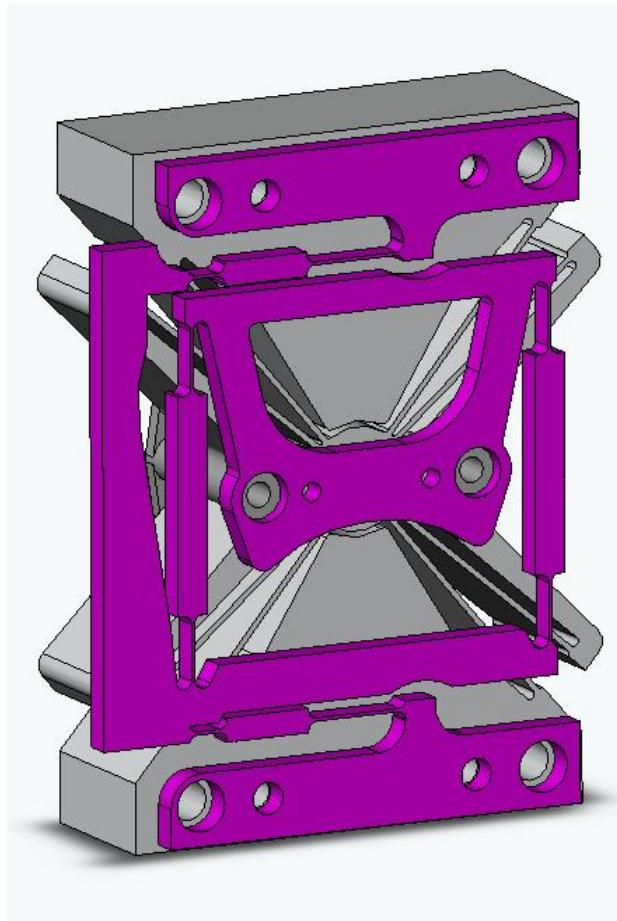


# FLEXURE-MECHANISM DESIGN PRINCIPLES

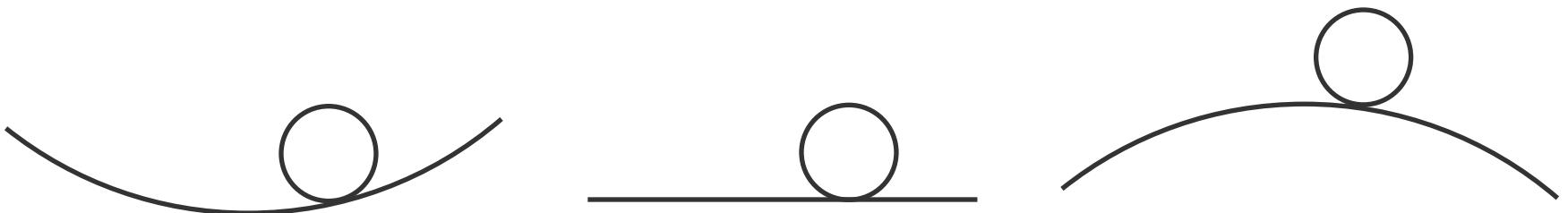


Prof. Simon Henein, Dr. Etienne Thalmann

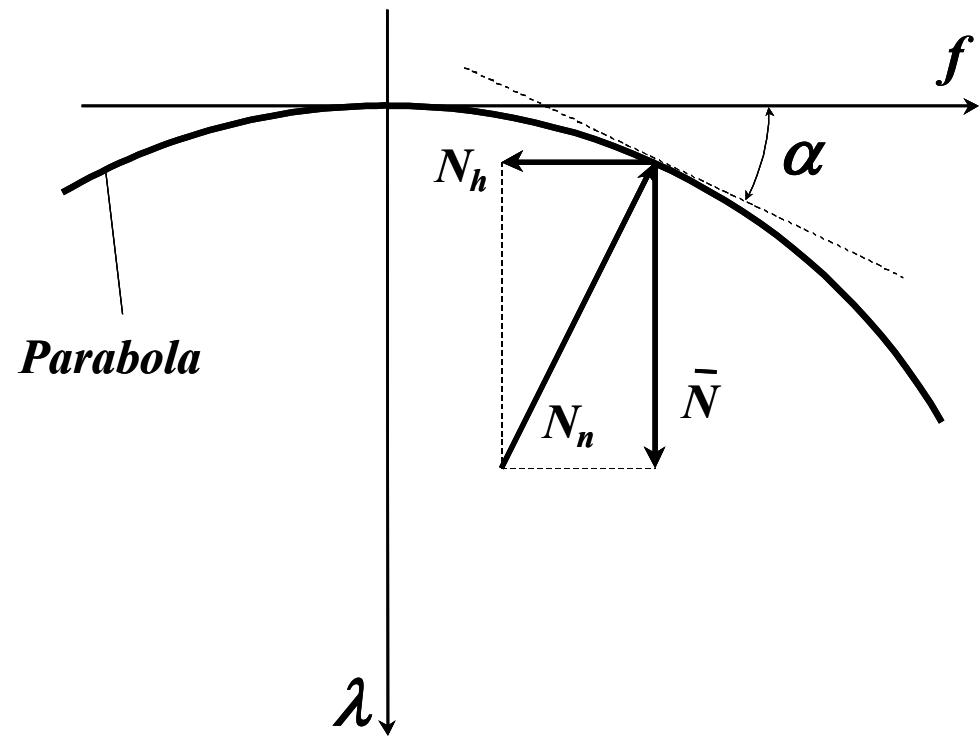
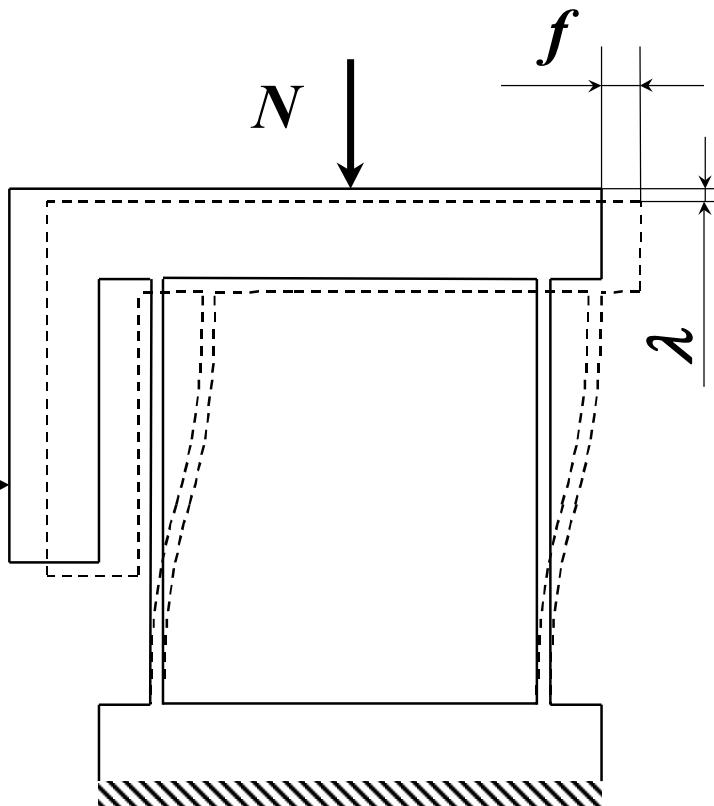
# Contents

- Flexure-based mechanisms
  - Stiffness compensation and bistability
  - Rectilinear & circular mechanisms
  - Flexures with large reduction factors
  - Elastic energy storage maximization
  - Examples of flexure-based parallel robots
- Prospects & conclusion

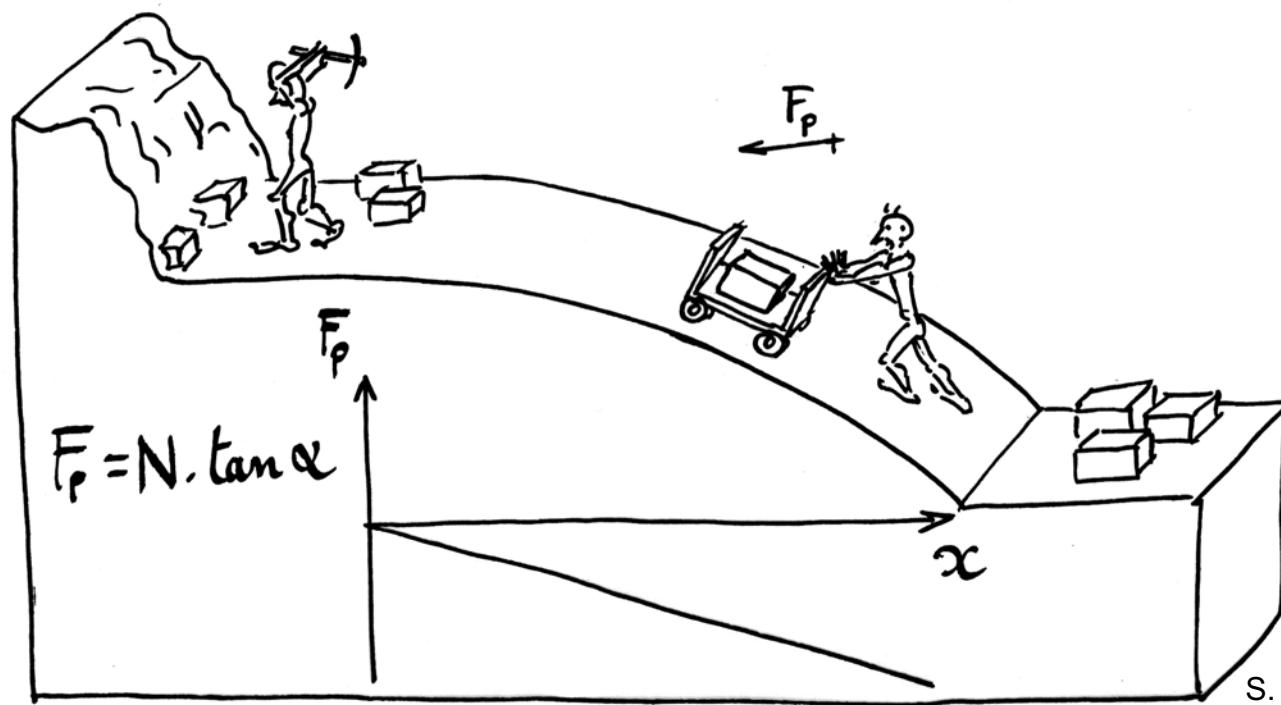
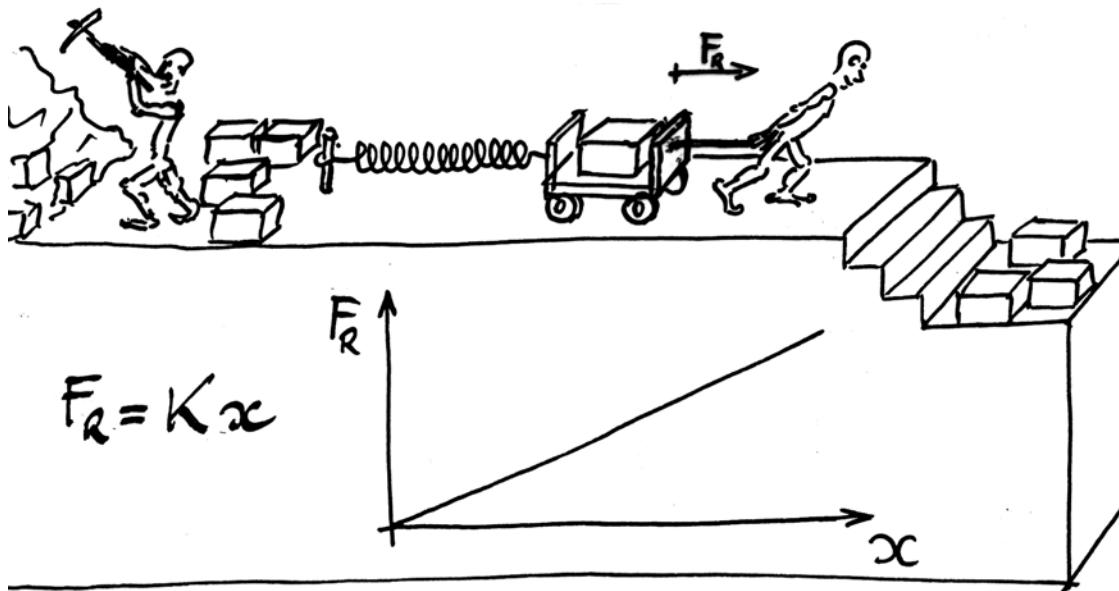
# **Stiffness compensation and bi-stability**



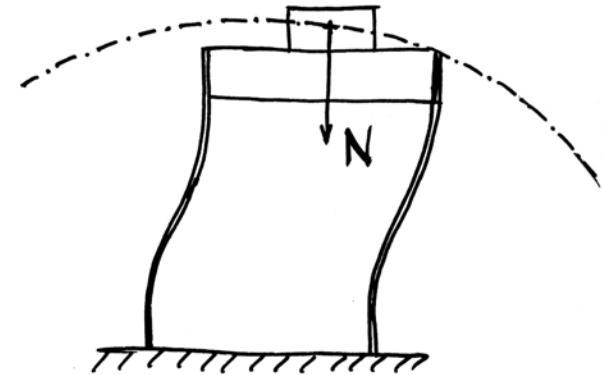
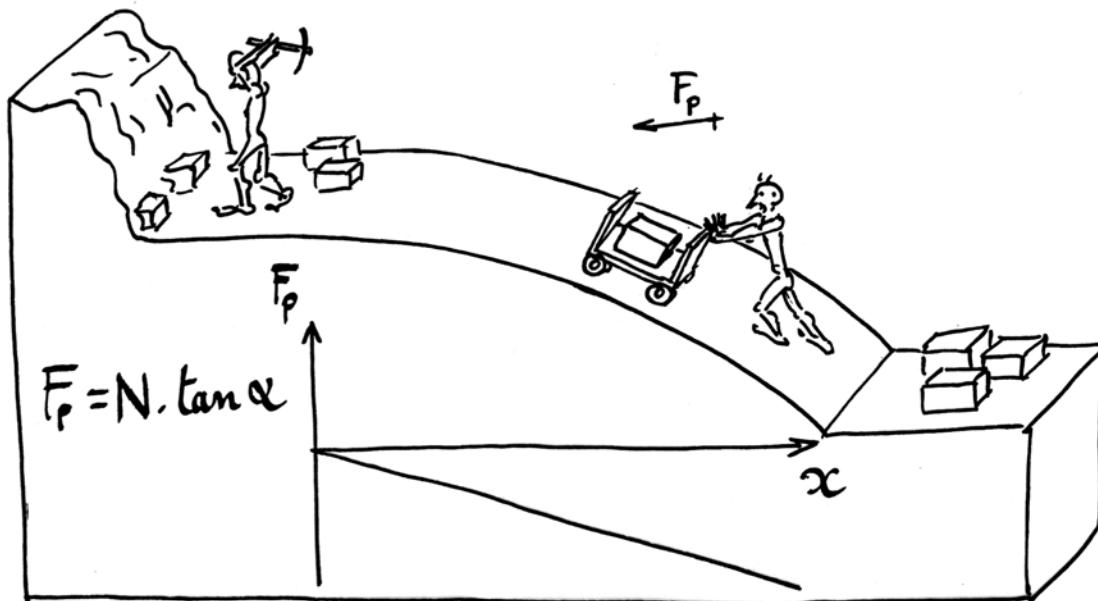
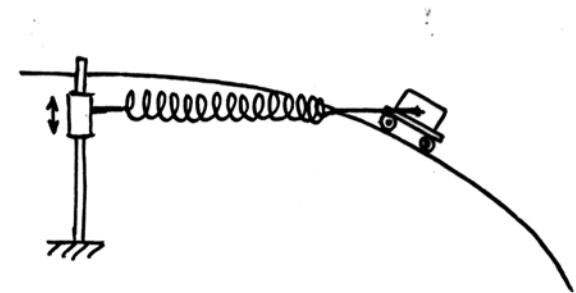
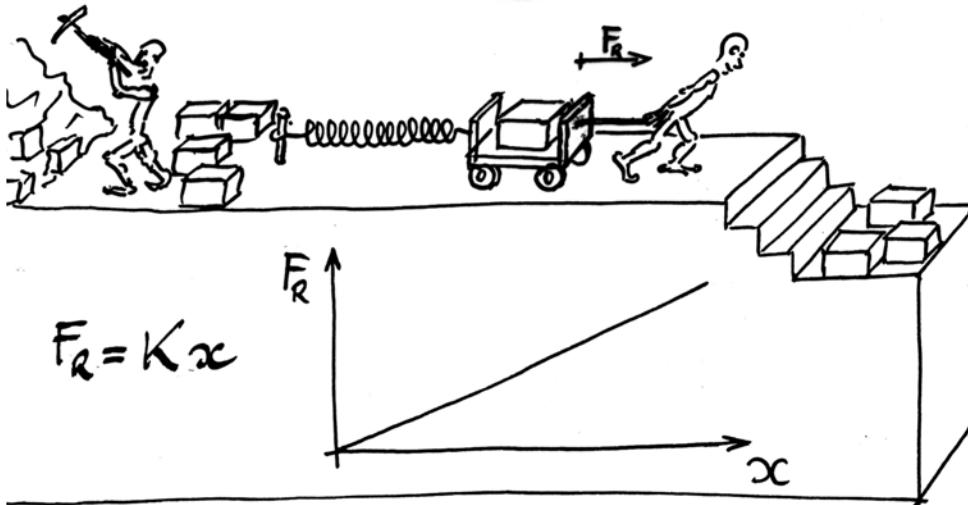
# Parallel spring stage with transverse load



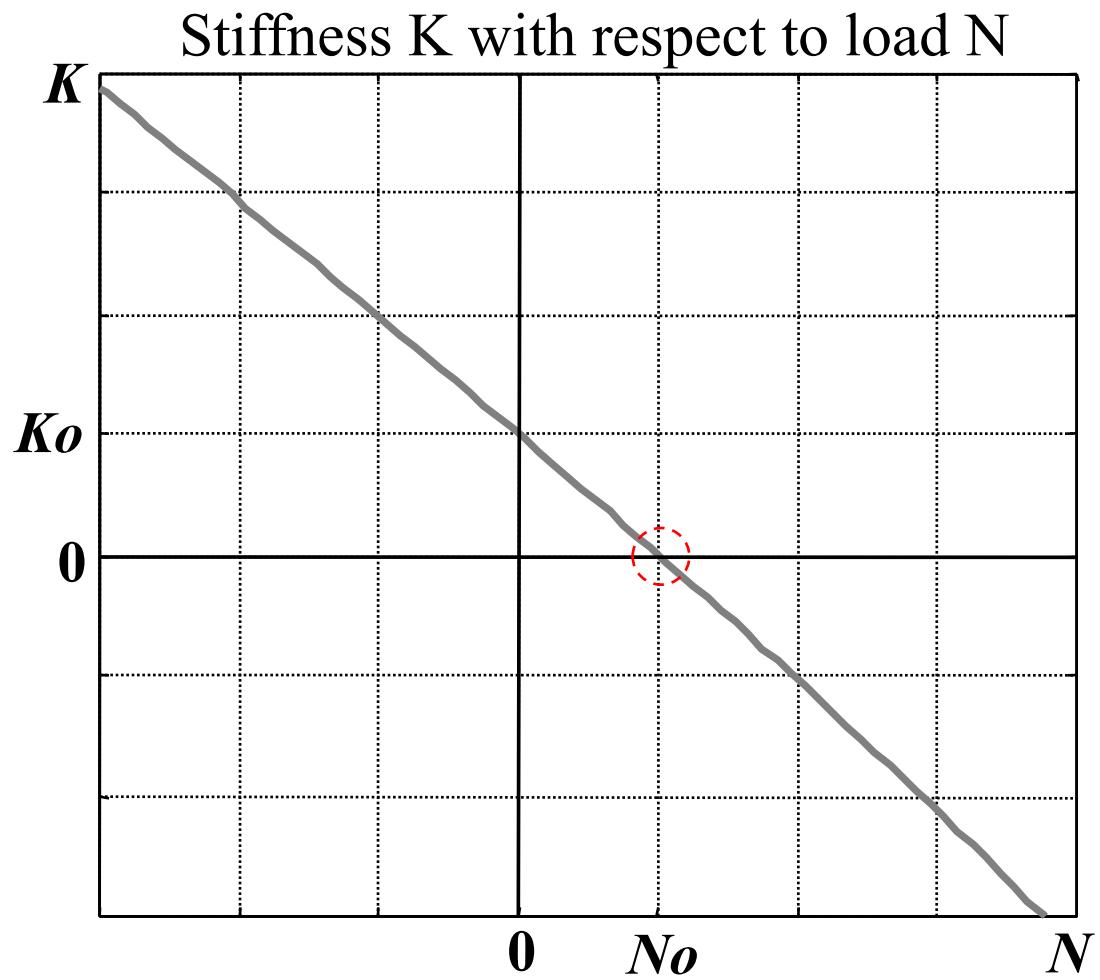
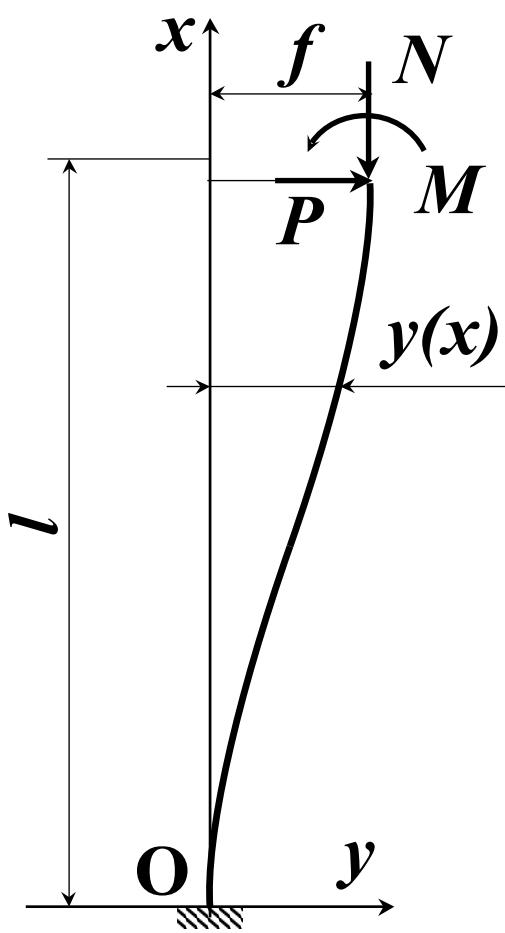
The stiffness  $K = \frac{P}{f}$  depends on the load  $N$

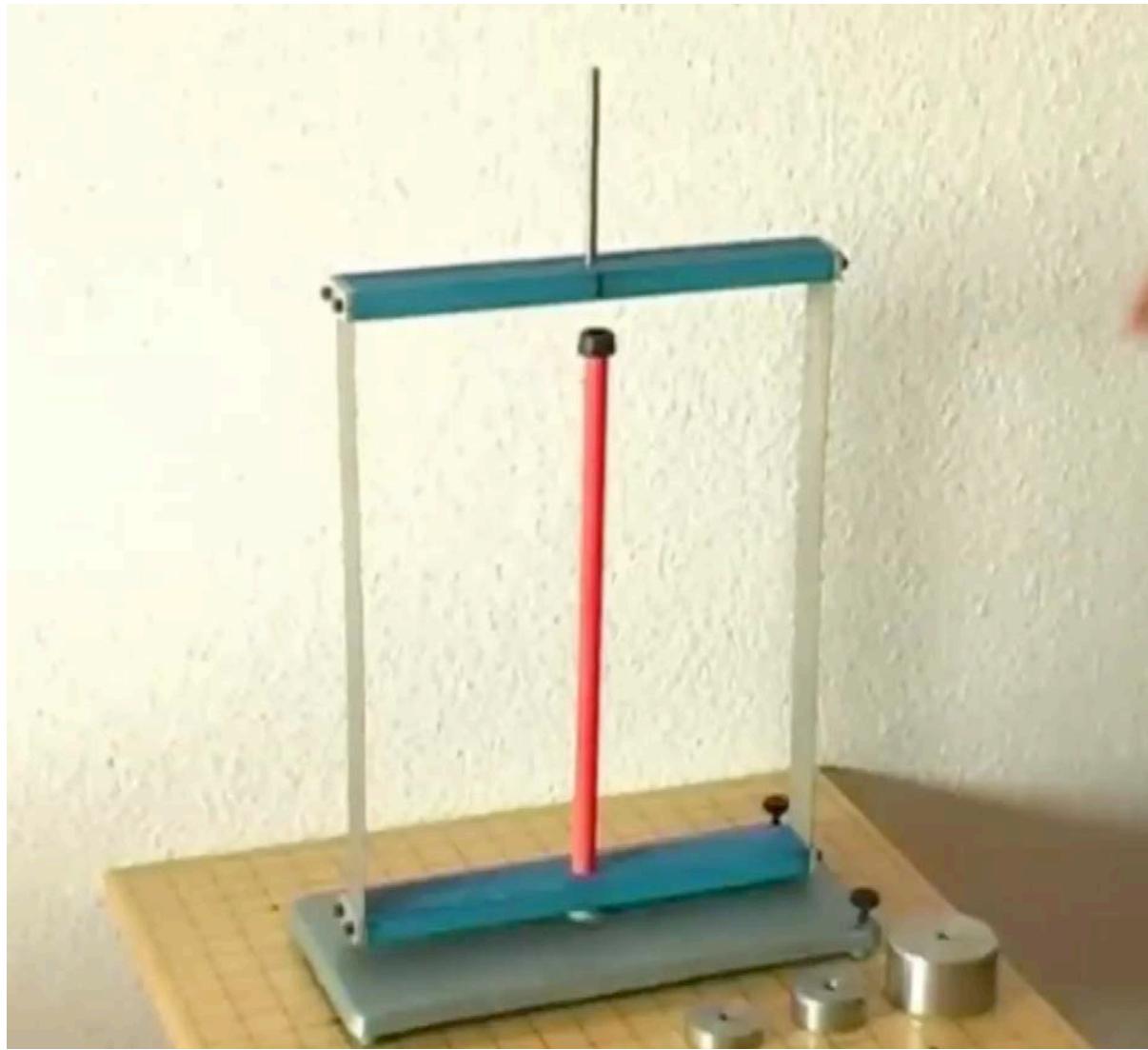


# Zero stiffness flexure

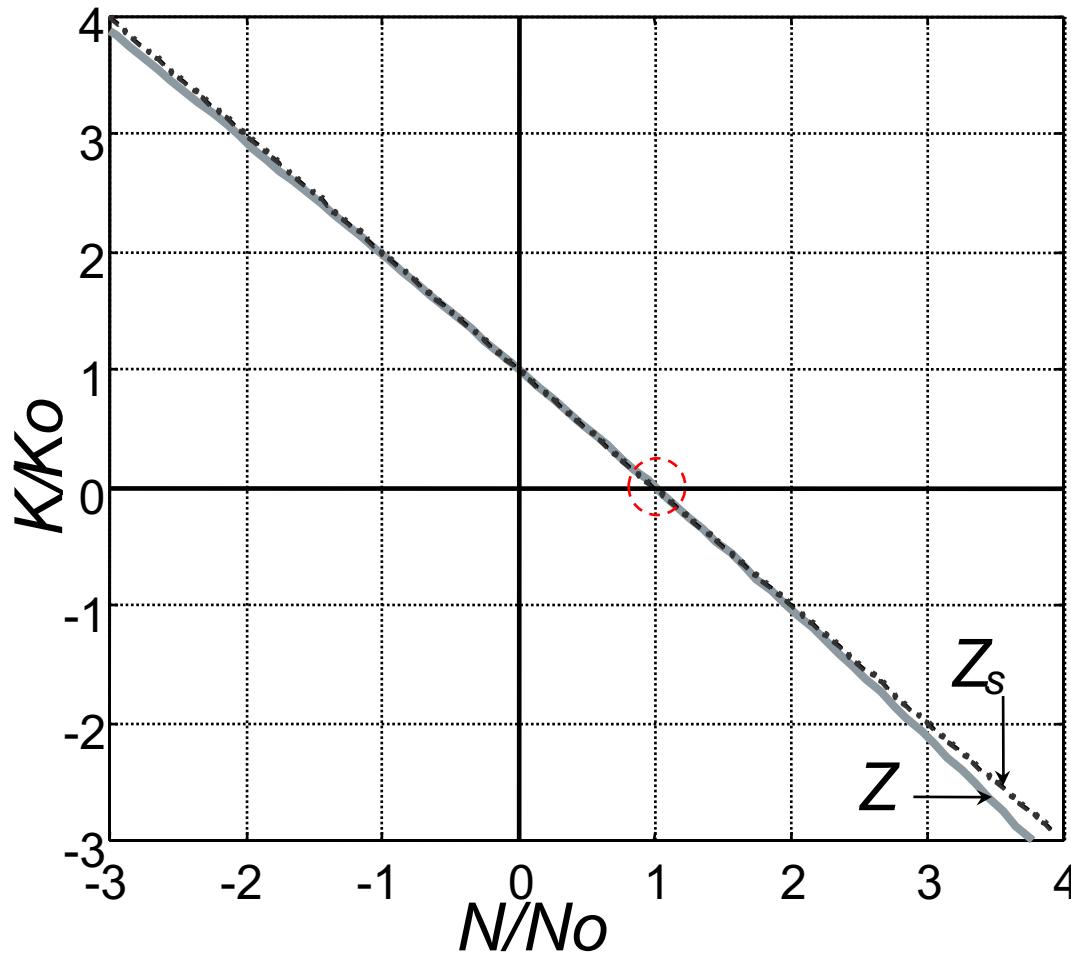


$$K = \frac{N}{\frac{2}{S} \tan \frac{Sl}{2} - l} \quad \text{with} \quad S = \sqrt{\frac{N}{EI}}$$





