Frisbee Trajectory

Lab Report

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Introduction

For our project, we decided to observe and graph the trajectory of a standard Frisbee in a distance vs height graph. We wanted to see how the launch angle (angle it was thrown with), the angle of attack (angle formed between the plane of the frisbee and the relative velocity vector), and the force it was thrown with. We wanted to see at what angle to throw the Frisbee with that will make it cover the most distance. To verify our results we are using data found in previous reports and comparing it with what we found. Our hypothesis is that if it is thrown at an angle between 45 and 30 degrees it will travel the furthest distance without going to high in the air. We believe that if thrown like this most of the force will be transferred to the x-axis and will be less affected by drag.

Scientific model of a Frisbee

There are two physical concepts that are used to explain the flight of a frisbee: the aerodynamic lift and the gyroscopic inertia. However, for our project we will disregard the gyroscopic inertia and only focus on the aerodynamic lift. The two main aerodynamic forces that will be applied on the frisbee will be drag and lift force. These forces are important because the lift force is the force that keeps the frisbee airborne and drag is the force that is opposing to the velocity of the frisbee. The drag force, F_d, is given by,

$$F_d = -\frac{C_D \rho \pi r^2 v^2}{2} = -\frac{C_D \rho A v^2}{2}.$$

The drag coefficient CD is given by a quadratic equation that only depends on the angle of attack

$$C_D = C_{D0} + C_{D\alpha}(\alpha - \alpha_0)^2.$$

The lift force is given by,

$$F_L = \frac{1}{2}\rho v^2 A C_L.$$

The coefficient CL is given as being a linear function of the angle of attack

$$C_L = C_{L0} + C_{L\alpha}\alpha,$$

$$C_{D0} = 0.08,\ C_{D\alpha} = 2.72,\ C_{L0} = .15,\ C_{L\alpha} = 1.4$$

The constants given above are constants that depend on the physical properties of the frisbee. We got these values from a paper where they did simulations and they used these values for the coefficients.

The lift force felt on the frisbee is similar to the one felt on airplane wings so we calculate the lift force the same way, using the Bernoulli principle. This principle states that the velocity of a fluid and the pressure or fluid potential energy are inversely proportional. An increase in speed means a decrease in pressure and increase in pressure means a decrease in speed. In the case of the frisbee, lower pressure over the frisbee is caused by the increase of its speed while the pressure is high under the frisbee. Since the pressure is higher below the wing it is pushed upwards. Since we are using a frisbee and not an airplane wing, we assume we could neglect the height difference between the air flowing above and below, we also assume that the velocity of the air above is directly proportional to the air flow below due to a constant path length.

This can be expressed by

$$\frac{C^2 {v_2}^2}{2} + \frac{p_1}{\rho} = \frac{{v_2}^2}{2} + \frac{p_2}{\rho}.$$

Numerical method

The dynamics of frisbee involve many variables that we have to take in account. We considered the drag force, lift force and gravity to be the three principle forces needed to be taken into account. Those forces were separated into horizontal and vertical components for the purpose of our project and Euler's method was applied to each component. All the variables have been used in a method in a Java program to graph the trajectory of a thrown frisbee, using numerical techniques.
When applying the Euler's method, we divided the time into small steps, Δt, and at each step of the method, the horizontal position and velocity are updated.

$$v_{i+1} = v_i + \Delta v,$$
$$x_{i+1} = x_i + \Delta x,$$

Here, Δv is the change in velocity and Δx is the change in position. For the vertical components, similar equations were used, replacing x with y.

The Δv's are obtained following these equations,

$$\begin{aligned} F_x &= F_D, \\ m\frac{\Delta v_x}{\Delta t} &= \frac{1}{2}\rho v_x^2 A C_D, \\ \Delta v_x &= \frac{1}{2m}\rho v_x^2 A C_D \Delta t, \end{aligned}$$

where F_D corresponds to the Drag Force.

Now, considering the forces acting on the y components, we get

$$\begin{aligned} F_y &= F_g + F_L \\ m\frac{\Delta v_y}{\Delta t} &= mg + \frac{1}{2}\rho v_x^2 A C_L \\ \Delta v_y &= \left(g + \frac{1}{2m}\rho v_x^2 A C_L\right)\Delta t \end{aligned}$$

Where F_g is the force of gravity on the frisbee. Also, the rate of change of x (Δv) and the rate of change of y (Δy) are related to time and velocity as

$$\begin{aligned} \Delta x &= v_x \Delta t \\ \Delta y &= v_y \Delta t \end{aligned}$$

For the purpose of our project, the interval of time used for every trial was Δt = 0.001s.

Test case

The papers we previously found show the results of a Frisbee trajectory using similar parameters for the simulation. Using their initial parameters, we are going to run our program and compare the results obtained with the ones from the papers.

So, the initial height of the frisbee was set to be 1m, the initial velocity of x was 14 m/s (standard velocity of a thrown frisbee) and the initial velocity of y as 0 m/s. The range of the angles of attack for every trial were changed within 0 to 45 degrees.

Using those initial parameters and playing with the angle of attack, we should see the frisbee travelling only a short distance (less than 20m) when the angle is less than 5 degrees. With larger angles, the distance should go up to 40m. Moreover, the maximum distance travelled should be obtained with an angle of approximately 12 degrees and a maximum height of 7.7m.