The Matching Function and Nonlinear Business Cycles*

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Abstract

The Cobb-Douglas matching function is ubiquitous in labor search and matching models, even though it imposes a constant matching elasticity that is unlikely to hold empirically. To examine the implications of this discrepancy, this paper uses a general constant returns to scale matching function to derive conditions that show how the cyclicality of the matching elasticity affects the shape of the job finding rate as a function of productivity and amplifies or dampens nonlinear labor market dynamics. It then shows that modest variation in the matching elasticity, consistent with recent estimates, significantly affects higher-order moments and optimal policy. This motivates research that can provide greater clarity on the matching function specification.

Keywords: Matching Function; Matching Elasticity; Nonlinear; Finding Rate; Unemployment

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1 INTRODUCTION

The matching function—the mapping from job seekers and vacancies into matches—is a core component of labor search and matching models. In particular, the elasticity of matches with respect to vacancies, which we refer to as the matching elasticity, is a key object in empirical and structural analysis. The Cobb-Douglas matching function is the most common specification, even though it imposes a constant matching elasticity that is unlikely to hold empirically. By relaxing the Cobb-Douglas assumption, this paper shows that modest cyclical variation in the matching elasticity, in line with recent empirical estimates, significantly affects higher-order moments and optimal policy.

To motivate our analysis, we first review the extensive empirical literature that estimates the matching elasticity. Although most of this work imposes the typical Cobb-Douglas specification, the wide range of estimates suggests that a fixed matching elasticity does not provide the best description of the data. Furthermore, Lange and Papageorgiou (2020) non-parametrically estimate the matching function and find support for a procyclical elasticity that fluctuates between 0.15 and 0.3. This motivates us to characterize the nonlinear effects of a general constant returns to scale matching function without *a priori* restrictions on the mean or cyclicality of the matching elasticity.

Using a simple model that permits a closed-form solution, we show the shape of the job finding rate as a function of productivity determines the nonlinearity in labor market dynamics. The convexity or concavity of the job finding rate function depends on the elasticity of substitution in the matching function, which controls the cyclicality of the matching elasticity. The job finding rate is convex when the matching elasticity is sufficiently procyclical and concave when the matching elasticity is sufficiently countercyclical. Intuitively, a procyclical matching elasticity increases the transmission of vacancies to matches when productivity increases, which amplifies the increase in the job finding rate and generates convexity. Likewise, decreases in the job finding rate are amplified with a countercyclical matching elasticity, generating concavity. The nonlinearity in the job finding rate then transmits to the unemployment rate through the law of motion for unemployment.

To quantify the mechanism and show that our analytical results are robust to a more general setting, we solve a textbook labor search and matching model nonlinearly using a constant elasticity of substitution (CES) matching function that nests the Cobb-Douglas specification. We find that cyclical variation in the matching elasticity, consistent with the estimates in Lange and Papageorgiou (2020), produces large differences in higher-order business cycle moments. For example, when holding the standard deviation of the unemployment rate fixed, switching from a countercyclical to a procyclical matching elasticity lowers the skewness of the unemployment rate from 2.37 to 0.29, nearly eliminating the nonlinear labor market dynamics emphasized in the literature.

We conclude by deriving the normative implications of a general matching function, thus extending the well-known results for the Cobb-Douglas specification. Away from this knife-edge case where the matching elasticity is constant, we show the cyclicality of the matching elasticity qualitatively affects the cyclicality of the vacancy tax that alleviates the externalities endemic to the frictional matching process. In addition, the differences in nonlinear unemployment dynamics that we document across matching functions transmit to consumption and hence to cyclical movements in the efficient real interest rate, which is a key ingredient of optimal monetary policy design. Understanding the true matching function is crucial for the conduct of optimal policy interventions.

The positive and normative implications of the matching function and uncertainty surrounding empirical estimates motivate additional research that can provide greater clarity on the specification of the matching function and the amount of variation in the matching elasticity. Until consensus is reached, it is important to consider alternative specifications of the matching function when assessing a model's ability to produce nonlinear features of the data and its optimal policy prescriptions.

Related Literature Our contribution is to uncover a general mechanism through which the matching function generates nonlinearities in the search and matching model, and to document its positive and normative implications. Our results build on a growing literature that uses the search and matching model to analyze business cycle asymmetries and nonlinearities (e.g., Abbritti and Fahr, 2013; Dupraz et al., 2019; Ferraro, 2018; Ferraro and Fiori, 2023; Petrosky-Nadeau and Zhang, 2017; Petrosky-Nadeau et al., 2018; Pizzinelli et al., 2020). While these papers focus on other mechanisms, we show that the matching function itself is a powerful source of nonlinear dynamics.

The literature has also examined the normative implications of search and matching frictions. Hairault et al. (2010) and Jung and Kuester (2011) studied how nonlinearities in the search and matching model affect the welfare cost of business cycles. While they derive conditions that determine how the shape of the job finding rate function affects welfare, they do not uncover the underlying mechanism, which we show depends on offsetting effects that the Cobb-Douglas restriction obscures. Others have shown how nonlinear search and matching frictions affect optimal policy, but almost exclusively under a Cobb-Douglas matching function (e.g., Cacciatore et al., 2016; Faia, 2009; Jung and Kuester, 2015; Lepetit, 2020; Ravenna and Walsh, 2011). A notable exception is Arseneau and Chugh (2012), which shows the optimal labor tax is highly variable using a general constant returns to scale matching function. We build on that result by tying the volatility of the optimal labor tax and optimal vacancy subsidy to the cyclicality of the matching elasticity, which depends on the elasticity of substitution in the matching function. We then show that the elasticity of substitution has meaningful effects on the responses of the efficient real interest rate to shocks.

Our analysis also sheds light on the properties of the matching function introduced by Den Haan et al. (2000, DRW), which is used in influential papers such as Hagedorn and Manovskii (2008) and Petrosky-Nadeau et al. (2018).[1] While that specification has been used interchangeably with

[1]Albertini et al. (2021), Bernstein et al. (2021), Dao and Delacroix (2018), Ferraro (2018), Ferraro (2018), Kandoussi and Langot (2022), Hashimzade and Ortigueira (2005), Mitman and Rabinovich (2015), Petrosky-Nadeau

the Cobb-Douglas matching function,[2] we show they have different nonlinear properties. In contrast with the estimates in Lange and Papageorgiou (2020), the DRW matching function generates countercyclical variation in the matching elasticity that introduces concavity in the job finding rate function and amplifies nonlinear labor market dynamics relative to the Cobb-Douglas specification. Our results indicate that a CES matching function with a procyclical matching elasticity would have significantly reduced the nonlinear labor market dynamics generated by these models.

Outline The rest of the paper proceeds as follows. Section 2 provides an overview of the key properties of the matching function and the empirical estimates of the matching elasticity. Section 3 lays out our search and matching model. Section 4 derives a closed-form solution and characterizes the sources of nonlinearity. Section 5 quantifies the nonlinearities and their effects on labor market dynamics. Section 6 shows the normative implications of the matching function. Section 7 concludes.

2 OVERVIEW OF MATCHING FUNCTIONS

To motivate our analytical and quantitative exercises, we briefly discuss some useful theoretical properties of matching functions and review the associated empirical literature that estimates them. We consider matching functions of the form $\mathcal{M}(u_t^s, v_t)$, where u_t^s measures the search effort of job seekers (often counts of unemployed workers) and v_t measures the recruitment effort of employers (often counts of vacancy postings). Throughout, we assume $\mathcal{M}(u_t^s, v_t)$ is strictly increasing, strictly concave, and twice differentiable in both arguments, and exhibits constant returns to scale (see Petrongolo and Pissarides (2001) for an overview of the evidence supporting constant returns).

A key object of theoretical and empirical interest is the elasticity of matches with respect to vacancies, which we denote by $\epsilon_t = \mathcal{M}_v(u_t^s, v_t)v_t/\mathcal{M}(u_t^s, v_t)$ and refer to as the matching elasticity. We note that due to constant returns to scale, the matching elasticity depends only on labor market tightness, $\theta_t = v_t/u_t^s$, and lies in the unit interval: $\epsilon(\theta_t) = \mathcal{M}_v(1, \theta_t)\theta_t/\mathcal{M}(1, \theta_t) \in (0, 1)$. Although some papers in the literature focus on the matching elasticity with respect to search effort, constant returns to scale also implies that $\mathcal{M}_u(u_t^s, v_t)u_t^s/\mathcal{M}(u_t^s, v_t) = 1 - \mathcal{M}_v(u_t^s, v_t)v_t/\mathcal{M}(u_t^s, v_t)$.

The goal of this paper is to uncover how the statistical properties of the matching elasticity (e.g., its mean, standard deviation, and cyclicality) affect nonlinear dynamics, given their recent emphasis in the search and matching literature. The following result establishes a benchmark that applies when restricting attention to linear models. All of the proofs are contained in Appendix A.

and Zhang (2017, 2021), Petrosky-Nadeau and Wasmer (2017), Shao and Silos (2013), and Sargent (2022) also use the DRW matching function. Arseneau and Chugh (2023) show the optimal policy results in Arseneau and Chugh (2012) are robust to the DRW matching function. Stevens (2007) provides a microfoundation for the DRW matching function.

[2] When comparing the Cobb-Douglas and Den Haan et al. (2000) matching functions Petrosky-Nadeau and Wasmer (2017) say the "business cycle moments of the model using either functional form are similar." The justification for using the Den Haan et al. (2000) specification is that it restricts the job filling and job finding rates to the unit interval.

Proposition 1. *To a first order, any constant returns to scale matching function is approximately equivalent to a Cobb-Douglas matching function, $\mathcal{M}(u_t^s, v_t) = \phi(u_t^s)^{1-\bar{\epsilon}} v_t^{\bar{\epsilon}}$, where $\phi > 0$ is matching efficiency and $0 < \bar{\epsilon} < 1$ is a fixed matching elasticity.*

The Cobb-Douglas matching function is a common assumption in business cycle research.[3] It imposes that the matching elasticity is invariant to labor market conditions. Proposition 1 shows that when we restrict attention to linear dynamics, this assumption is without loss of generality. Intuitively, in a linear model, only the value of the matching elasticity in the deterministic steady state affects dynamics. This value can be set as a parameter of a Cobb-Douglas matching function.

In this paper, we depart from the special linear case and shed light on the higher-order positive and normative consequences of the matching function. To lay the foundations, we first establish how the matching elasticity in general varies with labor market conditions, as measured by labor market tightness. To do so, it is useful to define the elasticity of substitution between vacancies and job seekers, $\sigma_t = \frac{d\ln(v_t/u_t^s)}{d\ln(\mathcal{M}_u(u_t^s,v_t)/\mathcal{M}_v(u_t^s,v_t))} \in (0,\infty)$, which also only depends on labor market tightness due to constant returns to scale in the matching function: $\sigma(\theta_t) = \frac{d\ln\theta_t}{d\ln(\mathcal{M}_u(1,\theta_t)/\mathcal{M}_v(1,\theta_t))}$.

Proposition 2. *The matching elasticity, $\epsilon_t = \epsilon(\theta_t)$, is increasing in θ_t when $\sigma_t = \sigma(\theta_t) > 1$, constant when $\sigma_t = 1$, and decreasing in θ_t when $\sigma_t < 1$.*

Recall that the matching elasticity is the marginal product of labor market tightness divided by the average product: $\epsilon(\theta_t) = \mathcal{M}_v(1,\theta_t)/(\mathcal{M}(1,\theta_t)/\theta_t)$. The effects of tightness on each term drive the matching elasticity in opposite directions. First, the average product is decreasing in tightness because a 1% increase in tightness yields a less than 1% increase in matches. This causes the matching elasticity to increase. Second, the marginal product is decreasing in tightness due to diminishing returns to vacancy creation. This causes the matching elasticity to decrease. The dominant effect depends on how quickly the marginal product declines, which is governed by the elasticity of substitution. When $\sigma(\theta_t) > 1$, high substitutability between vacancies and job seekers slows the decline in the marginal product, so the first effect dominates and $\epsilon(\theta_t)$ is increasing in tightness.