# Normalized stiffness versus normalized load (for 2 blades)



$$N_c = 4N_o$$

$$K \approx K_0 - \frac{K_0}{N_0} N$$

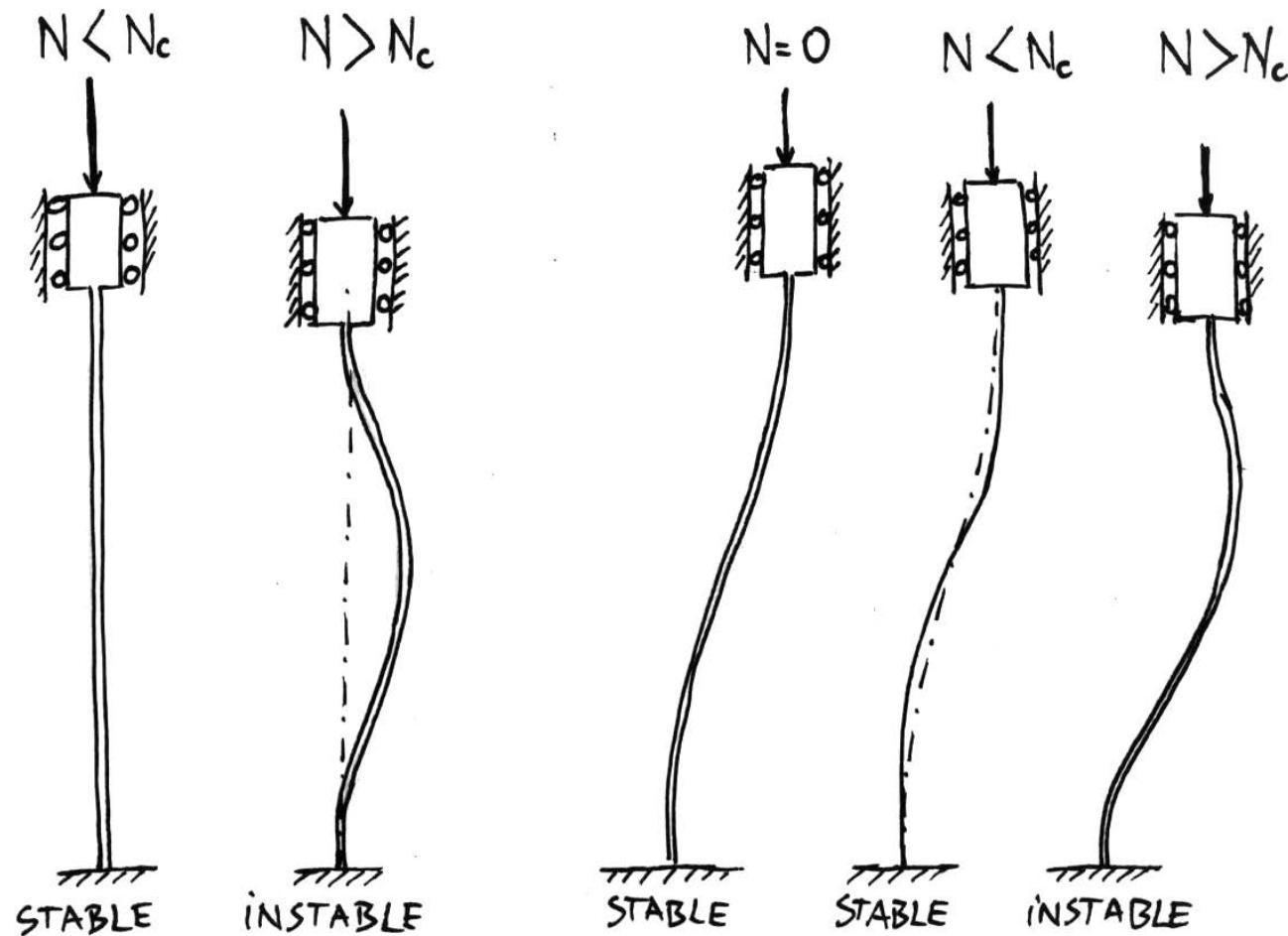
with

$$K_0 = \frac{24EI}{l^3}$$

and

$$N_0 = \frac{2\pi^2 EI}{l^2}$$

# Buckling of a single blade

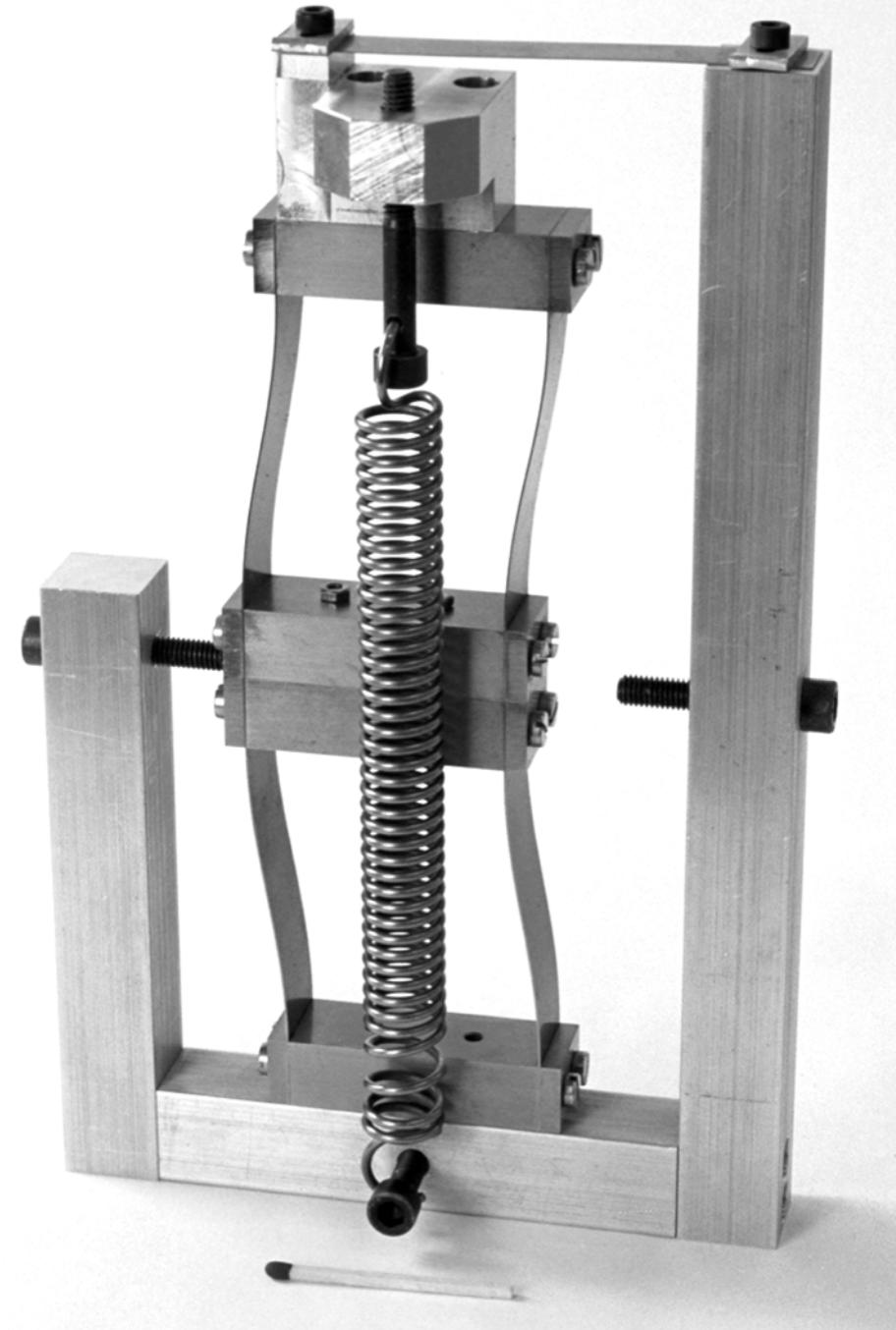


Critical Load:

$$N_c = \frac{4 \pi^2 EI}{l^2}$$

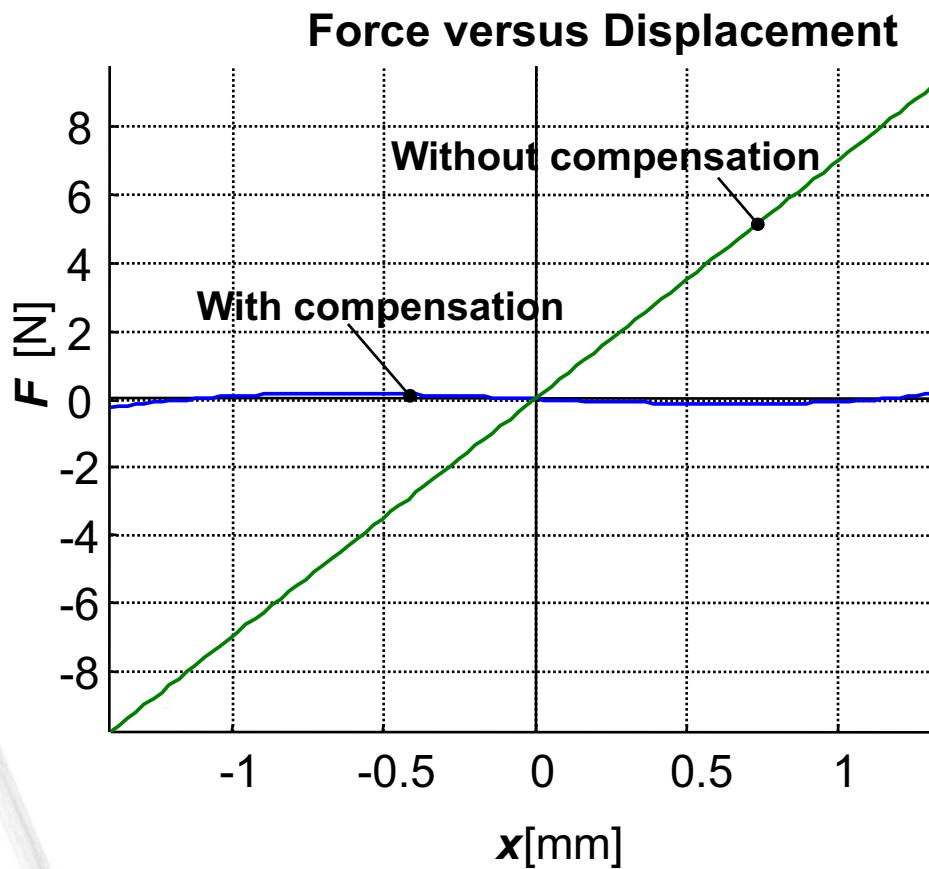
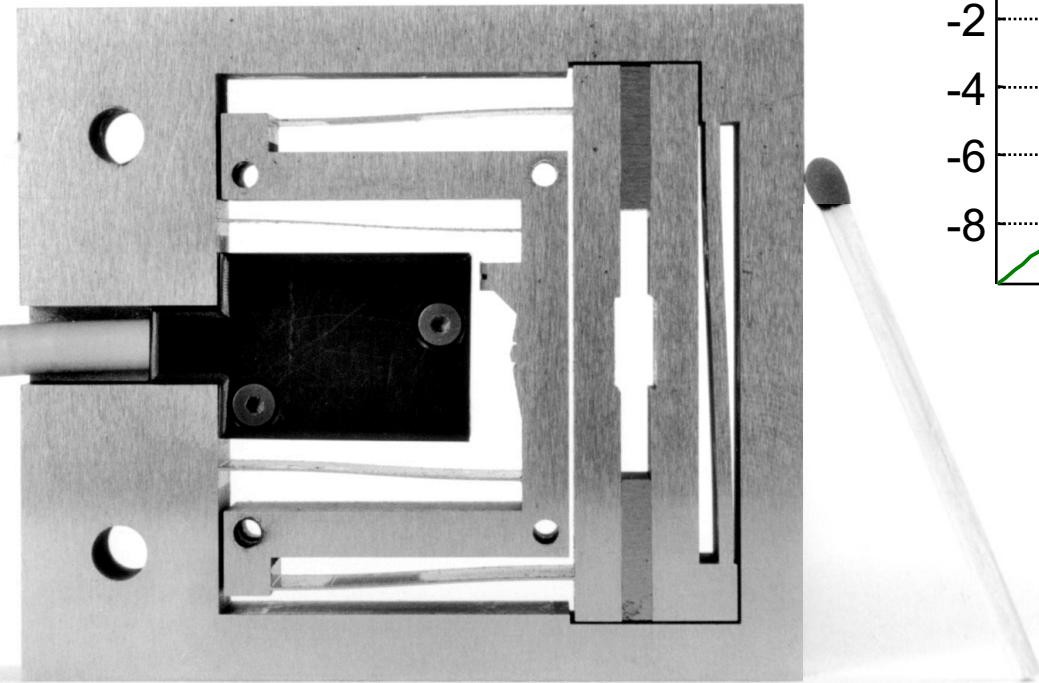
# Tunable stiffness translation bearing

Demonstrator

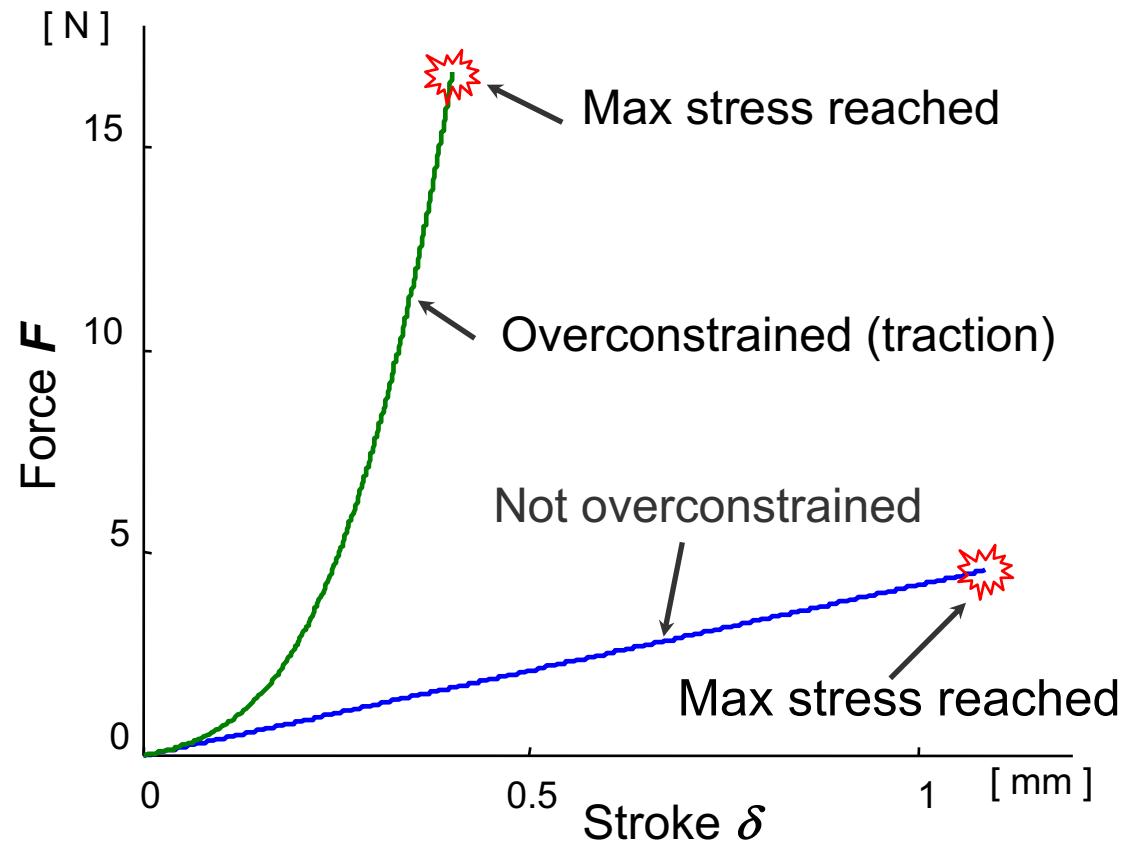
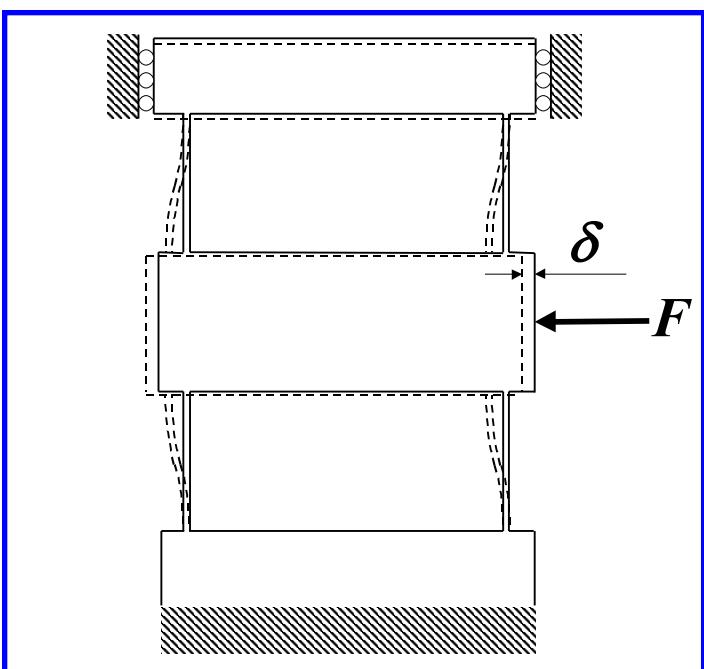
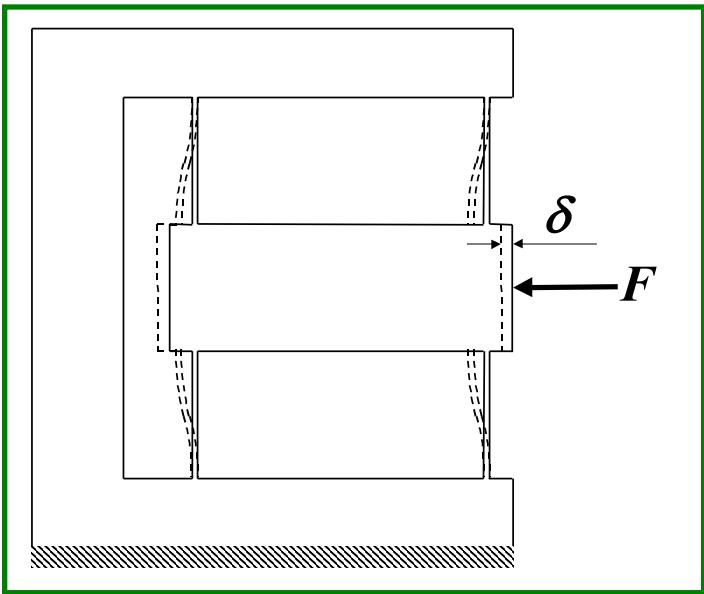


# Tunable stiffness translation bearing

Monolithic design



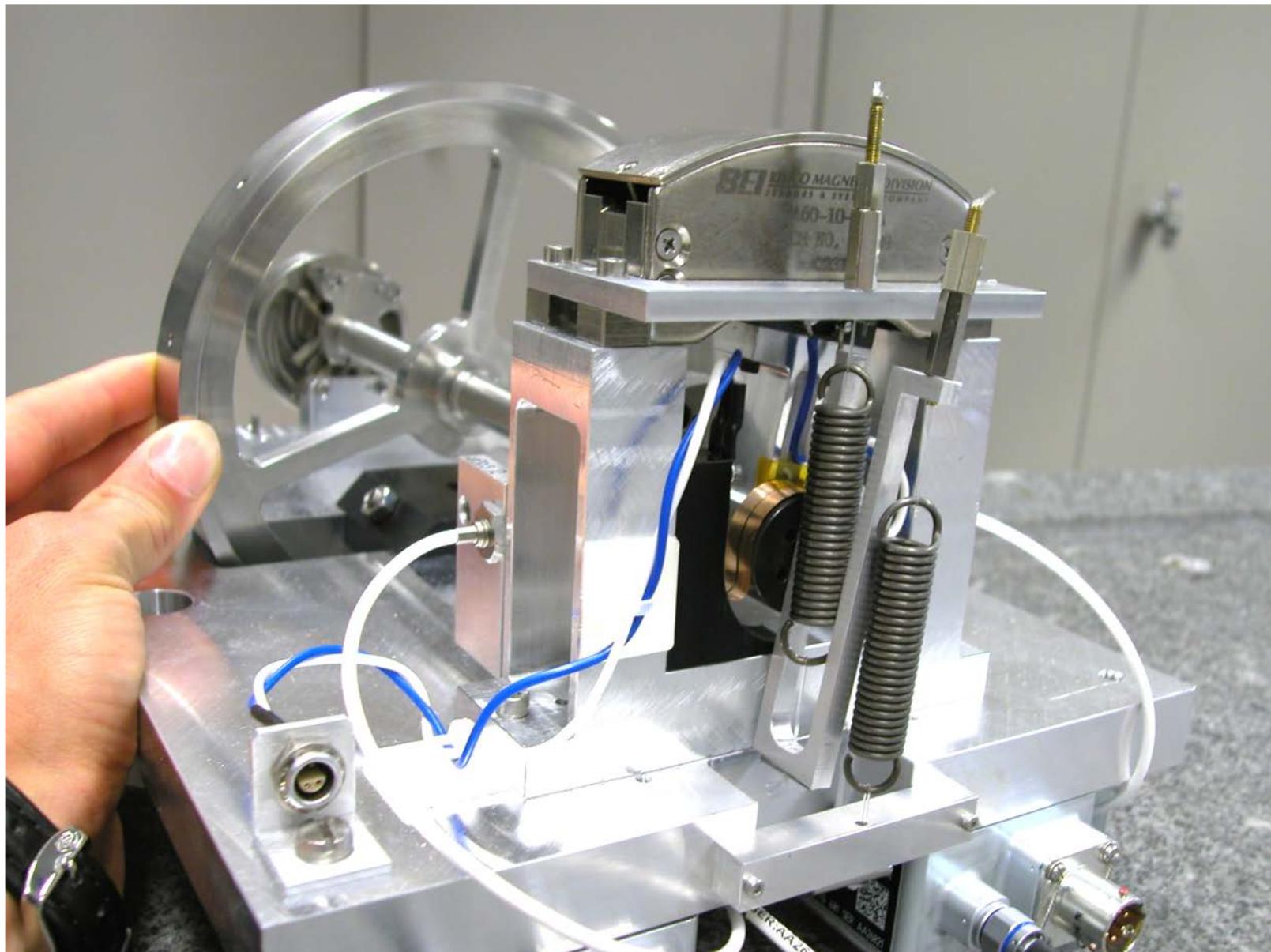
# Overconstrained linear stage (4 blades)



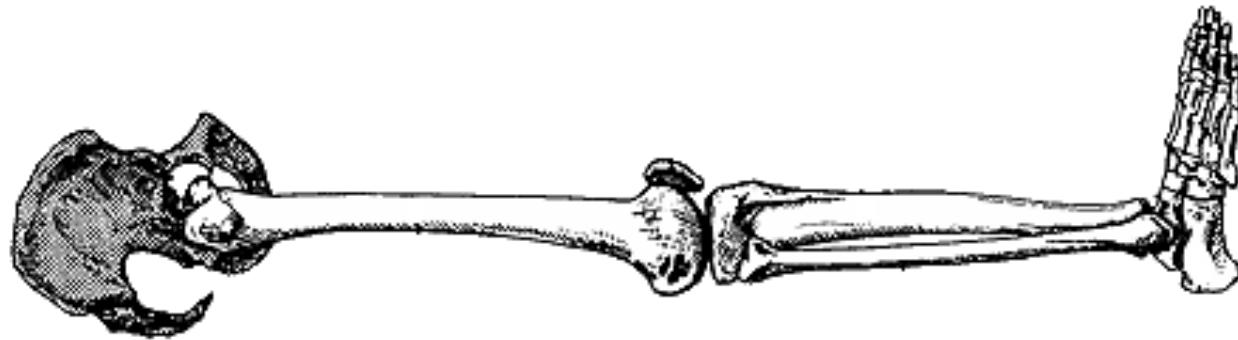
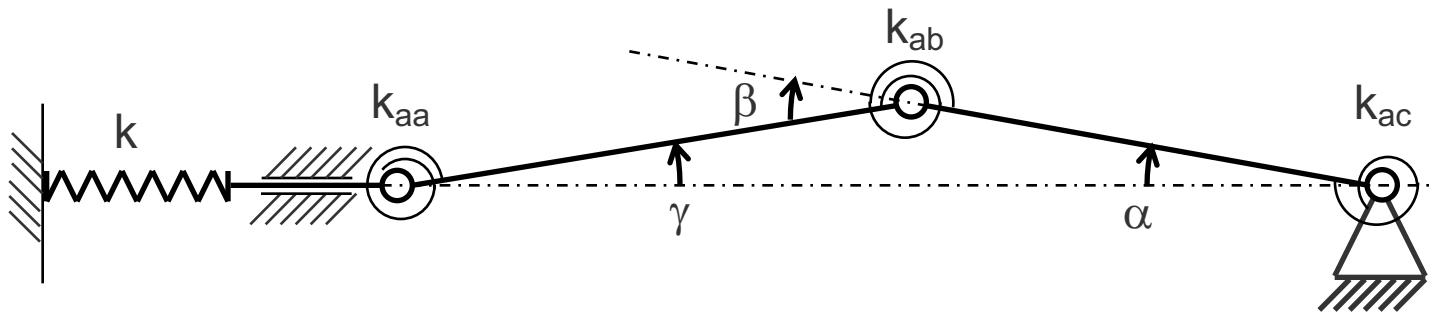
Overconstraints lead to:

