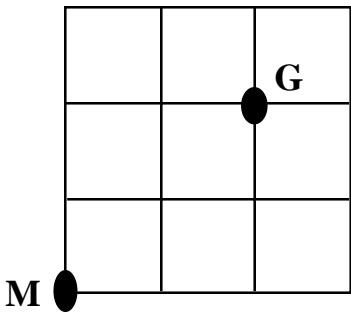


MathStars

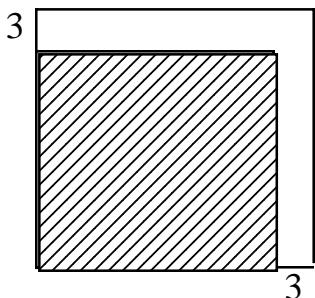
a problem solving newsletter

Vol. 7 No. 1

★★1. The miner located at point M has hidden his gold at point G. If the miner can only move up and to the right along the lines, how many different paths can he take from M to G?

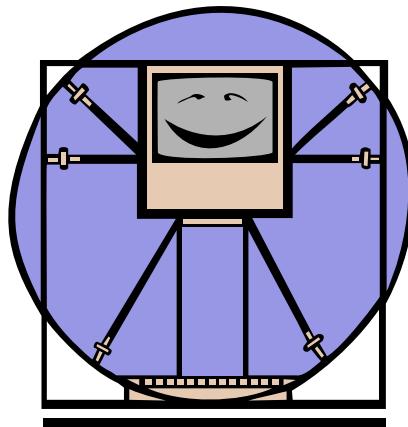


★★★2. If two sides of a square field were increased by three feet, as shown in the diagram, the area of the field would increase by 129 square feet. Find the area of the original field.



★★3. Mary is moving and wants photos to remember her six friends. If pictures are taken so there is a shot of every possible pair of friends - including Mary, how many pictures will be taken? Would one roll of 24 exposures be enough?

★4. GameKid Electronics doubled their sales of video games from 1985 to 1990, and then doubled sales again from 1990 to 1995. If they sold 1,842,100 video games in 1995, how many did they sell in 1985?



Strategy of the Month

Someone said, "A picture is worth a thousand words." Turning the words of a problem into a picture or a diagram can help you "see" the problem. By using the part of your brain that visualizes a situation or object, you may see relationships or information that helps you solve the problem. When someone tells you a story, try turning the words into a motion picture or a cartoon. When reading a description, try "seeing it in your mind's eye." If you can do these things, this strategy may be for you! Try using a picture or make a diagram to solve this problem:

Ten people met at a party. They all exchanged handshakes. How many handshakes were exchanged?

MathStars Home Hints

Every year you grow and change in many different ways. Get someone to help you measure and record these data about yourself. Be sure to save the information because we will measure again in two months!

How tall are you? _____

How much do you weigh? _____

What is the circumference of your head?

★★★★★ 5. Using exactly four 4's and any operations $[+, -, \times, \div, (), \sqrt{\quad}, !]$, find an expression to equal each of the following:

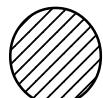
$$1 = \underline{\hspace{2cm}} \quad 4 = \underline{\hspace{2cm}}$$

$$2 = \underline{\hspace{2cm}} \quad 5 = \underline{\hspace{2cm}}$$

3 = Look for more of these
in later issues!!

★★6. Sandy dropped a basketball from the top of her Mom's office building which is 72 meters tall. She discovered that the ball bounced back 36 m on the first bounce and 18 m on the second bounce. If this pattern continues, how high will the ball be on the sixth bounce?

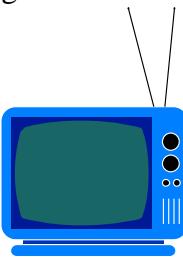
★★7. A bag contains red, blue and orange marbles. If the probability of randomly selecting a red marble is 0.4 and the probability of selecting a red or blue marble is 0.9, what is the probability of selecting a red or orange marble?



★★★8. Ben, Boykin, Bill and Boyd each like to watch a different type of TV show. The shows are on at 3, 5, 7, and 8 PM.

Use the clues below to find out the type of program that each boy watches and the time it is on.

- 1) Boykin likes science fiction.
- 2) Ben doesn't like wrestling, but his friend does.
- 3) Boykin's favorite show comes on later than the ones Boyd and Ben watch.
- 4) The news is on at 5:00, but that is not what Bill watches.
- 5) The movie is on at 8:00.
- 6) Boyd enjoys wrestling which is on before Ben's favorite show.



Program	Time
---------	------

Ben	<u> </u>	<u> </u>
Boykin	<u> </u>	<u> </u>
Bill	<u> </u>	<u> </u>
Boyd	<u> </u>	<u> </u>

Setting Personal Goals

Problem solving is what you do when you don't know what to do. Being a good problem solver will help you be ready to live and work in our changing world. Computers can do computations but people must tell the computers what to do. Good problem solvers know how to make plans and use many different strategies in carrying out their plans. They use all of their past experiences to help them in new situations. We learn to swim by getting in the water; we learn to be good problem solvers by solving problems!

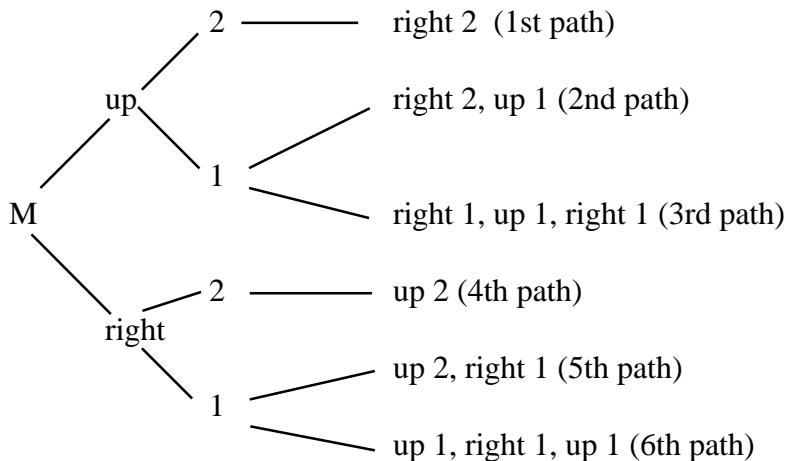
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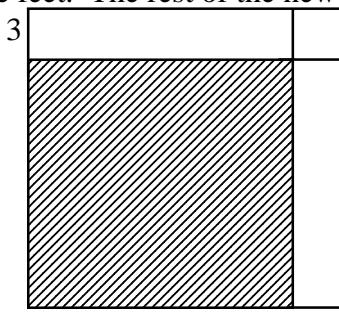
As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

1. (six paths) The easiest approach to this problem is a tree diagram showing the possibilities:



2.(400 sq.ft.) The corner square is 3 by 3 or nine square feet. The rest of the new or added space is 120 square feet. Each rectangle is 60 sq. ft. with dimensions 3 by 20. This gives the square a side of 20 feet. With a 20 by 20 square we have total area 400 square feet.



3

3.(21 pictures; one roll is sufficient) Mary will have her picture taken with each of six friends (six photos). Her first friend will pose with each of five friends (five more shots). The next friend will pose with the remaining four people (four more shots). Then three, two and finally one, for a grand total of $6 + 5 + 4 + 3 + 2 + 1 = 21$. Or AB, AC, AD, AE, AF, AG, BC, BD, BE, BF, BG, CD, CE, CF, CG, DE, DF, DG, EF, EG, FG which can be represented by the sides and diagonals of a heptagon.



4. (**460, 525**) If sales were 1,842,100 in 1995, then there were 921,050 in 1990 and 460,525 in 1985.

This is an excellent example of a working backwards strategy or moving from the "answer" to the question.

5. (**Solutions will vary**) You may wish to begin a "Four Fours" bulletin board to display student solutions. Each subsequent newsletter will contain a "four fours" problem for increasingly larger numbers.

6. (**1.125m**) The solution can be obtained by a series of divisions i.e. $72 \div 2 = 36$; $36 \div 2 = 18$ etc. or the expression $72(.5)^6 = 1.125$

7. (**0.5**) If $P(\text{red or blue}) = 0.9$, then $P(\text{orange}) = 0.1$. If $P(\text{red}) = 0.4$ then $P(\text{red or orange}) = 0.4 + 0.1$ or 0.5.

8.	Program	Time
Ben	News	5:00
Boykin	Science Fiction	7:00
Bill	Movie	8:00
Boyd	Wrestling	3:00

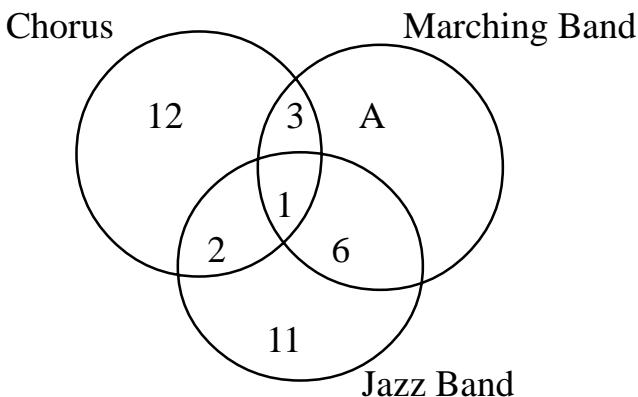


a problem solving newsletter

Vol. 7 No. 2

★★1. K, L, M, and N are integers and $K \times L = 6$; $L \times M = 20$; $M \times N = 15$; and $K \times N = 9$. What is the value of : $K \times L \times M \times N$?