Proposition 2 uncovers a tight relationship between variation in the matching elasticity and the elasticity of substitution that applies to a general matching function. We obtain further structure if we are willing to impose a functional form on the matching function. For example, it is common to assume the matching function is Cobb-Douglas, which is a special case of the general CES family,

$$\mathcal{M}(u_t^s, v_t^s) = \begin{cases} \phi\left(\vartheta(u_t^s)^{(\sigma-1)/\sigma}(1-\vartheta)v_t^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} & \sigma \neq 1, \\ \phi(u_t^s)^{\vartheta} v_t^{1-\vartheta} & \sigma = 1, \end{cases}$$

where $\phi > 0$ is matching efficiency and $\vartheta \in (0,1)$ is the importance of job seekers. Under this specification, the elasticity of substitution, $\sigma(\theta_t) = \sigma$, is fixed and we can strengthen Proposition 2.

[3]See, for example, Ljungqvist and Sargent (2017), Hall and Milgrom (2008), Pissarides (2009), and Shimer (2005).

Corollary 1. *Suppose the matching function is from the CES family. Then $\sigma > 1$ implies $\epsilon'(\theta_t) > 0$, $\sigma = 1$ implies $\epsilon'(\theta_t) = 0$, and $\sigma < 1$ implies $\epsilon'(\theta_t) < 0$ for all $\theta_t > 0$.*

Since tightness is procyclical in the data and in search and matching models, the choice of σ globally affects the cyclicality of the matching elasticity. When $\sigma = 1$, the matching function is Cobb-Douglas, and the matching elasticity is constant, $\epsilon_t = \bar{\epsilon} = 1 - \vartheta$. Away from this special case, higher substitutability ($\sigma > 1$) generates procyclical variation in the matching elasticity, while lower substitutability ($\sigma < 1$) implies countercyclical variation. Our analytical and quantitative exercises will shed light on how the cyclicality of ϵ_t translates into nonlinear labor market dynamics.

Table 1: Empirical estimates of the matching function

Author(s)	Method(s)	Sample	Parameter Estimates	
Cobb-Douglas			$\bar{\epsilon}$	
Blanchard and Diamond (1989)	OLS, AR1 residual	1968-1981	0.54	
Bleakley and Fuhrer (1997)	OLS with breakpoints	1979-1993	0.31-0.35	
Shimer (2005)	OLS, AR1 residual	1951-2003	0.28	
Hall (2005)	OLS	2000-2002	0.77	
Rogerson and Shimer (2011)	OLS, multiplicative noise	2001-2009	0.42	
Michaillat and Saez (2021)	OLS with breakpoints	1951-2019	0.51-0.61	
Cobb-Douglas with endogeneity correction			$\bar{\epsilon}$	
Borowczyk-Martins et al. (2013)	GMM IV	2000-2012	0.70	
Şahin et al. (2014)	OLS, GMM IV, varied data	2001-2012	0.24-0.66	
Barnichon and Figura (2015)	GMM IV	1968-2007	0.34	
Sedláček (2016)	OLS with non-unemployed	2000-2013	0.24	
Hall and Schulhofer-Wohl (2018)	OLS with aggregation	2001-2013	0.35	
CES			$\bar{\epsilon}$	σ
Blanchard and Diamond (1989)	NLS, AR1 residual	1968-1981	0.54	0.74
Shimer (2005)	NLS, AR1 residual	1951-2003	0.28	1.06
Şahin et al. (2014)	GMM IV, varied data	2001-2012	0.24-0.66	0.9-1.2
Non-parametric				
Lange and Papageorgiou (2020)	Non-parametric	2001-2017	$\epsilon_t \in (0.15, 0.3)$	Procyclical

Note: $\bar{\epsilon}$ is a fixed matching elasticity, σ is the elasticity of substitution in the matching function, and ϵ_t is a time-varying matching elasticity. *Cobb-Douglas* lists studies that impose a Cobb-Douglas matching function. *Cobb-Douglas with endogeneity correction* lists studies that account for endogeneity and impose a Cobb-Douglas matching function at the aggregate or job status level. *CES* lists studies that impose a CES matching function. *Non-parametric* lists studies that do not specify a particular matching function.

Empirical Evidence Table 1 summarizes the empirical literature that estimates either a fixed or time-varying matching elasticity. For brevity and to permit a cleaner comparison of the estimates, we focus on studies that use U.S. data and impose constant returns to scale in the matching function.

Early work used aggregate data on hires, vacancies, and unemployment to directly estimate a log-linear matching function with OLS. Following the logic of Proposition 1, this approach implicitly assumed a Cobb-Douglas matching function and estimated the fixed matching elasticity.

Due to differences in data sources and samples, the estimates ranged from around 0.3 in Bleakley and Fuhrer (1997) and Shimer (2005) up to 0.77 in Hall (2005). Furthermore, estimates based on data from the more recent JOLTS survey (Hall, 2005; Rogerson and Shimer, 2011) are higher than past estimates based on CPS flows data (Bleakley and Fuhrer, 1997) or the Shimer (2005) method. More recently, Michaillat and Saez (2021) develop a different approach in which they first estimate the elasticity of vacancies with respect to unemployment using OLS with breakpoints and then use a search and matching model to solve for the matching elasticity, which ranges from 0.51 to 0.61.

More recent work developed methods to deal with potential endogeneity due to unobserved variation in matching efficiency (the ϕ term in the Cobb-Douglas specification above), either by using instruments (Borowczyk-Martins et al., 2013) or by exploiting heterogeneity in job seekers (Hall and Schulhofer-Wohl, 2018). In addition, Sedláček (2016) proposed a latent-variable strategy to deal with unobserved job search by non-unemployed workers. These papers maintained the Cobb-Douglas assumption at either the aggregate or job status level and generated estimates in the same range as the estimates that did not correct for endogeneity. The broad range of estimates is again at least partially due to different data choices, with higher estimates generated by JOLTS data (Borowczyk-Martins et al., 2013; Şahin et al., 2014) and a lower estimates generated by CPS flows data (Barnichon and Figura, 2015) or industry-level hires from CPS data (Şahin et al., 2014).

A few papers relax the Cobb-Douglas assumption. Imposing the CES functional form, Blanchard and Diamond (1989) estimated an elasticity of substitution of 0.74. More recently, Shimer (2005) and Şahin et al. (2014) obtained estimates closer to 1 but with larger standard errors, indicating weak identification. Given Corollary 1, this suggests the cyclicality of the matching elasticity is highly uncertain. Finally, Lange and Papageorgiou (2020) propose a non-parametric identification strategy that allows for multiple types of job seekers and incorporates rich measures of search effort and recruiting intensity to account for possible endogeneity. They estimate a procyclical elasticity that fluctuates between 0.15 and 0.3, which is consistent with a CES matching function with $\sigma > 1$.

Outlook There is considerable uncertainty surrounding the matching elasticity, even when it is assumed to be fixed. Among papers relaxing that assumption, there is additional uncertainty about the elasticity of substitution between vacancies and job seekers and the cyclicality of the matching elasticity. The most recent and most general econometric specification finds that the matching elasticity is procyclical, in contrast with the Cobb-Douglas matching function. The lack of consensus and implausibility of a fixed matching elasticity shows why it is important to investigate the implications of a time-varying matching elasticity. As we will show, even modest variation has significant implications, which motivates empirical work that can provide greater clarity on the matching function.

3 ENVIRONMENT

To cleanly demonstrate our results, we use a textbook search and matching model. The one exception is that we use a general constant returns to scale matching function, rather than assuming a particular functional form. Each period denotes 1 month and the population (equal to the labor force) is normalized to unity. Business cycles are driven by shocks to labor productivity, a_t, which follows

$$a_{t+1} = \bar{a} + \rho_a(a_t - \bar{a}) + \sigma_a \varepsilon_{a,t+1},\ 0 \leq \rho_a < 1,\ \varepsilon_a \sim \mathbb{N}(0,1). \tag{1}$$

Search and Matching Entering period t, there are n_{t-1} employed workers and $u_{t-1} = 1 - n_{t-1}$ unemployed job seekers. In period t, firms post v_t vacancies, so the number of matches is given by

$$m_t = \min\{\mathcal{M}(u_{t-1}, v_t), u_{t-1}, v_t\}, \tag{2}$$

where $\mathcal{M}$ is a constant returns to scale matching function that satisfies the assumptions in Section 2.

Given the number of matches, the job finding rate, job filling rate, and laws of motion satisfy

$$f_t = m_t/u_{t-1}, \tag{3}$$

$$q_t = m_t/v_t, \tag{4}$$

$$n_t = (1-\bar{s})n_{t-1} + f_t u_{t-1}, \tag{5}$$

$$u_t = u_{t-1} + \bar{s}n_{t-1} - f_t u_{t-1}, \tag{6}$$

where $u_t = 1 - n_t$, $\bar{s} \in (0,1)$ is the exogenous separation rate, and (2) ensures that $f_t, q_t \in [0,1]$.

Firms A representative firm chooses vacancies and employment $\{v_t, n_t\}$ to solve

$$V_t = \max_{v_t, n_t} a_t n_t - w_t n_t - \kappa v_t + E_t[x_{t+1} V_{t+1}]$$

subject to $n_t = (1-\bar{s})n_{t-1} + q_t v_t$ and $v_t \geq 0$, where $\kappa > 0$ is the vacancy posting cost, w_t is the wage rate, and E_t is an expectation operator conditional on time-t information. The representative household's pricing kernel is $x_{t+1} = \beta(c_t/c_{t+1})^\gamma$, where c_t is consumption, $\beta \in (0,1)$ is the discount factor, and $\gamma \geq 0$ is the coefficient of relative risk aversion.[4] The optimality conditions imply

$$\tfrac{\kappa - \lambda_{v,t}}{q_t} = a_t - w_t + (1-\bar{s})E_t\left[x_{t+1}\tfrac{\kappa - \lambda_{v,t+1}}{q_{t+1}}\right], \tag{7}$$

$$\lambda_{v,t} v_t = 0, \quad \lambda_{v,t} \geq 0, \tag{8}$$

where $\lambda_{v,t}$ is the multiplier on the non-negativity constraint $v_t \geq 0$. Condition (7) sets the marginal cost of hiring, $(\kappa - \lambda_{v,t})/q_t$, equal to the marginal benefit of hiring, which consists of the flow

[4] When households are risk averse ($\gamma > 0$), we follow the business cycle literature and assume there is perfect consumption insurance for employed and unemployed workers (Andolfatto, 1996; Den Haan et al., 2000; Merz, 1995).

profits from the match, $a_t - w_t$, plus the savings from not having to post the vacancy in the future.

Wages As is common in the search and matching literature, wages are determined through Nash bargaining between employed workers and the firm. Following the steps in Appendix A, we obtain

$$w_t = \eta(a_t + \kappa E_t[x_{t+1}(v_{t+1}/u_t)]) + (1-\eta)b, \tag{9}$$

where $\eta \in (0,1)$ is the worker's bargaining power and $b > 0$ is the flow value of unemployment.

Equilibrium The aggregate resource constraint is given by

$$c_t + \kappa v_t = a_t n_t. \tag{10}$$

An equilibrium is infinite sequences of quantities $\{c_t, n_t, u_t, v_t, m_t, f_t, q_t\}_{t=0}^{\infty}$, prices $\{w_t, \lambda_{v,t}\}_{t=0}^{\infty}$, and productivity $\{a_t\}_{t=1}^{\infty}$ that satisfy (1)-(10) given the initial state $\{n_{-1}, a_{-1}\}$ and shocks $\{\varepsilon_{a,t}\}_{t=0}^{\infty}$.