- :(sad face) • Significantly reduced stroke
- :(sad face) • Poorly controlled stiffness
- :(sad face) • Amplified forces on the structures

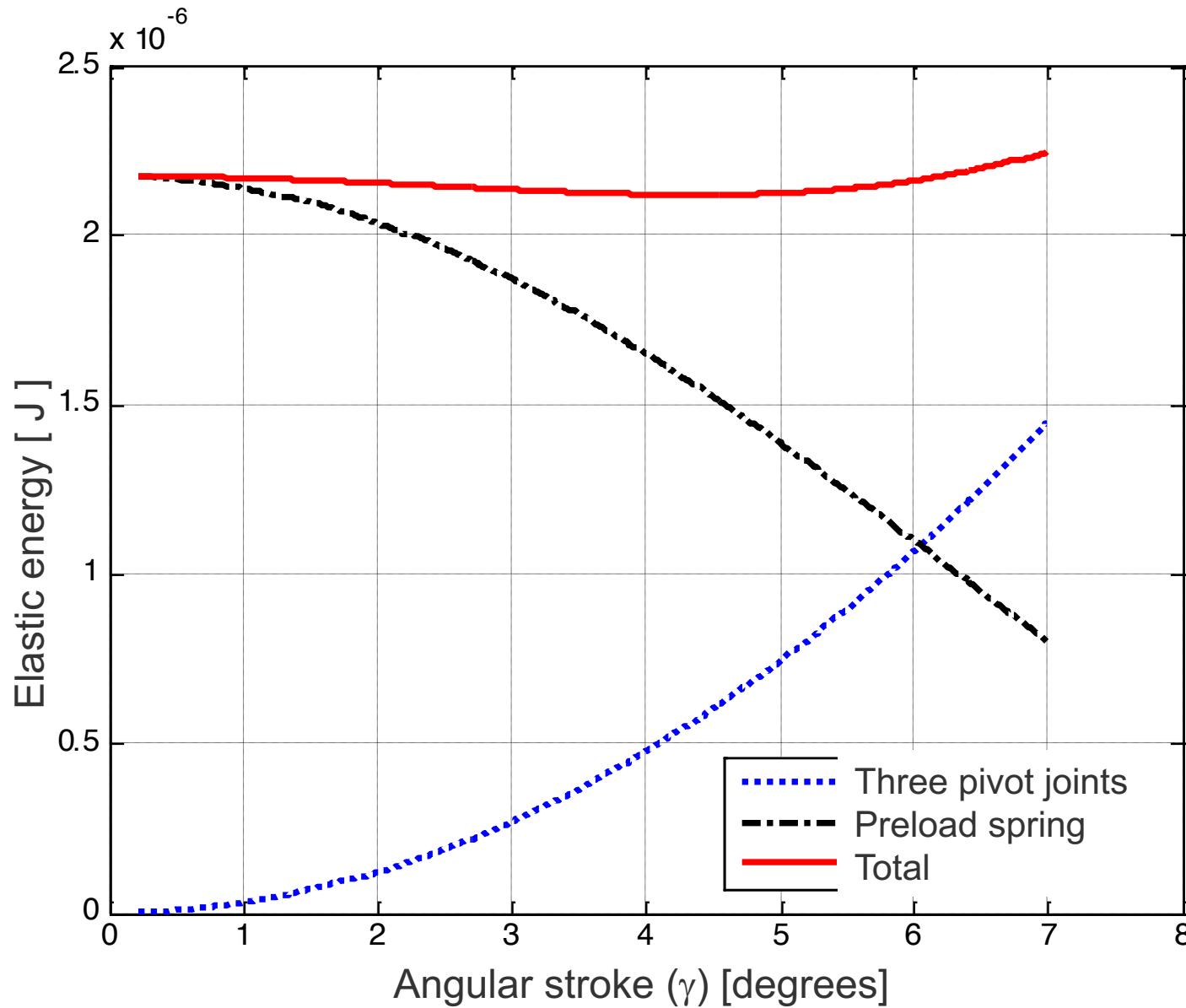
# Tunable stiffness pivot assembly



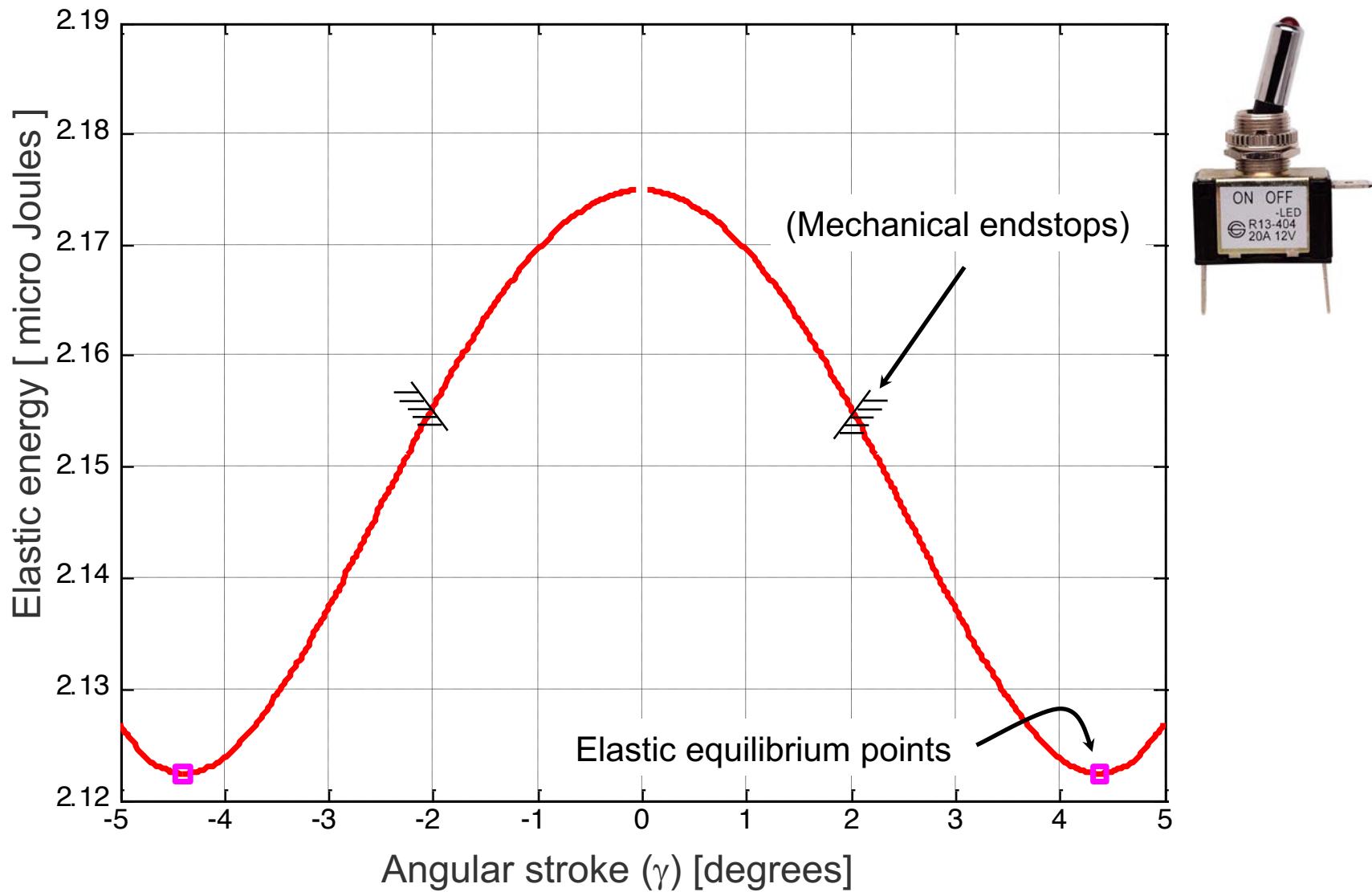
# Leg 2D kinematics (Toggle joint)



