1) $d(t) = \sqrt{t+1}$ $\quad V(t) = ?$

d en metros $\quad$ cuando

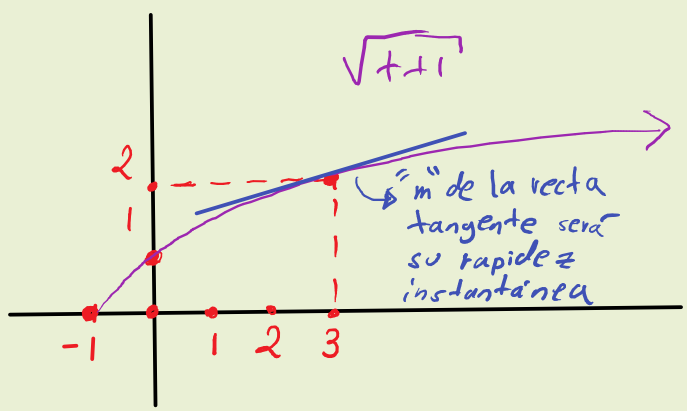
t en segundos $\quad t = 3s.$

En este caso, derivamos la función de posición y obtenemos que

$$d'(t) = v(t) = [\sqrt{t+1}]' = \frac{1}{2\sqrt{t+1}} \cdot (t+1)' = \frac{1}{2\sqrt{t+1}}$$

Luego: si $\boxed{t=3} \Rightarrow V(3) = \frac{1}{2\sqrt{3+1}} = \frac{1}{2\sqrt{4}} = \frac{1}{2\cdot 2} = \frac{1}{4} = 0.25$

R/ La velocidad del insecto a los 3 s es de 0.25 m/s



En este gráfico, la posición del insecto a los 3 s es de 2 m.

② P#1:

(↑ aumenta)

(↓ disminuye)

$t = 0s$ (balón desinflado) $t = 2s$ $t = 3s$

En este caso: $\boxed{r = 5 \text{ pulgadas}}$

$\underbrace{\frac{dV}{dt} \text{ cambia } \uparrow}_{=}$ no lo conocemos

$\underbrace{\frac{dr}{dt} \text{ cambia } \uparrow}_{=}$ $\frac{dr}{dt} = 0{,}2$ pulgadas/s (positiva)

P#2: $\boxed{V = \frac{4\pi r^3}{3}}$

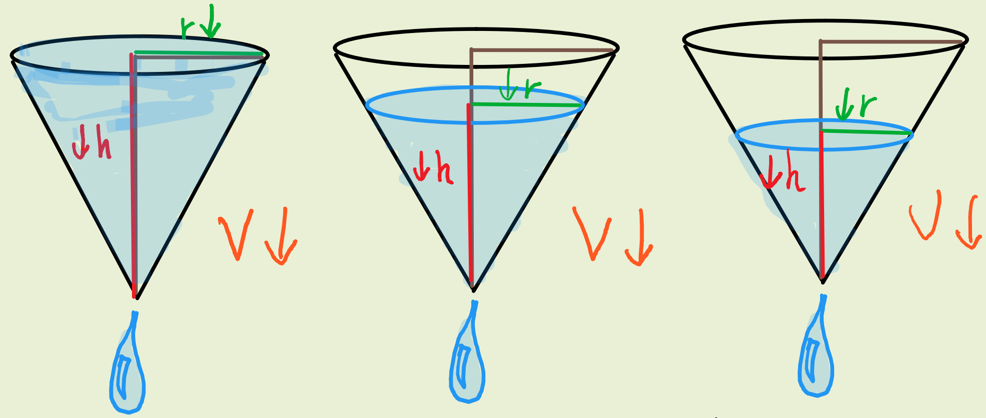
P#3: $[V]' = \left[\frac{4\pi r^3}{3}\right]' \iff V' = \frac{4}{3}\pi \cdot [r^3]' \iff V' = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \cdot r' \iff \boxed{\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}}$

P#4: $\frac{dV}{dt} = 4\pi \cdot (5)^2 \cdot (0{,}2) = 4\pi \cdot 25 \cdot \frac{2}{10} = 4\pi \cdot 25 \cdot \frac{1}{5} = 4 \cdot 5\pi = 20\pi \approx 62{,}83 \text{ pulgadas}^3/s$

