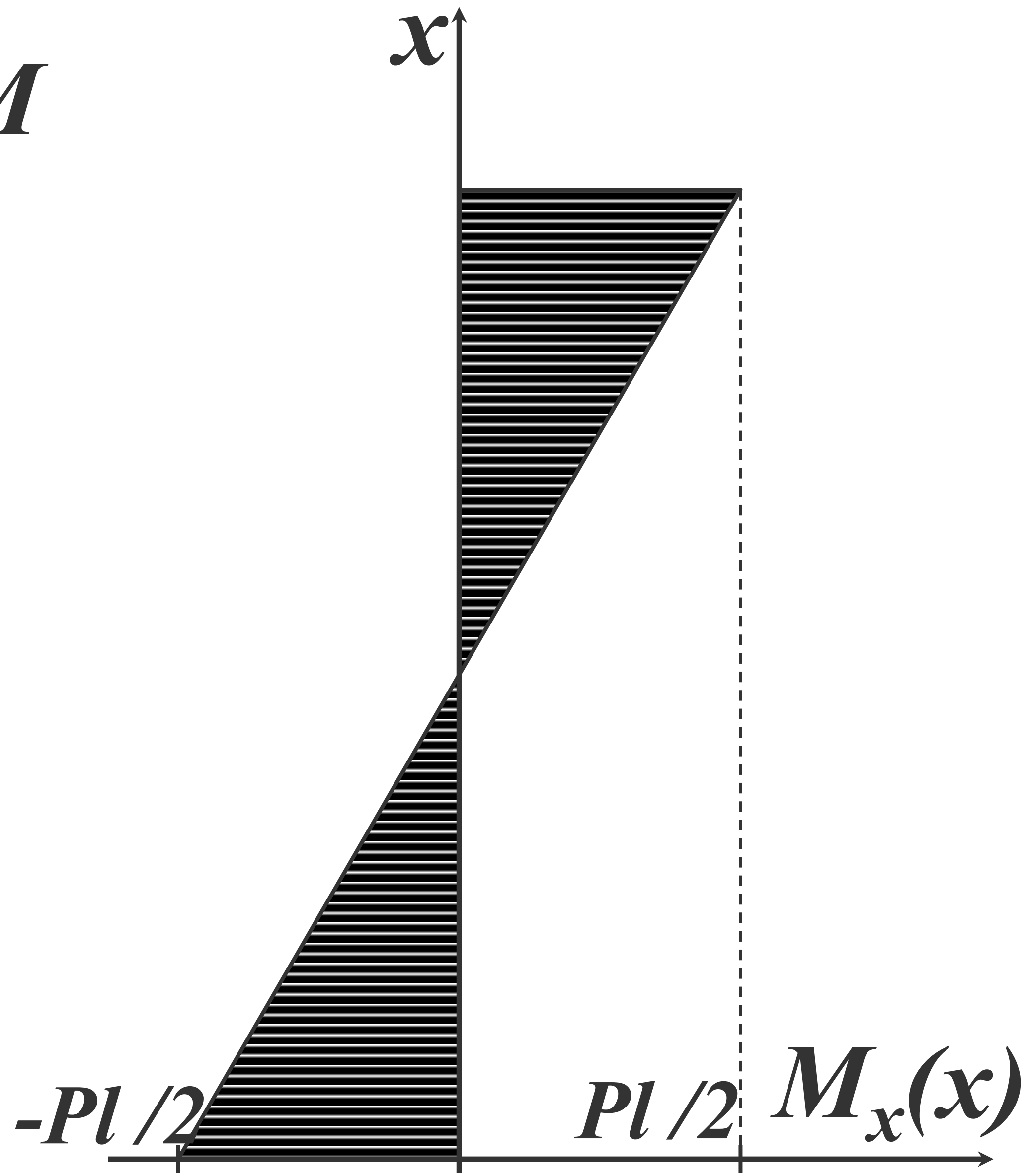
$\xi = 1$ 

$\xi \to 0$ 

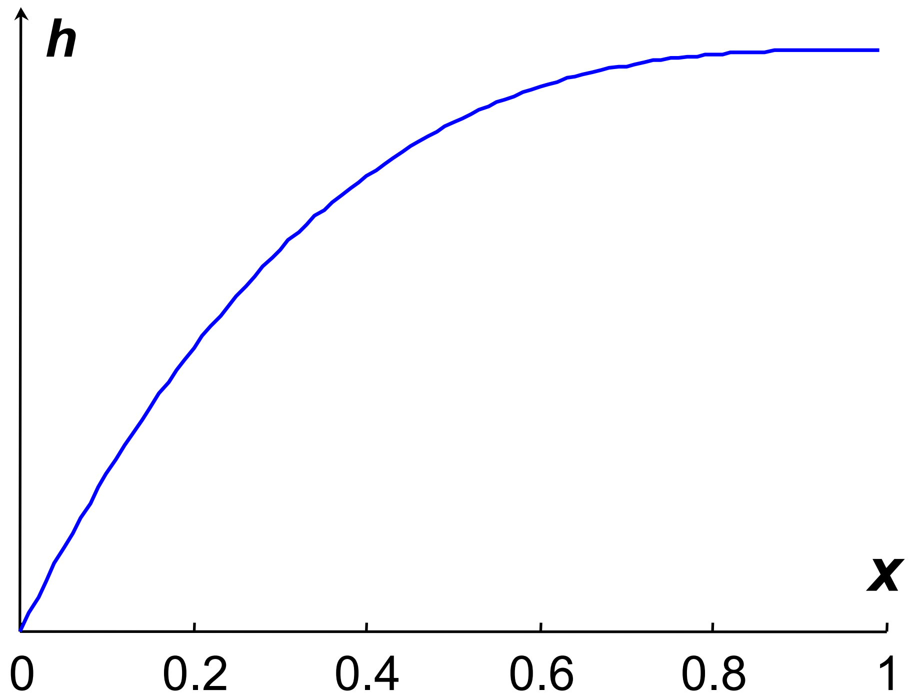
For a given stroke f and given outer dimensions l and b, what is the optimal hinge length l_c ?

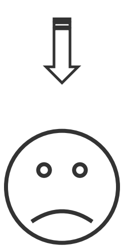
Bending moment distribution along the arm