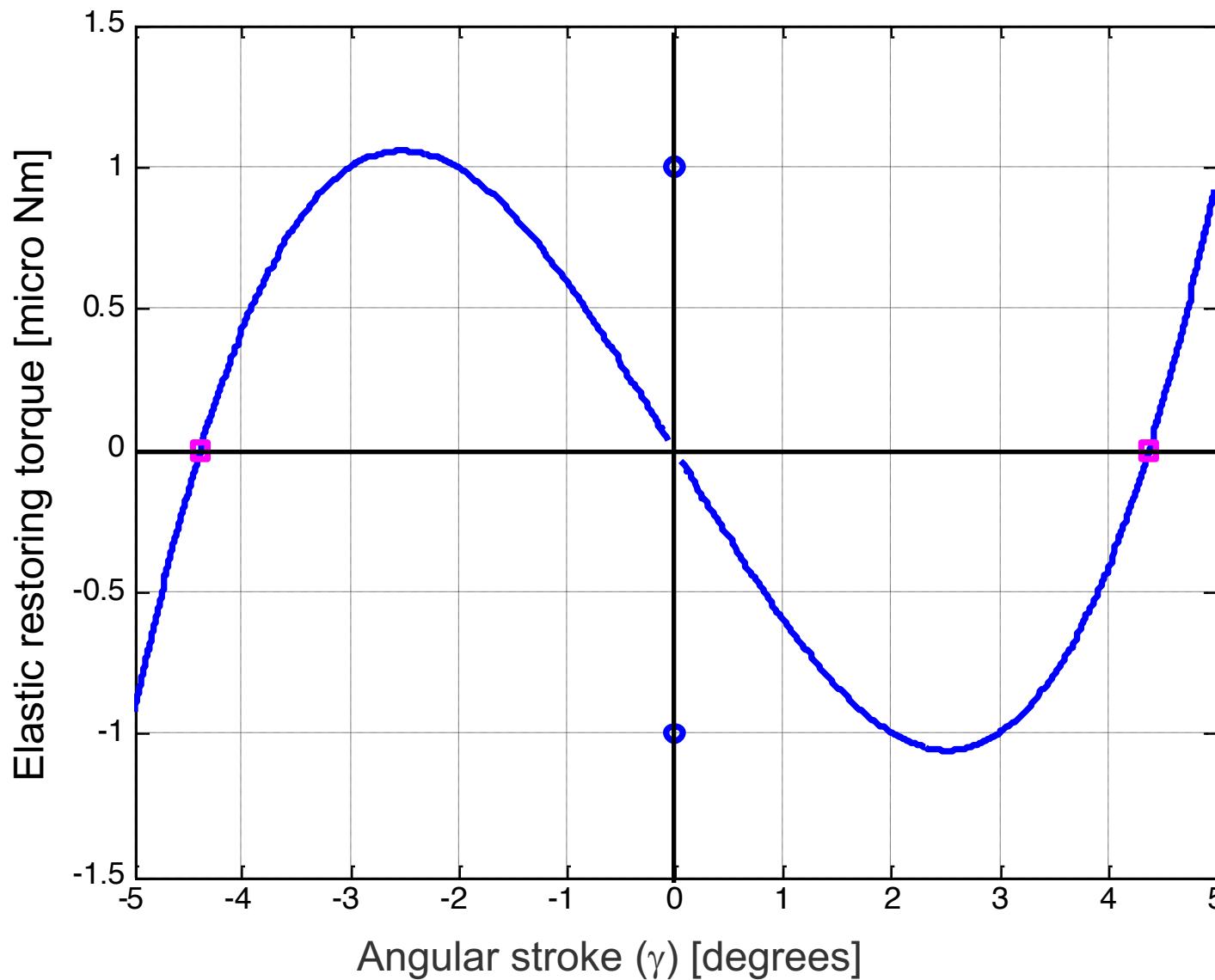
# Sum of elastic energies



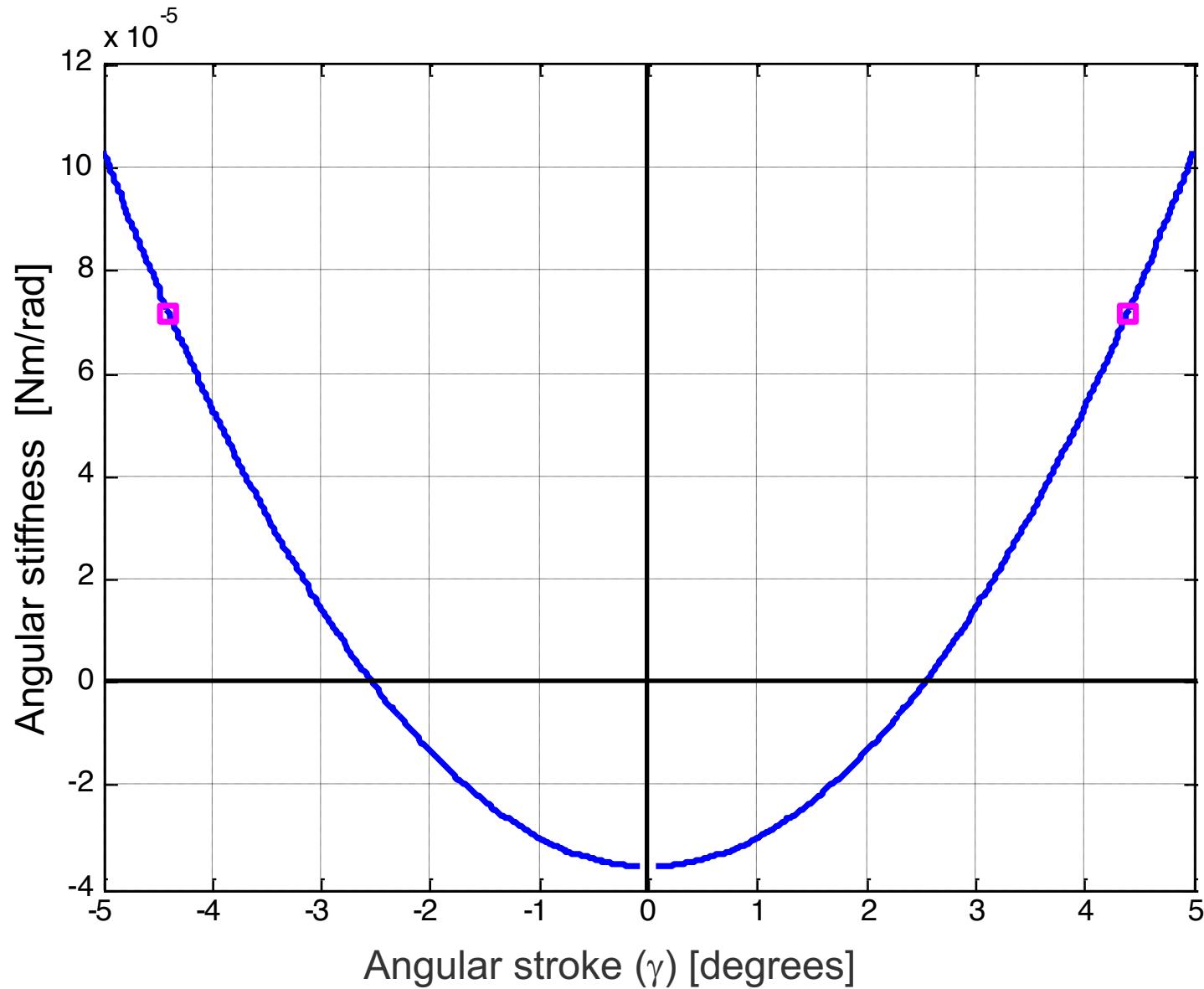
# Total elastic energy



# Elastic restoring torque



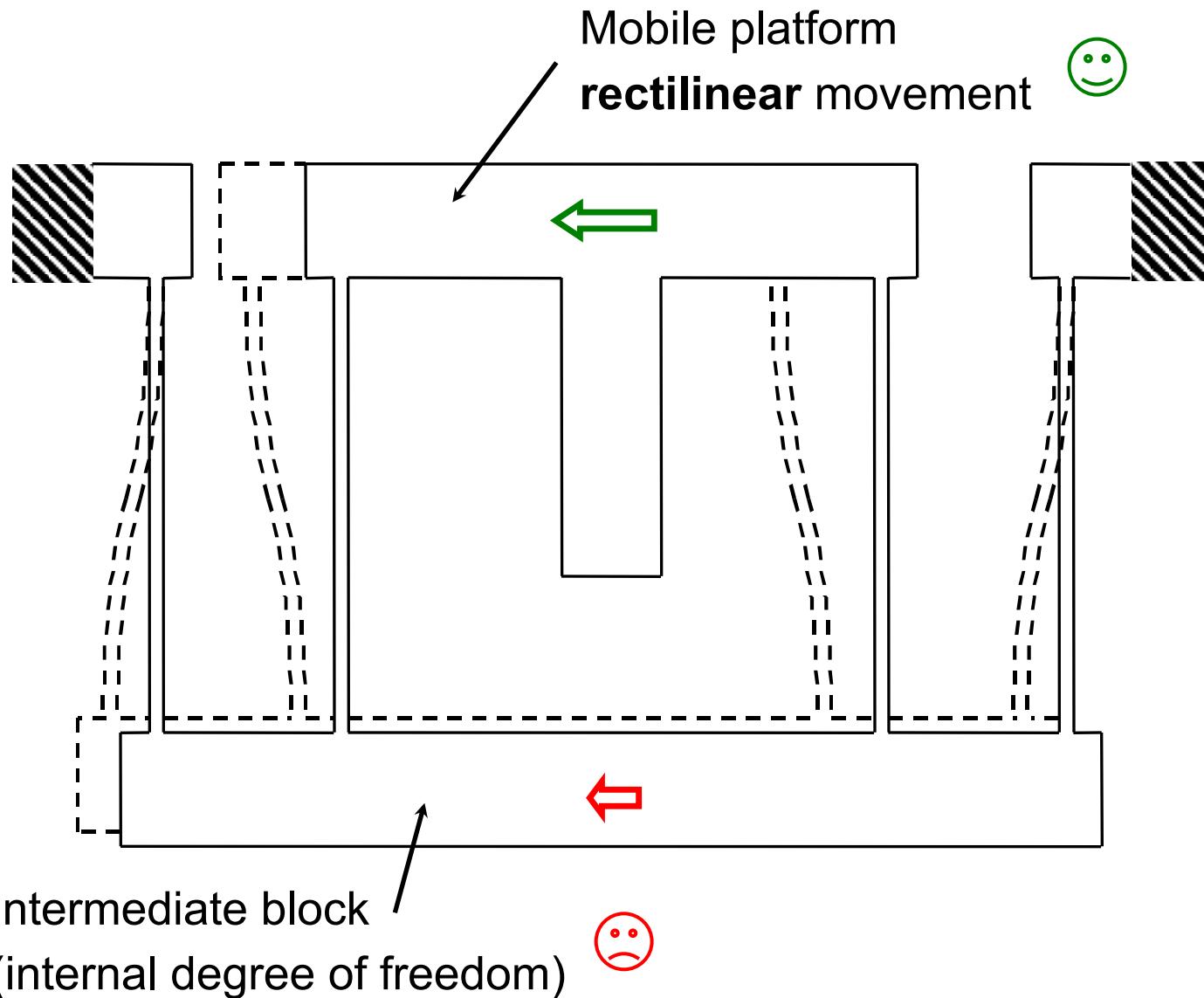
# Angular stiffness



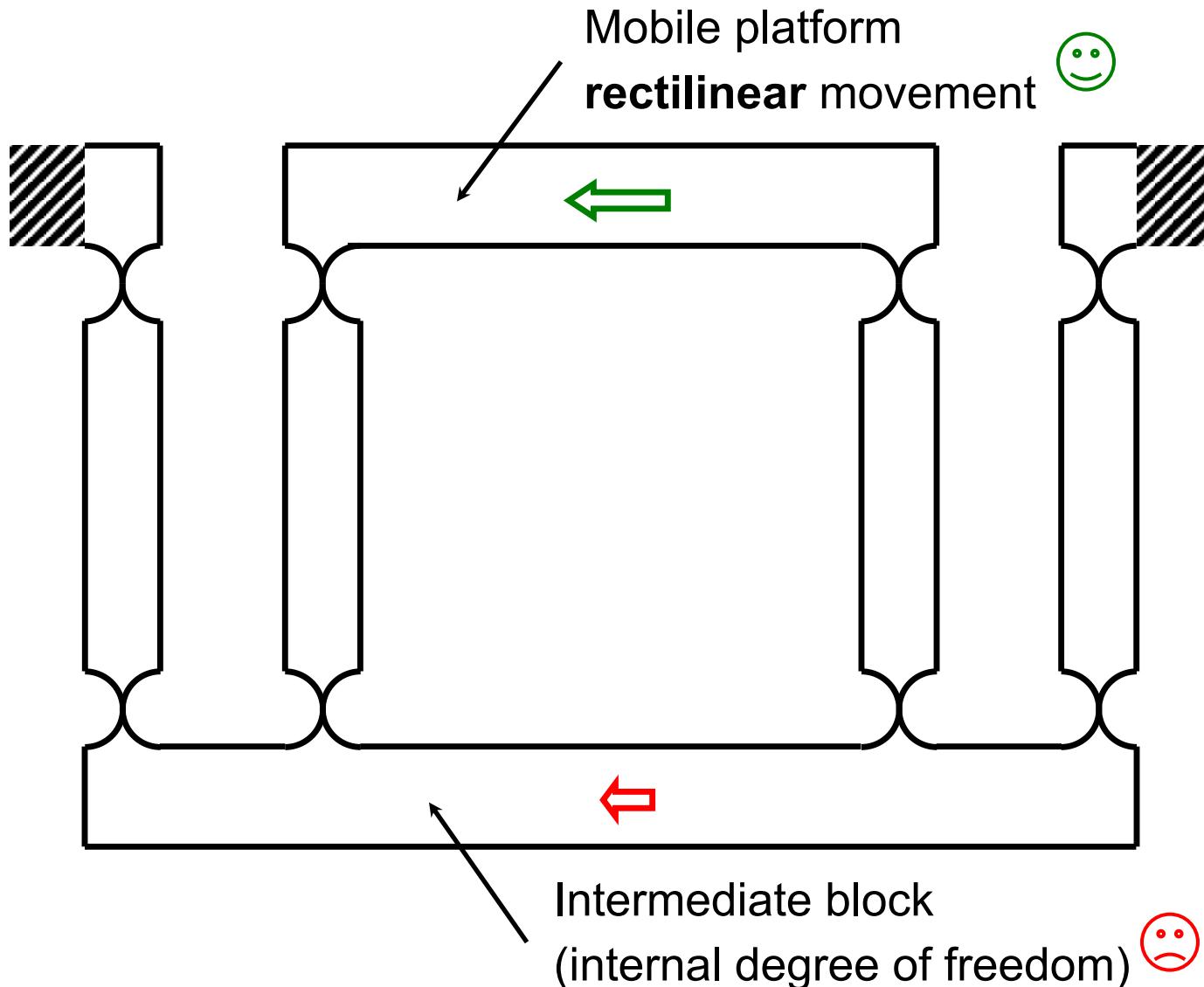
# Rectilinear and circular mechanisms



# Compound parallel spring stage (2 DOFs)

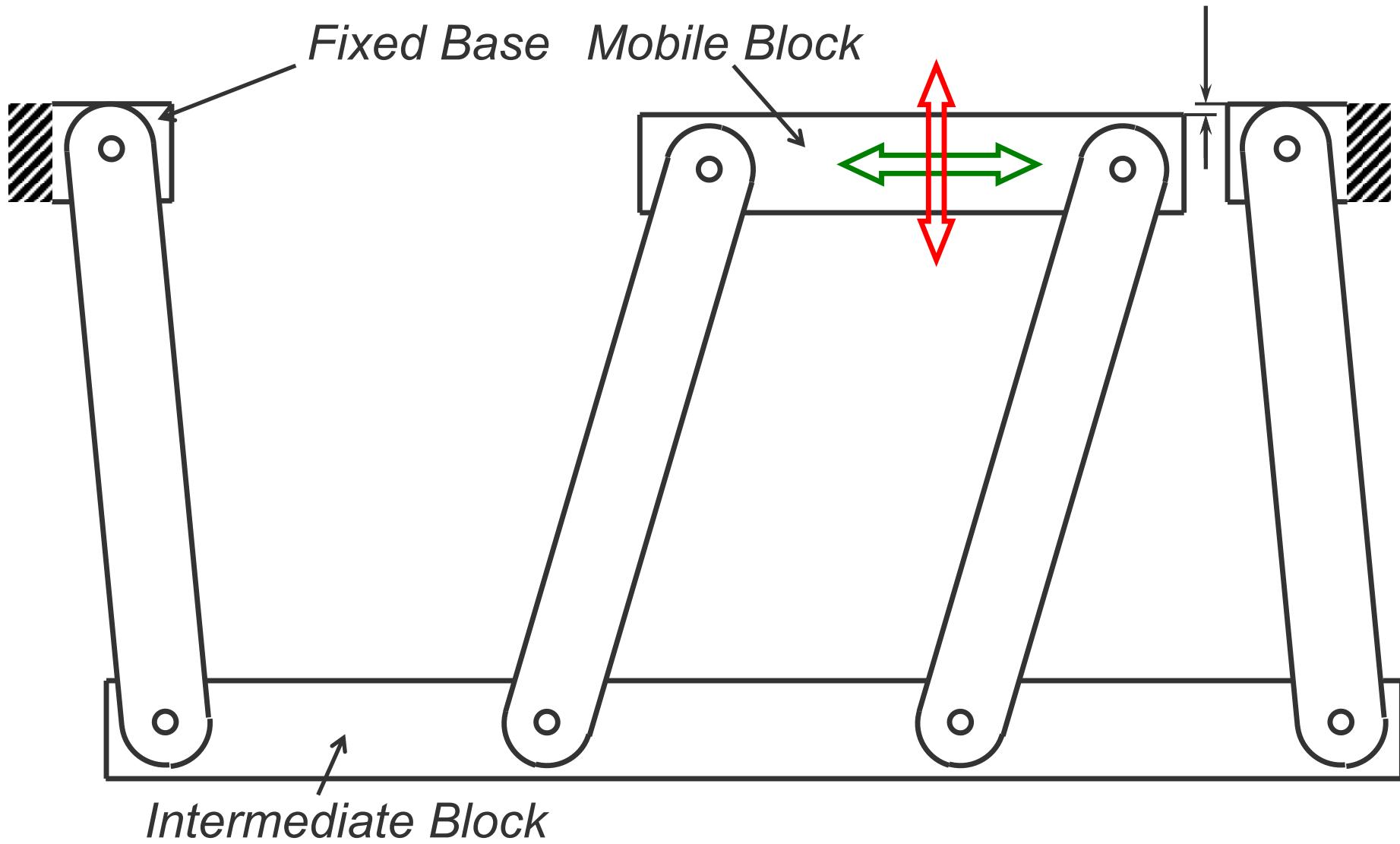


# Compound parallel spring stage (2 DOFs)

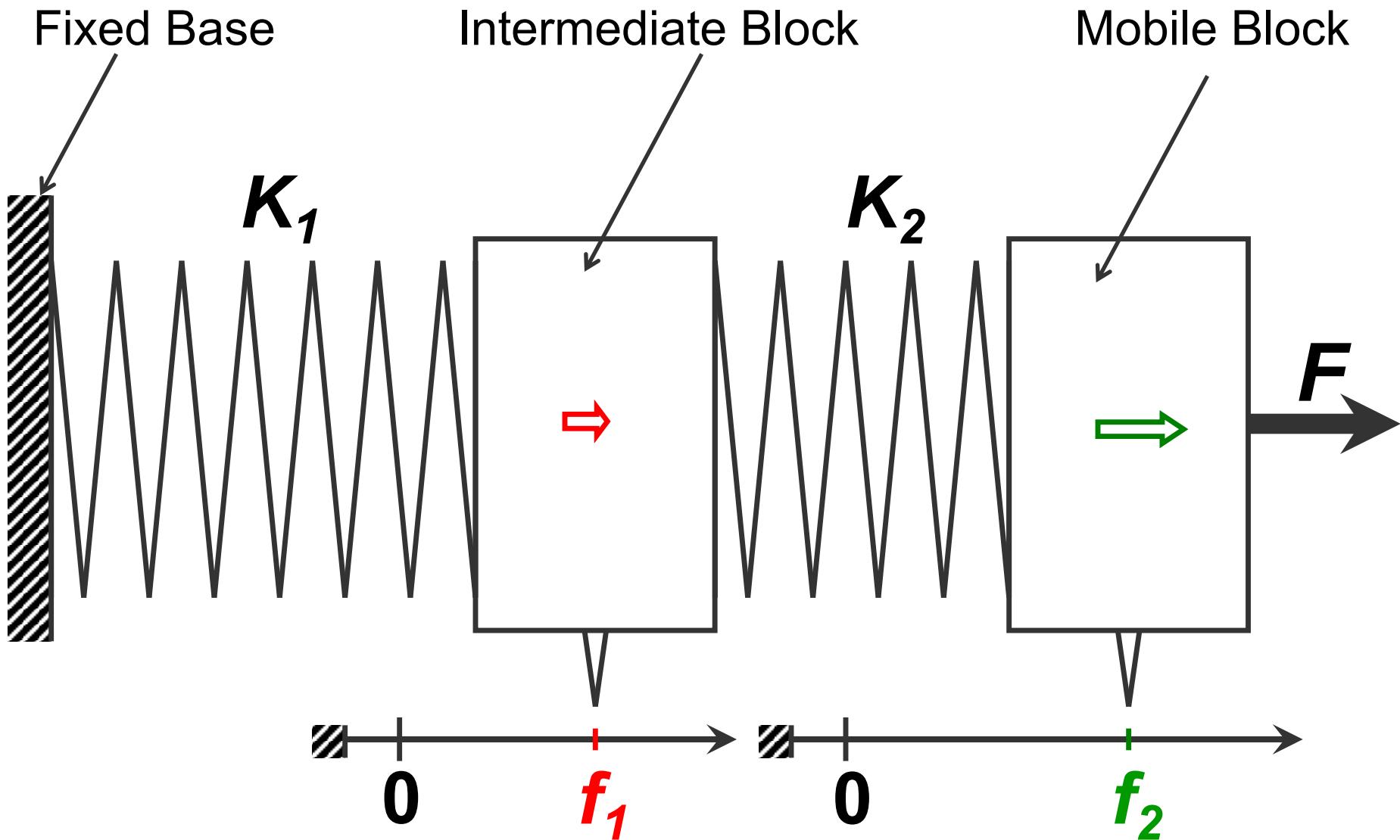


# Analog structure with ideal joints

2D: **DOF = 2** ;  $M = 8 - 3 \times 2 = 2$  ; DOH = 0

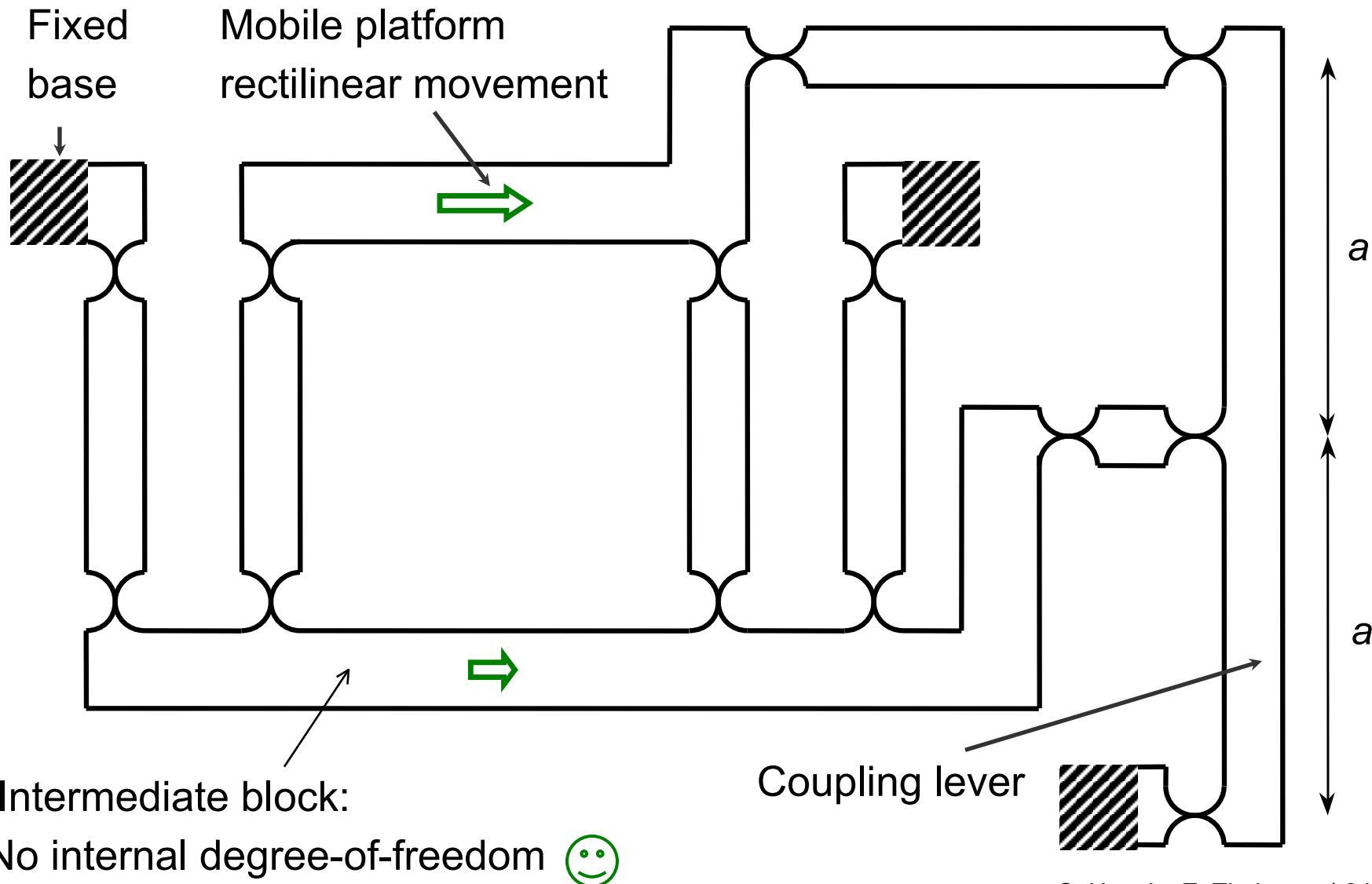


# Movement of the intermediate block



# Suppression of the internal degree-of-freedom

2D: **DOF = 1** ;  $M = 13 - 4 \times 3 = 1$  ; **DOH = 0**



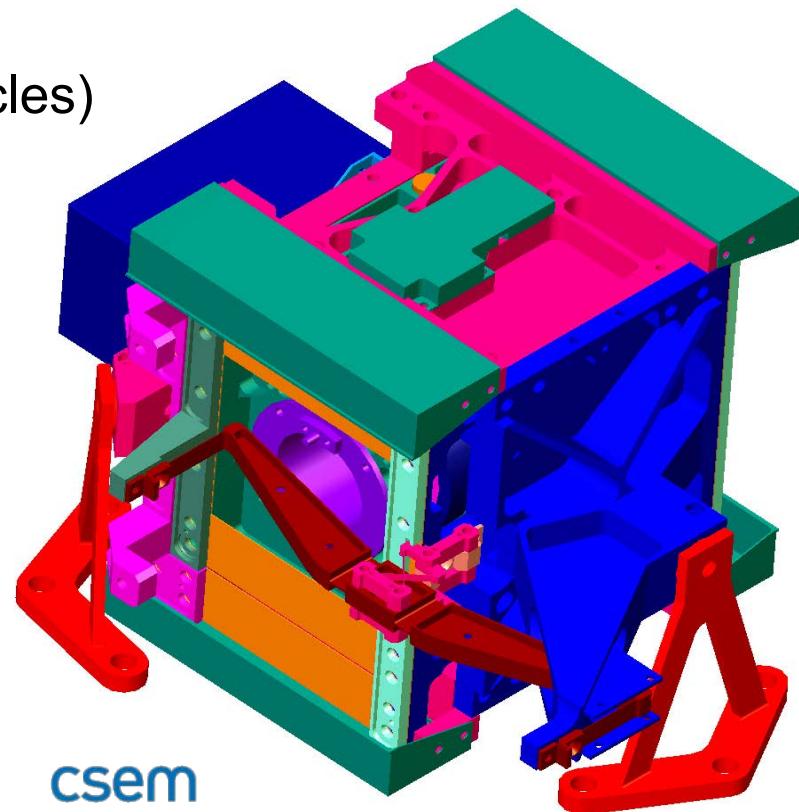
# Suppression of the internal degree-of-freedom

DOF = 1



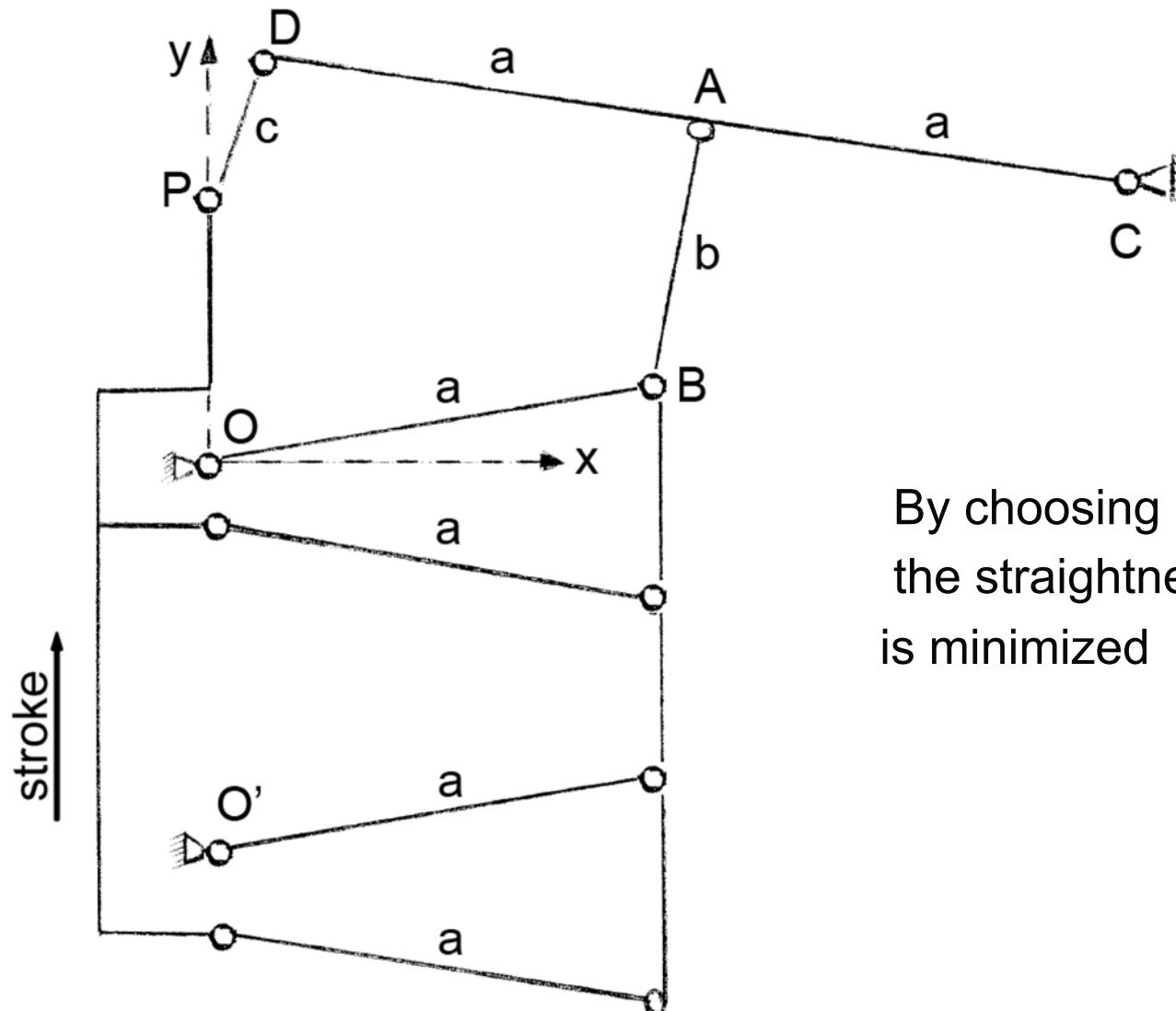
# Example: Corner Cube Mechanism

- Rectilinear guiding for interferometer scanner
- Stroke:  $\pm 12$  mm
- Lateral error off-axis  $<1 \mu\text{m}$
- 2.5 Hz constant velocity travel
- Lifetime : 5 years non-stop ( $5 \cdot 10^8$  cycles)



# Rectilinear mechanisms

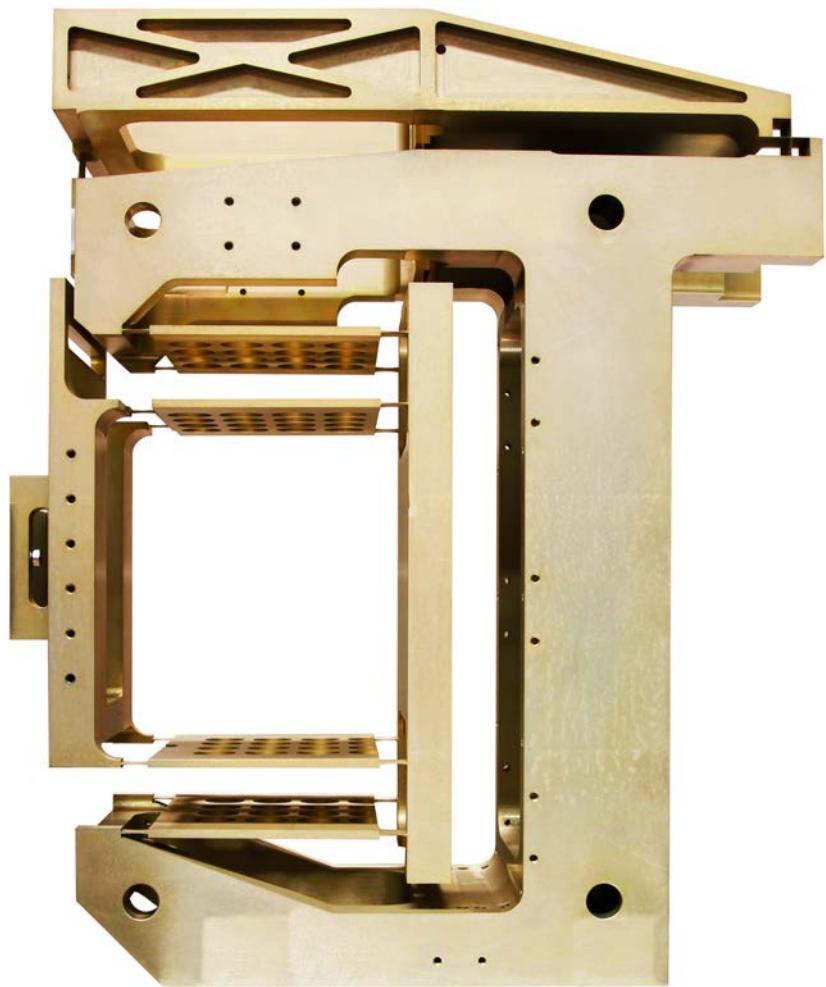
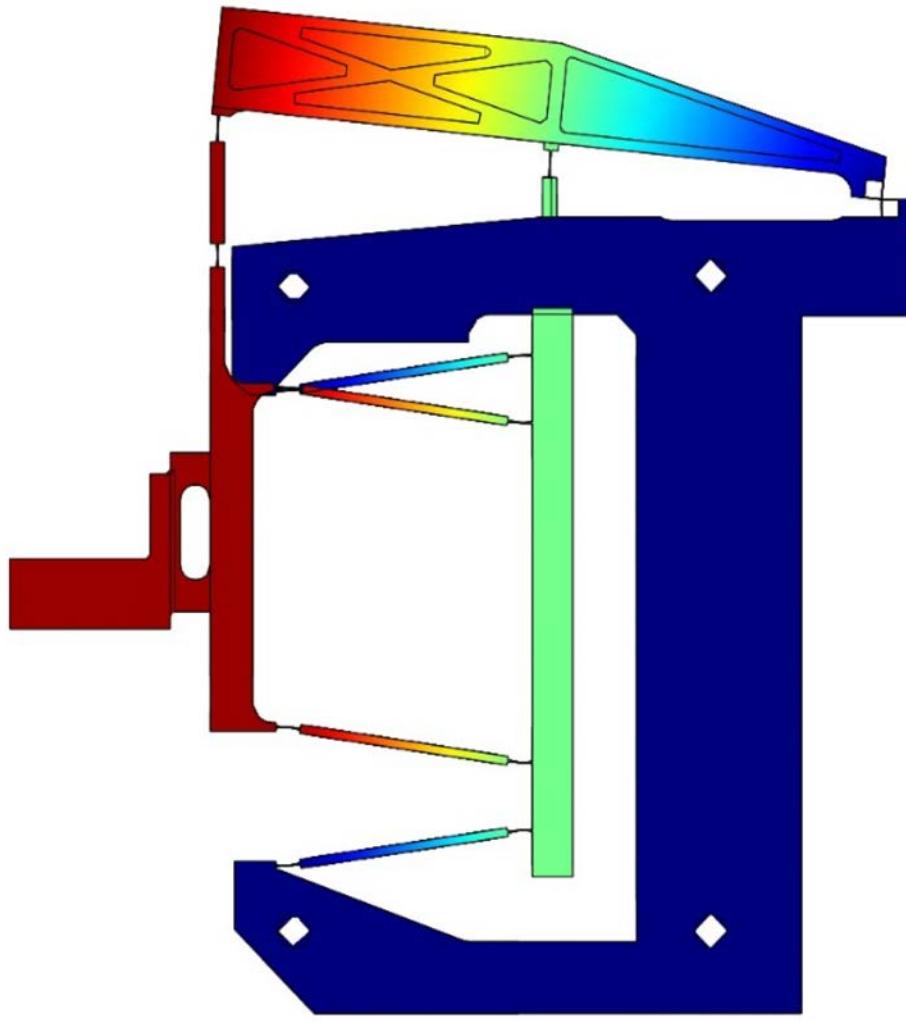
13 pivots optimized rectilinear mechanism



By choosing  $b = 2c$   
the straightness error  
is minimized

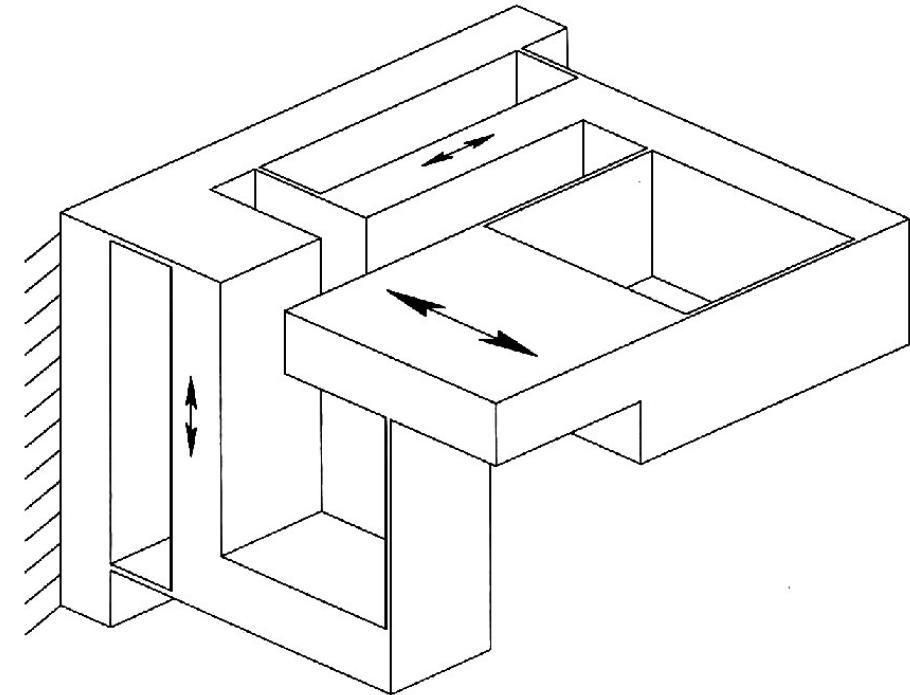
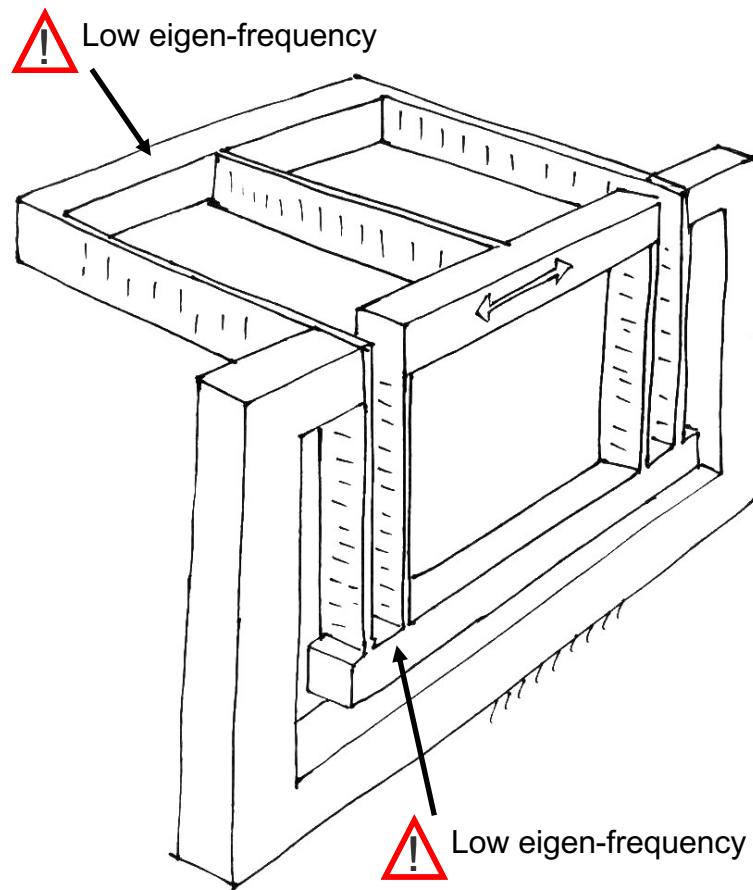
# Rectilinear mechanisms

