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The modeling of time series based on fuzzy information granules

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ABSTRACT

A lot of research has resulted in many time series models with high precision forecasting realized at the numerical level. However, in the real world, higher numerical precision may not be necessary for the perception, reasoning and decision-making of human. Model of time series with an ability of humans to perceive and process abstract entities (rather than numeric entities) is more adaptable for some problems of decision-making. With this regard, information granules and granular computing play a primordial role. Fox example, if change range (intervals) of stock prices for a certain period in the future is regarded as information granule, constructing model that can forecast change ranges (intervals) of stock prices for a period in the future is better able to help stock investors make reasonable decisions in comparison with those based upon specific forecasting numerical value of stock price. In this paper, we propose a new modeling approach to realize interval prediction, in which the idea of information granules and granular computing is integrated with the classical Chen's method. The proposed method is to segment an original numeric time series into a collection of time windows first, and then build fuzzy granules expressed as a certain fuzzy set over each time windows by exploiting the principle of justifiable granularity. Finally, fuzzy granular model can be constructed by mining fuzzy logical relationships of adjacent granules. The constructed model can carry out interval prediction by degranulation operation. Two benchmark time series are used to validate the feasibility and effectiveness of the proposed approach. The obtained results demonstrate the effectiveness of the approach. Besides, for modeling and prediction of large-scale time series, the proposed approach exhibit a clear advantage of reducing computation overhead of modeling and simplifying forecasting.

1. Introduction

Modeling and prediction of time series are classical issues which is widely researched. Researchers early exploited the linear system theory, the stochastic process theory and the black-box methodology to develop many classical *numeric* models of time series such as ARMA, ARIMA, ARARMA, ANN (artificial neural network) model and alike. These models have been widely applied to various domains, showing better forecasting performance at the numerical level. These models are difficult to comprehend due to their low interpretability. Fuzzy set theory (Zadeh, 1965) can be used to alleviate this shortcoming, while fuzzy reasoning offers a viable alternative to ensure robustness to the inherent uncertainty, which has also been involved into the modeling of time series.

Based on fuzzy set theory, the concept of fuzzy time series was originally proposed by Song and Chissom (1993b) to handle prediction problem in which the historical data are linguistic values. They have also developed two time series models — the time-invariant model (Song & Chissom, 1993a) and the time-variant model (Song & Chissom, 1994) to forecast the enrollments of the University of Alabama, which comprise four steps mainly for modeling of time series: (1) define and partition the universe of discourse; (2) define fuzzy sets on the universe of discourse and express historical data by using these fuzzy sets; (3) mine fuzzy logical relations; (4) perform prediction and defuzzify predicted results. Adhering to these four essential steps, researchers developed many improved models to predict time series coming from various domains such as stock prices, weather, temperature. Chen (1996) proposed a simplified model to reduce the computation overhead of Song's model (Song & Chissom, 1993a) by using simple arithmetic (interval) operations to replace max–min composition operations in the process of mining fuzzy logical relations and performing prediction. Huarng, Yu, and Hsu (2007) proposed a multivariate heuristic model to improve forecasting results of stock index time series by integrating with a multivariate heuristic function and Chen's models (1996). Chen (2002) presented a high-order fuzzy time series model to predict the enrollments of university, which can obtain better prediction accuracy than Chen's models (1996). Chen and Chung (2006) used genetic

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algorithms to enhance the high-order fuzzy time series model. Cheng, Chen, Teoh, and Chiang (2008) proposed a fuzzy time series model in which the adaptive expectation model is incorporated into forecasting processes to reduce forecasting errors. Cheng, Cheng, and Wang (2008) used fuzzy clustering technology to construct a multi-attribute fuzzy time series model. Chen and Tanuwijaya (2011) presented a multivariate fuzzy time series model with automatic clustering techniques. Egrioglu, Aladag, and Yolcu (2013) provided a fuzzy time series model to enhance forecasting accuracy by integrating fuzzy c-means clustering algorithm and artificial neural networks. Huarng and Yu (2005) proposed a Type-2 fuzzy time series model, in which extra data are used to enrich or to refine the fuzzy relationships obtained from Type 1 models (i.e. conventional fuzzy time series models such as Song's model, Chen's model (1996) etc.) for improving forecasting performance. Besides, Nilesh and Jerry (1999) showed also a model based on Type-2 fuzzy logic system to deal with the prediction of time series in the presence of noisy data.

The above mentioned diverse models of time series are developed from the perspective of numerical data (points), whose objective is to pursue higher prediction precision on the level of numerical value. However, higher precision is not absolutely necessary for the perception, reasoning and decision-making of human in the real world. Model of time series with an ability of humans to perceive and process abstract entities (rather than numeric entities) is more adaptable for some problems of decision-making. For example, model that can forecast change range of stock prices for a period in the future (interval) is more suitable to help stock investors make reasonable decisions than the models providing specific forecasting numerical value of stock price. Moreover, as a series of data ordered by time, time series is inherently associated with their large size. For the modeling of large-scale time series, most of the existing methods is time-consuming or invalid. Therefore, our objective is to develop a new time series modeling method to overcome the shortcomings of the existing models.

As representation of abstract entities, information granulation and granular computing seem to be a sound alternative to rectify the above-mentioned drawbacks. The role of information granulation is to organize detailed numerical data (points) into some meaningful, semantically sound entities (information granules). For the above-mentioned example, from the perspective of information granules, changes range of stock prices for a period in the future (interval) can be regards as a kind of information granule. Granular computing concentrates on processing these information granules and forms a unified conceptual and computing platform by exploiting theory of sets, fuzzy sets, rough sets and shadowed sets and others. In this study, bearing in mind the idea of information granules and granular computing, we propose a novel time series modeling method based on fuzzy information granules to construct fuzzy granular model of time series which can realize prediction at the granular level (i.e. interval prediction) instead of being focused on prediction completed at the numeric level. In what follows, we start with detailing a general method for developing granular model of time series, and then focus on a concrete method supporting the development of fuzzy granular model of time series and interval prediction.

The paper is organized as follows: Section 2 details a general approach for developing granular model of time series; Section 3 introduces the preparations for developing fuzzy granular model of time series; Section 4 presents a concrete method of developing fuzzy granular model of time series and realizing interval prediction; In Section 5, two benchmark time series are used to perform profound experiments and validate the feasibility and effectiveness of the proposed method. Besides, the impact of two parameters of the proposed modeling method on accuracy of the constructed fuzzy granular model is also discussed; Finally, Section 6 provides some conclusions.

2. A granular model framework of time series: A layered approach for the granulation modeling of time series

Information granulation and information granules play a key role in human cognitive and decision-making activities. Information granules are treated as collections of entities that are collected together because of their similarity, proximity, indistinguishability, functional closeness, which arise in the process of abstraction of data and derivation of knowledge from information. The aim of information granulation is to decompose complex problems into simple problems, which can help us capture some details of problems. Under the concept of granular computing, information granules can be expressed in the formalism of sets (interval analysis) (Bargiela, 2001; Hooman & Pedrycz, 2007), rough sets (Liang, Wang, & Qian, 2009; Qian, Liang, Yao, & Dang, 2010), fuzzy sets (Pedrycz & Vukovich, 2002; Zadeh, 1979), shadowed sets (Pedrycz, 1998) etc. and handled independently.