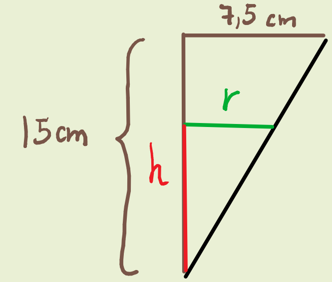
$\text{pulgadas}^2 \cdot \frac{\text{pulgadas}}{s} = \frac{\text{pulgadas}^3}{s}$

P#5: R/ El volumen del globlo crece a razón de $62{,}83 \text{ pulgadas}^3/s$. ■

③ P#1:



t=0s   t=3s   t=5s



$$\frac{15}{h} = \frac{7,5}{r}$$

$$15r = 7,5h$$

$$\frac{15r}{7,5} = h$$

$$\boxed{2r = h} \text{ ó } \boxed{r = \frac{h}{2}}$$

$$\boxed{h = 7,5\text{ cm}}$$

$$\Downarrow$$

$$\boxed{r = 3,75\text{cm}}$$

En este caso: $\underbrace{\frac{dV}{dt} \text{ cambia } \downarrow}_{=}$ $\underbrace{\frac{dr}{dt} \text{ cambia } \downarrow}_{=}$ $\underbrace{\frac{dh}{dt} \text{ cambia } \downarrow}_{=}$

$\frac{dV}{dt} = -5cm^3/s$

Como $2r = h \Rightarrow$

$2r' = h'$ o bien

$$\boxed{2\frac{dr}{dt} = \frac{dh}{dt}}$$

no lo conocemos

P#5: R/ El nivel del liquido des-ciende con una rapidez de 0.11 cm/s cuando h= 7,5 cm.

P#2: $V = \frac{1}{3}\pi r^2 \cdot h$

$2r' = h'$

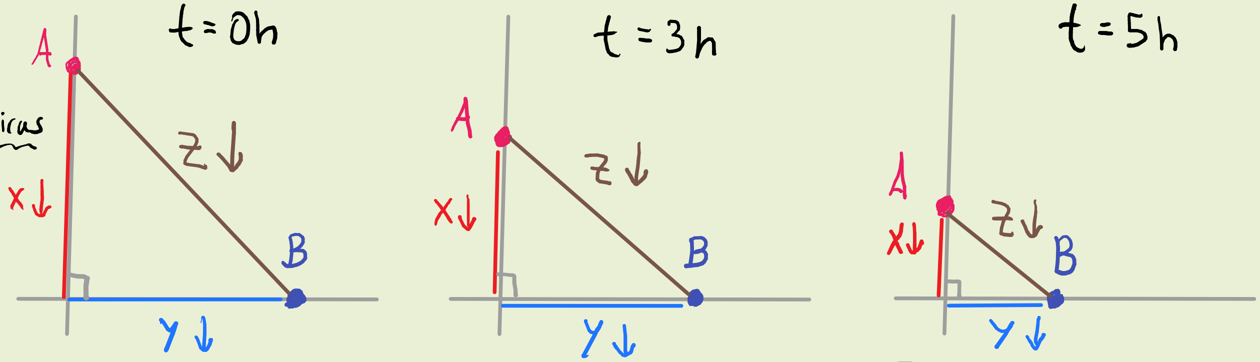
P#3: $[V]' = \left[\frac{1}{3}\pi r^2 h\right]' \Leftrightarrow V' = \frac{1}{3}\pi \cdot [r^2 \cdot h]' \Leftrightarrow V' = \frac{\pi}{3}\left[2r \cdot r'h + r^2 h'\right] \Leftrightarrow V' = \frac{\pi}{3}\left[h'rh + r^2 h'\right]$

P#4: $-5 = \frac{\pi}{3} \cdot \frac{dh}{dt}\left[(3,75) \cdot (7,5) + (3,75)^2\right] \Leftrightarrow -15 = \pi \cdot \frac{dh}{dt} \cdot 42.1875 \Leftrightarrow \frac{dh}{dt} \approx -0.11 \text{ cm/s}$

