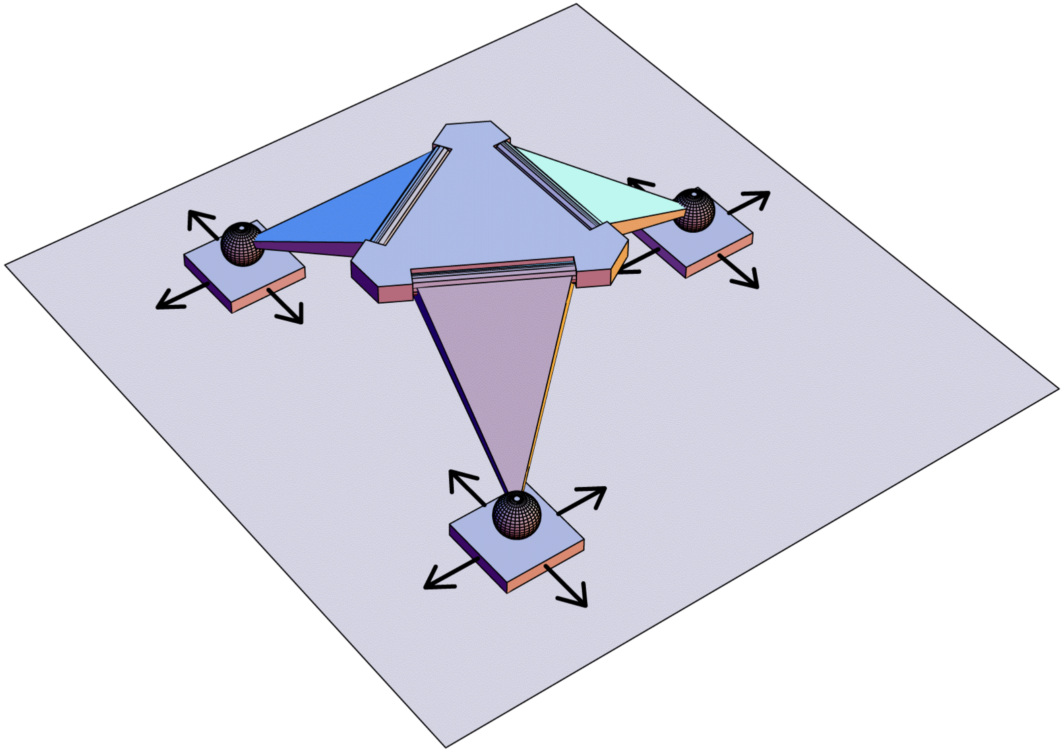
KINEMATICS OF ARTICULATED STRUCTURES



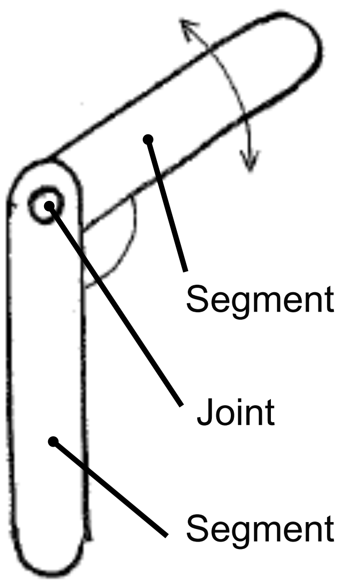
Prof. Simon Henein, Dr. Etienne Thalmann

Basic concepts of kinematics

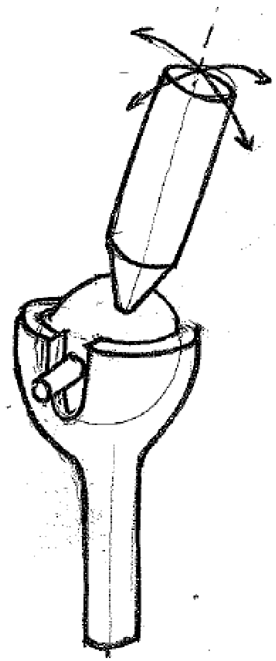
- **DOF** : Degree of Freedom
  - DOF of a **joint**
  - DOF of an **articulated structure**
  - DOS : Degree of **Spatiality**
  - **Internal** & **External** DOFs
- **M** : **Mobility** of an articulated structure (Grübler)
- **DOH** : Degree of Hyperstaticity

DOF of a joint

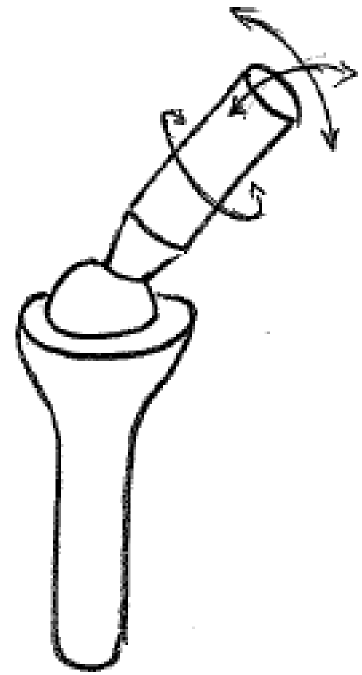
Simple examples of Degrees of **Freedom** of ideal joints



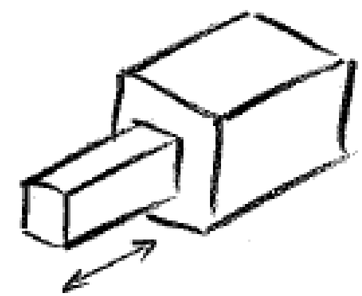
DOF = 1
Revolute joint



DOF = 2
Cardan or universal joint

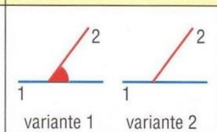
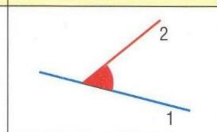
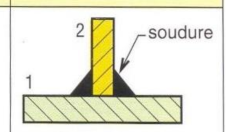
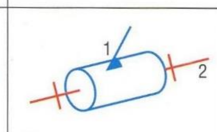
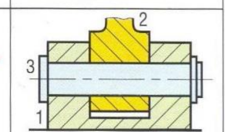
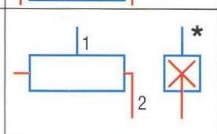
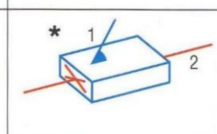
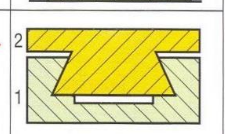
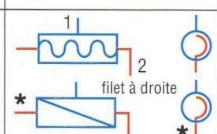
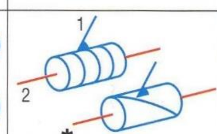
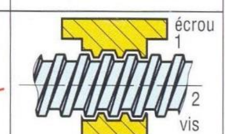
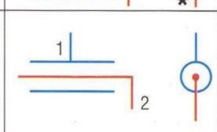
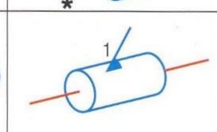
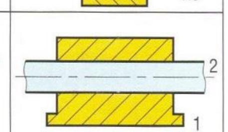
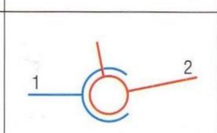
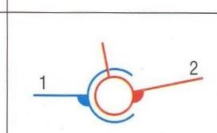
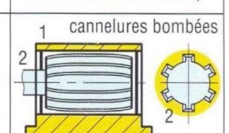


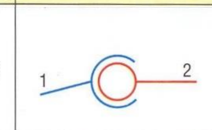
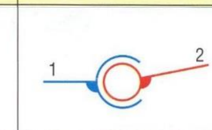
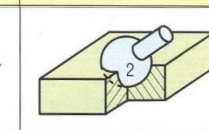
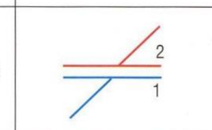
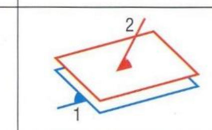
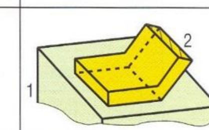
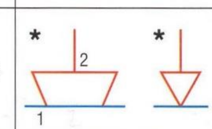
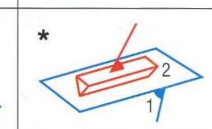
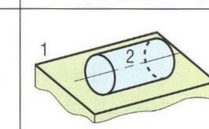
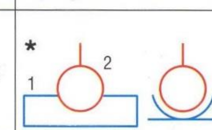
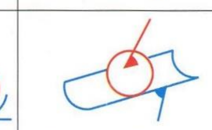
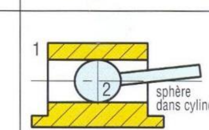
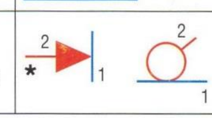
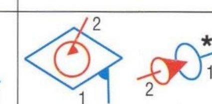
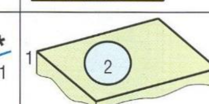
DOF = 3
Ball joint