In this section, we provide a layered approach to develop granular model of time series. We start with an overall view of the conceptual framework by stressing its key functionalities and a layered architecture and then focus on the concrete method (see Section 4).

The bird's eye view of the overall architecture supporting processing realized at several layers which stresses the associated functionality is displayed in Fig. 1. Let us elaborate in detail on the successive layers at which consecutive phases of processing are positioned:

The discretization of time series is to decompose the original numeric time series into a small number of homogeneous non-overlapping segments (it is also called time windows), such that the data in each time window can be described accurately by a sample model. The segmentation of time series is one crucial issue in the mining and analysis of time series. There are many approaches (Abonyi, Feil, Nemeth, & Arva, 2005; Fitzgibbon, Dowe, & Allison, 2002; Fu, lai Chung, & man Ng, 2006; Jiang, Zhang, & qing Wang, 2007; Oliver, Baxter, & Wallace, 1998) supporting time windows segmentation. For the modelling and prediction problem of time series, commonly, considering that convenience and simpleness of processing of time series, a fixed-length segmentation process method is adopted to segment evenly the original time series, i.e. use a width-fixed window to segment the original time series, so that the formed each subsequence includes the same number of samples.

The granulation of time series is to construct granules by carrying out granulation operation on the each time window that is generated by the discretization of time series. Each granules can serve as the representation of data in the corresponding time window. These granules can be expressed by any of the above-mentioned formalisms (intervals, fuzzy sets, rough sets and so on). Suppose that $X = \{x_1, x_2, \ldots, x_n\}$ is a certain time series and $p (1 \leqslant p \leqslant n)$ represents the number of time windows. If $p = 1$, the whole time series is regarded as one granule, and if $p = n$ means that every sample x_i is regarded as one granule. The number of time windows is clearly related to the ability to retain the inherent essence of the original time series. The more the time windows are, the more granules the granulation will produce. It is notable that the segmented time windows are associated with time-stamped, thus granules generated by granulating time windows spread over time window.

The linguistic description of granules is to reasonably assign linguistic term for every granule. Having a collection of linguistic terms $\mathbf{A} = \{A_1, A_2, \ldots, A_c\}$ where A_i is an information granule

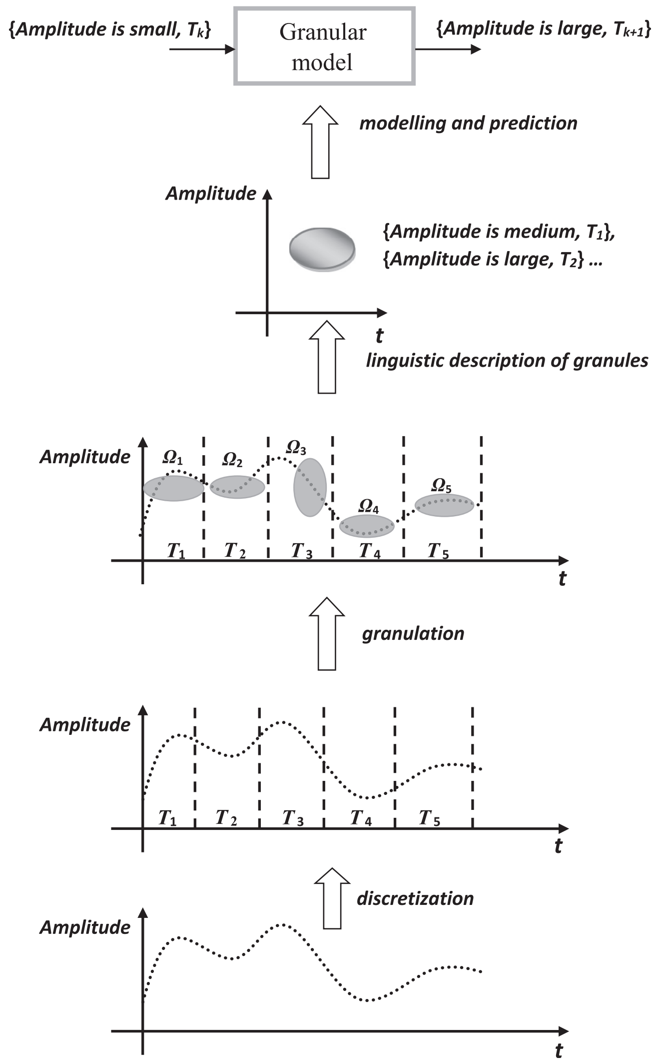


Fig. 1. Development of granular model of time series: discretization, granulation and forming semantics.

expressed in the framework of any of the formalisms presented before (intervals, fuzzy sets and so on). The linguistic description of granule is how to describe a new granule X in terms of the elements of **A**. Intuitively, one could envision a number of possible ways of describing X. A way which is quite appealing is to exploit the well-known matching measures encountered in fuzzy sets, say possibility or necessity measures.

With the operations above, time series can be describe as a string of the granular landmarks (for which the best matching has been accomplished). For instance, the granular landmark of T_i time window can be denoted as $\{amplitudesmall, T_i\}$. Each granular landmark comes with its own semantics (e.g., amplitude small, amplitude medium, etc.) and the associated time window. Once a series of granules associated with time-stamped are described by linguistic terms, it means that original time series can be transformed into *granule time series*.

The modelling and prediction of time series: The granular description of time series are useful vehicles to represent time series in a meaningful and easy to capture way. Essentially, the descriptors are not models such as standard constructs reported in time series analysis. They, however, deliver all components, which could be put together to form granular predictive models. Denoting that $A_1, A_2, \ldots, A_c$ is linguistic terms of describing granules, a crux of the predictive model is to determine (mine) relationships between the activation levels of the linguistic landmark of information granule presented for the current time window T_k and those levels encountered in the next time window T_{k+1}. The underlying form of the predictive mapping can be schematically expressed in the following way,

$$A_1(X_k), A_2(X_k), \ldots, A_c(X_k) \to A_1(X_{k+1}), A_2(X_{k+1}), \ldots, A_c(X_{k+1}),$$

where $A_i(X_k)$ stands for a level of activation (matching) observed between A_i and the current information granule X_k and $A_i(X_{k+1}(T_{k+1}))$ is the activation levels of the linguistic landmarks A_i caused by the predicted information granule X_{k+1}. The operational form of the predictive model can be realized in the form of a fuzzy relational equation

$$A(X_{k+1}) = A(X_k) \circ R$$

where "$\circ$" expresses a certain composition operator used in fuzzy sets (say, max–min or max-t composition where "t" stands for a certain t-norm) completed over information granules.

3. Preparations for developing fuzzy granular model of time series

3.1. Fuzzy time series

The definition of fuzzy time series was originally introduced by Song and Chissom (1993b, 1993a, 1994). Some definitions related with fuzzy time series are recalled here.