$\Leftrightarrow \frac{dV}{dt} = \frac{\pi}{3}\frac{dh}{dt}\left[rh + r^2\right]$

4 **P#1:**

m.n. = millas naúticas



$\frac{dx}{dt} = -442$ m.n./h $t = 0h$

$\frac{dy}{dt} = -481$ m.n./h $t = 1h$

$\boxed{\frac{dz}{dt} = ?}$ $t = 2h$

En este caso:

$\frac{dx}{dt}$ cambia ↓ (conocida)

$\frac{dy}{dt}$ cambia ↓ (conocida)

$\frac{dz}{dt}$ cambia ↓ (desconocida)

P#2: $x^2 + y^2 = z^2$

P#3: $[x^2+y^2]' = [z^2]' \iff 2x \cdot x' + 2y \cdot y' = 2z \cdot z' \iff x \cdot x' + y \cdot y' = z \cdot z'$

$\iff x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt} = z \cdot \frac{dz}{dt}$

P#4: Como $x^2+y^2=z^2$ y nos dicen que

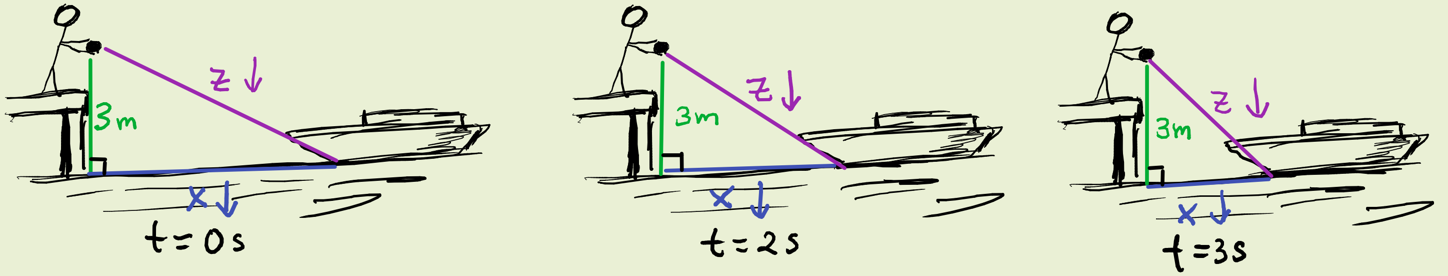
$\boxed{x = 5}$ y $\boxed{y = 12}$ $\Rightarrow$ $5^2 + 12^2 = z^2 \iff \boxed{z = 13}$. Con esto:

$5 \cdot (-442) + 12 \cdot (-481) = 13 \cdot \frac{dz}{dt} \iff -7482 = 13 \cdot \frac{dz}{dt} \iff \frac{dz}{dt} = -614$ m.n./h

P#5: Ambos aviones se aproximan con una rapidez de 614 millas naúticas por hora.

5

P#1:



En este caso: $\boxed{x = 4\,m}$

$\frac{dz}{dt} = -0.8\,m/s$

$\frac{dx}{dt} = ?$ (pero es negativa)

P#2: $3^2 + x^2 = z^2 \iff 9 + x^2 = z^2$

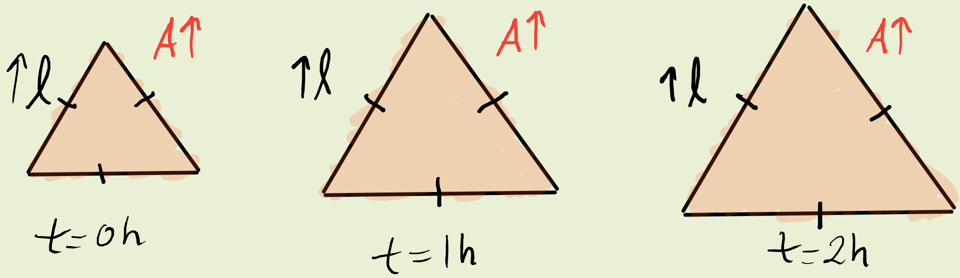
P#3: $[9 + x^2]' = [z^2]' \iff \cancel{2}x \cdot x' = \cancel{2}z \cdot z' \iff x \cdot x' = z \cdot z' \iff x' = \frac{z \cdot z'}{x} \iff \boxed{\frac{dx}{dt} = \frac{z \cdot \frac{dz}{dt}}{x}}$