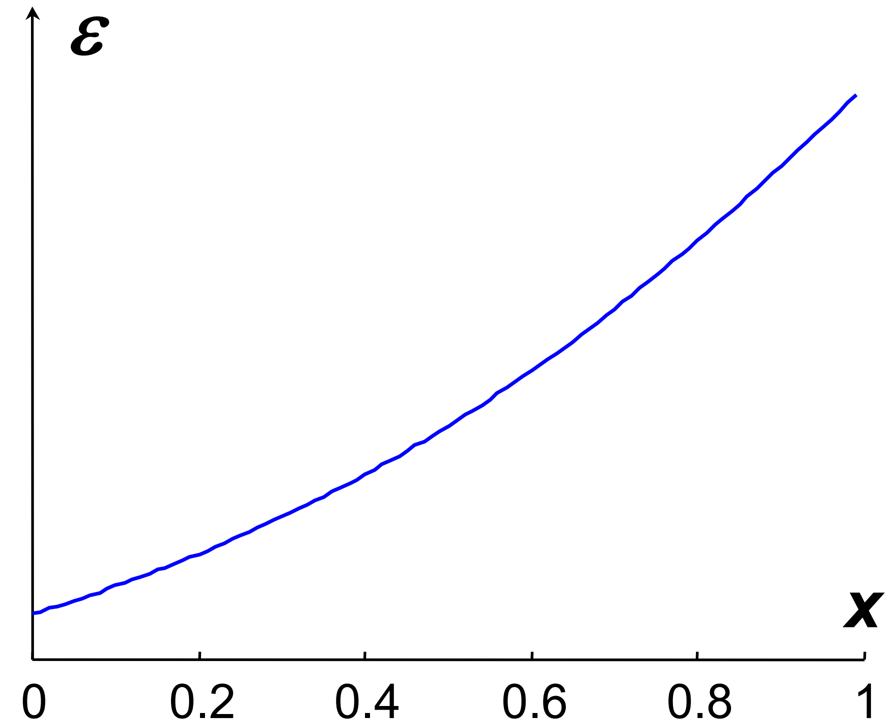
Thickness of the hinges

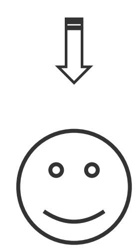




Sensitivity to manufacturing tolerances

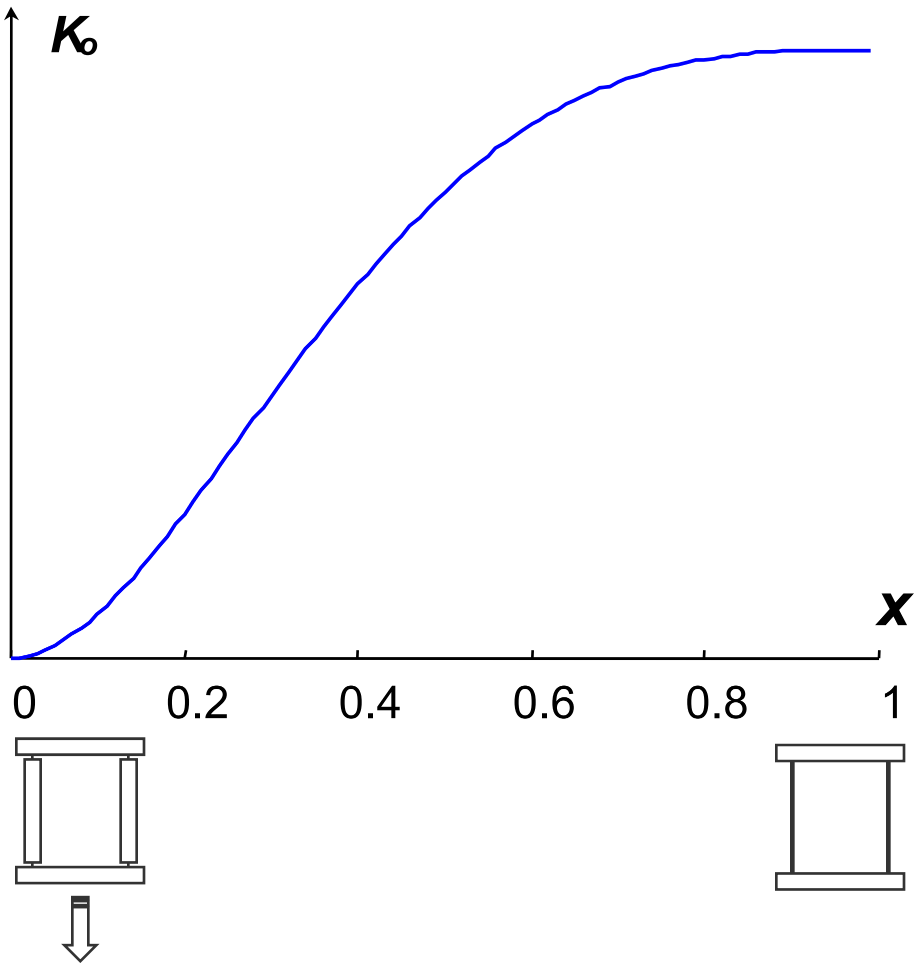
Aspect ratio of the hinges





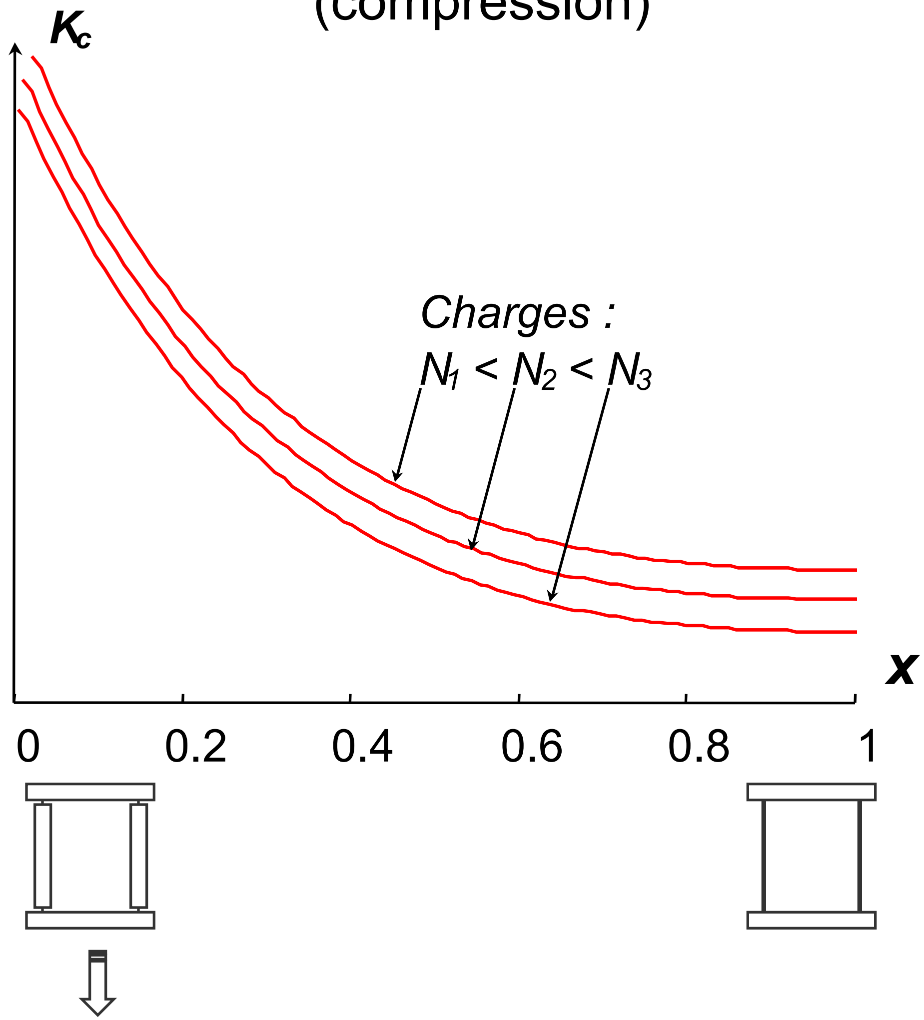
Easiness of manufacturing

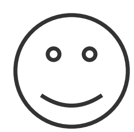
Natural stiffness



High stiffness-ratio

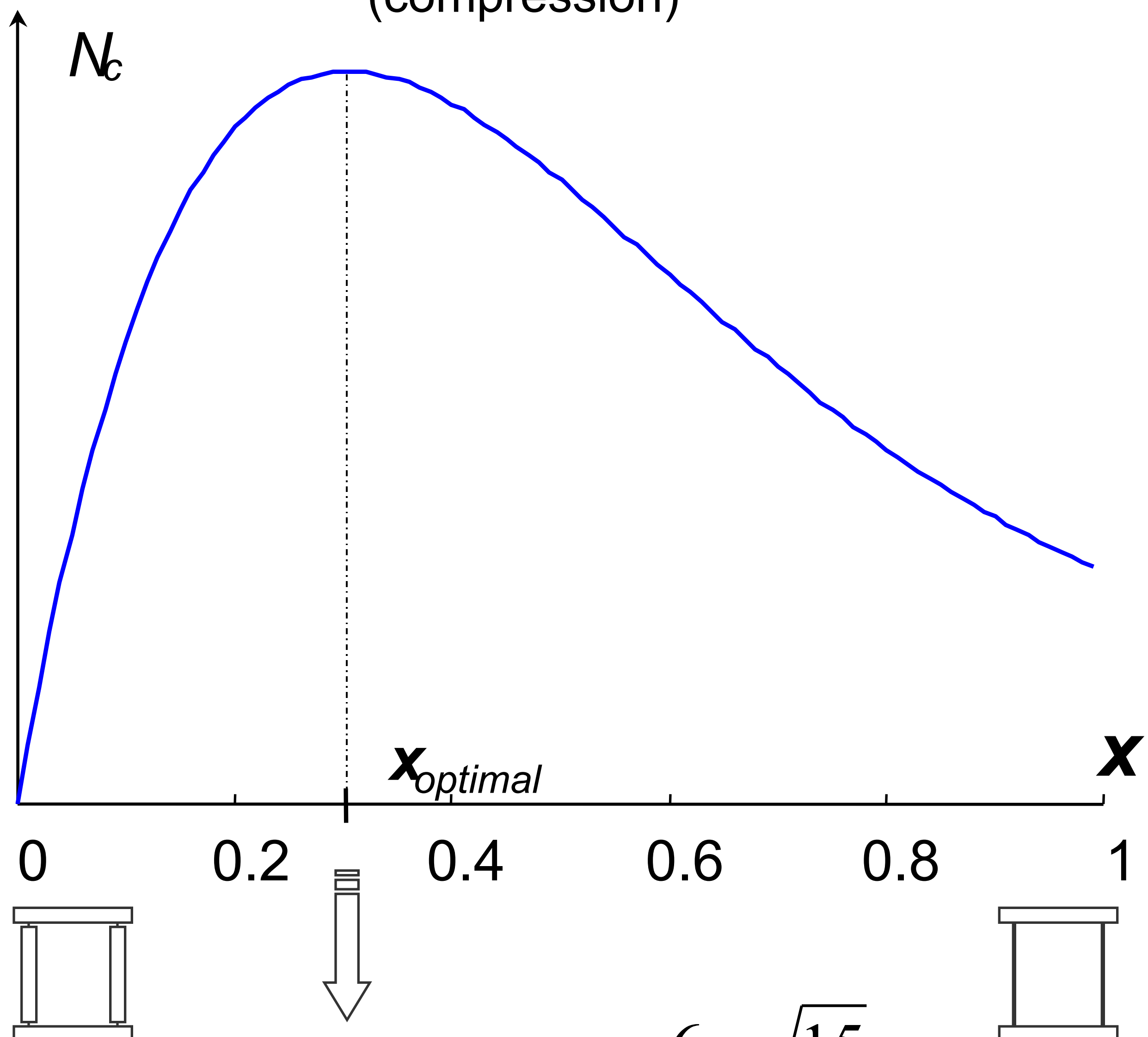
Transverse stiffness
(compression)





High stiffness-ratio

Critical load
(compression)

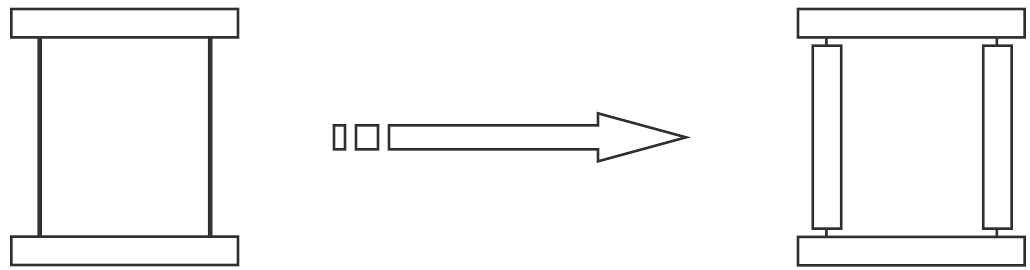


$$\xi_{optimal} = \frac{6-\sqrt{15}}{7} \cong 0{,}3$$

Linear stage with necked down hinges

Conclusion of the hinge length optimization

$\boxed{\xi = 1}$



$\boxed{\xi \to 0}$

Shortening of hinges (reduction of x) leads to:

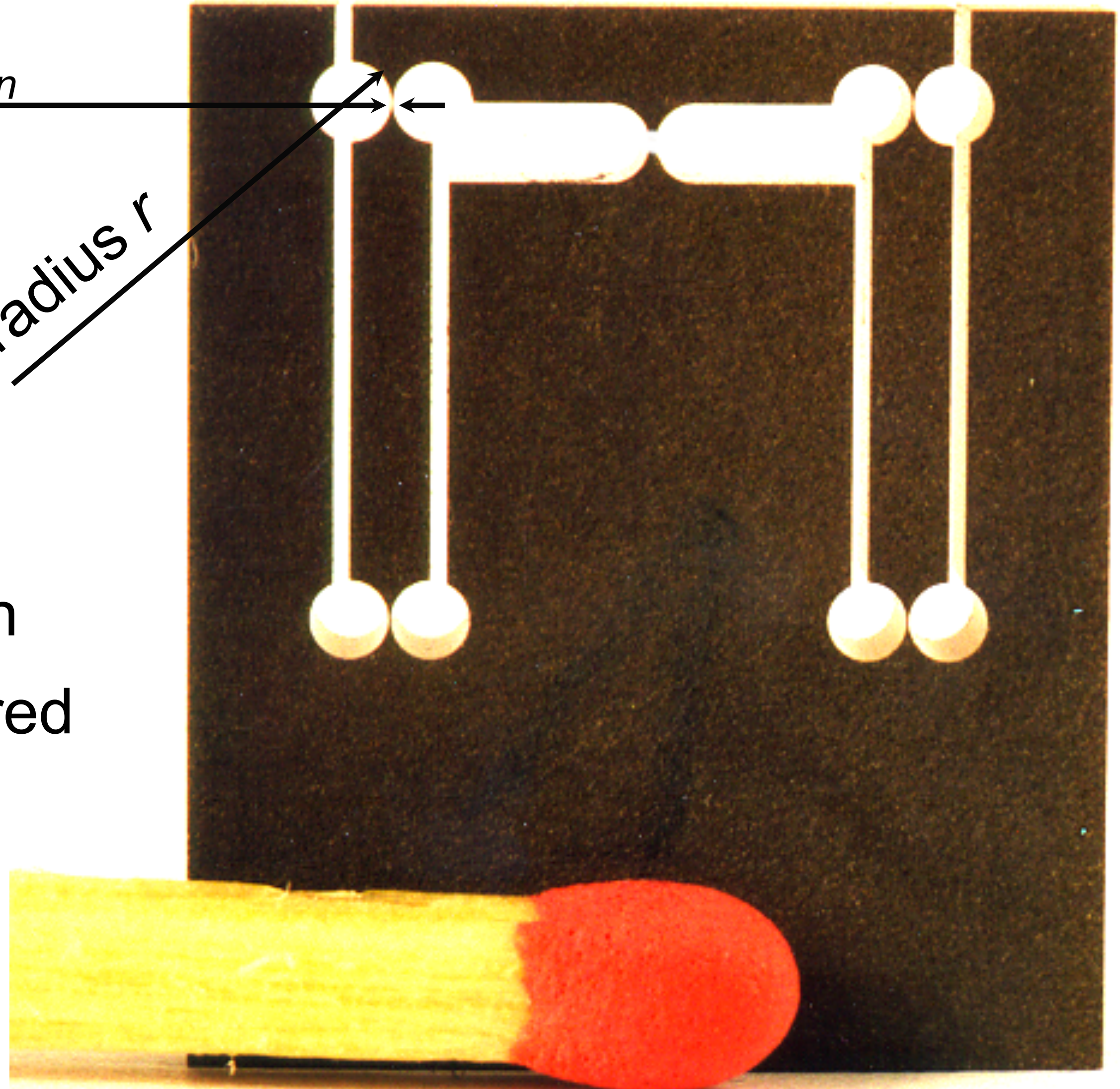
- Shorter aspect-ratio of the hinges
- Higher stiffness in tension-compression
- Lower natural stiffness
- Reduction of the thickness of the hinge