Let $U = \{u_1, u_2, \ldots, u_n\}$ be the universe of discourse. A fuzzy set A of U is defined as follows:

$$A = \frac{f_A(u_1)}{u_1} + \frac{f_A(u_2)}{u_2} + \cdots + \frac{f_A(u_n)}{u_n},$$

where f_A is the membership function of the fuzzy set A, $f_A : U \to [0,1]$, $f_A(u_i)(1 \leqslant i \leqslant n)$ is the degree of membership of u_i in the fuzzy set A.

Definition 1. Fuzzy time series. Let $Y(t)(t = \ldots, 0, 1, 2 \ldots)$, a subset of R^1, be the universe of discourse on which fuzzy sets $f_i(t)(i = 1, 2, \ldots)$ are defined. If $F(t)$ is a collection of $f_1(t), f_2(t), \ldots$ then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

From Definition 1, we can see that the main difference between the fuzzy time series and conventional time series is that the values of the former are fuzzy sets (linguistic terms) while the values of the latter are real numbers. $F(t)$ can be regarded as a linguistic variable and $f_i(t)(i = 1, 2, \ldots)$ can be viewed as possible linguistic values of $F(t)$, where $f_i(t)(i = 1, 2, \ldots)$ are represented by fuzzy sets. Song and Chissom (1993b) have used a example to explain the concepts of fuzzy time series: observe the weather of a certain place in North America, begin from the first day and ending with the last day of a year, where the common daily words (i.e., "good", "very good", "quite good", "very very good", "cool", "very cool", "quite cool", "hot", "very hot", "cold", "very cold", "quite cold", "very very cold", …, etc.) are used to describe the weather conditions and these words are represented by fuzzy sets.

It is worth noting that $F(t)$ is associated with time t, i.e., the values of $F(t)$ can be different at different times due to the fact that the universe of discourse can be different at different times. If $F(t)$ is caused by $F(t-1)$, then there is a certain fuzzy logical relationship between them. The definition of fuzzy logical relationship is given as follows.

Definition 2. Fuzzy logical relationship. If $F(t)$ is caused by $F(t-1)$ only, i.e., $F(t-1) \to F(t)$, then it can be expressed as $F(t) = F(t-1) \cdot R(t, t-1)$, where $R(t, t-1)$ is called the fuzzy logic relationship between $F(t-1)$ and $F(t)$, $\cdot$ is a predefined operator.

Relationships with the same fuzzy set on the left hand side can be further grouped into a fuzzy relationship group.

Definition 3. Fuzzy logical relationship group (Chen, 1996). Suppose there are fuzzy logical relationships such that $A_i \to A_{j1}$, $A_i \to A_{j2}, \ldots, A_i \to A_{jn}$, then they can be grouped into a fuzzy logical relationship group as follows: $A_i \to A_{j1}, A_{j2}, \ldots, A_{jn}$.

3.2. Use the principle of justifiable information granularity to construct fuzzy information granule

As noted so far, as the fundamental element of granular computing, information granules play a key role in the process of developing granular model of time series. In addition, granularity of information can help us focus on the most suitable level of detail. In some cases these details are essential, in others they are to be neglected as forming a significant overload burden and leading to computational inefficiency. The level of detail (which is represented in terms of the size of information granules) is an essential facet facilitating a way a hierarchical processing of information becomes realized.

How to design information granule is a crux of developing granular model of time series. Information granule can be expressed in any formalism mentioned above (sets, fuzzy sets, rough set and so on). In this paper, we concentrate on fuzzy information granule. Zadeh (1979) gave the general form of fuzzy information granule:

$$g = (x \text{ is } G) \text{ is } \lambda, \tag{1}$$

where x is a variable of a universe of discourse U, G is a convex fuzzy subset of U, and λ is the probability of x belonging to the subset G. Pedrycz and Vukovich (2001) introduced also a model of generalization and specialization of fuzzy information granules.

To design fuzzy information granule, there are various approaches supporting the design of fuzzy information granules such as clustering techniques (Bezdek, 1981). One of the most recent methods exploits the principle of justifiable granularity (Gacek, 2013). This information granulation method can form a *single* fuzzy information granule on basis of the available experimental evidence (data). In the following, we first introduce the principle of justifiable granularity, and then a illustrative example is showed how to use the principle of justifiable granularity for constructing fuzzy information granule.

3.2.1. The principle of justifiable information granularity

Let us consider a one-dimensional numerical data $D = \{x_1, x_2, \ldots, x_n\}$ and a single information granule $\Omega(x)$ that represented in the form of a certain family of fuzzy set (say triangular, trapezoidal, parabolic, gaussian and so on).

In the procedure of using the principle of justifiable granularity to construct information granule, there are two intuitively compelling requirements to be considered: one is justifiable granularity and the other is semantic meaning. For the first requirement, the constructed granule Ω should include as many data as possible, which can make granule become more legitimate (justifiable). The justifiable granularity can be quantified by calculating the sum of membership degrees of the data belonging to the fuzzy set Ω, denoted here by $\sum_{k=1}^{n}\Omega(x_k)$. Obviously, the higher the value of $\sum_{k=1}^{n}\Omega(x_k)$ is, the more the data included in granule is. More generally, we may consider an increasing function of this sum, say $f_1(\sum_{k=1}^{n}\Omega(x_k))$, where f_1 is an increasing function of its argument. For the second requirement, the constructed granule Ω should be as specific as possible that it comes with a well-defined semantics. This requires that the support of Ω (fuzzy set) should be as low as possible, Let $measure(supp(\Omega))$ be $(b-a)$, where "a" and "b" are the bounds of the support of Ω. It can serve as a sound indicator of the specificity of the information granule. We can define a continuous non-increasing function f_2 which is associated with the support of granule, it can be denoted as $f_2(measure(supp(\Omega))$. The higher the value of $f_2(measure(supp(\Omega))$ is, the better the satisfaction of the specificity requirement is.

It is evident that the above-mentioned two requirements are in conflict: the increase in the values of the requirement of experimental evidence (justifiable) brings about a deterioration of the specificity of the information granule (specific). Refer to Fig. 2 illustrating the character of these two requirements along with their conflicting nature: the granule Ω in Fig. 2(a) is very "specific" yet it does carry a very limited experimental evidence (note a limited number of data "embraced" by Ω). Fig. 2(b) reveals an opposite situation: we have an information granule of a large size (not being specific) but supported by a significant number of data points. As usual, we expect to find a sound compromise between these requirements. Based on the idea, the composite multiplicative index is considered for constructing independently the lower and upper bound of the support of granule, that is

$$max: V(b) = f_1(\sum_{m<x_k\leqslant b}\Omega(x_k)\} \times f_2\{|m-b|\}, \tag{2}$$

$$max: V(a) = f_1(\sum_{a\leqslant x_k<m}\Omega(x_k)\} \times f_2\{|m-a|\}, \tag{3}$$

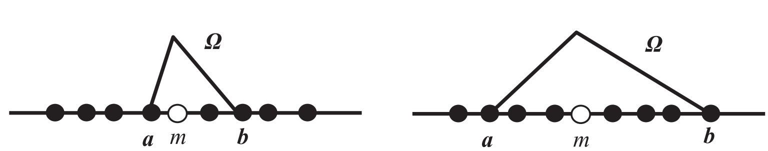
where "m" stand for the median of D that it can be regarded as a sound numerical representative of D. The median can also be taken as the core of the fuzzy set. By independently optimizing Eq. (2), we can obtain the optimal support upper bound of Ω, which is donated as b_{opt}, namely, $V(b_{opt}) = max_{b>m}V(b)$. In the same way, the optimal lower bound of Ω, which is donated as a_{opt}, can also be obtained by independently optimizing Eq. (3), that is $V(a_{opt}) = max_{a<m}V(a)$.