P#4: Como nos dicen que $\boxed{9 + x^2 = z^2}$ y $\boxed{x = 4}$ entonces $\boxed{z = 5}$. Con esto:

$$\frac{dx}{dt} = \frac{5 \cdot (-0.8)}{4} \iff \boxed{\frac{dx}{dt} = -1\ m/s}$$

P#5: R/ La lancha se aproxima al muelle a una velocidad de 1 m/s.

⑥ P#1:



En este caso:

$\frac{dA}{dt} = ?$ (es positiva)

$\frac{dl}{dt} = 2\,cm/h$

P#2: $A = \frac{l^2\sqrt{3}}{4}$ (área de un triángulo equilátero)

P#3: $[A]' = \left[\frac{l^2\sqrt{3}}{4}\right]' \iff A' = \frac{\sqrt{3}}{4}[l^2]' \iff A' = \frac{\sqrt{3}}{4}\cdot 2l\cdot l' \iff \boxed{\frac{dA}{dt} = \frac{\sqrt{3}}{2}\cdot l\cdot\frac{dl}{dt}}$

P#4: Recordando que $\boxed{l=8}$ se sigue

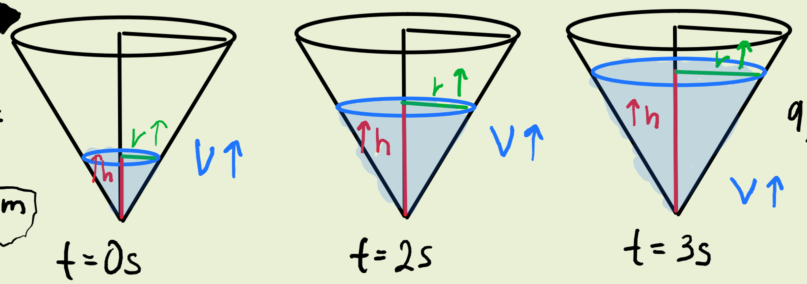
$$\frac{dA}{dt} = \frac{\sqrt{3}}{\cancel{2}}\cdot 8\cdot\cancel{2} \iff \frac{dA}{dt} = 8\sqrt{3} \approx 13.85\ cm^2/h$$

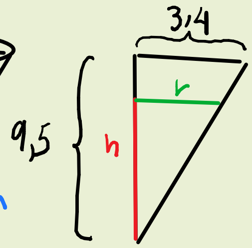
P#5: El área crece a una rapidez de 13.85 cm^2/h cuando $l = 8\,cm$.

⑦

diámetro = 6,8 cm

$\therefore r = 3{,}4$ cm





Aquí:

$$\frac{9{,}5}{h} = \frac{3{,}4}{r} \iff 9{,}5r = 3{,}4h$$

$$\boxed{r = \frac{34h}{95}}$$

Como $\boxed{h=5}$ entonces

$$r = \frac{34 \cdot 5}{95}$$

o bien

$$\boxed{r = \frac{34}{19}}$$

$$\frac{dV}{dt} = 2\ \text{cm}^3/s$$

$$\frac{dr}{dt} = \frac{34}{95} \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \ ?\ \text{(positiva)}$$

P#2: $V = \frac{1}{3}\pi \cdot r^2 \cdot h$ (volumen de un cono)

P#3: $[V]' = \left[\frac{1}{3}\pi r^2 \cdot h\right]' \iff V' = \frac{1}{3}\pi\,[r^2 \cdot h]' \iff V' = \frac{1}{3}\pi \cdot (2r \cdot r' \cdot h + r^2 \cdot h') \iff V' = \frac{\pi}{3}\left(2r \cdot \frac{34 \cdot h'}{95} \cdot h + r^2 \cdot h'\right)$

