

**1-1 Skills Practice*****Expressions and Formulas*****Find the value of each expression.**

1.  $18 \div 2 \times 3$

2.  $9 + 6 \div 2 + 1$

3.  $(3 - 8)^2(4) - 3$

4.  $5 + 3(2 - 12 \div 2)$

5.  $-\frac{1}{3}[-9 + 10(3)]$

6.  $\frac{6(7 - 5)}{4}$

7.  $(168 \div 7)3^2 - 4^3$

8.  $[3(5) - 128 \div 2^2]5$

**Evaluate each expression if  $r = -1$ ,  $s = 3$ ,  $t = 12$ ,  $v = 0$ , and  $w = -\frac{1}{2}$ .**

9.  $6r + 2s$

10.  $2st - 4rs$

11.  $w(s - r)$

12.  $s + 2r - 16v$

13.  $(4s)^2$

14.  $s^2r - wt$

15.  $2(3r + w)$

16.  $\frac{3v + t}{5s - t}$

17.  $-w[t + (t - r)]$

18.  $\frac{rv^3}{s^2}$

19.  $9r^2 + (s^2 - 1)t$

20.  $7s - 2v + \frac{2w}{r}$

**21. TEMPERATURE** The formula  $K = C + 273$  gives the temperature in kelvins (K) for a given temperature in degrees Celsius. What is the temperature in kelvins when the temperature is 55 degrees Celsius?

**22. TEMPERATURE** The formula  $C = \frac{5}{9}(F - 32)$  gives the temperature in degrees Celsius for a given temperature in degrees Fahrenheit. What is the temperature in degrees Celsius when the temperature is 68 degrees Fahrenheit?

**1-2 Skills Practice*****Properties of Real Numbers*****Name the sets of numbers to which each number belongs.**

1. 34

2.  $-525$

3. 0.875

4.  $\frac{12}{3}$

5.  $-\sqrt{9}$

6.  $\sqrt{30}$

**Name the property illustrated by each equation.**

7.  $3 \cdot x = x \cdot 3$

8.  $3a + 0 = 3a$

9.  $2(r + w) = 2r + 2w$

10.  $2r + (3r + 4r) = (2r + 3r) + 4r$

11.  $5y\left(\frac{1}{5y}\right) = 1$

12.  $15x(1) = 15x$

13.  $0.6[25(0.5)] = [0.6(25)]0.5$

14.  $(10b + 12b) + 7b = (12b + 10b) + 7b$

**Name the additive inverse and multiplicative inverse for each number.**

15. 15

16. 1.25

17.  $-\frac{4}{5}$

18.  $3\frac{3}{4}$

**Simplify each expression.**

19.  $3x + 5 + 2x - 3$

20.  $x - y - z + y - x + z$

21.  $-(3g + 3h) + 5g - 10h$

22.  $a^2 - a + 4a - 3a^2 + 1$

23.  $3(m - z) + 5(2m - z)$

24.  $2x - 3y - (5x - 3y - 2z)$

25.  $6(2 + v) - 4(2v + 1)$

26.  $\frac{1}{3}(15d + 3) - \frac{1}{2}(8 - 10d)$

# 1-3 Skills Practice

## Solving Equations

**Write an algebraic expression to represent each verbal expression.**

1. 4 times a number, increased by 7
2. 8 less than 5 times a number
3. 6 times the sum of a number and 5
4. the product of 3 and a number, divided by 9
5. 3 times the difference of 4 and a number
6. the product of  $-11$  and the square of a number

**Write a verbal expression to represent each equation.**

7.  $n - 8 = 16$
8.  $8 + 3x = 5$
9.  $b^2 + 3 = b$
10.  $\frac{y}{3} = 2 - 2y$

**Name the property illustrated by each statement.**

11. If  $a = 0.5b$ , and  $0.5b = 10$ , then  $a = 10$ .
12. If  $d + 1 = f$ , then  $d = f - 1$ .
13. If  $-7x = 14$ , then  $14 = -7x$ .
14. If  $(8 + 7)r = 30$ , then  $15r = 30$ .

**Solve each equation. Check your solution.**

15.  $4m + 2 = 18$
16.  $x + 4 = 5x + 2$
17.  $3t = 2t + 5$
18.  $-3b + 7 = -15 + 2b$
19.  $-5x = 3x - 24$
20.  $4v + 20 - 6 = 34$
21.  $a - \frac{2a}{5} = 3$
22.  $2.2n + 0.8n + 5 = 4n$

**Solve each equation or formula for the specified variable.**

23.  $I = prt$ , for  $p$
24.  $y = \frac{1}{4}x - 12$ , for  $x$
25.  $A = \frac{x + y}{2}$ , for  $y$
26.  $A = 2\pi r^2 + 2\pi rh$ , for  $h$

**1-4 Skills Practice*****Solving Absolute Value Equations***

Evaluate each expression if  $w = 0.4$ ,  $x = 2$ ,  $y = -3$ , and  $z = -10$ .

1.  $|5w|$

2.  $|-9y|$

3.  $|9y - z|$

4.  $-|17z|$

5.  $-|10z - 31|$

6.  $-|8x - 3y| + |2y + 5x|$

7.  $25 - |5z + 1|$

8.  $44 + |-2x - y|$

9.  $2|4w|$

10.  $3 - |1 - 6w|$

11.  $|-3x - 2y| - 4$

12.  $6.4 + |w - 1|$

Solve each equation. Check your solutions.

13.  $|y + 3| = 2$

14.  $|5a| = 10$

15.  $|3k - 6| = 2$

16.  $|2g + 6| = 0$

17.  $10 = |1 - c|$

18.  $|2x + x| = 9$

19.  $|p - 7| = -14$

20.  $2|3w| = 12$

21.  $|7x - 3x| + 2 = 18$

22.  $4|7 - y| - 1 = 11$

23.  $|3n - 2| = \frac{1}{2}$

24.  $|8d - 4d| + 5 = 13$

25.  $-5|6a + 2| = -15$

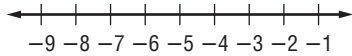
26.  $|k| + 10 = 9$

# 1-5 Skills Practice

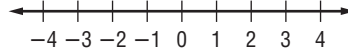
## Solving Inequalities

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

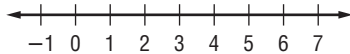
1.  $\frac{z}{-4} \geq 2$



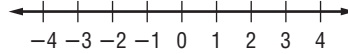
2.  $3a + 7 \leq 16$



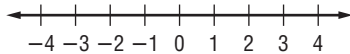
3.  $16 < 3q + 4$



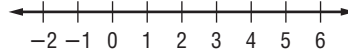
4.  $20 - 3s > 7s$



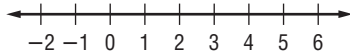
5.  $3x \geq -9$



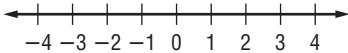
6.  $4b - 9 \leq 7$



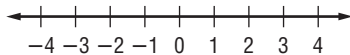
7.  $2z < -9 + 5z$



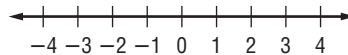
8.  $7f - 9 > 3f - 1$



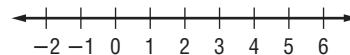
9.  $-3s - 8 \leq 5s$



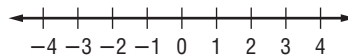
10.  $7t - (t - 4) \leq 25$



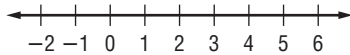
11.  $0.7m + 0.3m \geq 2m - 4$



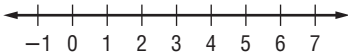
12.  $4(5x + 7) \leq 13$



13.  $1.7y - 0.78 > 5$



14.  $4x - 9 > 2x + 1$



Define a variable and write an inequality for each problem. Then solve.

15. Nineteen more than a number is less than 42.

16. The difference of three times a number and 16 is at least 8.

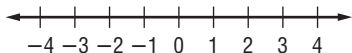
17. One half of a number is more than 6 less than the same number.

18. Five less than the product of 6 and a number is no more than twice that same number.

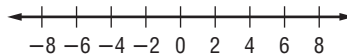
**1-6 Skills Practice****Solving Compound and Absolute Value Inequalities**

Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

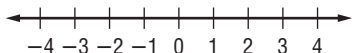
1. all numbers greater than or equal to 2 or less than or equal to  $-2$



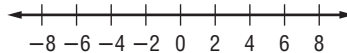
2. all numbers less than 5 and greater than  $-5$



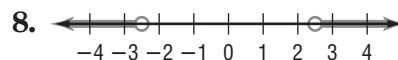
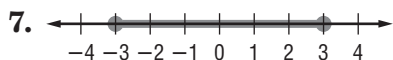
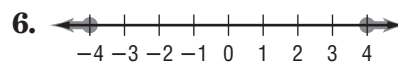
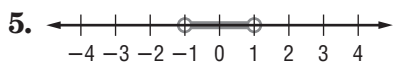
3. all numbers less than  $-1$  or greater than 1



4. all numbers between  $-6$  and 6

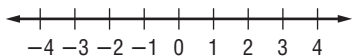


Write an absolute value inequality for each graph.

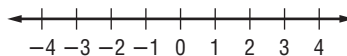


Solve each inequality. Graph the solution set on a number line.

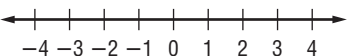
9.  $2c + 1 > 5$  or  $c < 0$



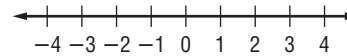
10.  $-11 \leq 4y - 3 \leq 1$



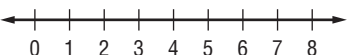
11.  $10 > -5x > 5$



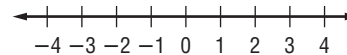
12.  $4a \geq -8$  or  $a < -3$



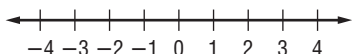
13.  $8 < 3x + 2 \leq 23$



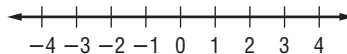
14.  $w - 4 \leq 10$  or  $-2w \leq 6$



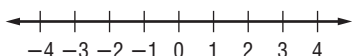
15.  $|t| \geq 3$



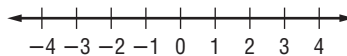
16.  $|6x| < 12$



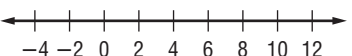
17.  $|-7r| > 14$



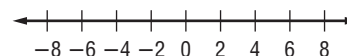
18.  $|p + 2| \leq -2$



19.  $|n - 5| < 7$



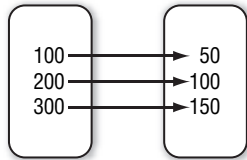
20.  $|h + 1| \geq 5$



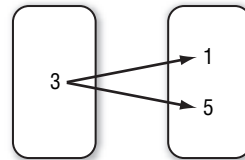
**2-1 Skills Practice****Relations and Functions**

Determine whether each relation is a function. Write *yes* or *no*.

1. Domain Range



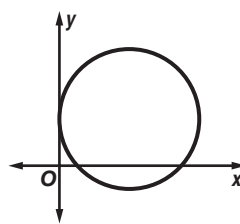
2. Domain Range



3.

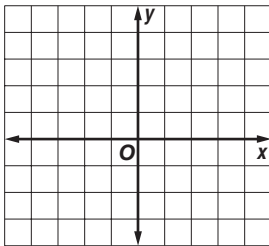
$x$	$y$
1	2
2	4
3	6

4.

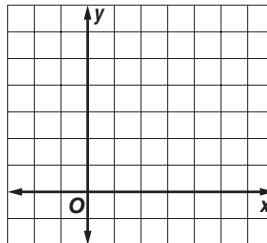


Graph each relation or equation and find the domain and range. Next determine if the relation is discrete or continuous. Then determine whether the relation or equation is a function.

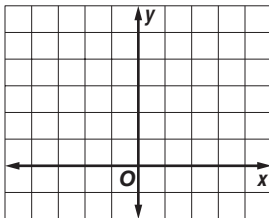
5.  $\{(2, -3), (2, 4), (2, -1)\}$



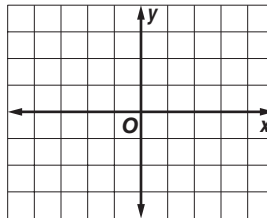
6.  $\{(2, 6), (6, 2)\}$



7.  $\{(-3, 4), (-2, 4), (-1, -1), (3, -1)\}$



8.  $x = -2$



Find each value if  $f(x) = 2x - 1$  and  $g(x) = 2 - x^2$ .

9.  $f(0)$

10.  $f(12)$

11.  $g(4)$

12.  $f(-2)$

13.  $g(-1)$

14.  $f(d)$

**2-2 Skills Practice****Linear Equations**

State whether each equation or function is linear. Write *yes* or *no*. If no, explain your reasoning.

1.  $y = 3x$

2.  $y = -2 + 5x$

3.  $2x + y = 10$

4.  $f(x) = 4x^2$

5.  $-\frac{3}{x} + y = 15$

6.  $\frac{1}{3}x = y + 8$

7.  $g(x) = 8$

8.  $h(x) = \sqrt{x} + 3$

Write each equation in standard form. Identify *A*, *B*, and *C*.

9.  $y = x$

10.  $y = 5x + 1$

11.  $2x = 4 - 7y$

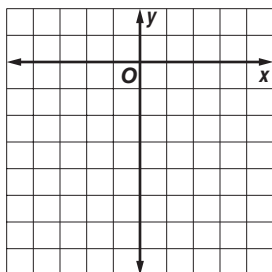
12.  $3x = -2y - 2$

13.  $5y - 9 = 0$

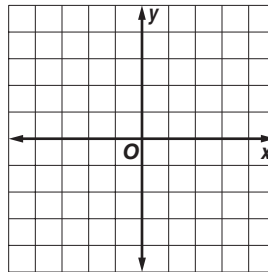
14.  $-6y + 14 = 8x$

Find the *x*-intercept and the *y*-intercept of the graph of each equation. Then graph the equation.

