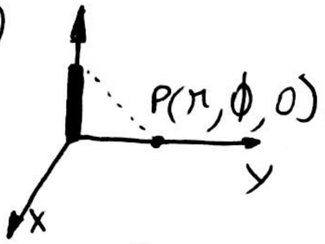
- FIO FINITO
- FIO INFINITO
- ESFERA CARREGADA
- ANEL
- DISCO
- FOLHA INFINITA

① 




POR COULOMB:

$$dE = \frac{dq}{4\pi\varepsilon_0 |R|^2}\hat{r}$$

$$\vec{R} = r\hat{r} - z\hat{z}$$

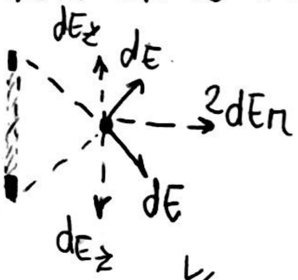
$$|R| = (r^2+z^2)^{1/2}$$

$$\hat{r} = \frac{r\hat{r} - z\hat{z}}{(r^2+z^2)^{1/2}}$$

$$dq = \rho_l\, dl = \rho_l\, dz$$

$$* \quad dE = \frac{\rho_l\, dz}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} (r\hat{r} - z\hat{z})$$

AS COMPONENTES $\hat{z}$ SE ANULAM




$$E = \frac{\rho_l}{4\pi\varepsilon_0}\int_{-L/2}^{L/2} \frac{r}{(r^2+z^2)^{3/2}}\, dz\ \hat{r}$$

$$= \frac{\rho_l \cancel{r}}{4\pi\varepsilon_0}\left[\frac{z}{r^2}\cdot\frac{1}{(r^2+z^2)^{1/2}}\right]_{-L/2}^{L/2}\hat{r}$$

$$E = \frac{\rho_l}{4\pi\varepsilon_0 r}\left[\frac{L}{2(r^2+L^2/4)^{1/2}} + \frac{L}{2(r^2+L^2/4)^{1/2}}\right]$$

$$E = \frac{\rho_l}{4\pi\varepsilon_0 r}\left(\frac{L}{(r^2+L^2/4)^{1/2}}\right)\hat{r}$$

FAZENDO $L \to \infty$

$$E = \frac{\rho_l}{4\pi\varepsilon_0 r}\left(\frac{1}{\left(\frac{r^2}{L^2}+\frac{1}{4}\right)^{1/2}}\right)$$

$$\lim_{L\to\infty} \quad \frac{r^2}{L^2} \to 0$$

$$\boxed{E = \frac{\rho_l}{2\pi\varepsilon_0 r}\hat{r}}$$

② FIO INFINITO

POR GAUSS:

$$\iint D\, ds = \int \rho_l\, dl$$

$$\int_0^{2\pi}\int_{-\infty}^{+\infty} (D\,\hat{r})(r\,d\phi\, dz\ \hat{r}) = \int_{-\infty}^{+\infty} \rho_l\, dz$$

$$D\, r\,(2\pi) = \rho_l$$

$$\varepsilon E\, r\,(2\pi) = \rho_l$$

$$\boxed{E = \frac{\rho_l}{\varepsilon\, r\, 2\pi}\hat{r}}$$

③ ESFERA CARREGADA

RAIO = a

PARA R (SUP. GAUSSIANA):

$$\boxed{R \leq a}$$

$$\iint D . ds = \iiint \rho_v\, dV$$

$$\iint (D\,\hat{r})\, R^2 \operatorname{sen}\theta\, d\phi\, d\theta = \iiint \rho_v\, dV$$

$$D R^2 \int_0^{2\pi}\int_0^{\pi} \operatorname{sen}\theta\, d\phi\, d\theta = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} \rho_v\, r^2 \operatorname{sen}\theta\, dr\, d\phi\, d\theta$$

$$D R^2 = \rho_v \left.\frac{r^3}{3}\right|_0^R$$

$$\varepsilon E = \frac{\rho_v R^{\cancel{3}}}{3 \cancel{R^2}}$$

$$\boxed{\vec{E} = \frac{\rho_v R}{3\varepsilon}\,\hat{r}}$$

$$\boxed{R \geq a}$$

$$\iint D \cdot dS = \iiint \rho_v\, dV$$

$$D \int_0^{2\pi}\int_0^{\pi} R^2 \operatorname{sen}\theta\, d\theta\, d\phi = \rho_v \int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2 \operatorname{sen}\theta\, d\theta\, dr\, d\phi$$