4 ANALYTICAL RESULTS

This section analytically shows the nonlinearity in labor market dynamics is determined by the curvature of the job finding rate, f_t, as a function of the productivity shock, a_t. To document this fact, we begin by solving the model under some simplifying assumptions to make it analytically tractable. Given that solution, we then show the curvature of labor market tightness, θ_t, as a function of a_t is determined by the elasticity of substitution in the matching function, σ, which controls the cyclicality of the matching elasticity, ϵ_t, according to Proposition 2. Finally, we show the combination of σ and ϵ_t determine the shape of the job finding rate function. Hence, there is a direct link between the specification of the matching function and the nonlinearity in labor market dynamics.

4.1 MODEL SOLUTION

To solve the model analytically, we make two simplifying restrictions.

Assumption 1. $\gamma = \eta = 0$.

These conditions imply that workers are risk neutral and have zero bargaining weight, so wages are sticky with $w_t = b$ (Hall, 2005).[5] We relax these restrictions in Section 5 for our quantitative exercises. Given these conditions, we obtain an analytical expression for the marginal cost of hiring.

Proposition 3. *Under Assumption 1, the marginal cost of hiring follows the stochastic process*

$$(\kappa - \lambda_{v,t})/q_t = \delta_0 + \delta_1(a_t - \bar{a}), \tag{11}$$

[5] An alternative assumption about wages would be to follow Jung and Kuester (2011) and Freund and Rendahl (2020) and use the ad-hoc linear wage rule $w_t = \eta a_t + (1-\eta)b$. Our qualitative results are unaffected by this choice.

where

$$\delta_0 = \frac{\bar{a} - b}{1 - \beta(1 - \bar{s})} > 0, \qquad \delta_1 = \frac{1}{1 - \beta(1 - \bar{s})\rho_a} > 0,$$

and $\lambda_{v,t} > 0$ *implies* $q_t = 1$.

In (11), δ_0 is the steady-state marginal cost of hiring, while δ_1 is the response of marginal cost to changes in productivity. Intuitively, δ_1 is increasing in the persistence of the productivity shock ρ_a.[6]

In the data, job finding and job filling rates are always strictly positive and strictly less than unity. In the model, the restriction $f_t, q_t \in (0, 1)$ implies $v_t > 0$, so $\lambda_{v,t} = 0$. Assuming shocks $\{a_t\}$ are such that this restriction always holds, we can invert (11) to obtain the job filling rate function,

$$q(a_t) = \kappa/(\delta_0 + \delta_1(a_t - \bar{a})), \tag{12}$$

which is decreasing and convex in productivity. Remarkably, this result does not depend on the matching function, and instead follows directly from firms' optimal vacancy creation and the definition of marginal cost. Intuitively, higher productivity increases vacancy creation, which reduces the probability of filling any given vacancy. Convexity in the job filling rate arises because as productivity increases, the probability declines at a slower rate since it is bounded below by zero.

In equilibrium, the job filling rate is determined by labor market tightness. To derive tightness as a function of productivity, it is convenient to define the auxiliary function $\mu_q(\theta) = \mathcal{M}(1, \theta)/\theta$, which is strictly decreasing in θ and therefore invertible. Recalling that $q_t = \mathcal{M}(1, \theta_t)/\theta_t$, we can implicitly define the equilibrium tightness function as $\mu_q(\theta(a_t)) = q(a_t)$, so differentiation implies

$$\theta'(a_t) = q'(a_t)/\mu_q'(\theta(a_t)) > 0. \tag{13}$$

Since $q'(a_t), \mu_q'(\theta(a_t)) < 0$, (13) confirms that labor market tightness is increasing in productivity.

Given the equilibrium tightness function, we can use the definitions from Section 2 to define

$$\epsilon_t = \frac{\mathcal{M}_v(1, \theta(a_t))\theta(a_t)}{\mathcal{M}(1, \theta(a_t))}, \quad \sigma_t = \frac{d \ln \theta(a_t)}{d \ln(\mathcal{M}_u(1, \theta(a_t))/\mathcal{M}_v(1, \theta(a_t)))},$$

where ϵ_t is the matching elasticity and σ_t is the elasticity of substitution between job seekers and vacancies. These definitions help us uncover the nonlinearity in equilibrium tightness from $\theta''(a_t)$.

Proposition 4. *Labor market tightness,* $\theta(a_t)$*, is convex at* a_t *when* $\sigma_t > 1/2$*, linear at* a_t *when* $\sigma_t = 1/2$*, and concave at* a_t *when* $\sigma_t < 1/2$*.*

To interpret these conditions, it is useful to write the slope of the tightness function as

$$\theta'(a_t) = \frac{\delta_1}{\kappa} \frac{\mathcal{M}(1, \theta(a_t))}{1 - \epsilon_t},$$

[6] Den Haan et al. (2021) independently developed a similar solution to shed light on the effects of volatility shocks.

which shows that productivity affects $\theta'(a_t)$ through two channels. First, higher productivity generates more matches, which raises $\mathcal{M}(1,\theta(a_t))$ and $\theta'(a_t)$. Second, higher productivity affects the matching elasticity. Given Proposition 2, an increase in productivity lowers the matching elasticity and $\theta'(a_t)$ when $\sigma_t < 1$. If $\sigma_t < 1/2$, this effect dominates the first channel, so tightness is concave in productivity. When $\sigma_t = 1/2$, the two channels exactly offset, so tightness is linear in productivity. Finally, when $\sigma_t > 1/2$, the first channel dominates, so tightness is convex in productivity.

Given the equilibrium dynamics of tightness, we can use the matching function to derive the dynamics of the job finding rate. Formally, $f(a_t) = \mathcal{M}(1,\theta(a_t))$, so it is immediate that the job finding rate is increasing in productivity. As with tightness, we analyze its nonlinearity through $f''(a_t)$.

Proposition 5. *The job finding rate, $f_t = f(a_t)$, is convex at a_t when $\sigma_t > 1/(2\epsilon_t)$, linear at a_t when $\sigma_t = 1/(2\epsilon_t)$, and concave at a_t when $\sigma_t < 1/(2\epsilon_t)$.*

To interpret these conditions, it is useful to write the slope of the job finding rate function as

$$f'(a_t) = \mathcal{M}_v(1,\theta(a_t))\theta'(a_t),$$

which shows that productivity also affects $f'(a_t)$ through two competing channels. First, higher productivity raises labor market tightness, which lowers its marginal product, $\mathcal{M}_v(1,\theta(a_t))$, due to diminishing returns to vacancy creation. This generates concavity in the job finding rate. Second, higher productivity affects the responsiveness of tightness itself through $\theta'(a_t)$. As Proposition 4 shows, this effect is positive when $\sigma_t > 1/2$, generating convexity. If $\sigma_t > 1/(2\epsilon_t) > 1/2$, it is strong enough to dominate the first channel, making the job finding rate convex in productivity. When $\sigma_t = 1/(2\epsilon_t)$, the two channels exactly offset, so the finding rate is linear in productivity. Finally, when $\sigma_t < 1/(2\epsilon_t)$, the first channel dominates, so the finding rate is concave in productivity.

More generally, the job finding rate is convex in productivity if the matching elasticity is sufficiently procyclical. Intuitively, a procyclical matching elasticity increases the transmission of vacancies to matches when productivity increases, which amplifies the finding rate response and generates convexity. Likewise, the finding rate is concave when the matching elasticity is sufficiently countercyclical, as positive shock responses are dampened by a falling matching elasticity.

If the job finding rate function is convex ($f''(a_t) > 0$), then a positive productivity shock at a_t will have a larger impact on the job finding rate than a negative shock, creating positive skewness in the ergodic distribution. Conversely, if the job finding rate function is concave ($f''(a_t) < 0$), then a negative shock will have a larger impact than a positive shock, creating negative skewness.

Any nonlinearity in the job finding rate will transmit to the unemployment rate. Differentiating (6) implies $\partial u_t/\partial a_t = -u_{t-1}f'(a_t)$. Therefore, a concave job finding rate function amplifies nonlinear unemployment dynamics since periods of high unemployment coincide with larger finding rate responses to productivity shocks. In contrast, a convex job finding function dampens the nonlinearity of unemployment because high unemployment occurs with smaller finding rate responses.

4.2 EXAMPLES Our results thus far are based on a general matching function. By considering specific functional forms, we can gather additional insights and draw connections to the literature.

CES Matching Function Recall the CES matching function first described in Section 2,

$$\mathcal{M}(u_{t-1}, v_t) = \begin{cases} \phi\left(\vartheta u_{t-1}^{(\sigma-1)/\sigma} + (1-\vartheta) v_t^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} & \sigma \neq 1, \\ \phi u_{t-1}^{\vartheta} v_t^{1-\vartheta} & \sigma = 1, \end{cases} \tag{14}$$

where $\phi > 0$ is matching efficiency and $\vartheta \in (0,1)$ governs the importance of unemployment. The elasticity of substitution, σ, is fixed, while the matching elasticity takes the specific form $\epsilon_t = (1-\vartheta)(\phi/q(a_t))^{(\sigma-1)/\sigma}$. In line with Corollary 1, ϵ_t is procyclical when $\sigma > 1$, constant when $\sigma = 1$ (Cobb-Douglas specification), and countercyclical when $\sigma < 1$. Using these properties, we can derive sufficient conditions for global convexity or concavity of the job finding rate function.

Corollary 2. *Suppose $\mathcal{M}(u_{t-1}, v_t)$ satisfies (14). Then $\sigma > \frac{1}{2(1-\vartheta)\phi^{(\sigma-1)/\sigma}} \geq 1$ implies that $f(a_t)$ is globally convex, $\sigma < \frac{1}{2(1-\vartheta)\phi^{(\sigma-1)/\sigma}} \leq 1$ implies that $f(a_t)$ is globally concave, and $\sigma = \frac{1}{2(1-\vartheta)} = 1$ implies that $f(a_t)$ is globally linear.*

Table 1 shows there is a wide range of estimates for the matching elasticity and large standard errors around the estimates for the elasticity of substitution. Corollary 2 shows that this level of uncertainty surrounding the parameters of the matching function implies a wide range of possible outcomes for the nonlinearity in labor market dynamics. Under the typical Cobb-Douglas matching function where $\sigma = 1$, there is very little nonlinearity. As σ gets further below unity, the job finding rate function becomes increasingly concave, creating negative skewness in labor market dynamics.

DRW Matching Function A small but influential set of papers (e.g., Hagedorn and Manovskii, 2008; Petrosky-Nadeau et al., 2018) use the function introduced by Den Haan et al. (2000, DRW):

$$\mathcal{M}(u_{t-1}, v_t) = u_{t-1} v_t / (u_{t-1}^{\iota} + v_t^{\iota})^{1/\iota}. \tag{15}$$

In this case, $\iota > 0$ and the elasticity of substitution is fixed at $1/(1+\iota) < 1$. The matching elasticity satisfies $\epsilon_t = q(a_t)^{\iota}$ and is always countercyclical according to Proposition 2. Thus, this specification is inconsistent with the empirical estimates in Lange and Papageorgiou (2020). While it is often justified by appealing to the fact that it guarantees bounded job finding and filling rates without the feasibility condition (2), it also has significant effects on nonlinear labor market dynamics.

Corollary 3. *Suppose $\mathcal{M}(u_{t-1}, v_t)$ satisfies (15). Then $\iota > 1$ implies $f(a_t)$ is globally concave.*

This shows that any model that uses the DRW matching function with $\iota > 1$ will generate concavity in the job finding rate function and nonlinear labor market dynamics. For example, Petrosky-Nadeau et al. (2018) set $\iota = 1.25$ and find that their model generates significant skewness and kurtosis in the unemployment rate. Replacing the DRW matching function with a CES specification that is consistent with the state-of-the art estimates in Lange and Papageorgiou (2020) would alter the nonlinear properties of the model. Given these implications, it is important to provide a strong justification for the functional form of the matching function and its underlying parameters.