13 pivots optimized rectilinear mechanism



# Rectilinear mechanisms

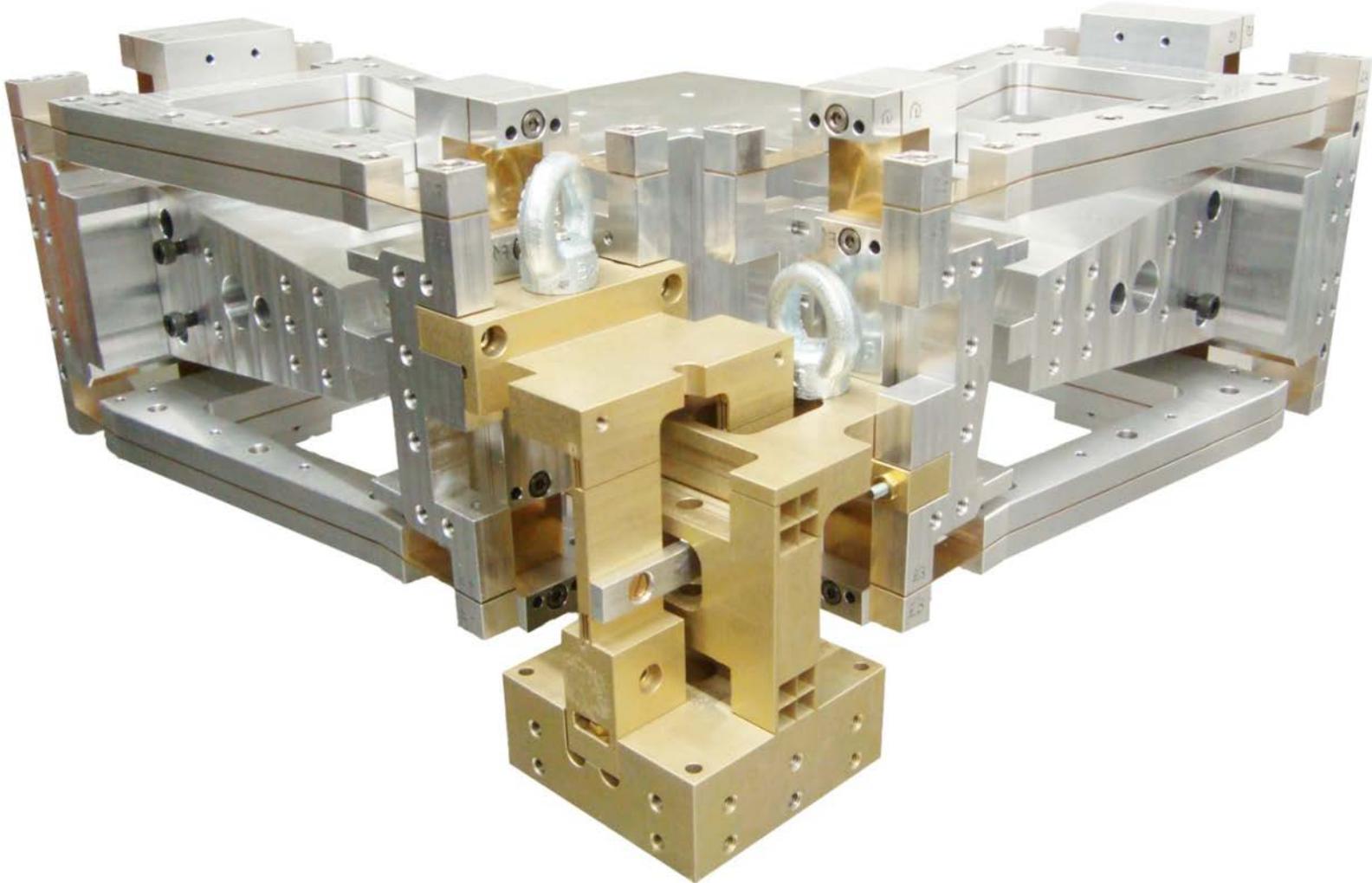
## Sarrus Mechanism



Flexure-based Sarrus mechanism

# Rectilinear mechanisms

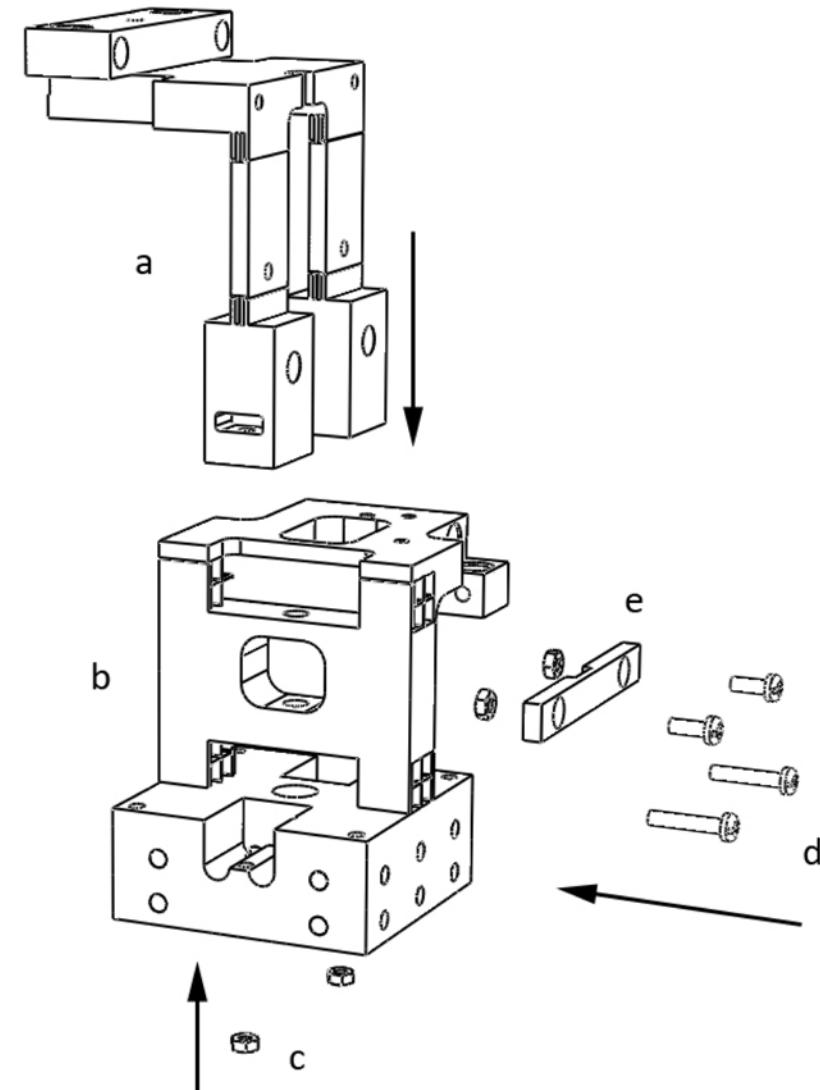
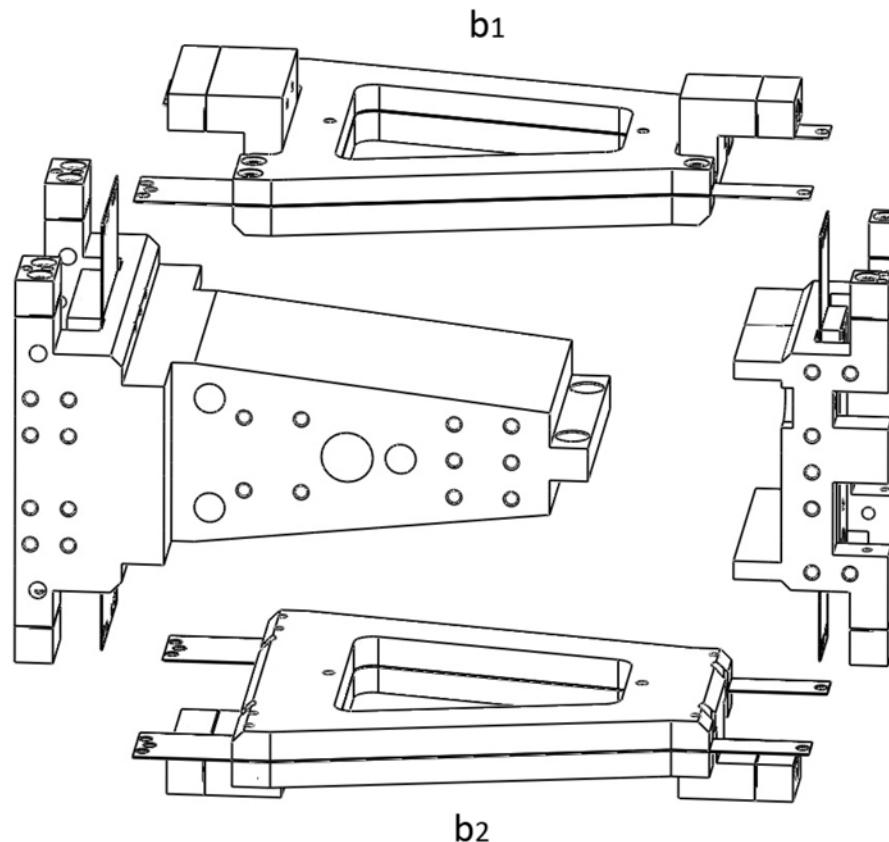
## Sarrus mechanism



F. Cosandier, Conception d'axes motorisés rectilignes d'ultra-haute précision, thèse EPFL n° 5665, 2012

# Rectilinear mechanisms

## Sarrus mechanism

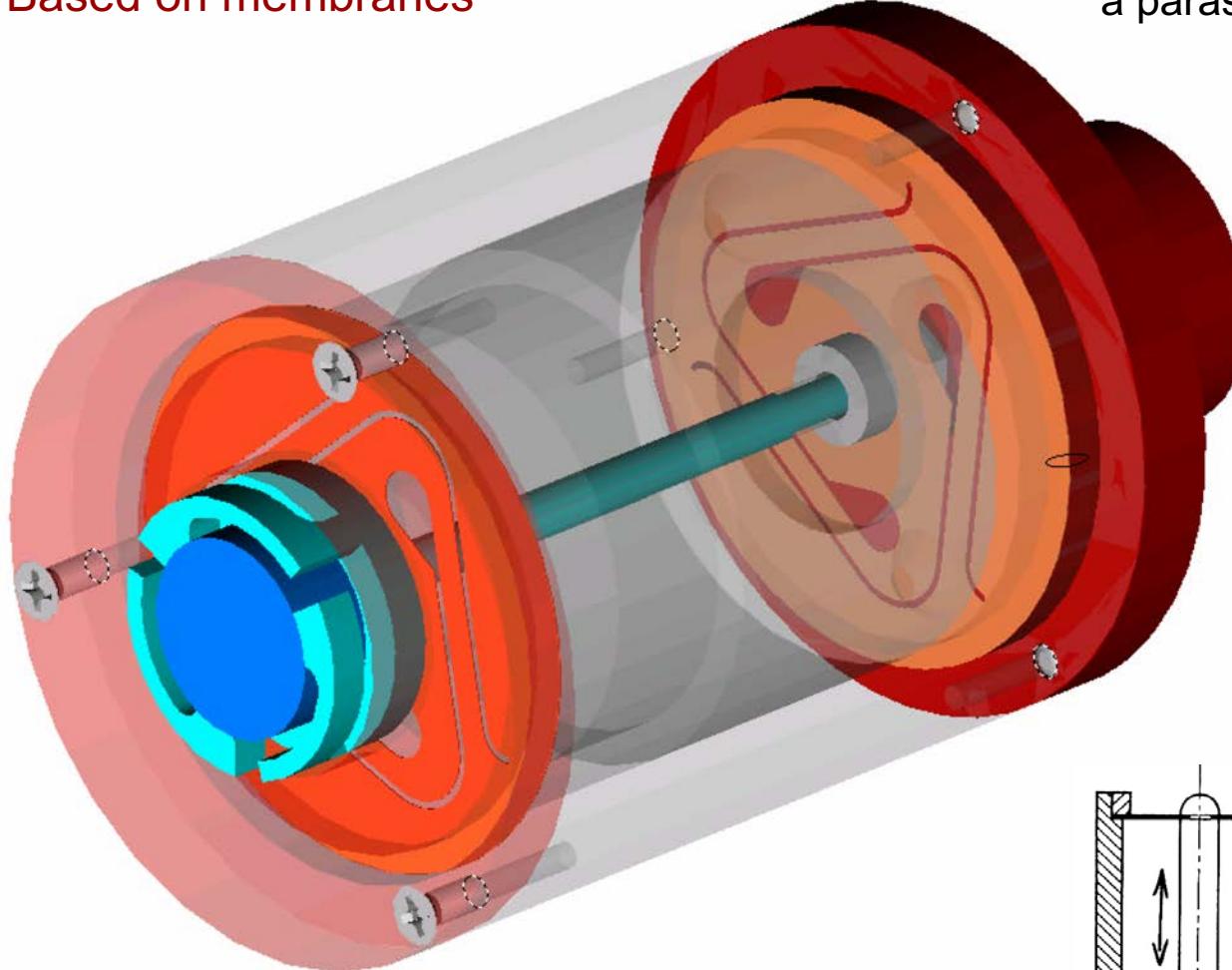


# Rectilinear mechanisms

Based on membranes

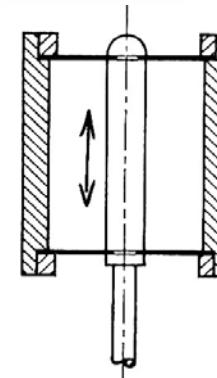


Rectilinear, but accompanied by  
a parasitic axial rotation

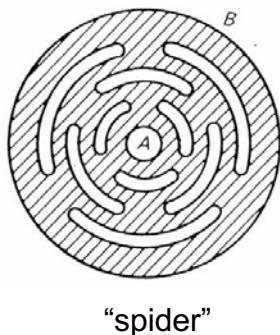


e.g. Interferometer:

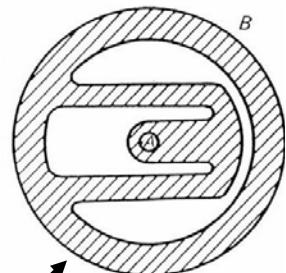
- Diameter: 34 mm
- Stroke: +/- 0.5 mm



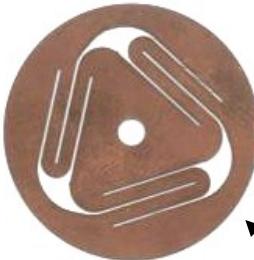
# Variety of membranes



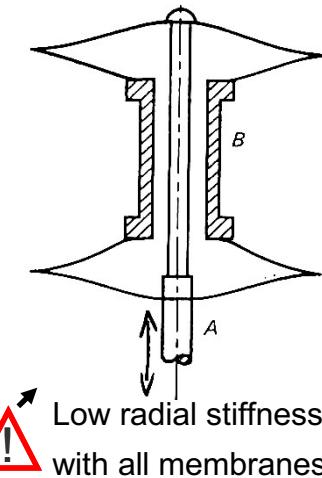
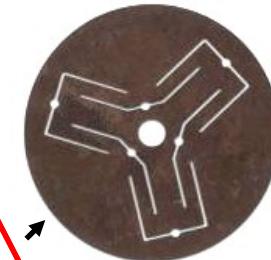
"spider"



! Not rectilinear

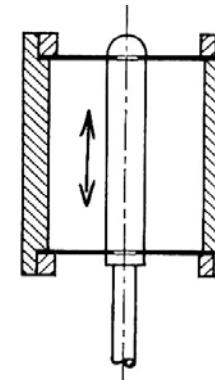
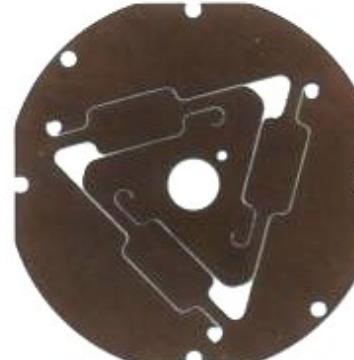


! Low radial stiffness



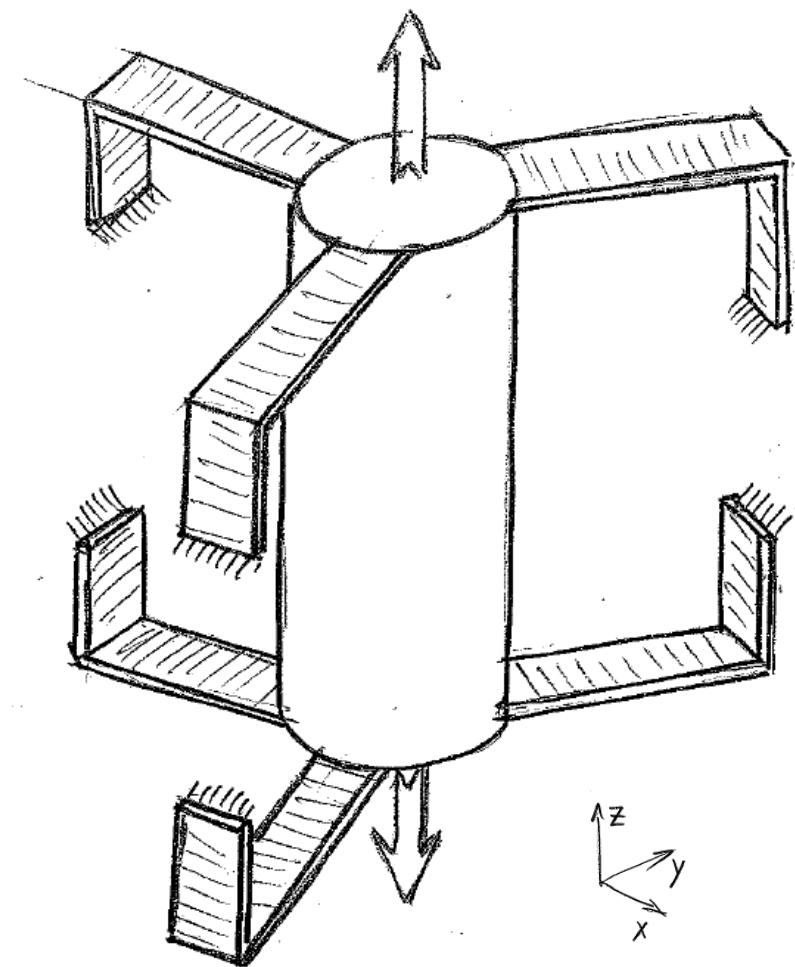
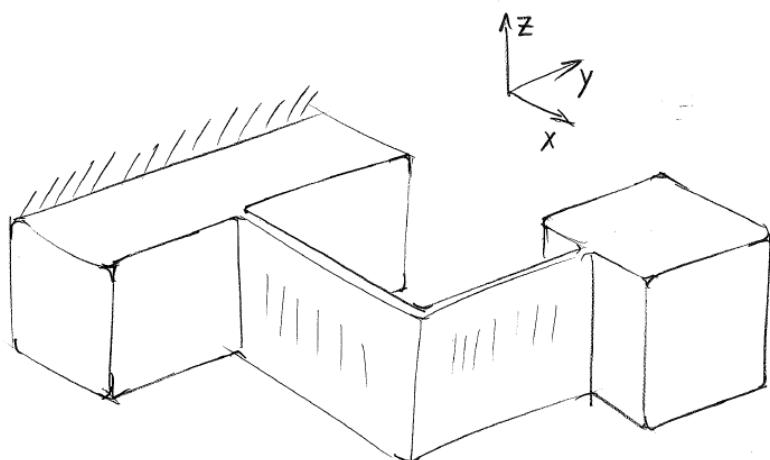
! Rectilinear,

but accompanied by parasitic axial rotation



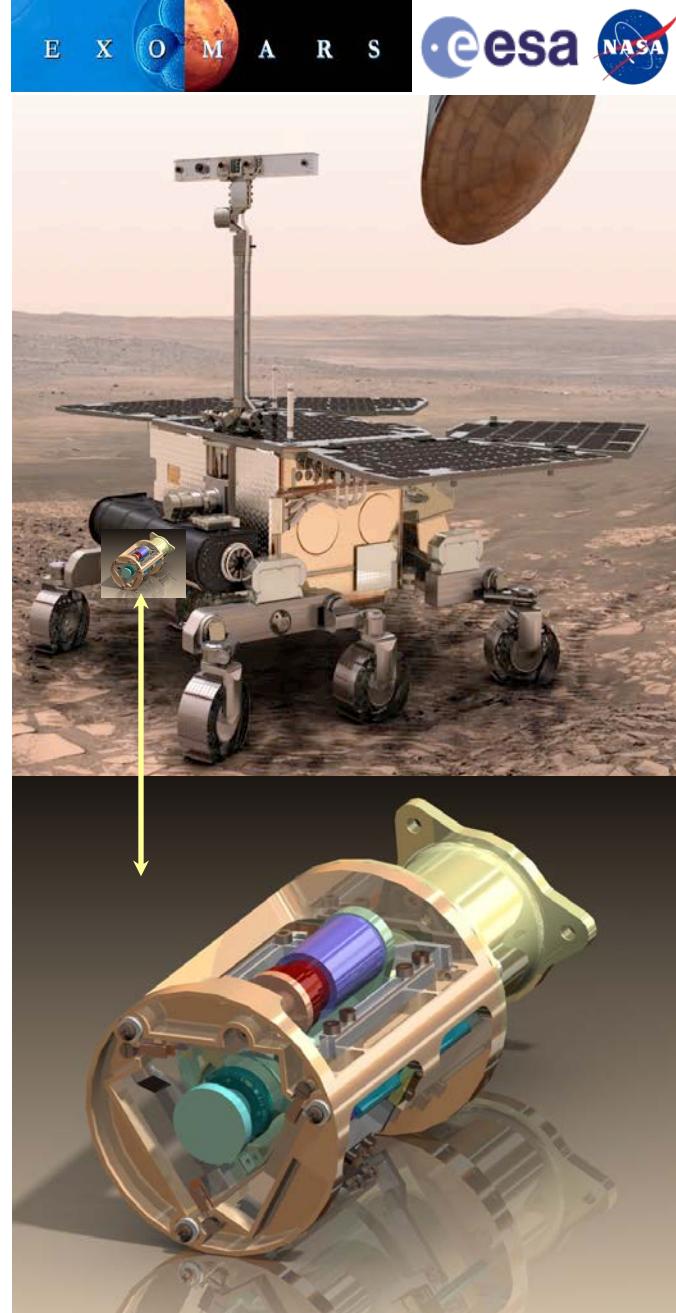
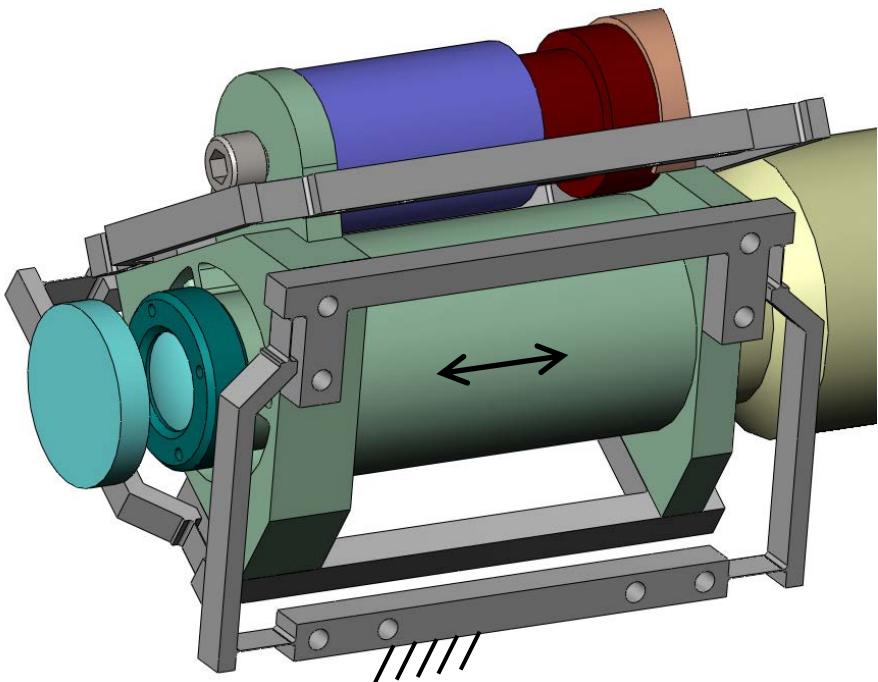
# Rectilinear mechanisms

Based on “corner-blades”



# Rectilinear mechanisms

Close-up Imager (CLUPI Instrument)  
European Mars Mission EXOMARS (ESA)



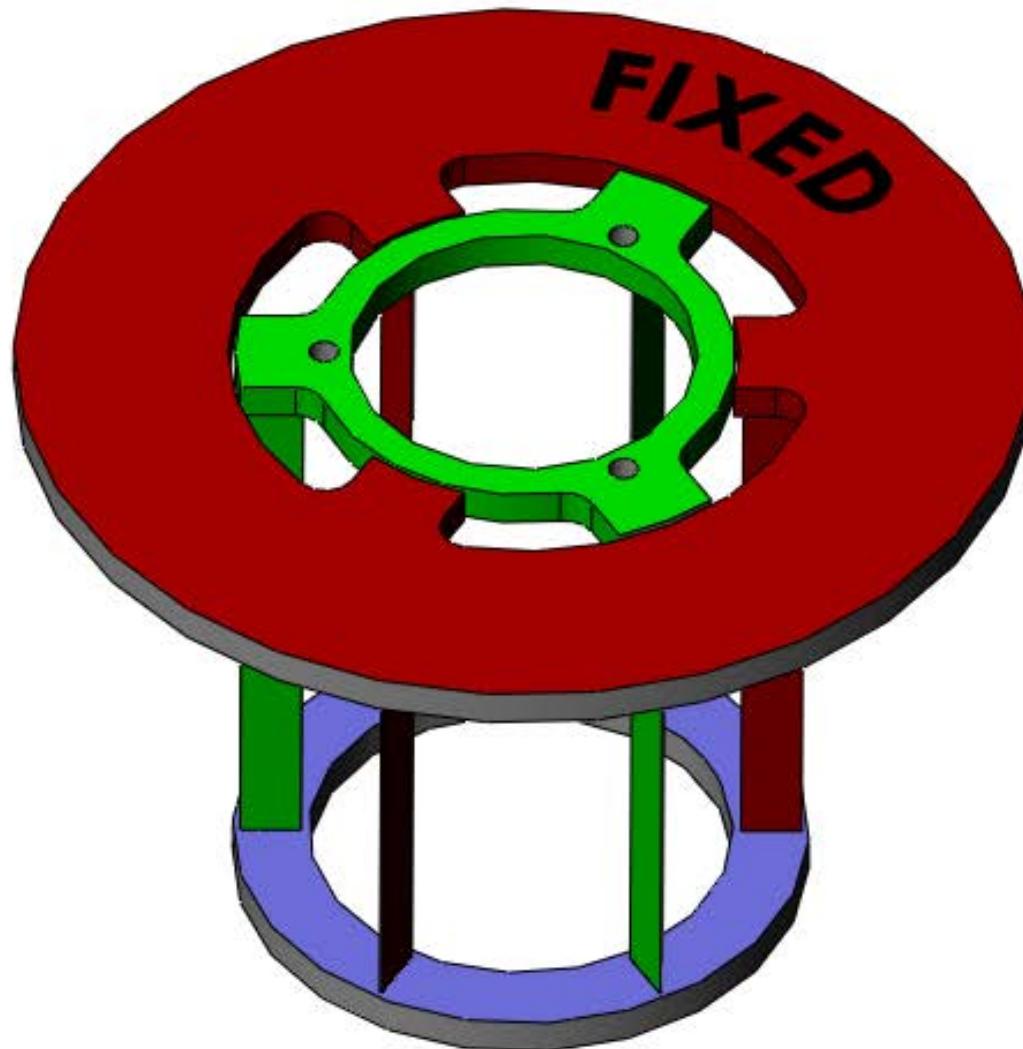
SPACE-X  
Space Exploration Institute  
Neuchâtel, Jean-Luc Josset  
[www.space-x.eu](http://www.space-x.eu)

csem

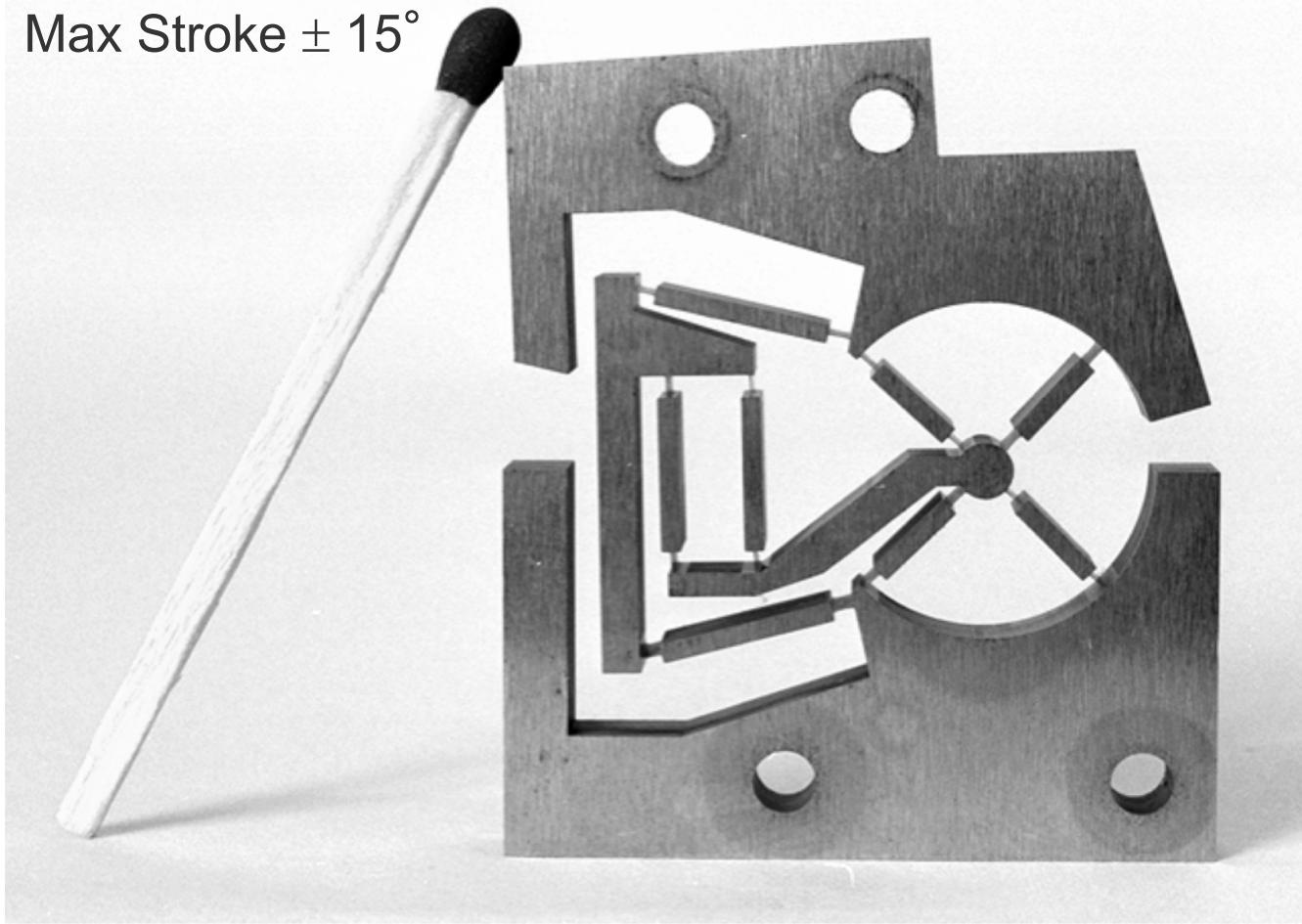
L. Giriens  
S. Henein  
Ph. Schwab  
P. Spanoudakis