DOF = 1
Prismatic joint

Normalized mechanical joints

Symboles des liaisons mécaniques NF EN 23952 / ISO 3952-1 NF EN ISO 3952-1						
Nom de la liaison	Translations	Rotations	Degrés de liberté	Principales représentations planes (orthogonales)	Représentation en perspective	Exemple
Encastrement ou liaison fixe	0	0	0			
Pivot	0	1	1			
Glissière	1	0	1			
Hélicoïdale	1 + 1 Combinées (fonction du pas)		1			
Pivot glissant	1	1	2			
Spérique ou rotule à doigt	0	2	2			

Symboles des liaisons mécaniques NF EN 23952 / ISO 3952-1 NF EN ISO 3952-1						
Nom de la liaison	Translations	Rotations	Degrés de liberté	Principales représentations planes (orthogonales)	Représentation en perspective	Exemple
Rotule ou sphérique	0	3	3			
Appui plan	2	1	3			
Linéaire rectiligne *	2	2	4			
Sphère cylindre ou linéaire annulaire	1	3	4			
Sphère-plan ou ponctuelle	2	3	5			

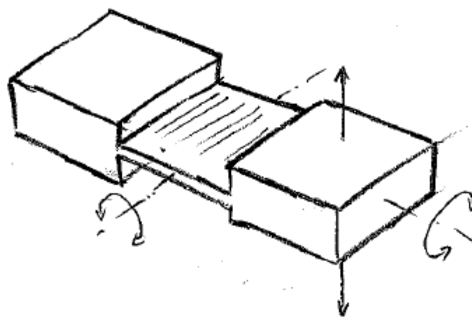
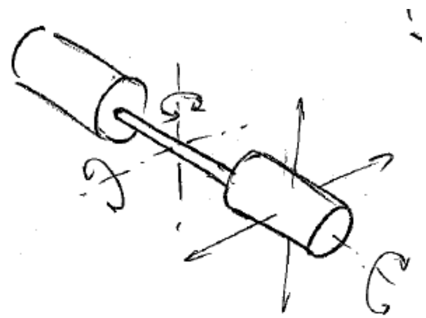
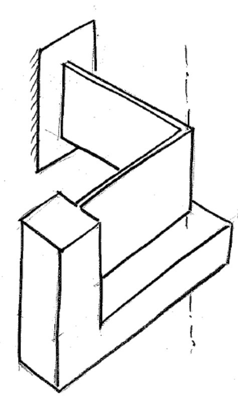
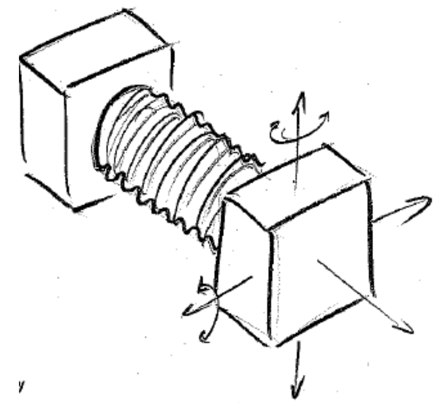
(*) ancienne normalisation NF E 04-015.

DOF of a flexure-joint

Simple examples of flexures as kinematical joints

Freedom

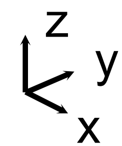
Ground

Name :	**Leaf spring**	**Rod**	**Corner-blade**	**Bellow**
Degrees of freedom:	**DOF = 3**	**DOF = 5**	**DOF = 5**	**DOF = 5**
Notation :	$\boldsymbol{L}_3{}^3$	$\boldsymbol{R}_1{}^5$	$\boldsymbol{C}_1{}^5$	$\boldsymbol{B}_1{}^5$

Remarks : - the sum of the *Freedom* and *Ground* indices is always equal to 6
- we assume that all loads are smaller than the critical loads (i.e. buckling is excluded)
- we assume that Hooke’s law and classical mechanics of material beam theory apply

Notation



- x, y, z : translations parallel to the respective axes
- rx, ry ,rz : rotations about axes parallel to the respective axes
- …^{F} : Free DOF
  - example : x^{F} = rectilinear translation along x
  - example: rx^{F} = pure rotation about an axis parallel to x
- …^{Fp}: Free DOF, accompanied with a parasitic motion
- …^{LF} : Local Degree of Freedom i.e. “free to vibrate”
- …^{LFp} : Local Degree of Freedom, with parasitic motion
- …$_{1}$: DOF blocked once
- …$_{2}$: DOF blocked twice, i.e. DOH = 1 (Degree of Hyperstaticity)
- …$_{3}$: DOF blocked thrice, i.e. DOH = 2

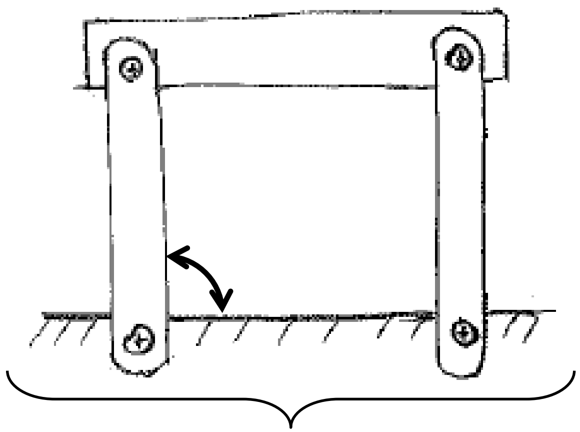
Freedom ⟶ *_ , F, Fp, LF, LFp*

Ground ⟶ *_ , 1, **2**, **3**, **4**, **5**,…*

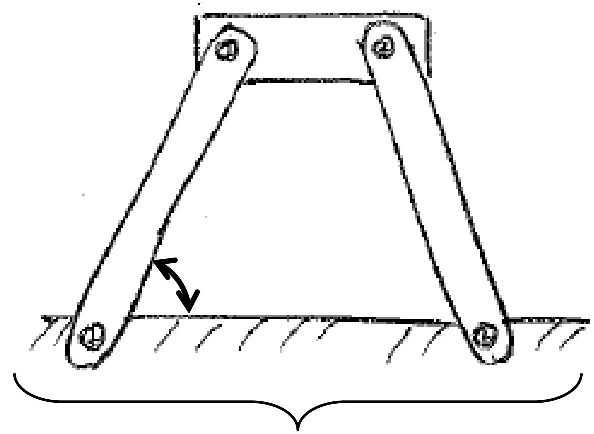
DOF of an articulated structure

Simple examples

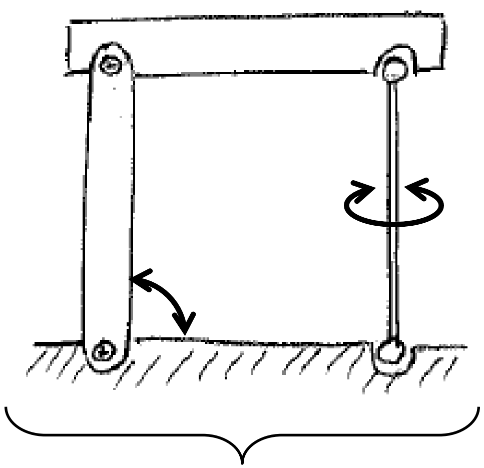
⊕	Pivot joints (1 DOF)
	Ball joints (3 DOF)