★ ★ ★ 2. The Venn diagram below shows the number of Johnson Middle School students who are in jazz band, marching band and/or chorus.



How many students take exactly two music classes?

If there are 22 more students in the marching band than in the jazz band, how many students are in only the marching band?

★ ★ ★ 3. In any year that is not a leap year, what will be the date of the 100th day?

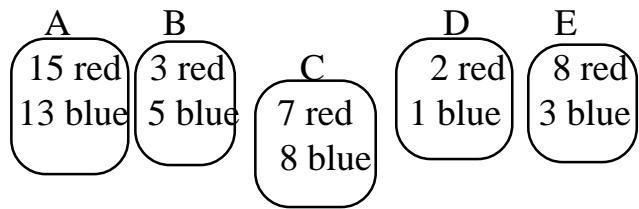
★ ★ 4. Determine the value for each letter that will make the following statements true:

$$(a) \text{ BE} \times \text{BE} = \text{BEE} \quad (b) \text{ AA} \times \text{AA} = \text{ADA}$$

A = B = C = D = E =

★ ★ ★ 5. Fran, Jessie, Chris, Joe, Terry and Sue went bowling on Wednesday. The average score for the first game was 148. As they were finishing, Neil joined them and scored 134. What is the average for the group now?

★ ★ ★ 6. The bags below contain red and blue marbles. A marble is to be drawn at random from one bag. For which bag is $P(\text{blue}) > P(\text{red})$ i.e. the probability of drawing blue greater than the probability of drawing red?



Strategy of the Month

*Your brain is an organizer. It organizes information as it stores that information. When a problem involves many pieces of information, your brain will have an easier time sorting through it if you make an organized list. A list helps you be sure you have thought of all of the possibilities without repeating any of them. Like drawing a picture or making a diagram, making an organized list helps your brain "see" the problem clearly and find a solution. Try **making an organized list** to solve this problem:*

You receive a penny on January 1st, two cents the next day, four cents the next and so on, doubling every day. How long will it take to be a millionaire? Suppose you started with a nickel and doubled, how long before you were worth a million dollars?

MathStars Home Hints

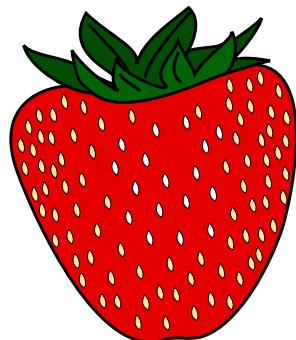
Sometimes the hardest part of solving a problem is just getting started. Having some steps to follow may help you.

1. Understand the information in the problem and what you are trying to find out.
2. Try a strategy you think might help you solve the problem.
3. Find the solution using that strategy or try another way until you solve the problem.
4. Check back to make certain your answer makes sense.

★★7. A real estate agent sold a house for \$230,000. The broker received a 4% commission and the agent received 60% of that commission. How much did the agent receive? _____



★★★ 8. Three pickers can harvest 30 gallons of strawberries in four hours. At this rate how long will it take five pickers to harvest 75 gallons of strawberries? _____



★★★ ★9. Using exactly four 4's and any operations [+,-, x, ÷, (), $\sqrt{}$, !], find an expression to equal each of the following:

$$6 = \underline{\hspace{2cm}}$$

$$7 = \underline{\hspace{2cm}}$$

$$8 = \underline{\hspace{2cm}}$$

$$9 = \underline{\hspace{2cm}}$$

$$10 = \underline{\hspace{2cm}}$$

★★★ 10. Roberto works for two hours. Lee works $2\frac{3}{4}$ as long as Roberto. Shaqua works $1\frac{1}{4}$ hours less than Jeff. Jeff works $2\frac{1}{2}$ times as long as Lee. How long does each person work?

Roberto _____

Lee _____

Shaqua _____

Jeff _____

Setting Personal Goals

Being able to ask good questions will help you in many ways. Use these to solve problems:

- What information do I know?
- What else do I need to find out?
- What question am I trying to answer?
- Have I missed anything?
- Does my answer make sense?

Set the goal of asking good questions!



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Discussion of the problems...

1. **(90)** $K \times L \times M \times N = (K \times L) \times (M \times N) = 6 \times 15 = 90$

2. **(11)** The students taking more than one course number $1 + 2 + 3 + 6 = 12$ but one student takes all three courses so 11 take two.

(32) The jazz band has 20 students; $(A + 10)$ students are in the marching band. If the marching band has 22 more students than the jazz band, then $(A + 10) = 22 + 20$ or $A = 32$.

3. **(April 10)**

4. **(A = 1, B = 1, D = 2, E = 0)** Calculating, $10 \times 10 = 100$ and $11 \times 11 = 121$.

5. **(146)** The first average was their total (T) divided by 6 or $148 = T \div 6$. The total then was 6×148 or 888. Adding Neil's score or $134 + 888 = 1022$. Computing, the new average then becomes $1022 \div 7$ or 146.

6. **(bag B)** Only two bags, B and C, have a greater probability of drawing blue than red. For bag B, $P(\text{blue}) = 5/8$ or 0.625; for bag C, $P(\text{blue}) = 8/15$ or 0.533.

7. **(\$5520)** The commission is computed as 60% of 4% of \$230,000 or $0.60 \times (0.04 \times 230,000) = 0.60 \times (9200) = 5520$.

8. **(six hours)** Three pickers harvest 30 gallons in four hours or each picker harvests 10 gallons in four hours. Ten gallons in four hours translates to 2.5 gallons each hour per picker. At this rate five pickers will harvest 12.5 gallons each hour. $75 \div 12.5 = 6$, or six hours are needed by five pickers to harvest 75 gallons of berries.



Commentary for Teachers

Vol. 7 No. 2

9. (Answers will vary)

Some sample responses:

$$6 = 4! \div 4 \times 4 \div 4; \quad 7 = 4 + 4 - 4 \div 4; \quad 8 = 4 \times 4 - (4 + 4); \quad 9 = 4 \div 4 + 4 + 4;$$

$$10 = 4 \times 4 - (4 + \sqrt{4})$$

10. (Roberto -- 2 hours; Lee -- 5.5 hours; Shaqua -- 12.5 hours; Jeff -- 13.75 hours)

MathStars

a problem solving newsletter

Vol. 7 No. 3

- ★1 What is the smallest integer that will make this expression true? The largest?

$$4 \times \square + 9 > 37$$

- ★2. If Tony had 18 fewer baseball cards he would have half as many as he has now.
How many cards does he have now?

- ★ ★ ★3. Three friends buy a motorcycle together. Fred pays half the cost while Georgio pays twice as much as Hal. What fraction of the bike does Georgio own?



- ★★4. In the problems below each letter stands for a different digit.

$$\begin{array}{r} ZX \\ + XZ \\ \hline YZY \end{array} \quad \begin{array}{r} LM \\ \times N \\ \hline LLL \end{array}$$

$$L = \underline{\hspace{1cm}} \quad M = \underline{\hspace{1cm}} \quad N = \underline{\hspace{1cm}}$$

$$X = \underline{\hspace{1cm}} \quad Y = \underline{\hspace{1cm}} \quad Z = \underline{\hspace{1cm}}$$

- ★ ★ ★ ★5. Using exactly four 4's and any operations [+,-, x, ÷, 0, $\sqrt{}$, !], find an expression to equal each of the following:

$$11 = \underline{\hspace{2cm}} \quad 12 = \underline{\hspace{2cm}}$$

$$13 = \underline{\hspace{2cm}} \quad 14 = \underline{\hspace{2cm}}$$

$$15 = \underline{\hspace{2cm}}$$

- ★ ★ ★ ★6. If this pattern continues, in which column will 100 appear?

A	B	C	D	E	F	G
1		2		3		4
	7		6		5	
8		9		10		11
	14		13		12	
15				

Strategy of the Month

Being a problem solver is something like being a detective! A detective has to solve crimes by guessing what happened and checking the guess to see if it fits the situation. For some problems, your best strategy may be to make a guess and then check to see if your answer fits the problem. If not, decide if your guess was too high or too low and then make a second "guesstimate." A good detective keeps records (usually some kind of chart) to help see any patterns and to narrow down the possibilities. You should do this too. The results of incorrect guesses can give you valuable clues to the correct solution. Guess and then check the solution to :

Jo, Ro & Bo have less than 20 tokens. Ro has more than Mo and Mo has more than Bo. Ro gives Bo three tokens and Mo gives Bo two. Now they each have the same number. What did they start with?

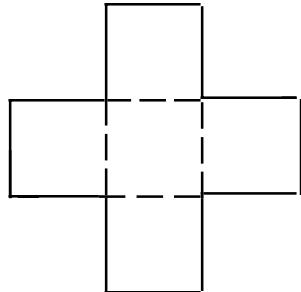
MathStars Home Hints

*Memorizing number facts will save you time.
Flash cards are one way to learn new facts, but
you also might try these ideas:*

- play dice or card games in which you need to add, subtract, multiply, or divide.
- learn new facts using ones you already know ($7+7=14$ so $7+8=15$).
- learn facts that are related to each other ($7 \times 6=42$, $6 \times 7=42$, $42 \div 6=7$, $42 \div 7=6$).
- make a list of the facts you need to memorize and learn 5 new facts each week.
- Spend 5-10 minutes every day practicing facts.