P#4: $2 = \frac{\pi}{3}\left[2\left(\frac{34}{19}\right) \cdot \frac{34}{95} \cdot h' \cdot 5 + \left(\frac{34}{19}\right)^2 \cdot h'\right]$

$$\frac{6}{\pi} = \frac{2312}{361} \cdot h' + \frac{1156}{361} h'$$

$$361 \cdot \frac{6}{\pi} = 2312 \cdot h' + 1156\, h'$$

$$\frac{2166}{\pi} = 3468 \cdot h'$$

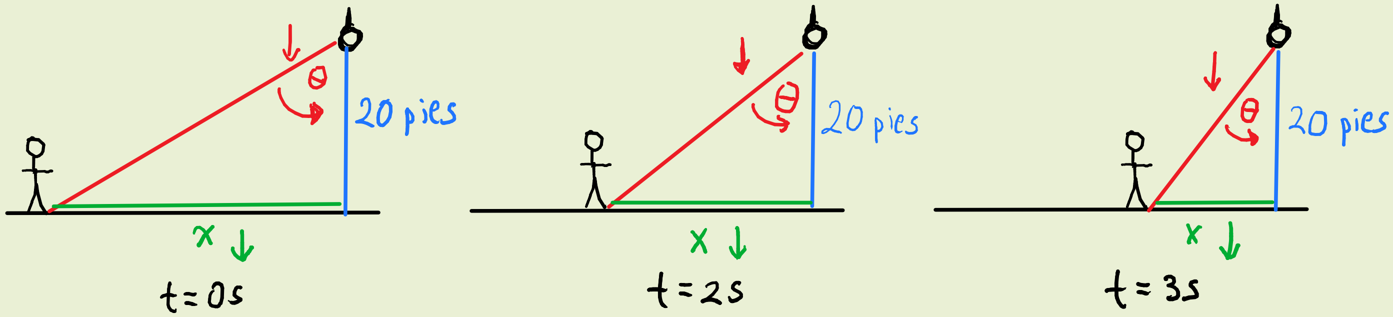
$$\frac{2166}{\pi\, 3468} = h'$$

$$\frac{361}{578\pi} = h'$$

$$\boxed{\frac{dh}{dt} \approx 0{,}1988\ \text{cm}/s}$$

P#5: R/ La rapidez del agua aumenta a razón de 0,1988 cm/s cuando $h = 5$ cm.

⑧ P#1:



En este caso:

$\frac{d\theta}{dt} = ?$ (disminuye)

$\frac{dx}{dt} = -4$ pies/s

$\boxed{x = 15 \text{ pies}}$

P#2: $\tan\theta = \frac{x}{20}$

P#3: $[\tan\theta]' = \left[\frac{x}{20}\right]' \Leftrightarrow \sec^2\theta \cdot \theta' = \frac{x'}{20} \Leftrightarrow \frac{\theta'}{\cos^2\theta} = \frac{x'}{20} \Leftrightarrow \theta' = \frac{x' \cdot \cos^2\theta}{20} \Leftrightarrow \boxed{\frac{d\theta}{dt} = \frac{\frac{dx}{dt} \cdot \cos^2\theta}{20}}$

P#4: Como $\boxed{\tan\theta = \frac{x}{20}}$ y $\boxed{x = 15}$ se sigue que $\tan\theta = \frac{15}{20} \Leftrightarrow \boxed{\theta = \arctan\left(\frac{15}{20}\right)}$

De este modo:

$$\frac{d\theta}{dt} = \frac{(-4) \cdot \cos^2\left(\arctan\left(\frac{15}{20}\right)\right)}{20}$$

$$\frac{d\theta}{dt} = \frac{-1}{5} \cos^2\left(\arctan\left(\frac{15}{20}\right)\right)$$