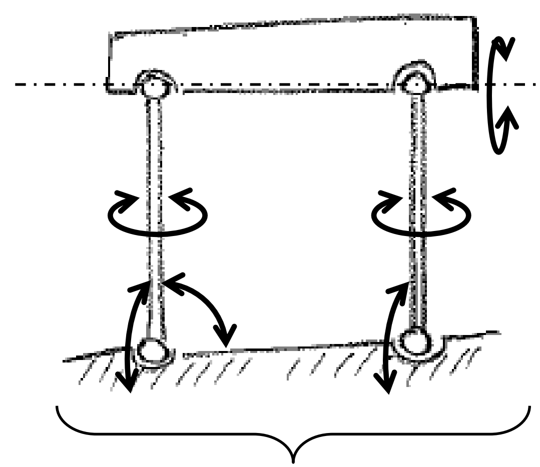
DOF = 1



DOF = 1



DOF = 2

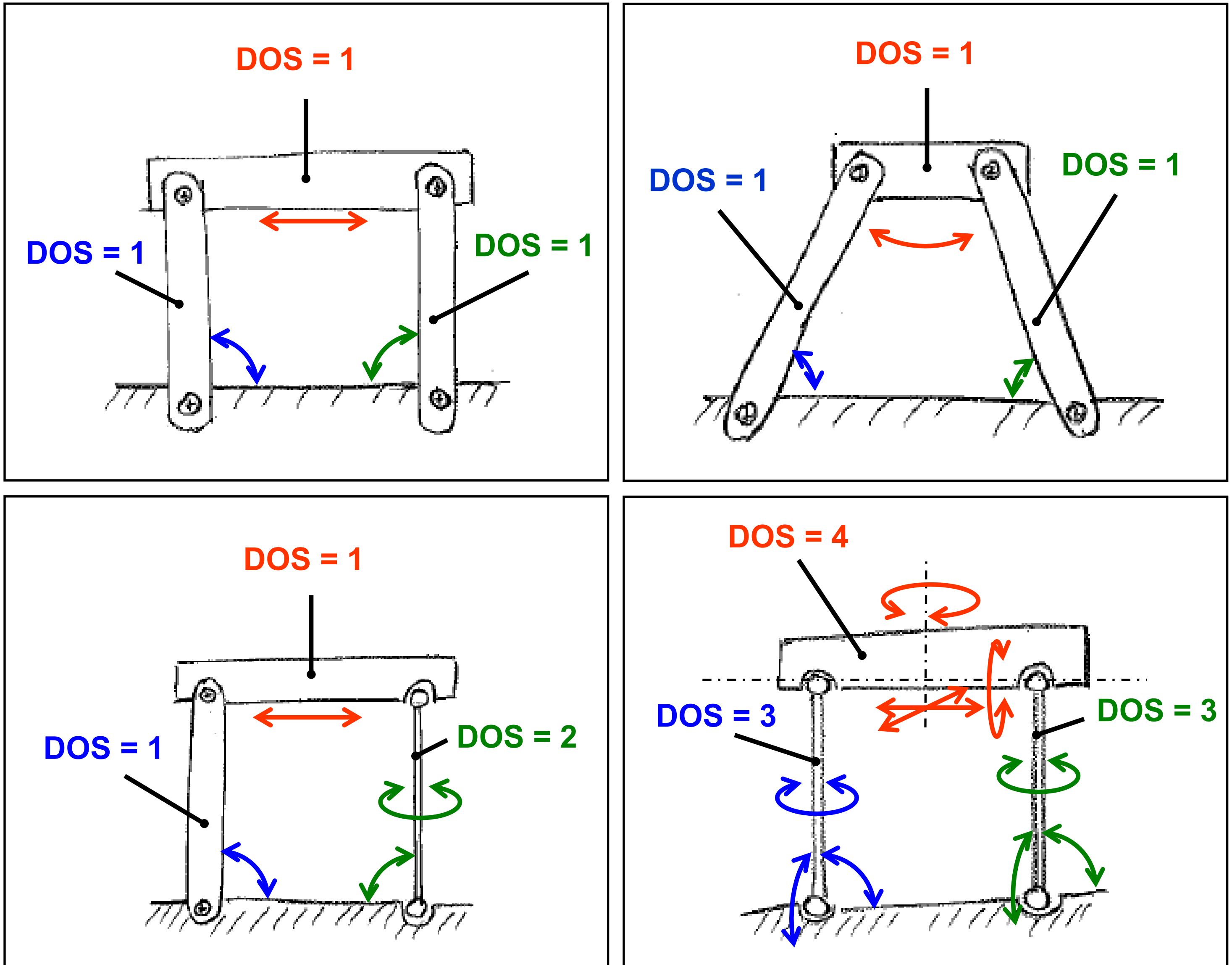


DOF = 6

DOS of a segment

Simple examples of Degrees of **Spatiality**

Remark : $0 \leq DOS \leq 6$

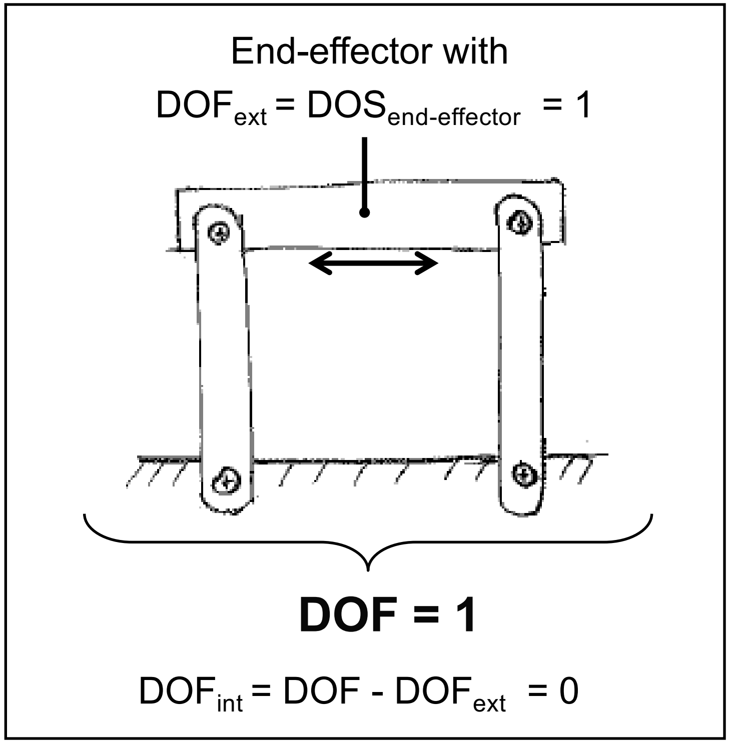


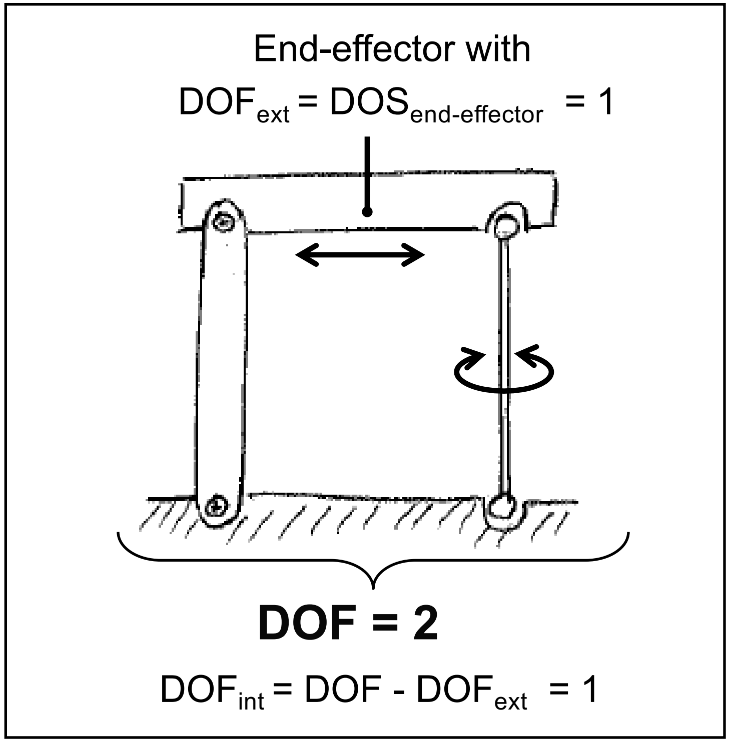
Internal and external DOFs

Applies to structures having an end-effector

- The number of external DOFs (DOF_{ext}) is equal to the number of Degrees of Spatiality of the end-effector : $\boxed{DOF_{ext} = DOS_{end\text{-}effector}}$
- The number of internal DOFs (DOF_{int}) is equal to the total number of DOFs of the structure to which the number of external DOFs is substracted:

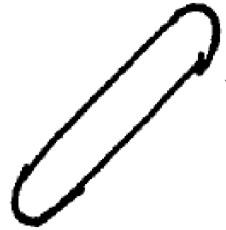
$$\boxed{DOF_{int} = DOF - DOF_{ext}}$$





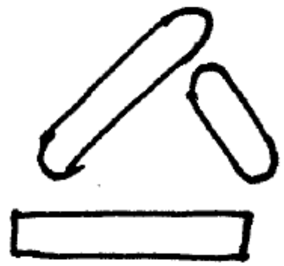
Martin Grübler's Mobility (1917) (1/3)

- 1 segment



$M = 6$

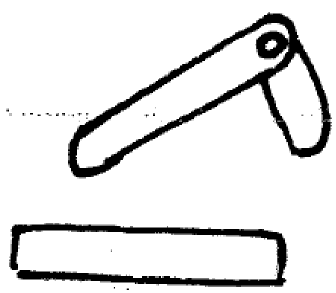
- n segments



$M = 6.n$

- n segments + 1 joint
  - d_i free DOFs of joints
  - f_i blocked DOFs of joints

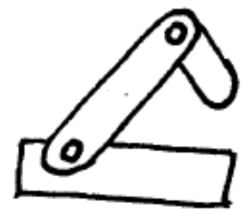
    $f_i = 6 - d_i$



$M = 6.n - (6 - d_i)$

Martin Grübler's Mobility (2/3)

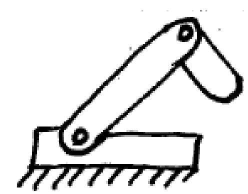
- n segments + k joints



$$M = 6.n - \sum_{i=1}^{k} (6 - d_i)$$

$$M = 6.n - 6.k + \sum d_i$$

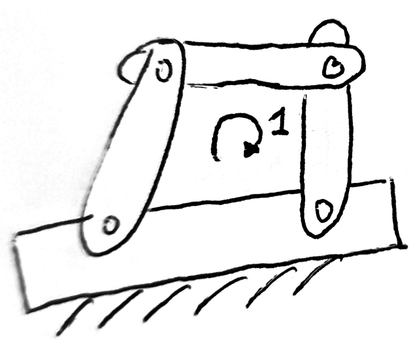
- If 1 segment is fixed (Fixed base)



$$M = 6.n - 6k + \sum d_i - 6$$

- Formulation using loops
  - b : number of closed loops

$$b = k - n + 1$$



$$M = 6n - 6k + \sum_{i=1}^{k} d_i - 6$$

$$M = \sum_{i=1}^{k} d_i - 6(k - n + 1)$$

$$M = \sum_{i=1}^{k} d_i - 6b$$

Martin Grübler's Mobility (3/3)