$\Rightarrow$ technological limit (machining of thin cross-sections)

Parallel stage with right-circular hinges

minimal thickness h_{min}

radius r



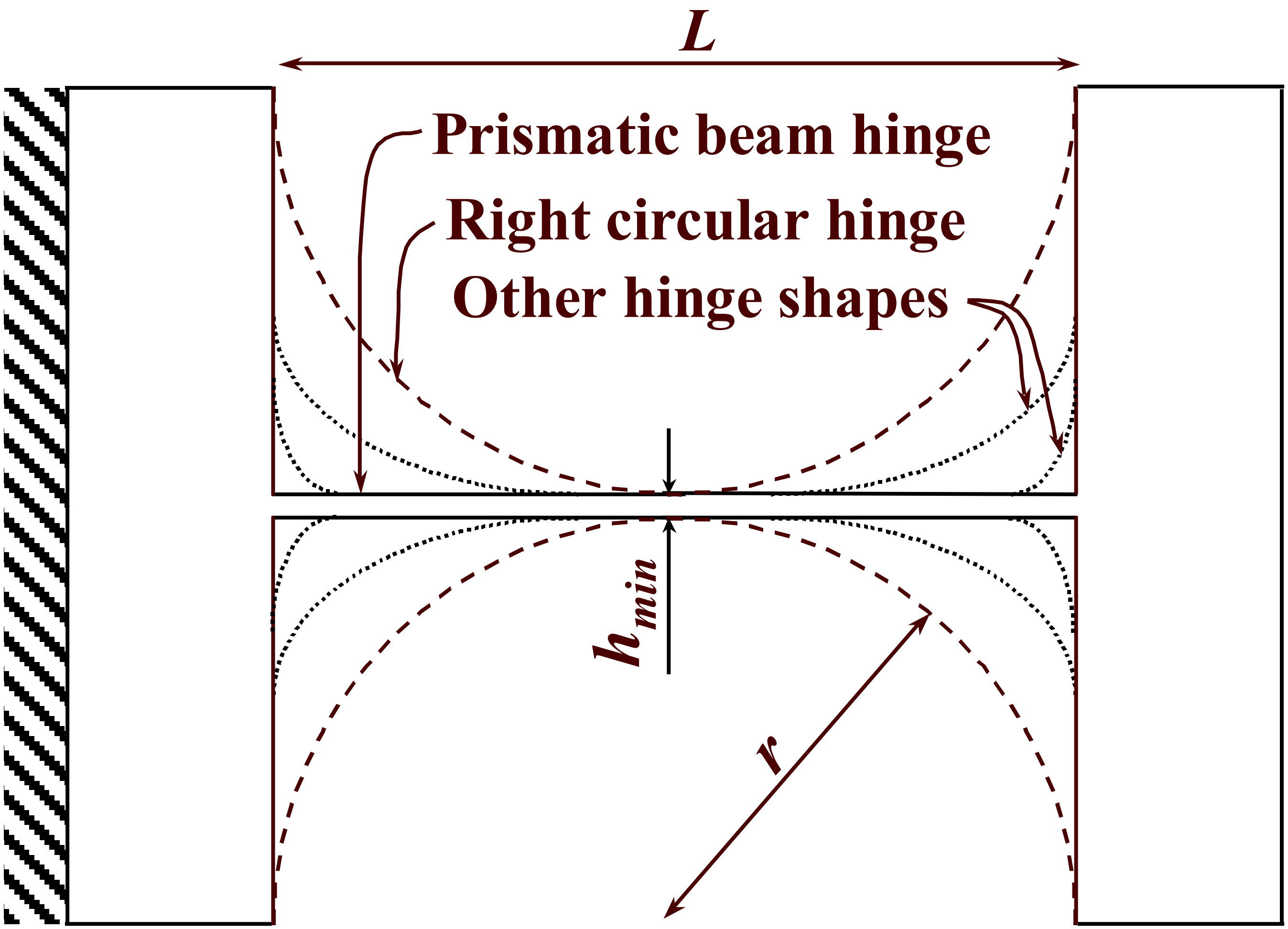
if $\boldsymbol{r < 5\, h_{min}}$,

stress concentration

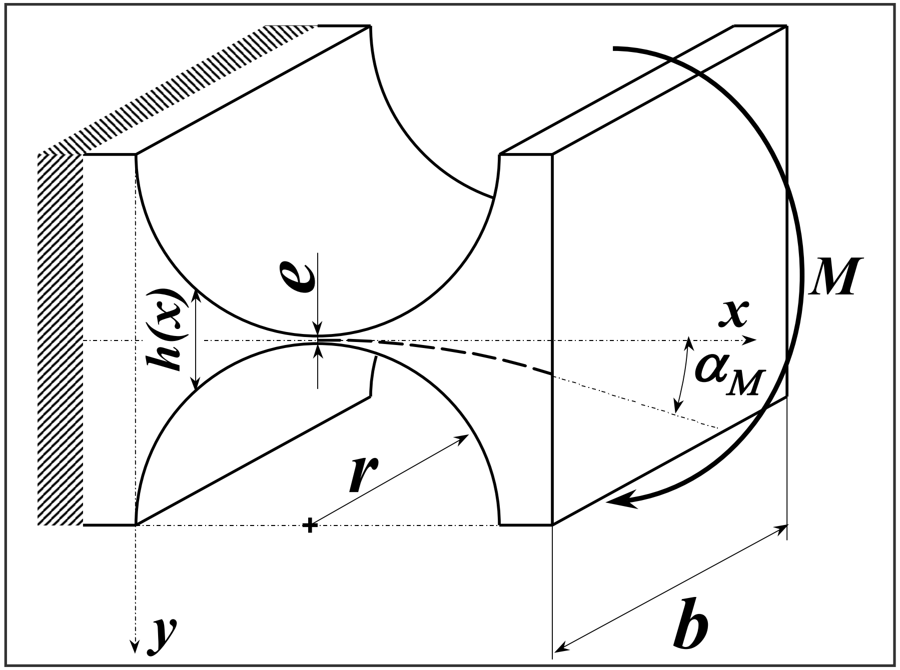
effects are considered

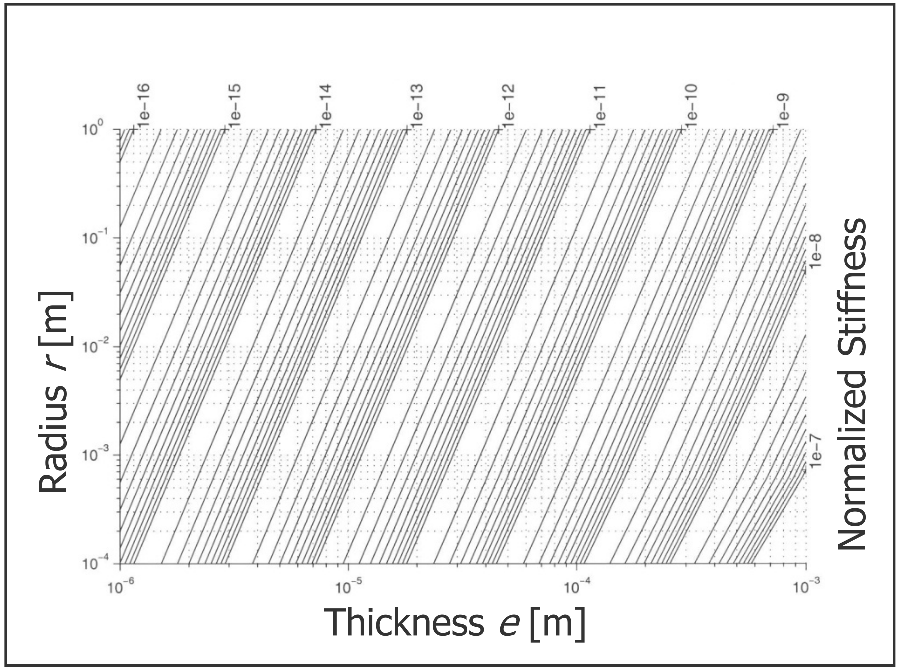
negligible

Necked down flexure-joints ("notch hinges")



Stiffness in the circular notch hinge





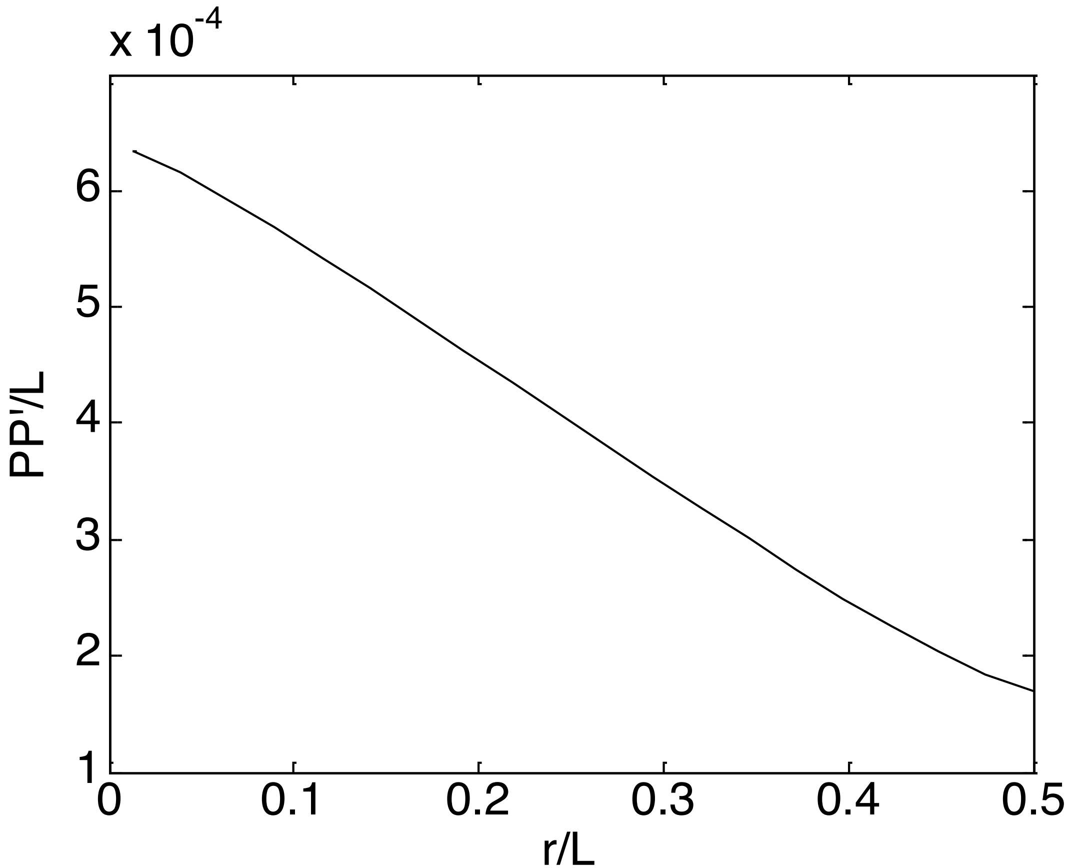
« Exact » formula : $K = M / y'(2r)$ with $y'' = M / (EI)$