ver nota

$$\frac{d\theta}{dt} = \frac{-1}{5} \cdot \frac{16}{25}$$ → con calculadora

$$\frac{d\theta}{dt} = \frac{-16}{125} = -0.128 \text{ rad/s}$$

o bien

$-7.33^{0}/s$

$$\tan^2 x + 1 = \sec^2 x$$

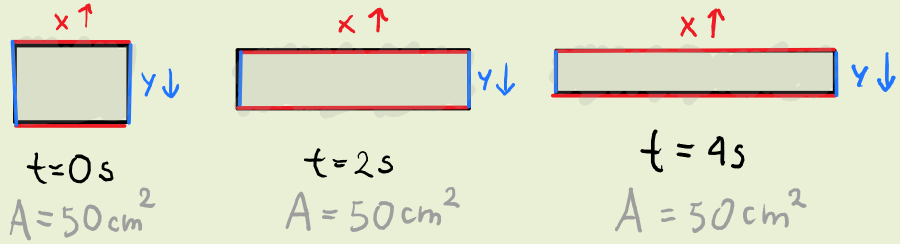
$$\tan^2 x + 1 = \frac{1}{\cos^2 x}$$

$$\cos^2 x = \frac{1}{\tan^2 x + 1}$$

$$\therefore \cos^2\left(\arctan\left(\frac{15}{20}\right)\right) = \frac{1}{\cancel{\tan^2}\left(\cancel{\arctan}\left(\frac{15}{20}\right)\right) + 1} = \frac{1}{\left(\frac{15}{20}\right)^2 + 1} = \frac{16}{25}$$

P#5: R/ El foco gira hacia su vertical con una velocidad de 0.128 rad/s.

(9) P#1:



En este caso: $\frac{dx}{dt} = 2$ cm/s, $\boxed{x = 5\text{cm}}$

(negativo) $\frac{dy}{dt} = ?$ $A = 50$ cm²

$\boxed{\frac{dP}{dt} = ?}$

P#2: $P = 2x + 2y$

P#2: $[P]' = [2x+2y]' \Leftrightarrow P' = 2x' + 2y' \Leftrightarrow \boxed{\frac{dP}{dt} = 2\cdot\frac{dx}{dt} + 2\frac{dy}{dt}}$

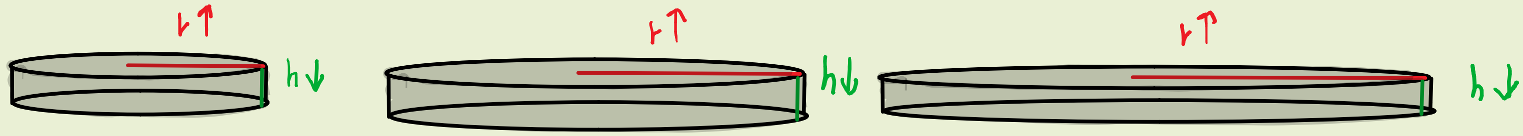
P#3: Como $\boxed{A = 50 = xy}$ entonces $\boxed{\frac{50}{x} = y}$. Dado que $\boxed{x = 5}$ se sigue que $\boxed{\frac{50}{5} = y}$, o bien $\boxed{10 = y}$. Además: $\left[\frac{50}{x}\right]' = [y]' \Rightarrow -\frac{50}{x^2}\cdot x' = y' \Leftrightarrow \boxed{-\frac{50}{x^2}\cdot\frac{dx}{dt} = \frac{dy}{dt}}$. Con $\boxed{x=5}$ y $\boxed{\frac{dx}{dt} = 2}$ se tiene:

$\frac{dy}{dt} = \frac{-50}{5^2}\cdot(2) \Leftrightarrow \boxed{\frac{dy}{dt} = -4 \text{ cm/s}}$ De esta forma:

$\frac{dP}{dt} = 2\cdot(2) + 2\cdot(-4) \Leftrightarrow \boxed{\frac{dP}{dt} = -4 \text{ cm/s}}$

P#3: R/ El perímetro del rectángulo decrece a razón de 4 cm/s cuando $x = 5$cm

(10) P#1:



$t = 0\,h$ $\quad V = 100\ m^3$ $\qquad t = 1\,h$ $\quad V = 100\ m^3$ $\qquad t = 3\,h$ $\quad V = 100\ m^3$

Se sabe que: $\boxed{r = 50m}$ $\quad \boxed{\dfrac{dh}{dt} = -0.1\ m/h}$

Se busca: $\boxed{\dfrac{dr}{dt} = ?\ (\text{positivo})}$

P#2: $100 = \pi \cdot r^2 \cdot h$

P#3: $[100]' = [\pi r^2 \cdot h]' \iff 0 = \pi [r^2 \cdot h]' \iff 0 = 2r \cdot r' h + r^2 \cdot h' \iff 0 = \cancel{r}(2r'h + rh')$

