Frisbee Trajectory

Lab Report

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Introduction

For our project, we decided to observe and graph the trajectory of a standard Frisbee in a distance vs height graph. We wanted to evaluate the effect of the launch angle (angle it was thrown with) of the frisbee on the distance travelled. We also decided to optimize the distance travelled based on the initial angle to throw the Frisbee. To verify our results, we are using data found in previous reports and comparing it with what we found. Our hypothesis is that if it is thrown at an angle between 30 and 45 degrees it will travel the furthest distance without going to high in the air. We believe that if thrown in that range for the launch angle, most of the force will be transferred to the x-axis and will be less affected by the drag force.

Scientific model of a Frisbee

There are two physical concepts that are used to explain the flight of a frisbee: the aerodynamic lift and the gyroscopic inertia. However, for our project we will disregard the gyroscopic inertia and only focus on the aerodynamic lift. The two main aerodynamic forces that are applied on the frisbee and help keep it airborne are drag and lift forces.

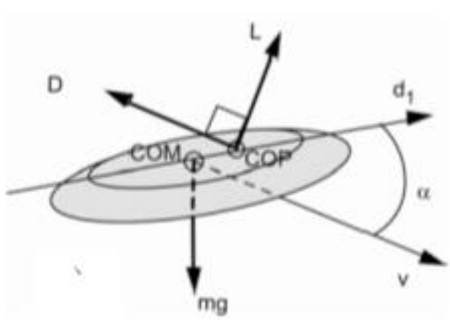

Figure 1.

The drag force is the resistance force caused by the motion of a body through a fluid, in our case it's the motion of the frisbee through the air. This force is opposite and parallel to the velocity vector and it is the force that slows down de frisbee. The drag force, F_d, has components on both the x and y axis, and it is given by,

$$F_d = -\frac{C_D \rho \pi r^2 v^2}{2} = -\frac{C_D \rho A v^2}{2}.$$

The drag coefficient C_D is given by a quadratic equation that only depends on the angle of attack.

$$C_D = C_{D0} + C_{D\alpha}(\alpha - \alpha_0)^2.$$

The lift force is the force that keeps the frisbee airborne and is always perpendicular to the drag force at any point during the flight of the frisbee.

The lift force is given by,

$$F_L = \frac{1}{2}\rho v^2 A C_L.$$

The coefficient C_L is given as being a linear function of the angle of attack.

$$C_L = C_{L0} + C_{L\alpha}\alpha,$$

$$C_{D0} = 0.08,\ C_{D\alpha} = 2.72,\ C_{L0} = .15,\ C_{L\alpha} = 1.4$$

The constants C_{L0} and $C_{L\alpha}$ given above (without units) are constants that depend on the physical properties of the frisbee. In Baumback's research paper on *The Aerodynamics of Frisbee Flight*, C_{L0} and $C_{L\alpha}$ depend on the physical properties of the Frisbee. Since we used the same dimensions and mass in our simulation, the constants we use have the same values as the one in

Baumback's research paper. C_{D0} is the form drag and $C_{D\alpha}$ is the induced drag, we also got these two values from Baumback's paper.

There are various angles that affect the flight of the frisbee, the launch angle and the angle of attack. the launch angle as the name entails is the angle the frisbee was thrown or launched with. The angle of attack is more interesting, it is the angle that lies between the angle of velocity and the tilt of the frisbee all from the point of reference. What is interesting about the angle of attack is that since it is dependant on the angle of velocity which is constantly changing due to the acceleration on the frisbee caused by the various forces acting on it, the angle of attack is constantly changing throughout the flight of the frisbee. And since the forces are dependant of the angle of attack, it in a way is responsible for its own change.

Numerical method

The flight of a frisbee depends on three forces: force of gravity, lift force and drag force. These forces were decomposed on the x and y axes. For the purpose of our project, Euler's method was applied to the velocity and position on both axes. All the variables have been used in methods in our Java program to graph the trajectory of a thrown frisbee, using numerical techniques. When applying Euler's method, we divided the time into small steps, $\Delta t = 0.001s$. We used a small delta t because the smaller the delta t the more iterations there are and the precise our results will be.

We used this delta t, because it is small enough to give us a precise end result, making the delta t any smaller barely changes the end result so we opted for 0.001s.

At each step of the method, the vertical and horizontal position and velocity are updated.

$$v_{i+1} = v_i + \Delta v,$$
$$x_{i+1} = x_i + \Delta x,$$

Here, Δv is the change in velocity and Δx is the change in position. For the vertical components, similar equations were used, replacing x with y.

Code

To plot the trajectory a frisbee thrown at the optimal angle that produces the furthest distance traveled, we used two different java codes. We used one java code to find the optimal angle and another one that prints out the position of a frisbee on the x and y axes as a function of time on a data file that we then turn into an excel file so that we can graph it. The first code we use is to find the optimal angle, to find the angle, the code uses the golden search method and returns the negative final position, since the golden search looks for the minimum value. After finding the optimal angle, we plug the value in our Frisbee Trajectory code to find the distance traveled by the frisbee and plot the trajectory.