$$D R^2 = \rho_v \left[\frac{r^3}{3}\right]_0^a$$

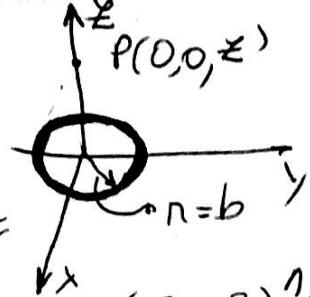
$$\boxed{E = \frac{\rho_v a^3}{3R^2\varepsilon}\,\hat{r}}$$

④ ANEL CARREGADO

POR COULOMB

$$dE = \frac{dq}{4\pi\varepsilon_0 |R|^2}\,\hat{r}$$

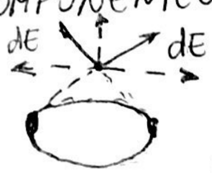
$$\vec{R} = z\hat{z} - b\hat{r}$$




- $|R| = (z^2 + b^2)^{1/2}$
- $r = b$
- $0 \leq \phi \leq 2\pi$
- $\hat{r} = \dfrac{z\hat{z} - b\hat{r}}{(z^2+b^2)^{1/2}}$
- $dq = \rho_l\, b\, d\phi$

$$dE = \frac{\rho_l\, b\, d\phi}{4\pi\varepsilon_0 (z^2+b^2)^{3/2}}\,(z\hat{z} - b\hat{r})$$

- AS COMPONENTES $\hat{r}$ SE CANCELAM

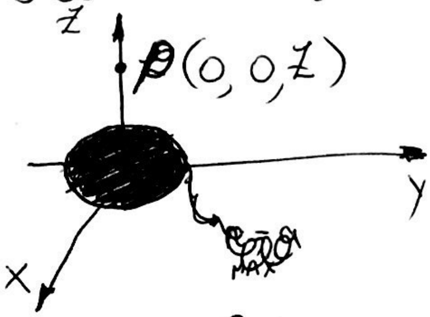



$$E = \frac{\rho_l\, b\, z}{4\pi\varepsilon_0 (z^2+b^2)^{3/2}} \int_0^{2\pi} d\phi\, \hat{z}$$

$$E = \frac{\rho_l\, b\, z}{4\pi\varepsilon_0 (z^2+b^2)^{3/2}}\,(2\pi)\,\hat{z}$$

$$\boxed{\vec{E} = \frac{\rho_l\, b\, z}{2\varepsilon_0 (z^2+b^2)^{3/2}}\,\hat{z}}$$

⑤ DISCO CARREGADO



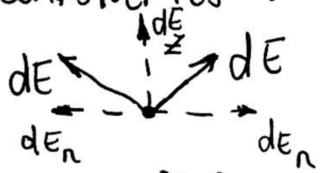

- $0 \leq r \leq a$
- $dq = \rho_s\, r\, dr\, d\phi$
- $\vec{R} = z\hat{z} - r\hat{r}$
- $|R| = (z^2 + r^2)^{1/2}$
- $\hat{r} = \dfrac{z\hat{z} - r\hat{r}}{(z^2+r^2)^{1/2}}$

POR COULOMB:

$$dE = \frac{dq}{4\pi\varepsilon_0 |R|^2}\,\hat{r}$$

$$dE = \frac{\rho_s\, r\, dr\, d\phi}{4\pi \varepsilon_0 (z^2+r^2)^{3/2}} (z\hat{z} - r\hat{r})$$

AS COMPONENTES $\hat{r}$ SE CANCELAM:




$$E = \frac{\rho_s}{4\pi\varepsilon_0}\int_0^{2\pi}\int_0^{a}\frac{z\, r\, dr\, d\phi}{(z^2+r^2)^{3/2}}\,\hat{z}$$

$$E = \frac{z\rho_s(2\pi)}{4\pi\varepsilon_0}\left[\frac{1}{z} - \frac{1}{(z^2+a^2)^{1/2}}\right]$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\left[1 - \frac{z}{(z^2+a^2)^{1/2}}\right]\hat{z}}$$