5 QUANTITATIVE RESULTS

Our analysis thus far highlights the importance of the matching elasticity and elasticity of substitution. To illustrate that the results derived in Section 4 generalize to a version of the model where the restrictive assumptions required to get an analytical solution are relaxed, we return to the original model in Section 3 and specify a CES matching function with $\bar{\epsilon} \in \{0.3, 0.7\}$ and $\sigma \in \{0.5, 1, 5\}$. Our choices for $\bar{\epsilon}$ capture the range of empirical estimates in Table 1. The choice of $\sigma = 0.5$ corresponds to $\iota = 1$ under the DRW matching function, which is comparable to values used in the literature, while $\sigma = 1$ is the Cobb-Douglas case. Lange and Papageorgiou (2020) estimate that the matching elasticity is procyclical with ϵ_t ranging from 0.15 to 0.3. We chose $\sigma = 5$ so the standard deviation of the matching elasticity in the model covers at most half the empirical range.

Each period in the model denotes 1 month, so the discount factor, β, is set to 0.9983, which corresponds to an average annual real interest rate of 2%. The coefficient of relative risk aversion, γ, is set to 1, consistent with log utility. The remaining parameters are based on U.S. data from 1955 to 2019. The steady-state job separation rate, $\bar{s}$, is set to its sample mean 0.0326, which we compute following Shimer (2012). The persistence ($\rho_a = 0.8826$) and standard deviation ($\sigma_a = 0.0062$) of productivity are set to match the autocorrelation and standard deviation of detrended productivity.

To isolate the impact of the matching function on higher-order labor market dynamics, we hold the mean and standard deviation of the unemployment rate fixed across $(\sigma, \bar{\epsilon})$ pairs. In particular, under each specification we estimate the vacancy posting cost, κ, and flow value of unemployment, b, to target the mean unemployment rate ($E(u) = 0.0589$) and the standard deviation of the detrended unemployment rate ($SD(u) = 11.79$) in our data sample. In addition, we estimate the bargaining power parameter, η, to target the wage-productivity elasticity ($Slope(w, a) = 0.47$) following Hagedorn and Manovskii (2008).[7] Each specification perfectly matches these empirical targets.

[7] The empirical targets are based on quarterly data. Each period in the model denotes 1 month, so we aggregate the simulated time series to a quarterly frequency to match the frequency of labor productivity in the data. To facilitate comparison with the literature, we detrend actual data using a Hodrick and Prescott (1997) filter with a smoothing parameter of $1{,}600$. We detrend simulated data by computing percent deviations from the short-sample time averages. The wage rate (w_t) is defined as the product of the labor share and labor productivity (a_t) in the nonfarm business sector (Hagedorn and Manovskii, 2008). The wage elasticity is the slope coefficient from regressing w_t on an intercept and a_t.

We set the steady-state job filling rate, $\bar{q}$, to 0.3306, which corresponds to a quarterly filling rate of 0.7 (Den Haan et al., 2000). The steady-state job finding rate, $\bar{f}$, is endogenously pinned down by the mean unemployment rate, $\bar{u}$, since $\bar{f} = \bar{s}(1-\bar{u})/\bar{u}$ and $\bar{u}$ is determined by the vacancy posting cost, κ. Given $\bar{q}$, $\bar{f}$, and a $(\sigma, \bar{\epsilon})$ pair, we pin down ϑ and ϕ using the steady-state restrictions

$$\phi = \begin{cases} \left(\bar{\epsilon}\bar{q}^{(\sigma-1)/\sigma} + (1-\bar{\epsilon})\bar{f}^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} & \sigma \neq 1, \\ \bar{q}^{\bar{\epsilon}}\bar{f}^{1-\bar{\epsilon}} & \sigma = 1, \end{cases}$$

$$\vartheta = (1-\bar{\epsilon})(\bar{f}/\phi)^{(\sigma-1)/\sigma}.$$

This ensures each matching function has similar first-order properties, in line with Proposition 1.

Solution Method To quantify the nonlinearities, we solve the model globally using the policy function iteration algorithm in Richter et al. (2014), which is based on the theoretical work in Coleman (1991). The algorithm minimizes the Euler equation errors on each node in the state space and computes the maximum change in the policy functions. It then iterates until the maximum change is below a specified tolerance criterion. Appendix B describes the solution method in more detail.

Table 2: Estimates and Implied Parameter Values

	Low Matching Elasticity ($\bar{\epsilon} = 0.3$)			High Matching Elasticity ($\bar{\epsilon} = 0.7$)		
	Low CES ($\sigma = 0.5$)	C-D ($\sigma = 1$)	High CES ($\sigma = 5$)	Low CES ($\sigma = 0.5$)	C-D ($\sigma = 1$)	High CES ($\sigma = 5$)
Vacancy Posting Cost (κ)	0.0794	0.0610	0.0507	0.3848	0.3493	0.3367
Flow Value of Unemployment (b)	0.9716	0.9777	0.9815	0.9243	0.9302	0.9328
Worker Bargaining Power (η)	0.1276	0.1320	0.1327	0.0515	0.0534	0.0535
Matching Efficiency (ϕ)	0.4540	0.4645	0.4683	0.3733	0.3811	0.3879
Unemployment Weight (ϑ)	0.5880	0.7000	0.7729	0.2096	0.3000	0.3841

Note: The choices for $\bar{\epsilon}$ capture the range of estimates in the literature. The choice of $\sigma = 0.5$ corresponds to $\iota = 1$ under the DRW matching function, $\sigma = 1$ is the Cobb-Douglas case, and $\sigma = 5$ is chosen so the standard deviation of the matching elasticity covers at most half the range in Lange and Papageorgiou (2020).

Estimates Table 2 reports the estimated parameters, (κ, b, η), and the implied matching function parameters, (ϕ, ϑ), given the steady-state matching elasticity, $\bar{\epsilon}$, and the elasticity of substitution, σ.[8] All of the parameter estimates are in line with values that are commonly used in the literature.

Job Finding Rate Function Section 4 showed the parameters of the matching function affect the shape of the job finding rate function. Figure 1 quantifies these effects using our choices of $\bar{\epsilon}$ and σ. Following Proposition 5, the nonlinearity around steady state depends on whether $\sigma \lesseqgtr 1/(2\bar{\epsilon})$.

[8] Consistent with Hagedorn and Manovskii (2008), the baseline model requires a b that is close to the marginal product of labor in order to generate realistic labor market volatility. Appendix C shows that if we introduce home production, we can set $b = 0.4$ so it resembles an unemployment benefit while achieving the same labor market volatility.

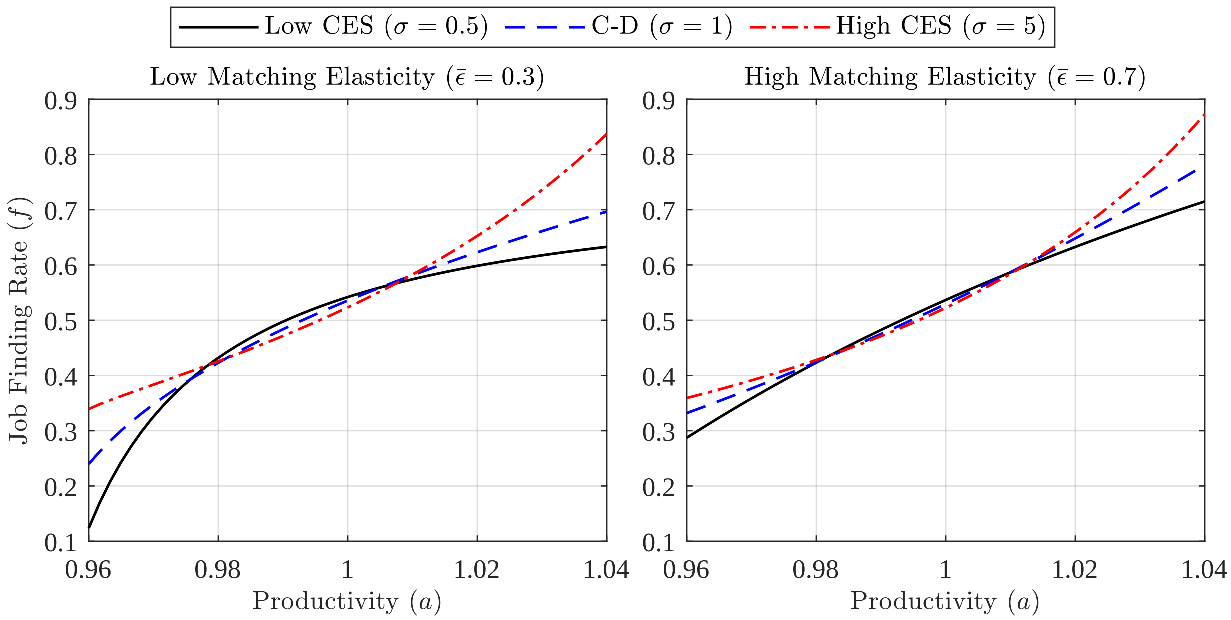

Figure 1: Nonlinearity of the job finding rate function.

When $\bar{\epsilon} = 0.3$, the threshold for convexity is relatively high, so the job finding rate is concave in the Cobb-Douglas case and features pronounced concavity when $\sigma = 0.5$. When $\bar{\epsilon} = 0.7$, the threshold is lower, which results in far weaker concavity when $\sigma = 0.5$ and mild convexity in the Cobb-Douglas case. When $\sigma = 5$, there is pronounced convexity for both values of $\bar{\epsilon}$. These results illustrate the importance of the matching function parameters for nonlinear labor market dynamics.

Higher-Order Moments Table 3 shows key untargeted moments across the $(\sigma, \bar{\epsilon})$ pairs. Consider first the specifications where $\bar{\epsilon} = 0.3$, which is close to recent estimates of the mean matching elasticity reported in Table 1. When $\sigma = 0.5$ and the matching elasticity is countercyclical, positive productivity shocks are dampened relative to negative shocks. As a result, job finding rate dynamics exhibit significant negative skewness (-1.4), which amplifies the positive skewness and kurtosis of the unemployment rate (2.37 and 9.78). These outcomes are flipped when $\sigma = 5$ and the matching elasticity is procyclical. The job finding rate becomes positively skewed (0.33), and the positive skewness and kurtosis of the unemployment rate are considerably weaker (0.29 and 0.04).

Qualitatively similar patterns emerge when $\bar{\epsilon} = 0.7$, though the differences across σ values are much less pronounced. In line with the logic from Proposition 5, a higher mean matching elasticity lowers the threshold that σ must exceed for the job finding rate function to be convex in productivity. Therefore, there is much less negative skewness when $\sigma = 0.5$ (-0.29), which results in less amplification of the positive skewness and kurtosis of the unemployment rate (0.95 and 1.55). When $\sigma = 5$, the job finding rate is even more positively skewed than when $\bar{\epsilon} = 0.3$ (0.49), which results in almost no skewness or kurtosis in the unemployment rate (0.15 and -0.08).

Table 3: Higher-Order Labor Market Moments

	Low Matching Elasticity ($\bar{\epsilon} = 0.3$)			High Matching Elasticity ($\bar{\epsilon} = 0.7$)		
	Low CES ($\sigma = 0.5$)	C-D ($\sigma = 1$)	High CES ($\sigma = 5$)	Low CES ($\sigma = 0.5$)	C-D ($\sigma = 1$)	High CES ($\sigma = 5$)
$Skew(f)$	−1.40	−0.58	0.33	−0.29	0.12	0.49
$Skew(u)$	2.37	1.35	0.29	0.95	0.49	0.15
$Kurt(f)$	3.35	0.75	0.15	0.08	−0.06	0.32
$Kurt(u)$	9.78	3.63	0.04	1.55	0.33	−0.08
$SD(\epsilon)$	0.07	0.00	0.07	0.04	0.00	0.03
$Corr(\epsilon, u)$	0.96	0.00	−0.98	0.97	0.00	−0.98

Note: The choices for $\bar{\epsilon}$ capture the range of estimates in the literature. The choice of $\sigma = 0.5$ corresponds to $\iota = 1$ under the DRW matching function, $\sigma = 1$ is the Cobb-Douglas case, and $\sigma = 5$ is chosen so the standard deviation of the matching elasticity covers at most half the range in Lange and Papageorgiou (2020). All $(\sigma, \bar{\epsilon})$ pairs have the same $E(u)$, $SD(u)$, and $Slope(w, a)$ to isolate the impact of the matching function.

