Linear Algebra

1. Representing a point in:
   2D -> p = [2,3], where 2 is x component and 3 is y component
   3D -> q = [2,3,5], where 2 is x, 3 is y and 5 is z component
   nD -> x = [2,3,4,1,5,………,n]
2. Distance of a point from origin:
   2D -> d = $\sqrt{a^2 + b^2}$
   3D -> d = $\sqrt{a^2 + b^2 + c^2}$
   nD -> d = $\sqrt{\sum_{i=1}^{n} a_i^2}$
3. Distance between 2 points:

   2D -> d = $\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$

   3D -> d = $\sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$

   nD -> d = $\sqrt{\sum_{i=1}^{n}(a_i - b_i)^2}$

4. Row and column vector:

   Row Vector: $A_{1\times n}$ -> here 1 is number of row and n is number of columns

   Column Vector: $B_{n\times 1}$ -> here n is number of row and 1 is number of column

5. Operations on vector:

   Addition: a = [a_1, a_2,……………………,a_n]

   b = [b_1, b_2……………………,b_n]

   c= a + b = [a_1+b_1, a_2+ b_2,………………………a_n+b_n]

Multiplication: dot product

a.b = a_1b_1+ a_2b_2+………………………+a_nb_n

= $a_{1\times n}$ $b_{n\times 1}$

a.b = $a^{\top}b$ = $\sum_{i=1}^{n} a_{i^{b}i}$

Geometrically,

a.b = $\|a\|\|b\| \cos\theta$, where $\|a\|$ is length of a -> distance of a from origin

$\|b\|$ is length of b -> distance of b from origin

θ = $\cos^{-1}\left\{\frac{a_1b_1+a_2b_2}{\|a\|\|b\|}\right\}$

Suppose, θ = 90^0

a·b = 0

for nD,

θ_{ab} = $\cos^{-1}\left(\frac{\sum_{i=1}^{n} a_i b_i}{\|a\|\|b\|}\right)$

a·a = $\|a\|^2$

6. Projection:

Projection of a on b:

d = $\frac{a \cdot b}{\|b\|}$

7. Unit Vector:

$\hat{a}$ = $\frac{a}{\|a\|}$, here $\hat{a}$ is in same direction as a and $\|a\|$ = 1

8. Line in 2D (2-dimension):

Y = mx + c

In general form,

ax + by + c = 0

we can write it as,

$w_1x_1 + w_2x_2 + w_0 = 0$

9. Plane in 3D (3 - dimension):

ax + by + cz + d = 0

we can write it as,

$w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$

10. Hyper-plane in nD (n - dimension):

$w_0 + \sum_{i=1}^{n} w_ix_i = 0$

Vector notation,

$\pi: \quad W_0 + W_{n\times1}^{T}X_{n\times1} = 0$

11. Equation of a plane passing through origin:

Line -> 2D: $w_1x_1 + w_2x_2 = 0$

Plane -> 3D: $w_1x_1 + w_2x_2 + w_3x_3 = 0$

Hyper-plane -> nD: $\sum_{i=1}^{n} w_ix_i = 0$ => $w^Tx = 0$

So,

π_n: $w^Tx = 0$, if $w \perp \pi$

Then, $w \cdot x_i = 0 \quad \forall\, x_i \in \pi$

12. Distance of a point from a plane:

If $\|w\|$ = 1, d = $\frac{w^Tp}{\|w\|}$ = $\frac{w \cdot p}{\|w\|}$

A line, plane or hyper-plane divides an space into 2 halves, which is called half-spaces

Now,

d = $\frac{w \cdot p}{\|w\|}$ = +ve -> p is in same direction as w

d = $\frac{w \cdot p}{\|w\|} = -\mathrm{ve}$ -> p is in opposite direction from w

13. Circle: 2D

Eqn of a circle,

$$x^2 + y^2 = r^2$$

If centre of circle is not in the origin,

C(h,k); $(x-h)^2 + (y-k)^2 = r^2$

14. Sphere: 3D

P(x_1, x_2, x_3),

$x_1^2 + x_2^2 + x_3^2 = r^2$

15. Hypersphere: nD

P(x_1, x_2, x_3,…………., x_n),

$\sum_{i=1}^{n} x_i^2 = r^2$

$\sum_{i=1}^{n} x_i^2 < r^2$ -> p lies inside the hypersphere

$\sum_{i=1}^{n} x_i^2 > r^2$ -> p lies outside the hypersphere

$\sum_{i=1}^{n} x_i^2 = r^2$ -> p lies on the hypersphere

16. Ellipse: 2D

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

17. Ellipsoid: 3D

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

18. Hyper-ellipsoid: nD

$$\sum_{i=1}^{n} \frac{x_i^2}{w_i^2} = 1$$