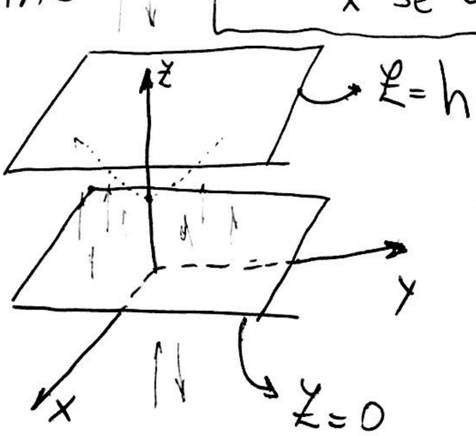
⑥ FOLHA INFINITA.

FAZENDO $a \to \infty$

$$\lim_{a\to\infty}\frac{\rho_s}{2\varepsilon_0}\left[1 - \cancel{\frac{z}{(z^2+a^2)^{1/2}}}\right] \to 0$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\varepsilon_0}\hat{z}}$$

⑦ DUAS FOLHAS INFINITAS




PARA $0 \leq P \leq h$ • $P(0,0,z)$

$$\vec{E}_1 = \frac{\rho_s}{2\varepsilon_0}\hat{z} \qquad \vec{E}_2 = \frac{\rho_s}{2\varepsilon_0}\hat{z}$$

$$\boxed{\vec{E} = 2\frac{\rho_s}{2\varepsilon_0} = \frac{\rho_s}{\varepsilon_0}\hat{z}}$$

PARA $\boxed{P \geq h}$

$$\vec{E} = 0$$

PARA $\boxed{P \leq h}$

$$\vec{E} = 0$$

~~7~~ DUAS LINHAS INFINITAS DE CARGA


DE GAUSS

$$E = \frac{\rho_l}{2\pi\varepsilon_0 r}\hat{r}$$

- $r = (1^2 + y^2)^{1/2}$
- ~~$\hat{r} = $~~
- $\hat{r} = \dfrac{-\hat{x} + y\hat{y}}{(1+y^2)^{1/2}}$
- AS COMPONENTES $\hat{x}$ SE CANCELAM.

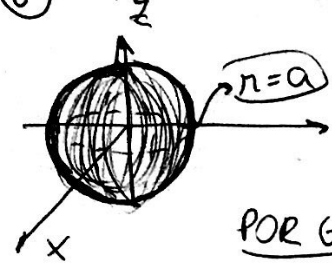
$$\vec{E}_1 = \frac{\rho_l\, y}{2\pi\varepsilon_0(1+y^2)}\hat{y}$$

POR SUPERPOSIÇÃO:

$$\vec{E} = 2.\vec{E}_1$$

$$\boxed{\vec{E} = \frac{\rho_l\, y}{\pi\varepsilon_0(1+y^2)}\hat{y}}$$

## (8) CONCHA ESFÉRICA CARREGADA




POR GAUSS

$$\iiint D \cdot ds = \iint \rho_s \, ds$$

PARA $\boxed{R<a}$

$$\vec{E} = \vec{D} = 0$$

PARA $\boxed{R>a}$

$$\int_0^{2\pi}\int_0^{\pi} D\,\hat{n} \cdot R^2 \,sen\theta\, d\theta\, d\phi = \rho_s \int_0^{2\pi}\int_0^{\pi} a^2\, sen\theta\, d\theta\, d\phi$$

$$D R^2 = \rho_s a^2$$

$$\boxed{\vec{E} = \frac{\rho_s a^2}{\varepsilon R^2}\hat{n}}$$

## (9) POTENCIAL DEVIDO CARGA PONTUAL

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 |R|^2}\hat{R} \qquad |R| = R$$