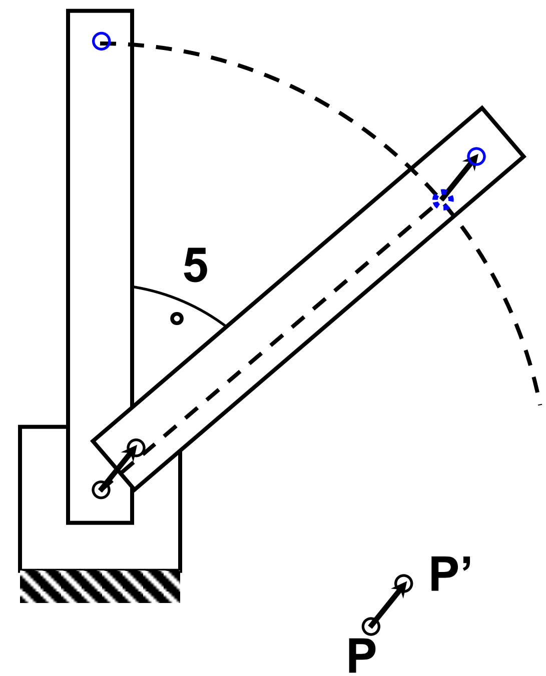
Simplified formula : $K \approx \mathbf{A}\, E\, b\, e^{\mathbf{B}}\, r^{\mathbf{C}}$ $\Rightarrow$ $\boxed{K \approx \mathbf{0.07}\, E\, b\, e^{\mathbf{2.5}}\, r^{\mathbf{-0.5}}}$

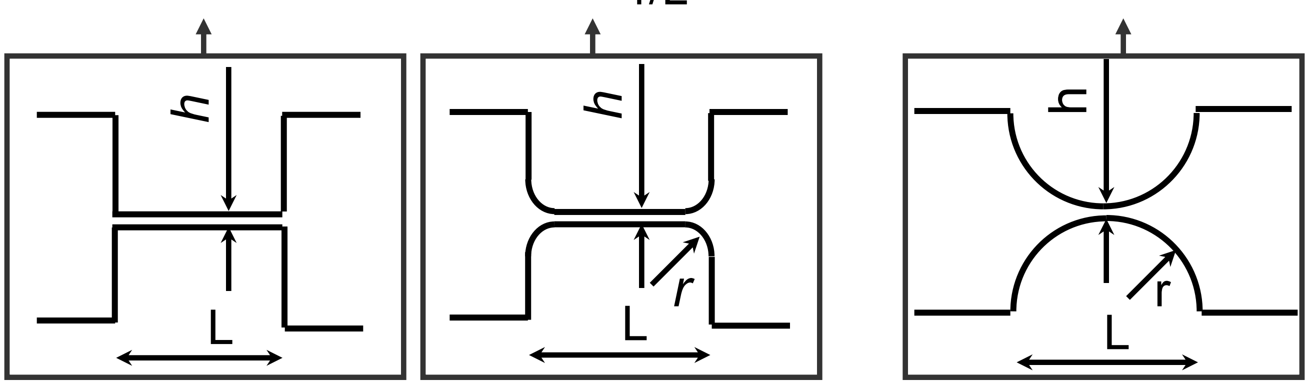
Scaling laws : $K^* \approx b^*$; $K^* \approx E^*$; $K^* \approx e^{*\,\mathbf{2.5}}$; $K^* \approx r^{*\,\mathbf{(-0.5)}}$

Standard Hinge : $e = 50\mu m$; $r = 3mm$
Validity domain : $1\mu m < e < 1mm$; $0.1mm < r < 1m$
The error of the simplified formula over the validity domain is smaller than 2.6%

Parasitic shift of the circular-fillet hinge for a 5° rotation





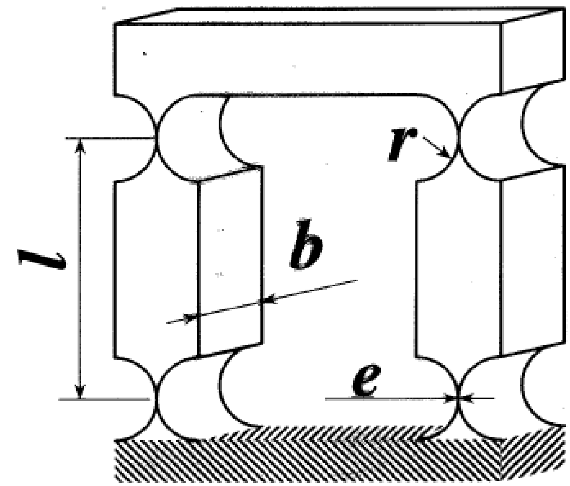


Example

Calculation of the admissible stroke a translation stage

Book pages: 94 & 137

$e = 100\ \mu m$
$r = 4\ mm$
$b = 10\ mm$
$l = 50\ mm$
$E = 210\ GPa$
$\sigma_{adm} = 800\ MPa$



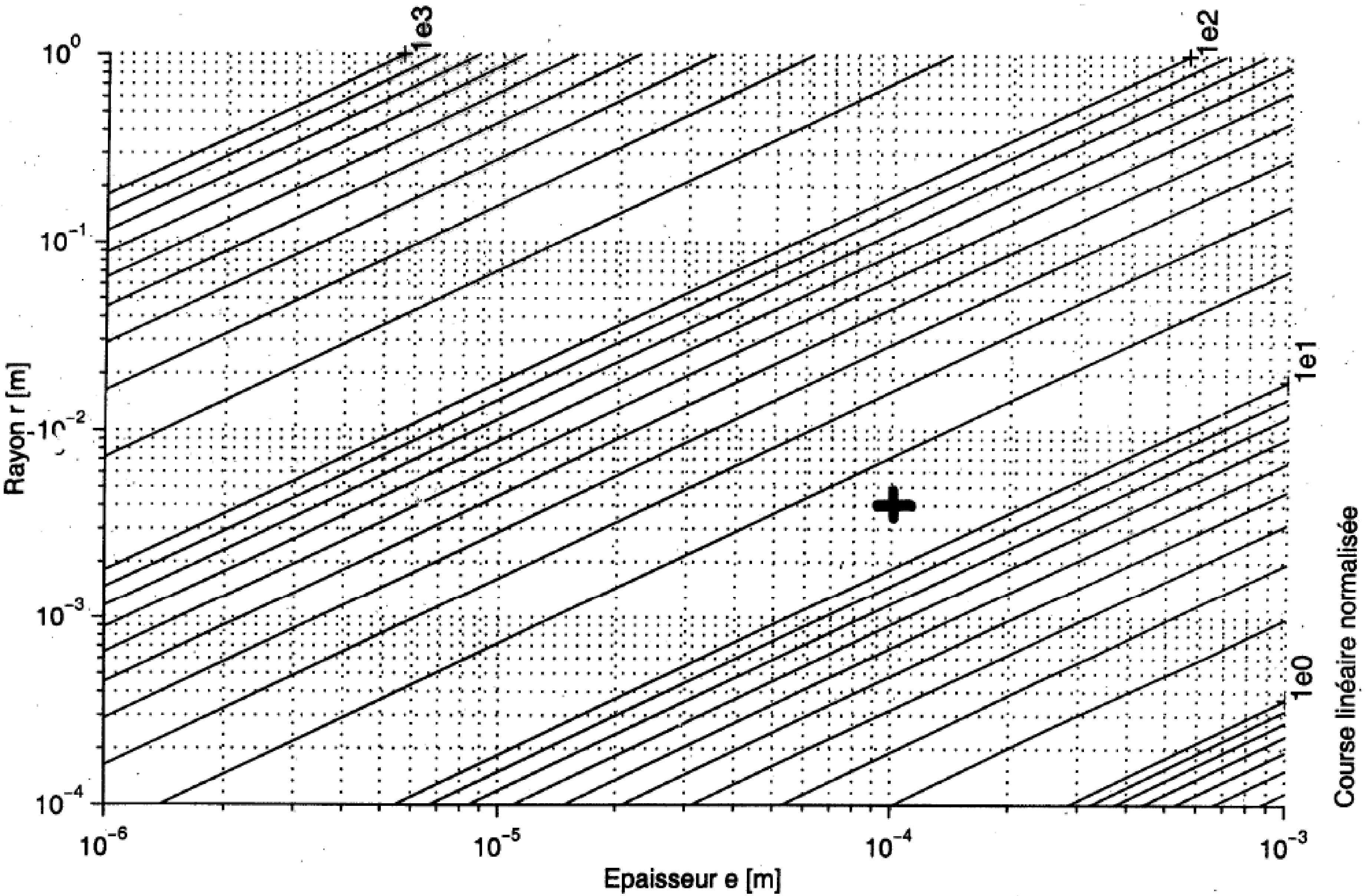
Abaque : $f_{max\ n} \simeq 15 \quad \Rightarrow \quad f_{max} = f_{max\ n}\, \sigma_{adm}\, l / E \simeq 2.86\ mm$

Formule simplifiée : $f_{max} \simeq \dfrac{3\,\pi\,\sigma_{adm}\, r^{0.5}\, l}{4\, E\, e^{0.5}} = 2.84\ mm$

Valeur théorique exacte : $f_{max} = 2.83\ mm$ (Programme Matlab)

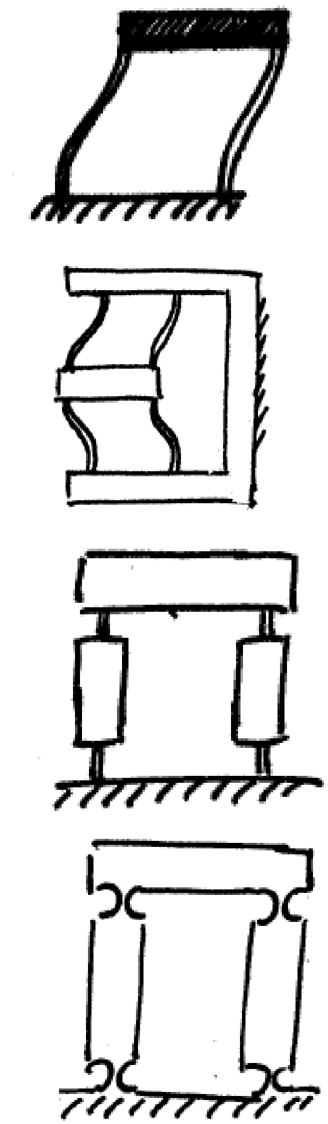
Lois de similitude : $e \longrightarrow e/2 \quad \Rightarrow \quad f_{max} \longrightarrow \sqrt{2}\, f_{max}$