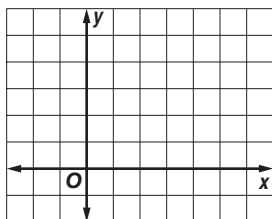
15.  $y = 3x - 6$



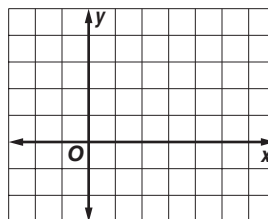
16.  $y = -2x$



17.  $x + y = 5$



18.  $2x + 5y = 10$





**2-3 Skills Practice****Slope****Find the slope of the line that passes through each pair of points.**

1.  $(1, 5), (-1, -3)$

2.  $(0, 2), (3, 0)$

3.  $(1, 9), (0, 6)$

4.  $(8, -5), (4, -2)$

5.  $(-3, 5), (-3, -1)$

6.  $(-2, -2), (10, -2)$

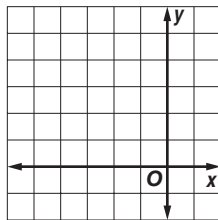
7.  $(4, 5), (2, 7)$

8.  $(-2, -4), (3, 2)$

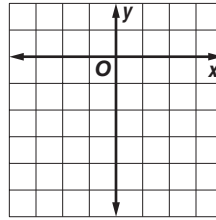
9.  $(5, 2), (-3, 2)$

**Graph the line passing through the given point with the given slope.**

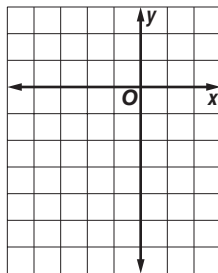
10.  $(0, 4), m = 1$



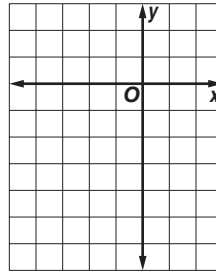
11.  $(2, -4), m = -1$



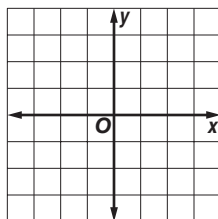
12.  $(-3, -5), m = 2$



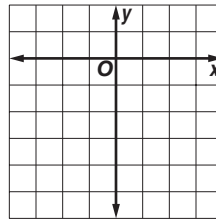
13.  $(-2, -1), m = -2$

**Graph the line that satisfies each set of conditions.**

14. passes through  $(0, 1)$ , perpendicular to a line whose slope is  $\frac{1}{3}$



15. passes through  $(0, -5)$ , parallel to the graph of  $y = 1$



**16. HIKING** Naomi left from an elevation of 7400 feet at 7:00 A.M. and hiked to an elevation of 9800 feet by 11:00 A.M. What was her rate of change in altitude?

**2-4 Skills Practice****Writing Linear Equations**

State the slope and y-intercept of the graph of each equation.

1.  $y = 7x - 5$

2.  $y = -\frac{3}{5}x + 3$

3.  $y = \frac{2}{3}x$

4.  $3x + 4y = 4$

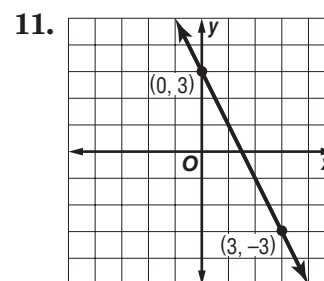
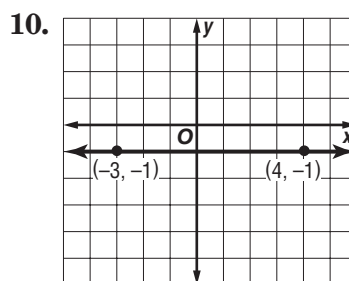
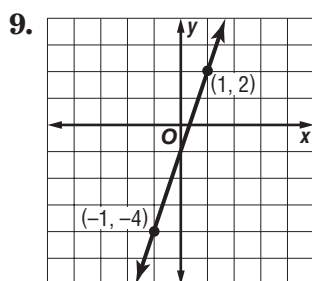
5.  $7y = 4x - 7$

6.  $3x - 2y + 6 = 0$

7.  $2x - y = 5$

8.  $2y = 6 - 5x$

Write an equation in slope-intercept form for each graph.



Write an equation in slope-intercept form for the line that satisfies each set of conditions.

12. slope 3, passes through  $(1, -3)$

13. slope  $-1$ , passes through  $(0, 0)$

14. slope  $-2$ , passes through  $(0, -5)$

15. slope 3, passes through  $(2, 0)$

16. passes through  $(-1, -2)$  and  $(-3, 1)$

17. passes through  $(-2, -4)$  and  $(1, 8)$

18. x-intercept 2, y-intercept  $-6$

19. x-intercept  $\frac{5}{2}$ , y-intercept 5

20. passes through  $(3, -1)$ , perpendicular to the graph of  $y = -\frac{1}{3}x - 4$ .

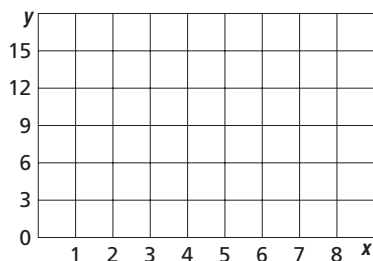
**2-5 Skills Practice*****Modeling Real-World Data: Using Scatter Plots***

For Exercises 1–3, complete parts a–c for each set of data.

- Draw a scatter plot.
- Use two ordered pairs to write a prediction equation.
- Use your prediction equation to predict the missing value.

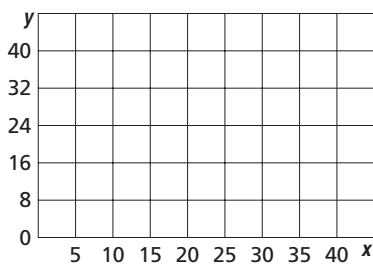
1.

$x$	$y$
1	1
3	5
4	7
6	11
7	12
8	15
10	?



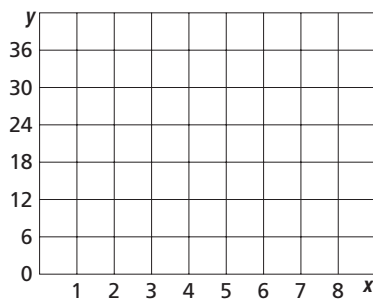
2.

$x$	$y$
5	9
10	17
20	22
25	30
35	38
40	44
50	?



3.

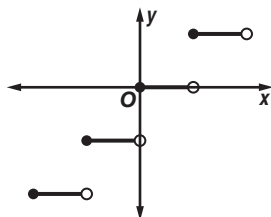
$x$	$y$
1	16
2	16
3	?
4	22
5	30
7	34
8	36



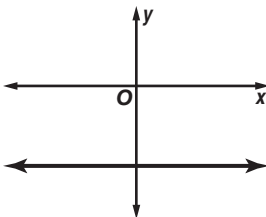
**2-6 Skills Practice*****Special Functions***

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

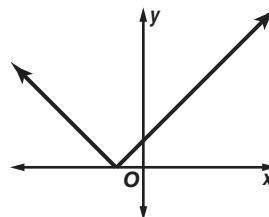
1.



2.

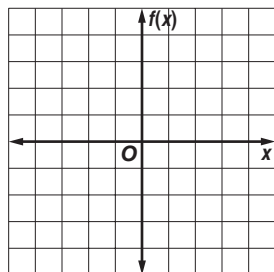


3.

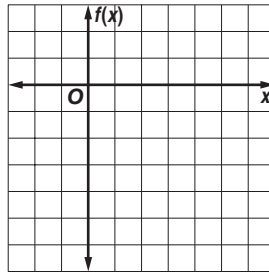


Graph each function. Identify the domain and range.

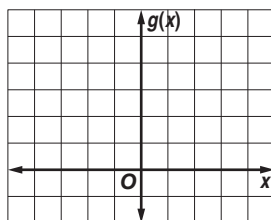
4.  $f(x) = \llbracket x + 1 \rrbracket$



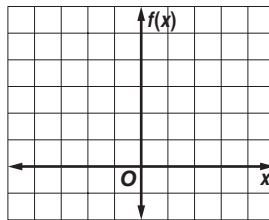
5.  $f(x) = \llbracket x - 3 \rrbracket$



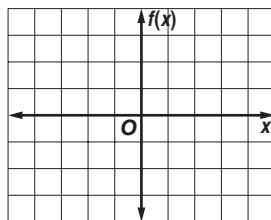
6.  $g(x) = 2|x|$



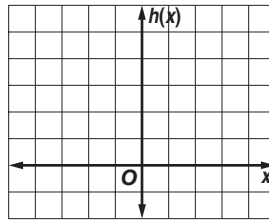
7.  $f(x) = |x| + 1$



8.  $f(x) = \begin{cases} x & \text{if } x < 0 \\ 2 & \text{if } x \geq 0 \end{cases}$



9.  $h(x) = \begin{cases} 3 & \text{if } x < -1 \\ x + 1 & \text{if } x > 1 \end{cases}$

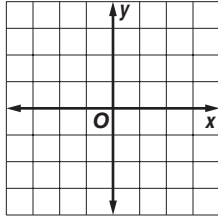


# 2-7 Skills Practice

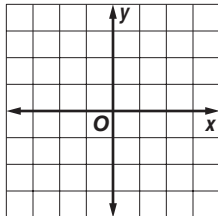
## Graphing Inequalities

Graph each inequality.

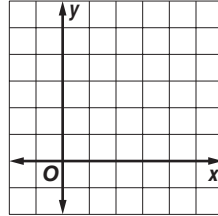
1.  $y > 1$



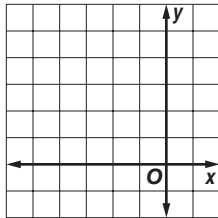
2.  $y \leq x + 2$



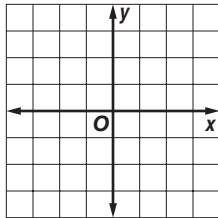
3.  $x + y \leq 4$



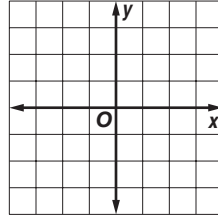
4.  $x + 3 < y$



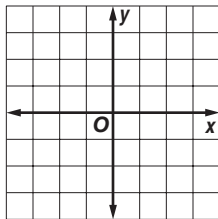
5.  $2 - y < x$



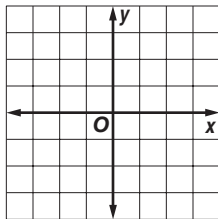
6.  $y \geq -x$



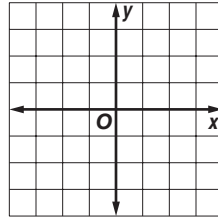
7.  $x - y > -2$



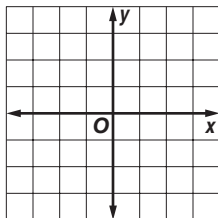
8.  $9x + 3y - 6 \leq 0$



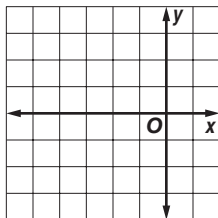
9.  $y + 1 \geq 2x$



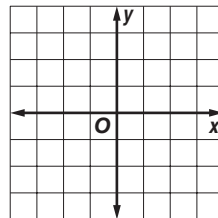
10.  $y - 7 \leq -9$



11.  $x > -5$



12.  $y > |x|$

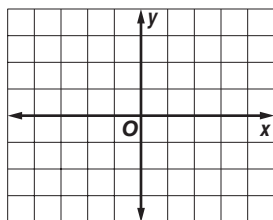


**3-1 Skills Practice****Solving Systems of Equations By Graphing**

Solve each system of equations by graphing.

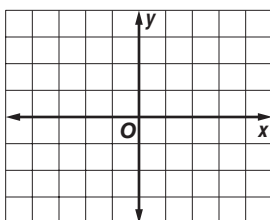
1.  $x = 2$

$y = 0$



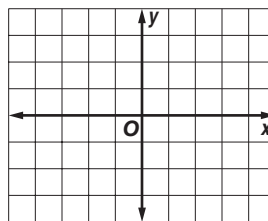
2.  $y = -3x + 6$

$y = 2x - 4$



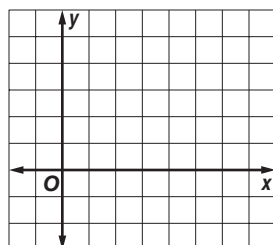
3.  $y = 4 - 3x$

$y = -\frac{1}{2}x - 1$



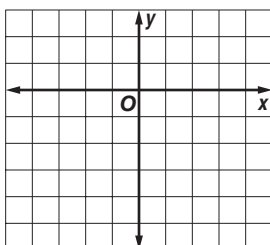
4.  $y = 4 - x$

$y = x - 2$



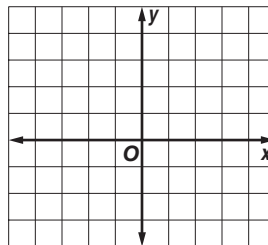
5.  $y = -2x + 2$

$y = \frac{1}{3}x - 5$



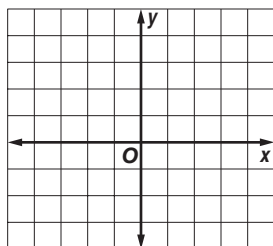
6.  $y = x$

$y = -3x + 4$



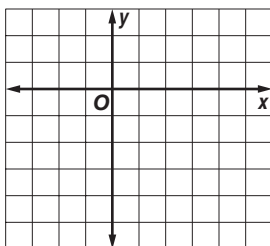
7.  $x + y = 3$

$x - y = 1$



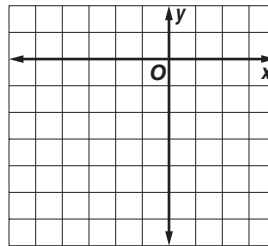
8.  $x - y = 4$

$2x - 5y = 8$



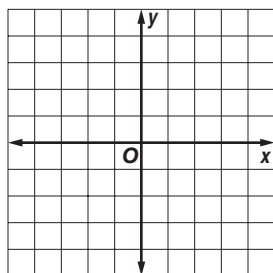
9.  $3x - 2y = 4$

$2x - y = 1$

Graph each system of equations and describe it as *consistent and independent*, *consistent and dependent*, or *inconsistent*.