$$V = -\int_{\infty}^{R} E \cdot dl$$

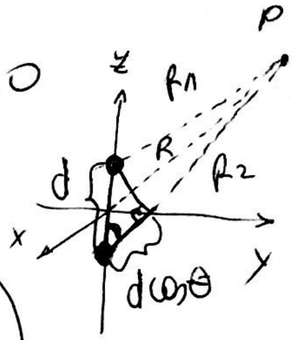
$$= -\int_{\infty}^{R} \left(\frac{q}{4\pi\varepsilon_0 |R|^2}\right) dR \qquad R^2$$

$$= \frac{q}{4\pi\varepsilon_0 R} \quad V$$

## (10) POTENCIA DEVIDO DISTRIBUIÇÃO DE CARGA PONTUAIS

$$V = \frac{1}{4\pi\varepsilon_0}\sum_{0}^{n}\frac{q}{|R_n|} \quad V$$

## (11) CAMPO ELÉTRICO DO DIPOLO

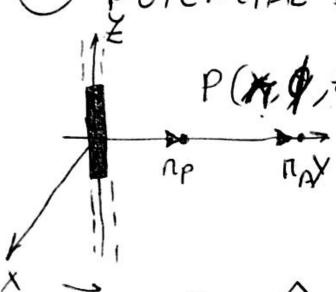



$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1} + \frac{(-q)}{R_2}\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{d\cos\theta}{R^2}\right)$$

## (12) POTENCIAL DEVIDO FIO INFINITO




$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon_0 r}\hat{r}$$

$$\int_{V_A}^{V_P} dV = -\int^{r_P} E \cdot dl$$

$$\int_{V_A}^{V_P} dV = -\int_{r_a}^{r_P} \frac{\rho_l}{2\pi\varepsilon_0 r}\, dr$$

$$V_P - V_A = \frac{\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{r_P}{r_a}\right)$$

## 13 POTENCIAL DEVIDO ESFERA CARREGADA

$$\int dV = -\int E\,dl$$

$$\rho_v = \frac{Q}{\frac{4}{3}\pi r^3}$$

$$\vec{E} = \frac{\rho_v r}{3\varepsilon}\hat{r}$$

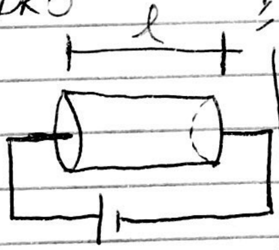
$$V = -\int_{\infty}^{R} E \cdot dr$$

$$V = -\int \frac{\rho_v r}{3\varepsilon_0}\,dr$$

$$V = -\int_{\infty}^{R} \frac{Q}{4\pi r^2 \varepsilon_0}\,dr = \frac{Q}{4\pi r \varepsilon_0}$$

$$\boxed{V = \frac{Q}{4\pi r \varepsilon_0} \quad [V]}$$

## 14 RESISTÊNCIA CILINDRO




$$R = \frac{-\int E \cdot dl}{\int J \cdot ds}$$

$$V = -E\int_{x_2}^{x_1} dR = El$$

$$I = \int J \cdot ds = \int (E\sigma\hat{y})(ds\,\hat{y})$$

$$= E\sigma A$$

$$R = \frac{El}{E\sigma A}$$

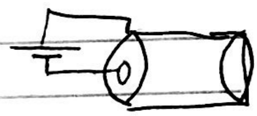


$$\boxed{R = \frac{l}{\sigma A} \quad [\Omega]}$$

## 15 RESISTOR COAXIAL

$$V = -\int E\,dl$$

Ex:




$$J = \frac{I}{2\pi r l}\hat{r}$$

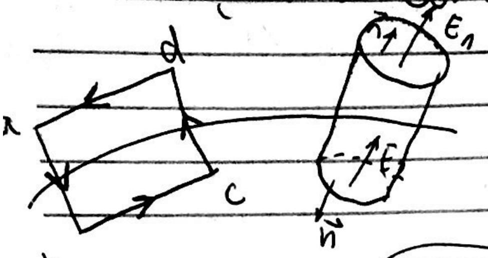
$$E = \frac{I}{2\pi r \sigma l}\hat{r}$$