Crucially, the large variation in higher-order labor market moments is driven by modest cyclical movements in the matching elasticity. When $\sigma \neq 1$, the standard deviation of the matching elasticity ranges from 0.03 to 0.07. This modest variation implies that the matching elasticity would rarely leave the range of estimates in Table 1, given a mean in that range. It also aligns with the direct evidence of cyclical variation provided by Lange and Papageorgiou (2020). They find the matching elasticity is procyclical, varying between 0.15 and 0.30 with a standard deviation of 0.04. These estimates imply far less nonlinearity in labor market dynamics than the literature has recently emphasized (e.g., Ferraro, 2018; Petrosky-Nadeau and Zhang, 2017; Petrosky-Nadeau et al., 2018).[9]

The dependence of the higher-order moments on the elasticity of substitution suggests we could identify σ by adding them as empirical targets. We explored this strategy but did not find it compelling for two reasons. First, the estimate for σ was sensitive to the targeted higher-order moments (e.g., $Skew(u)$ or $Skew(f)$) and the steady-state matching elasticity $\bar{\epsilon}$. Second, identifying σ using higher-order moments assumes that cyclical variation in the matching elasticity is the only driver of nonlinear dynamics. This contradicts existing work such as Dupraz et al. (2019), who show how downward wage rigidity can also create nonlinear labor market dynamics. Thus, the estimate of σ would also be sensitive to the inclusion of model ingredients that affect higher-order moments. The macro implications of the matching function, instead, motivate additional microeconometric work.

Impulse Responses A growing literature uses the search and matching model as a lens for understanding deep recessions and business cycle asymmetries (e.g., Dupraz et al., 2019; Petrosky-Nadeau and Zhang, 2017; Petrosky-Nadeau et al., 2018). Our analysis shows the matching function specification plays a crucial role in this setting. While the skewness and kurtosis moments capture some of this effect, Figure 2 provides further context by plotting generalized impulse responses of

[9] Appendix D shows these results are robust to introducing endogenous labor participation into the baseline model.

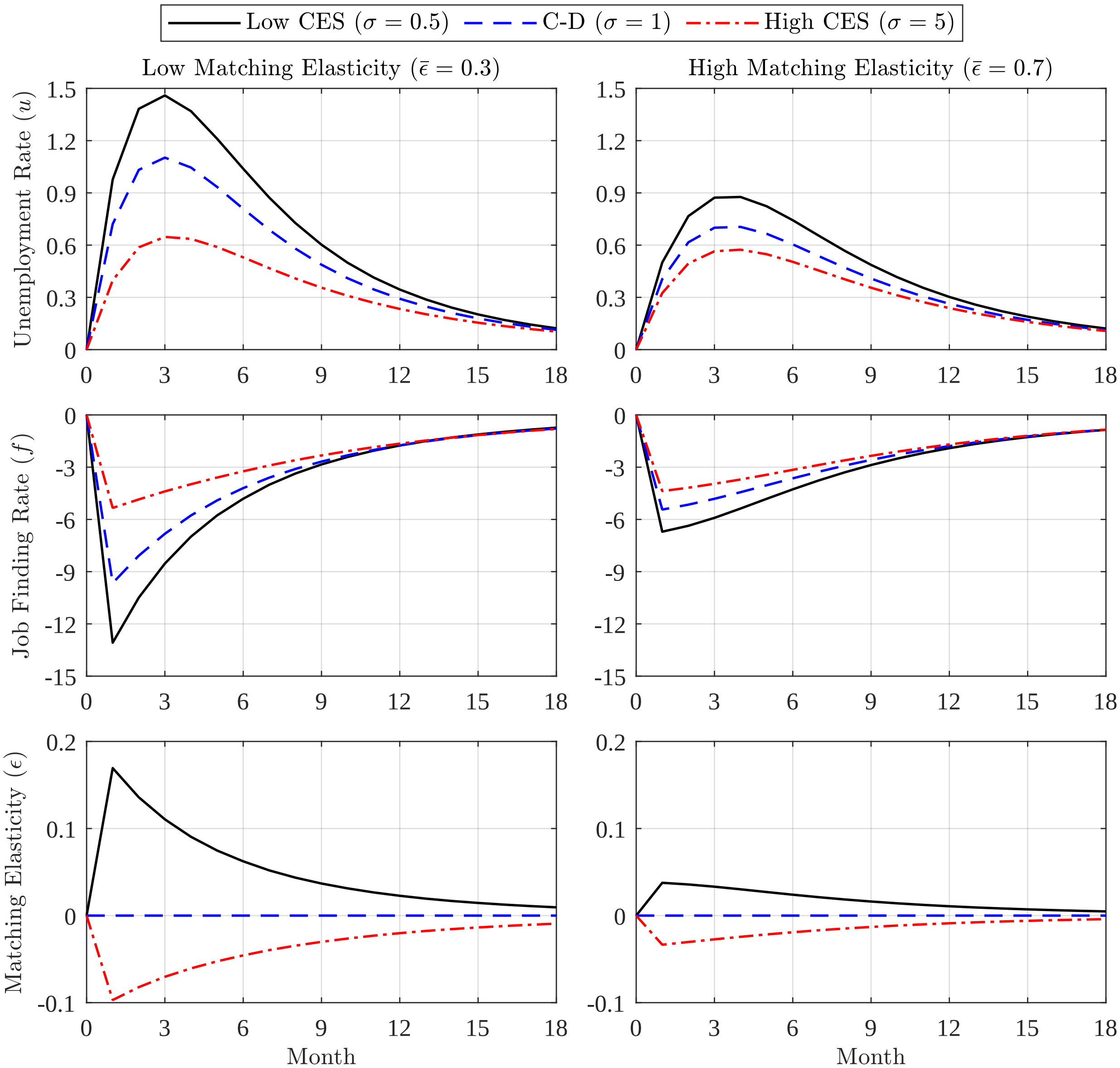

Figure 2: Generalized impulse responses to a -2 SD shock initialized in a recession ($u_0 = 7.5\%$). The job finding and unemployment rates are percentage point changes and the matching elasticity is a level change.

the unemployment and job finding rates to a 2 standard deviation negative productivity shock.[10] We allow for state-dependence by initializing the simulations in a recession ($u_0 = 7.5\%$). When we alternatively initialize the simulations at steady state ($u_0 = 5.9\%$), the responses are similar across matching function specifications. This intuitively follows from the fact that our parameter calibration strategy ensures that all matching function specifications generate similar first-order dynamics.

Large differences in the impulse responses emerge when the shock hits in a recessionary state

[10]Following Koop et al. (1996), the response of x_{t+h} over horizon h is given by $\mathcal{G}_t(x_{t+h}|\varepsilon_{a,t+1} = -2, \mathbf{z}_t) = E_t[x_{t+h}|\varepsilon_{a,t+1} = -2, \mathbf{z}_t] - E_t[x_{t+h}|\mathbf{z}_t]$, where $\mathbf{z}_t$ is a vector of initial states and -2 is the shock size in period $t+1$.

and the mean matching elasticity is low ($\bar{\epsilon} = 0.3$). When $\sigma = 0.5$ and the job finding rate is a concave function of productivity, the matching function generates an unemployment rate response that is more than double the response when $\sigma = 5$. The larger response is driven by a larger decline in the job finding rate, which follows from the countercyclical increase in the matching elasticity. If the mean matching elasticity is higher ($\bar{\epsilon} = 0.7$), the differences in the responses across σ are still apparent, but not as pronounced. This again shows that the average level and cyclicality of the matching elasticity are important to account for when studying nonlinear business cycle dynamics.

6 NORMATIVE IMPLICATIONS

This section shows the cyclicality of the matching elasticity has normative implications, which affect the wedges that restore efficiency and the response of the efficient real interest rate to shocks.

6.1 EFFICIENT FISCAL POLICY The equilibrium of a search and matching model is generally inefficient due to two externalities in the matching process (Hosios, 1990). First, when a firm posts a new vacancy, it imposes a positive externality on unemployed workers who face a higher job finding rate. Second, the same vacancy posting imposes a negative externality on other firms who face lower job filling rates and a higher marginal cost of vacancy creation today and in the future.

To show how the cyclicality of the matching elasticity affects the externalities in the matching process, we follow Arseneau and Chugh (2012) and derive the solution to a planning problem in which both externalities are internalized using a general matching function, $\mathcal{M}(u_{t-1}, v_t)$. The planner's problem and solution are shown in Appendix A. The key optimality condition is given by

$$\frac{\kappa - \lambda_{v,t}}{\mathcal{M}_v(u_{t-1}, v_t)} = a_t - b + E_t\left[x_{t+1}\frac{\kappa - \lambda_{v,t+1}}{\mathcal{M}_v(u_t, v_{t+1})}(1 - \bar{s} - \mathcal{M}_u(u_t, v_{t+1}))\right], \tag{16}$$

which determines the optimal level of vacancy creation by setting the social marginal cost (SMC) of a vacancy equal to its social marginal benefit (SMB). We then compare this solution to the private equilibrium shown in Section 3. The gaps between the SMC and SMB and the private marginal cost (PMC) and private marginal benefit (PMB) reflect inefficiencies of the equilibrium.

To characterize these gaps, we follow the public finance literature and solve for the wedges—state-dependent, linear taxes—that equate the two solutions. Let $\tau_{v,t}$ denote a tax on vacancy creation, v_t, and $\tau_{n,t}$ a tax on a firm's payroll, n_{t-1}, so that the firm's flow profits are given by $(a_t - w_t)n_t - (1+\tau_{v,t})\kappa v_t - \tau_{n,t}n_{t-1}$.[11] Then the firm's optimal vacancy creation choice is given by

$$\frac{\kappa - \lambda_{v,t}}{q_t} = \frac{1-\eta}{1+\tau_{v,t}}(a_t - b) + E_t\left[\tilde{x}_{t+1}\frac{\kappa - \lambda_{v,t+1}}{q_{t+1}}\left(1 - \bar{s} - \frac{1}{1+\tau_{v,t+1}}\frac{q_{t+1}}{\kappa - \lambda_{v,t+1}}(\kappa\eta\theta_{t+1} + \tau_{n,t+1})\right)\right],$$

where $\tilde{x}_{t+1} \equiv x_{t+1}(1+\tau_{v,t+1})/(1+\tau_{v,t})$. We can now solve for the wedges that restore efficiency.

[11] Placing a wedge on n_t would be equivalent. We put the wedge on n_{t-1} since it is easier to compute and interpret.

Proposition 6. *The efficiency-restoring wedges are given by*

$$\tau_v(\theta_t) = (1-\eta)/\epsilon(\theta_t) - 1,$$
$$\tau_n(\theta_t) = \theta_t((\kappa - \lambda_{v,t})\tau_v(\theta_t) - \eta\lambda_{v,t}),$$

where each wedge is evaluated at the solution to the planning problem. Furthermore, $\tau_v'(\theta_t) > 0$ *when* $\sigma_t < 1$*, and* $\tau_n'(\theta_t) > 0$ *when* $\sigma_t < \frac{1-\eta}{\eta}\frac{1-\epsilon_t}{\epsilon_t}$ *for all* $\theta_t > 0$*.*

The expression for $\tau_{v,t}$ shows how the vacancy tax balances the externalities. Note that $1-\eta$ is the ratio of the period-t PMB, $(1-\eta)(a_t - b)$, to the period-t SMB, $a_t - b$. The matching elasticity $\epsilon_t = \frac{\kappa/q_t}{\kappa/\mathcal{M}_v(u_{t-1},v_t)}$ is the ratio of the PMC to the SMC. The sign of the wedge depends on which ratio is larger. For example, $\tau_{v,t} > 0$ when $\epsilon_t < 1-\eta$ and the marginal cost gap is smaller than the marginal benefit gap. In this case, there is inefficiently high private vacancy creation and the negative externality on firms dominates the positive externality on workers. A positive vacancy wedge dampens the incentive for private vacancy creation, restoring efficiency of the equilibrium.