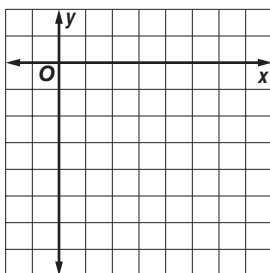
10.  $y = -3x$

$y = -3x + 2$



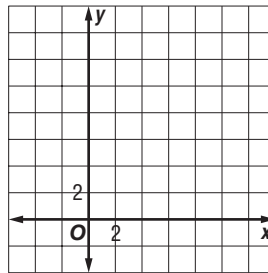
11.  $y = x - 5$

$-2x + 2y = -10$



12.  $2x - 5y = 10$

$3x + y = 15$



**3-2 Skills Practice*****Solving Systems of Equations Algebraically*****Solve each system of equations by using substitution.**

$$\begin{aligned} 1. \quad m + n &= 20 \\ m - n &= -4 \end{aligned}$$

$$\begin{aligned} 2. \quad x + 3y &= -3 \\ 4x + 3y &= 6 \end{aligned}$$

$$\begin{aligned} 3. \quad w - z &= 1 \\ 2w + 3z &= 12 \end{aligned}$$

$$\begin{aligned} 4. \quad 3r + s &= 5 \\ 2r - s &= 5 \end{aligned}$$

$$\begin{aligned} 5. \quad 2b + 3c &= -4 \\ b + c &= 3 \end{aligned}$$

$$\begin{aligned} 6. \quad x - y &= -5 \\ 3x + 4y &= 13 \end{aligned}$$

**Solve each system of equations by using elimination.**

$$\begin{aligned} 7. \quad 2p - q &= 17 \\ 3p + q &= 8 \end{aligned}$$

$$\begin{aligned} 8. \quad 2j - k &= 3 \\ 3j + k &= 2 \end{aligned}$$

$$\begin{aligned} 9. \quad 3c - 2d &= 2 \\ 3c + 4d &= 50 \end{aligned}$$

$$\begin{aligned} 10. \quad 2f + 3g &= 9 \\ f - g &= 2 \end{aligned}$$

$$\begin{aligned} 11. \quad -2x + y &= -1 \\ x + 2y &= 3 \end{aligned}$$

$$\begin{aligned} 12. \quad 2x - y &= 12 \\ 2x - y &= 6 \end{aligned}$$

**Solve each system of equations by using either substitution or elimination.**

$$\begin{aligned} 13. \quad -r + t &= 5 \\ -2r + t &= 4 \end{aligned}$$

$$\begin{aligned} 14. \quad 2x - y &= -5 \\ 4x + y &= 2 \end{aligned}$$

$$\begin{aligned} 15. \quad x - 3y &= -12 \\ 2x + y &= 11 \end{aligned}$$

$$\begin{aligned} 16. \quad 2p - 3q &= 6 \\ -2p + 3q &= -6 \end{aligned}$$

$$\begin{aligned} 17. \quad 6w - 8z &= 16 \\ 3w - 4z &= 8 \end{aligned}$$

$$\begin{aligned} 18. \quad c + d &= 6 \\ c - d &= 0 \end{aligned}$$

$$\begin{aligned} 19. \quad 2u + 4v &= -6 \\ u + 2v &= 3 \end{aligned}$$

$$\begin{aligned} 20. \quad 3a + b &= -1 \\ -3a + b &= 5 \end{aligned}$$

$$\begin{aligned} 21. \quad 2x + y &= 6 \\ 3x - 2y &= 16 \end{aligned}$$

$$\begin{aligned} 22. \quad 3y - z &= -6 \\ -3y - z &= 6 \end{aligned}$$

$$\begin{aligned} 23. \quad c + 2d &= -2 \\ -2c - 5d &= 3 \end{aligned}$$

$$\begin{aligned} 24. \quad 3r - 2s &= 1 \\ 2r - 3s &= 9 \end{aligned}$$

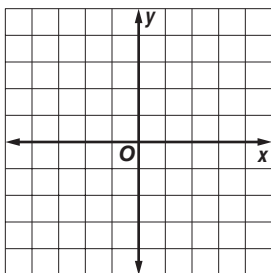
**25.** The sum of two numbers is 12. The difference of the same two numbers is  $-4$ . Find the numbers.

**26.** Twice a number minus a second number is  $-1$ . Twice the second number added to three times the first number is 9. Find the two numbers.

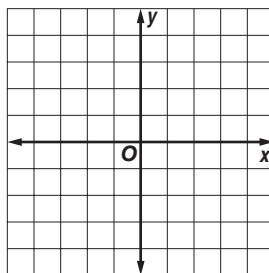
**3-3 Skills Practice****Solving Systems of Inequalities by Graphing**

Solve each system of inequalities by graphing.

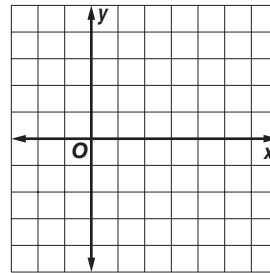
$$\begin{aligned} 1. \quad & x < 1 \\ & y \geq -1 \end{aligned}$$



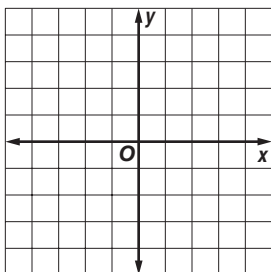
$$\begin{aligned} 2. \quad & x \geq -3 \\ & y \geq -3 \end{aligned}$$



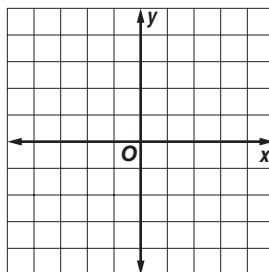
$$\begin{aligned} 3. \quad & x \leq 2 \\ & x > 4 \end{aligned}$$



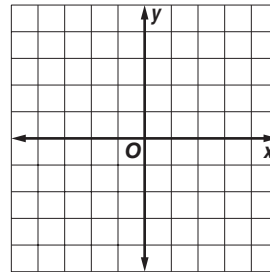
$$\begin{aligned} 4. \quad & y \geq x \\ & y \geq -x \end{aligned}$$



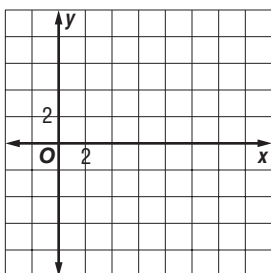
$$\begin{aligned} 5. \quad & y < -4x \\ & y \geq 3x - 2 \end{aligned}$$



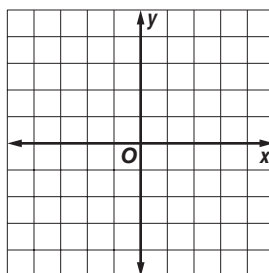
$$\begin{aligned} 6. \quad & x - y \geq -1 \\ & 3x - y \leq 4 \end{aligned}$$



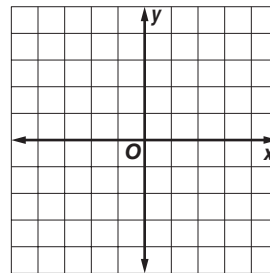
$$\begin{aligned} 7. \quad & y < 3 \\ & x + 2y < 12 \end{aligned}$$



$$\begin{aligned} 8. \quad & y < -2x + 3 \\ & y \leq x - 2 \end{aligned}$$



$$\begin{aligned} 9. \quad & x - y \leq 4 \\ & 2x + y < 4 \end{aligned}$$



Find the coordinates of the vertices of the figure formed by each system of inequalities.

$$\begin{aligned} 10. \quad & y < 0 \\ & x < 0 \\ & y \geq -x - 1 \end{aligned}$$

$$\begin{aligned} 11. \quad & y < 3 - x \\ & y \geq 3 \\ & x > -5 \end{aligned}$$

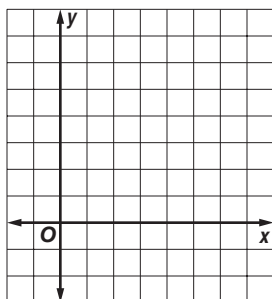
$$\begin{aligned} 12. \quad & x \geq -2 \\ & y > x - 2 \\ & x + y \leq 2 \end{aligned}$$



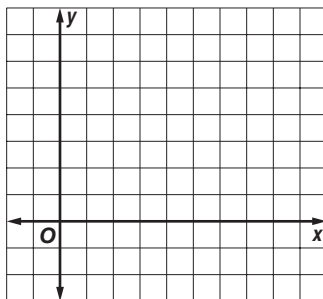
**3-4 Skills Practice****Linear Programming**

Graph each system of inequalities. Name the coordinates of the vertices of the feasible region. Find the maximum and minimum values of the given function for this region.

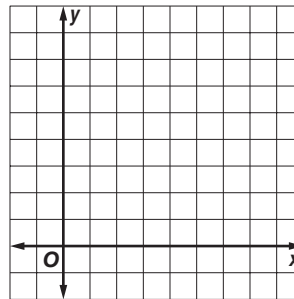
1.  $x \geq 2$   
 $x \leq 5$   
 $y \geq 1$   
 $y \leq 4$   
 $f(x, y) = x + y$



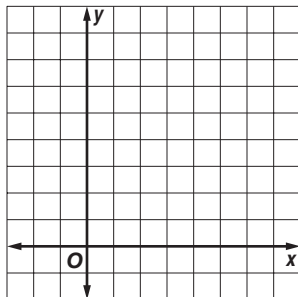
2.  $x \geq 1$   
 $y \leq 6$   
 $y \geq x - 2$   
 $f(x, y) = x - y$



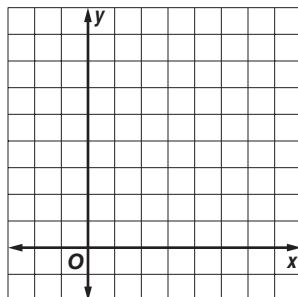
3.  $x \geq 0$   
 $y \geq 0$   
 $y \leq 7 - x$   
 $f(x, y) = 3x + y$



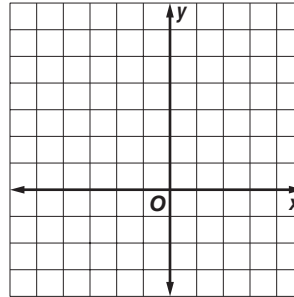
4.  $x \geq -1$   
 $x + y \leq 6$   
 $f(x, y) = x + 2y$



5.  $y \leq 2x$   
 $y \geq 6 - x$   
 $y \leq 6$   
 $f(x, y) = 4x + 3y$



6.  $y \geq -x - 2$   
 $y \geq 3x + 2$   
 $y \leq x + 4$   
 $f(x, y) = -3x + 5y$



7. **MANUFACTURING** A backpack manufacturer produces an internal frame pack and an external frame pack. Let  $x$  represent the number of internal frame packs produced in one hour and let  $y$  represent the number of external frame packs produced in one hour. Then the inequalities  $x + 3y \leq 18$ ,  $2x + y \leq 16$ ,  $x \geq 0$ , and  $y \geq 0$  describe the constraints for manufacturing both packs. Use the profit function  $f(x, y) = 50x + 80y$  and the constraints given to determine the maximum profit for manufacturing both backpacks for the given constraints.

**3-5 Skills Practice*****Solving Systems of Equations in Three Variables***

Solve each system of equations.

1.  $2a + c = -10$

$b - c = 15$

$a - 2b + c = -5$

2.  $x + y + z = 3$

$13x + 2z = 2$

$-x - 5z = -5$

3.  $2x + 5y + 2z = 6$

$5x - 7y = -29$

$z = 1$

4.  $x + 4y - z = 1$

$3x - y + 8z = 0$

$x + 4y - z = 10$

5.  $-2z = -6$

$2x + 3y - z = -2$

$x + 2y + 3z = 9$

6.  $3x - 2y + 2z = -2$

$x + 6y - 2z = -2$

$x + 2y = 0$

7.  $-x - 5z = -5$

$y - 3x = 0$

$13x + 2z = 2$

8.  $-3r + 2t = 1$

$4r + s - 2t = -6$

$r + s + 4t = 3$

9.  $x - y + 3z = 3$

$-2x + 2y - 6z = 6$

$y - 5z = -3$

10.  $5m + 3n + p = 4$

$3m + 2n = 0$

$2m - n + 3p = 8$

11.  $2x + 2y + 2z = -2$

$2x + 3y + 2z = 4$

$x + y + z = -1$

12.  $x + 2y - z = 4$

$3x - y + 2z = 3$

$-x + 3y + z = 6$

13.  $3x - 2y + z = 1$

$-x + y - z = 2$

$5x + 2y + 10z = 39$

14.  $3x - 5y + 2z = -12$

$x + 4y - 2z = 8$

$-3x + 5y - 2z = 12$

15.  $2x + y + 3z = -2$

$x - y - z = -3$

$3x - 2y + 3z = -12$

16.  $2x - 4y + 3z = 0$

$x - 2y - 5z = 13$

$5x + 3y - 2z = 19$

17.  $-2x + y + 2z = 2$

$3x + 3y + z = 0$

$x + y + z = 2$

18.  $x - 2y + 2z = -1$

$x + 2y - z = 6$

$-3x + 6y - 6z = 3$

19. The sum of three numbers is 18. The sum of the first and second numbers is 15, and the first number is 3 times the third number. Find the numbers.

**4-1 Skills Practice*****Introduction to Matrices*****State the dimensions of each matrix.**

1.  $\begin{bmatrix} 3 & 2 & 4 \\ -1 & 4 & 0 \end{bmatrix}$

2.  $[0 \quad 15]$

3.  $\begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}$

4.  $\begin{bmatrix} 6 & 1 & 2 \\ -3 & 4 & 5 \\ -2 & 7 & 9 \end{bmatrix}$

5.  $\begin{bmatrix} 9 & 3 & -3 & -6 \\ 3 & 4 & -4 & 5 \end{bmatrix}$

6.  $\begin{bmatrix} -1 \\ -1 \\ -1 \\ -3 \end{bmatrix}$

**Solve each equation.**

7.  $[5x \quad 3y] = [15 \quad 12]$

8.  $[3x - 2] = [7]$

9.  $\begin{bmatrix} 7x \\ 14 \end{bmatrix} = \begin{bmatrix} -14 \\ 2y \end{bmatrix}$

10.  $[2x \quad -8y \quad z] = [10 \quad 16 \quad -1]$

11.  $\begin{bmatrix} 8 - x \\ 2y - 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

12.  $\begin{bmatrix} 56 - 20 \\ 56 - 6y \end{bmatrix} = \begin{bmatrix} 10x \\ 32 \end{bmatrix}$

13.  $\begin{bmatrix} 5x \\ 24 \end{bmatrix} = \begin{bmatrix} -20 \\ 8y \end{bmatrix}$

14.  $\begin{bmatrix} 3x + 2 \\ 7y - 2 \end{bmatrix} = \begin{bmatrix} 5x + 2 \\ 3y - 10 \end{bmatrix}$

15.  $\begin{bmatrix} 4x - 1 \\ 9y + 5 \end{bmatrix} = \begin{bmatrix} 3x \\ y - 3 \end{bmatrix}$

16.  $\begin{bmatrix} 3x + 1 & 18 \\ 12 & 4z \end{bmatrix} = \begin{bmatrix} 7 & 2y - 4 \\ 12 & 28 \end{bmatrix}$

17.  $\begin{bmatrix} x \\ 16 \\ 3z \end{bmatrix} = \begin{bmatrix} 9 \\ 4y \\ 9 \end{bmatrix}$

18.  $\begin{bmatrix} 4y - 5x \\ 8z \end{bmatrix} = \begin{bmatrix} 4x + 1 \\ 13 \\ 4z \end{bmatrix}$

19.  $\begin{bmatrix} x \\ y + 2 \end{bmatrix} = \begin{bmatrix} 6y \\ x \end{bmatrix}$

20.  $\begin{bmatrix} x \\ 3y \end{bmatrix} = \begin{bmatrix} 4y \\ x - 3 \end{bmatrix}$

**4-2 Skills Practice****Operations with Matrices**

Perform the indicated matrix operations. If the matrix does not exist, write *impossible*.