The trajectory of the frisbee was plotted using numerical methods and all the constants we use are from Baumback's paper, which they got from Morrison's experiment. The mass and radius we used are fixed dimensions of a standard frisbee which are 0.175kg and 0.13m respectively. The air density used, is the air density at ocean level which is 1.23kg/m^3. Our code was split into two parts: the launch and the flight.

In the first part of the code, the launch, we initialized all the variables. The initial position on x and y are 0 and 1 in m respectively. We consider the tilt of the frisbee to be a constant and equal to the launch angle. We take the initial velocity to be 14 m/s and the initial angle of velocity to be equal to the launch angle as well. Since the initial velocity angle and the tilt of the frisbee are equal to the launch angle, the initial angle of attack is zero. We obtain V_x and V_y by multiplying the velocity by cos and sin of the current velocity angle. We use newton's second law: **F** = **ma** to calculate the acceleration on the x and y axes. All the angles we use are always taken between the direction of the vector and the x axis.

$$a_x = \sum\left(\frac{F_{dx} - F_{lx}}{m}\right)$$

$$a_y = \sum\left(\frac{F_{dy} - F_{ly} - mg}{m}\right)$$

In the second part of the code, the flight, a time increment of 0.001s was used in Euler's method and kinematics equations to calculate the changes in the velocity and position on both x and y axes caused by the change of the angle of attack.

$$Vx1 = Vx0 + a_x * \text{delta } t$$

$$Vy1 = Vy0 + a_y * \text{delta } t$$

$$X_1 = X_0 + Vx0 * \text{ delta } t + (a_x * \text{delta } t\text{^}2)/2$$

$$Y_1 = Y_0 + Vy0 * \text{ delta } t + (a_y * \text{delta } t\text{^}2)/2$$

After finding the new velocity on x and y axes, we use the vector magnitude formula

$$V = \sqrt{Vx^2 + Vy^2}$$

to find the magnitude of the velocity and use arctan(Vy/Vx) to find the new angle of velocity. The new angle of attack is found by taking the difference of the tilt of the frisbee which is a constant and the new angle of velocity. After finding the new angle of attack, we find the new lift force and drag force since the forces depend on the angle of attack. After finding the new values of the forces on the x and y axes, we use newton's second law **F** = **ma** to find the new acceleration on both axes. We then use Euler's method and kinematics to find new velocity and position on the axes. And we repeat the same process until the frisbee hits the ground (y = 0). When the loop ends the position on the x and y axes at each time increment are sent to a data file, we turn into a an excel file and graph the trajectory of the frisbee.

Results

Coming back to the goal of the simulation, we wanted to optimize the distance travelled by finding the ideal launch angle, given a specific initial height and velocity, in our case 1m and 14m/s respectively. After we run the Golden Ratio code, the optimal angle printed was approximately 10.29 degrees. We then plugged that value in the Frisbee code as the new launch angle. After we run the code, new positions of the frisbee on the x and y axes are printed after each dt until the y value reaches approximately 0m (the ground), giving a maximum distance on the x-axis of 19.19m, as seen in Figure 2.

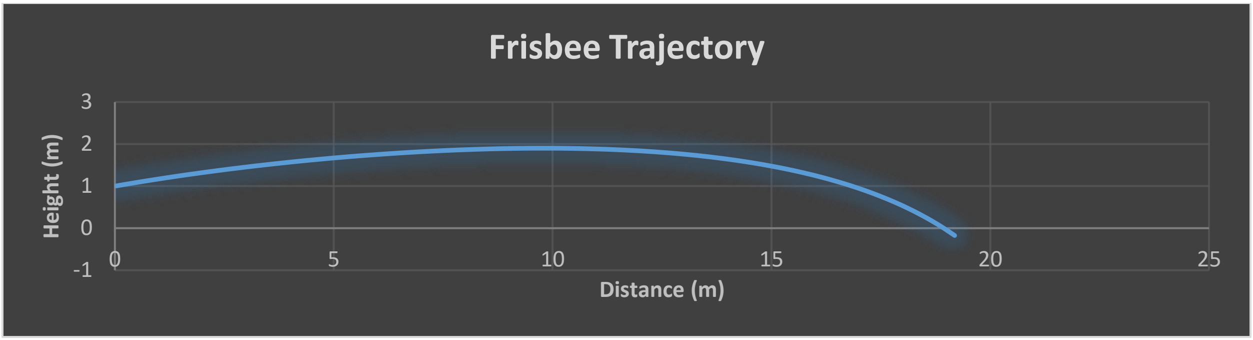

Figure 2.

To confirm that the optimal angle was truly the one given by the Golden Ratio code, we also conducted different trials with a different launch angle for each trial to see the effect on the distance travelled. In Figure 3, the different angles we used as well as the corresponding distance travelled are presented. This confirms that whenever the angle is either slightly smaller or slightly bigger than the optimal angle, the total distance covered by distance will be smaller than if the launch angle is 10.19 degrees.

Angle (degrees)	Distance (M)
10,29	19,19
12	19,07
15	18,5
8	18,68
5	17,05

Figure 3.