★ ★ ★ 7. On Monday Frannie Farmer put 20 pumpkins in her produce stand and some of them were sold. On Tuesday she doubled the number that were unsold and sold the same amount as Monday. On Wednesday she tripled the number that were unsold and again sold the same number as Monday, leaving none left over. How many pumpkins did she sell each day?

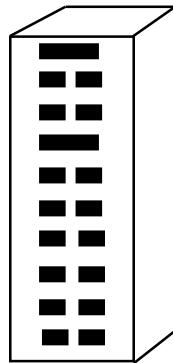
★★★ 8. A piece of posterboard is cut into the shape below consisting of five congruent squares with a total area of 180 square centimeters.



If the shape is folded along the dotted lines to make an open box, what is its volume in cubic centimeters? _____

★★ 9. An auto mechanic earns \$14.75 per hour for a 40 hour week. She pays 25% of her earnings in taxes and 5.5% for Social Security. What is her net or take-home pay? _____

★ 10. Jim works in a ten story office building. He gets on at the fourth floor and rides to the top floor. He then goes down seven floors, rides up four realizes he is lost and rides down five more floors. Where is Jim now? _____



Setting Personal Goals

Communicating mathematically means that you are able to share your ideas and understandings with others orally and in writing. Because there is a strong link between language and the way we understand ideas, you should take part in discussions, ask questions when you do not understand, and think about how you would explain to someone else the steps you use in solving problems.

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Discussion of the problems...

1. **(8 , none)** Guess and check is one strategy students may employ remembering the definition of integer. In pursuing a largest integer, students should note that all integers greater than seven will satisfy the expression.
2. **(36)** If 18 fewer cards would leave Tony with half the number he has now, then half is 18. He must have 36 now.
3. **(1/3)** Georgio and Hal own half the bike. Their share can be viewed as $1/2 = 3/6 = 2/6 + 1/6$. The share that Georgio owns ($2/6$) is twice Hal's ($1/6$).
4. **(L = 3, M = 7, N = 9, X = 9, Y = 1, Z = 2)** Since Y is the "carry" for Z + X, the largest and only possible value for Y is one. Therefore, X + Z must be 11. The possibilities are 9 + 2, 8 + 3, 7 + 4, 6 + 5 the second column is 11 + 1 or 12 which makes Z = 2 and X = 9.
5. **(Answers will vary.)**
6. **(C)** Dividing by seven (A-G), $100 \div 7 = 14$ with remainder 2. Multiples of seven are in Column B. From B, numbers continue left and down to the next row. Proceeding, 99 will be in column A and 100 in column C.
7. **(12)** Another good problem for the guess and check strategy.
8. **(216 cc)** Each of the congruent squares has area $1/5(180)$ or 36 square centimeters. The edge measure is therefore 6 centimeters. The volume of the box becomes $6 \times 6 \times 6$ or 216 cc.



9. (**\$410.05**) Gross salary $40 \times \$14.75 = \590.00 while deductions for taxes and Social Security total $\$179.95$ $\{25\% \text{ of } \$590.00 + 5.5\% \text{ of } \$590.00 = 30.5\% \text{ of } \$590.00 = \$147.50 + \$32.45\}$

10. (**2nd floor**) Poor Jim! Using integers students may write: $4 + 6 - 7 + 4 - 5 =$ present location or 2.

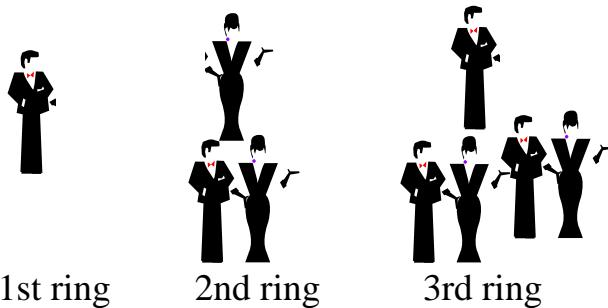
MathStars

a problem solving newsletter

Vol. 7 No. 4

★★★★★ 1. The evil warlord, Medar, has locked the beautiful (and brainy) Princess Almedia in a dungeon. To be released, she must find the first perfect square number greater than 100 whose digits do not sum to a perfect square! Can you help her find the number?

★★★ 2. The Murphys love to give parties. Last Friday they gave a party and the doorbell rang 15 times. At the first ring, one guest arrived. Each time the doorbell rang after that, two more guests arrived than the time before.

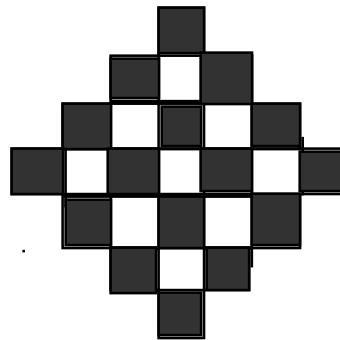


On Saturday they had another party. At the first ring of the doorbell a single guest arrived, at the second ring two guests appeared, at the third ring three guests and so on. If the doorbell rang 20 times Saturday night, how many guests attended? Was this party bigger than Friday's? How do you know?

★★ 3. Steve, Adam, Jose and Don are going camping. Steve brought \$5 worth of snacks. Adam bought hot dogs and buns for \$4, Jose spent \$8 on the s'mores and other goodies while Don bought sodas for \$3. If expenses are to be shared equally, how much should each pay the other?

★★★★★ 4. A hiking club of 34 adults and 27 children are on a mountain excursion when they come upon a swift and deep river. A canoe which can hold one adult or two children rests on the riverbank. How many trips will it take to get the party across the river if everyone can row the canoe?

★★★ 5. How many squares are in this figure?



Strategy of the Month

Noticing patterns helps people solve problems at home, at work, and especially in math class! Math has been called "the study of patterns," so it makes sense to look for a pattern when you are trying to solve a problem. Recognizing patterns helps you to see how things are organized and to make predictions. If you think you see a pattern, try several examples to see if using the pattern will fit the problem situation. Looking for patterns is helpful to use along with other strategies such as make a list or guess and check. How can finding a pattern help you solve this problem?

If $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, the products have 9, 7, 1 respectively in the one's place. If you compute 3^{15} , what number will be in one's place?

Is there a similar pattern for $4^2, 4^3, 4^4 \dots$?

MathStars Home Hints

Set aside a special time each day to study. This should be a time to do homework, to review, or to do extra reading. Be organized and have a special place in which to work. This place needs to have a good light and to be a place where you can concentrate. Some people like to study with quiet music; others like to sit at the kitchen table. You need to find what works for you!

Remember that when you are reviewing or working on solving problems it may help to study in a group.

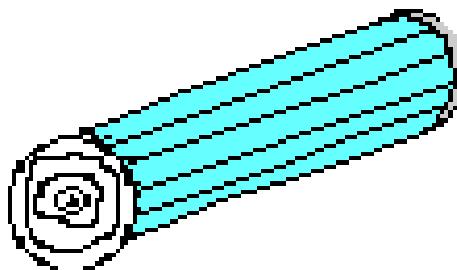
★★★★★ 6. Using exactly four 4's and any operations [$+$, $-$, \times , \div , $()$, $\sqrt{\quad}$, $!$], find an expression to equal each of the following:

$$16 = \underline{\hspace{2cm}} \quad 17 = \underline{\hspace{2cm}}$$

$$18 = \underline{\hspace{2cm}} \quad 19 = \underline{\hspace{2cm}}$$

$$20 = \underline{\hspace{2cm}}$$

★ 7. How many cuts are needed to cut a log into four pieces? into eight pieces? into 20 pieces?



★ 8. Write the next five numbers in this pattern:
64, 32, 16, , , , ,

9. Find the mystery numbers:

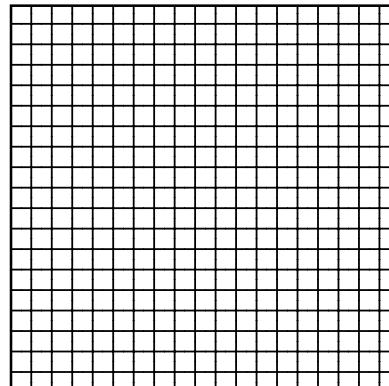
- (a) It is a two-digit prime;
its digits sum to eight;
the tens digit is greater than the ones digit;
the product of the digits is one less than 4^2 .

The number is

- (b) Two numbers whose sum is 111 and whose difference is 25.

The numbers are and

★★ 10. Given the points A(3,1), B(3,6), C(7,1), what are the coordinates of D if ABCD is a rectangle?



Setting Personal Goals

If your goal is to become a more responsible student, it means that you

- actively participate in class.
- complete your assignments.
- have everything you need in class.
- ask for help when you do not understand.
- be willing to investigate new ideas.



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Discussion of the problems...

1. **(256)** The first perfect square after 100 is 121, (11^2), but the sum of its digits is 2². Continuing, the first square that fits the criteria is 256, (16^2), and the sum of its digits is 13, definitely not a perfect square.

2. **(210; no; 225 > 210)** Friday $1 + 3 + 5 + \dots + 29 = 225$ Saturday $1 + 2 + 3 + \dots + 20 = 210$ This might be a good place to introduce little Gauss and his technique for summing an arithmetic progression.

