1 translation, rotation, scaling

We consider a left-handed coordinate system.

$$v' \quad = \quad v \, * \, M$$

- v' : output vertex
- v : input vertex
- M : matrix (translation, rotation, scaling, ...)

1.1 translation matrix

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ tx & ty & tz & 1 \end{bmatrix}$$

1.2 scaling matrix

$$S = \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.3 rotation matrix around x-axis

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(n) & sin(n) & 0 \\ 0 & -sin(n) & cos(n) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.4 rotation matrix around y-axis

$$R_y = \begin{bmatrix} cos(n) & 0 & -sin(n) & 0 \\ 0 & 1 & 0 & 0 \\ sin(n) & 0 & cos(n) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.5 rotation matrix around z-axis

$$R_z = \begin{bmatrix} cos(n) & sin(n) & 0 & 0 \\ -sin(n) & cos(n) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2 camera and clip space

2.1 translation matrix

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -x & -y & -z & 1 \end{bmatrix}$$

(x, y, z) : camera position

2.2 rotation matrix

$$R = \begin{bmatrix} r.x & u.x & f.x & 0 \\ r.y & u.y & f.y & 0 \\ r.z & u.z & f.z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- r.x : x coordinates of r vector
- r : right camera orientation $(1, 0, 0, 1)$
- u : up camera orientation $(0, 1, 0, 1)$
- f : forward camera orientation $(0, 0, 1, 1)$

2.3 camera matrix

$$CM \quad = \quad T \ * \ R$$

2.4 projection matrix

$$P = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & 0 \\ 0 & \frac{2}{top-bottom} & 0 & 0 \\ 0 & 0 & \frac{far+near}{far-near} & 1 \\ 0 & 0 & \frac{-2.near.far}{far-near} & 0 \end{bmatrix}$$

- $h_{fov} = \pi/3$
- $v_{fov} = h_{fov} * \frac{height_{screen}}{width_{screen}}$
- near plane $= 0.1$
- far plane $= 100$
- right $= tan(h_{fov}/2)$
- left = - right
- top $= tan(v_{fov}/2)$
- bottom = - top

3 normalized clipping space (NDC) projection

$$v_p \quad = \quad v \; * \; CM \; * \; P$$

$$v_{pn} \quad = \quad v_p \; / \; v_p.w$$

- v : vertex
- v_p : vertex projected
- v_{pn} : vertex projected and normalized

if one of the x,y,z coordinates of the v_{pn} is outside $[-1, 1]$, then the vertex is off screen and must not be rendered.

4 screen projection

4.1 screen matrix

$$SC = \begin{bmatrix} \frac{width_{screen}}{2} & 0 & 0 & 0 \\ 0 & -\frac{height_{screen}}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{width_{screen}}{2} & \frac{height_{screen}}{2} & 0 & 1 \end{bmatrix}$$

$$v_s \quad = \quad v_{pn} \; * \; SC$$