# Circular mechanisms

Compound squirrel cage



# Suppression of internal degrees-of-freedom

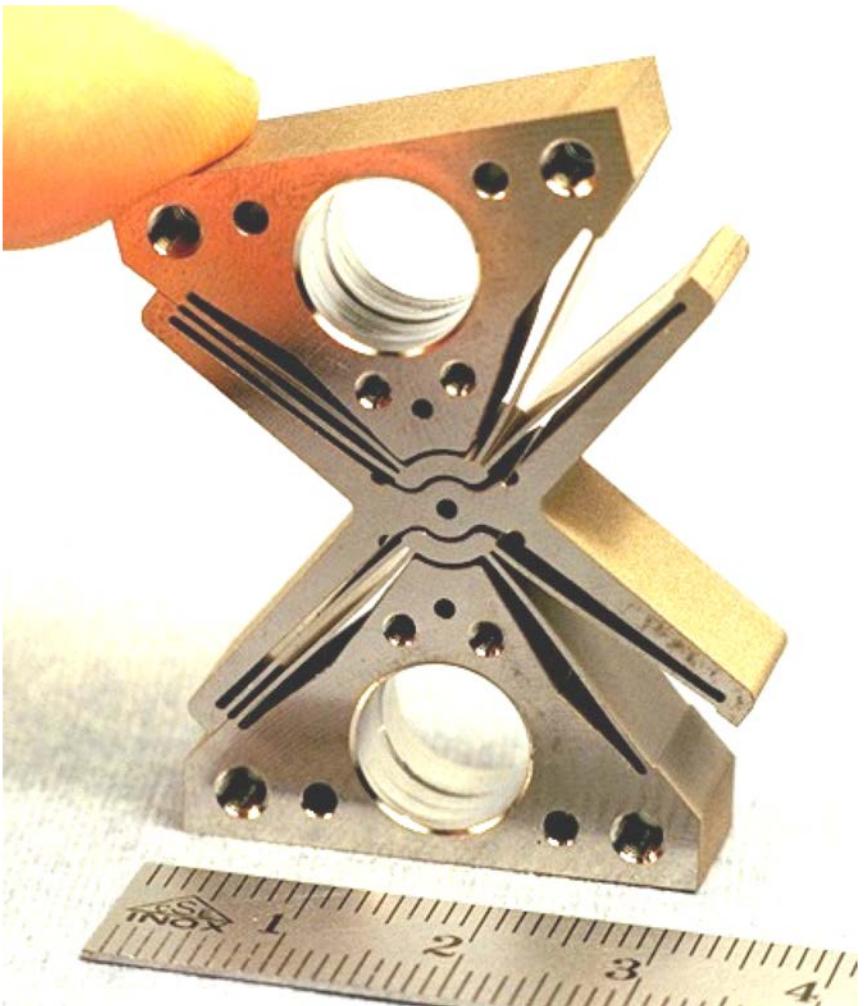


- Radial stiffness **with** coupling kinematic chain: 5.3 N/um
  - Radial stiffness **without** coupling kinematic chain: 0.6 N/um
- } Improvement factor  $\approx 9$

[S.Henein et al. (1998), Pivot flexible à grande course angulaire et à rigidité élevée,  
European Patent N° 98123953.6, Holder: Sysmelec SA]

# Circular Mechanisms

## Butterfly Pivot

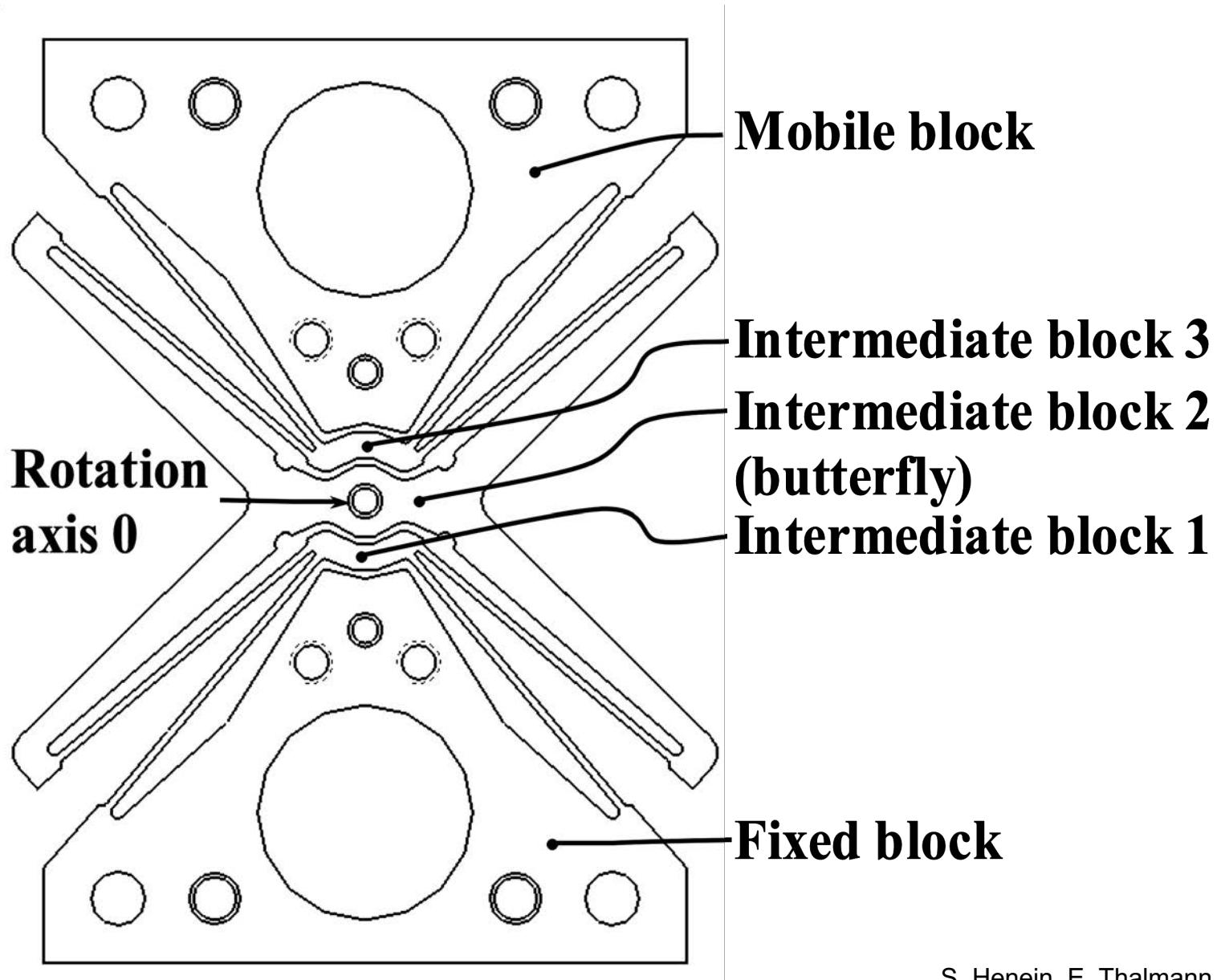


csem

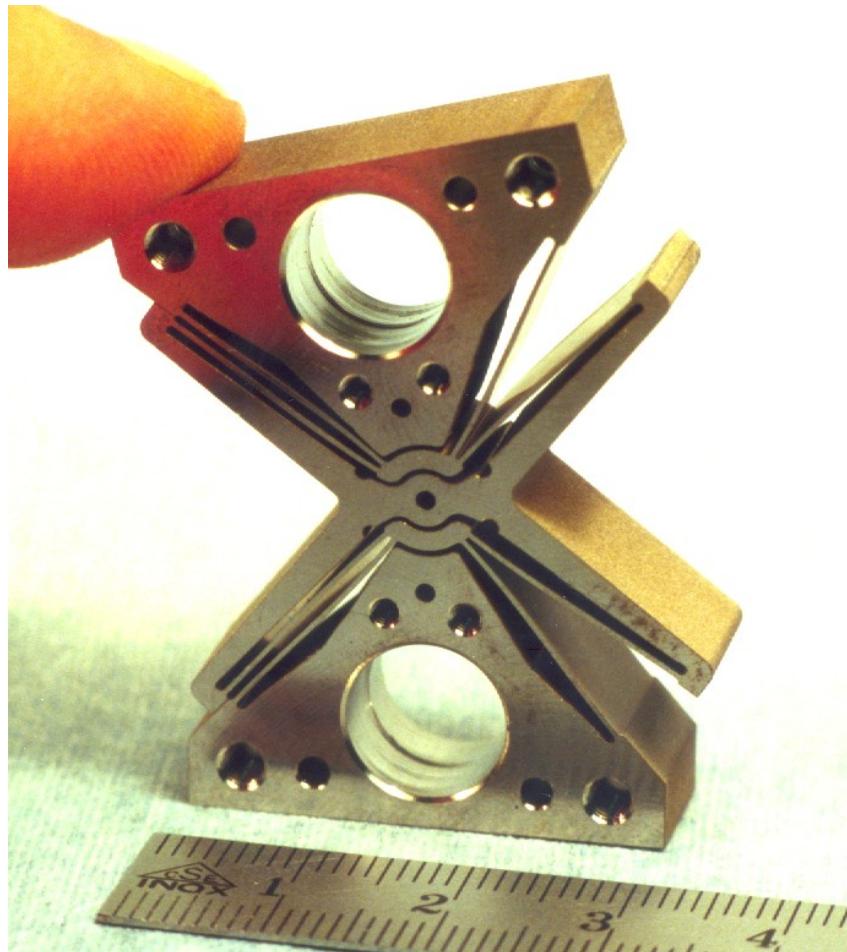
[S.Henein et al. (2003), Flexure Pivot for Aerospace Mechanisms, ESA SP-524, Proc. 10th European Space Mechanisms & Tribology Symp.]

MOTION											
Angular range	$\pm 10^\circ$										
Centre parasitic shift at $10^\circ$	$1 \mu\text{m}$										
Pivot characteristics											
Material	Ti-6Al-4V										
Monolithic construction											
Manufacturing Process	Wire EDM										
Dimensions	<table><thead><tr><th>Height</th></tr></thead><tbody><tr><td>43 mm</td></tr><tr><th>Width</th></tr><tr><td>30 mm</td></tr><tr><th>Thickness</th></tr><tr><td>10 mm</td></tr><tr><th>Blade length</th></tr><tr><td>15 mm</td></tr><tr><th>Blade thickness</th></tr><tr><td>0.35 mm</td></tr></tbody></table>	Height	43 mm	Width	30 mm	Thickness	10 mm	Blade length	15 mm	Blade thickness	0.35 mm
Height											
43 mm											
Width											
30 mm											
Thickness											
10 mm											
Blade length											
15 mm											
Blade thickness											
0.35 mm											
STIFFNESS											
Rotary stiffness ( $\theta_x$ )	1.0 Nm/rad										
Rotational stiffness ( $\theta_y$ )	5000 Nm/rad										
Rotational stiffness ( $\theta_z$ )	5000 Nm/rad										
Axial (translation) stiffness ( $K_x$ )	6.5 Nm/ $\mu\text{m}$										
Lateral (radial) stiffness ( $K_y K_z$ )	0.7 Nm/ $\mu\text{m}$										

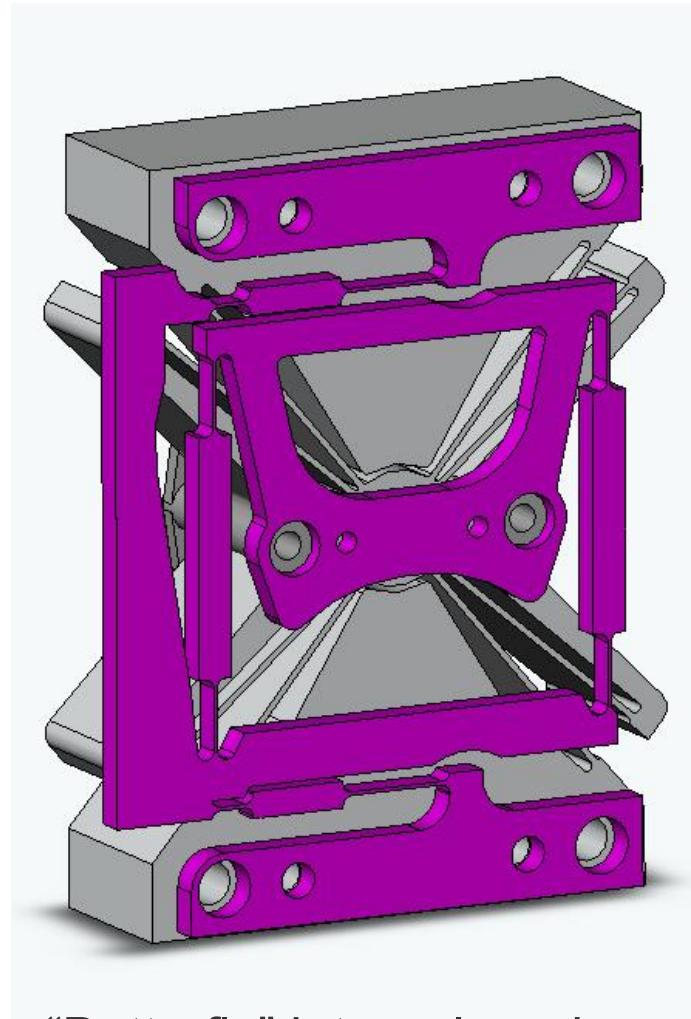
# *Butterfly Pivot : Topology and Kinematics*



# Suppression of internal degrees-of-freedom



“Butterfly” internal mode  
frequency 320 Hz

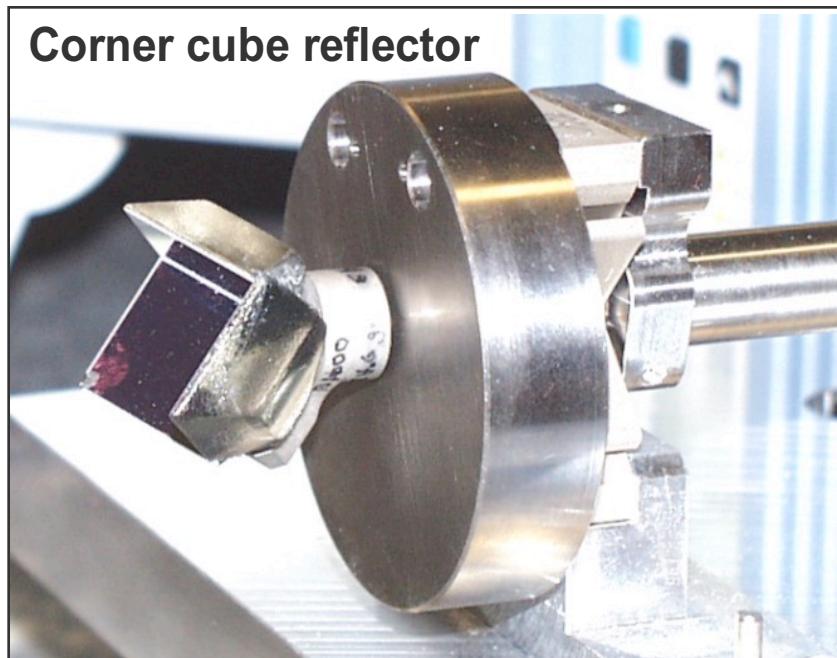
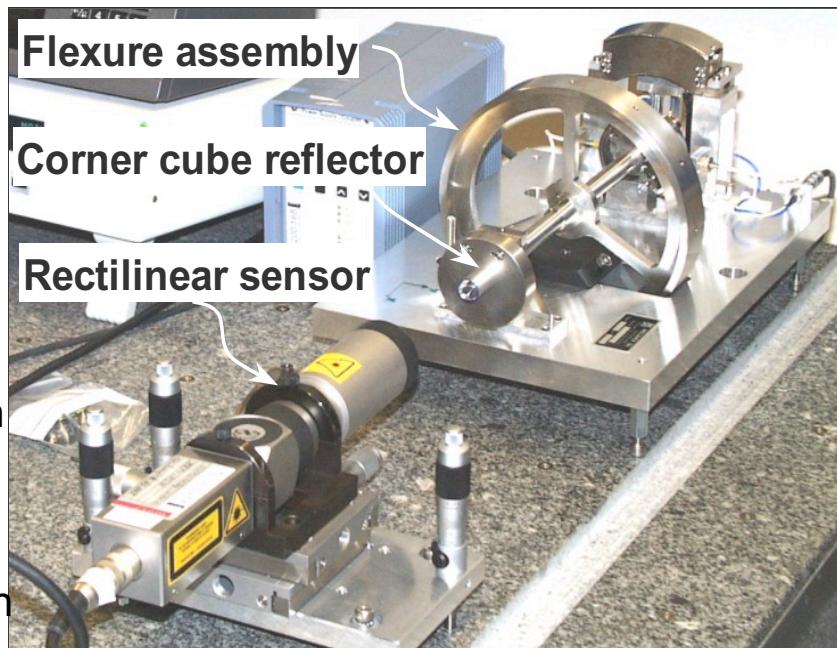
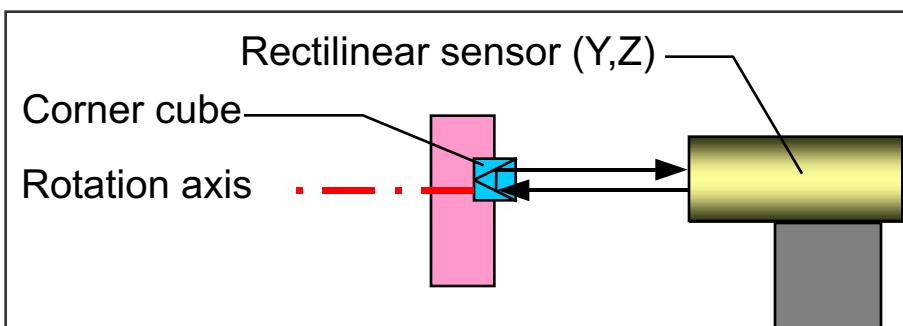
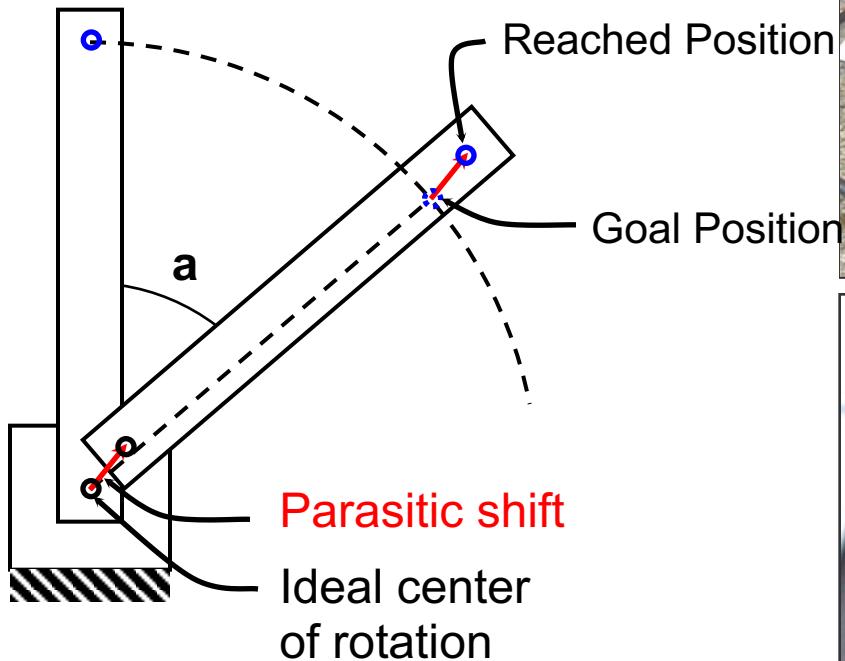


“Butterfly” internal mode  
frequency above 2000 Hz

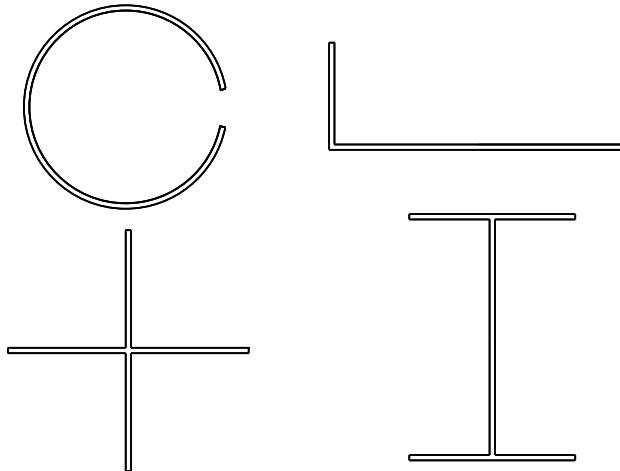
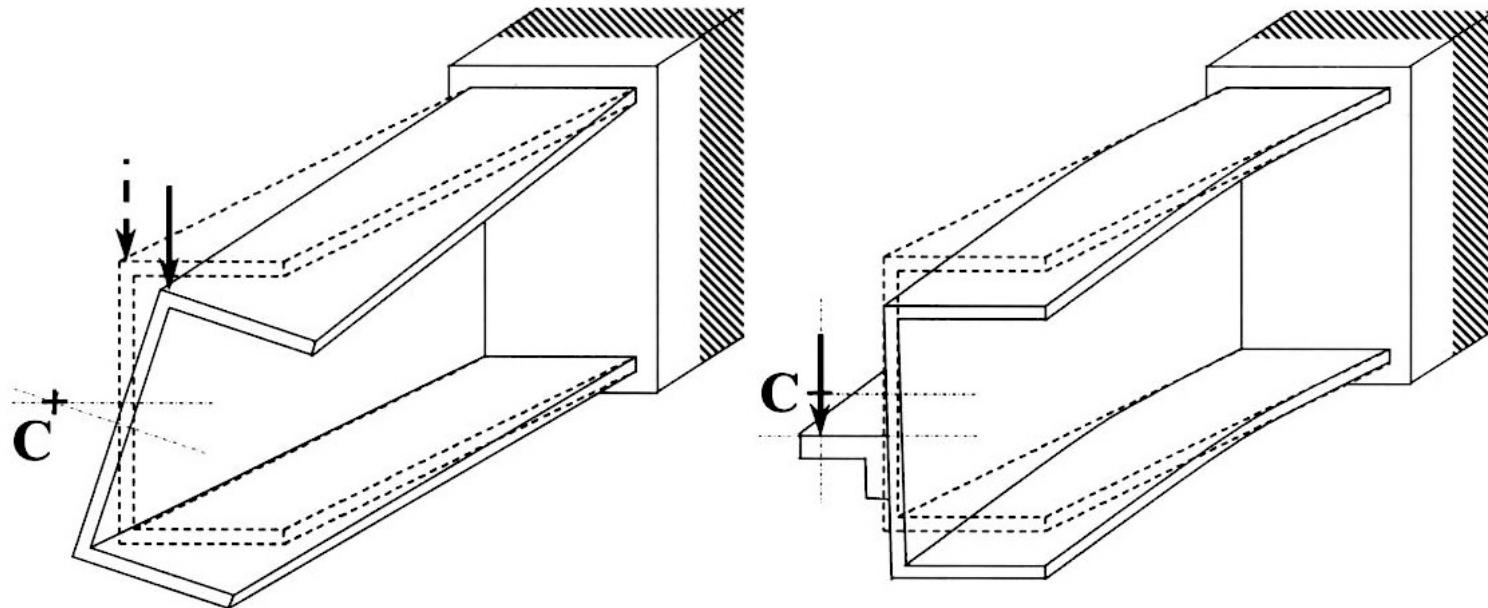
# Parasitic center shift

Butterfly pivot < 2 microns  
for  $\pm 10^\circ$  stroke.

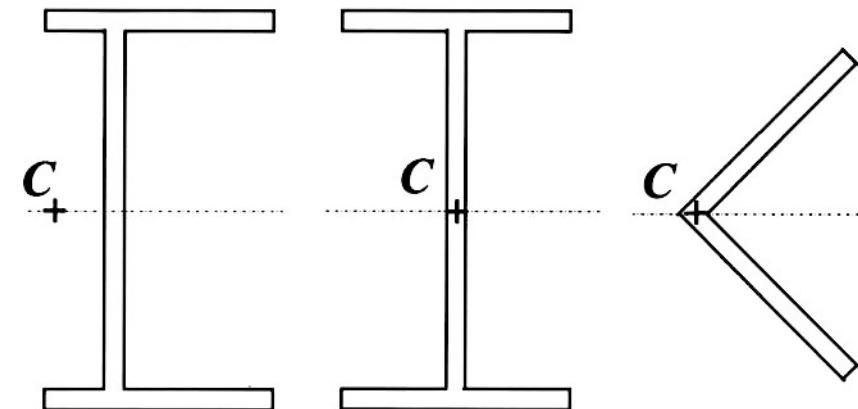
(Classical cross spring pivot of similar size  
produces 75 microns shift for  $\pm 10^\circ$  stroke)



# Torsion bars



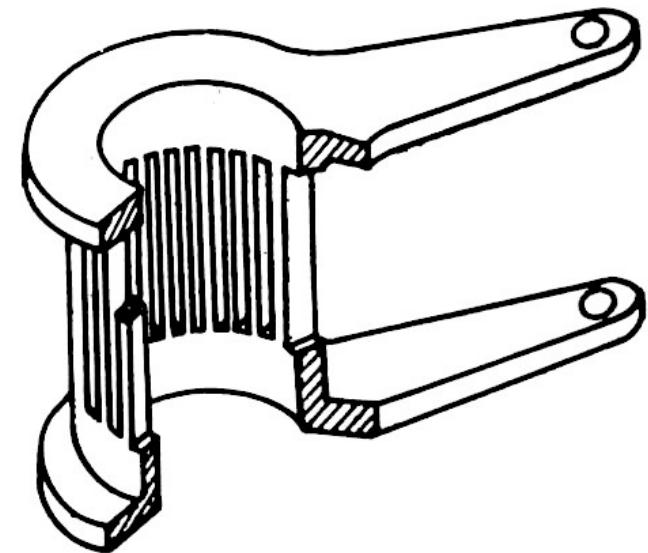
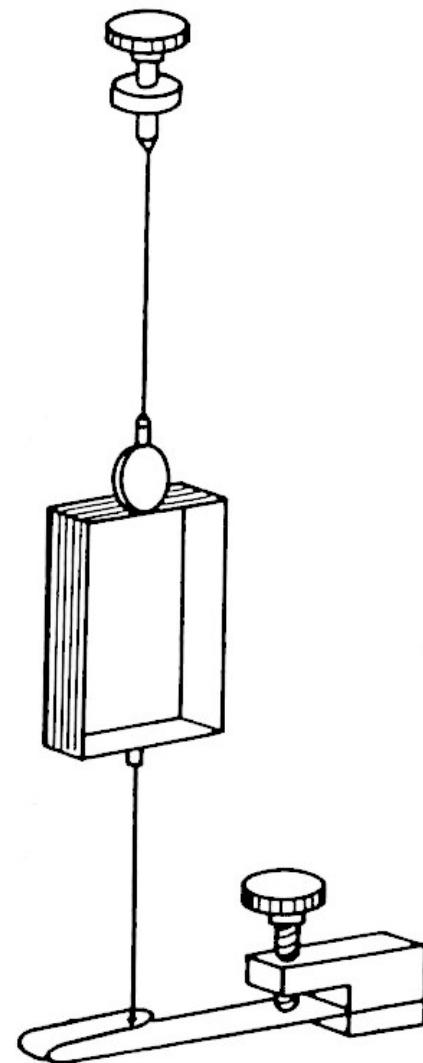
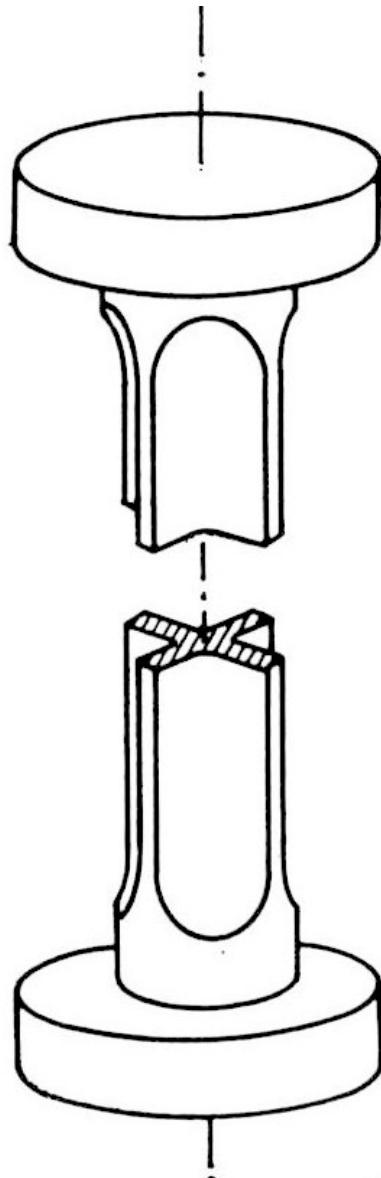
Shapes with equal total cross-section and thin walls have approx. equal torsion stiffness