3. **(Adam owes Jose \$1 and Don owes him) \$2** The total amount spent was \$20. Each person's share is therefore \$5. Steve spent that amount and owes no one. Jose spent \$3 more than his share. Adam spent only \$4 and Don spent only \$3, thus they owe Jose \$1 and \$2, respectively.

4. **(187 trips)** Hint: two children can row over but one must return to allow an adult to cross.

5. **(42)** 25 - 1x1 squares, 12 - 2x2 squares, 5 - 3x3 squares.

6. **(Answers will vary.)**

7. **(3, 7, 19)** One less cut than the number of pieces required.

8. **(8, 4, 2, 1, .5)**

9. **(53; 43 and 68)** (a) Students should follow the clues and eliminate those primes which do not fit.
(b) A guess and check strategy can be employed for this problem.

10. **(D (7,6))** Students should use the definition of a rectangle, and the grid to determine the fourth vertex. Establishing right angles and/or equal, opposite sides will enable them to solve the problem.



a problem solving newsletter

Vol. 7 No. 5

- ★★★ 1. Find three different ways to fill in operations in the boxes below to make the equations true.

$$6 \square 1 \square 2 \square 2 = 5$$

$$6 \square 1 \square 2 \square 2 = 5$$

$$6 \square 1 \square 2 \square 2 = 5$$

- ★★★ 2. Which number(s) between 50 and 60 can be expressed both as the sum of two consecutive integers and the sum of three consecutive integers?
-

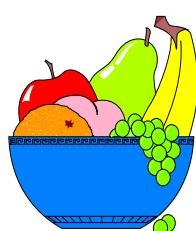
- ★★★★ 3. Using exactly four 4's and any operations [+,-, x, ÷, (), √, !], find an expression to equal each of the following:

$$21 = \underline{\hspace{2cm}} \quad 22 = \underline{\hspace{2cm}}$$

$$23 = \underline{\hspace{2cm}} \quad 24 = \underline{\hspace{2cm}}$$

$$25 = \underline{\hspace{2cm}}$$

- ★★ 4. If two apples and three bananas cost \$0.51, while four apples and seven bananas cost \$1.11, what is the cost of one apple and five bananas?
-



- ★★ 5. Katherine's Closet is having a red dot sale - "Take an extra $\frac{1}{3}$ off the already reduced price!!" When Jennifer was about to buy a dress that had been reduced from \$78 to \$52 the clerk advised her to wait a day when a new sale would announce 50% off the original price. Should Jennifer wait or buy the dress now? Why?



- ★ 6. Name the mystery number.

- less than 11^2
- greater than 4^3
- a prime number
- the sum of the digits is 11

Strategy of the Month

Sometimes mathematical ideas are hard to think about without something to look at or to move around. Drawing a picture or using objects or models helps your brain "see" the details, organize the information, and carry out the action in the problem. Beans, pennies, toothpicks, pebbles, or cubes are good manipulatives to help you model a problem. You can use objects as you guess and check or look for patterns. Try **using objects** to help you solve this problem: Place the first six counting numbers in the circles so that the sum on each side of the triangle is nine. Is it possible to use the numbers four to nine to make the sum of each side the same?

MathStars Home Hints

Remember when you had "Show and Tell" in kindergarten? Now you have a great deal to share in mathematics. Talk to the folks at home about what you are learning. Show them your papers and tell them about what is happening in your math class. Let them see that you are doing problems in class similar to these. Each week choose an assignment that you are proud of and display it somewhere in your house.

★★ 7.

Justin's Car Rental

\$30 a day + 8¢ a mile

OR

\$200 a week + 8¢ a mile

Kelly's Car Rental

\$25 a day + 10¢ a mile

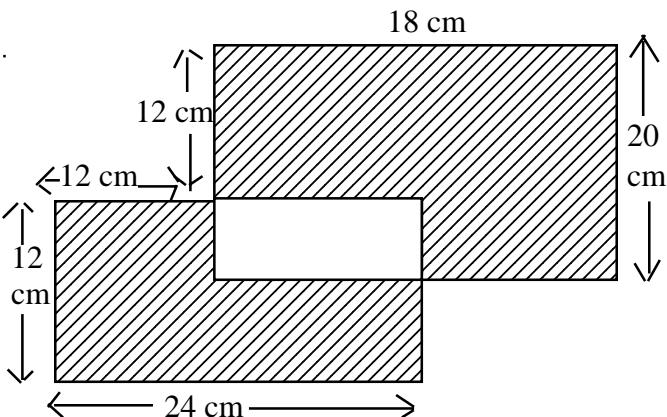
OR

\$140 a week + 10¢ a mile

You are taking an eight-day, 3,500 mile trip. Which car rental will give you the best deal? How do you know?

★★★★ 8. Twenty one people tied for first place in a contest and were eligible for the grand prize. The contest manager invited all the finalists to sit at a large round table where he devised a plan to choose the winner. He counted each person around the table and every other person was sent away. (The first person stayed, the second was sent away, the third person stayed, the next was sent away, and so on) He continued in this manner until only one person was left. If you were a finalist where should you sit to win the grand prize?

★★ 9. Find the area of the shaded region.



★★ 10. The Scenic View Tour Company offers tours of the Rainbow Gardens Preserve every weekday. The Magnolia Tour leaves every 15 minutes and takes $2\frac{1}{2}$ hours to complete. It passes through Crystal Falls halfway through the tour. The Camelia Tour also leaves every 15 minutes and takes one hour to complete. It passes Crystal Falls $\frac{1}{4}$ of the way through the tour. If Fredric takes the Magnolia Tour at 2:30 PM, when should Ricardo start his tour to arrive at Crystal Falls at the same time as Fredric?

Setting Personal Goals

Mathematics is all around us. We use it every day in personal living and in all of our school work. When we read graphs in social studies, gather and use data in science investigations, or count in music or physical education, we are using mathematics. We make connections in our math classes also; for example, measurement skills help us in solving many geometry problems and classification skills help us in organizing data. We use computation in many different situations. You will become a stronger mathematics student by making connections.



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As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

1. Answers will vary. Here are four solutions: $6 \times 1 - 2 \div 2$; $6 - 1 \times 2 \div 2$; $6 \div 1 - 2 \div 2$; $6 \times 1 \div 2 + 2$. Students need to be made aware of the application of order of operations in determining correct expressions.
2. [51 and 57] The sum of two consecutive integers will always be odd i.e. $x + (x + 1) = 2x + 1$. The sum of three consecutive integers i.e. $x + (x + 1) + (x + 2) = 3x + 3 = 3(x + 1)$ will be divisible by three. The only odd integers divisible by three between 50 and 60 are 51 and 57.
3. Answers will vary.
4. [\$0.57] Let a = the cost of an apple and b = the cost of a banana. If $2a + 3b = 51$, then $4a + 6b = 102$. We are given that $4a + 7b = 111$. Comparing the two expressions tells us that one banana costs 9¢. Using the first equation as a base $2a + 3 \times 9 = 51$ or $2a = 24$. Therefore, an apple costs 12¢. One apple and five bananas will cost $\$0.12 + 5 \times \0.09 or \$0.57.
5. [She should not wait; \$34.67 < \$39] Today the dress will cost $2/3$ of \$52 or \$34.67. Tomorrow the dress will cost
1/2 the original price or \$39.
6. [83] The process of elimination, as each clue is determined, will yield the solution.
7. [Justin's; \$510 < \$515] Justin's best price is $\$200 + \$30 + \$0.08 \times 3500 = \510 . At Kelly's the charges are $\$140 + \$25 + \$0.10 \times 3500 = \515 .
8. [the 11th chair] The first round eliminates those in chairs 2, 4, 6, 8, 10, 12, 14, 16, 18, 20. The second round eliminates those in chairs 1, 5, 9, 13, 17, 21. The third round eliminates those in chairs 7 and 15. The fourth round eliminates 3 and 19, leaving chair 11 with the grand prize winner!



9. [504 sq. cm] The area of larger shaded rectangle is 18 cm x 20 cm or 360 sq. cm, while the area of the smaller shaded rectangle is 12 cm x 24 cm or 288 sq.cm. Each rectangle loses 72 sq. cm for a total of 144 sq. cm. This makes the total shaded area 360 sq. cm + 288 sq. cm - 144 sq. cm or 504 sq. cm.

10. [3:30 PM] The Magnolia Tour passes Crystal Falls at 3:45 PM. The Camelia Tour passes the Falls 15 minutes into the tour (1/4 of an hour), so Ricardo should take the 3:30 Camelia Tour to arrive at the Falls at 3:45 PM.



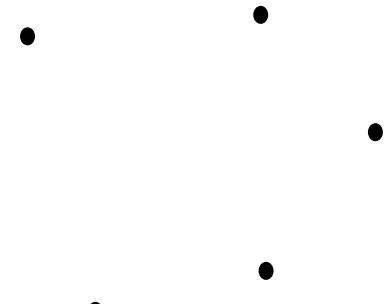
a problem solving newsletter

Vol. 7 No. 6

1. ★★ Mr. Williams used the stem and leaf plot below to display scores on a recent test. What is the median score?

9	1 3 6 7 8
8	3 4 4 7 8
7	1 2 3 5 8 9
6	5 8 9 9

2. ★★ How many different line segments can be drawn to connect these points?



3. ★★★ Eight workers of the Signal Paving Company took nine days to pave a stretch of country road. The crew usually has 12 workers, but three were sick and one was on vacation. If the entire crew of 12 was available, and all worked at the same rate, how long would the job have taken?