Example:



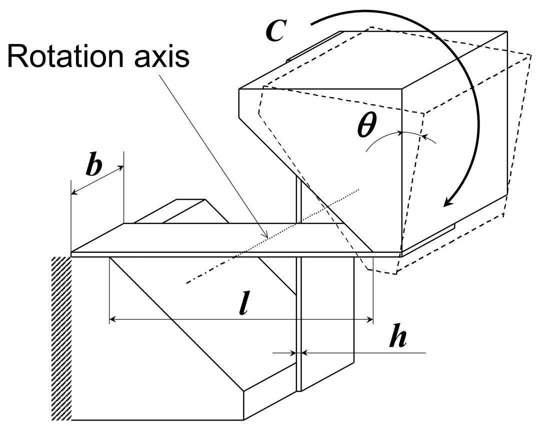
1e3
1e2
1e1
1e0
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
Rayon r [m]
10^{-6}
10^{-5}
10^{-4}
10^{-3}
Epaisseur e [m]
Course linéaire normalisée

Translation bearings : summary

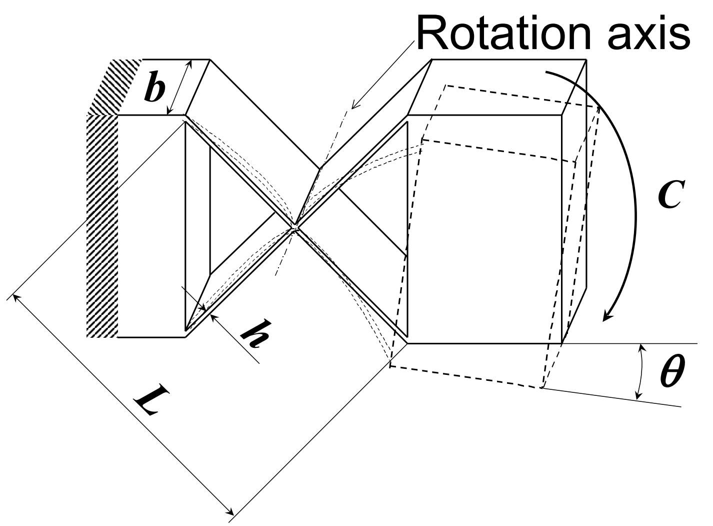


- Parallel spring stage
  - Parabolic translation
  - Driving point at mid-height
  - Spreading of the leaf springs
  - Calculation of stiffness and stoke
- Necked down flexure hinges
  - High stiffness ratios☺
  - Smaller aspect ratio☺
  - Reduced hinge thickness → technological limit
- Hinge shape “optimisation”

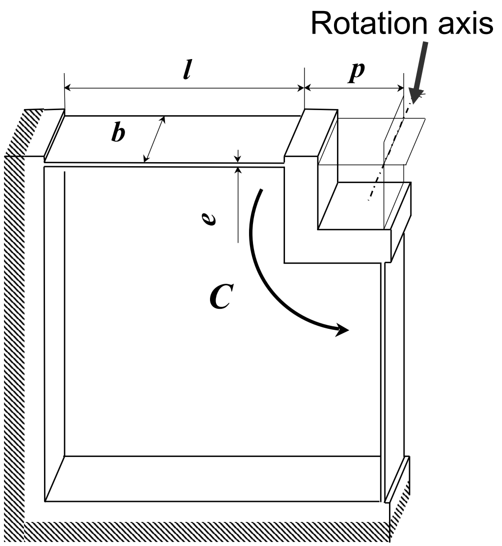
Flexure pivots



Cross spring pivot

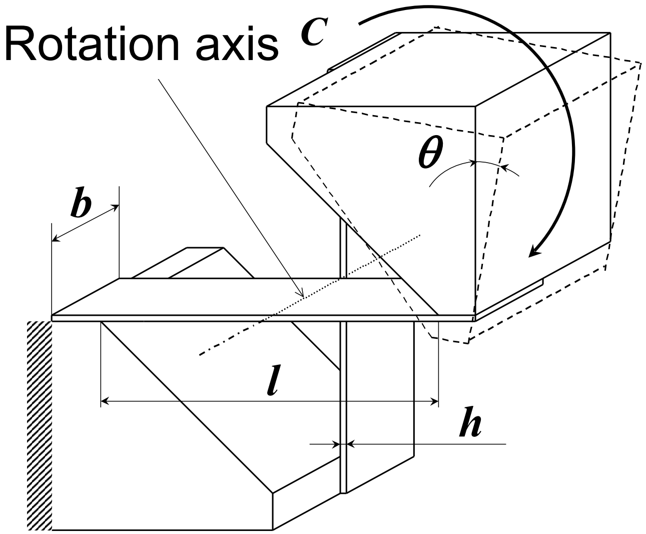


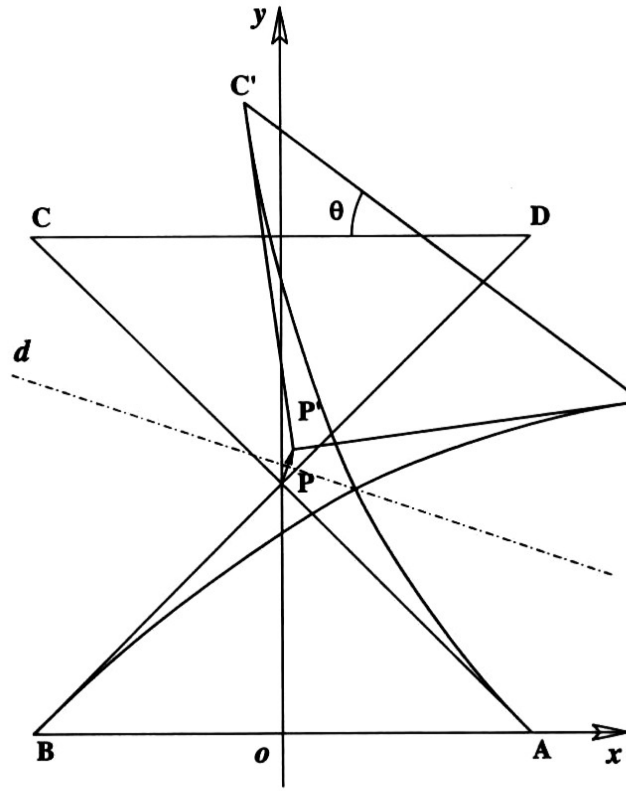
Unseparated cross spring pivot

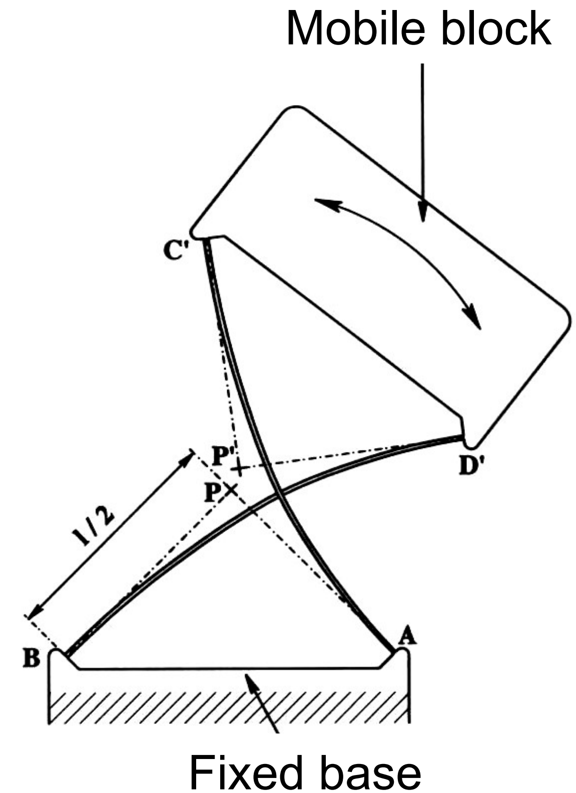


Remonte-Center-Compliance Pivot (RCC)