The location of the torsion axis depends on the shape of the cross-section

# Torsion bars

(Bassiére et Gaignebet, 1966)

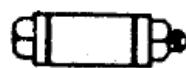
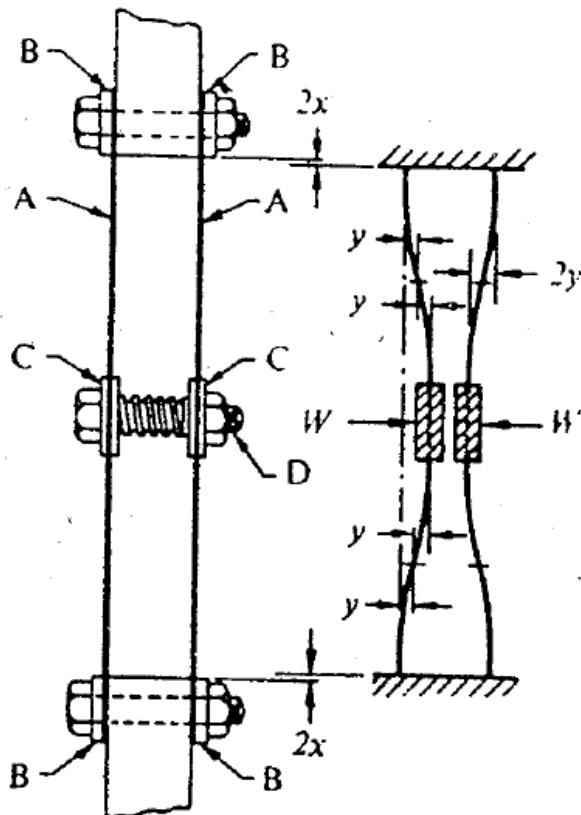


## Flexures with large reduction factors



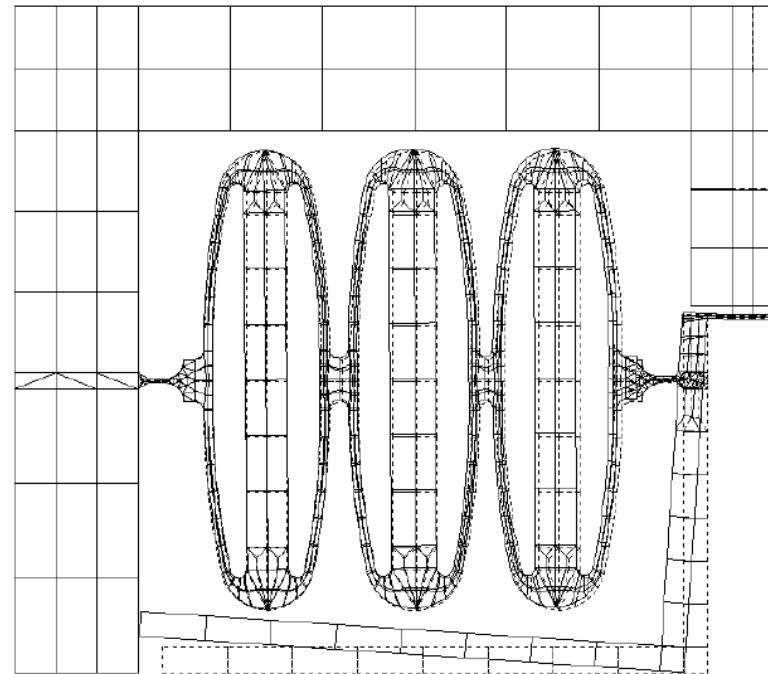
# Examples of reduction and amplification mechanisms

Adjustable Strut



*Application of spring strips to instrument design,*  
E. M. Eden, Notes of Applied Science No.15, 1956

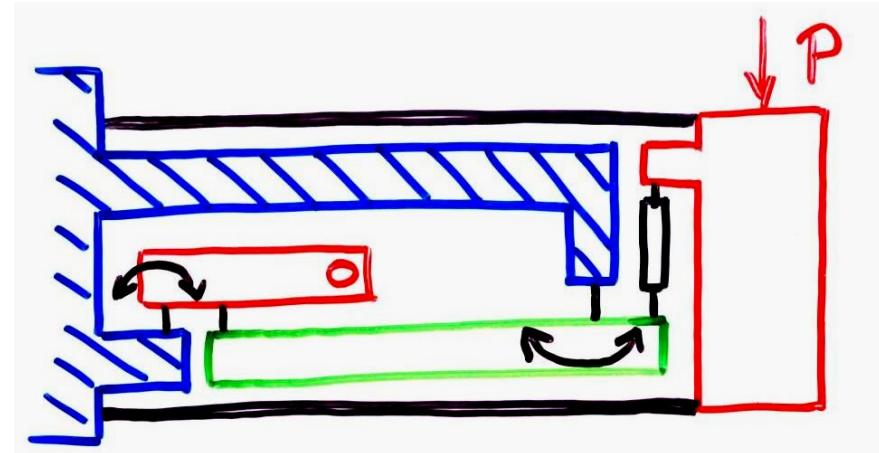
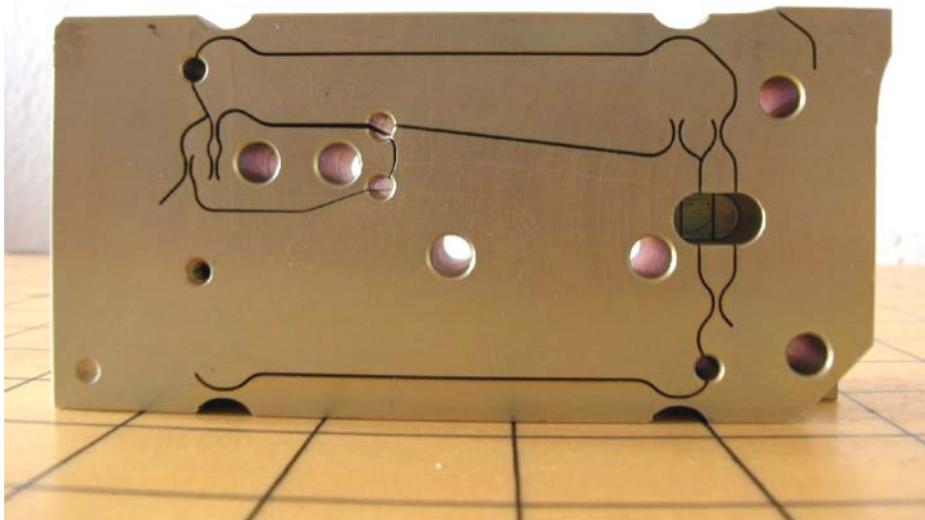
Elliptical motion amplifier



*Super Amplified Piezoelectric Actuator,*  
N. Lehrmet, Flux Magazine,  
Cedrat-Technologies, Jan. 2000

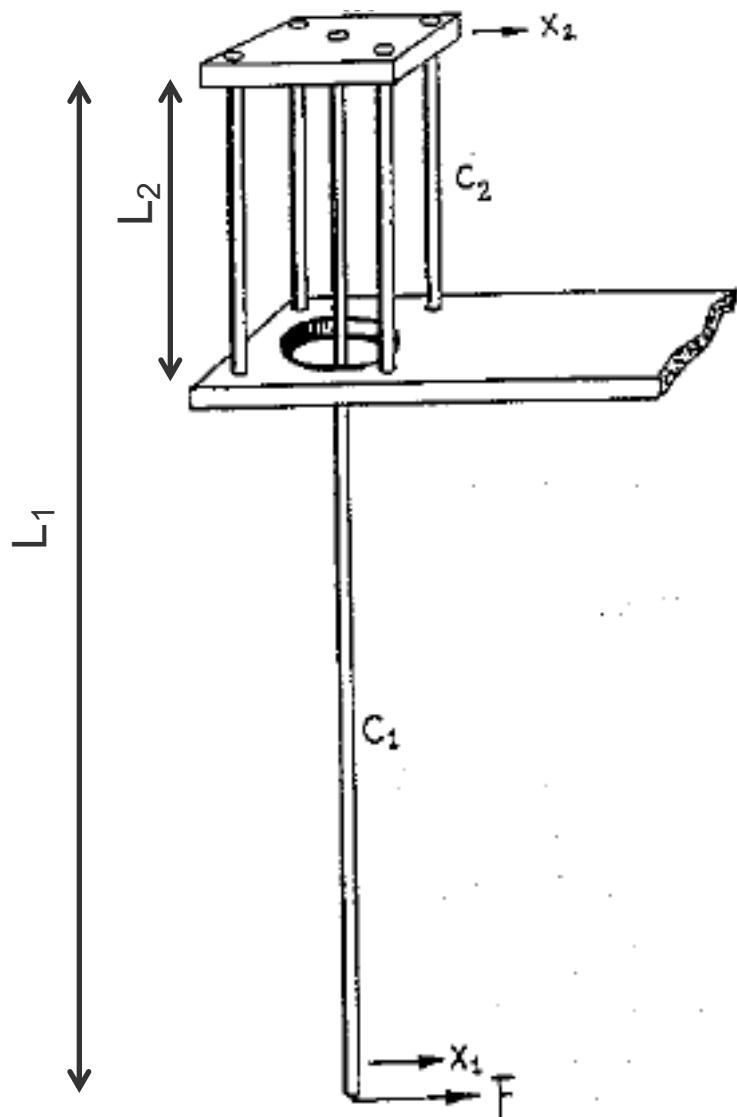
# Examples of reduction and amplification mechanisms

Flexible Cell for Precision Weighing Scales (Mettler-Toledo)



[E.Hungerbühler et al. (1994), *Device for reducing the force in a force measuring apparatus, in particular a scale*, US Patent 5,340,952]

# Elastic reduction



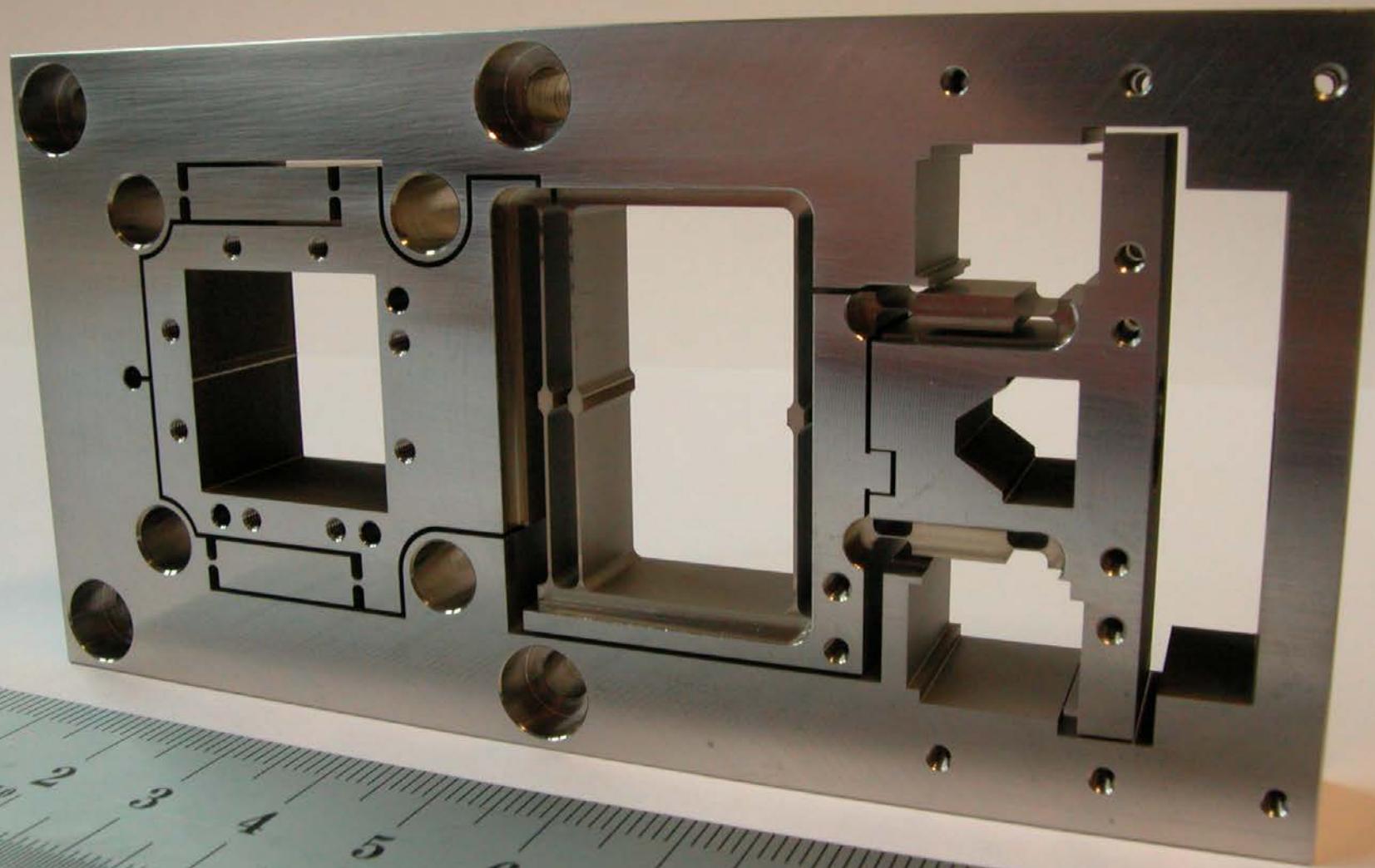
$$L_1 = 3.97 \cdot L_2$$



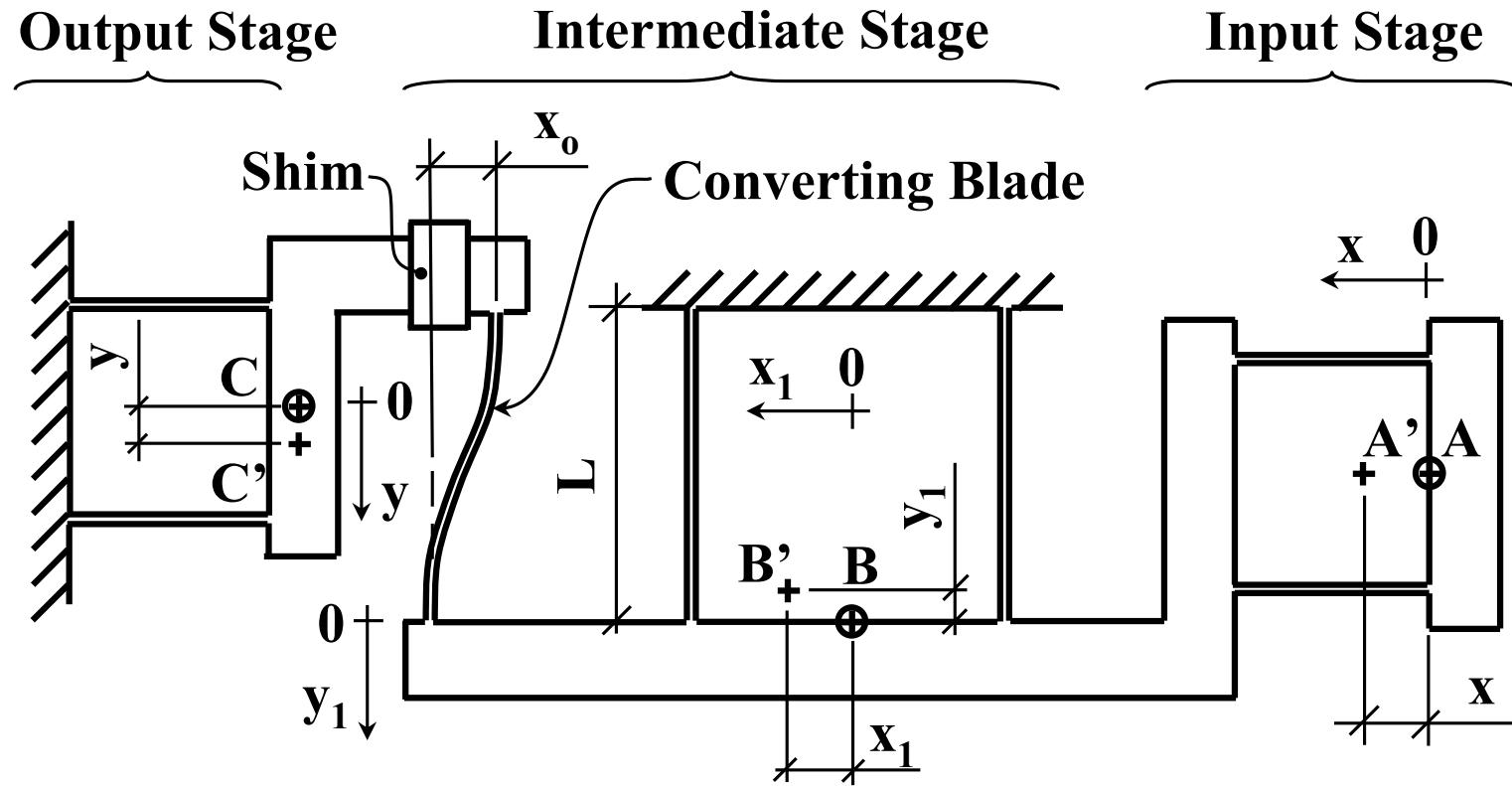
$$x_2 = x_1 / 1000$$

[M.P. Koster, Constructieprincipes, 1996]

# Nanoconverter : « *Converting $\mu m$ into nanometers* »



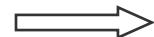
# Nanoconverter : working principle



$$y_1 = -3x_1^2/(5L)$$

$$x_1 \approx x$$

$$y = \frac{3(x+x_0)^2}{5L} - \frac{3x^2}{5L} = \frac{6x_0}{5L}x + \frac{3x_0^2}{5L}$$



Reduction factor:

$$i = \frac{x}{y} = \frac{5L}{6x_0}$$

[Henein, S. (2006), European Patent EP06021785, Device for converting a first motion into a second motion responsive to said first motion under a demagnification scale, Holder: Paul Scherrer Institut]

# Nanoconverter : key characteristics

## Main blades

Length: 30 mm

Thickness: 0.35 mm

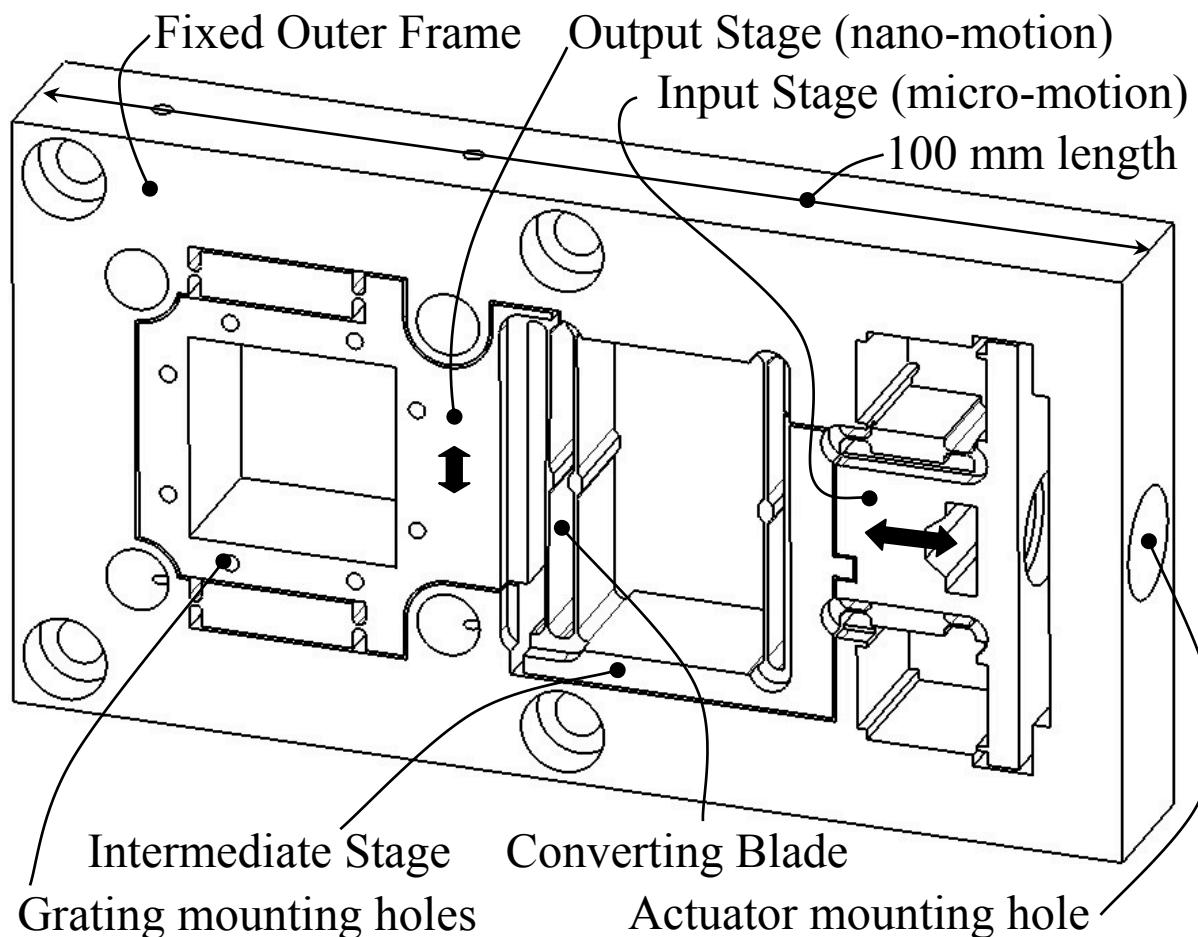
Width: 10 mm

Offset: 0.25 mm

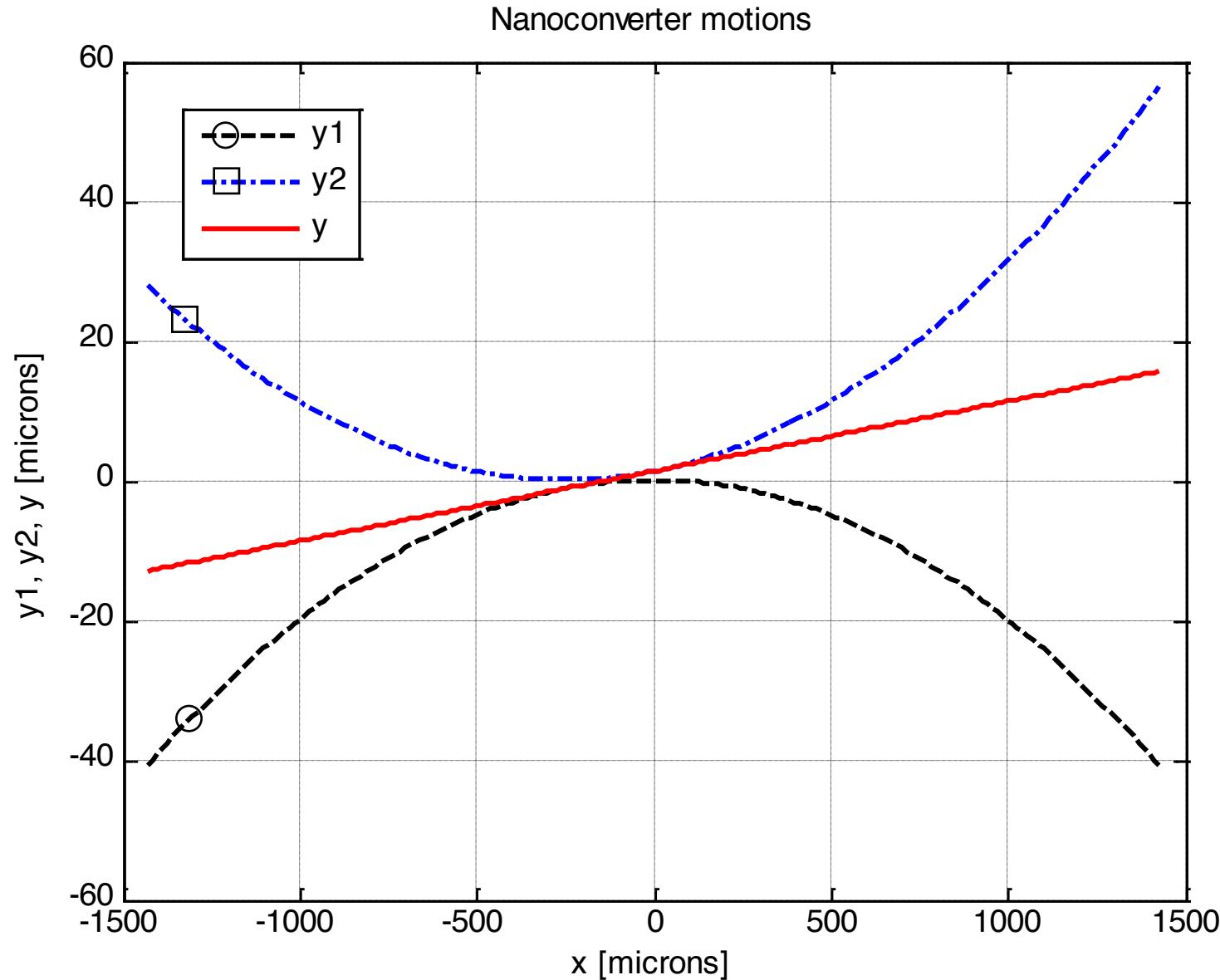
Reduction factor: 100

Material : Stainless Steel  
Böhler W720

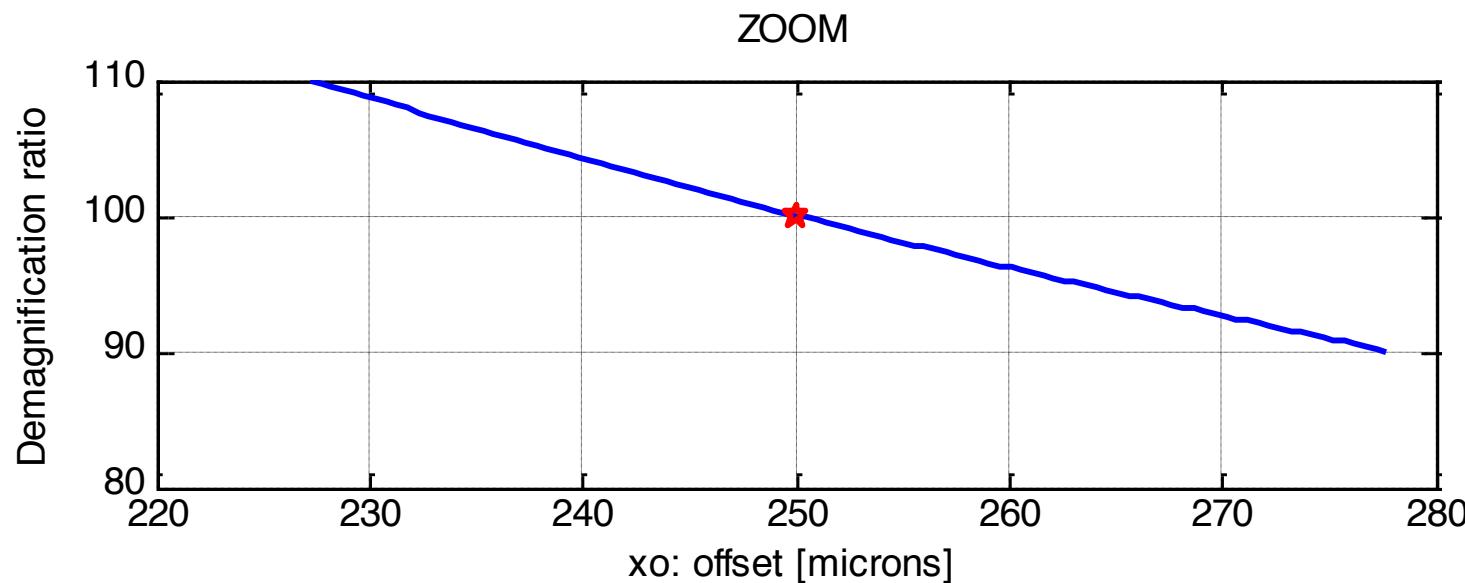
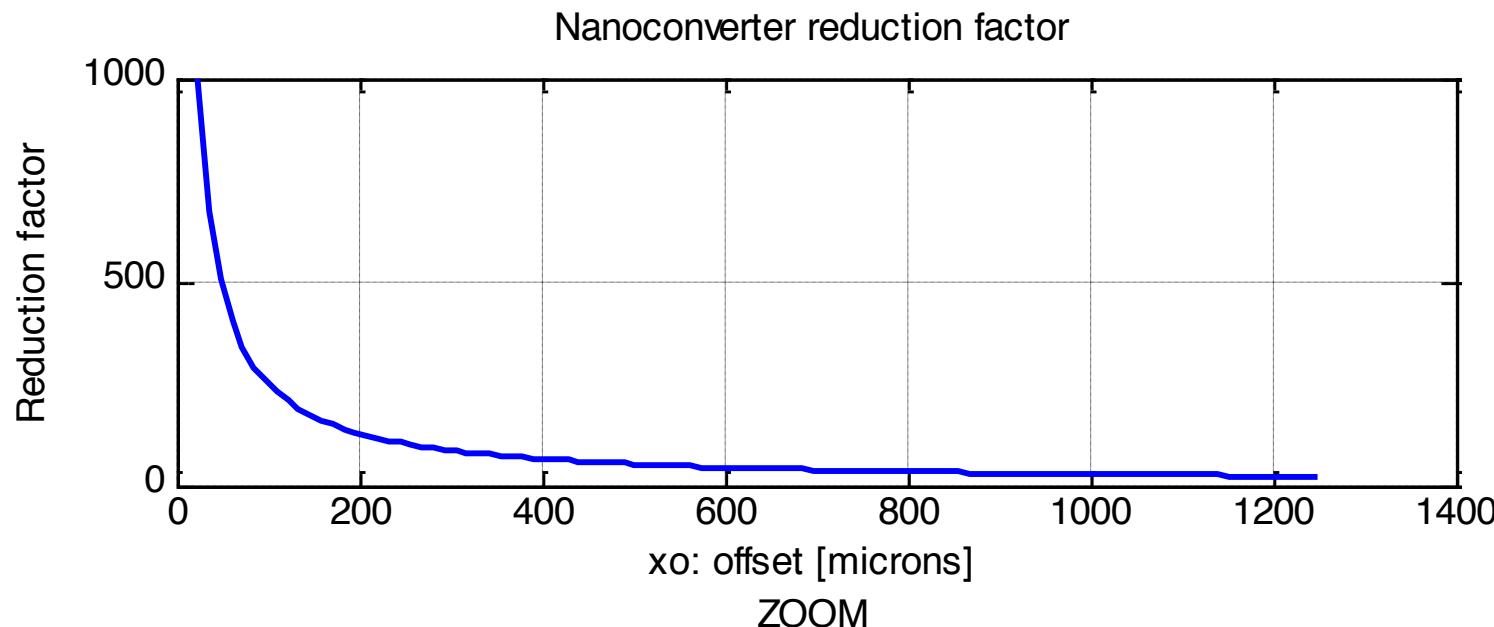
Manufacturing: Wire-EDM  
(Monolithical)