Crucially, $\tau_{v,t}$ co-moves negatively with the matching elasticity, indicating its time-varying strength. For example, if the matching function is CES, then the matching elasticity is countercyclical and $\tau_{v,t}$ is procyclical when $\sigma < 1$ because the gap between private and efficient vacancy creation is larger in booms. In contrast, $\tau_{v,t}$ is countercyclical when $\sigma > 1$ because the gap is larger in recessions. Finally, in the knife-edge case where $\sigma = 1$, the efficient vacancy tax is constant. Thus, the matching function specification is crucial for implementing efficient taxes on vacancy creation.

The payroll tax accounts for the gap between the period-$t+1$ SMB and PMB. Intuitively, private vacancy creation boosts employment today, which lowers u_t and raises the marginal cost of vacancy creation in the future. A payroll tax is necessary to limit private vacancy creation in period t and undo the negative externality. Restricting attention to $\theta_t > 0$ so that $\lambda_{v,t} = 0$ and $\tau_{n,t} = \kappa\theta_t\tau_{v,t}$, the variation in the optional labor tax is determined by two forces. The first is procyclical variation in labor market tightness, θ_t. The second is variation in $\tau_{v,t}$, which is decreasing in labor market tightness when $\sigma_t > 1$. As long as $\sigma_t < \frac{1-\eta}{\eta}\frac{1-\epsilon_t}{\epsilon_t}$, this force is dominated by or amplifies the first channel so that $\tau_{n,t}$ is procyclical. Under the typical calibration suggested by Hagedorn and Manovskii (2008), the Nash bargaining weight, η, tends to be relatively small. Therefore, even if the elasticity of substitution is greater than one, the optimal labor tax will be highly volatile due to the procyclical variation in labor market tightness, consistent with Arseneau and Chugh (2012).

6.2 OPTIMAL MONETARY POLICY

When the real allocation is efficient, the corresponding real interest rate, r_t^*, serves as the key target for monetary policy in the presence of nominal rigidities.[12]

[12]The optimality of targeting r_t^* requires appropriate fiscal policies to correct for the matching externalities described above and for the inefficient markups created by price-setting power. See Lepetit (2020) for an example of optimal monetary policy without fiscal policies in a search and matching model with the Cobb-Douglas matching function.

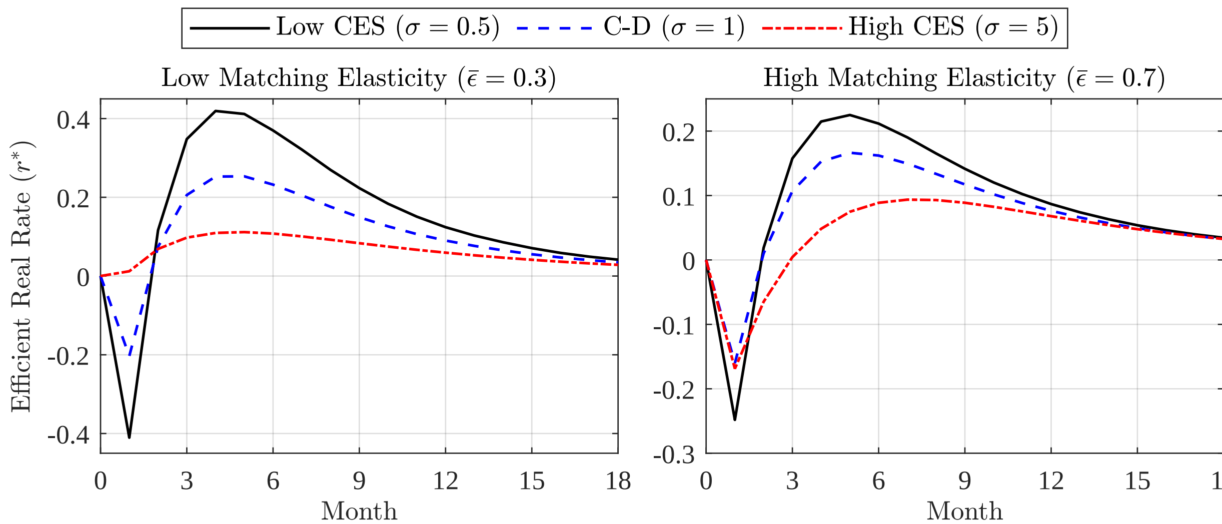

Figure 3: Percentage point responses to a -2 SD shock initialized in a recession ($u_0 = 7.5\%$).

To understand how the nonlinearities in the matching function impact the optimal monetary policy response to productivity shocks, Figure 3 plots generalized impulse responses of r_t^* to a 2 standard deviation negative labor productivity shock when the economy begins in a recession ($u_0 = 7.5\%$).[13]

The responses of r_t^* are driven by expected changes in consumption growth. Since the consumption response largely follows the negative of the unemployment rate response in Figure 2, r_t^* inherits its nonlinear dynamics, which are affected by the matching function specification. Consider the responses when $\bar{\epsilon} = 0.3$. When $\sigma = 0.5$, the higher peak unemployment response leads to a larger decline in consumption and a more volatile r_t^* response than when $\sigma = 5$. The initial decline in r_t^* occurs because consumption growth first declines in response to the shock, before increasing as the shock dissipates. This effect disappears when $\sigma = 5$ due to the weaker unemployment response. Similar results emerge when $\bar{\epsilon} = 0.7$, except the differences in the r^* responses are muted with less curvature from the matching function. Just like the optimal wedges, these results show the importance of knowing the matching function for the conduct of optimal monetary policy.

7 CONCLUSION

The Cobb-Douglas matching function is ubiquitous in search and matching models, even though it imposes a constant elasticity of matches with respect to vacancies that is unlikely to hold empirically. To examine the implications of this discrepancy, we use a general constant returns to scale matching function to derive conditions that show how the cyclicality of the matching elasticity affects the shape of the job finding rate as a function of productivity and amplifies or dampens nonlin-

[13]Following the approach in Section 5, we set the vacancy posting cost, κ, and flow value of unemployment, b, in the efficient equilibrium so that the mean and standard deviation of the unemployment rate are fixed across $(\sigma, \bar{\epsilon})$ pairs.

ear labor market dynamics. We then show these effects are quantitatively large and driven by modest cyclical variation in the matching elasticity that is consistent with recent empirical estimates.

While richer models could affect the strength of the nonlinearities, the Cobb-Douglas matching function is not without loss of generality. The cyclicality of the matching elasticity that ensues when deviating from Cobb-Douglas would feed into to job finding and unemployment rate dynamics in any search and matching model, so it is important for future research to show how alternative matching functions affect their results. Furthermore, we hope our analysis motivates empirical work that provides additional clarity on the true nature of the matching frictions in the labor market.

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A DERIVATIONS AND PROOFS

A.1 WAGES To derive the wage rate under Nash bargaining, consider the household's problem:

$$J_t = \max_{c_t} c_t^{1-\gamma}/(1-\gamma) + \beta E_t J_{t+1}$$

subject to

$$\begin{aligned} c_t &= w_t n_t + d_t + b u_t - \tau_t, \\ n_t &= (1-\bar{s})n_{t-1} + f_t u_{t-1}, \\ u_t &= u_{t-1} + \bar{s} n_{t-1} - f_t u_{t-1}, \end{aligned}$$

where τ_t is a lump-sum tax and d_t are lump-sum dividends from firm ownership. The marginal values of employment and unemployment relative to the marginal utility of consumption are given by

$$\begin{aligned} J_{n,t}^H &= w_t + E_t[x_{t+1}((1-\bar{s})J_{n,t+1}^H + \bar{s} J_{u,t+1}^H)], \\ J_{u,t}^H &= b + E_t[x_{t+1}(f_{t+1} J_{n,t+1}^H + (1-f_{t+1})J_{u,t+1}^H)]. \end{aligned}$$

Similarly, use the firm's problem to define the marginal value of employment to the firm,

$$J^F_{n,t} = a_t - w_t + (1-\bar{s})E_t[x_{t+1}J^F_{n,t+1}] = \tfrac{\kappa-\lambda_{v,t}}{q_t}.$$

Define the total surplus of a new match as $\Lambda_t = J^F_{n,t} + J^H_{n,t} - J^H_{u,t}$. The equilibrium wage maximizes $(J^H_{n,t} - J^H_{u,t})^\eta (J^F_{n,t})^{1-\eta}$. Optimality implies $J^H_{n,t} - J^H_{u,t} = \eta\Lambda_t$ and $J^F_{n,t} = (1-\eta)\Lambda_t$. Combining the optimality conditions with $J^H_{n,t}$, $J^H_{u,t}$, and $J^F_{n,t}$, and defining tightness as $\theta_t = v_t/u_{t-1}$, we obtain

$$w_t = \eta(a_t + \kappa E_t[x_{t+1}\theta_{t+1}]) + (1-\eta)b.$$

A.2 THE EFFICIENT ALLOCATION To solve for the efficient allocation, we imagine that the frictional labor market is controlled by a central planner who posts vacancies on behalf of firms, so it internalizes the two externalities associated with vacancy creation. The central planner solves

$$W_t = \max_{c_t, n_t, v_t} c_t^{1-\gamma}/(1-\gamma) + \beta E_t W_{t+1}$$

subject to

$$c_t = a_t n_t - \kappa v_t + b(1-n_t) - \tau_t,$$
$$n_t = (1-\bar{s})n_{t-1} + \mathcal{M}(1-n_{t-1}, v_t),$$
$$v_t \geq 0,$$

which imposes $u_t = 1 - n_t$. The efficient allocation is characterized by (1), (8), (10), and

$$\tfrac{\kappa-\lambda_{v,t}}{\mathcal{M}_v(1-n_{t-1},v_t)} = a_t - b + E_t[x_{t+1}\tfrac{\kappa-\lambda_{v,t+1}}{\mathcal{M}_v(1-n_t,v_{t+1})}(1-\bar{s}-\mathcal{M}_u(1-n_t,v_{t+1}))], \tag{A.1}$$
$$n_t = (1-\bar{s})n_{t-1} + \mathcal{M}(1-n_{t-1}, v_t). \tag{A.2}$$

A.3 PROOFS Recall $\mathcal{M}(u^s_t, v_t)$ is strictly increasing, strictly concave, and twice differentiable in both arguments, and it exhibits constant returns to scale. We use the following standard results:

Lemma 1. $\mathcal{M}_{vv}(1,\theta_t)\theta_t = -\mathcal{M}_{uv}(1,\theta_t)$.

Lemma 2. *The elasticity of substitution has the equivalent representation*

$$\sigma(\theta_t) = \frac{\mathcal{M}_v(1,\theta_t)\mathcal{M}_u(1,\theta_t)}{\mathcal{M}_{vu}(1,\theta_t)\mathcal{M}(1,\theta_t)}.$$

Proposition 1 A constant returns to scale matching function, $\mathcal{M}(u^s_t, v_t)$, has linear approximation

$$\mathcal{M}(u^s_t, v_t) \approx \mathcal{M}(\bar{u}^s, \bar{v}) + \mathcal{M}_u(\bar{u}^s, \bar{v})(u^s_t - \bar{u}^s) + \mathcal{M}_v(\bar{u}^s, \bar{v})(v_t - \bar{v}),$$

where $(\bar{u}^s, \bar{v})$ is the point of approximation (e.g., a model's deterministic steady state). By constant returns to scale, Euler's theorem implies $\bar{m} \equiv \mathcal{M}(\bar{u}^s, \bar{v}) = \mathcal{M}_u(\bar{u}^s, \bar{v})\bar{u}^s + \mathcal{M}_v(\bar{u}^s, \bar{v})\bar{v}$. Combining these results and converting the steady-state partial derivatives into matching elasticities yields

$$\mathcal{M}(u_t^s, v_t) \approx (1-\bar{\epsilon})\tfrac{\bar{m}}{\bar{u}^s}u_t^s + \bar{\epsilon}\tfrac{\bar{m}}{\bar{v}}v_t, \tag{A.3}$$

where $\bar{\epsilon}$ is the matching elasticity evaluated at the approximation point. However, (A.3) is also the first-order approximation of a Cobb-Douglas matching function $\mathcal{M}(u_t^s, v_t) = \phi(u_t^s)^{\alpha}v_t^{1-\alpha}$ with $\alpha = 1-\bar{\epsilon}$. Thus, using the Cobb-Douglas specification is without loss of generality up to first order.