1.  $[5 \ -4] + [4 \ 5]$

2.  $\begin{bmatrix} 8 & 3 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -7 \\ 6 & 2 \end{bmatrix}$

3.  $[3 \ 1 \ 6] + \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

4.  $\begin{bmatrix} 5 & -1 & 2 \\ 1 & 8 & -6 \end{bmatrix} + \begin{bmatrix} 9 & 9 & 2 \\ 4 & 6 & 4 \end{bmatrix}$

5.  $3[9 \ 4 \ -3]$

6.  $[6 \ -3] - 4[4 \ 7]$

7.  $-2\begin{bmatrix} -2 & 5 \\ 5 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

8.  $3\begin{bmatrix} 8 \\ 0 \\ -3 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}$

9.  $5\begin{bmatrix} -4 & 6 \\ 10 & 1 \\ -1 & 1 \end{bmatrix} + 2\begin{bmatrix} 6 & 5 \\ -3 & -2 \\ 1 & 0 \end{bmatrix}$

10.  $3\begin{bmatrix} 3 & 1 & 3 \\ -4 & 7 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & -1 & 5 \\ 6 & 6 & -3 \end{bmatrix}$

Use  $A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix}$ , and  $C = \begin{bmatrix} -3 & 4 \\ 3 & 1 \end{bmatrix}$  to find the following.

11.  $A + B$

12.  $B - C$

13.  $B - A$

14.  $A + B + C$

15.  $3B$

16.  $-5C$

17.  $A - 4C$

18.  $2B + 3A$

**4-3 Skills Practice*****Multiplying Matrices***

Determine whether each matrix product is defined. If so, state the dimensions of the product.

1.  $A_{2 \times 5} \cdot B_{5 \times 1}$

2.  $M_{1 \times 3} \cdot N_{3 \times 2}$

3.  $B_{3 \times 2} \cdot A_{3 \times 2}$

4.  $R_{4 \times 4} \cdot S_{4 \times 1}$

5.  $X_{3 \times 3} \cdot Y_{3 \times 4}$

6.  $A_{6 \times 4} \cdot B_{4 \times 5}$

Find each product, if possible.

7.  $\begin{bmatrix} 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

8.  $\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$

9.  $\begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

10.  $\begin{bmatrix} 3 \\ -2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -1 & 1 \end{bmatrix}$

11.  $\begin{bmatrix} -3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

12.  $\begin{bmatrix} -1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 & -2 \end{bmatrix}$

13.  $\begin{bmatrix} 5 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 8 \end{bmatrix}$

14.  $\begin{bmatrix} 2 & -2 \\ 4 & 5 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 3 & 0 \end{bmatrix}$

15.  $\begin{bmatrix} -4 & 4 \\ -2 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -3 \\ 0 & 2 \end{bmatrix}$

16.  $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

Use  $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & 2 \\ 5 & 1 \end{bmatrix}$ ,  $C = \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}$ , and scalar  $c = 2$  to determine whether the following equations are true for the given matrices.

17.  $(AC)c = A(Cc)$

18.  $AB = BA$

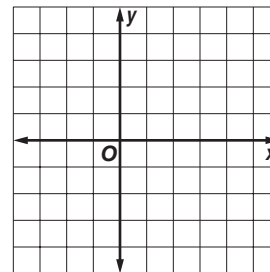
19.  $B(A + C) = AB + BC$

20.  $(A - B)c = Ac - Bc$

**4-4 Skills Practice****Transformations with Matrices****For Exercises 1–3, use the following information.**

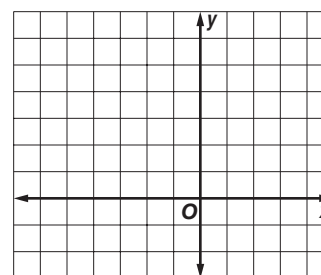
Triangle  $ABC$  with vertices  $A(2, 3)$ ,  $B(0, 4)$ , and  $C(-3, -3)$  is translated 3 units right and 1 unit down.

1. Write the translation matrix.
2. Find the coordinates of  $\triangle A'B'C'$ .
3. Graph the preimage and the image.

**For Exercises 4–6, use the following information.**

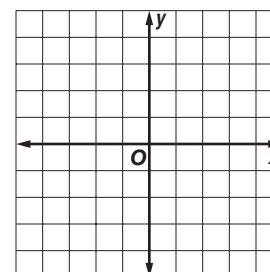
The vertices of  $\triangle RST$  are  $R(-3, 1)$ ,  $S(2, -1)$ , and  $T(1, 3)$ . The triangle is dilated so that its perimeter is twice the original perimeter.

4. Write the coordinates of  $\triangle RST$  in a vertex matrix.
5. Find the coordinates of the image  $\triangle R'S'T'$ .
6. Graph  $\triangle RST$  and  $\triangle R'S'T'$ .

**For Exercises 7–10, use the following information.**

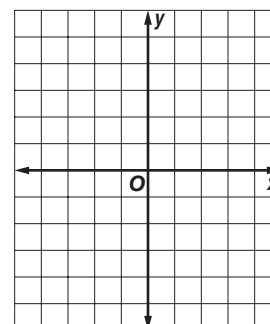
The vertices of  $\triangle DEF$  are  $D(4, 0)$ ,  $E(0, -1)$ , and  $F(2, 3)$ . The triangle is reflected over the  $x$ -axis.

7. Write the coordinates of  $\triangle DEF$  in a vertex matrix.
8. Write the reflection matrix for this situation.
9. Find the coordinates of  $\triangle D'E'F'$ .
10. Graph  $\triangle DEF$  and  $\triangle D'E'F'$ .

**For Exercises 11–14, use the following information.**

Triangle  $XYZ$  with vertices  $X(1, -3)$ ,  $Y(-4, 1)$ , and  $Z(-2, 5)$  is rotated  $180^\circ$  counterclockwise about the origin.

11. Write the coordinates of the triangle in a vertex matrix.
12. Write the rotation matrix for this situation.
13. Find the coordinates of  $\triangle X'Y'Z'$ .
14. Graph the preimage and the image.



**4-5 Skills Practice*****Determinants*****Find the value of each determinant.**

1.  $\begin{vmatrix} 5 & 2 \\ 1 & 3 \end{vmatrix}$

2.  $\begin{vmatrix} 10 & 9 \\ 5 & 8 \end{vmatrix}$

3.  $\begin{vmatrix} 1 & 6 \\ 1 & 7 \end{vmatrix}$

4.  $\begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix}$

5.  $\begin{vmatrix} 0 & 9 \\ 5 & 8 \end{vmatrix}$

6.  $\begin{vmatrix} 3 & 12 \\ 2 & 8 \end{vmatrix}$

7.  $\begin{vmatrix} -5 & 2 \\ 8 & -6 \end{vmatrix}$

8.  $\begin{vmatrix} -3 & 1 \\ 8 & -7 \end{vmatrix}$

9.  $\begin{vmatrix} 9 & -2 \\ -4 & 1 \end{vmatrix}$

10.  $\begin{vmatrix} 1 & -5 \\ 1 & 6 \end{vmatrix}$

11.  $\begin{vmatrix} 1 & -3 \\ -3 & 4 \end{vmatrix}$

12.  $\begin{vmatrix} -12 & 4 \\ 1 & 4 \end{vmatrix}$

13.  $\begin{vmatrix} 3 & -5 \\ 6 & -11 \end{vmatrix}$

14.  $\begin{vmatrix} -1 & -3 \\ 5 & -2 \end{vmatrix}$

15.  $\begin{vmatrix} -1 & -14 \\ 5 & 2 \end{vmatrix}$

16.  $\begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix}$

17.  $\begin{vmatrix} 2 & 2 \\ -1 & 4 \end{vmatrix}$

18.  $\begin{vmatrix} -1 & 6 \\ 2 & 5 \end{vmatrix}$

**Evaluate each determinant using expansion by minors.**

19.  $\begin{vmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 2 & 3 & -2 \end{vmatrix}$

20.  $\begin{vmatrix} 6 & -1 & 1 \\ 5 & 2 & -1 \\ 1 & 3 & -2 \end{vmatrix}$

21.  $\begin{vmatrix} 2 & 6 & 1 \\ 3 & 5 & -1 \\ 2 & 1 & -2 \end{vmatrix}$

**Evaluate each determinant using diagonals.**

22.  $\begin{vmatrix} 2 & -1 & 6 \\ 3 & 2 & 5 \\ 2 & 3 & 1 \end{vmatrix}$

23.  $\begin{vmatrix} 3 & -1 & 2 \\ 1 & 0 & 4 \\ 3 & -2 & 0 \end{vmatrix}$

24.  $\begin{vmatrix} 3 & 2 & 2 \\ 1 & -1 & 4 \\ 3 & -1 & 0 \end{vmatrix}$

**4-6 Skills Practice*****Cramer's Rule*****Use Cramer's Rule to solve each system of equations.**

$$\begin{aligned} 1. \quad & 2a + 3b = 6 \\ & 2a + b = -2 \end{aligned}$$

$$\begin{aligned} 2. \quad & 3x + y = 2 \\ & 2x - y = 3 \end{aligned}$$

$$\begin{aligned} 3. \quad & 2m + 3n = -6 \\ & m - 3n = 6 \end{aligned}$$

$$\begin{aligned} 4. \quad & x - y = 2 \\ & 2x + 3y = 9 \end{aligned}$$

$$\begin{aligned} 5. \quad & 2x + y = 4 \\ & 7x - 2y = 3 \end{aligned}$$

$$\begin{aligned} 6. \quad & 3r - s = 7 \\ & 5r - 2s = 8 \end{aligned}$$

$$\begin{aligned} 7. \quad & 4g + 5h = 1 \\ & g + 3h = 2 \end{aligned}$$

$$\begin{aligned} 8. \quad & 7x + 5y = -8 \\ & 9x + 2y = 3 \end{aligned}$$

$$\begin{aligned} 9. \quad & 3x - 4y = 2 \\ & 4x - 3y = 12 \end{aligned}$$

$$\begin{aligned} 10. \quad & 2x - y = 5 \\ & 3x + y = 5 \end{aligned}$$

$$\begin{aligned} 11. \quad & 3p - 6q = 18 \\ & 2p + 3q = 5 \end{aligned}$$

$$\begin{aligned} 12. \quad & x - 2y = -1 \\ & 2x + y = 3 \end{aligned}$$

$$\begin{aligned} 13. \quad & 5c + 3d = 5 \\ & 2c + 9d = 2 \end{aligned}$$

$$\begin{aligned} 14. \quad & 5t + 2v = 2 \\ & 2t + 3v = -8 \end{aligned}$$

$$\begin{aligned} 15. \quad & 5a - 2b = 14 \\ & 3a + 4b = 11 \end{aligned}$$

$$\begin{aligned} 16. \quad & 65w - 8z = 83 \\ & 9w + 4z = 0 \end{aligned}$$

**17. GEOMETRY** The two sides of an angle are contained in the lines whose equations are  $3x + 2y = 4$  and  $x - 3y = 5$ . Find the coordinates of the vertex of the angle.

**Use Cramer's Rule to solve each system of equations.**

$$\begin{aligned} 18. \quad & a + b + 5c = 2 \\ & 3a + b + 2c = 3 \\ & 4a + 2b - c = -3 \end{aligned}$$

$$\begin{aligned} 19. \quad & x + 3y - z = 5 \\ & 2x + 5y - z = 12 \\ & x - 2y - 3z = -13 \end{aligned}$$

$$\begin{aligned} 20. \quad & 3c - 5d + 2e = 4 \\ & 2c - 3d + 4e = -3 \\ & 4c - 2d + 3e = 0 \end{aligned}$$

$$\begin{aligned} 21. \quad & r - 4s - t = 6 \\ & 2r - s + 3t = 0 \\ & 3r - 2s + t = 4 \end{aligned}$$



**4-7 Skills Practice*****Identity and Inverse Matrices*****Determine whether each pair of matrices are inverses.**

1.  $X = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, Y = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

2.  $P = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, Q = \begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$

3.  $M = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}, N = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$

4.  $A = \begin{bmatrix} -2 & 5 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}$

5.  $V = \begin{bmatrix} 0 & 7 \\ -7 & 0 \end{bmatrix}, W = \begin{bmatrix} 0 & -\frac{1}{7} \\ \frac{1}{7} & 0 \end{bmatrix}$

6.  $X = \begin{bmatrix} -1 & 4 \\ 1 & 2 \end{bmatrix}, Y = \begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$

7.  $G = \begin{bmatrix} 4 & -3 \\ 1 & 2 \end{bmatrix}, H = \begin{bmatrix} \frac{2}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{4}{11} \end{bmatrix}$

8.  $D = \begin{bmatrix} -4 & -4 \\ -4 & 4 \end{bmatrix}, E = \begin{bmatrix} -0.125 & -0.125 \\ -0.125 & -0.125 \end{bmatrix}$

**Find the inverse of each matrix, if it exists.**

9.  $\begin{bmatrix} 0 & 2 \\ 4 & 0 \end{bmatrix}$

10.  $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

11.  $\begin{bmatrix} 9 & 3 \\ 6 & 2 \end{bmatrix}$

12.  $\begin{bmatrix} -2 & -4 \\ 6 & 0 \end{bmatrix}$

13.  $\begin{bmatrix} 1 & -1 \\ 3 & 3 \end{bmatrix}$

14.  $\begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix}$

15.  $\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$

16.  $\begin{bmatrix} -4 & 5 \\ 1 & 2 \end{bmatrix}$

17.  $\begin{bmatrix} 0 & -7 \\ -7 & 0 \end{bmatrix}$

18.  $\begin{bmatrix} 10 & 8 \\ 5 & 4 \end{bmatrix}$

19.  $\begin{bmatrix} 10 & 8 \\ 10 & -8 \end{bmatrix}$

20.  $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

**4-8 Skills Practice*****Using Matrices to Solve Systems of Equations*****Write a matrix equation for each system of equations.**

$$\begin{aligned} 1. \quad x + y &= 5 \\ 2x - y &= 1 \end{aligned}$$

$$\begin{aligned} 2. \quad 3a + 8b &= 16 \\ 4a + 3b &= 3 \end{aligned}$$

$$\begin{aligned} 3. \quad m + 3n &= -3 \\ 4m + 3n &= 6 \end{aligned}$$

$$\begin{aligned} 4. \quad 2c + 3d &= 6 \\ 3c - 4d &= 7 \end{aligned}$$

$$\begin{aligned} 5. \quad r - s &= 1 \\ 2r + 3s &= 12 \end{aligned}$$

$$\begin{aligned} 6. \quad x + y &= 5 \\ 3x + 2y &= 10 \end{aligned}$$

$$\begin{aligned} 7. \quad 6x - y + 2z &= -4 \\ -3x + 2y - z &= 10 \\ x + y + z &= 3 \end{aligned}$$

$$\begin{aligned} 8. \quad a - b + c &= 5 \\ 3a + 2b - c &= 0 \\ 2a + 3b &= 8 \end{aligned}$$

**Solve each matrix equation or system of equations by using inverse matrices.**

$$9. \begin{bmatrix} 1 & 3 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} w \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ -1 \end{bmatrix}$$

$$10. \begin{bmatrix} 4 & 3 \\ 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$

$$11. \begin{bmatrix} 5 & 8 \\ 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$$

$$12. \begin{bmatrix} 7 & -3 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 15 \\ 23 \end{bmatrix}$$

$$13. \begin{bmatrix} 3 & 12 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 25 \\ 12 \end{bmatrix}$$

$$14. \begin{bmatrix} 5 & 6 \\ 12 & -6 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \end{bmatrix}$$

$$\begin{aligned} 15. \quad p - 3q &= 6 \\ 2p + 3q &= -6 \end{aligned}$$

$$\begin{aligned} 16. \quad -x - 3y &= 2 \\ -4x - 5y &= 1 \end{aligned}$$

$$\begin{aligned} 17. \quad 2m + 2n &= -8 \\ 6m + 4n &= -18 \end{aligned}$$

$$\begin{aligned} 18. \quad -3a + b &= -9 \\ 5a - 2b &= 14 \end{aligned}$$

**5-1 Skills Practice****Graphing Quadratic Functions**

For each quadratic function, find the  $y$ -intercept, the equation of the axis of symmetry, and the  $x$ -coordinate of the vertex.