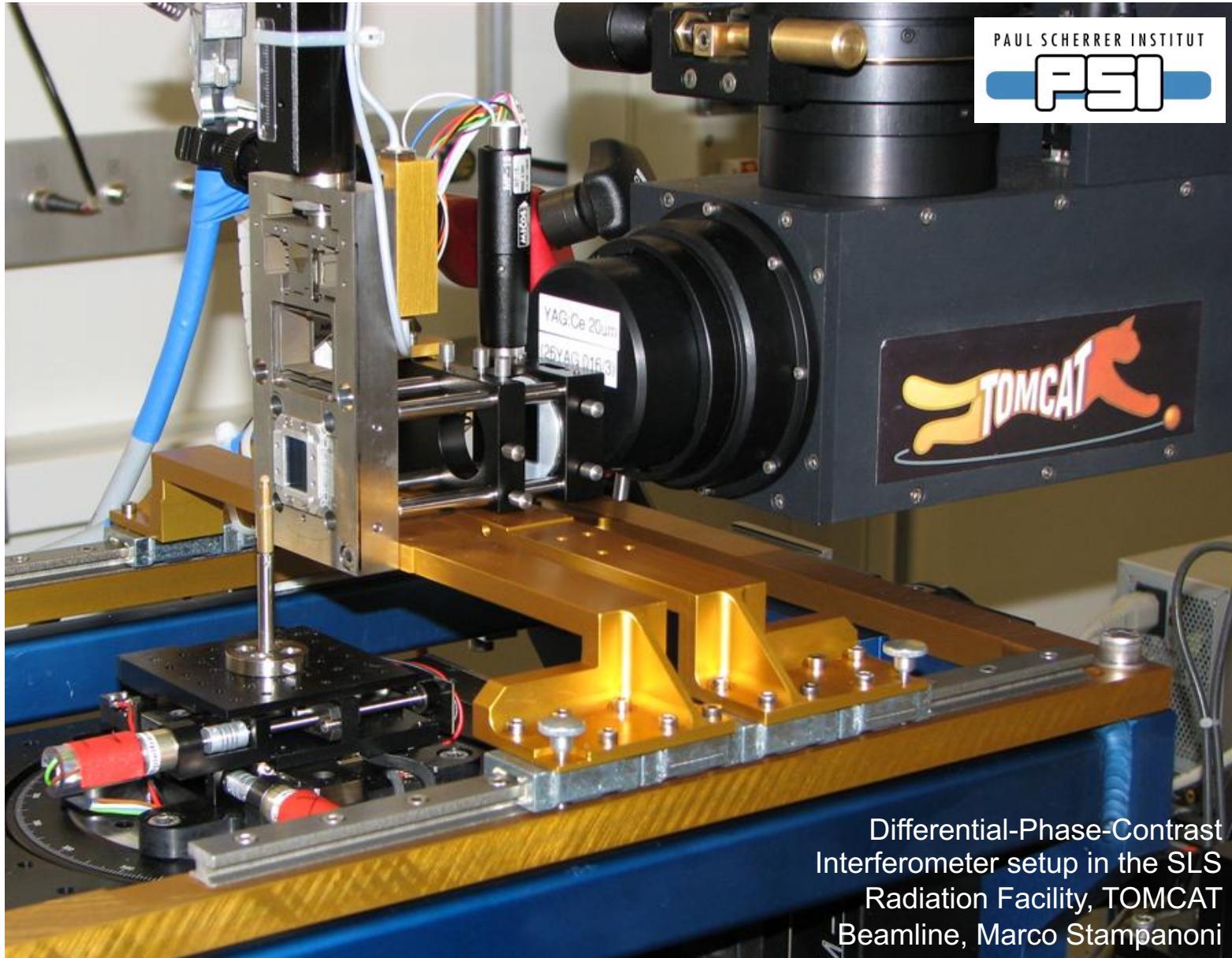
# Nanoconverter : differential linear conversion



## Reduction factor as a function of the offset (e.g. for L=30 mm)

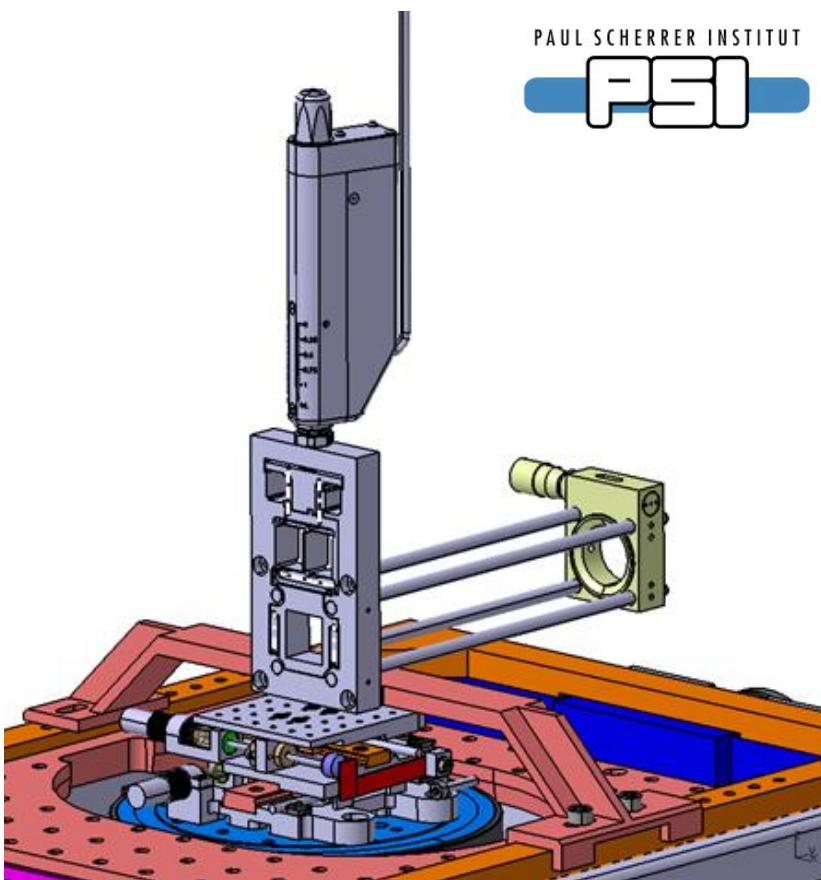


# Differential Phase Contrast setup

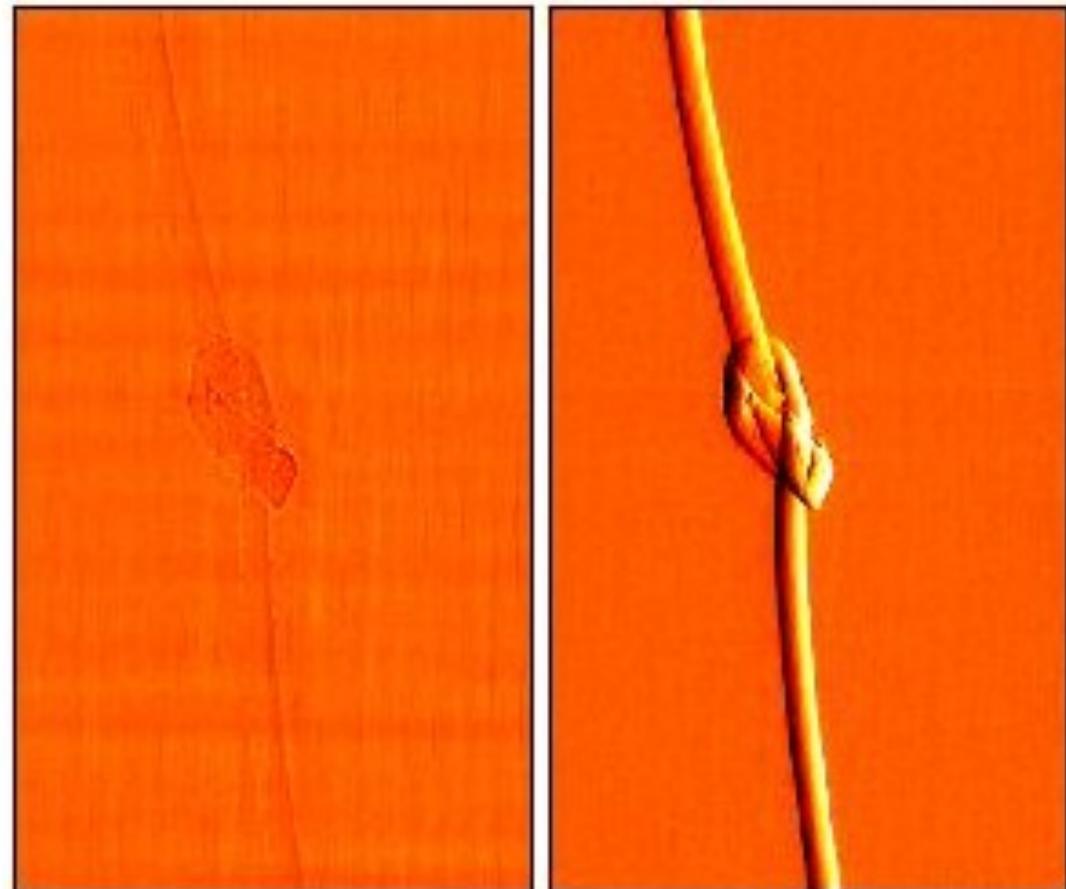


Differential-Phase-Contrast  
Interferometer setup in the SLS  
Radiation Facility, TOMCAT  
Beamline, Marco Stampanoni

# Application: Differential Phase Contrast (DPC) Interferometry



Differential-Phase-Contrast Setup



Phase gradient image (right) of a human hair with knot compared to classical absorption image (left)

*Trends in synchrotron-based tomographic imaging: the SLS experience, M. Stampanoni et al., Proceedings of SPIE, Vol. 6318, Developments in X-Ray Tomography V, Ulrich Bonse, Editor, 2006*

# Nanoconverter : conclusions

## Key advantages

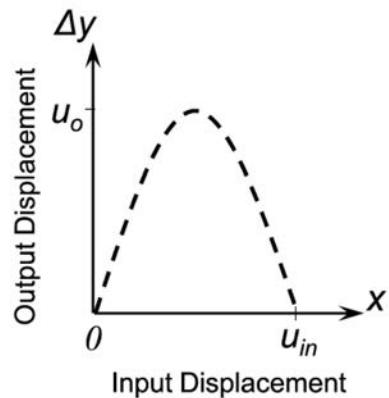
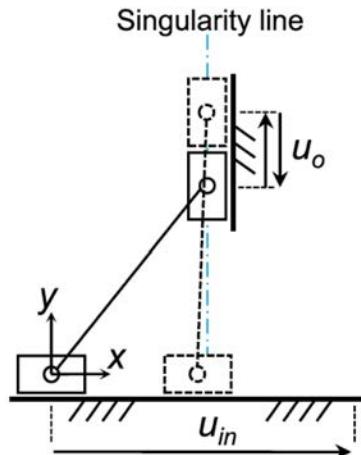
- Wide range of reduction factor achievable: typically 20 to 1000
- Linear conversion (reduction factor fixed over the full motion range)
- Planar design (2D), monolithically manufacturable by EDM, Laser, Etching...etc

[Henein S. (2006), *Device for converting a first motion into a second motion responsive to said first motion under a demagnification scale, Patent EP06021785, Holder: Paul Scherrer Institut*]

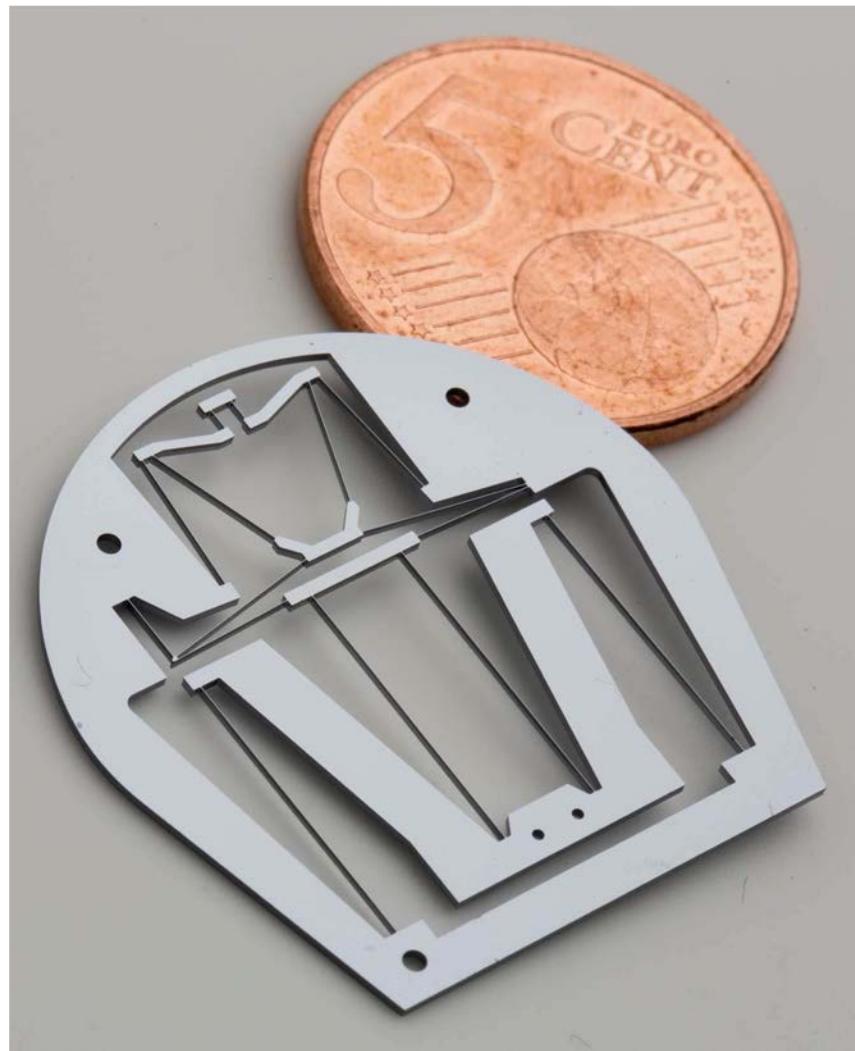
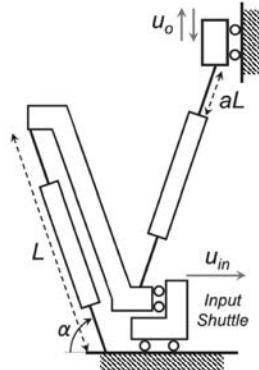
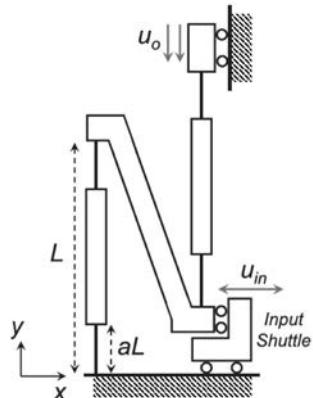
[Henein, S. et al. (2007), *The Nanoconverter: a novel flexure-based mechanism to convert microns into nanometers, Proc. of the 7th EUSPEN International Conference, Bremen*]

# Frequency quadrupler

Frequency doubling using parasitic motion



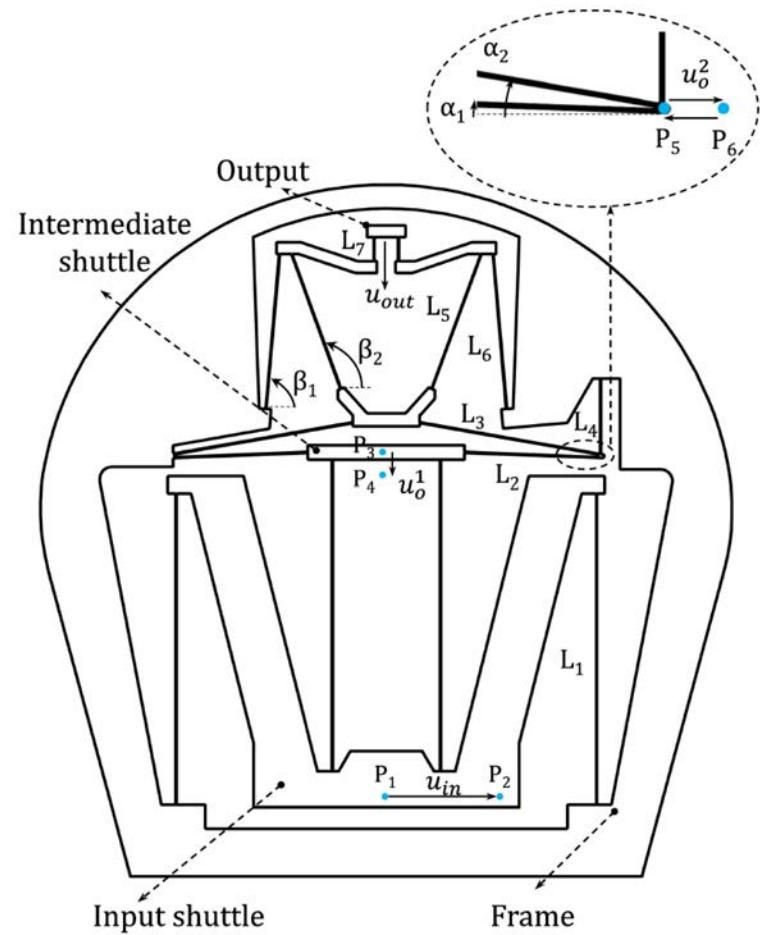
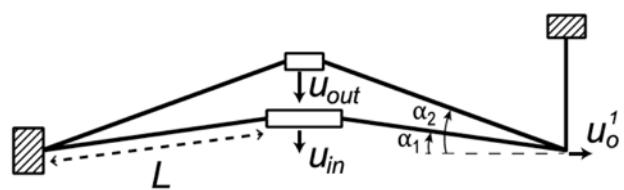
Flexure implementation



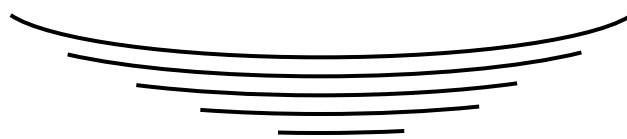
Machekposhti, D. F., Herder, J. L., Sémon, G., & Tolou, N. (2018). A Compliant Micro Frequency Quadrupler Transmission Utilizing Singularity. *Journal of Microelectromechanical Systems*, 27(3), 506–512.

# Frequency quadrupler

Combination with stroke amplifier



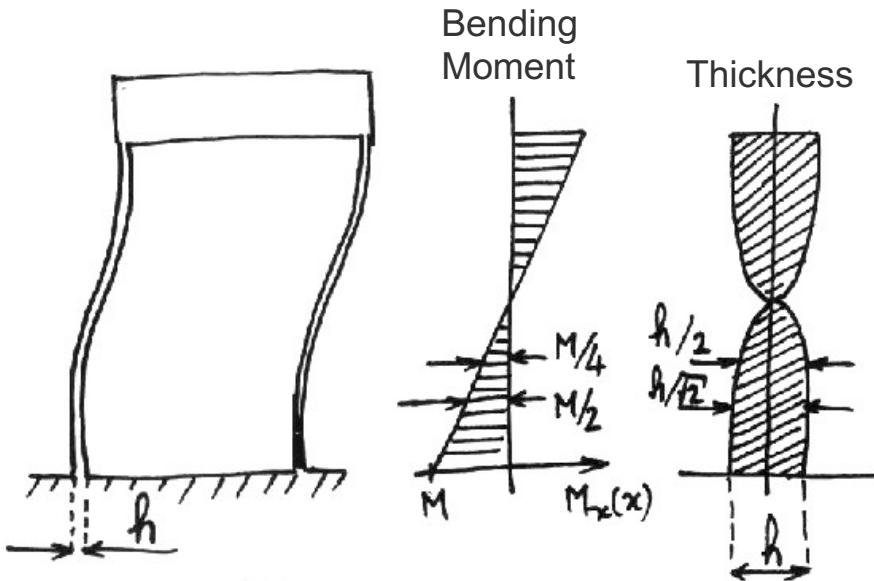
# Elastic energy storage maximization



# Parabolic blades

$$E = K x^2 / 2$$

$$E = F x / 2$$

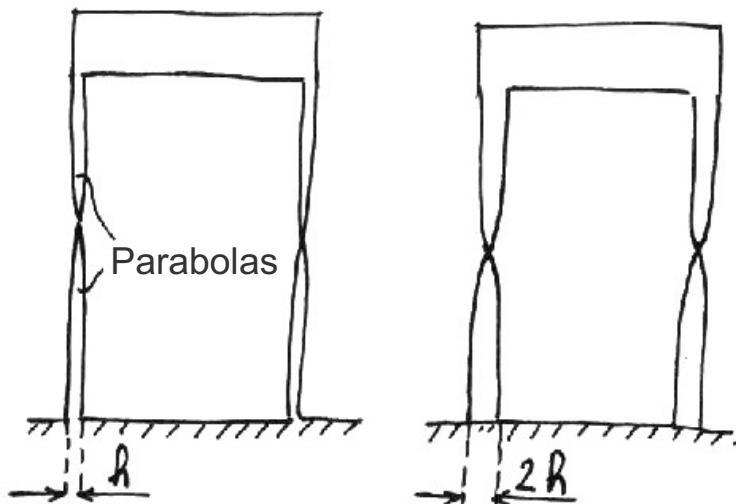


**Stroke** :  $x_{\text{adm}}$

**Stiffness** :  $K$

**Force** :  $F_{\text{maxc}}$

**Energy** :  $E$  ( $E = \frac{1}{2} K x_{\text{adm}}^2 = \frac{1}{2} x_{\text{adm}} F_{\text{maxc}}$ )



$2 \cdot x_{\text{adm}}$

$K/2$

$F_{\text{maxc}}$

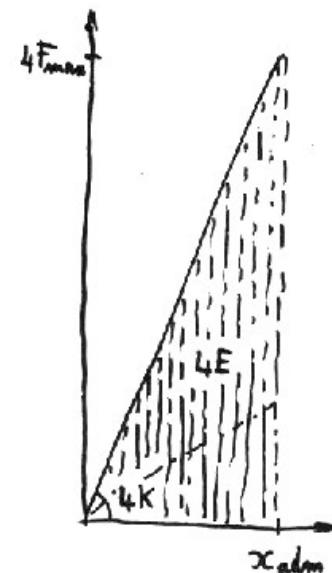
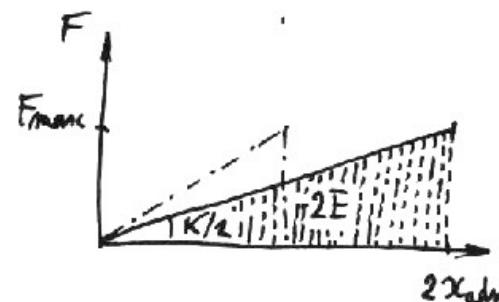
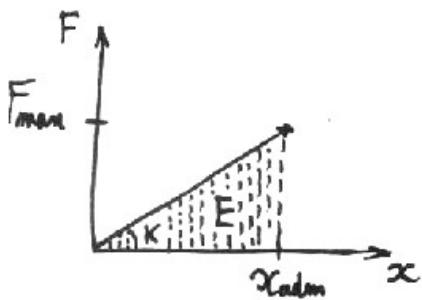
$2E$

$x_{\text{adm}}$

$4K$

$4F_{\text{maxc}}$

$4E$



# Elastic energy storage maximization

Example of a Power Counter for Bicycles



[S.Henein et al. (2000). Power Sensing Device, European Patent N° 001109196.6]

Structure optimized to get the **maximal angular stroke** for a given torque

