به نام خداوندی که تکیه

انتگرال توابع برداری

$$\vec{r}(t) = f(t)\,\vec{i} + g(t)\,\vec{j} + h(t)\,\vec{k}$$

$$\int \vec{r}(t)\,dt = \int f(t)\,dt\,\vec{i} + \int g(t)\,dt\,\vec{j} + \int h(t)\,dt\,\vec{k}$$

$$\int_a^b \vec{r}(t)\,dt = \int_a^b f(t)\,dt\,\vec{i} + \int_a^b g(t)\,dt\,\vec{j} + \int_a^b h(t)\,dt\,\vec{k}$$

$$\int \vec{r}(t)\,dt = \int \left(\cos 2t\,\vec{i} + \sin 4t\,\vec{j} + \cos t\,\vec{k}\right) dt$$

$$= \frac{1}{2}\sin 2t\,\underset{+c}{\vec{i}} - \frac{1}{4}\cos 4t\,\underset{+c'}{\vec{j}} + \sin t\,\underset{+c''}{\vec{k}}$$

تکمیل برنامه

$$\int_0^1 \left(t^2 i + 3t^3 j + \sqrt{t}\, k\right) dt \qquad \sqrt{t} = t^{\frac{1}{2}}$$

$$= \left(\frac{t^3}{3}\vec{i} + \frac{3t^4}{4}\vec{j} + \frac{t^{\frac{1}{2}+1}}{\frac{3}{2}}\vec{k}\right)_0^1$$

$$= \left(\frac{1}{3}\right)\vec{i} + \frac{3}{4}\vec{j} + \frac{2}{3}\vec{k}$$

$$\int_0^{\pi} \left(\cos t\, \vec{i} - 2\sin t\, \vec{j} + 5\vec{k}\right) dt = \left(\sin t\, i + 2\cos t\, j + 5t k\right)_0^{\pi}$$

$$= (\underbrace{\sin\pi}_{0} - \underbrace{\sin 0}_{0})\vec{i} + (\underbrace{2\underbrace{\cos\pi}_{-1}}_{-2} - \underbrace{2\underbrace{\cos 0}_{1}}_{-2})\vec{j} + (5\pi - 0)\vec{k}$$

$$= 0\vec{i} - 4\vec{j} + 5\pi\vec{k}$$

$$\ln(t-1)\Big|_2^3 \Rightarrow \ln(3-1)-\ln(2-1)$$

$$\ln(2)-\cancel{\ln(1)}^{0} = \boxed{\ln(2)}$$

$$\frac{x^3}{3}\Big|_2^3 \Rightarrow \frac{27}{3}-\frac{8}{3}=\boxed{\frac{19}{3}}$$

$$\ln(2)\, i + \frac{19}{3} j + \frac{2}{\pi} k$$

به نام خداوندی که کسی برنامش

$$\int_2^3 \frac{1}{u}du\, i + \left(\frac{t^3}{3} j - \frac{2}{\pi}\sin\frac{t\pi}{2}\right)_2^3 = \overset{\ln u}{\ln(t-1)}\, i + \frac{t^3}{3} j - \frac{2}{\pi}\sin\frac{t\pi}{2}k\Big)_2^3$$

$t-1=u$

$du=dt$

$$= (\ln 2 - \cancel{\ln 1})\, i + \frac{27-8}{3} j - \frac{2}{\pi}\Big(\overset{-1}{\sin\frac{3\pi}{2}} - \overset{0}{\sin\frac{2\pi}{2}}$$

$$= \ln 2\, i + \frac{19}{3} j - \frac{2}{\pi}(-1)k$$

$$+\frac{2}{\pi}k$$

$$\int_2^3\left(\frac{1}{t-1} i + t^2 j - \cos\frac{t\pi}{2} k\right)dt$$

$$\ln(t-1)\Big|_2^3\, i + \frac{t^3}{3} - \frac{2}{\pi}\sin\left(\frac{\pi t}{2}\right)\Big|_2^3$$

$$\frac{2\sin\left(\frac{3\pi}{2}\right)}{\pi} - \frac{2\sin\left(\frac{2\pi}{2}\right)}{\pi} = \frac{-2}{\pi}$$

$$\frac{١}{٢}\int_{0}^{١}\left[٤(٢t+١)^{٣}\,i+٢\sqrt{٢t+١}\;j\right]dt$$

$$=\frac{١}{٢}\int_{١}^{٣} u^{٣}\,i+\underbrace{\sqrt{u}}_{u^{\frac{١}{٢}}}\;j$$

$u=٢t+١$

$du=٢dt$

$t=0\longrightarrow u=١$

$t=١\longrightarrow u=٣$

$$=\frac{١}{٢}\left(\frac{u^{٤}}{٤}\,i+\frac{u^{\frac{٣}{٢}}}{\frac{٣}{٢}}\,j\right)\Bigg|_{١}^{٣}$$

$$=\cancel{\frac{١}{٢}}\left(\overbrace{\left(\frac{٨١}{٤}-\frac{١}{٤}\right)}^{\frac{٨٠}{٤}=٢٠}i+\frac{\cancel{٢}}{٣}\Big(\underbrace{٣^{\frac{٣}{٢}}}_{\sqrt{٣^{٣}}=٣\sqrt{٣}}-١^{\frac{٣}{٢}}\Big)j\right)$$

$$=١٠\,i+\frac{١}{٣}\left(٣\sqrt{٣}-١\right)j$$

به نام خداوندی که کتیبه برنامه

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\frac{2}{u} \qquad 4t^{-3}$$

$$\int_1^3 \left[(t-1)^3\vec{i} + \frac{2}{t+1}\vec{j} + \frac{4}{t^3}\vec{k}\right]dt$$

$$= \left[\frac{(t-1)^4}{4}i + 2\ln(t+1)j + \frac{\cancel{4}\,t^{-2}}{\cancel{-2}}\vec{k}\right]_1^3$$

$$\ln\frac{4}{2} \qquad \frac{1}{3^2} - \frac{1}{1} = \frac{1-9}{9} = -\frac{8}{9} \qquad -2$$

$$= \left(\frac{2^4}{4} - 0\right)i + 2(\ln 4 - \ln 2)j - 2(3 - 1)\vec{k}$$

$$= 4\vec{i} + 2\ln 2\,\vec{j} + \frac{16}{9}\vec{k}$$