4. ★★ Suppose you roll two number cubes with faces labeled zero to five. What are the chances of rolling a seven?

5. ★ How many days are there in a month which starts on Monday and has five Wednesdays?

6. ★★★★ Using exactly four 4's and any operations $[+, -, \times, \div, (), \sqrt{\quad}, !]$, find an expression to equal each of the following:

$$26 = \underline{\hspace{2cm}} \quad 27 = \underline{\hspace{2cm}}$$

$$28 = \underline{\hspace{2cm}} \quad 29 = \underline{\hspace{2cm}}$$

$$30 = \underline{\hspace{2cm}}$$

Strategy of the Month

When a problem involves data with more than one characteristic, **making a table, chart, or graph** is a very good way to organize the information. It helps your brain to identify patterns and to discover any missing data. Tables help you record data without repeating yourself. Making a table or chart is especially useful for certain problems about probability and for some logic problems. Sometimes tables and charts are included in your information and you need to read through them carefully to understand the data you need to solve your problem. Creating a graph is also a good way to organize and visualize information. **Make a table** to solve this problem: A certain factory makes bikes and trikes. Seats come in boxes of six. The foreman hates to have any leftovers at the end of a shift. If 59 wheels are sent by the home office how many boxes of seats should be ordered?

MathStars Home Hints

Everyone learns from sharing, and you can continue to learn by teaching others about the new mathematics ideas you are learning. Become a teacher and help a younger student. Explain what you have learned and what else you want to know. Good teachers set goals and evaluate the progress made toward reaching these goals. You will continue to be a learner whenever you become a teacher.

7. ★★ There are three partners in the Empty Arms Hotel. Mrs. Jones owns a $\frac{5}{8}$ share, and Mr. Kelly owns twice as much as Ms Lane. What percent of the hotel does Mr. Kelly own? _____

8. ★★ Matthew has an average of 86 after four math tests. His next exam is on Friday. What is the highest possible average he can have after Friday? _____

9. ★★ Leonardo of Pisa (Fibonacci) is credited with the following sequence:
1, 1, 2, 3, 5, 8, 13, ... What are the next five numbers in the sequence?

_____, _____, _____, _____, _____,

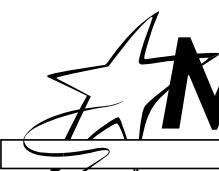
10. ★★★ a) Draw two squares using five straight lines.

- b) Draw two squares using six straight lines.

- c) Draw two squares using seven straight lines.

Setting Personal Goals

Perseverance means that you do not give up easily. Good problem solvers try different strategies when they are stumped and are not discouraged when they cannot find an answer quickly. They stick to the task, using all of their previous experiences to make connections with what they know and the problem they are trying to solve. If something does not work, they discard the unsuccessful idea and try again using a different strategy.



MathStars

Commentary for Teachers

Vol. 7 No. 6

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Discussion of the problems...

1. [81] There are 20 scores so the median is the average of the 10th and 11th scores or $(79 + 83) \div 2$.

2. [15] From the first point five segments can be drawn, from the second point four segments, from the third three and so on yielding $5 + 4 + 3 + 2 + 1 = 15$. Another approach is the number of combinations of six things taken two at a time, ${}_n C_r$ or $6! \div (2! 4!)$.

3. [six days] The job requires 9×8 or 72 days. If 12 workers are available $72 \div 12 = 6$ days needed to complete the task.

4. [$P(7) = 1/9$]

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

5. [31 days] The first Wednesday would be the 3rd, the second Wednesday the 10th, subsequent Wednesdays would be the 17th, 24th and 31st.

6. Answers will vary.

MathStars

Commentary for Teachers

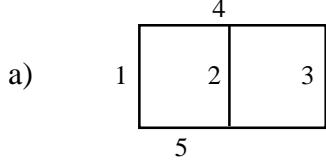
Vol. 7 No. 6

7. [25 %] Mr. Kelly and Ms Lane own $3/8$ of the property and $3/8 = 2/8 + 1/8$ or Kelly's share plus Lane's share.

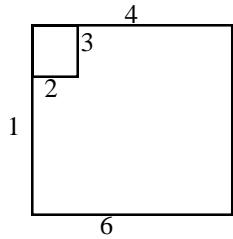
8. [88.8 or 89] His total points before Friday are 86×4 or 344. If he scores 100 on the next test his new average will be $444 \div 5$.

9. [21, 34, 55, 89, 144] Each term is generated by adding the two previous terms i.e. $T_n = T_{n-1} + T_{n-2}$

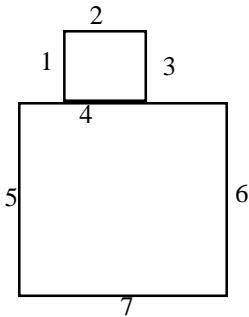
10. Answers will vary. Samples:



b)



c)





a problem solving newsletter

Vol. 7 No. 7

1. ★ The circumference of a truck wheel is 9.5 meters. Find, to the nearest meter, the diameter of the wheel.

_____ m

2. ★★★ The following data set represents the height in centimeters of a group of students:

Student	Height in cm
Tim	145
Liz	136
Latoya	154
Paloma	178
Sean	173
Julio	164
Andrea	144
Allie	170
Tyrone	183
Juanita	144

What is the mode of the set? _____

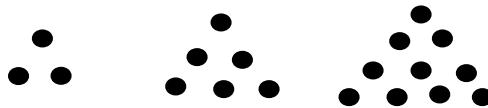
What is the range of the set? _____

Whose height is closest to the median height for the set? _____

Whose height is closest to the mean height for the set? _____

3. ★★★ A new candle is 21 centimeters long. After it has burned for 15 minutes, it is 17.5 cm long. If it continues to burn at this rate, how long will it take to be completely consumed? _____

4. ★★ Triangular numbers are numbers that can be represented by a triangular arrangement of dots. The first three triangular numbers are shown here. Draw the next three triangular numbers.



Strategy of the Month

*Some problems are difficult to "see" even if you draw a picture. For these problems, it can be helpful to actually **act out the problem**. When you role play with friends or people at home, you may discover the solution as you act out the problem. Or you may recognize another strategy that will help you find the answer. Sometimes "acting out" a problem can be done with manipulative materials. To find the solution to the problem below, become the director and choose your cast to act this out: Joey's little sister, Ella, must take four steps for every three steps Joey takes. Suppose one of Joey's steps covers 32 centimeters. How far will Ella travel when she has taken 12 steps?*

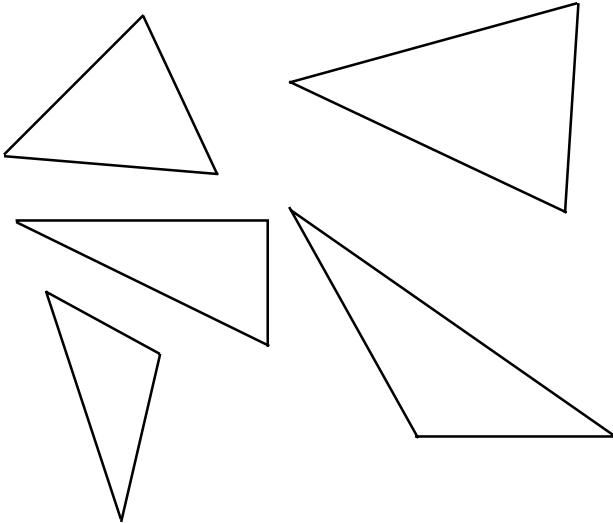
MathStars Home Hints

Calculators are important tools. They do not replace mathematical thinking; you must tell the calculator what numbers and operations to use. Calculators allow students to focus their energies on solving problems and to easily try alternative solutions. They also allow students to solve problems that were too difficult for pencil and paper. Number sense and good estimation skills are important when students use technology to carry out computations. Explore some "what if" situations with the calculator. "What if the cost of gas goes up 4¢... What if we build the patio 2 feet wider..."

5. ★★ Find the mystery numbers:
When added their sum is 19, when multiplied their product is 48, and their difference is 13.
The numbers are _____ and _____.

6. ★ Two angles of a triangle measure 39° and 114° . What is the measure of the third angle? _____

- What kind of triangle is it? _____



7. ★★★★ Complete the pattern by filling in the missing numbers in the table. Then write the rule for finding any number.

X	Y
3	7
5	23
7	47
9	_____
11	_____

8. ★★★★ Jamal is playing "Pick-A-Fish" at the carnival. There are ten fish in the pond. Two of the fish are worth 15 points, three are worth ten points and five are eight-point fish. The prize he would like requires two 15-point fish. What is the probability he will win the prize with only two picks?

9. ★★★★ Using exactly four 4's and any operations $[+, -, \times, \div, (), \sqrt{}, !]$, find an expression to equal each of the following:

$$31 = \text{_____} \quad 32 = \text{_____}$$

$$33 = \text{_____} \quad 34 = \text{_____}$$

$$35 = \text{_____}$$

Setting Personal Goals

Accuracy is very important to everyone. Pharmacists must always measure accurately when preparing prescriptions and carpenters must cut supporting boards precisely to fit. Careless mistakes may be avoided in the classroom by computing carefully, checking back over work, and writing numbers clearly and neatly. Remember: If work is worth doing, it is worth doing well.

