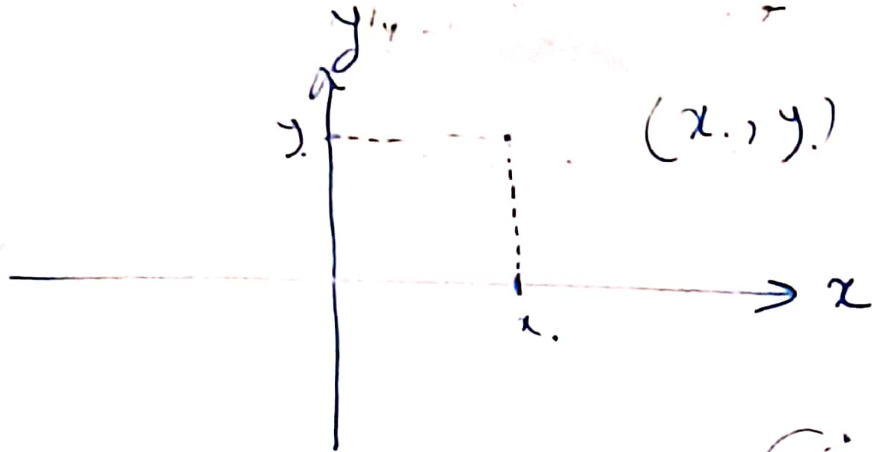
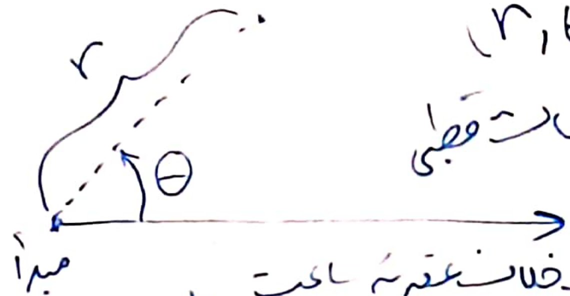
$0 < x < 6m$

x^2

$\Sigma F_x = 0 \qquad \underline{N = 0}$



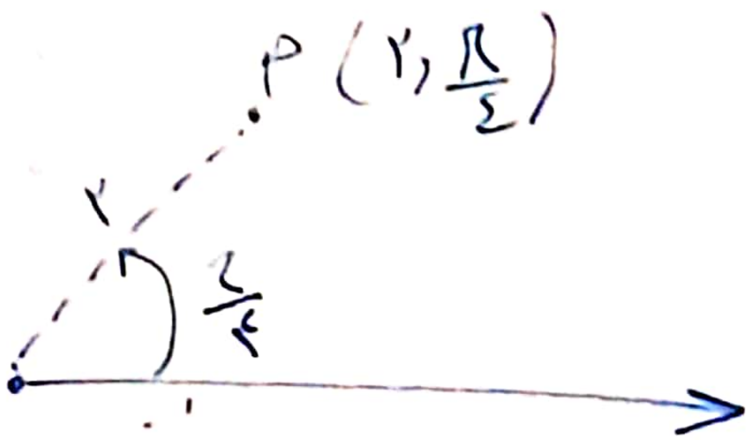


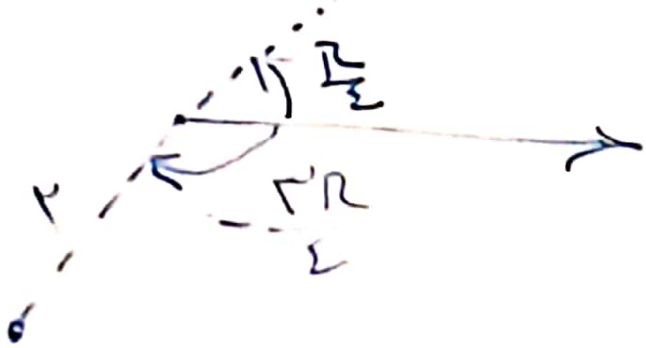
(r, θ)