- Mobility of 3D mechanisms

$$M = \sum_{i=1}^{k} d_i - 6.b$$

number of DOFs of the structure

sum of the DOFs of all joints

number of closed loops

- Mobility of 2D mechanisms

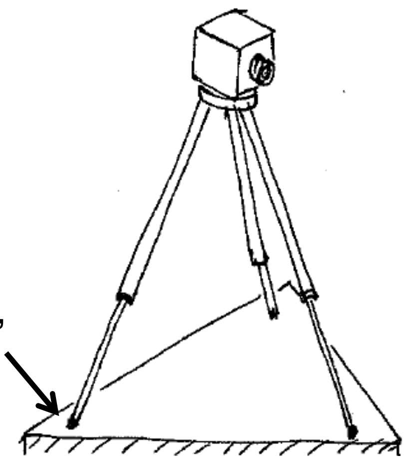
$$M = \sum_{i=1}^{k} d_i - 3.b$$

Degree Of Hyperstaticity : $\boxed{DOH = DOF - M}$

Simple examples

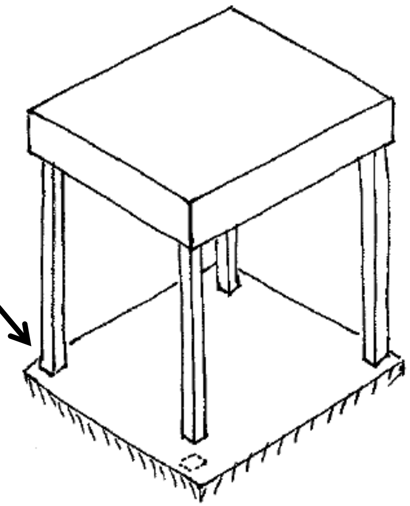
- 3D

5 DOF “joint”



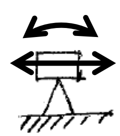
DOF = 3 ; M = 3 ; DOH = 0
(b = 2 loops)

5 DOF “joint”

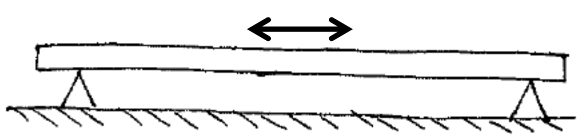


DOF = 3 ; M = 2 ; DOH = 1
(b = 3 loops)

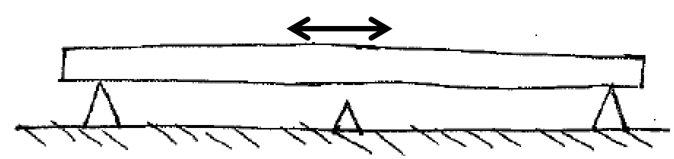
- 2D



2 DOF “joint”



DOF = 1 ; M = 1 ; DOH = 0
(b = 1 loop)

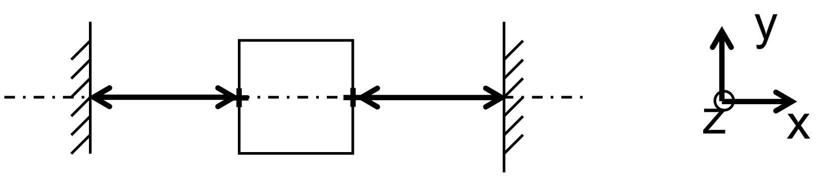


DOF = 1 ; M = 0 ; DOH = 1
(b = 2 loops)

Degree Of Hyperstaticity (overconstraint)

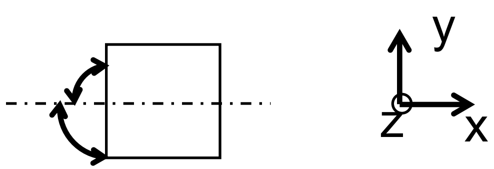
Causes

- Redundant translation constraint of two points of a rigid body both located on an axis parallel to the translation axis



x_2 (DOH = 1)

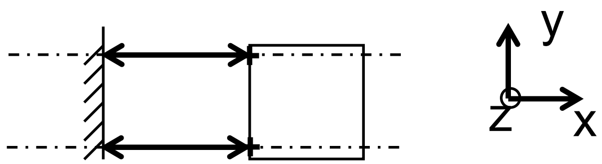
- Redundant rotation constraint of a rigid body about a given rotation axis



rz_2 (DOH = 1)

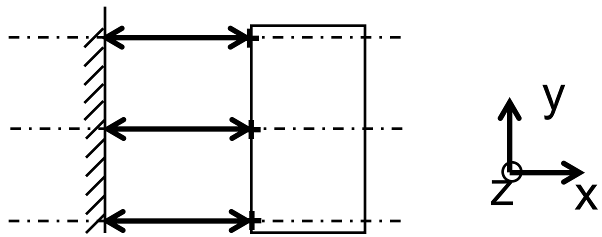
DOH : Remarks (1/2)

- 2 parallel translation-constraints of two points of a rigid body that are located on two distinct axes block one translation and one rotation DOF (no hyperstaticity).



$x_1\ rz_1$ (DOH = 0)

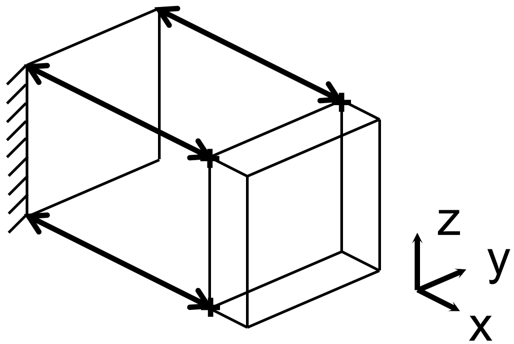
- 3 parallel translation-constraints of three points of a rigid body that are located in the same plane parallel to the translation axis lead to a hyperstaticity in rotation.



$x_1\ rz_2$ (DOH = 1)

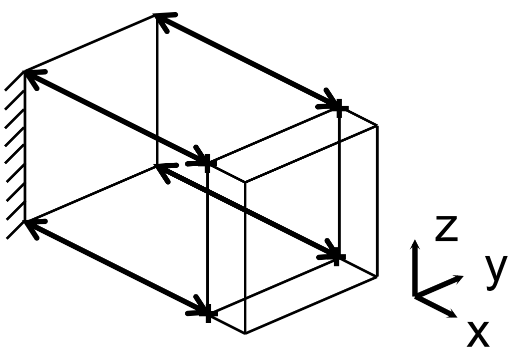
DOH : Remark (2/2)

- 3 parallel translation-constraints of three points located on distinct axes lead to a blocking of two rotations (no hyperstaticity).



$x_1\ y^F\ z^F\ rx^F\ ry_1\ rz_1$ (DOH = 0)

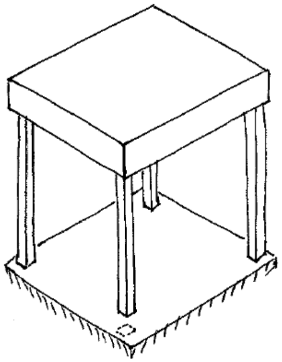
- 4 parallel translation-constraints of 4 points located on 4 distinct axes lead to a hyperstaticity in rotation.



$x_1\ y^F\ z^F\ rx^F\ ry_2\ rz_1$ (DOH = 1)
or
$x_1\ y^F\ z^F\ rx^F\ ry_1\ rz_2$ (DOH = 1)

DOH : Degree Of Hyperstaticity

Consequences

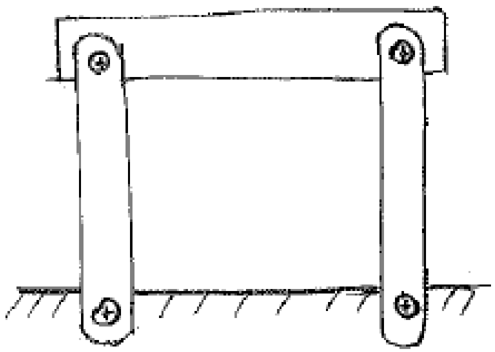


Geometrical imperfections (e.g. manufacturing tolerances or thermal expansions) lead to significant stresses in the joints and segments:

- Parasitic deformation of optical surfaces
- Stress stiffening effects (elastic energy loading and releasing) lead to a poorly predictable stiffness characteristic
- Problems during the assembly
- Possible reduction of the lifetime (fatigue)
- High stresses due to thermal effects (dilation)

