$x = y^2 - 2 \qquad x = y$

$y = y^2 - 2 \rightarrow y^2 - y - 2 = 0$

$(y-2)(y+1) = 0 \rightarrow \begin{cases} y = 2 \\ y = -1 \end{cases}$

$$\int_{-1}^{2}\int_{y^2-2}^{y} dx\,dy = \int_{-1}^{2} (x)\Big|_{y^2-2}^{y} dy = \int_{-1}^{2} \underbrace{y - (y^2-2)}_{-y^2+y+2}\, dy$$

$$= \left(\frac{y^2}{2} - \frac{y^3}{3} + 2y\right)\Big|_{-1}^{2} = \frac{4}{2} - \frac{8}{3} + 4 - \left(\frac{1}{2} + \frac{1}{3} - 2\right)$$

$$= 4 - \frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 2 = 8 - 3 - \frac{1}{2} = 4.5$$

$y = \sin x \qquad x = 0$

$y = x + 1 \qquad x = \frac{\pi}{2}$

$\cos x \rightarrow -\sin x$

$$\int_{0}^{\frac{\pi}{2}}\int_{\sin x}^{x+1} dy\,dx = \int_{0}^{\frac{\pi}{2}} (y)\Big|_{\sin x}^{x+1} dx = \int_{0}^{\frac{\pi}{2}} (x+1) - \sin x \, dx$$

$$= \left(\frac{x^2}{2} + x + \cos x\right)\Big|_{0}^{\frac{\pi}{2}}$$

$$= \frac{\frac{\pi^2}{4}}{2} + \frac{\pi}{2} + \cancel{\cos\frac{\pi}{2}} - \cancel{\cos 0}$$

$$\frac{\pi^2}{8} + \frac{\pi}{2} - 1$$

$xy = ١ \qquad y = ٠ \qquad x = \frac{١}{٤} \qquad x = ٤$

$$\downarrow$$

$$y = \frac{١}{x}$$

$$\int_{\frac{١}{٤}}^{٤} \int_{٠}^{\frac{١}{x}} dy\, dx = \int_{\frac{١}{٤}}^{٤} \left(y \right)_{٠}^{\frac{١}{x}} dx = \int_{\frac{١}{٤}}^{٤} \frac{١}{x}\, dx = \left(\ln |x| \right)_{\frac{١}{٤}}^{٤} = \ln ٤ - \ln \frac{١}{٤}^{-١}$$

$$\ln ٤ + \ln ٤$$

$$= ٢ \ln ٤ = \ln ١٦$$

$\left. ٣ - \frac{١}{٢} \right| = ٤٫٥$

حجم مستقیم از صفحه زیر سطح $z=x^2+y^2$ و بالای دایره $x^2+y^2=4$

$z=r^2$

در صفحه xy است را بیابید. (به روش قطبی)

$$\iint_R f \underbrace{dA}_{r\,dr\,d\theta}$$

$$R: \begin{cases} 0\le r\le 2 \\ 0\le\theta\le 2\pi \end{cases}$$

$$\int_0^{2\pi}\int_0^2 \underbrace{r^2}_{r^2} r\,dr\,d\theta = \int_0^{2\pi}\left(\frac{r^4}{4}\right)_0^2 d\theta = \int_0^{2\pi} 4\,d\theta = (4\theta)_0^{2\pi} = 8\pi$$

$xy=1 \quad y=0 \quad x=\frac{1}{4} \quad x=4$

$y=\frac{1}{x}$

$$\int_{1/4}^{4}\int_0^{1/x} dy\,dx = \int_{1/4}^{4}(y)_0^{1/x}dx = \int_{1/4}^{4}\frac{1}{x}dx = (\ln|x|)_{1/4}^{4} = \ln 4 - \ln\frac{1}{4}$$

$\ln 4 + \ln 4$

$= 2\ln 4 = \ln 16$

$$\log_c a + \log_c b = \log_c ab$$

$$\log_c a^n = \log_c a\cdot a\cdots a = \log_c a + \log_c a + \cdots + \log_c a = n\log_c a$$

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$$1)\; f(x,y) \begin{cases} \dfrac{x^{٢}-y^{٢}}{x+y} & (x,y) \neq (1,-1) \\ ٣ & (x,y) = (1,-1) \end{cases}$$

$$f_{(1,1)} = ٣$$

$$\lim_{(x,y)(1,-1)} f = \frac{x^{٢}-y^{٢}}{x+y} = \frac{\cancel{(x+y)}(x-y)}{\cancel{x+y}} = 1-(-1) = ٢$$

$$f(x,y)\; \frac{x^{٣}+y^{٣}}{x^{٢}-y^{٢}} \qquad (-1,1)$$

$$\lim_{(x,y)\to(1,1)} \frac{\cancel{(x+y)}(x^{٢}-xy+y^{٢})}{(x-y)\;\cancel{(x+y)}} = \lim_{(x,y)\to(-1,1)} \frac{x^{٢}-xy+y^{٢}}{(x-y)} =$$

$$\frac{(-1)^{٢}-(-1)(1)+(1)^{٢}}{-1-1} = \frac{1+1+1}{-٢} = -\frac{٣}{٢}$$

$\cos u = -u' \sin u$

1) $w = \cos xy - \sin z \rightarrow dw = -y \sin xy \, dx + (-x \sin xy \, dy) - \cos z \, dz$

2) $z = x^3 + x^2 y^2 - y^4 \rightarrow dz = (3x^2 + 2xy^2) \, dx + (2y x^2 - 4y^3) \, dy$

$f(x,y) \quad \dfrac{x^3 + y^3}{x^2 - y^2} \quad (-1,1)$

$\lim\limits_{(x,y) \to (-1,1)} \dfrac{\cancel{(x+y)}(x^2 - xy + y^2)}{(x-y) \cancel{(x+y)}} = \lim\limits_{(x,y) \to (-1,1)} \dfrac{x^2 - xy + y^2}{(x-y)} =$

$\dfrac{(-1)^2 - (-1)(1) + (1)^2}{-1-1} = \dfrac{1+1+1}{-2} = -\dfrac{3}{2}$

3 - $f(x,y,z) = x^2 + y^2 - 2z^2 \quad P(1,1,1) \quad V = -2\vec{i} + 3\vec{j} + 6\vec{k}$

$|\vec{u}| = \sqrt{4+9+36} = \sqrt{49} = 7$

$D_u f = -2 \times \frac{2}{7} + 2 \times \frac{3}{7} + (-4 \times \frac{6}{7}) = \frac{-4}{7} + \frac{6}{7} - \frac{24}{7} = \frac{-22}{7}$

$u_u = \frac{1}{7}(-2\vec{i} + 3\vec{j} + 6\vec{k}) = \frac{-2}{7}\vec{i} + \frac{3}{7}\vec{j} + \frac{6}{7}\vec{k}$

$f_x = 2x \quad f_x(1,1,1) = 2$

$f_y = 2y \quad f_y = (1,1,1) = 2$

$f_z = -4z \quad f_z = (1,1,1) = -4$