مختصات قطبی

در جهت خلاف عقربه ساعت +

عقربه ساعت −





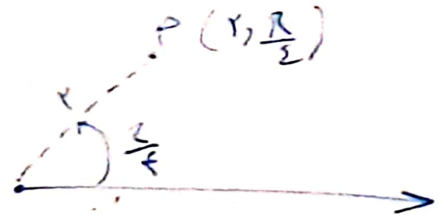
$(-۲, \frac{\pi}{۴})$

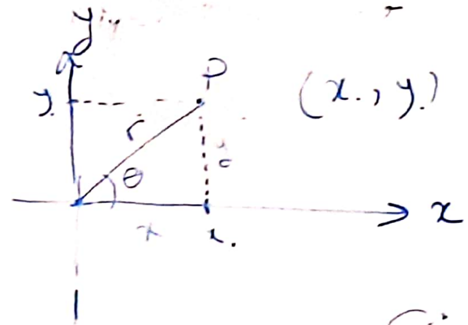
$(۲, -\frac{۳\pi}{۴})$

۲۹ ۱۴۰۲/۳

$0 < x < 6\pi$

$\sum F_y = 0 \qquad N = 0$



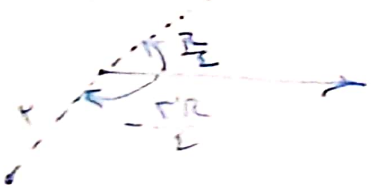


تبدیل قطبی به دکارتی

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

تبدیل دکارتی به قطبی

$$\begin{cases} r = \sqrt{x^2 + y^2} \\ tg\theta = \frac{y}{x} \end{cases}$$



$(-2, \frac{\pi}{4})$

$(2, -\frac{3\pi}{4})$

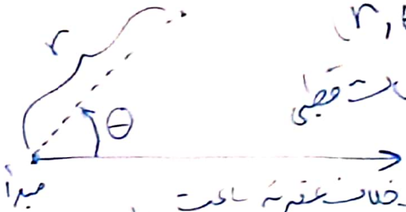
$\cos\theta = \frac{x}{r}$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

$\sin\theta = \frac{y}{r} \Rightarrow y = r\sin\theta$

$\cos\theta = \frac{x}{r} \Rightarrow x = r\cos\theta$



(r, θ)

مختصات قطبی

+ در جهت خلاف عقربه ساعت

− عقربه ساعت

$0 < x < 2\pi$

$(2, \frac{\pi}{3})$

$x = r\cos\theta = 2\times\cos\frac{\pi}{3} = 2\times\frac{1}{2} = 1$

$y = r\sin\theta = 2\times\sin\frac{\pi}{3} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$

$(1, \sqrt{3})$

$\tan\theta = \frac{y}{x}$

$r^2 = x^2 + y^2$

$r = \sqrt{x^2 + y^2}$

$\sin\theta = \frac{y}{r} \to y =$

$\cos\theta = \frac{x}{r} \to x =$

$(-2, \frac{5\pi}{3})$ $\quad (2\pi - \frac{\pi}{3})$

$x = -2\cos\frac{5\pi}{3} = -2\cos\frac{\pi}{3} = -1$

منفی

$y = -2\sin\frac{5\pi}{3} = -\cancel{2}\times -\frac{\sqrt{3}}{\cancel{2}} = +\sqrt{3}$

$(-1, \sqrt{3})$

$r>0 \,,\quad 0 \le \theta < 2\pi$

$\left(2\sqrt{2}, \frac{7\pi}{4}\right)$

$P(-3, 0)$

$r = \sqrt{9+0} = 3$

$(3, \pi)$

$\theta = \frac{0}{-3} = \quad \theta = 0$

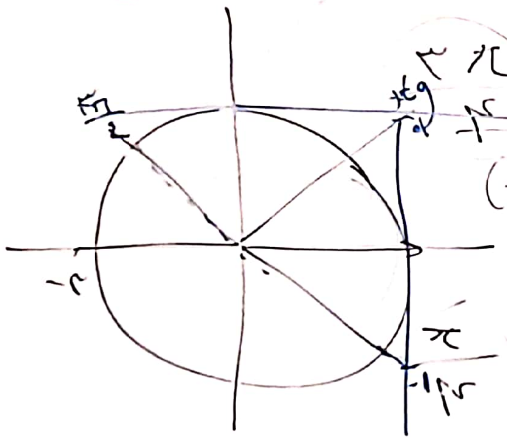
$\theta = \pi$

$\left(2, \frac{\pi}{3}\right)$

$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2 \times \frac{1}{2} = 1$

$y = r\sin\theta = 2 \times \sin\frac{\pi}{3} = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

$(1, \sqrt{3})$



$\left(-2, \frac{5\pi}{3}\right)$

$x = -2\cos\frac{5\pi}{3} = -2\cos\frac{\pi}{3} = -1$

$y = -2\sin\frac{5\pi}{3} = -2 \times -\frac{\sqrt{3}}{2} = \sqrt{3}$

$(-1, \sqrt{3})$

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$0 < x < 2\pi$

$r^2 = x^2 + y^2$

$\tan\theta = \frac{y}{x}$

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$dx\,dy = r\,dr\,d\theta$

$(-\sqrt{3}, -1)$

$r = \sqrt{x^2 + y^2} = \sqrt{\sqrt{3}^2 + (-1)^2} = \sqrt{4} = 2$

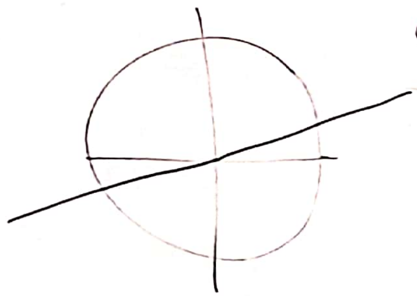
$tg\ \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{7\pi}{6}$

