1 2D rigid body

To model the 2d rigid body, we will start from a 2D polygon made of n points.
Each point will have a position and a mass.
We can get the total mass of the rigid body by summing the mass of its points :

$$m = \sum m_i$$

We can get the center of mass of the rigid body as such :

$$\bar{R} = \frac{\sum m_i \bar{r}_i}{\sum m_i}$$

We can get the moment of inertia of the rigid body as such :

$$I = \sum m_i r_i^2$$

Now we can give initial conditions to the rigid body, namely :

- $\bar{r}_0$: initial position
- $\bar{v}_0$: initial velocity
- θ_0 : initial angle
- ω_0 : initial angular velocity

We have the following relationships :

in the continous domain

$$\bar{a} = \frac{d\bar{v}}{dt}$$

$$\bar{v} = \frac{d\bar{r}}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

$$\omega = \frac{d\theta}{dt}$$

in the discrete domain

$$\bar{a} = \frac{\Delta\bar{v}}{\Delta t}$$

$$\bar{v} = \frac{\Delta\bar{r}}{\Delta t}$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Each cycle of the physics loop, we can apply forces on the rigid body :

$$\bar{F} = m\bar{a}$$

If the force is applied elsewhere than the center of mass, it will also provoque a torque on the rigid body :

$$\bar{\tau} = \bar{r} \times \bar{F}$$

$$\bar{\tau} = I\bar{\alpha}$$

Each cycle of the physics loop, we can know the acceleration on the rigid body, thus its velocity, thus its position as such :

$$\bar{a} = \frac{\sum \bar{F}}{m}$$

$$\bar{v}_n = \bar{v}_{n-1} + \bar{a}\Delta t$$

$$\bar{r}_n = \bar{r}_{n-1} + \bar{v}_n\Delta t$$

Each cycle of the physics loop, we can know the angular acceleration on the rigid body, thus its angular velocity, thus its angular position as such :

$$\bar{\alpha} = \frac{\sum \bar{\tau}}{I}$$

$$\bar{\omega}_n = \bar{\omega}_{n-1} + \bar{\alpha}\Delta t$$

$$\bar{\theta}_n = \bar{\theta}_{n-1} + \bar{\omega}_n\Delta t$$