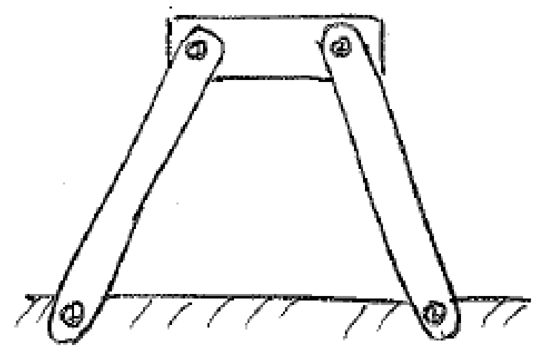
Martin Grübler's Mobility

Simple 2D and 3D examples



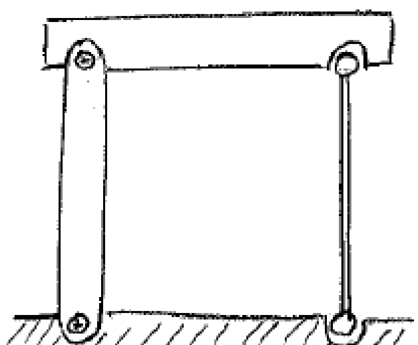
2D : **DOF = 1**; M = 1; DOH = 0

3D : **DOF = 1**; M = -2; DOH = 3



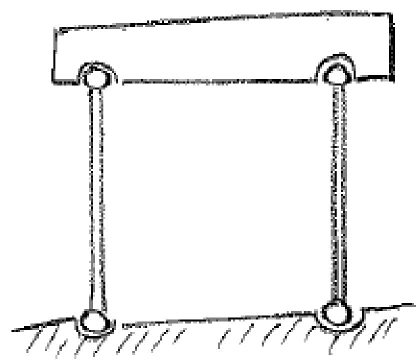
2D : **DOF = 1**; M = 1; DOH = 0

3D : **DOF = 1**; M = -2; DOH = 3



3D : **DOF = 2**; M = 2; DOH = 0

(**1 external DOF** + 1 internal DOF)



3D : **DOF = 6**; M = 6; DOH = 0

(**4 external DOF** + 2 internal DOF)

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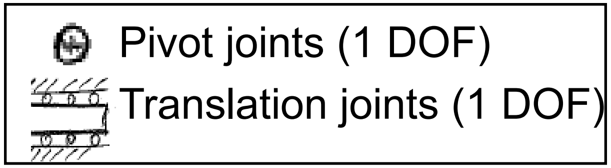
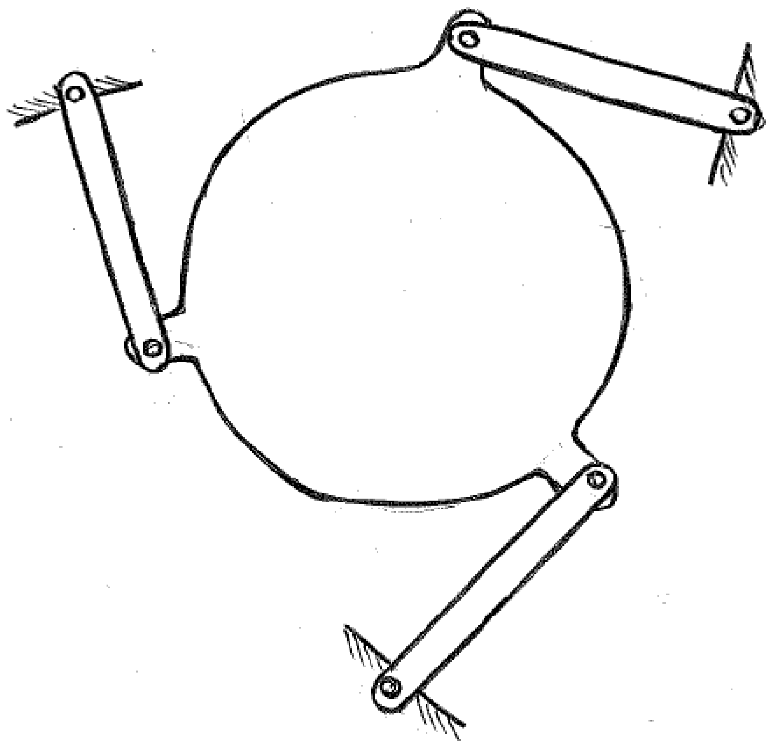
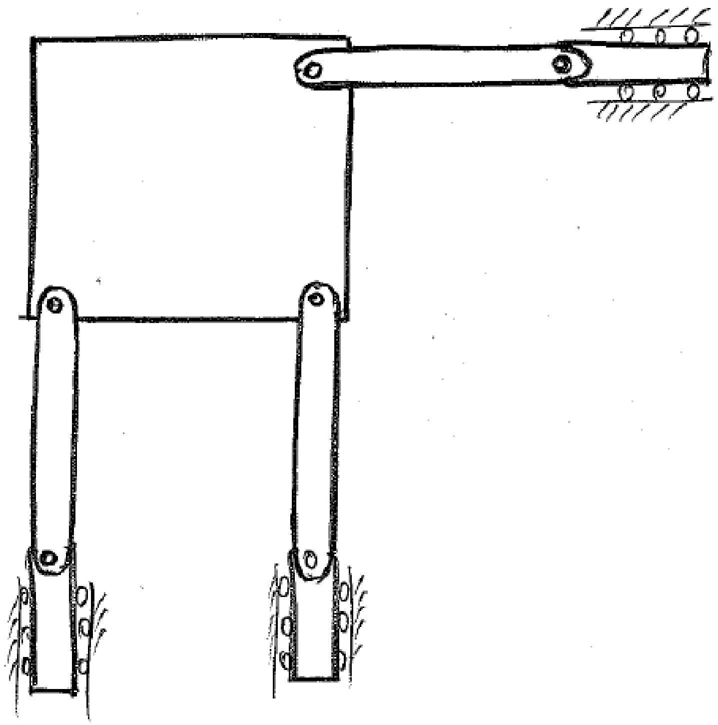
2D Example







2D : DOF = … ; M = … ; DOH = …

3D : DOF = …; M = …, DOH = …

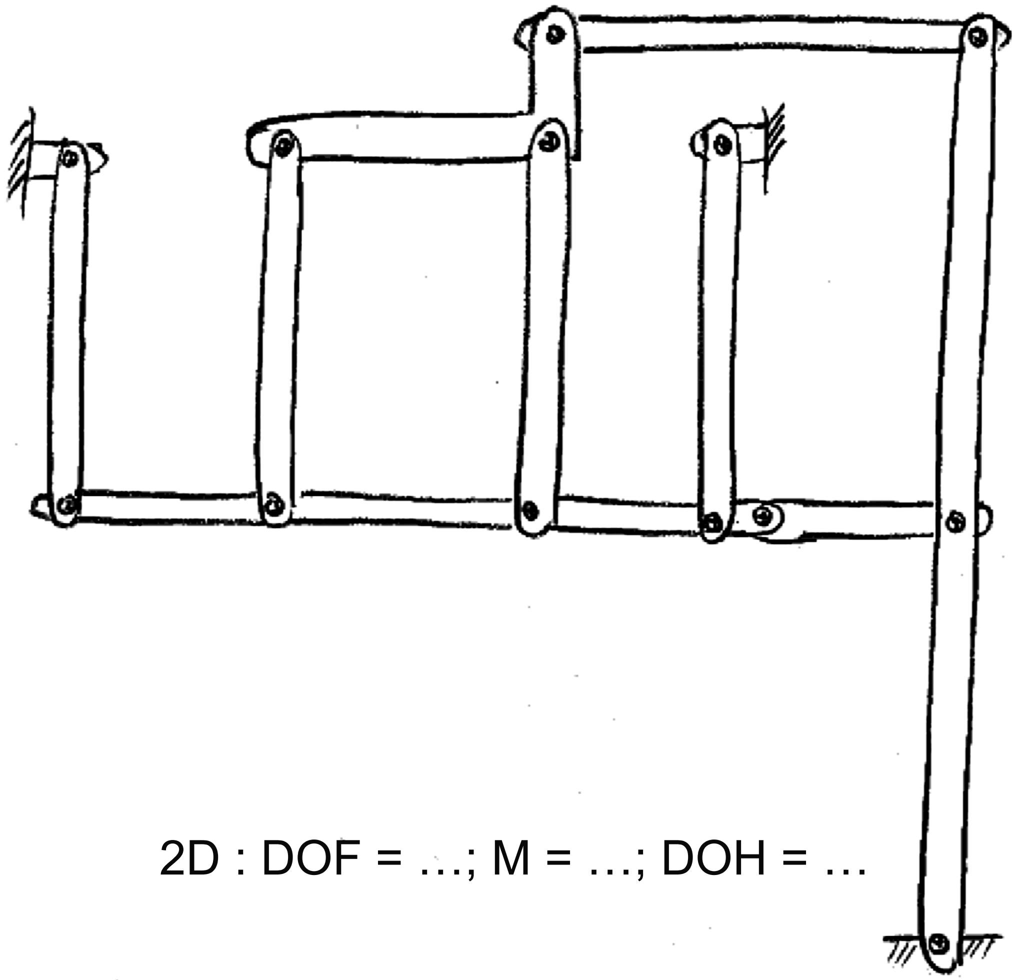
2D : DOF = …; M = …; DOH = …

3D : DOF = …; M = …; DOH = …

Martin Grübler's Mobility

2D example

Pivot joints (1 DOF)