MathStars

Commentary for Teachers

Vol. 7 No. 7

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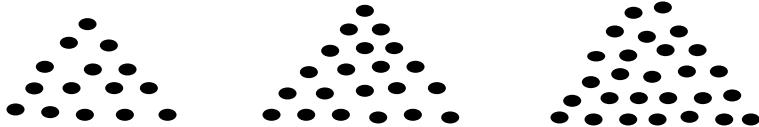
Discussion of the problems...

1. [3 meters] Students can use the relationship $C = \pi d$, where C represents the circumference, d represents the diameter and π can be approximated by 3.14 or 22/7. The ratio C/π will give a value greater than 3.02 which rounds to three meters.

2. [mode is 144; range is 47; Latoya is closest to the median(159); and also closest to the mean (159.1)] This problem gives a good opportunity to review the concepts and strategies for the mean, median, mode and range.

3. [90 minutes] Students can set up a proportion i.e. $3.5 : 15 :: 21 : x$, they can reason that 3.5 cm burn in 15 minutes ($21 - 17.5 = 3.5$). And $21 \div 3.5 = 6$ or there are six 3.5 cm segments in the candle each of which takes 15 minutes to burn which yields 90 total minutes to burn the entire candle.

4. [3, 6, 10, 15, 21, 28, ...]



The sequence progresses with the addition of consecutive counting numbers,
i.e. $3 + \textcircled{3} = 6$

$$6 + \textcircled{4} = 10$$

$$10 + \textcircled{5} = 15$$

$$15 + \textcircled{6} = 21$$

$$21 + \textcircled{7} = 28$$



5. [16 and 3] Students can use any clue to begin their search. Knowing that the product is 48 will yield a set of factors whose sums and differences can be computed and checked against the clues.

Using the fact that the sum is 19, a series of addends can be written and their products and differences computed and checked. The same is true for the fact that the difference is 13. Students might then be asked, "Which strategy is most efficient?" or "Which strategy gives the smallest set to be examined?" The groundwork for algebraic, or symbolic, expression is laid when students express the clues as equations and then examine the results. The non-commutivity of subtraction makes for a unique assignment of variables.

6. [27°, scalene or obtuse] This problem affords a good opportunity to review some vocabulary and basic facts about triangles.

7. [79, 119; $Y = X^2 - 2$] Students need to look for a pattern; what happens to X to transform it to Y?

8. [2/90] There are ten fish in the pond. The probability of drawing the first 15-pointer is 2/10; the probability of drawing the second 15-pointer is 1/9, which gives the probability of two in a row as $(2/10)(1/9)$ or 2/90.

9. Answers will vary.

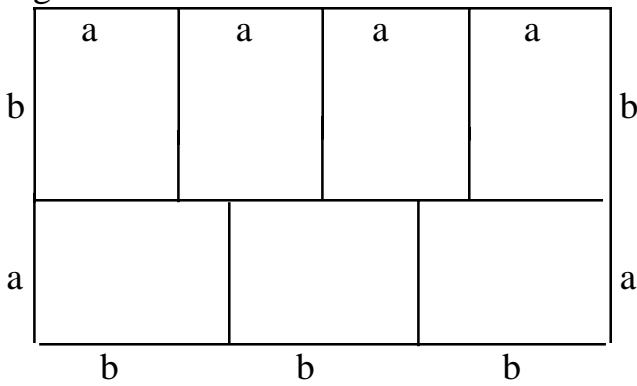


a problem solving newsletter

Vol. 7 No. 8

1. ★★ Using the digits 1, 2, 3, and 4 you can make 24 different four-digit numbers. What is the sum of these numbers?

2. ★★★ The seven families that live in Melville Towers want to plant a garden behind their building. They have laid out a rectangular region with seven congruent plots, one for each family, as shown in the diagram below.

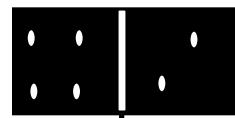
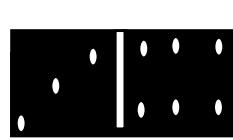


- Each family's plot is 48 square meters. If a fence is to be built around the entire garden along the outside boundary, how much fencing is needed?

3. ★ Builders have observed that residence windows are most attractive if they have the width to length ratio 3:5. If a window is to be 48 inches wide, what should its length be for the most attractive appearance?

4. ★★ Al's Awesome Autos advertised a special sale on cars - Dealer cost plus 5%!! Jack and Margaret bought a luxury sedan for \$23,727.90. What was the dealer's cost?

5. ★★ A craftsman wanted to make a large set of wooden dominoes for his niece. Each of the two halves of a domino would be marked with zero to nine dots. If he is to make a complete set with one domino for each possible combination, how many blocks of wood will he need?



Strategy of the Month

What do you do if you have a problem that seems to be very complicated? It may have a lots of large numbers, too much information, or multiple conditions. One approach is to create a simpler problem like the one you need to solve. As you solve the easier problem, you may see the way to solve the more difficult one. Or you may discover a different process that will work with the harder problem. The trick is to be sure that your simpler problem is enough like the original one that the patterns or process you use will help you with the harder situation. Make a simpler problem first as you solve this: What is the sum of the first 500 counting numbers? i.e. The sum of $1 + 2 + 3 + 4 + \dots + 496 + 497 + 498 + 499 + 500$? How about only the odds : $1 + 3 + 5 + 7 + \dots + 493 + 495 + 497 + 499$?

MathStars Home Hints

Math skills develop as you apply concepts learned in school to real life situations.

Which product is the best buy? How many tiles will it take to cover the kitchen floor? What time should we start baking the turkey so that we can have dinner at 7p.m.? What do the statistics tell us about the two baseball players?

6. ★★★★ Using exactly four 4's and any operations [+,-, x, ÷, (), √, !], find an expression to equal each of the following:

$$36 = \underline{\hspace{2cm}} \quad 37 = \underline{\hspace{2cm}}$$

$$38 = \underline{\hspace{2cm}} \quad 39 = \underline{\hspace{2cm}}$$

$$40 = \underline{\hspace{2cm}}$$

7. ★★ Stephanee's great aunt gave her 1¢ on the first of June, 2¢ on the second of the month, 4¢ on the third, 8¢ on the fourth day and so on. If Stephanee saves all the money her great aunt gives her, how much will she have at the end of the month?

\$

8. ★★ What is the difference between the sum of the first 15 odd numbers and the sum of the first 15 even numbers?

9. ★★★ The human heart beats an average of 72 times a minute. At this rate, how many times will a person's heart beat in a lifetime of 78 years?

10. ★★★★ Amy, Connor, Jalissa, Stella and Gonzo live in apartments A, B, C, D, E in the same building. Their ages are 9, 10, 11, 13, and 14. They each get up at a different time each morning, either 6:00, 7:00, 7:30, 8:00, or 8:15. Using the clues below, determine the apartment, age and rising time for each student.

- Connor gets up at 6:00 and is older than the student in Apt. B.
- Amy is 11 and gets up two hours later than the boy in Apt. A.
- The ten-year old girl lives in Apt. B and it isn't Stella.
- The youngest student, a girl, doesn't live in Apt. E.
- The person in Apt. B gets up later than everyone else.
- Gonzo is younger than Conner and he lives in Apt. D.
- The shades go up at 7:30 in Apt. C.

	TIME	APT.	AGE
Amy	<u> </u>	<u> </u>	<u> </u>
Connor	<u> </u>	<u> </u>	<u> </u>
Jalissa	<u> </u>	<u> </u>	<u> </u>
Stella	<u> </u>	<u> </u>	<u> </u>
Gonzo	<u> </u>	<u> </u>	<u> </u>

Setting Personal Goals

Confidence means that you believe in yourself. You can become a more confident problem solver by learning to use a variety of strategies. If your first idea does not work, don't give up just try another way! Working with a buddy also helps. You need to remember that there is usually more than one way to solve a problem and that practice always helps us learn.



Commentary for Teachers

Vol. 7 No. 8

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Discussion of the problems...

1. **[66,660]** Each digit appears six times in each of the four places. Therefore the sum of the ones places is $6(1 + 2 + 3 + 4)$ or 60. Each subsequent column will total 66 (60 plus the regrouped 6 from the previous column).

2. **[76 meters]** One plot is 6 by 8 meters i.e. $a = 6$ meters and $b = 8$ meters. (These are the only factors of 48 that fit the diagram.) The perimeter is $6a + 5b$ or $36 + 40$ or 76 meters.

3. **[80 inches]** The ratio $3 : 5 :: 48 : x$ will yield the desired dimension. You may wish to note that the most attractive ratio, 3 by 5, is a pair of consecutive Fibonacci numbers, as are 5 by 8, 8 by 13 and other often-used ratios. The Golden Rectangle and Golden Mean could also be explored at this time.

4. **[\$22,598.00]** Students can explore the relationship $105\% \text{ of } ?? = \$23,727.90$ or examine the concept of 105% as 21 parts ($20/20 + 1/20$). The dealer makes $\$23727.90 \div 21$ or $\$1,129.90$, while the car costs 20 times that amount or \$22,598.00.

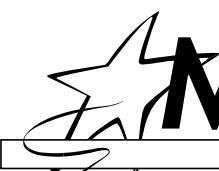
5. **[55 pieces]** $0,0; 0,1, 1,1; 0,2, 1,2, 2,2; 0,3, 1,3, 2,3, 3,3; \dots 0,9, 1,9, 2,9, 3,9, 4,9, 5,9, 6,9, 7,9, 8,9, 9,9$

1 + 2 + 3 + 4 + ... + 10

6. Answers will vary.

7. **[1,073,741,823¢ or \$10,737,418.23]** A calculator with memory function is especially useful to solve this problem. Also making a chart will help students see the sum of the 30 addends as a function of the earlier day's amount i.e.

