معادله دایره $(x-\alpha)^2+(y-\beta)^2=r^2$ (α,β) مرکز ، شعاع r

مرکز مبدأ مختصات $x^2+y^2=r^2$

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}(x^3+xy^2)\,dx\,dy=\int_0^{2\pi}\int_0^1\left(r^3\cos^3\theta+r\cos\theta\, r^2\sin^2\theta\right)r\,dr\,d\theta=$$

$$\int_0^{2\pi}\int_0^1\left(\cancel{r^3\cos^3\theta}+r^3\cos\theta\left(1-\cos^2\theta\right)\right)r\,dr\,d\theta \qquad +r^3\cos\theta-\cancel{r^3\cos^3\theta}$$

$$\int_0^{2\pi}\int_0^1 r^4\cos\theta\,dr\,d\theta=\int_0^{2\pi}\left(\frac{r^5}{5}\cos\theta\right)\Big|_0^1 d\theta$$

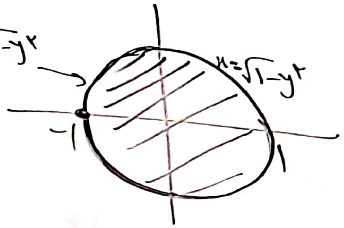
$$\frac{1}{5}\int_0^{2\pi}\cos\theta\,d\theta=\frac{1}{5}\sin\theta\Big|_0^{2\pi}=0$$

$-\sqrt{1-y^2}\leq x\leq\sqrt{1-y^2}$ $x=\sqrt{1-y^2}$

$-1\leq y\leq 1$

$x=\sqrt{1-y^2}$

$x^2=1-y^2\rightarrow y^2+x^2=1$



$$\begin{cases}0\leq r\leq 1\\ 0\leq\theta\leq 2\pi\end{cases}$$

حجم زیر صفحه $5z+4y=20$ که تصویر آن بر صفحه xy مستطیل $0 \le y \le 3$, $0 \le x \le 2$ می‌باشد را بیابید.

$$V = \iint_R f(x,y)\, dA \qquad z = f(x,y)$$

$$5z = 20 - 4y$$

$$z = \frac{20 - 4y}{5}$$

$$\int_0^2 \int_0^3 \frac{\cancel{20}^{\,4}}{\cancel{5}} - \frac{4y}{5}\, dy\, dx = \int_0^2 \left(4y - \frac{4y^2}{10} \right)\Big|_0^3 dx$$

$$= \int_0^2 \underbrace{12 - \frac{36}{10}}_{8.4} dx = \left(8.4\, x \right)\Big|_0^2 = 16.8$$

حجم زیر سطح استوانه $z = x^2 + 2$ بالای مثلث $y \le x \le 2$ و $0 \le y \le 2$

$$\int_0^2 \int_y^2 (x^2+2)\,dx\,dy = \int_0^2 \left(\frac{x^3}{3} + 2x\right)_y^2 dy$$

$$= \int_0^2 \underbrace{\frac{8}{3} + \frac{4\times 3}{1\times 3}}_{20/3} - \left(\frac{y^3}{3} + 2y\right) dy$$

$$\left(\frac{20}{3}y - \frac{y^4}{12} - \frac{\cancel{2}y^2}{\cancel{2}}\right)_0^2 = \frac{40}{3} - \frac{\overset{4}{\cancel{16}}}{\underset{3}{\cancel{12}}} - \frac{4\times 3}{1\times 3} \quad \frac{40 \overbrace{-4-12}^{-16}}{3} = \frac{24}{3} = 8$$

زیر $z=1-x^2-y^2$ و بالای ناحیه $x+y\le 1$ و $x\geq 0$, $y\geq 0$

$$\begin{cases} 0\le y < -x+1 \\ y\geq 0 \qquad 0\le x < 1 \end{cases}$$

$$\int_0^1\int_0^{-x+1}(1-x^2-y^2)\,dy\,dx = \int_0^1 \left. y - x^2y - \frac{y^3}{3}\right|_0^{-x+1}$$

$z=xy$

(۲,۰)

(۰,۲)

(۰,۰)

$m=\frac{\quad}{\quad}=-1$

$y=-x+2$

$0\le y\le -x+2$

$0\le x\le 2$

$$\int_0^2\int_0^{-x+2} xy\,dy\,dx = \int_0^2 \left.\frac{xy^2}{2}\right|_0^{-x+2} = \int_0^2 \frac{x(-x+2)^2}{2}\,dx = \int_0^2 \frac{x(x^2-4x+4)}{2}\,dx$$

$$= \frac12\int_0^2 (x^3-4x^2+4x)\,dx = \frac12\left(\frac{x^4}{4}-\frac{4x^3}{3}+\frac{4x^2}{2}\right)\Big|_0^2 = \frac12\left(\frac{16}{4}-\frac{32}{3}+\frac{16}{2}\right) = \frac12\times\frac{4}{3}=\frac{2}{3}$$

$\frac{12}{1}-\frac{32}{3}$

زیر $z=1-x^2-y^2$ و بالای ناحیه $x+y\le 1$ و $x\ge 0$ ، $y\ge 0$

$$\begin{cases} 0\le y < -x+1 \\ 0<x<1 \end{cases} \qquad y\ge 0$$

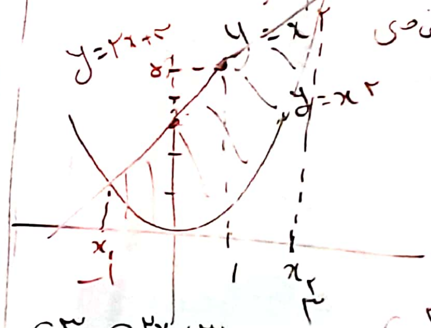
$$\int_0^1\int_0^{-x+1}(1-x^2-y^2)\,dy\,dx = \int_0^1 y - x^2y - \frac{y^3}{3}\Big|_0^{-x+1}$$

$$\frac{9+9}{18}-\frac{27}{8}-\left(\frac{1-3}{-2}+\frac{1}{3}\right)=\frac{9}{1}+\frac{5}{2}=\frac{32}{3}$$

$$V=\iint f(x,y)\,dA$$

$$S=\iint dA$$

مثال: مساحت ناحیه محصور بین منحنی‌های $y=2x+3$ و $y=x^2$



$x^2=2x+3$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=3$

$x=-1$

$$\int_{-1}^{3}\int_{x^2}^{2x+3} dy\,dx = \int_{-1}^{3}(y)\Big|_{x^2}^{2x+3}dx$$

$$=\int_{-1}^{3}(2x+3-x^2)\,dx=\left(\frac{2x^2}{2}+3x-\frac{x^3}{3}\right)\Big|_{-1}^{3}$$