1.  $f(x) = 3x^2$

2.  $f(x) = x^2 + 1$

3.  $f(x) = -x^2 + 6x - 15$

4.  $f(x) = 2x^2 - 11$

5.  $f(x) = x^2 - 10x + 5$

6.  $f(x) = -2x^2 + 8x + 7$

Complete parts a–c for each quadratic function.

a. Find the  $y$ -intercept, the equation of the axis of symmetry, and the  $x$ -coordinate of the vertex.

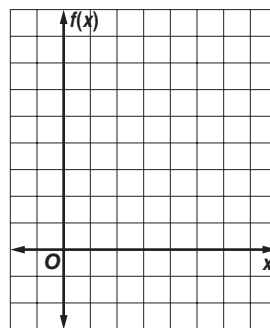
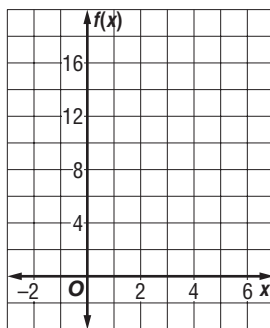
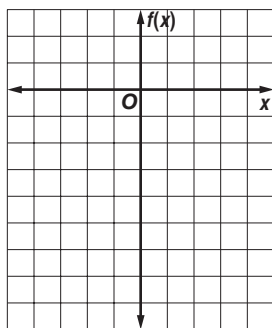
b. Make a table of values that includes the vertex.

c. Use this information to graph the function.

7.  $f(x) = -2x^2$

8.  $f(x) = x^2 - 4x + 4$

9.  $f(x) = x^2 - 6x + 8$



Determine whether each function has a maximum or a minimum value, and find the maximum or minimum value. Then state the domain and range of the function.

10.  $f(x) = 6x^2$

11.  $f(x) = -8x^2$

12.  $f(x) = x^2 + 2x$

13.  $f(x) = x^2 + 2x + 15$

14.  $f(x) = -x^2 + 4x - 1$

15.  $f(x) = x^2 + 2x - 3$

16.  $f(x) = -2x^2 + 4x - 3$

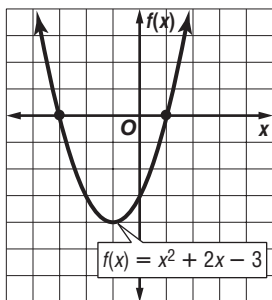
17.  $f(x) = 3x^2 + 12x + 3$

18.  $f(x) = 2x^2 + 4x + 1$

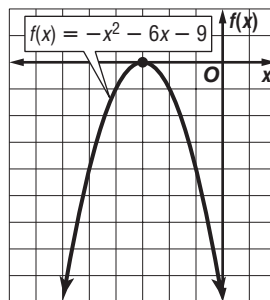
**5-2 Skills Practice****Solving Quadratic Equations By Graphing**

Use the related graph of each equation to determine its solutions.

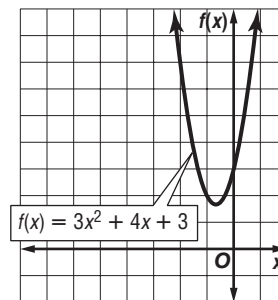
1.  $x^2 + 2x - 3 = 0$



2.  $-x^2 - 6x - 9 = 0$

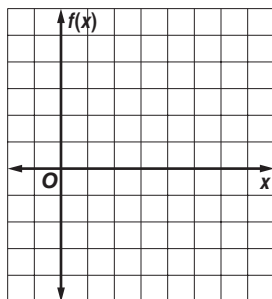


3.  $3x^2 + 4x + 3 = 0$

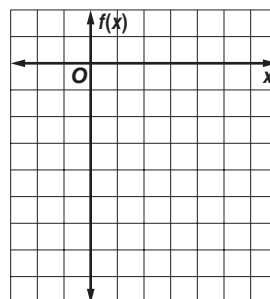


Solve each equation by graphing. If exact roots cannot be found, state the consecutive integers between which the roots are located.

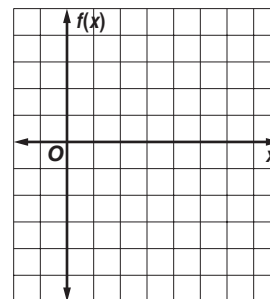
4.  $x^2 - 6x + 5 = 0$



5.  $-x^2 + 2x - 4 = 0$

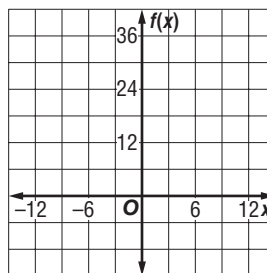
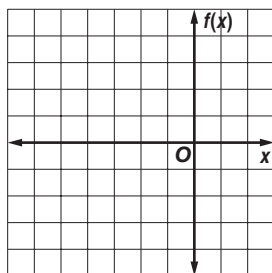


6.  $x^2 - 6x + 4 = 0$



Use a quadratic equation to find two real numbers that satisfy each situation, or show that no such numbers exist.

7. Their sum is  $-4$ , and their product is  $0$ .      8. Their sum is  $0$ , and their product is  $-36$ .



**5-3 Skills Practice*****Solving Quadratic Equations by Factoring***

Write a quadratic equation with the given roots. Write the equation in standard form.

1. 1, 4

2. 6, -9

3. -2, -5

4. 0, 7

5.  $-\frac{1}{3}$ , -3

6.  $-\frac{1}{2}$ ,  $\frac{3}{4}$

Factor each polynomial.

7.  $m^2 + 7m - 18$

8.  $2x^2 - 3x - 5$

9.  $4z^2 + 4z - 15$

10.  $4p^2 + 4p - 24$

11.  $3y^2 + 21y + 36$

12.  $c^2 - 100$

Solve each equation by factoring.

13.  $x^2 = 64$

14.  $x^2 - 100 = 0$

15.  $x^2 - 3x + 2 = 0$

16.  $x^2 - 4x + 3 = 0$

17.  $x^2 + 2x - 3 = 0$

18.  $x^2 - 3x - 10 = 0$

19.  $x^2 - 6x + 5 = 0$

20.  $x^2 - 9x = 0$

21.  $x^2 - 4x = 21$

22.  $2x^2 + 5x - 3 = 0$

23.  $4x^2 + 5x - 6 = 0$

24.  $3x^2 - 13x - 10 = 0$

**25. NUMBER THEORY** Find two consecutive integers whose product is 272.

**5-4****Skills Practice****Complex Numbers****Simplify.**

1.  $\sqrt{99}$

2.  $\sqrt{\frac{27}{49}}$

3.  $\sqrt{52x^3y^5}$

4.  $\sqrt{-108x^7}$

5.  $\sqrt{-81x^6}$

6.  $\sqrt{-23} \cdot \sqrt{-46}$

7.  $(3i)(-2i)(5i)$

8.  $i^{11}$

9.  $i^{65}$

10.  $(7 - 8i) + (-12 - 4i)$

11.  $(-3 + 5i) + (18 - 7i)$

12.  $(10 - 4i) - (7 + 3i)$

13.  $(7 - 6i)(2 - 3i)$

14.  $(3 + 4i)(3 - 4i)$

15.  $\frac{8 - 6i}{3i}$

16.  $\frac{3i}{4 + 2i}$

**Solve each equation.**

17.  $3x^2 + 3 = 0$

18.  $5x^2 + 125 = 0$

19.  $4x^2 + 20 = 0$

20.  $-x^2 - 16 = 0$

21.  $x^2 + 18 = 0$

22.  $8x^2 + 96 = 0$

**Find the values of  $m$  and  $n$  that make each equation true.**

23.  $20 - 12i = 5m + 4ni$

24.  $m - 16i = 3 - 2ni$

25.  $(4 + m) + 2ni = 9 + 14i$

26.  $(3 - n) + (7m - 14)i = 1 + 7i$

**5-5 Skills Practice****Completing the Square**

Solve each equation by using the Square Root Property.

1.  $x^2 - 8x + 16 = 1$

2.  $x^2 + 4x + 4 = 1$

3.  $x^2 + 12x + 36 = 25$

4.  $4x^2 - 4x + 1 = 9$

5.  $x^2 + 4x + 4 = 2$

6.  $x^2 - 2x + 1 = 5$

7.  $x^2 - 6x + 9 = 7$

8.  $x^2 + 16x + 64 = 15$

Find the value of  $c$  that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

9.  $x^2 + 10x + c$

10.  $x^2 - 14x + c$

11.  $x^2 + 24x + c$

12.  $x^2 + 5x + c$

13.  $x^2 - 9x + c$

14.  $x^2 - x + c$

Solve each equation by completing the square.

15.  $x^2 - 13x + 36 = 0$

16.  $x^2 + 3x = 0$

17.  $x^2 + x - 6 = 0$

18.  $x^2 - 4x - 13 = 0$

19.  $2x^2 + 7x - 4 = 0$

20.  $3x^2 + 2x - 1 = 0$

21.  $x^2 + 3x - 6 = 0$

22.  $x^2 - x - 3 = 0$

23.  $x^2 = -11$

24.  $x^2 - 2x + 4 = 0$

**5-6 Skills Practice*****The Quadratic Formula and the Discriminant***

Complete parts a–c for each quadratic equation.

a. Find the value of the discriminant.

b. Describe the number and type of roots.

c. Find the exact solutions by using the Quadratic Formula.

1.  $x^2 - 8x + 16 = 0$

2.  $x^2 - 11x - 26 = 0$

3.  $3x^2 - 2x = 0$

4.  $20x^2 + 7x - 3 = 0$

5.  $5x^2 - 6 = 0$

6.  $x^2 - 6 = 0$

7.  $x^2 + 8x + 13 = 0$

8.  $5x^2 - x - 1 = 0$

9.  $x^2 - 2x - 17 = 0$

10.  $x^2 + 49 = 0$

11.  $x^2 - x + 1 = 0$

12.  $2x^2 - 3x = -2$

Solve each equation by using the method of your choice. Find exact solutions.

13.  $x^2 = 64$

14.  $x^2 - 30 = 0$

15.  $x^2 - x = 30$

16.  $16x^2 - 24x - 27 = 0$

17.  $x^2 - 4x - 11 = 0$

18.  $x^2 - 8x - 17 = 0$

19.  $x^2 + 25 = 0$

20.  $3x^2 + 36 = 0$

21.  $2x^2 + 10x + 11 = 0$

22.  $2x^2 - 7x + 4 = 0$

23.  $8x^2 + 1 = 4x$

24.  $2x^2 + 2x + 3 = 0$

**25. PARACHUTING** Ignoring wind resistance, the distance  $d(t)$  in feet that a parachutist falls in  $t$  seconds can be estimated using the formula  $d(t) = 16t^2$ . If a parachutist jumps from an airplane and falls for 1100 feet before opening her parachute, how many seconds pass before she opens the parachute?



**5-7 Skills Practice****Analyzing Graphs of Quadratic Functions**

Write each quadratic function in vertex form, if not already in that form. Then identify the vertex, axis of symmetry, and direction of opening.

1.  $y = (x - 2)^2$

2.  $y = -x^2 + 4$

3.  $y = x^2 - 6$

4.  $y = -3(x + 5)^2$

5.  $y = -5x^2 + 9$

6.  $y = (x - 2)^2 - 18$

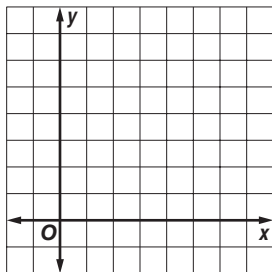
7.  $y = x^2 - 2x - 5$

8.  $y = x^2 + 6x + 2$

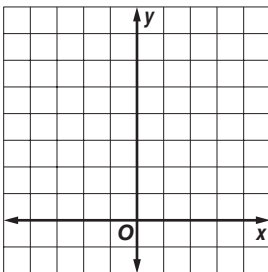
9.  $y = -3x^2 + 24x$

Graph each function.

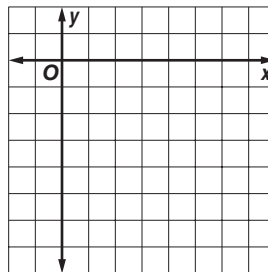
10.  $y = (x - 3)^2 - 1$



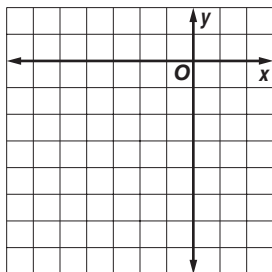
11.  $y = (x + 1)^2 + 2$



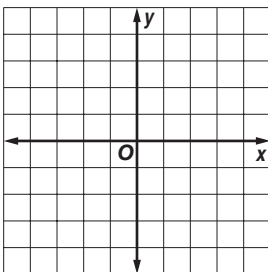
12.  $y = -(x - 4)^2 - 4$



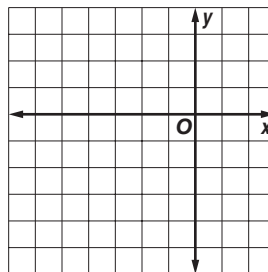
13.  $y = -\frac{1}{2}(x + 2)^2$



14.  $y = -3x^2 + 4$



15.  $y = x^2 + 6x + 4$



Write an equation for the parabola with the given vertex that passes through the given point.