Day.	1	2	3	4	5	6	7	8	9	10	11	12
Gift	.01	.02	.04	.08	.16	.32	.64	1.28	2.56	5.12	10.24	20.48
Total	.01	.03	.07	.15	.31	.63	1.27	2.55	etc.			



MathStars

Commentary for Teachers

Vol. 7 No. 8

8. [15] The odds: $1 + 3 + 5 + \dots + 29 = 225$ while the evens: $2 + 4 + 6 + \dots + 30 = 240$. Their difference is 15. Or, for each odd number, the corresponding even number is one greater i.e. $2 - 1 = 1$, $4 - 3 = 1$, $6 - 5 = 1 \dots$. Since there are 15 such pairs, the total difference is, again, 15.

9. [2,951,769,600 or 2,953,739,520 or 2,953,843,200] Computing for a non-leap year the results are:
72 beats/min. \times 60 min./hr. \times 24 hr./day \times 365 days/yr. \times 78 yrs = 2,951,769,600 beats.

Now for the leap years. Depending upon when the subject was born, s/he could experience 19 or 20 leap years ($78 \div 4 = 19.5$).

For 19 leap years we compute: 72 beats/min. \times 60 min./hr. \times 24 hr./day \times 19 days = 1,969,920 additional beats for a total of 2,953,739,520.

For 20 leap years we compute : 72 beats/min. \times 60 min./hr. \times 24 hr./day \times 20 days = 2,073,600 additional beats for a total of 2,953,843,200.

10.	TIME	APT.	AGE
Amy	8:00	E	11
Connor	6:00	A	14
Jalissa	8:15	B	10
Stella	7:30	C	9
Gonzo	7:00	D	13



MathStars

a problem solving newsletter

Vol. 7 No. 9

1. ★★★ Students in Mrs. Cook's class were asked to each draw a triangle. Fifteen of the triangles were right triangles, 12 were isosceles, and four were neither right nor isosceles. If there are 29 students in the class, how many drew isosceles right triangles?

2. ★★★★ Working alone, the master painter finished a room in three hours. Her apprentice required seven hours to complete an identical room. If there is a third room exactly like the first two, how long will the job take if they work together?

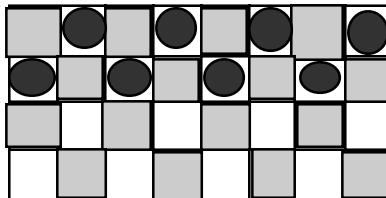
3. ★★ At the Seeds-Be-Gone Watermelon Stand, Farmer Green charges a fixed amount for the first three watermelons you buy and an extra amount for each additional watermelon. If seven melons cost \$16.00 and ten melons are \$22.00, what is the fixed price for the first three watermelons?

4. ★★ In the addition problem below, if M and N are one-digit positive integers, what is the greatest possible value of N?

$$8 + M + N = 15$$

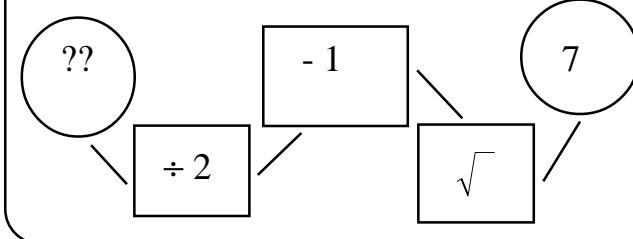
5. ★ At Donna's Deli, salads are \$0.21 an ounce and an empty plate weighs three ounces. What is the cost if the scale reads 1.25 pounds?

6. ★ How many possible first moves are there on a checkerboard?



Strategy of the Month

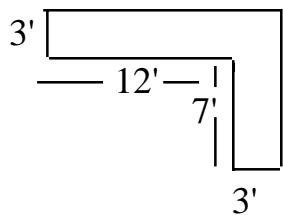
What if you know the result of a situation, but you don't know the beginning? For example, you might know that you end up with thirteen baseball cards after doing a certain number of trades and you want to figure out how many cards you had before the trading started. In that case you need to work backwards; you have to think about your actions in reverse order. This strategy works for any sequence of actions when you know the end result rather than the starting place. Try working backwards to find the starting number on this flow chart:



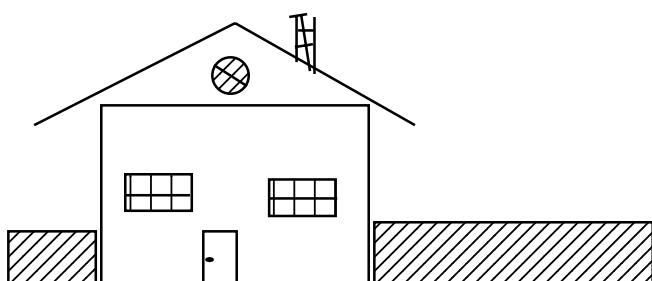
MathStars Home Hints

Mathematics can make life easier for you when you become a good estimator. Spatial estimation helps you plan how you will rearrange your furniture or how far to jump to cross a puddle of water. Using estimation helps you know whether you have enough money for your purchases before you get to the check-out line. We become good estimators by practicing. Use your number sense and spatial sense to think about what the answers to problems will be before you start to solve them.

7. ★★★ Four bricks will make a square foot of a brick walkway. How many bricks are needed to make the walkway pictured here? _____



8. ★★★★ Alphonso's parents are building a new home. The house costs \$65 a square foot, the deck is \$7.50 a square foot, the driveway \$3.00 a square foot and the fence is \$8.60 a linear foot. What is the total cost of a 1,900 square foot house with a 12 by 15 foot deck, a 720 square foot driveway and 420 feet of fencing? _____



9. ★★★★ Using exactly four 4's and any operations [+,-, x, ÷, () , $\sqrt{}$, !], find an expression to equal each of the following:

$$41 = \underline{\hspace{2cm}} \quad 42 = \underline{\hspace{2cm}}$$

$$43 = \underline{\hspace{2cm}} \quad 44 = \underline{\hspace{2cm}}$$

$$45 = \underline{\hspace{2cm}}$$

10. ★★ Sixteen players are in a single elimination tournament. When a player loses a match s/he is out of the tournament. How many matches will be played? _____

Setting Personal Goals

When you encounter a new situation, you use all of your previous experiences to figure out the current problem. Reasoning mathematically means using your brain power to think logically and sequentially, to put prior knowledge with new information. Set the goal of developing mathematical power and use your thinking power to achieve the goal!

MathStars

Commentary for Teachers

About these newsletters...

Vol. 7 No.9

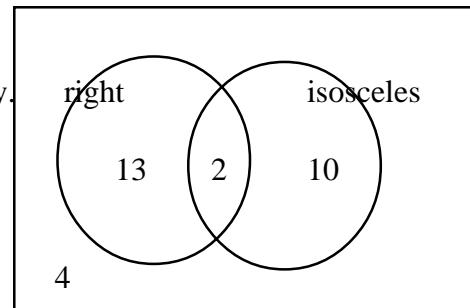
The purpose of the MathStars Newsletters is to challenge students beyond the classroom setting. Good problems can inspire curiosity about number relationships and geometric properties. It is hoped that in accepting the challenge of mathematical problem solving, students, their parents, and their teachers will be led to explore new mathematical horizons.

As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

1.[2] A Venn diagram is one strategy that students could employ.

Another method, logic, notes that 31 triangles are counted, but only 29 students are in the class. This implies a duplication of 2.



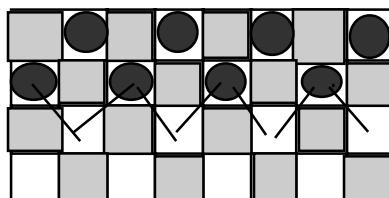
2. [126 minutes] The master painter finishes $\frac{1}{3}$ of the room in one hour, the apprentice does $\frac{1}{7}$ in that amount of time. Together, $\frac{10}{21}$ ($\frac{1}{3} + \frac{1}{7}$) is completed in an hour. If $\frac{10}{21}$ takes an hour, (60 minutes) then $\frac{1}{21}$ of the job should take 6 minutes. The whole job ($\frac{21}{21}$) would therefore take 21×6 or 126 minutes.

3. [\$8.00] Seven melons for \$16.00, ten melons for \$22.00 implies that $10 - 7$ or 3 melons will cost \$6.00 or \$2.00 each. The breakdown for seven is then $3 @$ fixed price + $4 @ \$2.00$ each = \$16.00 or $\$8.00 + \8.00 . The argument is similar for ten melons : \$8.00 (fixed price) for the first three plus \$14.00 ($7 @ \2.00 each) for the next seven.

4. [6] Since M and N are positive integers, M must be at least one. The largest value for N is then six.

5. [\$3.57] The scale reads 1.25 lbs. or 20 ounces, three for the plate and 17 for the salad. At \$.21 an ounce the cost becomes $17 \times .21$ or \$3.57.