2D : DOF = …; M = …; DOH = …

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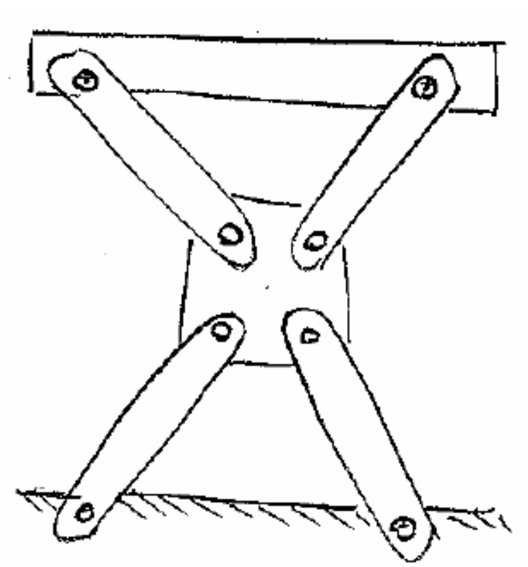
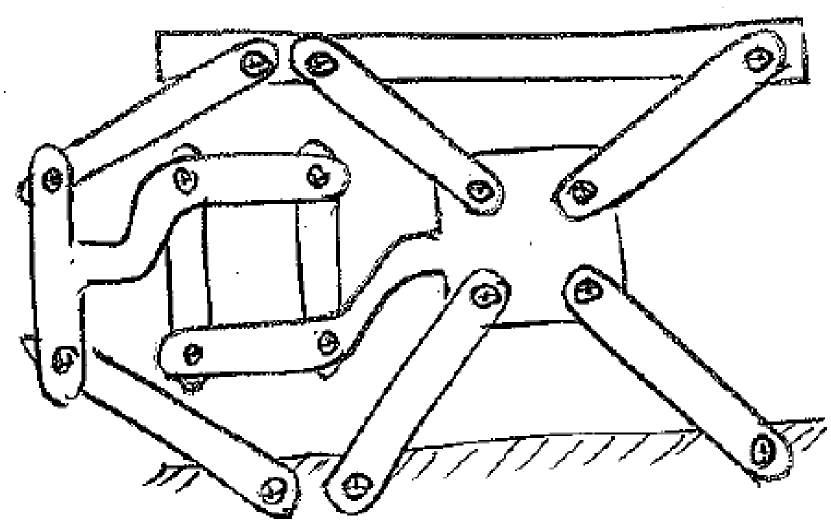
2D examples

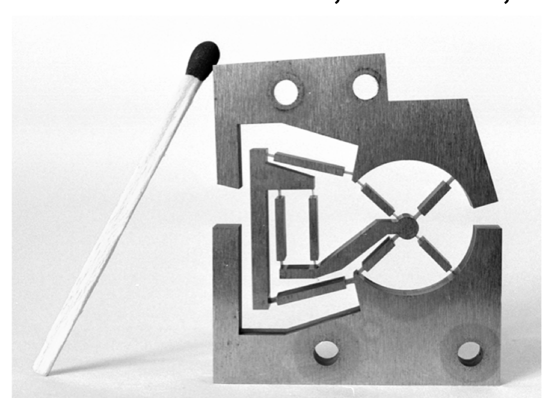
Pivot joints (1 DOF)



2D : DOF = …; M = …; DOH = …



2D : DOF = …; M = …; DOH = …



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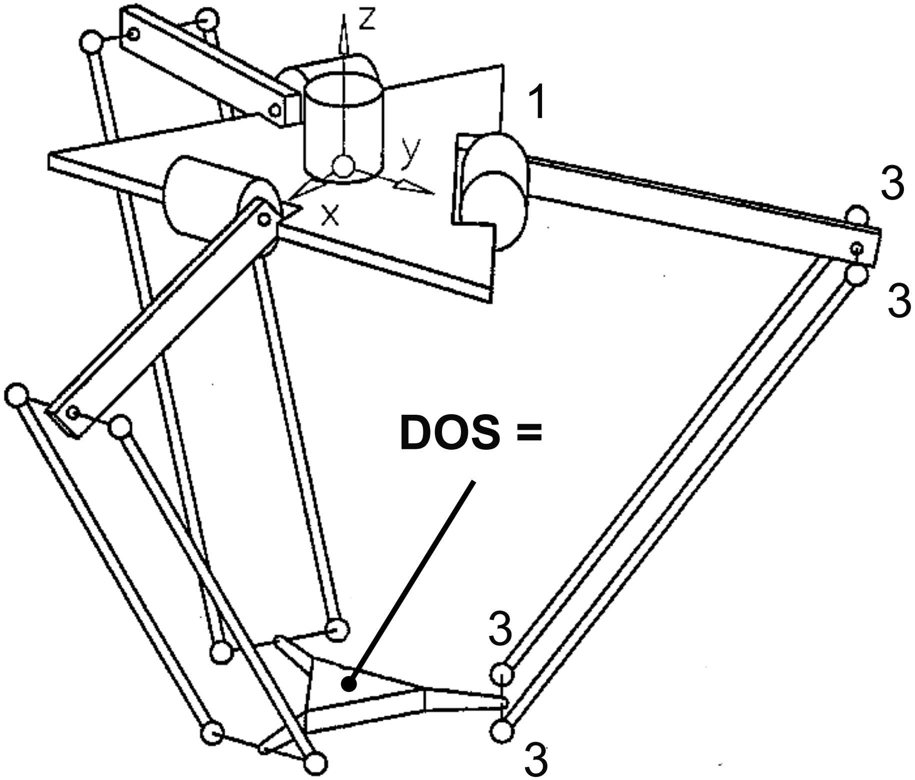
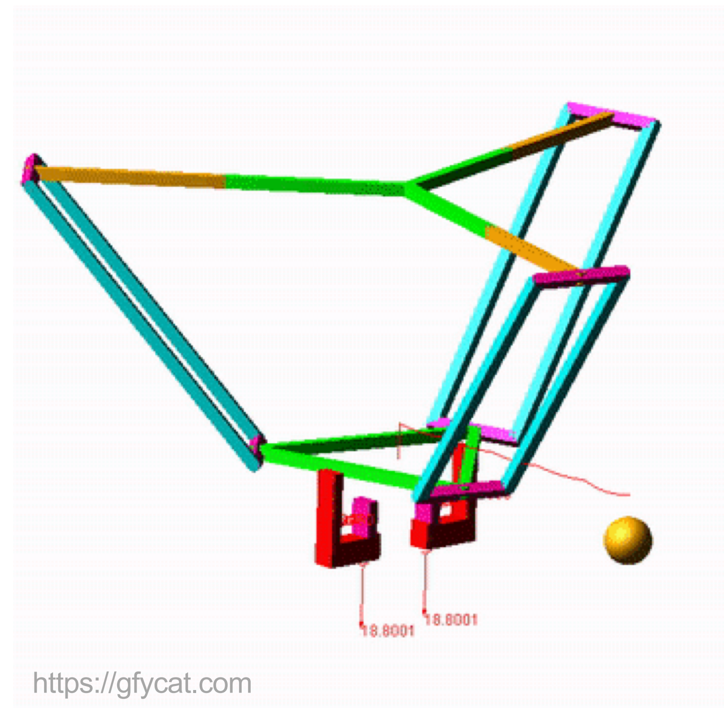
Examples of the Delta robot





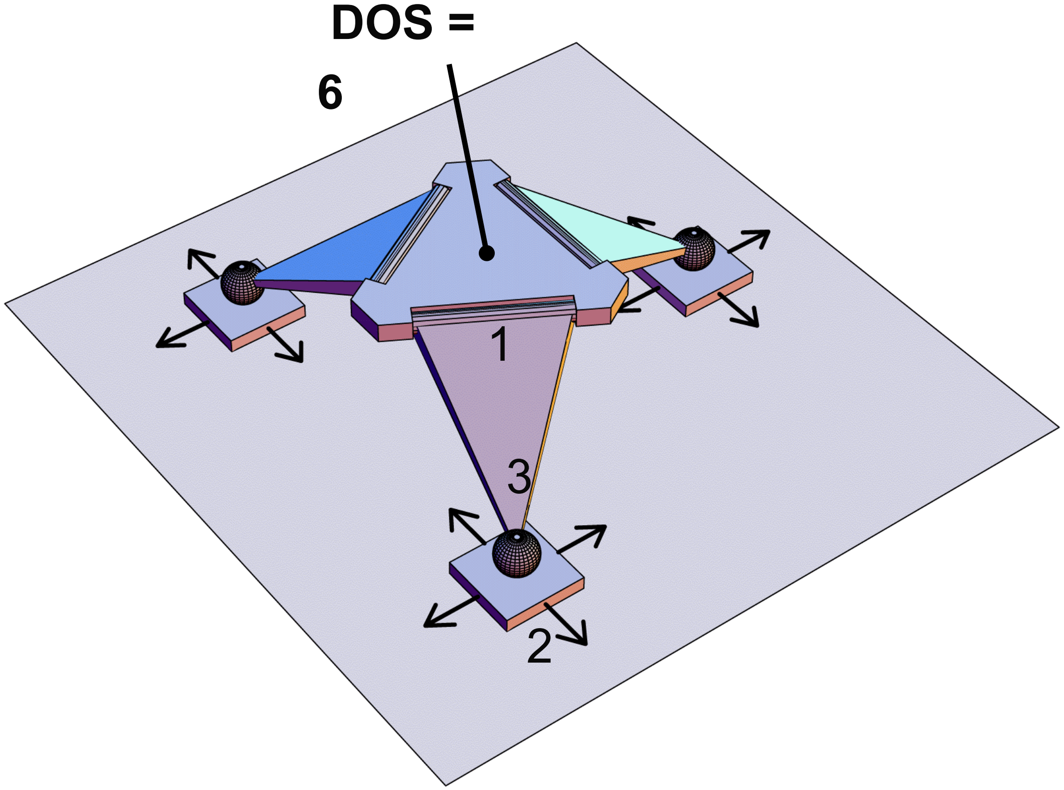
M = ...