$\left(2, \frac{7\pi}{6}\right)$

$\left(2, \frac{\pi}{3}\right)$

$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\times\frac{1}{2} = 1$

$y = r\sin\theta = 2 \times \sin\frac{\pi}{3} = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$



$(1, \sqrt{3})$

$2\times -\frac{\sqrt{3}}{2} = -\sqrt{3}$

$= \frac{7\pi}{6}$

$0 < x < 2\pi$

$r^2 = x^2+y^2$

$\tan\theta = \frac{y}{x}$

$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$dxdy = r\,dr\,d\theta$

دایره‌ای به مرکز (۰،۰) و شعاع ۱

به مرکز (۰،۰) و شعاع ۲

R ناحیه محدود به دو دایره $x^2+y^2=1$ و $x^2+y^2=4$ در ناحیه

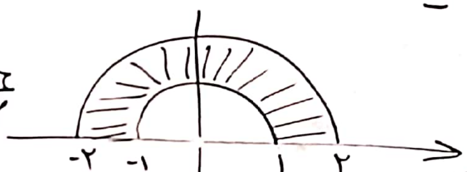
اول و دوم می‌باشد. انتگرال $\iint_R e^{x^2+y^2}\,dxdy$ را به شکل قطبی بنویسید و سپس آن را محاسبه کنید.

$(-\sqrt{3}, -1)$

$r = \sqrt{x^2+y^2} = \sqrt{\sqrt{3}^2+(-1)^2} = \sqrt{4} = 2$

$\operatorname{tg}\ \frac{y}{x} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{7\pi}{6}$

$\left(2, \frac{7\pi}{6}\right)$



$R: \begin{cases} 1 \leq r \leq 2 \\ 0 \leq \theta \leq \pi \end{cases}$

$r^2 = u$

$2r\,dr = du$

$\begin{cases} u = 1 \\ u = 4 \end{cases}$

$\frac{1}{2}\int_0^\pi\int_1^2 e^{r^2}\,2r\,dr\,d\theta = \frac{1}{2}\int_0^\pi\int_1^4 e^u\,du\,d\theta = \frac{1}{2}\int_0^\pi \left(e^u\right)_1^4 d\theta = \frac{1}{2}\left(e^4-e^1\right)\theta\Big|_0^\pi$

$e^4 - e^1$

$= \frac{1}{2}\left(e^4-e^1\right)\pi$

$0 < x < 6\pi$

$\iint_R \sqrt{9-x^2-y^2}\,dx\,dy$ انتگرال، R دایره‌ای به مرکز (۰،۰) و شعاع ۲ باشد

$9-(x^2+y^2)$

$dx\,dy = r\,dr\,d\theta$

$$R:\begin{cases} 0 \le \theta \le 2\pi \\ 0 \le r \le 2 \end{cases}$$

$$-\frac{1}{2}\int_0^{2\pi}\int_0^{2} \sqrt{9-r^2}\,(-2)r\,dr\,d\theta = -\frac{1}{2}\int_0^{2\pi}\int_9^{5}\sqrt{u}\,du\,d\theta = -\frac{1}{2}\int_0^{2\pi} \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\Big|_9^5\,d\theta$$

$u = 9-r^2$, $du = -2r\,dr$, $\begin{cases} r=0 \to u=9 \\ r=2 \to u=5 \end{cases}$

$$= -\frac{1}{3}\left(5^{\frac{3}{2}} - 9^{\frac{3}{2}}\right)\int_0^{2\pi} d\theta = -\frac{1}{3}\left(\underbrace{5^{\frac{3}{2}}}_{5\sqrt5} - \underbrace{9^{\frac{3}{2}}}_{27}\right)2\pi = -\frac{2\pi}{3}\left(5\sqrt{5}-27\right)$$

R ناحیه محدود به دو دایره $x^2+y^2=1$ ، $x^2+y^2=4$ در ناحیه اول و دوم می‌باشد. انتگرال $\iint_R e^{x^2+y^2}\,dx\,dy$ را به شکل قطبی بنویسید سپس آن را حساب کنید.

دایره‌ای به مرکز (۰،۰) و شعاع ۱

به مرکز (۰،۰) و شعاع ۲

$$R:\begin{cases} 1 \le r \le 2 \\ 0 \le \theta \le \pi \end{cases}$$

$$\int_0^{\pi}\int_1^{2} e^{r^2}\,2r\,dr\,d\theta \cdot \frac{1}{2} = \frac{1}{2}\int_0^{\pi}\int_1^{4} e^{u}\,du\,d\theta = \frac{1}{2}\int_0^{\pi} \left(e^{u}\right)\Big|_1^4\,d\theta = \frac{1}{2}\left(e^4-e^1\right)\theta\Big|_0^{\pi} = \frac{1}{2}\left(e^4-e^1\right)\pi$$