Symmetrical cross spring pivot







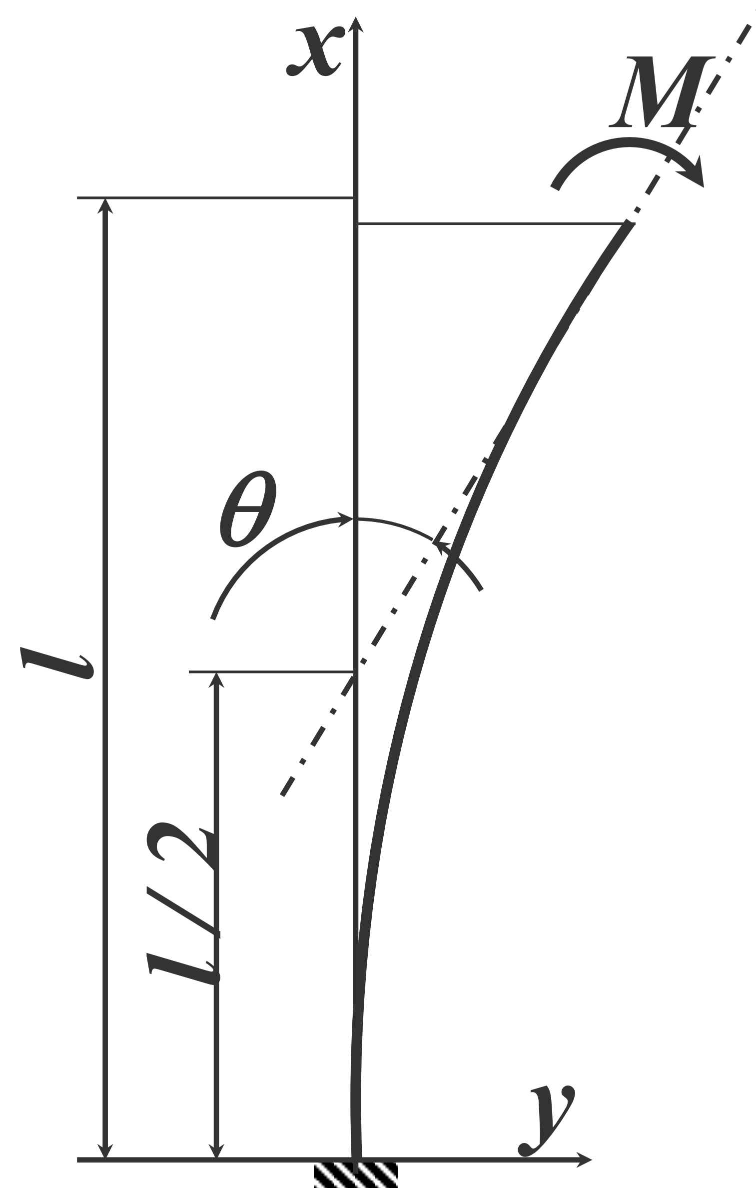
☺ Largest stroke

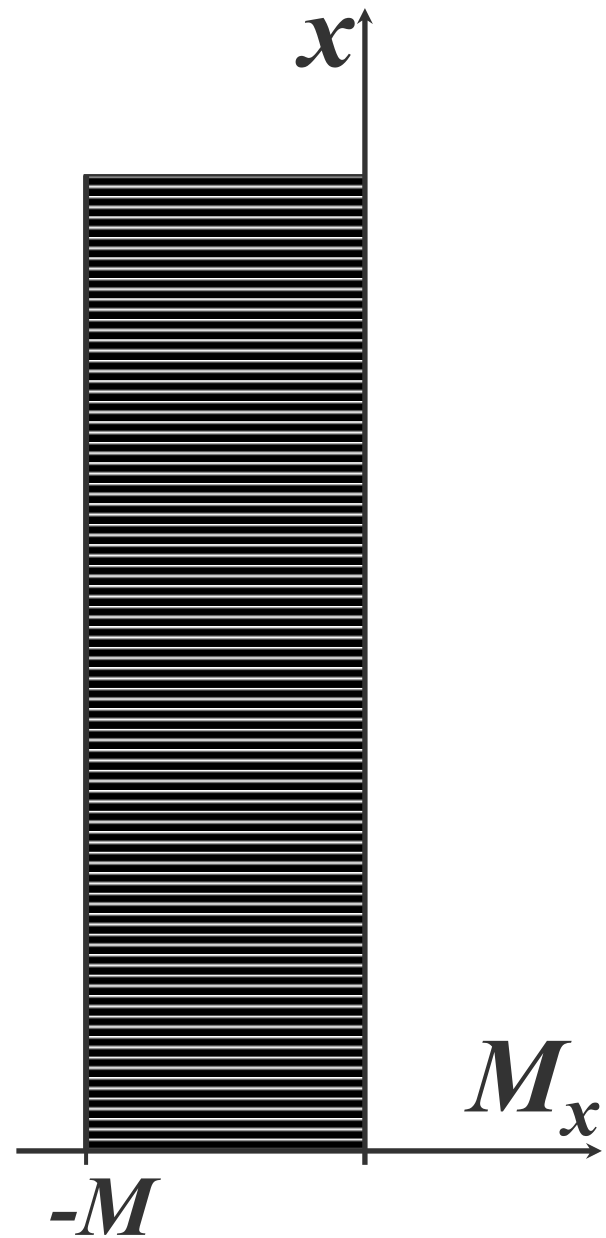
☺ Available of-the-shelf

☹ Large parasitic shift

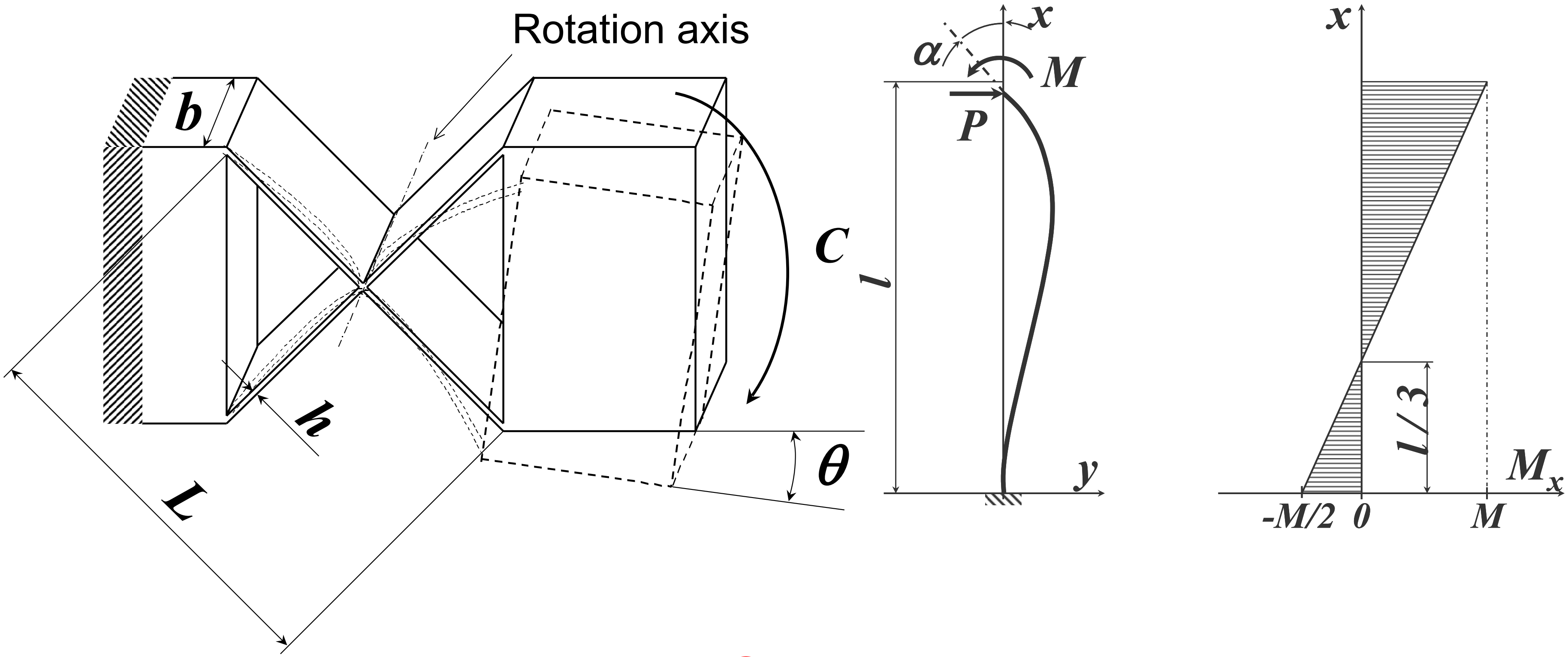
☹ Three-dimensional (difficult to manufacture)

Bending moment distribution along the blade





Unseparated cross spring pivot

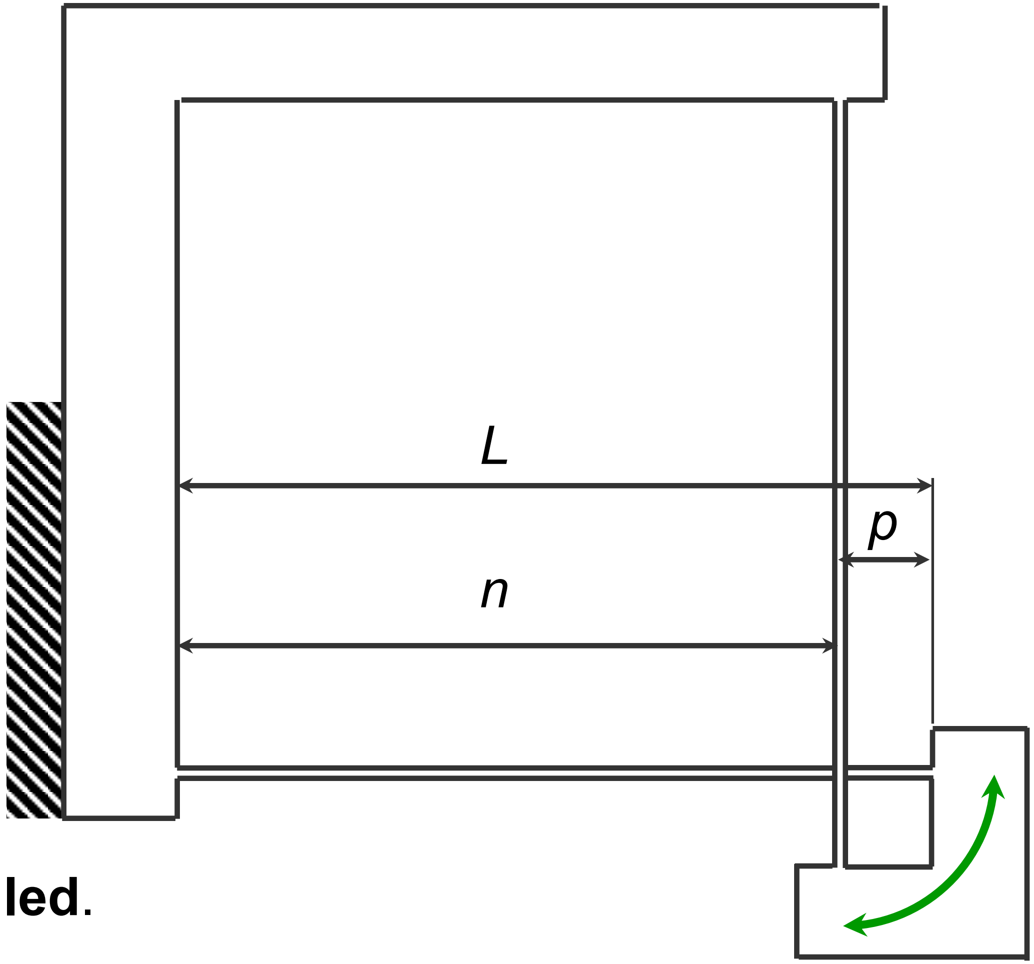


☺ Planar structure (easy to manufacture monolithically)

☹ 4 times less stroke than with identical separated blades

☹ 4 times stiffer than with identical separated blades

Asymmetrical cross spring pivot



For $p = 0.127\,L$

and $n = 0.873\,L$,

☺ **the parasitic shift is cancelled**.