Proposition 2 Differentiating the matching elasticity function $\epsilon(\theta_t) = \frac{\mathcal{M}_v(1,\theta_t)\theta_t}{\mathcal{M}(1,\theta_t)}$ yields

$$\epsilon'(\theta_t) = \left(\frac{\mathcal{M}_{vv}(1,\theta_t)\theta_t}{\mathcal{M}_v(1,\theta_t)} + 1 - \epsilon(\theta_t)\right)\frac{\mathcal{M}_v(1,\theta_t)}{\mathcal{M}(1,\theta_t)}.$$

Use Lemma 1 and Lemma 2 to obtain

$$\epsilon'(\theta_t) = \left(-\frac{1}{\sigma(\theta_t)}\frac{\mathcal{M}_u(1,\theta_t)}{\mathcal{M}(1,\theta_t)} + 1 - \epsilon(\theta_t)\right)\frac{\mathcal{M}_v(1,\theta_t)}{\mathcal{M}(1,\theta_t)}.$$

Replace $\frac{\mathcal{M}_u(1,\theta_t)}{\mathcal{M}(1,\theta_t)} = 1 - \epsilon(\theta_t)$ and rearrange to obtain

$$\epsilon'(\theta_t) = \frac{\sigma(\theta_t)-1}{\sigma(\theta_t)}(1-\epsilon(\theta_t))\frac{\mathcal{M}_v(1,\theta_t)}{\mathcal{M}(1,\theta_t)}. \tag{A.4}$$

Hence the sign of $\epsilon'(\theta_t)$ has the same sign as $\sigma(\theta_t) - 1$.

Corollary 1 Combine Proposition 2 with the fact that $\sigma(\theta_t) = \sigma$ for all $\theta_t > 0$.

Proposition 3 After imposing Assumption 1, (7) simplifies to

$$\frac{\kappa - \lambda_{v,t}}{q_t} = a_t - b + \beta(1-\bar{s})E_t\left[\frac{\kappa - \lambda_{v,t+1}}{q_{t+1}}\right].$$

We can guess and verify a unique solution of the form $\frac{\kappa-\lambda_{v,t}}{q_t} = \delta_0 + \delta_1(a_t - \bar{a})$, where

$$\delta_0 = \frac{\bar{a} - b}{1-\beta(1-\bar{s})}, \qquad \delta_1 = \frac{1}{1-\beta(1-\bar{s})\rho_a}.$$

If $\lambda_{v,t} > 0$ then $v_t = 0$. Since $m_t = v_t$ and $q_t = 1$ for v_t arbitrarily close to 0, we have $q_t = 1$ when $\lambda_{v,t} > 0$ by continuity. Therefore, if productivity is such that $\kappa/(\delta_0 + \delta_1(a_t - \bar{a})) \in [0, 1)$, then $q(a_t) = \kappa/(\delta_0 + \delta_1(a_t - \bar{a}))$ and $\lambda_{v,t} = 0$. Otherwise, $q_t = 1$ and $\lambda_{v,t} = \kappa - \delta_0 - \delta_1(a_t - \bar{a})$.

Proposition 4 Differentiate $\mu_q(\theta) = \mathcal{M}(1,\theta)/\theta$ to obtain $\mu_q'(\theta) = -\frac{1-\epsilon(\theta)}{\theta}\frac{\mathcal{M}(1,\theta)}{\theta}$. Hence

$$\theta'(a_t) = -\frac{q'(a_t)}{1-\epsilon_t}\frac{\theta(a_t)^2}{\mathcal{M}(1,\theta(a_t))}.$$

Use $q'(a_t) = -q(a_t)^2\delta_1/\kappa$ and $q(a_t)\theta(a_t) = \mathcal{M}(1,\theta(a_t))$, to obtain

$$\theta'(a_t) = \frac{\delta_1}{\kappa}\frac{\mathcal{M}(1,\theta(a_t))}{1-\epsilon_t} > 0.$$

Differentiate and use (A.4) to obtain

$$\theta''(a_t) = \frac{\delta_1}{\kappa}\frac{2\sigma_t - 1}{\sigma_t}\frac{\mathcal{M}_v(1,\theta(a_t))\theta'(a_t)}{1-\epsilon_t}. \tag{A.5}$$

Hence the sign of $\theta''(a_t)$ has the same sign as $\sigma_t - 1/2$.

Proposition 5 Differentiate $f'(a_t) = \mathcal{M}_v(1,\theta(a_t))\theta'(a_t)$ to obtain

$$f''(a_t) = \mathcal{M}_{vv}(1,\theta(a_t))\theta'(a_t)^2 + \mathcal{M}_v(1,\theta(a_t))\theta''(a_t).$$

Use Lemma 1 and Lemma 2 to obtain

$$f''(a_t) = \left(\theta''(a_t) - \frac{1}{\sigma(\theta)}\frac{\mathcal{M}_u(1,\theta)}{\mathcal{M}(1,\theta)}\frac{\theta'(a_t)^2}{\theta(a_t)}\right)\mathcal{M}_v(1,\theta(a_t)).$$

Replace $\frac{\mathcal{M}_u(1,\theta_t)}{\mathcal{M}(1,\theta_t)} = 1 - \epsilon(\theta_t)$ and use (A.5) to obtain

$$f''(a_t) = \left(\frac{\delta_1}{\kappa}\frac{2\sigma_t - 1}{\sigma_t}\frac{\mathcal{M}_v(1,\theta(a_t))}{1-\epsilon_t} - \frac{1-\epsilon_t}{\sigma_t}\frac{\theta'(a_t)}{\theta(a_t)}\right)\theta'(a_t)\mathcal{M}_v(1,\theta(a_t)).$$

Use $\theta'(a_t) = \frac{\delta_1}{\kappa}\frac{\mathcal{M}(1,\theta(a_t))}{1-\epsilon_t}$ and $\epsilon_t = \frac{\mathcal{M}_v(1,\theta_t)\theta_t}{\mathcal{M}(1,\theta_t)}$ to obtain

$$f''(a_t) = \frac{2\sigma_t\epsilon_t - 1}{\sigma_t}\frac{\mathcal{M}_v(1,\theta(a_t))(\theta'(a_t))^2}{\theta(a_t)}.$$

Hence the sign of $f''(a_t)$ is the same as the sign of $\sigma_t\epsilon_t - 1/2$.

Corollary 2 Recall that $q(a_t) \in (0,1)$. When the matching function is CES, we have $\sigma_t = \sigma$ and $\epsilon_t = (1-\vartheta)(\phi/q(a_t))^{(\sigma-1)/\sigma}$. By Proposition 5, the sign of $f''(a_t)$ depends on whether

$$\mathcal{F}_t \equiv 2\sigma(1-\vartheta)(\phi/q(a_t))^{(\sigma-1)/\sigma} \lesseqgtr 1.$$

Case 1 ($\sigma > 1$): $(\phi/q(a_t))^{(\sigma-1)/\sigma} \in \left(\phi^{(\sigma-1)/\sigma}, \infty\right)$, so $\mathcal{F}_t > 2\sigma(1-\vartheta)\phi^{(\sigma-1)/\sigma}$ for all feasible $q(a_t)$. Thus, $\sigma > \frac{1}{2(1-\vartheta)\phi^{(\sigma-1)/\sigma}} \geq 1$ implies $f''(a_t) > 0$ for all a_t such that $q(a_t) \in (0,1)$.

Case 2 ($\sigma < 1$): $(\phi/q(a_t))^{(\sigma-1)/\sigma} \in \left(0, \phi^{(\sigma-1)/\sigma}\right)$, so $\mathcal{F}_t < 2\sigma(1-\vartheta)\phi^{(\sigma-1)/\sigma}$ for all feasible $q(a_t)$. Thus, $\sigma < \frac{1}{2(1-\vartheta)\phi^{(\sigma-1)/\sigma}} \leq 1$ implies $f''(a_t) < 0$ for all a_t such that $q(a_t) \in (0,1)$.

Case 3 ($\sigma = 1$): $\sigma = 2(1-\vartheta) = 1$ implies $f''(a_t) = 0$ for all a_t such that $q(a_t) \in (0,1)$.

Corollary 3 Given the Den Haan et al. (2000) matching function, we have $\sigma_t = 1/(1+\iota)$ and $\epsilon_t = q(a_t)^\iota$. By Proposition 5, the sign of $f''(a_t)$ depends on whether

$$\mathcal{F}_t = 2q(a_t)^\iota/(1+\iota) \lesseqgtr 1.$$

Since $\iota > 0$, we have $2q(a_t)^\iota/(1+\iota) < 2/(1+\iota)$ for all feasible $q(a_t)$. Therefore $\iota > 1$ implies that $f''(a_t) < 0$ for all a_t such that $q(a_t) \in (0,1)$.

Proposition 6 Given wedges $\{\tau_{v,t}, \tau_{n,t}\}$, the firm's optimal vacancy creation condition becomes

$$\frac{\kappa-\lambda_{v,t}}{q_t} = \frac{1-\eta}{1+\tau_{v,t}}(a_t-b) + E_t\left[\tilde{x}_{t+1}\frac{\kappa-\lambda_{v,t+1}}{q_{t+1}}\left(1-\bar{s}-\frac{1}{1+\tau_{v,t+1}}\frac{q_{t+1}}{\kappa-\lambda_{v,t+1}}(\kappa\eta\theta_{t+1}+\tau_{n,t+1})\right)\right],$$

where $\tilde{x}_{t+1} \equiv x_{t+1}(1+\tau_{v,t+1})/(1+\tau_{v,t})$. Setting

$$\tau_v(\theta_t) = (1-\eta)/\epsilon(\theta_t) - 1,$$
$$\tau_n(\theta_t) = \theta_t((\kappa-\lambda_{v,t})\tau_v(\theta_t) - \eta\lambda_{v,t}),$$

aligns the private optimality condition with the efficient condition (A.1). Differentiating yields

$$\tau_v'(\theta_t) = -(1-\eta)\epsilon'(\theta_t)/\epsilon(\theta_t)^2,$$
$$\tau_n'(\theta_t) = \kappa(\theta_t\tau_v'(\theta_t) + \tau_v(\theta_t)) = \kappa\left[\frac{1-\eta}{\epsilon(\theta_t)} - 1 - \frac{1-\eta}{\epsilon(\theta_t)^2}\theta_t\epsilon'(\theta_t)\right].$$

Since (A.4) implies $\epsilon'(\theta_t)\theta_t/\epsilon(\theta_t) = (\sigma_t-1)(1-\epsilon(\theta_t))/\sigma_t$, we obtain

$$\tau_v'(\theta_t) = -\left(\frac{1-\eta}{\theta_t}\right)\left(\frac{\sigma_t-1}{\sigma_t}\right)\left(\frac{1-\epsilon(\theta_t)}{\epsilon(\theta_t)}\right),$$
$$\tau_n'(\theta_t) = \kappa\left[\frac{1-\eta}{\epsilon(\theta_t)} - 1 - (1-\eta)\left(\frac{\sigma_t-1}{\sigma_t}\right)\left(\frac{1-\epsilon(\theta_t)}{\epsilon(\theta_t)}\right)\right].$$

Hence, $\tau_v'(\theta_t) > 0$ when $\sigma_t < 1$ and $\tau_n'(\theta_t) > 0$ when $\sigma_t < \frac{1-\eta}{\eta}\frac{1-\epsilon_t}{\epsilon_t}$ for all $\theta_t > 0$.