As one of the possible design options, the function f_1 and f_2 can be respectively chosen as the following form:

$$f_1(u) = u, \tag{4}$$

$$f_2(u) = \exp(-\alpha u), \tag{5}$$

where α is a positive parameter offering some flexibility when optimizing the information granule Ω. Its role is to calibrate an impact of the specificity requirement on the constructed information granule. Higher values of α stress the increasing importance of the specificity of constructed information granule. Sufficiently high values of α promote very confined, numeric-like information granules. If the value of α is equal to zero, then the specificity criterion is ignored.



(a) The specificity requirement (b) The legitimation requirement

Fig. 2. Two intuitively requirements for the design of fuzzy information granule.

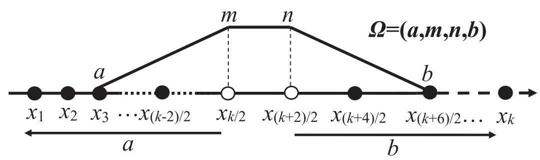


Fig. 3. The optimization process of support of the trapezoidal granules (where k is even number).

3.2.2. Illustrative example

As an illustrative brief example, let D be $\{2.5, 1.95, 0.2, 2, -0.5, 1.7, 1.9, 1.1, 0.9, 0.1, 1.75, 2.25, -0.65, -0.32\}$. In what follows, we illustrate how to use the principle of justifiable granularity to construct information granule expressed in form of the *trapezoidal* fuzzy set, which denoted by $\Omega(\alpha) = (a, m, n, b)$, where (m, n) is the core of the trapezoidal fuzzy set and (a, b) is the bound of support of the trapezoidal fuzzy set.

We first arrange the data in an increasing order $x_1 \leqslant x_2 \leqslant \cdots \leqslant x_k$ and then compute its median. The median is the core of the constructed fuzzy set. For the trapezoidal fuzzy set, when n is odd number, let m and n all be $x_{(k+1)/2}$, and whereas when n is even number, let m and n be $x_{k/2}$ and $x_{(k+2)/2}$, respectively. In here, for D, m and n are equal to 1.1 and 1.7, respectively. Once the core of the fuzzy set is determined, it splits the data into two subsets (refer to Fig. 3) that are processed separately leading to the computations of the left-hand (the portion that is less than m) and right-hand (the portion that is greater than n) portion of the membership function of Ω. Next, for a given α we sweep through all data points considering each of them to be a potential value of the parameter of the membership function. The one that maximizes the performance index $V(a)$ (refer to Eq. (3)) forms the solution to the problem $V(a_{opt}) = max_{a<m} V(a)$. Note that in the above formulation we were dealing with the increasing portion of the membership function (the left-hand portion of the membership function). Evidently, the same process is carried out for the decreasing portion of the membership function (the right-hand portion of the membership function).

Fig. 4 shows the constructed trapezoidal information granule on D at the different value of α. To construct granules expressed in the other form of the fuzzy set such as triangular fuzzy set, Gaussian fuzzy set and so on, we can also refer to above method.

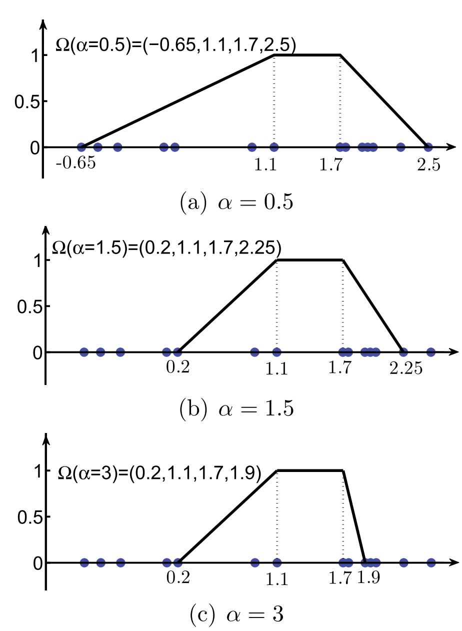


Fig. 4. The constructed trapezoidal information granule on D at the different value of α.

4. A method of building fuzzy granular model of time series: the augmented version of Chen's method

In this section, we provide a method of building fuzzy granular model of time series, in which the general method of building granular model of time series (see Section 2) is integrated with Chen's model (Chen, 1996) so that Chen's model is extended from the level of numeric to the level of granule. Compared with Chen's model, our model is constructed on the basis of granules (time windows), and the predicted result is presented in the form of intervals instead of a single point (numeric). In other words, our constructed fuzzy granular model can predict the change range of value of time series for *a period in the future*, namely, prediction is performed along with time window. In here, the principle of justifiable information granularity is applied to construct fuzzy granules.

Given that $X = \{x_1, x_2, \ldots, x_n\}$ is time series and w is the length of time window, our proposed method is now presented as follows:

Step 1. Define the universe of discourse U and partition it into n equal length intervals.
The universe of discourse U is defined as $[D_{min} - D_1, D_{max} + D_2]$, where D_{min} is the minimum of historical data of time series, D_{max} is the maximum of historical data of time series, and D_1, D_2 are trim factors. Dividing U into n equal length intervals $u_1, u_2, \ldots, u_n$.
Step 2. Define fuzzy sets $A_1, A_2, \ldots, A_k$ on the universe of discourse U, and then assign linguistic term into every defined fuzzy set. These linguistic term will be used to describe information granules. The form of definition of fuzzy sets used in Chen's model (Chen, 1996) is adopted in here, viz.,:

$$\begin{aligned} A_1 &= \tfrac{a_{11}}{u_1} + \tfrac{a_{12}}{u_2} + \cdots + \tfrac{a_{1m}}{u_m}, \\ A_2 &= \tfrac{a_{21}}{u_1} + \tfrac{a_{22}}{u_2} + \cdots + \tfrac{a_{2m}}{u_m}, \\ &\vdots \\ A_k &= \tfrac{a_{k1}}{u_1} + \tfrac{a_{k2}}{u_2} + \cdots + \tfrac{a_{km}}{u_m}, \end{aligned} \tag{6}$$

where $a_{ij} \in [0, 1]$, $1 \leqslant i \leqslant k$, and $1 \leqslant j \leqslant m$. The value of a_{ij} represents the membership degree of the crisp interval u_j in the fuzzy set A_i.