$\iff 0 = 2r'h + rh'$

$\iff \dfrac{-rh'}{2h} = r'$

$\iff \boxed{\dfrac{-r \cdot \frac{dh}{dt}}{2 \cdot h} = \dfrac{dr}{dt}}$

P#4: Como $\boxed{100 = \pi \cdot r^2 \cdot h}$ y $\boxed{r = 50}$ se sigue que:

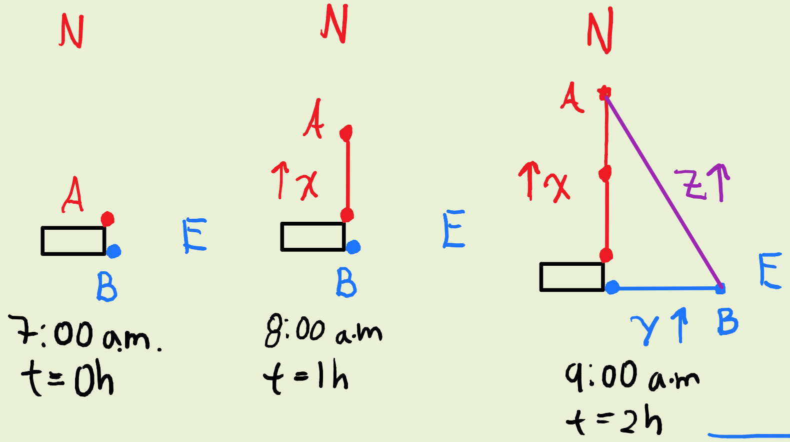
$100 = \pi \cdot (50)^2 \cdot h \iff \dfrac{\cancel{100}}{\pi \cdot 25\cancel{00}} = h \iff \boxed{\dfrac{1}{25\pi} = h}$

De esta forma:

$$\frac{0.1)}{2 \cdot \left(\frac{1}{25\pi}\right)} = \frac{dr}{dt} \iff \frac{\frac{5}{1}}{\frac{2}{25\pi}} = \frac{dr}{dt} \iff \boxed{\frac{125\pi}{2} = \frac{dr}{dt}} \text{ ó } \boxed{\frac{dr}{dt} = 196.34\ m/h}$$

P#5: R/ El radio aumenta a razón de 196.34 m/h.

⑪ P#1:



Se sabe:

$\frac{dx}{dt} = 45\ Km/h$

$\frac{dy}{dt} = 60\ Km/h$

Determinar:

$\frac{dz}{dt} = ?$

9:00 a.m ($t=2h$)

Se debe aclarar que cuando $t=2h \Rightarrow \boxed{y = 60\,Km}, \boxed{x = 90\,Km}$

P#2: $z^2 = x^2 + y^2$

P#3: $[z^2]' = [x^2+y^2]' \Leftrightarrow \cancel{2}z\cdot z' = \cancel{2}x\cdot x' + \cancel{2}y\cdot y' \Leftrightarrow z\cdot z' = x\cdot x' + y\cdot y' \Leftrightarrow \boxed{z\cdot\frac{dz}{dt} = x\cdot\frac{dx}{dt} + y\cdot\frac{dy}{dt}}$

P#4: Como $\boxed{z^2 = x^2 + y^2}$ y $\boxed{y=60}$, $\boxed{x=90}$ entonces $\boxed{z = 30\sqrt{13}}$. Así:

$30\sqrt{13}\cdot\frac{dz}{dt} = 90(45) + 60\cdot(60) \Leftrightarrow 30\sqrt{13}\,\frac{dz}{dt} = 7650 \Leftrightarrow \frac{dz}{dt} = \frac{765\cancel{0}}{3\cancel{0}\sqrt{13}}$

$\Leftrightarrow \boxed{\frac{dz}{dt} = \frac{255}{\sqrt{13}}\ Km/h}$

ó

$\boxed{\frac{dz}{dt} = 70.72\ Km/h}$

P#5: R/ Los barcos se separan a razón de 70.72 Km/h cuando han transcurrido 2 horas.