Figure 4 shows the trajectory of a frisbee with an angle smaller than the optimal launch angle, and Figure 5 shows the trajectory with a bigger angle. As seen in these two graphs, with larger launch angle, the lift force becomes larger, causing the frisbee to go higher but travels a smaller distance. In the same way, when the launch angle is smaller, the distance travelled also becomes smaller than using the optimal angle.

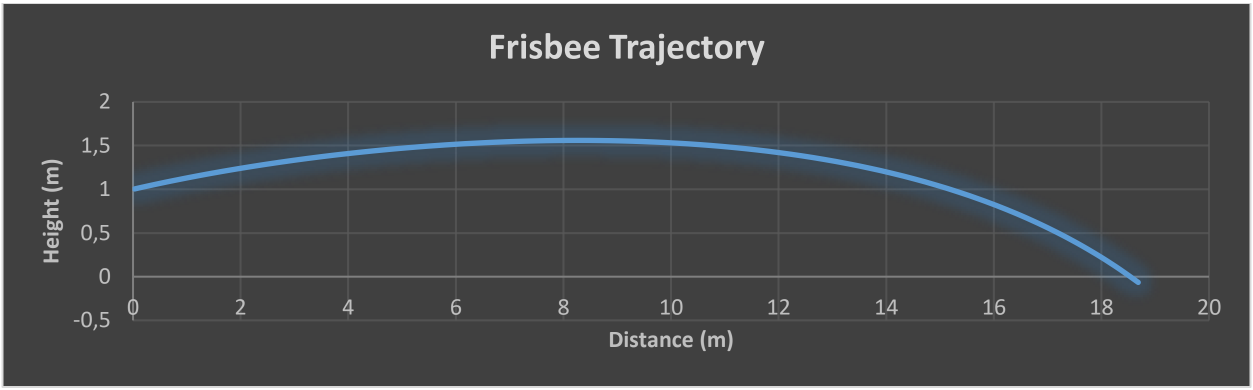

Figure 4.

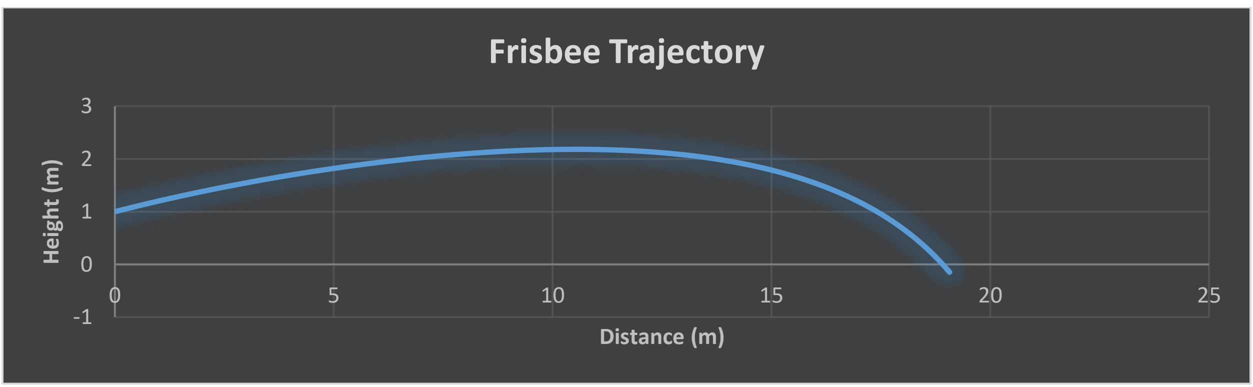

Figure 5.

As you can see, the optimal 10.29 degrees was very far from what the hypothesized the optimal launch angle would be which was between 30 and 45 degrees. The angle in this range actually gives a very short distance travelled.

In Baumback's report, they simulated a frisbee thrown with an initial angle of attack of 10 degrees and an initial velocity of 14m/s travelling a maximum distance of 26.23m which was not so close to the results we found using our code.

Discussion

The reason there is a discrepancy between our results and the ones from the papers we used is caused by the fact that they disregard and assume in places where they shouldn't have if they wanted precise results. For instance, they disregarded the velocity on the y axis, because they believe that the velocity on the x axis is a lot larger than the one on the y axis, which isn't always the case, especially when the frisbee is travelling mostly downward. They never mention a launch angle and tilt of the frisbee, they only mention an angle of attack and never define it. They get the velocity on both axes by multiplying the velocity by the sin and cos of the angle of attack. In this paper we are using to compare the results they don't really explain what the angle of attack is or what they consider it to be so it is not fully clear why they use it this way, however in other papers, the angle of attack is the difference between the tilt of the frisbee and the angle of velocity, which is the way we defined it in our term project. On top of that they only consider the force of lift on the y axis and the force of drag on the x axis, when they should consider both forces on both axes. These differences in the we went about graphing the trajectory of a frisbee are probably why we have a discrepancy between our values and the ones found in the paper.

References

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Baumback, K. "The Aerodynamics of Frisbee Flight." Undergraduate Journal of Mathematical Modeling: One + Two: Vol. 3: ISS. 1, Article 29. DOI: http://dx.doi.org/10.5038/2326-3652.3.1.31. 2010.

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