DOF = ...

DOH = ...

Martin Grübler's Mobility

Examples of the Tribias structure



M = …

DOF = …

DOH = …

DOFs : summary

- **DOF** : Degree of Freedom
  - DOF of a **joint** : number of independent variables required to fully define the relative position of the two segments connected by the joint
  - DOF of an **articulated structure** : number of independent variables required to fully define the position of all the segments of an articulated structure composed of several segments and joints mounted in parallel or in series. While counting the number of DOF, one assumes that the geometry of the joints and segments is ideal, i.e. potential hyperstaticity issues are not taken into account. Singularity issues are ignored as well.
- **DOS** : Degree of Spatiality
  - Degree of Freedom of a rigid body attached to a fixed base by one or several chains or segments and joints.
- **M** : Mobility of an articulated structure (Grübler):
  - Sum of the DOF of all the individual joints of the structure minus 6 times the number of closed loops
- **DOH** : Degree Of Hyperstaticity of an articulated structure
  - DOH = DOF – M

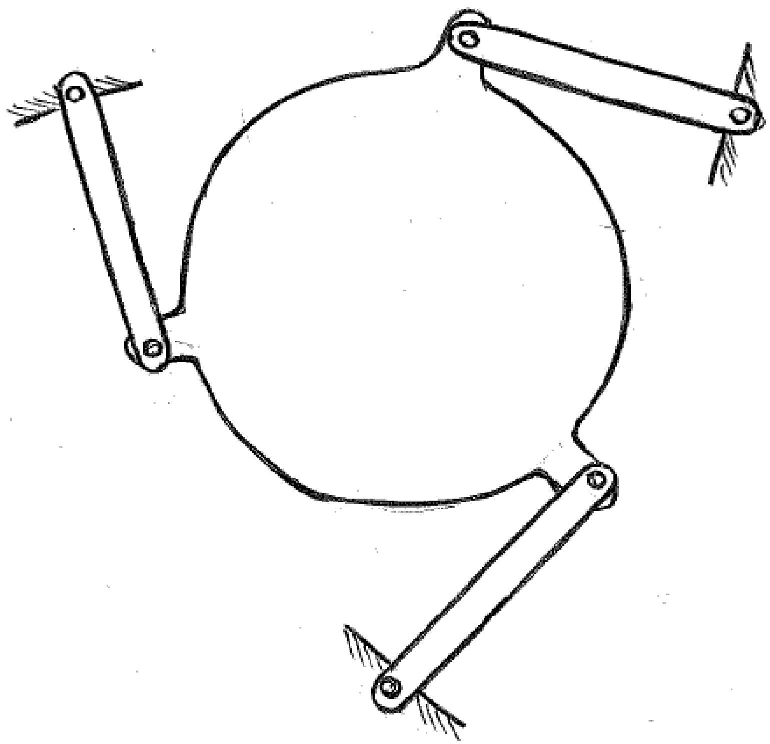
Solutions

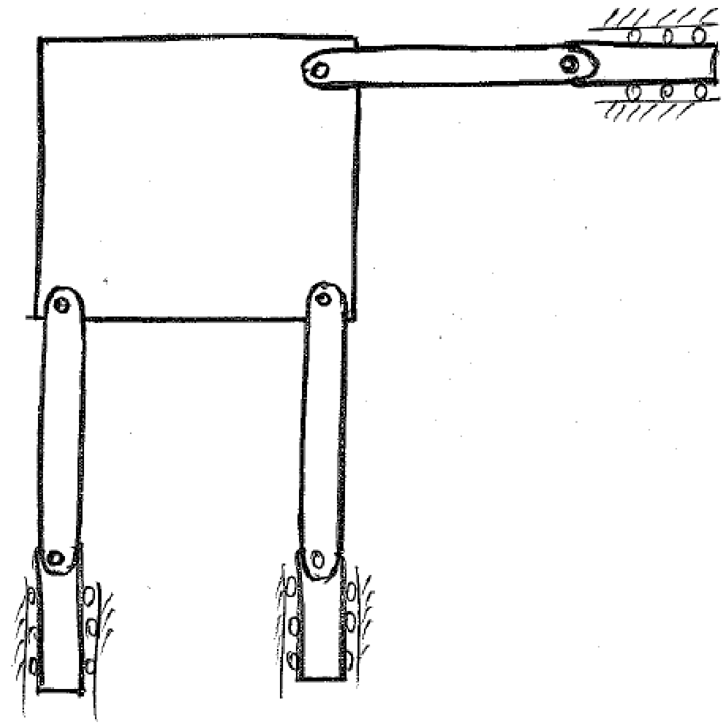
Martin Grübler's Mobility

2D Example

Pivot joints (1 DOF)

Translation joints (1 DOF)





2D : **DOF = 0**; M = 0; DOH = 0

3D : **DOF = 0**; M = -6, DOH = 6

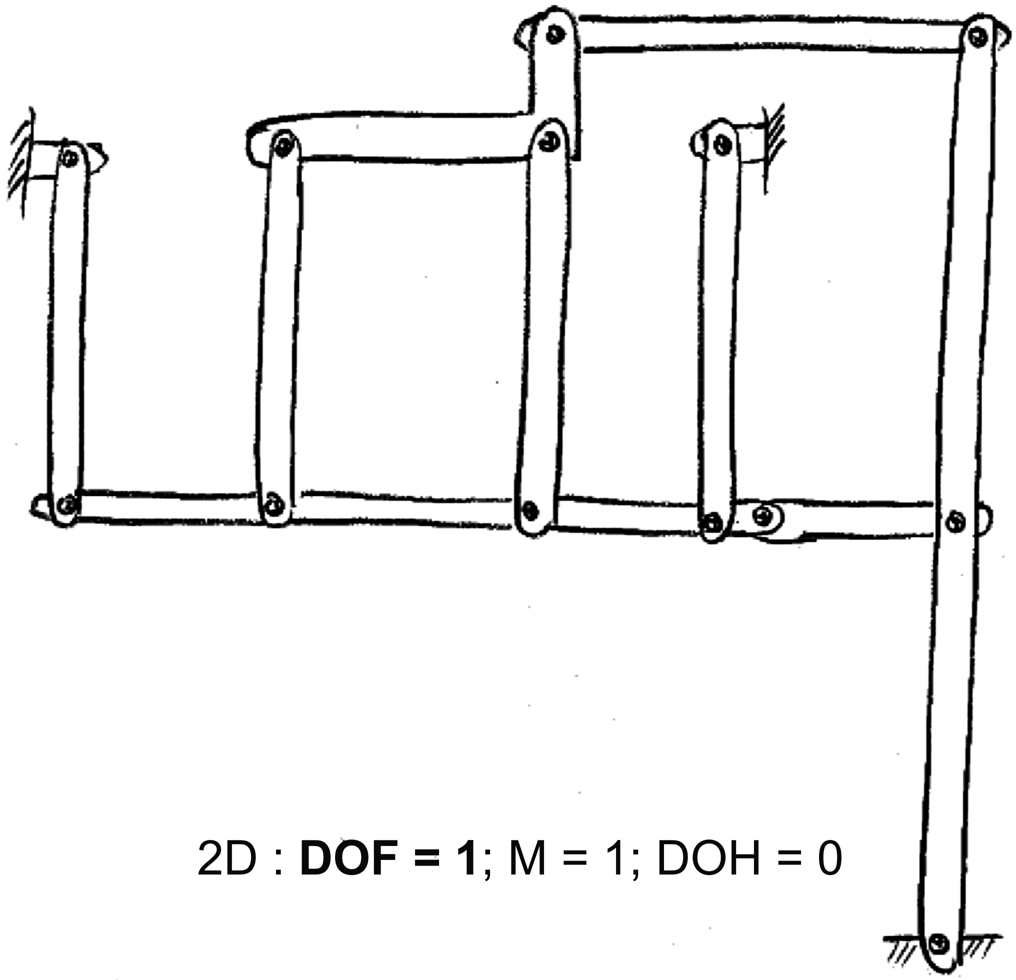
2D : **DOF = 3**; M = 3; DOH = 0

3D : **DOF = 3**; M = -3; DOH = 6

Martin Grübler's Mobility

2D example

Pivot joints (1 DOF)