$$V = \int_a^b \frac{I}{2\pi\sigma l}\cdot\frac{dr}{r}$$

$$\bullet\ V = \frac{I}{2\pi\sigma l}\ln\left(\frac{b}{a}\right)$$

$$\boxed{R = \frac{V}{I} = \frac{\ln(b/a)}{2\pi\sigma l} \quad [\Omega]}$$

## 15 CONDIÇÕES DE CONTORNO.




$$\oint E\,dl = 0$$ CAMPO CONSERVATIVO

$$\int_b^c E\,dl + \int_d^a E\,dl = 0$$

$$E_{1t}\Delta L - E_{2t}\Delta L = 0$$

$$\boxed{E_{1t} = E_{2t}}$$

GAUSS

$$\oint D \cdot ds = \iiint \rho_s ds$$

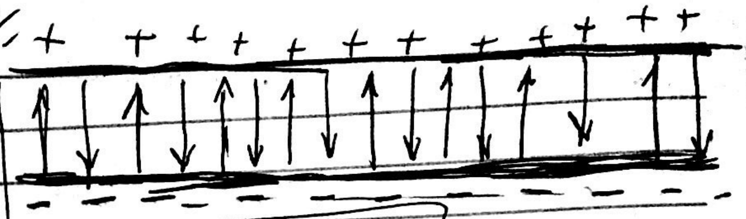
$$\int D_1 \hat{n}\, ds + \int D_2(-\hat{n})\, ds = \rho_s \Delta S$$

$$D_{1N}\Delta S - D_2 \Delta S = \rho_s \Delta S$$

$$\boxed{D_{1N} - D_{2N} = \rho_s}$$

$$\boxed{\varepsilon_1 E_{1n} - E_{2n}\varepsilon_2 = \rho_s}$$

## 16 CONDUTOR E DIELÉTRICO



$$\boxed{E_{1t} = E_{2t} = 0}$$

$$D_{1N} - \cancel{D_{2N}}^{0} = \rho_s$$

$$\boxed{D_{1N} = \rho_s}$$

$$\boxed{\varepsilon_1 E_1 = \rho_s}$$

## 17 FRONTEIRA ENTRE DOIS CONDUTORES (NÃO PERFEITOS)

$$E_{1t} = E_{2t}$$

$$\boxed{\frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2}}$$

$$\frac{\varepsilon_1 J_{1t}}{\sigma_1} - \frac{J_{2t}\varepsilon_2}{\sigma_2} = \rho_s$$

NA ELETRSTÁTICA: $J_{1t} = J_{2t}$

$$\boxed{J_{1t}\left(\frac{\varepsilon_1}{\sigma_1} - \frac{\varepsilon_2}{\sigma_2}\right) = \rho_s}$$

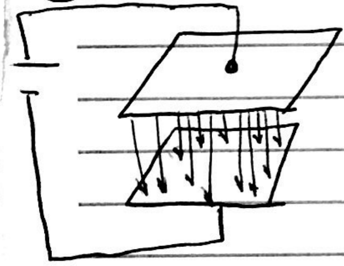
D S T Q Q S S

## (18) CAPACITÂNCIA

$$C = \frac{Q}{V} = \frac{\int \rho_s \, dS}{-\int E \, dl}$$

$$= \frac{\int \varepsilon_1 E \cdot dS}{-\int_l E \, dl}$$

## (19) CAPACITOR PLACAS PARALELAS



$$Q = \int \varepsilon_1 E \cdot dS = \varepsilon_1 E A$$

$$V = -\int_l E \cdot dl = Ed$$

$$\boxed{C = \frac{\varepsilon_1 E A}{E d} = \frac{\varepsilon A}{d} \quad [F]}$$

## (20) CAPACITÂNCIA LINHA COAXIAL

$$\varepsilon E = \rho_s$$

$$E = \frac{\rho_s}{\varepsilon} = \frac{Q}{2\pi r \varepsilon l}$$

$$V = \int_a^b E \, dl$$

$$= \int_a^b \left(\frac{Q}{2\pi \varepsilon l} \hat{r}\right)\left(\frac{dr}{r} \hat{r}\right)$$

$$= \frac{Q}{2\pi \varepsilon l} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{\frac{Q}{2\pi \varepsilon l} \ln\left(\frac{b}{a}\right)}$$

$$\boxed{C = \frac{2\pi \varepsilon l}{\ln\left(\frac{b}{a}\right)} \quad [F]}$$