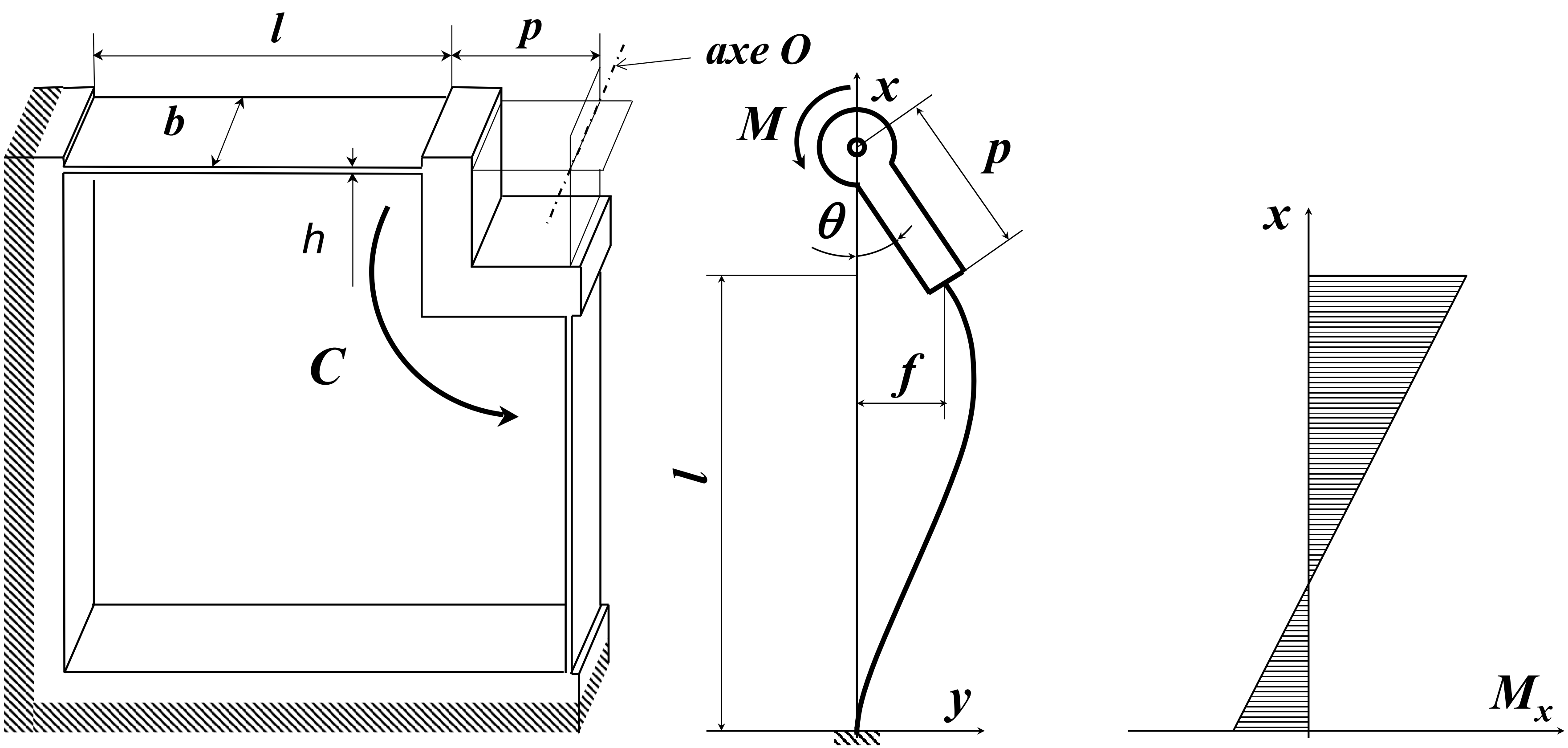
But, compared to the symmetric cross spring pivot,

☹ the angular stiffness is multiplied approx. by a factor 3.3

☹ the angular stroke is reduced approx. by a factor 3.2.

[Wittrick, W.H. (1951), The properties of crossed flexure pivots and the influence of the point at which strips cross, The Aeronautic Quartely, 1951, II (4), p. 272-292]

Remote-Center-Compliance Pivot (RCC)



- ☺ Planar structure
- ☺ “Virtual pivot”
- ☹ Limited stroke, especially if *p* is large
- ☹ High stiffness, especially if *p* is large

Generalized cross spring pivot dimensioning

- Rotational stiffness:

  $K = \frac{8EI}{L}(3\delta^2+3\delta+1)$, E is Young's modulus,

  $I = \frac{bh^3}{12}$, $\delta = \frac{p}{L}$

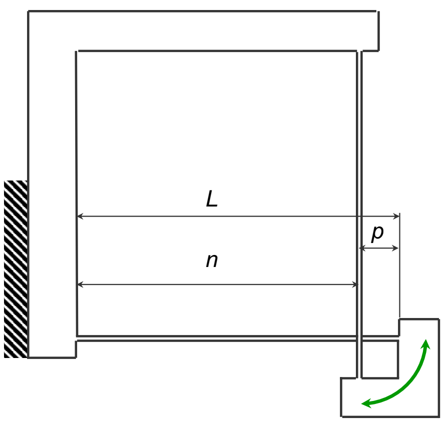
- Admissible stroke:

  $\theta = \frac{\sigma_{adm}L}{Eh(2+3\delta)}$, σ_{adm} is the admissible stress in the flexures

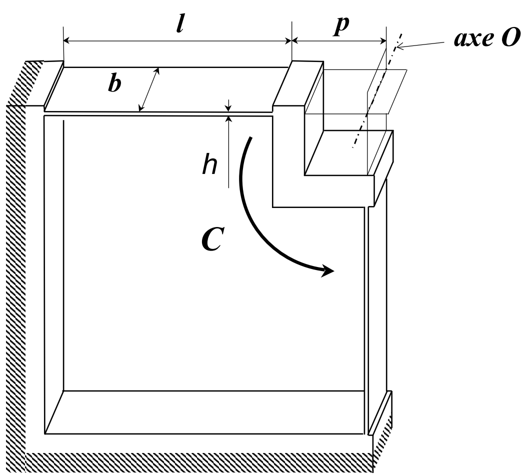
- Parasitic shift:

  $|PP'| \approx \left|\frac{(9\delta^2+9\delta+1)\theta^2 L}{15\cos(\alpha)}\right|$, α is the half angle between the flexures

$\delta < 0$



$\delta > 0$



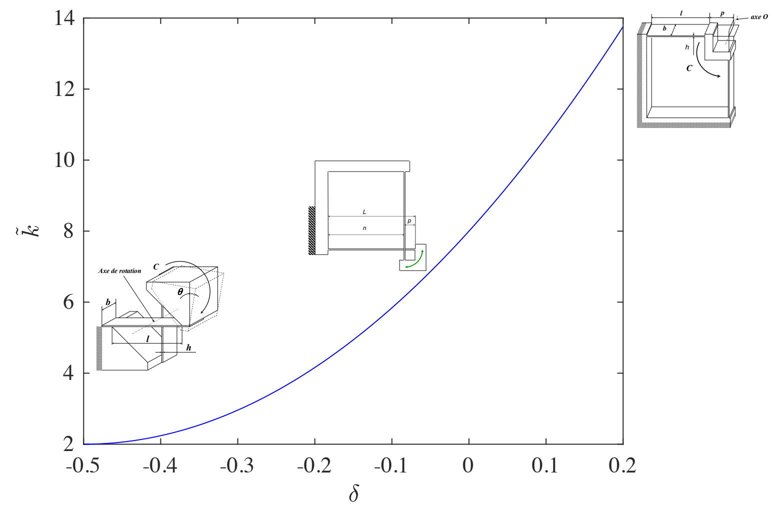
Zhao, H., & Bi, S. (2010). Accuracy characteristics of the generalized cross-spring pivot. *Mechanism and Machine Theory*, *45*(10)

Zhao, H., & Bi, S. (2010). Stiffness and stress characteristics of the generalized cross-spring pivot. *Mechanism and Machine Theory*, *45*(3)

Generalized cross spring pivot dimensioning

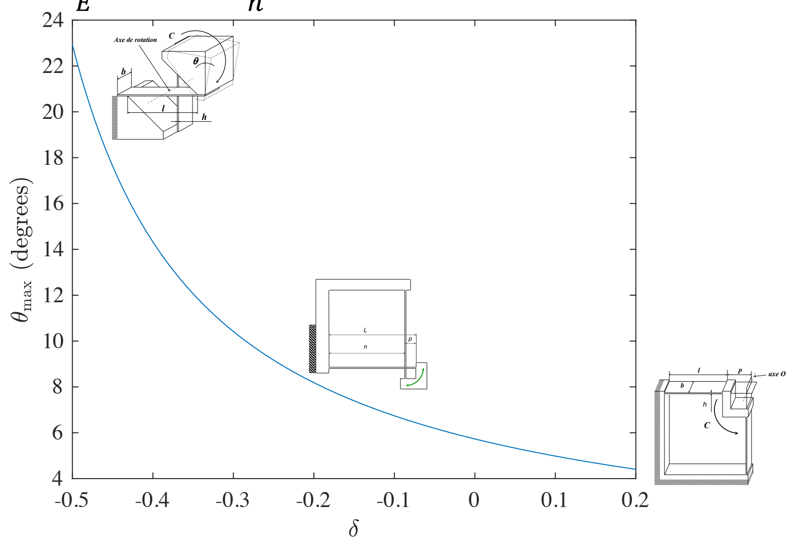
Influence of parameter δ on a cross spring pivot with constant flexure dimensions.