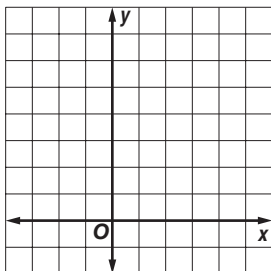
16. vertex:  $(4, -36)$   
point:  $(0, -20)$

17. vertex:  $(3, -1)$   
point:  $(2, 0)$

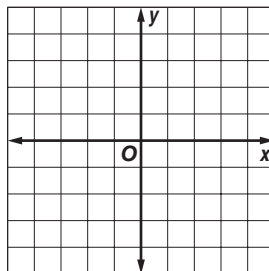
18. vertex:  $(-2, 2)$   
point:  $(-1, 3)$

**5-8 Skills Practice****Graphing and Solving Quadratic Inequalities****Graph each inequality.**

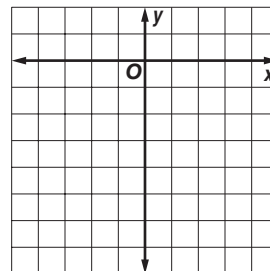
1.  $y \geq x^2 - 4x + 4$



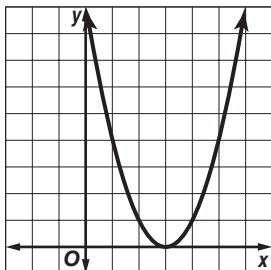
2.  $y \leq x^2 - 4$



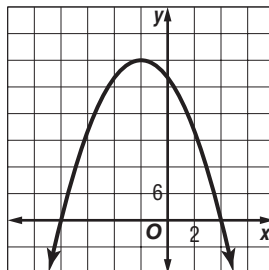
3.  $y > x^2 + 2x - 5$

**Use the graph of its related function to write the solutions of each inequality.**

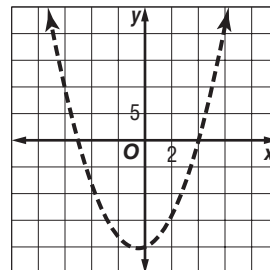
4.  $x^2 - 6x + 9 \leq 0$



5.  $-x^2 - 4x + 32 \geq 0$



6.  $x^2 + x - 20 > 0$

**Solve each inequality algebraically.**

7.  $x^2 - 3x - 10 < 0$

8.  $x^2 + 2x - 35 \geq 0$

9.  $x^2 - 18x + 81 \leq 0$

10.  $x^2 \leq 36$

11.  $x^2 - 7x > 0$

12.  $x^2 + 7x + 6 < 0$

13.  $x^2 + x - 12 > 0$

14.  $x^2 + 9x + 18 \leq 0$

15.  $x^2 - 10x + 25 \geq 0$

16.  $-x^2 - 2x + 15 \geq 0$

17.  $x^2 + 3x > 0$

18.  $2x^2 + 2x > 4$

19.  $-x^2 - 64 \leq -16x$

20.  $9x^2 + 12x + 9 < 0$

# 6-1 Skills Practice

## *Properties of Exponents*

**Simplify. Assume that no variable equals 0.**

1.  $b^4 \cdot b^3$

2.  $c^5 \cdot c^2 \cdot c^2$

3.  $a^{-4} \cdot a^{-3}$

4.  $x^5 \cdot x^{-4} \cdot x$

5.  $(g^4)^2$

6.  $(3u)^3$

7.  $(-x)^4$

8.  $-5(2z)^3$

9.  $-(-3d)^4$

10.  $(-2t^2)^3$

11.  $(-r^7)^3$

12.  $\frac{s^{15}}{s^{12}}$

13.  $\frac{k^9}{k^{10}}$

14.  $(-3f^3g)^3$

15.  $(2x)^2(4y)^2$

16.  $-2gh(g^3h^5)$

17.  $10x^2y^3(10xy^8)$

18.  $\frac{24wz^7}{3w^3z^5}$

19.  $\frac{-6a^4bc^8}{36a^7b^2c}$

20.  $\frac{-10pq^4r}{-5p^3q^2r}$

**Express each number in scientific notation.**

21. 53,000

22. 0.000248

23. 410,100,000

24. 0.00000805

**Evaluate. Express the result in scientific notation.**

25.  $(4 \times 10^3)(1.6 \times 10^{-6})$

26.  $\frac{9.6 \times 10^7}{1.5 \times 10^{-3}}$

**6-2 Skills Practice****Operations with Polynomials**

Determine whether each expression is a polynomial. If it is a polynomial, state the degree of the polynomial.

1.  $x^2 + 2x + 2$

2.  $\frac{b^2c}{d^4}$

3.  $8xz + \frac{1}{2}y$

**Simplify.**

4.  $(g + 5) + (2g + 7)$

5.  $(5d + 5) - (d + 1)$

6.  $(x^2 - 3x - 3) + (2x^2 + 7x - 2)$

7.  $(-2f^2 - 3f - 5) + (-2f^2 - 3f + 8)$

8.  $(4r^2 - 6r + 2) - (-r^2 + 3r + 5)$

9.  $(2x^2 - 3xy) - (3x^2 - 6xy - 4y^2)$

10.  $(5t - 7) + (2t^2 + 3t + 12)$

11.  $(u - 4) - (6 + 3u^2 - 4u)$

12.  $-5(2c^2 - d^2)$

13.  $x^2(2x + 9)$

14.  $2q(3pq + 4q^4)$

15.  $8w(hk^2 + 10h^3m^4 - 6k^5w^3)$

16.  $m^2n^3(-4m^2n^2 - 2mnp - 7)$

17.  $-3s^2y(-2s^4y^2 + 3sy^3 + 4)$

18.  $(c + 2)(c + 8)$

19.  $(z - 7)(z + 4)$

20.  $(a - 5)^2$

21.  $(2x - 3)(3x - 5)$

22.  $(r - 2s)(r + 2s)$

23.  $(3y + 4)(2y - 3)$

24.  $(3 - 2b)(3 + 2b)$

25.  $(3w + 1)^2$

**6-3 Skills Practice*****Dividing Polynomials*****Simplify.**

1.  $\frac{10c + 6}{2}$

2.  $\frac{12x + 20}{4}$

3.  $\frac{15y^3 + 6y^2 + 3y}{3y}$

4.  $\frac{12x^2 - 4x - 8}{4x}$

5.  $(15q^6 + 5q^2)(5q^4)^{-1}$

6.  $(4f^5 - 6f^4 + 12f^3 - 8f^2)(4f^2)^{-1}$

7.  $(6j^2k - 9jk^2) \div 3jk$

8.  $(4a^2h^2 - 8a^3h + 3a^4) \div (2a^2)$

9.  $(n^2 + 7n + 10) \div (n + 5)$

10.  $(d^2 + 4d + 3) \div (d + 1)$

11.  $(2s^2 + 13s + 15) \div (s + 5)$

12.  $(6y^2 + y - 2)(2y - 1)^{-1}$

13.  $(4g^2 - 9) \div (2g + 3)$

14.  $(2x^2 - 5x - 4) \div (x - 3)$

15.  $\frac{u^2 + 5u - 12}{u - 3}$

16.  $\frac{2x^2 - 5x - 4}{x - 3}$

17.  $(3v^2 - 7v - 10)(v - 4)^{-1}$

18.  $(3t^4 + 4t^3 - 32t^2 - 5t - 20)(t + 4)^{-1}$

19.  $\frac{y^3 - y^2 - 6}{y + 2}$

20.  $\frac{2x^3 - x^2 - 19x + 15}{x - 3}$

21.  $(4p^3 - 3p^2 + 2p) \div (p - 1)$

22.  $(3c^4 + 6c^3 - 2c + 4)(c + 2)^{-1}$

**23. GEOMETRY** The area of a rectangle is  $x^3 + 8x^2 + 13x - 12$  square units. The width of the rectangle is  $x + 4$  units. What is the length of the rectangle?

**6-4 Skills Practice****Polynomial Functions**

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1.  $a + 8$

2.  $(2x - 1)(4x^2 + 3)$

3.  $-5x^5 + 3x^3 - 8$

4.  $18 - 3y + 5y^2 - y^5 + 7y^6$

5.  $u^3 + 4u^2v^2 + v^4$

6.  $2r - r^2 + \frac{1}{r^2}$

Find  $p(-1)$  and  $p(2)$  for each function.

7.  $p(x) = 4 - 3x$

8.  $p(x) = 3x + x^2$

9.  $p(x) = 2x^2 - 4x + 1$

10.  $p(x) = -2x^3 + 5x + 3$

11.  $p(x) = x^4 + 8x^2 - 10$

12.  $p(x) = \frac{1}{3}x^2 - \frac{2}{3}x + 2$

If  $p(x) = 4x^2 - 3$  and  $r(x) = 1 + 3x$ , find each value.

13.  $p(a)$

14.  $r(2a)$

15.  $3r(a)$

16.  $-4p(a)$

17.  $p(a^2)$

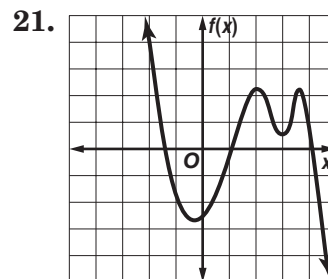
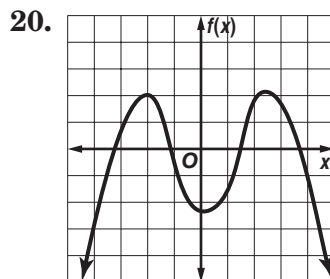
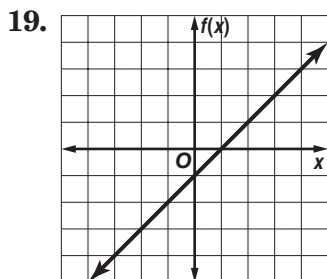
18.  $r(x + 2)$

For each graph,

a. describe the end behavior,

b. determine whether it represents an odd-degree or an even-degree polynomial function, and

c. state the number of real zeroes.



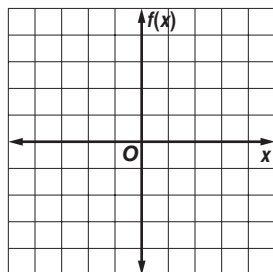
**6-5 Skills Practice****Analyze Graphs of Polynomial Functions**

Complete each of the following.

**a. Graph each function by making a table of values.****b. Determine consecutive values of  $x$  between which each real zero is located.****c. Estimate the  $x$ -coordinates at which the relative maxima and minima occur.**

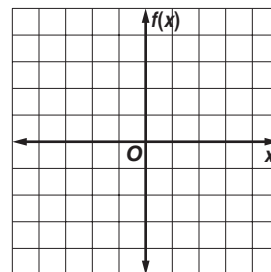
1.  $f(x) = x^3 - 3x^2 + 1$

$x$	$f(x)$
-2	
-1	
0	
1	
2	
3	
4	



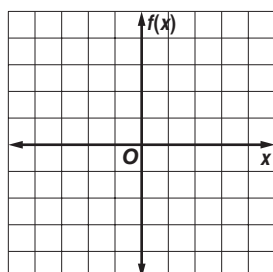
2.  $f(x) = x^3 - 3x + 1$

$x$	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



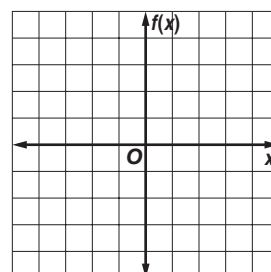
3.  $f(x) = 2x^3 + 9x^2 + 12x + 2$

$x$	$f(x)$
-3	
-2	
-1	
0	
1	



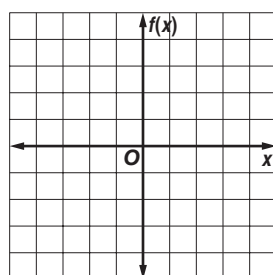
4.  $f(x) = 2x^3 - 3x^2 + 2$

$x$	$f(x)$
-1	
0	
1	
2	
3	



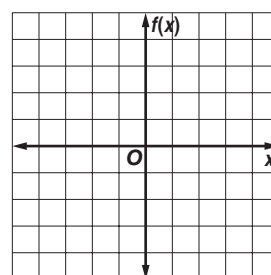
5.  $f(x) = x^4 - 2x^2 - 2$

$x$	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



6.  $f(x) = 0.5x^4 - 4x^2 + 4$

$x$	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	



**6-6 Skills Practice*****Solving Polynomial Equations*****Factor completely. If the polynomial is not factorable, write *prime*.**

1.  $7x^2 - 14x$

2.  $19x^3 - 38x^2$

3.  $21x^3 - 18x^2y + 24xy^2$

4.  $8j^3k - 4jk^3 - 7$

5.  $a^2 + 7a - 18$

6.  $2ak - 6a + k - 3$

7.  $b^2 + 8b + 7$

8.  $z^2 - 8z - 10$

9.  $4f^2 - 64$

10.  $d^2 - 12d + 36$

11.  $9x^2 + 25$

12.  $y^2 + 18y + 81$

13.  $n^3 - 125$

14.  $m^4 - 1$

**Write each expression in quadratic form, if possible.**

15.  $5x^4 + 2x^2 - 8$

16.  $3y^8 - 4y^2 + 3$

17.  $100a^6 + a^3$

18.  $x^8 + 4x^4 + 9$

19.  $12x^4 - 7x^2$

20.  $6b^5 + 3b^3 - 1$

**Solve each equation.**

21.  $a^3 - 9a^2 + 14a = 0$

22.  $x^3 = 3x^2$

23.  $t^4 - 3t^3 - 40t^2 = 0$

24.  $b^3 - 8b^2 + 16b = 0$



**6-7 Skills Practice*****The Remainder and Factor Theorems*****Use synthetic substitution to find  $f(2)$  and  $f(-1)$  for each function.**