Step 3. Split the original time series by time windows evenly into the non-overlapping subseries, i.e., the original time series X is split into $p = [\frac{n}{w}]$ subseries — $T_1 = \{x_1, x_2, \ldots, x_w\}$, $T_2 = \{x_{w+1}, x_{w+2}, \ldots, x_{w+w}\}, \ldots, T_p = \{x_{(p-1)w+1}, x_{(p-1)w+2}, \ldots, x_{(p-1)w+w}\}$, where "$[x]$" represents the maximal integer not more than x.
In here, the roles of length of time windows w have two folds. One is to split time series into a series of subsets, and the other is to predict the change range (interval) of data of the future w steps by exploiting the past w data in the process of performing prediction using the constructed fuzzy granular model. The selection of length of time windows is depend on the task of prediction. If the demanded prediction results are finer, the length of time windows can be selected as a smaller value; if the demanded prediction results are rougher, the length of time windows can be selected as a larger value.
Step 4. Given a expressed form of granule (it can be any of the above-mentioned forms such as triangle granules, trapezoidal granules etc.) and a parameter α, we apply the principle of justifiable granularity (refer to subSection 3.2) to granulate each time window $T_1, T_2, \ldots, T_p$ so that the corresponding fuzzy

granule sequence $\Omega(\alpha) = \{\Omega_1, \Omega_1, \ldots, \Omega_p\}$ is formed at α granularity.

Step 5. Obtain the linguistic description of granular time series, that is, we use the element of A defined previously to describe each granule Ω_i formed by Step 4. To reach it, intuitively, we can exploit possibility measure $Poss(\Omega, A_i)$ that can describe the relationship between granule Ω and A_i in terms of coincidence (overlap) of these two, where $Poss(\Omega, A_i)$ is defined as follows:

$$Poss(\Omega, A_i) = sup_{x\in\Omega}[\Omega(x)tA_i(x)], \tag{7}$$

where "t" is some *t-norm*.

For each granule Ω_i, it relative to each $A_i(i = 1, 2, \ldots, k)$ exists a overlap degree which can be computed by Eq. (7). Further, the linguistic description of granule Ω_i can be determined by maximum overlap degree in them. Once the granules associated with time-stamped are described by linguistic terms, it implies that the original time series is transformed into the granular time series.

Step 6. Minie fuzzy logical relationships from the granular time series according to Definition 2, and then group the fuzzy logical relationships into the fuzzy logical relationship groups according to Definition 3.

Step 7. Perform reasoning along with time windows and degranulation operation according to the following rules.

Assume that the linguistic term of granule Ω_{i-1} is A_j on the $(i-1)$th time window T_{i-1} and the fuzzy logical relationship group of A_j is $A_j \to A_{j_1}, A_{j_2}, \ldots, A_{j_p}$, then the value of lower bound a_i and the value of upper bound b_i for the ith time window T_i to be forecasted are determined by the following rules:

Rule 1. If the data of T_i time window has *only uptrend* relative to T_{i-1} time window, i.e., there is $j_1, j_2, \ldots, j_p > j$ in $A_j \to A_{j_1}, A_{j_2}, \ldots, A_{j_p}$, then the predicted lower bound value a_i at time window T_i is equal to m_j and the predicted upper bound value b_i at time window T_i is equal to $(m_{j_1} + m_{j_2} + \cdots + m_{j_p})/p$, where $m_{j_i}(i = 1, 2, \ldots, p)$ is the midpoint of the interval $u_{j_i}(i = 1, 2, \ldots, p)$ and m_j is the midpoint of interval u_j.

Rule 2. If the data of T_i time window has *only downtrend* relative to T_{i-1} time window, i.e., there is $j_1, j_2, \ldots, j_p \leqslant j$ in $A_j \to A_{j_1}, A_{j_2}, \ldots, A_{j_p}$, then the predicted lower bound value a_i at time window T_i is equal to $(m_{j_1} + m_{j_2} + \cdots + m_{j_p})/p$ and the predicted upper bound value b_i at time window T_i is equal to m_j, where $m_{j_i}(i = 1, 2, \ldots, p)$ is the midpoint of the interval $u_{j_i}(i = 1, 2, \ldots, p)$ and m_j is the midpoint of interval u_j.

Rule 3. If the data of T_i time window has *both downtrend and uptrend* relative to T_{i-1} time window, i.e., there are $j_1, j_2, \ldots, j_k \leqslant j$ and $j_{k+1}, \ldots, j_p > j(k < p)$ in $A_j \to A_{j_1}, A_{j_2}, \ldots, A_{j_p}$, then the predicted lower bound value a_i at time window T_i is equal to $(m_{j_1} + m_{j_2} + \cdots + m_{j_k})/k$ and the predicted upper bound value b_i at time window T_i is equal to $(m_{j_{k+1}} + m_{j_{k+2}} + \cdots + m_{j_p})/(p-k)$, where $m_{j_i}(i = 1, 2, \ldots, p)$ is the midpoint of the interval $u_{j_i}(i = 1, 2, \ldots, p)$.

5. Experimental study

In this section, two time series, the enrollments of University of Alabama time (Chen, 2002; Chen, 1996; Chen & Chung, 2006; Huarng et al., 2007;Song & Chissom, 1993a, 1994) and the daily value of Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) (Huarng & Yu, 2005), are used to carry out experiment, respectively. The former is used to detail how to apply our proposed method to construct fuzzy granular model and realize interval prediction, whereas the latter is used to show the impacts of parameters on accuracy of model. Its worth noting that the predicted results (outputs) of the proposed model are intervals, which represent the change range of time series in a period in the future (time windows).

5.1. The evaluation of fuzzy granular model

In order to verify the effectiveness of building fuzzy granular model of time series, we propose a performance index (P for short) to quantify accuracy of time series granular model.

We introduce two performance criteria first. One is the coverage criterion (denoted by Q), which is used to measure the quantity of the predicted interval including samples; the other is the specificity criterion (denoted as V), which is used to measure the width of the predicted interval.

Assume that the $X_i = \{x_1^i, x_2^i, \ldots, x_w^i\}$ are real numerical values included in the ith time window T_i and the predicted interval on T_i is $[a_i^+, b_i^+]$, then the coverage criterion (Q_i) and the specificity criterion (V_i) are respectively defined as follows:

$$Q_i\% = \sum_{j=1}^{w} \frac{f_i(j)}{w} \cdot 100 \tag{8}$$

$$f_i(j) = \begin{cases} 1 & x_j^i \in [a_i^+, b_i^+] \\ 0 & x_j^i \notin [a_i^+, b_i^+] \end{cases} \quad j = 1, 2, \ldots, w, \tag{9}$$

$$V_i = b_i^+ - a_i^+. \tag{10}$$

For time window T_i, if the value of Q_i is the same, the smaller the value of V_i is, the more accurate the predicted results of the constructed fuzzy granular model are; if the value of V_i is same, the larger the value of Q_i is, the more accurate the predicted results of the constructed fuzzy granular model are, which can be illustrated in Fig. 5. If the number of time windows is p, we have:

$$P\% = \frac{\sum_{i=1}^{p} \frac{Q_i}{V_i}}{p} \cdot 100 \tag{11}$$

Note that the larger the value of Q_i is and the smaller the value of V_i is in each time window, the larger of the value of P is and the more accurate the constructed model is.