B SOLUTION METHOD

The equilibrium system of the model is summarized by $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1})|\mathbf{z}_t, \mathcal{P}] = 0$, where g is a vector-valued function, $\mathbf{x}_t$ is a vector of variables, ε is a vector of productivity shocks, $\mathbf{z}_t$ is a vector of states, and $\mathcal{P}$ is a vector of parameters. There are many ways to discretize the productivity process. We use the Markov chain in Rouwenhorst (1995), which Kopecky and Suen (2010) show outperforms other methods for approximating autoregressive processes. The bounds on the state variable n_{t-1} are set to $[0.85, 0.98]$, which contains over 99% of the ergodic distribution. We discretize a_t and n_{t-1} into 7 and 21 evenly-spaced points, respectively. The product of the points in each dimension, D, is the total nodes in the state space ($D = 147$). The realization of $\mathbf{z}_t$ on node d is denoted $\mathbf{z}_t(d)$. The Rouwenhorst method provides integration weights, $\phi(m)$, for $m \in \{1, \ldots, M\}$.

Since vacancies $v_t \geq 0$, we introduce an auxiliary variable, μ_t, such that $v_t = \max\{0, \mu_t\}^2$ and $\lambda_{0,t} = \max\{0, -\mu_t\}^2$, where $\lambda_{v,t}$ is the Lagrange multiplier on the non-negativity constraint. If $\mu_t \geq 0$, then $v_t = \mu_t^2$ and $\lambda_{v,t} = 0$. When $\mu_t < 0$, the constraint is binding, $v_t = 0$, and $\lambda_{v,t} = \mu_t^2$. Therefore, the constraint on v_t is transformed into a pair of equalities (Garcia and Zangwill, 1981).

The following steps outline our nonlinear policy function iteration algorithm:

1. Use Sims's (2002) `gensys` algorithm to solve the linearized model. Then map the solution for the policy functions to the discretized state space. This provides an initial conjecture.
2. On iteration $j \in \{1, 2, \ldots\}$ and each node $d \in \{1, \ldots, D\}$, use Chris Sims's `csolve` to find $\mu_t(d)$ to satisfy $E[g(\cdot)|\mathbf{z}_t(d), \mathcal{P}] \approx 0$. Guess $\mu_t(d) = \mu_{j-1}(d)$. Then apply the following:
   (a) Solve for all variables dated at time t, given $\mu_t(d)$ and $\mathbf{z}_t(d)$.
   (b) Linearly interpolate the policy function, μ_{j-1}, at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mu_{t+1}(m)$ on every integration node, $m \in \{1, \ldots, M\}$.
   (c) Given $\{\mu_{t+1}(m)\}_{m=1}^{M}$, solve for the other elements of $\mathbf{x}_{t+1}(m)$ and compute
   $$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1})|\mathbf{z}_t(d), \mathcal{P}] \approx \textstyle\sum_{m=1}^{M} \phi(m) g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$

   Set $\mu_j(d) = \mu_t(d)$ when `csolve` converges.
3. Repeat step 2 until $\text{maxdist}_j < 10^{-7}$, where $\text{maxdist}_j \equiv \max\{|\mu_j - \mu_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate nonlinear solution.

The algorithm is programmed in Fortran with Open MPI and run on the BigTex supercomputer.

C HOME PRODUCTION

In the baseline model, we set b to target the standard deviation of unemployment in our sample. This section shows we can equivalently set b externally as an unemployment benefit, and instead use home production to target unemployment volatility by following Petrosky-Nadeau et al. (2018).

The household derives utility from the consumption of the final market good $c_{m,t}$ and home production $c_{h,t}$. It has log utility over composite consumption $c_t = (\omega c_{m,t}^e + (1-\omega)c_{h,t}^e)^{1/e}$, where $\omega \in (0, 1)$ is the preference weight on the final market good and $e \leq 1$ governs the elasticity of substitution $1/(1-e)$. The home production technology is $c_{h,t} = a_h u_t$, where $a_h > 0$ is productivity.

Household optimization yields the pricing kernel $x_{t+1} = \beta(c_{m,t}/c_{m,t+1})^{1-e}(c_t/c_{t+1})^e$. The flow value of unemployment becomes $z_t = a_h((1-\omega)/\omega)(c_{m,t}/c_{h,t})^{1-e} + b$, so the Nash wage satisfies

$$w_t = \eta((1-\alpha)y_t/n_t + \kappa(1-\chi\bar{s})E_t[x_{t+1}\theta_{t+1}]) + (1-\eta)z_t.$$

The other equilibrium conditions are unchanged from the baseline model described in Section 3.

We set $b = 0.4$ to reflect the value of unemployment benefits (Shimer, 2005), and set $a_h = 1$ to steady-state labor productivity in final good production. We then set $e = 1$, in line with existing calibrations and estimates (Benhabib et al., 1991; Petrosky-Nadeau et al., 2018). In this case, $z_t = (1-\omega)/\omega + b$, so ω determines the level of z, and hence the volatility of unemployment following the fundamental surplus arguments in Ljungqvist and Sargent (2017). Thus, we can set ω in each model to generate the same unemployment volatility and quantitative results as the baseline model.

D LABOR FORCE PARTICIPATION

In our baseline search and matching model in Section 3, the labor force is equal to the population, as is common in the literature. This section examines the robustness of our results by extending the baseline model to include endogenous labor force participation, which is an important component of labor market flows. Following Arseneau and Chugh (2012), the representative household solves

$$J_t = \max_{c_t, n_t, u_t} \tfrac{c_t^{1-\gamma}}{1-\gamma} - \tfrac{\zeta}{1+\nu}(n_t + u_t)^{1+\nu} + \beta E_t J_{t+1}$$

subject to

$$c_t = w_t n_t + d_t + b u_t - \tau_t,$$
$$n_t = (1-\bar{s}) n_{t-1} + f_t u_{t-1},$$
$$n_t + u_t \leq 1,$$

where $1/\nu$ is the Frisch elasticity of labor force participation with respect to the real wage and ζ is a preference parameter that determines the mean participation rate. The optimality conditions imply

$$\xi_t = w_t - b + E_t[x_{t+1}\xi_{t+1}(1 - \bar{s} - f_{t+1})], \tag{D.1}$$
$$\zeta \ell_t^{\nu} c_t^{\gamma} + \lambda_{\ell,t} c_t^{\gamma} = b + E_t[x_{t+1}\xi_{t+1} f_{t+1}], \tag{D.2}$$
$$\lambda_{\ell,t}(1-\ell_t) = 0, \quad \lambda_{\ell,t} \geq 0, \tag{D.3}$$

where $\ell_t \equiv n_t + u_t$ is the labor force participation rate, $\lambda_{\ell,t}$ is the multiplier on the labor force constraint, and ξ_t is the marginal value of employment relative to the marginal utility of consumption. Note the unemployment rate is given by $ur_t \equiv u_t/\ell_t$, since the labor force is no longer equal to 1.

The firm's problem is unchanged with optimality conditions given by (7) and (8). Nash bargaining between employed workers and the firm leads to the same equilibrium wage rate given by (9).

Endogenous labor force participation introduces two new parameters, ζ and ν, which are estimated to match the mean labor force participation rate ($E(\ell) = 0.633$) and the standard deviation of the detrended labor force participation rate ($SD(\ell) = 0.35$). All other parameters are pinned down using the strategy in Section 5. Each specification perfectly matches the six empirical targets.

Table 4: Endogenous Labor Force Participation Results

	Low Matching Elasticity ($\bar{\epsilon} = 0.3$)			High Matching Elasticity ($\bar{\epsilon} = 0.7$)		
	Low CES ($\sigma = 0.5$)	C-D ($\sigma = 1$)	High CES ($\sigma = 5$)	Low CES ($\sigma = 0.5$)	C-D ($\sigma = 1$)	High CES ($\sigma = 5$)
Vacancy Posting Cost (κ)	0.0811	0.0629	0.0528	0.3850	0.3659	0.3535
Flow Value of Unemployment (b)	0.9709	0.9770	0.9808	0.9223	0.9268	0.9294
Worker Bargaining Power (η)	0.1283	0.1323	0.1329	0.0545	0.0537	0.0538
Labor Force Utility Weight (ζ)	5.8060	5.9786	6.0398	5.7848	5.4791	5.4860
Labor Force Elasticity (ν)	2.7171	2.7804	2.8023	2.7220	2.5959	2.5982
Matching Efficiency (ϕ)	0.4534	0.4638	0.4680	0.3726	0.3809	0.3877
Unemployment Weight (ϑ)	0.5886	0.7000	0.7728	0.2110	0.3000	0.3838

(a) Estimated and implied parameter values.

	Low Matching Elasticity ($\bar{\epsilon} = 0.3$)			High Matching Elasticity ($\bar{\epsilon} = 0.7$)		
	Low CES ($\sigma = 0.5$)	C-D ($\sigma = 1$)	High CES ($\sigma = 5$)	Low CES ($\sigma = 0.5$)	C-D ($\sigma = 1$)	High CES ($\sigma = 5$)
$Skew(f)$	-1.34	-0.55	0.32	-0.29	0.12	0.47
$Skew(ur)$	2.35	1.33	0.31	0.95	0.49	0.16
$Kurt(f)$	3.11	0.66	0.14	0.09	-0.05	0.32
$Kurt(ur)$	9.96	3.60	0.07	1.61	0.34	-0.06
$SD(\epsilon)$	0.07	0.00	0.07	0.03	0.00	0.03
$Corr(\epsilon, ur)$	0.98	0.00	-1.00	0.99	0.00	-1.00

(b) Higher-order labor market moments.

Note: The choices for $\bar{\epsilon}$ capture the range of estimates in the literature. The choice of $\sigma = 0.5$ corresponds to $\iota = 1$ under the DRW matching function, $\sigma = 1$ is the Cobb-Douglas case, and $\sigma = 5$ is chosen so the standard deviation of the matching elasticity covers at most half the range in Lange and Papageorgiou (2020). All $(\sigma, \bar{\epsilon})$ pairs have the same $E(ur)$, $E(\ell)$, $SD(ur)$, $SD(\ell)$, and $Slope(w, a)$ to isolate the impact of the matching function. The unemployment rate is $ur = u/\ell$, since the labor force is no longer equal to 1.

Table 4 shows the estimated parameters and untargeted higher-order labor market moments, the analogous results to those shown Table 2 and 3. The estimates for the vacancy posting cost, κ, flow value of unemployment, b, and worker bargaining power, η, are very similar to the baseline model, while the estimates for ζ and ν are in line with the values reported in Arseneau and Chugh (2012).

Crucially, the patterns of the higher-order moments across $(\sigma, \bar{\epsilon})$ pairs are unchanged. When the steady-state matching elasticity, $\bar{\epsilon}$, and elasticity of substitution in the matching function, σ, are low, the matching elasticity is countercyclical, which creates significant skewness and kurtosis in the job finding and unemployment rates. Conversely, when σ is high and the matching elasticity is procyclical, these nonlinear dynamics are weaker. With a higher $\bar{\epsilon}$, changes in σ generate the same dynamics, but they are much less pronounced since the job finding rate function has less curvature.

These results show that our baseline results in Section 5 are robustness to endogenous labor force participation. This striking similarity in the predictions follows from the fact that the key conditions of nonlinearity derived in Section 4 are unchanged. If we impose the same assumptions we previously used for tractability ($\eta = \gamma = 0$), the solution for the job filling rate in Proposition 3 is identical. The curvature of the job finding rate function is also unchanged, since it only depends on the specification of the matching function and the curvature of labor market tightness that is pinned down by the solution for the job filling rate. In other words, the labor market dynamics we are focused on are effectively divorced from the labor force participation decision of the household.