**1.**  $f(x) = x^2 + 6x + 5$

**2.**  $f(x) = x^2 - x + 1$

**3.**  $f(x) = x^2 - 2x - 2$

**4.**  $f(x) = x^3 + 2x^2 + 5$

**5.**  $f(x) = x^3 - x^2 - 2x + 3$

**6.**  $f(x) = x^3 + 6x^2 + x - 4$

**7.**  $f(x) = x^3 - 3x^2 + x - 2$

**8.**  $f(x) = x^3 - 5x^2 - x + 6$

**9.**  $f(x) = x^4 + 2x^2 - 9$

**10.**  $f(x) = x^4 - 3x^3 + 2x^2 - 2x + 6$

**11.**  $f(x) = x^5 - 7x^3 - 4x + 10$

**12.**  $f(x) = x^6 - 2x^5 + x^4 + x^3 - 9x^2 - 20$

**Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.**

**13.**  $x^3 + 2x^2 - x - 2; x + 1$

**14.**  $x^3 + x^2 - 5x + 3; x - 1$

**15.**  $x^3 + 3x^2 - 4x - 12; x + 3$

**16.**  $x^3 - 6x^2 + 11x - 6; x - 3$

**17.**  $x^3 + 2x^2 - 33x - 90; x + 5$

**18.**  $x^3 - 6x^2 + 32; x - 4$

**19.**  $x^3 - x^2 - 10x - 8; x + 2$

**20.**  $x^3 - 19x + 30; x - 2$

**21.**  $2x^3 + x^2 - 2x - 1; x + 1$

**22.**  $2x^3 + x^2 - 5x + 2; x + 2$

**23.**  $3x^3 + 4x^2 - 5x - 2; 3x + 1$

**24.**  $3x^3 + x^2 + x - 2; 3x - 2$

**6-8 Skills Practice*****Roots and Zeros*****Solve each equation. State the number and type of roots.**

1.  $5x + 12 = 0$

2.  $x^2 - 4x + 40 = 0$

3.  $x^5 + 4x^3 = 0$

4.  $x^4 + 625 = 0$

5.  $4x^2 - 4x - 1 = 0$

6.  $x^5 - 81x = 0$

**State the possible number of positive real zeros, negative real zeros, and imaginary zeros of each function.**

7.  $g(x) = 3x^3 - 4x^2 - 17x + 6$

8.  $h(x) = 4x^3 - 12x^2 - x + 3$

9.  $f(x) = x^3 - 8x^2 + 2x - 4$

10.  $p(x) = x^3 - x^2 + 4x - 6$

11.  $q(x) = x^4 + 7x^2 + 3x - 9$

12.  $f(x) = x^4 - x^3 - 5x^2 + 6x + 1$

**Find all the zeros of each function.**

13.  $h(x) = x^3 - 5x^2 + 5x + 3$

14.  $g(x) = x^3 - 6x^2 + 13x - 10$

15.  $h(x) = x^3 + 4x^2 + x - 6$

16.  $q(x) = x^3 + 3x^2 - 6x - 8$

17.  $g(x) = x^4 - 3x^3 - 5x^2 + 3x + 4$

18.  $f(x) = x^4 - 21x^2 + 80$

**Write a polynomial function of least degree with integral coefficients that has the given zeros.**

19.  $-3, -5, 1$

20.  $3i$

21.  $-5 + i$

22.  $-1, \sqrt{3}, -\sqrt{3}$

23.  $i, 5i$

24.  $-1, 1, i\sqrt{6}$

**6-9 Skills Practice*****Rational Zero Theorem*****List all of the possible rational zeros of each function.**

**1.**  $n(x) = x^2 + 5x + 3$

**2.**  $h(x) = x^2 - 2x - 5$

**3.**  $w(x) = x^2 - 5x + 12$

**4.**  $f(x) = 2x^2 + 5x + 3$

**5.**  $q(x) = 6x^3 + x^2 - x + 2$

**6.**  $g(x) = 9x^4 + 3x^3 + 3x^2 - x + 27$

**Find all of the rational zeros of each function.**

**7.**  $f(x) = x^3 - 2x^2 + 5x - 4$

**8.**  $g(x) = x^3 - 3x^2 - 4x + 12$

**9.**  $p(x) = x^3 - x^2 + x - 1$

**10.**  $z(x) = x^3 - 4x^2 + 6x - 4$

**11.**  $h(x) = x^3 - x^2 + 4x - 4$

**12.**  $g(x) = 3x^3 - 9x^2 - 10x - 8$

**13.**  $g(x) = 2x^3 + 7x^2 - 7x - 12$

**14.**  $h(x) = 2x^3 - 5x^2 - 4x + 3$

**15.**  $p(x) = 3x^3 - 5x^2 - 14x - 4$

**16.**  $q(x) = 3x^3 + 2x^2 + 27x + 18$

**17.**  $f(x) = 3x^3 - 7x^2 + 4$

**18.**  $f(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$

**19.**  $p(x) = x^4 - 5x^3 - 9x^2 - 25x - 70$

**20.**  $n(x) = 16x^4 - 32x^3 - 13x^2 + 29x - 6$

**Find all of the zeros of each function.**

**21.**  $f(x) = x^3 + 5x^2 + 11x + 15$

**22.**  $q(x) = x^3 - 10x^2 + 18x - 4$

**23.**  $m(x) = 6x^4 - 17x^3 + 8x^2 + 8x - 3$

**24.**  $g(x) = x^4 + 4x^3 + 5x^2 + 4x + 4$

**7-1 Skills Practice****Operations on Functions**

Find  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(f \cdot g)(x)$ , and  $\left(\frac{f}{g}\right)(x)$  for each  $f(x)$  and  $g(x)$ .

1.  $f(x) = x + 5$

$g(x) = x - 4$

2.  $f(x) = 3x + 1$

$g(x) = 2x - 3$

3.  $f(x) = x^2$

$g(x) = 4 - x$

4.  $f(x) = 3x^2$

$g(x) = \frac{5}{x}$

For each set of ordered pairs, find  $f \circ g$  and  $g \circ f$  if they exist.

5.  $f = \{(0, 0), (4, -2)\}$

$g = \{(0, 4), (-2, 0), (5, 0)\}$

6.  $f = \{(0, -3), (1, 2), (2, 2)\}$

$g = \{(-3, 1), (2, 0)\}$

7.  $f = \{(-4, 3), (-1, 1), (2, 2)\}$

$g = \{(1, -4), (2, -1), (3, -1)\}$

8.  $f = \{(6, 6), (-3, -3), (1, 3)\}$

$g = \{(-3, 6), (3, 6), (6, -3)\}$

Find  $[g \circ h](x)$  and  $[h \circ g](x)$ .

9.  $g(x) = 2x$

$h(x) = x + 2$

10.  $g(x) = -3x$

$h(x) = 4x - 1$

11.  $g(x) = x - 6$

$h(x) = x + 6$

12.  $g(x) = x - 3$

$h(x) = x^2$

13.  $g(x) = 5x$

$h(x) = x^2 + x - 1$

14.  $g(x) = x + 2$

$h(x) = 2x^2 - 3$

If  $f(x) = 3x$ ,  $g(x) = x + 4$ , and  $h(x) = x^2 - 1$ , find each value.

15.  $f[g(1)]$

16.  $g[h(0)]$

17.  $g[f(-1)]$

18.  $h[f(5)]$

19.  $g[h(-3)]$

20.  $h[f(10)]$

21.  $f[h(8)]$

22.  $[f \circ (h \circ g)](1)$

23.  $[f \circ (g \circ h)](-2)$

**7-2 Skills Practice*****Inverse Functions and Relations*****Find the inverse of each relation.**

1.  $\{(3, 1), (4, -3), (8, -3)\}$

2.  $\{(-7, 1), (0, 5), (5, -1)\}$

3.  $\{(-10, -2), (-7, 6), (-4, -2), (-4, 0)\}$

4.  $\{(0, -9), (5, -3), (6, 6), (8, -3)\}$

5.  $\{(-4, 12), (0, 7), (9, -1), (10, -5)\}$

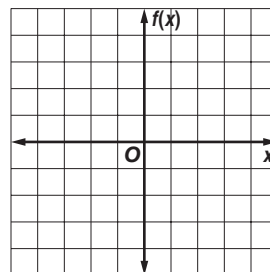
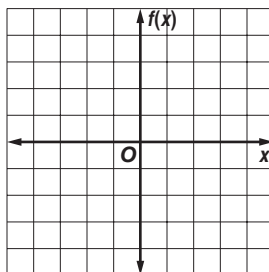
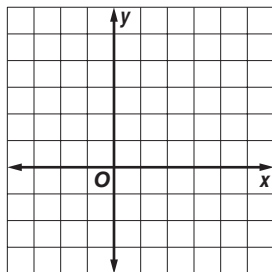
6.  $\{(-4, 1), (-4, 3), (0, -8), (8, -9)\}$

**Find the inverse of each function. Then graph the function and its inverse.**

7.  $y = 4$

8.  $f(x) = 3x$

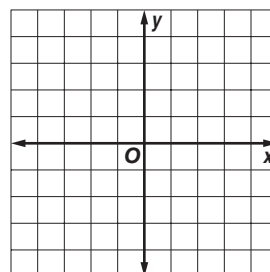
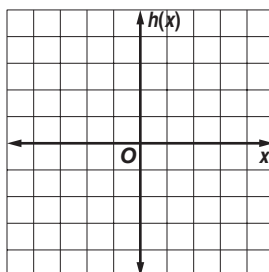
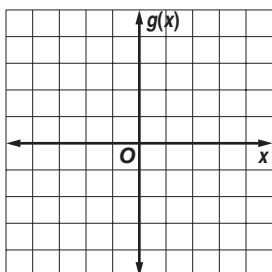
9.  $f(x) = x + 2$



10.  $g(x) = 2x - 1$

11.  $h(x) = \frac{1}{4}x$

12.  $y = \frac{2}{3}x + 2$

**Determine whether each pair of functions are inverse functions.**

13.  $f(x) = x - 1$   
 $g(x) = 1 - x$

14.  $f(x) = 2x + 3$   
 $g(x) = \frac{1}{2}(x - 3)$

15.  $f(x) = 5x - 5$   
 $g(x) = \frac{1}{5}x + 1$

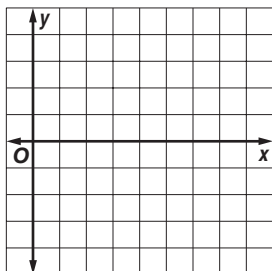
16.  $f(x) = 2x$   
 $g(x) = \frac{1}{2}x$

17.  $h(x) = 6x - 2$   
 $g(x) = \frac{1}{6}x + 3$

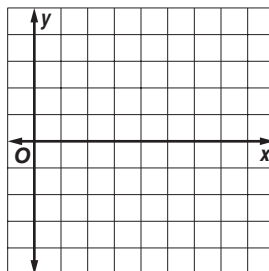
18.  $f(x) = 8x - 10$   
 $g(x) = \frac{1}{8}x + \frac{5}{4}$

**7-3 Skills Practice*****Square Root Functions and Inequalities*****Graph each function. State the domain and range of each function.**

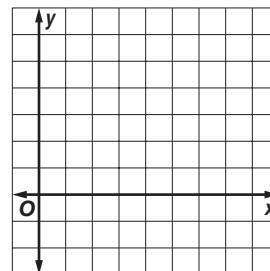
1.  $y = \sqrt{2x}$



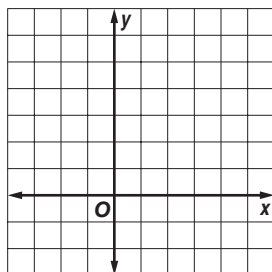
2.  $y = -\sqrt{3x}$



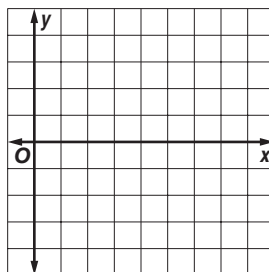
3.  $y = 2\sqrt{x}$



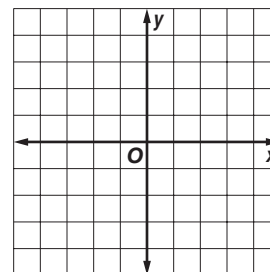
4.  $y = \sqrt{x + 3}$



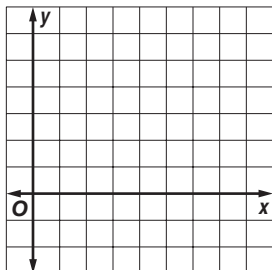
5.  $y = -\sqrt{2x - 5}$



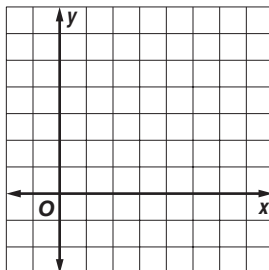
6.  $y = \sqrt{x + 4} - 2$

**Graph each inequality.**

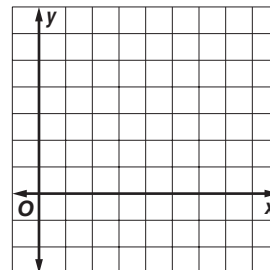
7.  $y < \sqrt{4x}$



8.  $y \geq \sqrt{x + 1}$



9.  $y \leq \sqrt{4x - 3}$



**7-4 Skills Practice*****n*th Roots**

Use a calculator to approximate each value to three decimal places.