6. [7 moves]

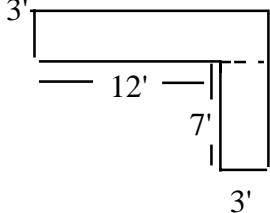


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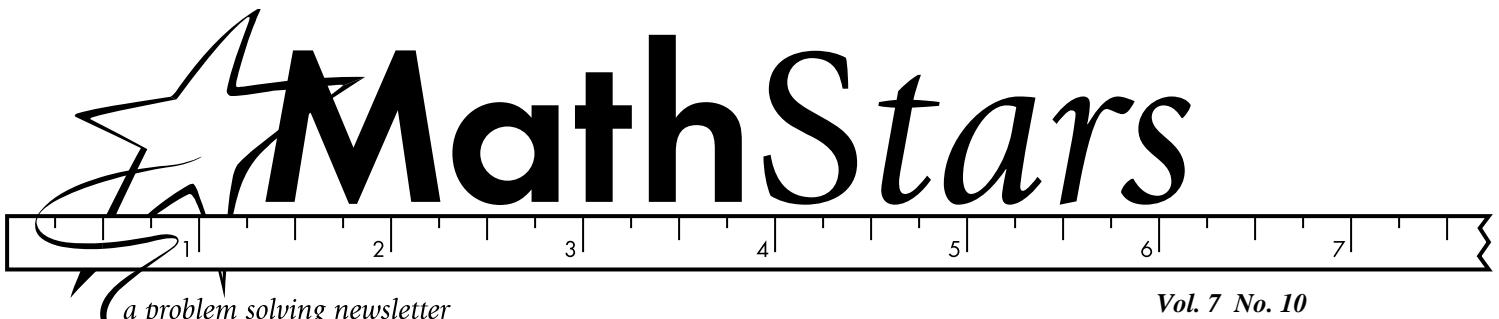
7. [264 bricks] Two rectangles, 3 by 15 and 7 by 3 yield an area of $45 + 21$ or 66 square feet. At four bricks a square foot the total number of bricks is 66×4 or 264 bricks.



8. [\$130,622]	House	at \$65 x 1900 =	\$123,500
	Deck	at \$7.50 x (12 x 15) =	\$1,350
	Driveway	at \$3.00 x 720 =	\$2,160
	Fencing	at \$8.60 x 420 =	\$3,612
	Total:	<hr/>	
		\$130,622	

9. Answers will vary.

10. [15] Each match eliminates a player, and there are 16 players. Since all but one is to be eliminated, 15 matches are necessary.



a problem solving newsletter

Vol. 7 No. 10

1. ★★ In which column will 1,002 appear if this pattern continues? _____

A	B	C	D	E
3		6		
15		12		9
	18		21	
30		27		24
	33		...	

2. ★★★ What are the 10th and 50th terms of the sequences below:

10th term 50th term

- a) 5, 8, 11, 14, . . . _____ _____
- b) 3, $\frac{7}{2}$, 4, $\frac{9}{2}$, . . . _____ _____
- c) 8, 8.4, 8.8, 9.2, . . . _____ _____

3. ★★★★ In the equation below, each letter represents a different digit. What is the value of each letter?

$$\sqrt{\text{RADAR}} = \text{RAR}$$

$$R = \underline{\quad} A = \underline{\quad} D = \underline{\quad}$$

4. ★★★★ Using exactly four 4's and any operations $[+, -, \times, \div, (), \sqrt{\quad}, !]$, find an expression to equal each of the following:

$$46 = \underline{\quad} \qquad 47 = \underline{\quad}$$

$$48 = \underline{\quad} \qquad 49 = \underline{\quad}$$

$$50 = \underline{\quad}$$

5. ★★ How can the dogcatcher place 33 dogs in four pens so that the number of dogs in each pen is odd?

Strategy of the Month

You have tried many ways to solve problems this year. Already you know that when one strategy does not lead you to a solution, you back up and try something else. Sometimes you can find a smaller problem inside the larger one that must be solved first. Sometimes you need to think about the information that is missing rather than what is there. Sometimes you need to read the problem again and look for a different point of view. Sometimes you need to tell your brain to try to think about the problem in an entirely different way - perhaps a way you have never used before. Looking for different ways to solve problems is like brainstorming. Try to solve this problem. You may need to **change your point of view**. Bob and his dad visited the hardware store on Saturday. They observed the following transactions: Mr. Harris bought 4 for \$0.50, Mrs. Clarke bought 21 for \$1.00, Mr. Montaro bought 1134 for \$2.00 and Ms Park bought 450 for \$1.50. What were they buying?

MathStars Home Hints

Identifying the mathematics that is all around you can be lots of fun. Think about the geometry and spatial visualization you use in playing video games or when you play golf or basketball. When your parents parallel park, they are using their spatial skills too. When you track a hurricane, you use coordinates. When you check the stock market or read the latest sports statistics, you are using mathematics. With your family or friends go on a math scavenger hunt. Who can identify mathematics is the most unusual places?

6. ★★★ Campers at Camp Crafty made three legged stools and four legged stools in arts and crafts class. They used a total of 106 legs to make 31 stools. How many of each did they make? _____

7. ★ What are the next four letters in this pattern?

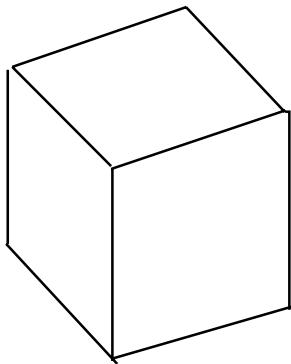
O, T, T, F, F, S, S, ___, ___, ___, ___, ...

8. ★★★ A class of 28 students stood in a circle and counted off by ones. The multiples of four sat down. The remaining students counted by ones again and this time multiples of three sat down. Those still standing again counted by ones and the multiples of two took a seat. When this third count off was completed, how many students were sitting?



9. ★★★ A wooden cube, with a side four inches long, is painted red and then cut into one inch cubes. How many one inch cubes will have:

- a) three red faces? _____
b) exactly one red face? _____
c) no red faces? _____



10. ★★★ Andi, John, Rhonda, Kay and Tommie collect coins. Andi has $\frac{1}{3}$ as many coins as John and twice as many as Rhonda. Kay has 15 fewer than Rhonda. Tommie and Kay each have 25 coins. How many coins do the others have?

Andi _____ John _____ Rhonda _____

Setting Personal Goals

Students who recognize the value of mathematics are well on their way to becoming mathematically powerful citizens. Valuing mathematics means that we appreciate the richness, power, and usefulness of mathematics. Without math there would be no roads or bridges, computers or movies, banks or fast food restaurants. How can you become mathematically powerful?

About these newsletters...

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As with all good problems, the solutions and strategies suggested are merely a sample of what you and your students may discover. Enjoy!!

Discussion of the problems...

1. [C] The difference between consecutive numbers in any column is 15. Every number in a given column will yield the same remainder when divided by 15. When 1002 is divided by 15 the remainder is 12 i.e. $1002 = 66 \times 15 + 12$. Therefore 1002 will appear in the same column as 12 (division reveals $12 = 15 \times 0 + 12$) or in column C.

Observing multiples of 15 in column A is also an effective strategy. Noting that $15 \times 67 = 1005$, then 1002 or three less will appear in column C as the arrangement winds from left to right to left.

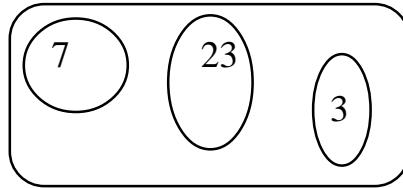
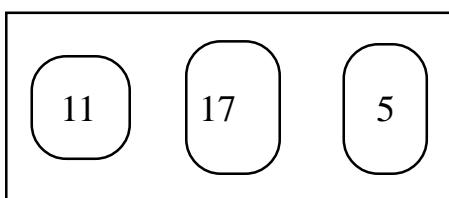
2. [a) 31, 152; b) 15/2, 55/2; c) 11.6, 27.6] In each case there are nine intervals from the first to the tenth terms. The tenth term is nine times the common difference, d, plus the first term, a, or the 10th = $a + 9d$.

Likewise there are 49 terms between the first and fiftieth so that the 50th = $a + 49d$.

3. [R = 1; A = 0; D = 2] Guess and check is a good strategy for alphanumerics. A calculator is an excellent tool that promotes efficiency as well as speed.

4. Answers will vary.

5. Answers will vary. Since the sum of four odd number will always be even, the pens must not be completely distinct. Here are some possibilities:





6. [18 three-legged stools and 13-four legged stools] Students can use a guess and check strategy, organize a table or chart to examine the possibilities or the following logic: All stools use at least three legs or $31 \times 3 = 93$ legs. This leaves $106 - 93$ or 13 legs for the four legged stools.
7. [E, N, T, E] One, Two, Three, Four, Five, Six, Seven, Eight, Nine, Ten, Eleven.
8. [21 students] After the first count off 1/4 or seven students sat down. The 21 left then did another count and this time 1/3 or seven more took seats. With 14 remaining on their feet, the last count off removed 1/2 or seven more from the group so the seated students number 21.
9. [a) 8 cubes; b) 24 cubes; c) 8 cubes] Students may find using unifix cubes a useful tool in modelling this problem.
10. [Andi 80 coins; John 240 coins; Rhonda 40 coins] Rhonda = Kay + 15 or $25 + 15 = 40$
Andi = $2 \times$ Rhonda or $2 \times 40 = 80$
John = $3 \times$ Andi or $3 \times 80 = 240$