Normalized rotational stiffness $\widetilde{K} = \frac{KL}{EI}$



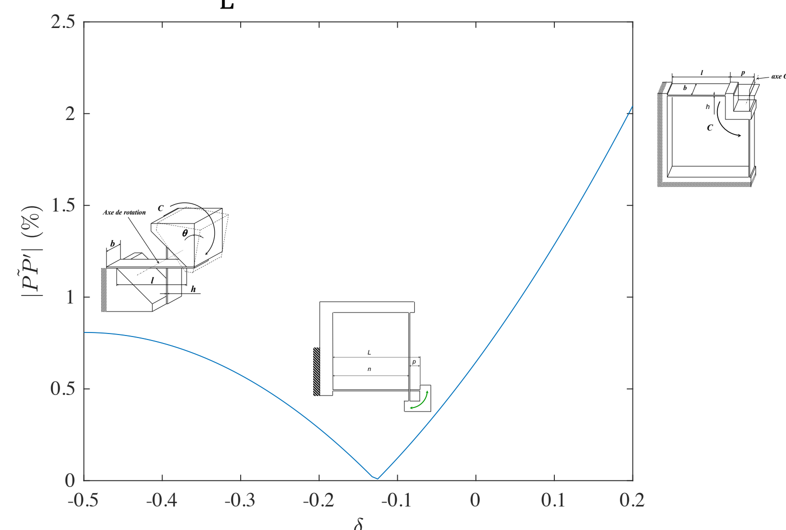
Example of admissible stroke θ_{max}

for $\frac{\sigma}{E} = 0.4\%$, $\frac{L}{h} = 50$

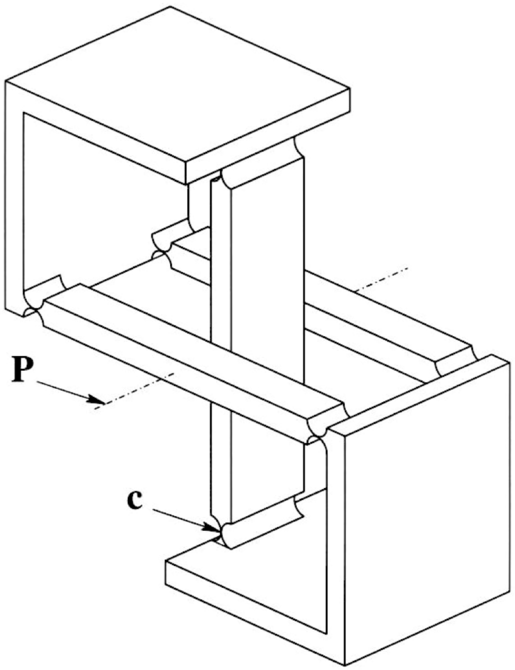


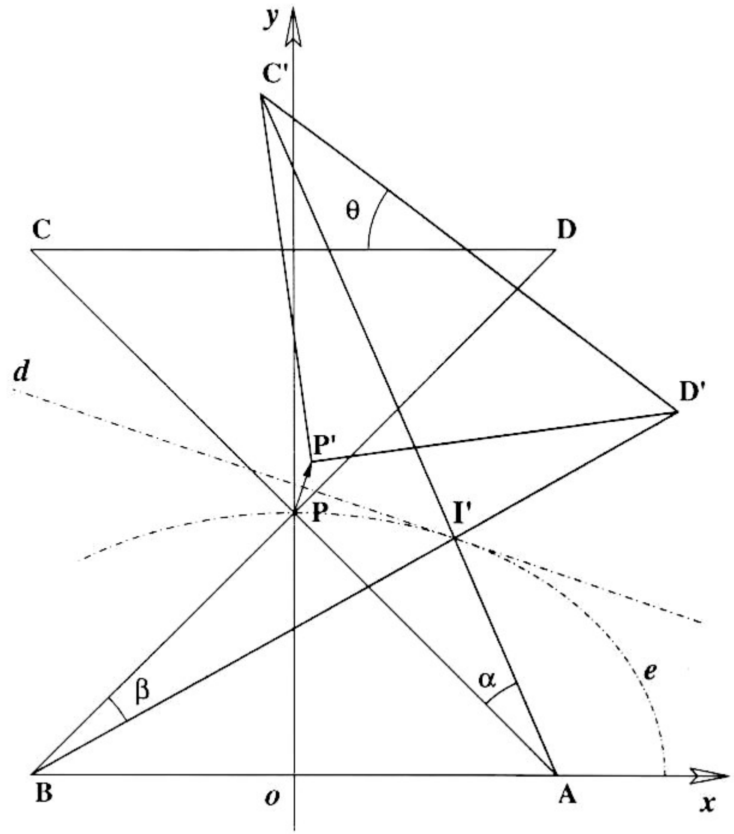
Example of normalized parasitic shift

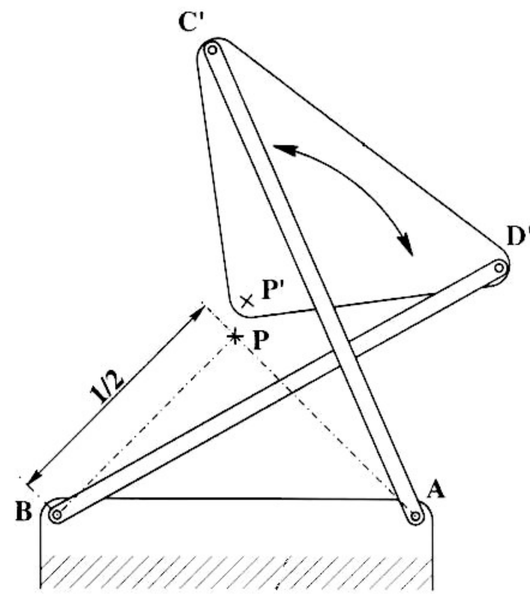
$|\widetilde{PP'}| = \frac{|PP'|}{\mathrm{L}}$ for $\theta = 15°, \alpha = 45°$



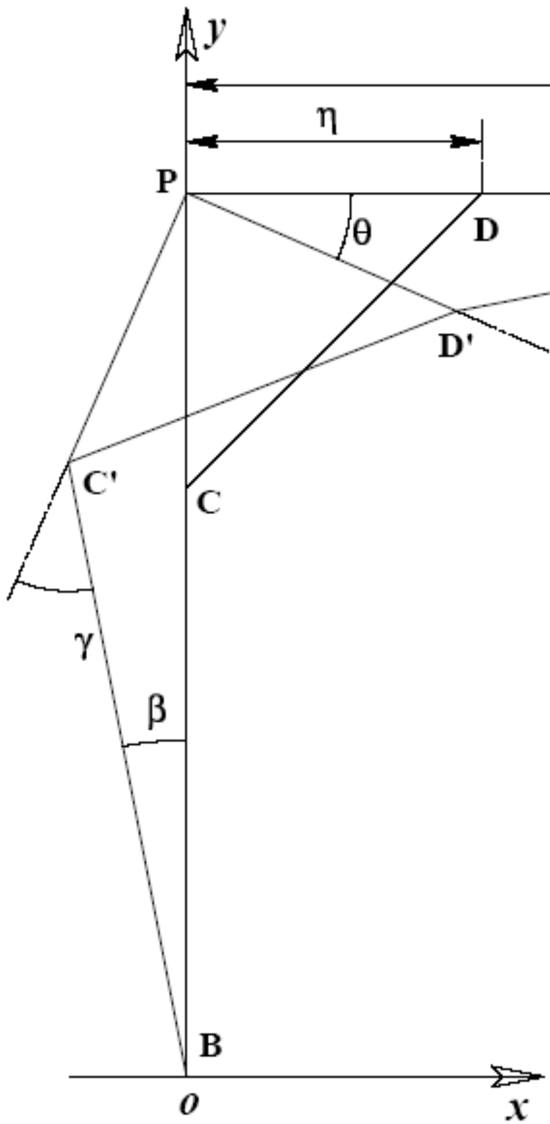
Cross pivot with necked-down hinges

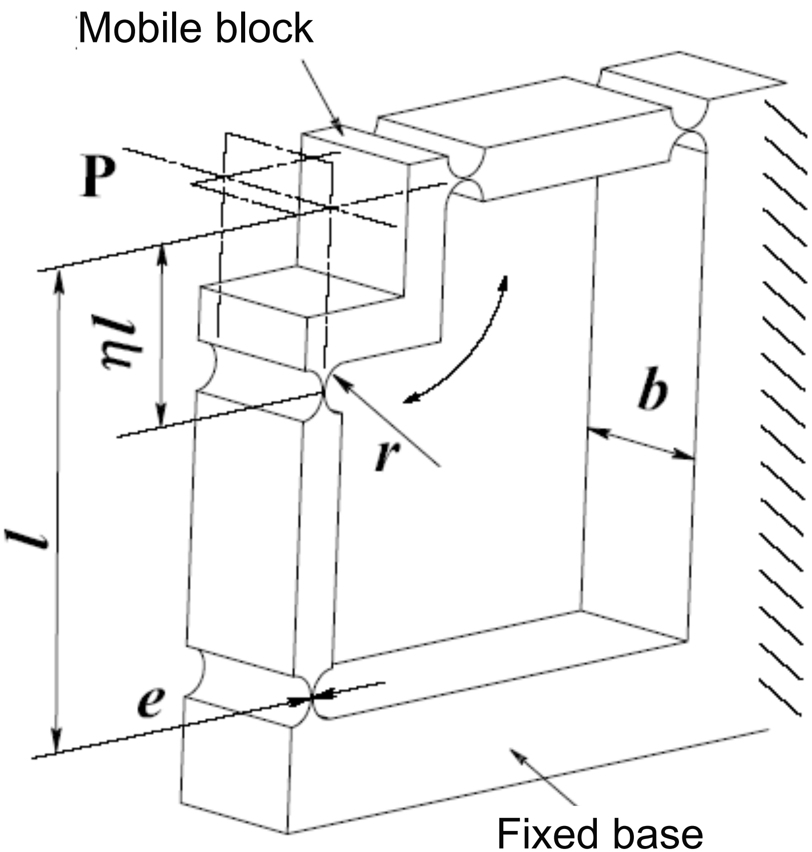




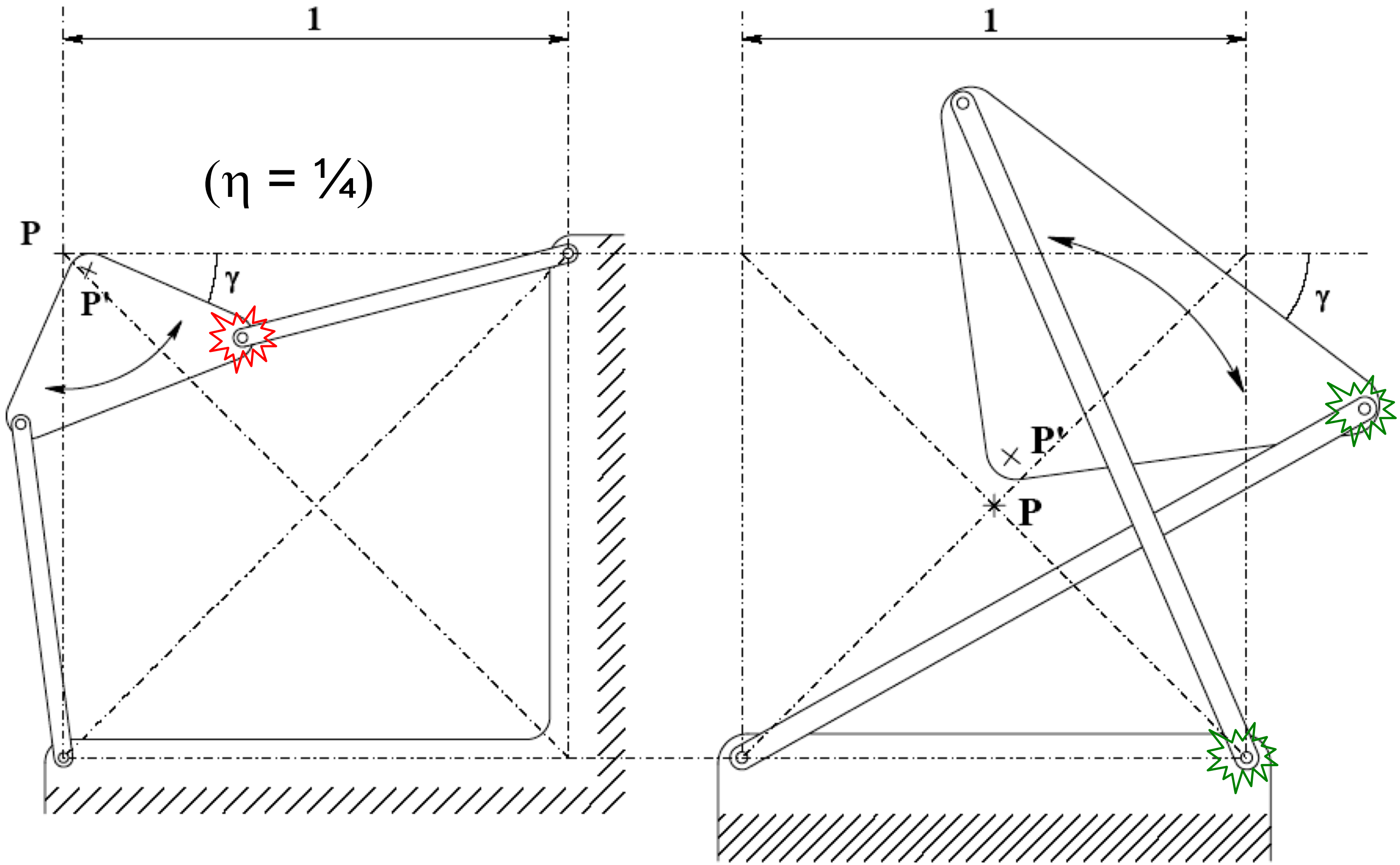


Remote-Center-Compliance pivot with hinges





Comparison of RCC and Cross pivots



☹ Max pivot rotation > γ

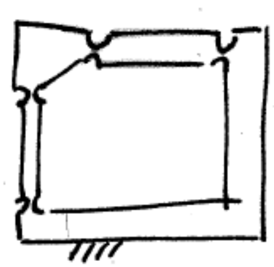
☺ Max pivot rotation ≃ γ / 2

😐 Parasitic shifts [PP'] are approx. equal for both

Rotation bearings : summary

- Cross-spring pivots
  - Separated blades (3D, long stroke)
  - Un-separated blades (2D, short stroke)

- Remote-Centre-Compliance Pivots



- Parasitic shift
- Torsion bars