5.2. The enrollments time series

The yearly data of enrollments of university of Alabama from 1971 to 1992 are commonly used to validate the prediction method of fuzzy time series. In this paper, the same data set is also used

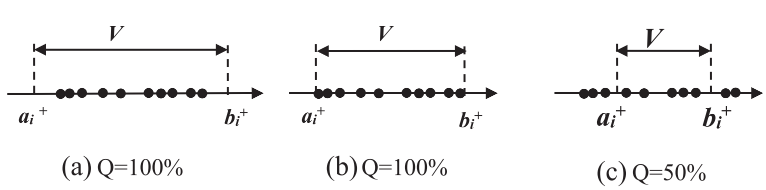


Fig. 5. The coverage criterion (V) and the specificity criterion (Q) of evaluating fuzzy granular model of time series.

to validate the constructed fuzzy granular model. The original data of enrollments is showed in the 2nd column of Table 2.

The detailed process of building fuzzy granular model of enrollments time series is showed as follows:

Step 1. Define and partition the universe of discourse U.

From the second column Table 2, we obtain $D_{min} = 13055$ and $D_{max} = 19337$. Let D_1 and D_2 be 55 and 663, respectively, the universe of discourse $U = [13000, 20000]$ can be obtained. Dividing U into seven intervals of equal length – $u_1, u_2, u_3, u_4, u_5, u_6$ and u_7, we have – $u_1 = [13000, 14000], u_2 = [14000, 15000], u_3 = [15000, 16000], u_4 = [16000, 17000], u_5 = [17000, 18000], u_6 = [18000, 19000]$, and $u_7 = [19000, 20000]$.

Step 2. Define the linguistic terms of granules on the U.

For the enrollment time series, the following linguistic terms $A_1 = (notmany), A_2 = (nottoomany), A_3 = (many), A_4 = (manymany), A_5 = (verymany), A_6 = (toomany)$ and $A_7 = (toomanymany)$ are adopted from Chen (1996), where:

$$A_1 = \frac{1}{u_1} + \frac{0.5}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$
$$A_2 = \frac{0.5}{u_1} + \frac{1}{u_2} + \frac{0.5}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$
$$A_3 = \frac{0}{u_1} + \frac{0.5}{u_2} + \frac{1}{u_3} + \frac{0.5}{u_4} + \frac{0}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$
$$A_4 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0.5}{u_3} + \frac{1}{u_4} + \frac{0.5}{u_5} + \frac{0}{u_6} + \frac{0}{u_7},$$
$$A_5 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0.5}{u_4} + \frac{1}{u_5} + \frac{0.5}{u_6} + \frac{0}{u_7},$$
$$A_6 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0.5}{u_5} + \frac{1}{u_6} + \frac{0.5}{u_7},$$
$$A_7 = \frac{0}{u_1} + \frac{0}{u_2} + \frac{0}{u_3} + \frac{0}{u_4} + \frac{0}{u_5} + \frac{0.5}{u_6} + \frac{1}{u_7}.$$

Step 3. Determine length of time window w and use it to split evenly the original time series.

For the enrollment time series including 22 samples $X = \{x_1, x_2, \ldots, x_{22}\}$, if w is selected as 3, the original time series data is split into 7 subsets evenly–$T_1 = \{x_1, x_2, x_3\}$, $T_2 = \{x_4, x_5, x_6\}, \ldots, T_7 = \{x_{19}, x_{20}, x_{21}, x_{22}\}$. In here, the meaning of $w = 3$ is to predict the change range (interval) of enrollments of the future three years by exploiting the change range of enrollments of the past three years.

Step 4. Determine a expressed form of fuzzy granule (triangle fuzzy set, trapezoidal fuzzy set, etc.) and a parameter α, and then granulate each time window by the principle of justifiably granularity detailed in Section 3.2.

In here, the expression of fuzzy granule is selected as *trapezoidal* fuzzy set and the parameter α is selected as 1, we can obtain a series trapezoidal granules $\Omega(\alpha) = [a, m, n, b]$, where "$a$" and "$b$" is support of trapezoidal granule and "m" and "n" is core of the trapezoidal granule. The obtained trapezoidal granules at $\alpha = 1$ are reported as follows: $\Omega_1(\alpha = 1) = [13055, 13563, 13563, 13867]$, $\Omega_2(\alpha = 1) = [14696, 15311, 15311, 15460]$, $\Omega_3(\alpha = 1) = [15603, 15861, 15861, 16807]$, $\Omega_4(\alpha = 1) = [15433, 16388, 16388, 16919]$, $\Omega_5(\alpha = 1) = [15145, 15163, 15163, 15497]$, $\Omega_6(\alpha = 1) = [15984, 16859, 16859, 18150]$, $\Omega_7(\alpha = 1) = [18876, 18970, 19328, 19337]$. These formed fuzzy granules is showed also in the 4th column of Table 2.

Step 5. Determine the linguistic description of each fuzzy granules generated by step 4. For a granule Ω_j $(j = 1, 2, \ldots, 7)$, we calculate the overlap degree between it and $A_i (i = 1, 2, \ldots, 7)$ by Eq. (7), say:

$$Poss(\Omega_j, A_i) = max\{min(\Omega_j(x_k), A_i(x_k))\}, x_k \in T_i. \tag{12}$$

A_i corresponding to the maximum overlap degree is regarded as the linguistic description of the granule Ω_j.

As an example, let us determine the linguistic description of granule $\Omega_1(\alpha = 1)$ coming from the first time window and $\Omega_7(\alpha = 1)$ coming from the seventh time window, respectively. According to Eq. (12), the overlap degrees of granule $\Omega_1(\alpha = 1)$ versus $A_i (i = 1, 2, \ldots, 7)$ are $1, 0.5, 0, 0, 0, 0, 0$, respectively. Since the maximum overlap degree between $\Omega_1(\alpha = 1)$ and A_i occurs at A_1, the linguistic description of granule $\Omega_1(\alpha = 1)$ is A_1. For granule $\Omega_7(\alpha = 1)$, the overlap degrees of it versus $A_i (i = 1, 2, \ldots, 7)$ are $0, 0, 0, 0, 0.5, 1, 1$ respectively. The maximum overlap degree between $\Omega_7(\alpha = 1)$ and A_i occurs at A_6 and A_7, in this case, $0.5A_6 + 0.5A_7$ is taken regards as the linguistic description of granule $\Omega_7(\alpha = 1)$. The linguistic description of all seven granules, in turn, is reported as follows — A_1, A_3, A_3, A_4, A_3, A_4, $0.5A_6 + 0.5A_7$.

Step 6. Mine the fuzzy logical relationships between adjacent granules and then group them into the fuzzy logical relationship groups.

From 5th column of Table 2, the fuzzy logical relationships between the adjacent granules in granular time series, in turn, are $A_1 \to A_3$, $A_3 \to A_3$, $A_3 \to A_4$, $A_4 \to A_3$, $A_3 \to A_4$ and $A_4 \to 0.5A_6 + 0.5A_7$. Exploiting the Definition 3, we can obtain a completed fuzzy logical relationships groups, which is shown in Table 1.

Step 7. Perform reasoning and degranulation operation by the given rules.