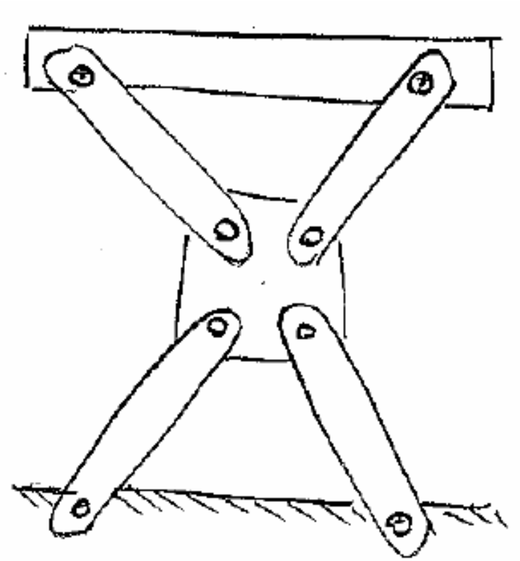


2D : **DOF = 1**; M = 1; DOH = 0

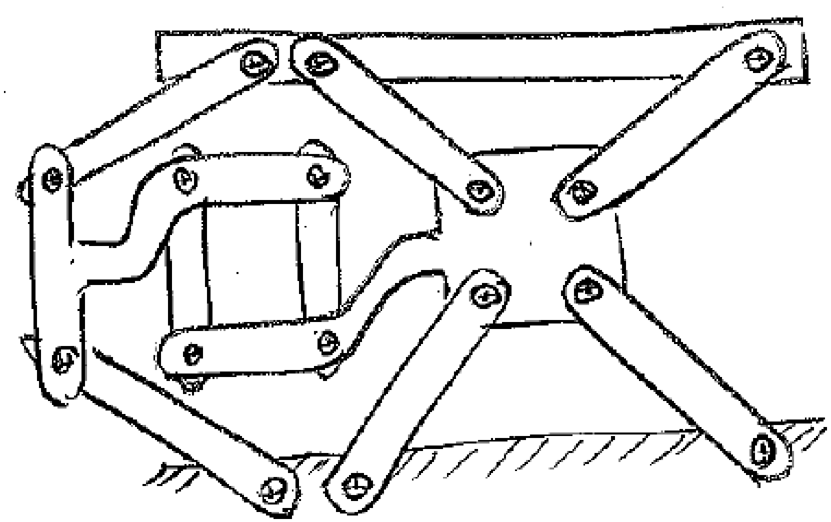
Pivot joints (1 DOF)

Martin Grübler's Mobility

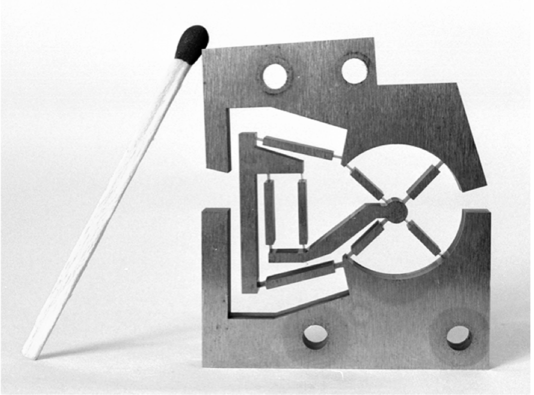
2D examples



2D : **DOF = 2**; M = 2; DOH = 0

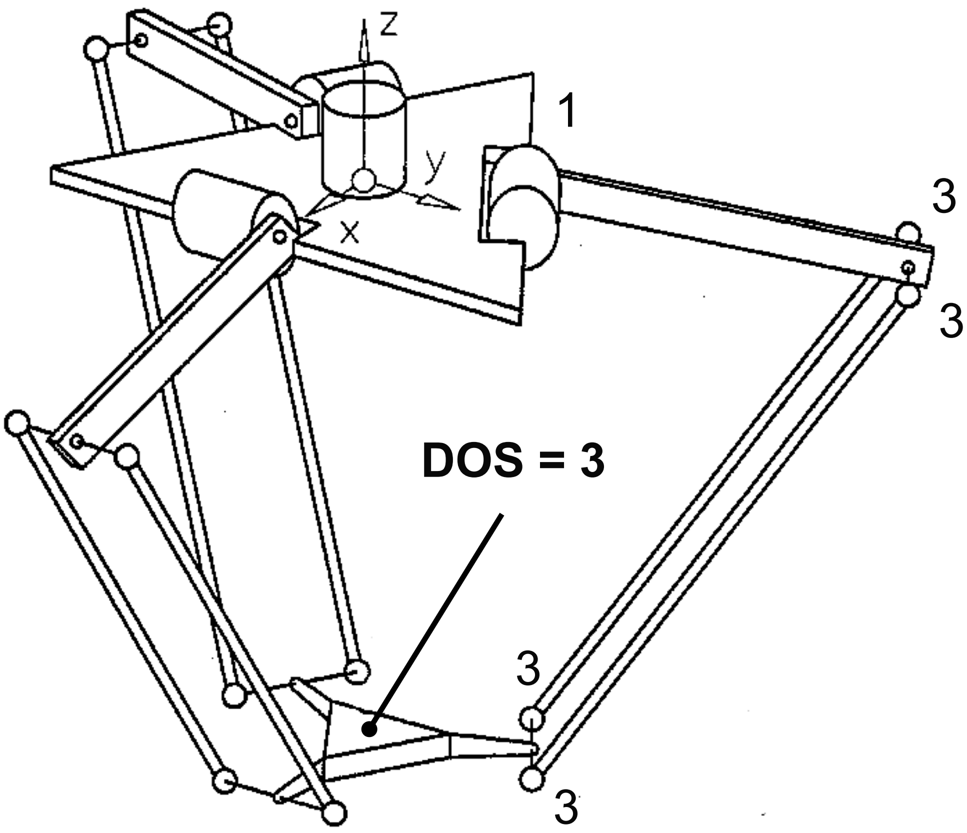


2D : **DOF = 1**; M = 1; DOH = 0



Martin Grübler's Mobility

Examples of the Delta robot



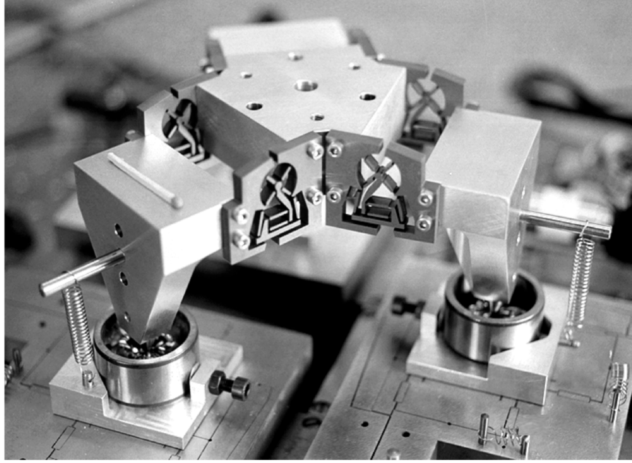
$M = 13 \times 3 - 6 \times 5 = 39 - 30 = 9$

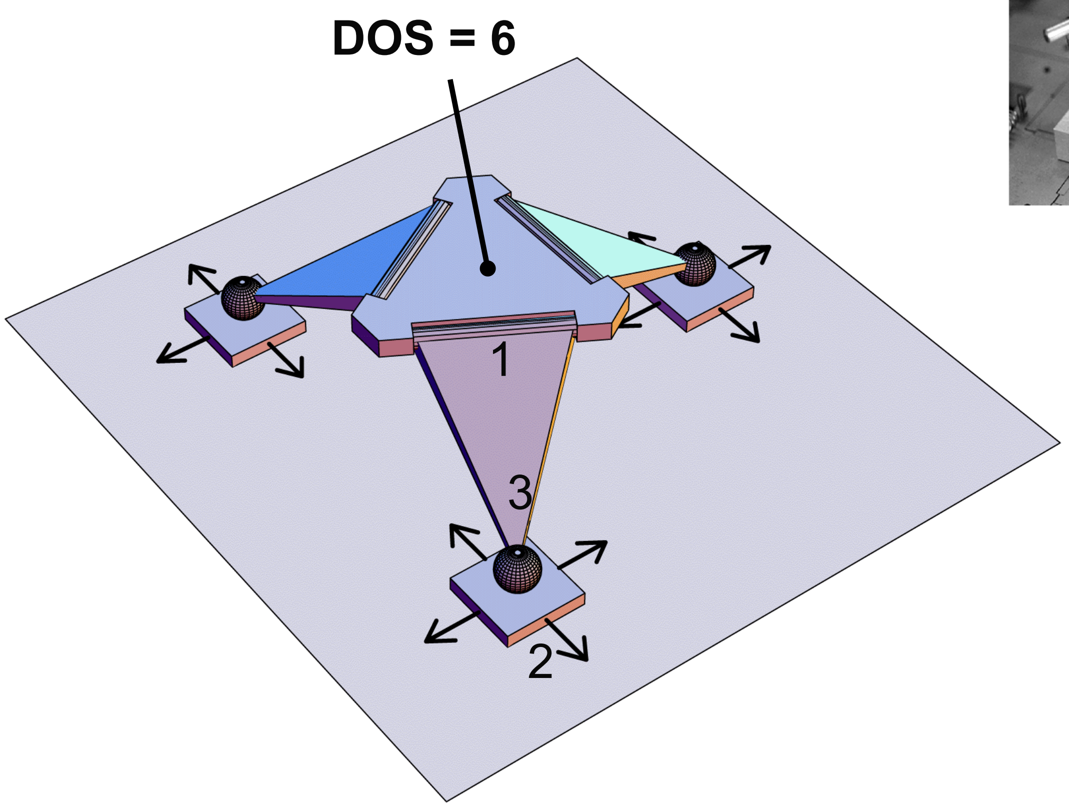
DOF = 9 (3 external + 6 internal)

DOH = 0

Martin Grübler's Mobility

Examples of the Tribias structure





M = 6 x 3 – 6 x 2 = 18 – 12 = 6

DOF = 6

DOH = 0