For example, we use the first time window (T_1) to forecast the change range (interval) of data included in the second time window T_2, that is we exploit the change range of enrollments during year 1971 to year 1973 to forecast the change range of enrollments in the next three years (from year 1974 to year 1976). For time window T_1, the linguistic terms of the corresponding fuzzy granule is A_1. From Table 1 it can be seen that A_1 is associated with A_3, which implies the data included in T_2 time window has only *uptrend* relative to T_1 time window. Using *Rule 1*, for T_2 time window, the forecasted lower bound is equal to the midpoint of interval $u_1 = [13000, 14000]$ — 13500, and the forecasted upper bound is the midpoint of interval $u_3 = [15000, 16000]$ — 15500, i.e., the forecasted interval is [13500, 15500]. The meaning of the predicted result is that the maximum (upper bound) of enrollments from year 1974 to year 1976 is 15500 and the minimum (lower bound) of enrollments from year 1974 to year 1976 is 13500. The change range of data included in T_3 time window is forecasted by exploiting T_2 time window. The linguistic term of fuzzy granule corresponding to T_2 is A_3, we have fuzzy logical relationships groups $A_3 \to A_3, A_4$. Using *Rule 3*, for T_3 time window, the forecasted lower bound is equal to the midpoint of interval $u_3 = [15000, 16000]$ — 15500, and the forecasted upper bound is the midpoint of interval $u_4 = [16000, 17000]$ — 16500, namely, the forecasted interval is [15500, 16500] for T_3 time window. Similarly, we can obtain the predicted result of other time windows. All forecasted results are also shown in the last column of Table 2. Compared with the 3th column and the last column in Table 2, the predicted results that is obtained by our constructed granular model are intuitively appealing.

5.3. The TAIEX time series

The TAIEX time series, including 242 observations, recorded the daily value of TAIEX from January 1, 2000 to December 30, 2000. The time series is used to illustrate the impacts of parameters of

Table 1
The fuzzy logical relationships groups for fuzzy granular model of enrollment time series ($w = 3, \alpha = 1$).

Group no.	Fuzzy logical relationship groups
Group 1	$A_1 \to A_3$
Group 2	$A_3 \to A_3, A_4$
Group 3	$A_4 \to A_3, 0.5A_6 + 0.5A_7$

Table 2
The forecasted results of fuzzy granular model of enrollment time series ($w = 3, \alpha = 1$).

Time window	Year: enrollments	Actual change range (interval)	the constructed trapezoidal granules	Linguistic description of granules	Predicted linguistic description	Predicted change range (interval)
T_1	1971 : 13055 1972 : 13563 1973 : 13867	$[13055, 13867]$	$[13055, 13563, 13563, 13867]$	A_1	No forecasted	No forecasted
T_2	1974 : 14696 1975 : 15460 1976 : 15311	$[14696, 15460]$	$[14696, 15311, 15311, 15460]$	A_3	$\uparrow A_3$	[13500,15500]
T_3	1977 : 15603 1978 : 15861 1979 : 16897	$[15603, 16807]$	$[15603, 15861, 15861, 16807]$	A_3	$\uparrow A_4$ $\downarrow A_3$	[15500,16500]
T_4	1980 : 16919 1981 : 16388 1982 : 15433	$[15433, 16919]$	$[15433, 16388, 16388, 16919]$	A_4	$\uparrow A_4$ $\downarrow A_3$	[15500,16500]
T_5	1983 : 15497 1984 : 15145 1985 : 15163	$[15145, 15497]$	$[15145, 15163, 15163, 15497]$	A_3	$\uparrow 0.5A_6 + 0.5A_7$ $\downarrow A_3$	[15500,19000]
T_6	1986 : 15984 1987 : 16859 1988 : 18150	$[15984, 18150]$	$[15984, 16859, 16859, 18150]$	A_4	$\uparrow A_4$ $\downarrow A_3$	[15500,16500]
T_7	1989 : 18970 1990 : 19328 1991 : 19337 1992 : 18876	$[18876, 19337]$	$[18876, 18970, 19328, 19337]$	$0.5A_6 + 0.5A_7$	$\uparrow 0.5A_6 + 0.5A_7$ $\downarrow A_3$	[15500,19000]

"↓" represent that the data in T_i has *downtrend* versus T_{i-1}. "↑" represent that the data of T_i has *uptrend* versus T_{i-1}.

our proposed modeling method on the performance of the constructed fuzzy granular model of time series. In what follows, the setup and results of experiments for TAIEX time series is discussed first, and then the analysis of experimental results is presented.

5.3.1. Setup and results of experiments

The proposed modeling method have two adjustable parameters, i.e. the length of time window w and the level of granularity α. The goal of experiments is to investigate the impact on the performance of the constructed fuzzy granular model being brought by the two parameters. Let $D_{min} = 4614.63$ and $D_{max} = 10202.02$, we define the universe of discourse $U = [4600, 10300]$ and evenly divide it into 57 intervals with the same length, that is $u_1 = [4600, 4700]$, $u_2 = [4700, 4800], \ldots, u_{57} = [10200, 10300]$. The same definition form of fuzzy sets with Huarng's model (Huarng & Yu, 2005) is adopted in here. Information granules is defined as *trapezoidal* fuzzy sets.

Next, two experiments are carried out separately. One experiment, which is called *w-experiments*, is used to test the impact of length of time windows w on the performance of the constructed fuzzy granular model. The other experiment, which is called *α-experiments*, is used to test the impact of the level of granularity α on the performance of the model. For all experiments, the TAIEX time series is split into two subset—the training subset and the test subset. The former includes the former 195 observations of TAIEX time series (data from 2000/1/4 to 2000/10/31), which is used to construct fuzzy granular model and evaluate the modeling performance. The latter includes the latter 47 observations of TAIEX time series (data from 2000/11/1 to 2000/12/30), which is used to perform interval prediction and evaluate the prediction performance of model.

For *w-experiments*, the experiment scheme is designed as follows: (1) fix the value of $\alpha = 1$; (2) let the values of the length of time window w be respectively be $3, 5, 7$ and 8; (3) for each w, which is used to divide evenly the training subset into p_1 time windows. These time windows are used to construct the fuzzy granular model of TAIEX time series according to Section 4 and quantify the modeling performance by computing the performance index

Table 3
The results of w–experiments at the level of granularity $\alpha = 1$.

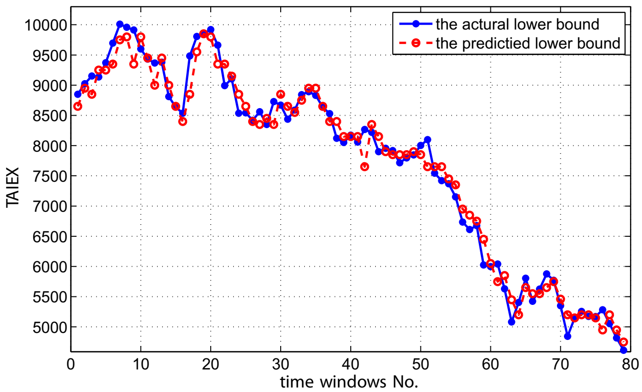
w	m	Modeling performance		Prediction performance	
		p_1	P_1 (%)	p_2	P_2 (%)
3	39	65	12.95	15	12.57
5	24	39	11.66	9	11.23
7	17	28	8.56	6	7.85
8	15	24	8.25	6	7.18

(P) (see subSection 5.1); (4) the test subset is divided evenly into p_2 time windows by use same w, and then build fuzzy granules for each time window formed by the step and obtain the linguistic description of them (see the 5th step proposed method); (5) exploit the fuzzy granular model which is constructed by the step 3 to perform interval prediction (see the 6th step proposed method) and quantify the prediction performance by computing P. All results of experiments are reported in Table 3, where "w" represents the length of time windows, "m" represents the number of fuzzy relation groups included in the corresponding fuzzy granular model, "p_1" is the number of time windows generated by using w divided the training subset and "p_2" is the number of time windows generated by using w divided the test subset. Besides, in table, P_1 and P_2 represent respectively the performance index for the modeling performance and the prediction performance of model. Fig. 6 shows a comparison of predicted results for TAIEX time series in case when $w = 3$ and $\alpha = 1$.

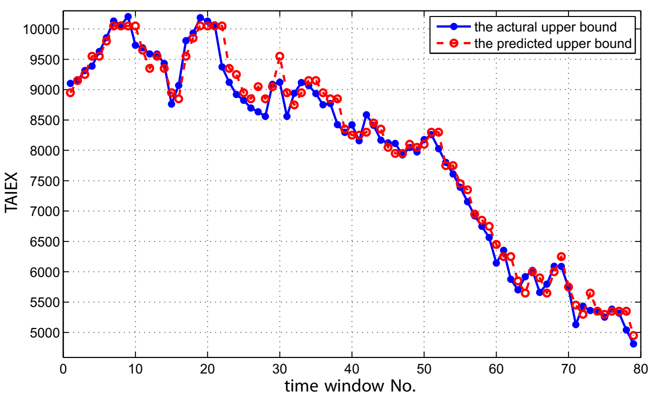
In a similar way we can carry out *α-experiments*. Here we first fix the value of $w = 8$ and let the value of α be respectively 0.5, 1, 1.7, 3 and 5. The next, experiments are carried out according to the 3rd–5th step mentioned above. All results of experiments are presented in Table 4. Fig. 7 shows a comparison of predicted results for TAIEX time series in case of $w = 8$ and $\alpha = 1.7$.

5.3.2. Analysis of experimental results

Now we discuss mainly the impact of two parameters of the proposed modeling method on the accuracy of the constructed fuzzy granular model.



(a) Comparison of the predicted lower bounds and the actual lower bounds



(b) Comparison of the predicted upper bounds and the actual bounds

Fig. 6. Comparison plots of the predicted intervals and the actual intervals for TAIEX time series ($w = 3, \alpha = 1$).

Table 4
The results of α–experiments at the length of time window $w = 8$.

α	m	Modeling performance	Prediction performance
		P_1 (%)	P_2 (%)
0.5	26	8.25	7.18
1	26	8.25	7.18
1.7	26	8.48	7.42
3	24	7.93	6.85
5	24	7.93	6.85

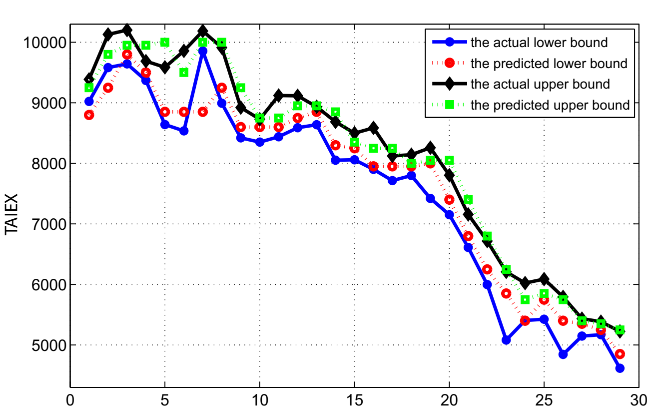


Fig. 7. A comparison plot of the predicted intervals and the actual intervals for TAIEX time series ($w = 8, \alpha = 1.7$).

1) *The impact of the length of time window w.* As the show of Table 3, we can see that the accuracy of the constructed fuzzy granular model increase with decrement of w — the value of performance index of modeling and prediction of model (namely P_1 and P_2) are respectively 8.25% and 7.18% in case of $w = 8$, and whereas 12.95% and 12.57% in case of $w = 3$. Additional, we notice that the smaller length of time window can reach the more number of fuzzy logical relation groups (FLRGs for short), which results in higher accuracy of prediction — the number of the formed FLRGs (m) is 39 in case of $w = 3$, 15 in case of $w = 8$. The findings can be interpreted easily: the decreasing length of time windows give more granules (time windows), and the more the number of granules is, the more fine the linguistic description of time series (the more number of the formed FLGRs), thus the prediction has higher accuracy. When the length of time windows w reaches 1, in this case, each point in time series is regarded as one granule (viz. each time window includes only one point), the constructed fuzzy granular model degenerates into the classical Chen's model (Chen, 1996) and the interval prediction degenerates into single point prediction.
2) *The impact of the level of granularity α.* From Table 4, we can see that the level of granularity α has important effects on accuracy of prediction — the value of P_1 and P_2 get higher with the increasing of the value of α however the value of it becomes reducing when going beyond a certain value of α, i.e. 1.7. In other words, the prediction accuracy of model is not continuously increase with the increasing value of α. The explanation for this is that when the value of α is lower, the granules formed by the principle of justifiable granularity include more data, and a small portion of these data is redundant (i.e., they have no important effect on the future trends of time series) and cause the lost of specificity of granules, which reduces accuracy of prediction. But the value of α is larger, the formed granules has include less data (i.e. the formed granules has strong specificity), which not sufficient to represent data included in time window so that the prediction has lower accuracy. Obviously, a justifiable granularity is crucial to the modeling of time series.
3) *The impact of other factors.* Actually, our constructed fuzzy granular model is the augmented version of Chen's model (Chen, 1996). We extend the Chen's model to the level of granular, and thus the reasonable partition of universe of discourse U is important to accuracy of granular model. What more, our proposed modeling method transforms single point operation in Chen's model into granular operation over time windows, which can reduce the computational overhead of modeling. The clear advantage of the proposed modeling method is more adaptable for modeling of long-scale time series.

6. Conclusions

In this paper, a new modeling method of time series is proposed, in which concept of information granules and granular computing is integrated into the classical Chen's model (Chen, 1996). It segments time series into a series of time windows first and then exploits the justified information granularity principle to build fuzzy granules for each time window, and then the model is constructed on the level of granules. The constructed fuzzy granular model can realize the forecasting of intervals of time series, which is more in accordance with human cognition and can provide basis for some decision-making. Besides, the method can also simplify the forecasting problem (from the traditional single point prediction to the intervals prediction) and reduce computational overhead of modeling, which is more adaptable for modeling of long-scale time series. Two benchmark time series are considered to validate the feasibility and effectiveness of the proposed method, whose results can draw some conclusions as follows: (i) the length of time window impacts significantly accuracy of the constructed fuzzy granular model. When the length of time window is 1, the constructed fuzzy granular model reduces to the Chen's model (Chen, 1996); (ii) the justifiable level of granularity is also crucial to the prediction accuracy; (iii) similarly to the "traditional" fuzzy time series modeling and prediction, the partition of universe of discourse is important to accuracy of fuzzy granular model.

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