1.  $\sqrt{230}$

2.  $\sqrt{38}$

3.  $-\sqrt{152}$

4.  $\sqrt{5.6}$

5.  $\sqrt[3]{88}$

6.  $\sqrt[3]{-222}$

7.  $-\sqrt[4]{0.34}$

8.  $\sqrt[5]{500}$

**Simplify.**

9.  $\pm\sqrt{81}$

10.  $\sqrt{144}$

11.  $\sqrt{(-5)^2}$

12.  $\sqrt{-5^2}$

13.  $\sqrt{0.36}$

14.  $-\sqrt{\frac{4}{9}}$

15.  $\sqrt[3]{-8}$

16.  $-\sqrt[3]{27}$

17.  $\sqrt[3]{0.064}$

18.  $\sqrt[5]{32}$

19.  $\sqrt[4]{81}$

20.  $\sqrt{y^2}$

21.  $\sqrt[3]{125s^3}$

22.  $\sqrt{64x^6}$

23.  $\sqrt[3]{-27a^6}$

24.  $\sqrt{m^8n^4}$

25.  $-\sqrt{100p^4q^2}$

26.  $\sqrt[4]{16w^4v^8}$

27.  $\sqrt{(-3c)^4}$

28.  $\sqrt{(a + b)^2}$

**7-5 Skills Practice*****Operations with Radical Expressions*****Simplify.**

1.  $\sqrt{24}$

2.  $\sqrt{75}$

3.  $\sqrt[3]{16}$

4.  $-\sqrt[4]{48}$

5.  $4\sqrt{50x^5}$

6.  $\sqrt[4]{64a^4b^4}$

7.  $\sqrt[3]{-\frac{1}{8}d^2f^5}$

8.  $\sqrt{\frac{25}{36}s^2t}$

9.  $-\sqrt{\frac{3}{7}}$

10.  $\sqrt[3]{\frac{2}{9}}$

11.  $\sqrt{\frac{2g^3}{5z}}$

12.  $(3\sqrt{3})(5\sqrt{3})$

13.  $(4\sqrt{12})(3\sqrt{20})$

14.  $\sqrt{2} + \sqrt{8} + \sqrt{50}$

15.  $\sqrt{12} - 2\sqrt{3} + \sqrt{108}$

16.  $8\sqrt{5} - \sqrt{45} - \sqrt{80}$

17.  $2\sqrt{48} - \sqrt{75} - \sqrt{12}$

18.  $(2 + \sqrt{3})(6 - \sqrt{2})$

19.  $(1 - \sqrt{5})(1 + \sqrt{5})$

20.  $(3 - \sqrt{7})(5 + \sqrt{2})$

21.  $(\sqrt{2} - \sqrt{6})^2$

22.  $\frac{3}{7 - \sqrt{2}}$

23.  $\frac{4}{3 + \sqrt{2}}$

24.  $\frac{5}{8 - \sqrt{6}}$



**7-6 Skills Practice*****Rational Exponents*****Write each expression in radical form.**

1.  $3^{\frac{1}{6}}$

2.  $8^{\frac{1}{5}}$

3.  $12^{\frac{2}{3}}$

4.  $(s^3)^{\frac{3}{5}}$

**Write each radical using rational exponents.**

5.  $\sqrt{51}$

6.  $\sqrt[3]{37}$

7.  $\sqrt[4]{15^3}$

8.  $\sqrt[3]{6xy^2}$

**Evaluate each expression.**

9.  $32^{\frac{1}{5}}$

10.  $81^{\frac{1}{4}}$

11.  $27^{-\frac{1}{3}}$

12.  $4^{-\frac{1}{2}}$

13.  $16^{\frac{3}{2}}$

14.  $(-243)^{\frac{4}{5}}$

15.  $27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}$

16.  $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

**Simplify each expression.**

17.  $c^{\frac{12}{5}} \cdot c^{\frac{3}{5}}$

18.  $m^{\frac{2}{9}} \cdot m^{\frac{16}{9}}$

19.  $\left(q^{\frac{1}{2}}\right)^3$

20.  $p^{-\frac{1}{5}}$

21.  $x^{-\frac{6}{11}}$

22.  $\frac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$

23.  $\frac{y^{-\frac{1}{2}}}{y^{\frac{1}{4}}}$

24.  $\frac{n^{\frac{1}{3}}}{n^{\frac{1}{6}} \cdot n^{\frac{1}{2}}}$

25.  $\sqrt[12]{64}$

26.  $\sqrt[8]{49a^8b^2}$

**7-7 Skills Practice*****Solving Radical Equations and Inequalities*****Solve each equation or inequality.**

1.  $\sqrt{x} = 5$

2.  $\sqrt{x} + 3 = 7$

3.  $5\sqrt{j} = 1$

4.  $v^{\frac{1}{2}} + 1 = 0$

5.  $18 - 3y^{\frac{1}{2}} = 25$

6.  $\sqrt[3]{2w} = 4$

7.  $\sqrt{b - 5} = 4$

8.  $\sqrt{3n + 1} = 5$

9.  $\sqrt[3]{3r - 6} = 3$

10.  $2 + \sqrt{3p + 7} = 6$

11.  $\sqrt{k - 4} - 1 = 5$

12.  $(2d + 3)^{\frac{1}{3}} = 2$

13.  $(t - 3)^{\frac{1}{3}} = 2$

14.  $4 - (1 - 7u)^{\frac{1}{3}} = 0$

15.  $\sqrt{3z - 2} = \sqrt{z - 4}$

16.  $\sqrt{g + 1} = \sqrt{2g - 7}$

17.  $\sqrt{x - 1} = 4\sqrt{x + 1}$

18.  $5 + \sqrt{s - 3} \leq 6$

19.  $-2 + \sqrt{3x + 3} < 7$

20.  $-\sqrt{2a + 4} \geq -6$

21.  $2\sqrt{4r - 3} > 10$

22.  $4 - \sqrt{3x + 1} > 3$

23.  $\sqrt{y + 4} - 3 \geq 3$

24.  $-3\sqrt{11r + 3} \geq -15$

**8-1 Skills Practice*****Multiplying and Dividing Rational Expressions*****Simplify each expression.**

1.  $\frac{21x^3y}{14x^2y^2}$

2.  $\frac{5ab^3}{25a^2b^2}$

3.  $\frac{(x^6)^3}{(x^3)^4}$

4.  $\frac{8y^2(y^6)^3}{4y^{24}}$

5.  $\frac{18}{2x - 6}$

6.  $\frac{x^2 - 4}{(x - 2)(x + 1)}$

7.  $\frac{3a^2 - 24a}{3a^2 + 12a}$

8.  $\frac{3m}{2n} \cdot \frac{n^3}{6}$

9.  $\frac{24e^3}{5f^2} \cdot \frac{10(ef)^3}{8e^5f}$

10.  $\frac{5s^2}{s^2 - 4} \cdot \frac{s + 2}{10s^5}$

11.  $\frac{7g}{y^2} \div 21g^3$

12.  $\frac{80y^4}{49z^5v^7} \div \frac{25y^5}{14z^{12}v^5}$

13.  $\frac{3x^2}{x + 2} \div \frac{3x}{x^2 - 4}$

14.  $\frac{q^2 + 2q}{6q} \div \frac{q^2 - 4}{3q^2}$

15.  $\frac{w^2 - 5w - 24}{w + 1} \cdot \frac{w^2 - 6w - 7}{w + 3}$

16.  $\frac{t^2 + 19t + 84}{4t - 4} \cdot \frac{2t - 2}{t^2 + 9t + 14}$

17.  $\frac{x^2 - 5x + 4}{2x - 8} \div (3x^2 - 3x)$

18.  $\frac{16a^2 + 40a + 25}{3a^2 - 10a - 8} \div \frac{4a + 5}{a^2 - 8a + 16}$

19.  $\frac{\frac{c^2}{2d^2}}{-\frac{c^6}{5d}}$

20.  $\frac{\frac{a^2 - b^2}{4a}}{\frac{a + b}{2a}}$

**8-2 Skills Practice*****Adding and Subtracting Rational Expressions*****Find the LCM of each set of polynomials.**

1.  $12c, 6c^2d$

2.  $18a^3bc^2, 24b^2c^2$

3.  $2x - 6, x - 3$

4.  $5a, a - 1$

5.  $t^2 - 25, t + 5$

6.  $x^2 - 3x - 4, x + 1$

**Simplify each expression.**

7.  $\frac{3}{x} + \frac{5}{y}$

8.  $\frac{3}{8p^2q} + \frac{5}{4p^2q}$

9.  $\frac{2c - 7}{3} + 4$

10.  $\frac{2}{m^2n} + \frac{5}{n}$

11.  $\frac{12}{5y^2} - \frac{2}{5yz}$

12.  $\frac{7}{4gh} + \frac{3}{4h^2}$

13.  $\frac{2}{a + 2} - \frac{3}{2a}$

14.  $\frac{5}{3b + d} - \frac{2}{3bd}$

15.  $\frac{3}{w - 3} - \frac{2}{w^2 - 9}$

16.  $\frac{3t}{2 - x} + \frac{5}{x - 2}$

17.  $\frac{m}{m - n} - \frac{m}{n - m}$

18.  $\frac{4z}{z - 4} + \frac{z + 4}{z + 1}$

19.  $\frac{1}{x^2 + 2x + 1} + \frac{x}{x + 1}$

20.  $\frac{2x + 1}{x - 5} - \frac{4}{x^2 - 3x - 10}$

21.  $\frac{n}{n - 3} + \frac{2n + 2}{n^2 - 2n - 3}$

22.  $\frac{3}{y^2 + y - 12} - \frac{2}{y^2 + 6y + 8}$

**8-3 Skills Practice****Graphing Rational Functions**

Determine the equations of any vertical asymptotes and the values of  $x$  for any holes in the graph of each rational function.

1.  $f(x) = \frac{3}{x^2 - 2x - 8}$

2.  $f(x) = \frac{10}{x^2 - 13x + 36}$

3.  $f(x) = \frac{x + 12}{x^2 + 10x - 24}$

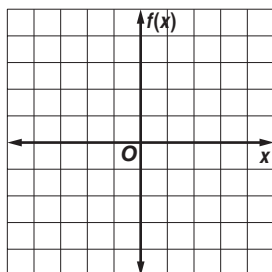
4.  $f(x) = \frac{x - 1}{x^2 - 4x + 3}$

5.  $f(x) = \frac{x^2 + 8x + 12}{x + 2}$

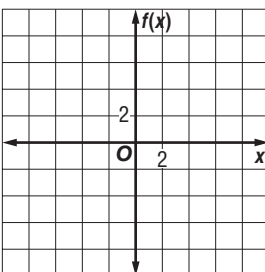
6.  $f(x) = \frac{x^2 + x - 12}{x - 3}$

Graph each rational function.

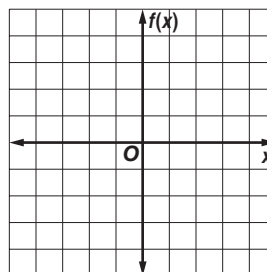
7.  $f(x) = \frac{-3}{x}$



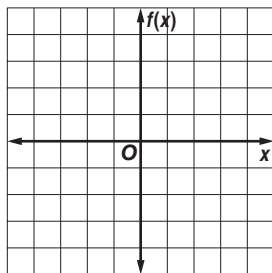
8.  $f(x) = \frac{10}{x}$



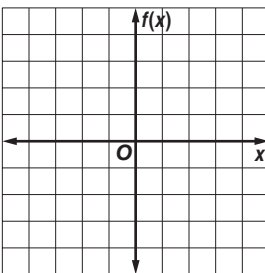
9.  $f(x) = \frac{-4}{x}$



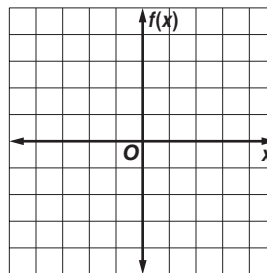
10.  $f(x) = \frac{2}{x - 1}$



11.  $f(x) = \frac{x}{x + 2}$



12.  $f(x) = \frac{x^2 - 4}{x - 2}$



**8-4 Skills Practice*****Direct, Joint, and Inverse Variation***

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation.

1.  $c = 12m$

2.  $p = \frac{4}{q}$

3.  $A = \frac{1}{2}bh$

4.  $rw = 15$

5.  $y = 2rst$

6.  $f = 5280m$

7.  $y = 0.2s$

8.  $vz = -25$

9.  $t = 16rh$

10.  $R = \frac{8}{w}$

11.  $\frac{a}{b} = \frac{1}{3}$

12.  $C = 2\pi r$

Find each value.

13. If  $y$  varies directly as  $x$  and  $y = 35$  when  $x = 7$ , find  $y$  when  $x = 11$ .

14. If  $y$  varies directly as  $x$  and  $y = 360$  when  $x = 180$ , find  $y$  when  $x = 270$ .

15. If  $y$  varies directly as  $x$  and  $y = 540$  when  $x = 10$ , find  $x$  when  $y = 1080$ .

16. If  $y$  varies directly as  $x$  and  $y = 12$  when  $x = 72$ , find  $x$  when  $y = 9$ .

17. If  $y$  varies jointly as  $x$  and  $z$  and  $y = 18$  when  $x = 2$  and  $z = 3$ , find  $y$  when  $x = 5$  and  $z = 6$ .

18. If  $y$  varies jointly as  $x$  and  $z$  and  $y = -16$  when  $x = 4$  and  $z = 2$ , find  $y$  when  $x = -1$  and  $z = 7$ .

19. If  $y$  varies jointly as  $x$  and  $z$  and  $y = 120$  when  $x = 4$  and  $z = 6$ , find  $y$  when  $x = 3$  and  $z = 2$ .

20. If  $y$  varies inversely as  $x$  and  $y = 2$  when  $x = 2$ , find  $y$  when  $x = 1$ .

21. If  $y$  varies inversely as  $x$  and  $y = 6$  when  $x = 5$ , find  $y$  when  $x = 10$ .

22. If  $y$  varies inversely as  $x$  and  $y = 3$  when  $x = 14$ , find  $x$  when  $y = 6$ .

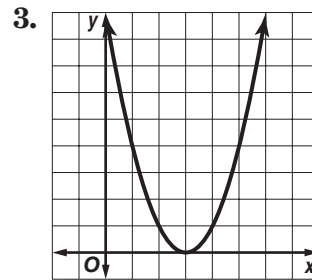
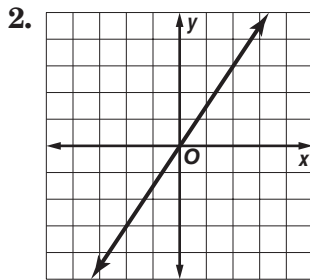
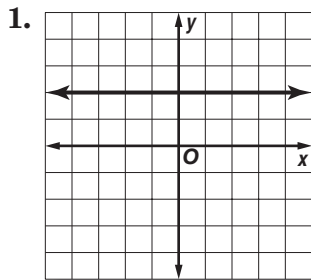
23. If  $y$  varies inversely as  $x$  and  $y = 27$  when  $x = 2$ , find  $x$  when  $y = 9$ .

24. If  $y$  varies directly as  $x$  and  $y = -15$  when  $x = 5$ , find  $x$  when  $y = -36$ .

# 8-5 Skills Practice

## Classes of Functions

Identify the type of function represented by each graph.



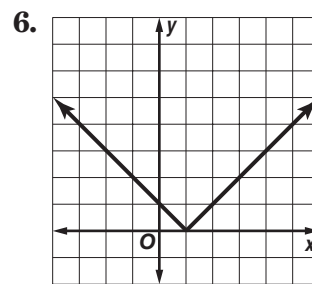
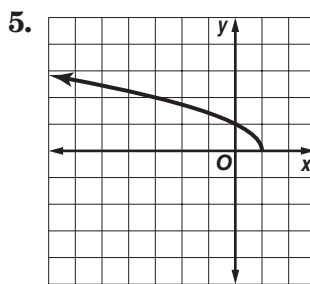
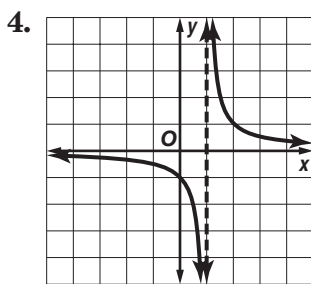
Match each graph with an equation below.

A.  $y = |x - 1|$

B.  $y = \frac{1}{x - 1}$

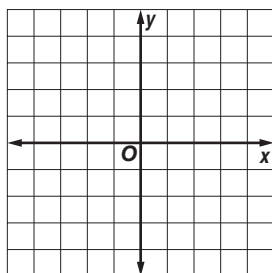
C.  $y = \sqrt{1 - x}$

D.  $y = \lfloor x \rfloor - 1$

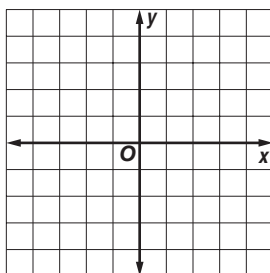


Identify the type of function represented by each equation. Then graph the equation.

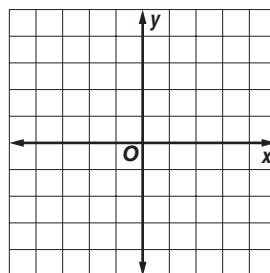
7.  $y = \frac{2}{x}$



8.  $y = 2\lfloor x \rfloor$



9.  $y = -3x$



**8-6 Skills Practice*****Solving Rational Equations and Inequalities*****Solve each equation or inequality. Check your solutions.**

1.  $\frac{x}{x-1} = \frac{1}{2}$

2.  $2 = \frac{4}{n} + \frac{1}{3}$

3.  $\frac{9}{3x} = \frac{-6}{2}$

4.  $3 - z = \frac{2}{z}$

5.  $\frac{2}{d+1} = \frac{1}{d-2}$

6.  $\frac{s-3}{5} = \frac{8}{s}$

7.  $\frac{2x+3}{x+1} = \frac{3}{2}$

8.  $-\frac{12}{y} = y - 7$

9.  $\frac{x-2}{x+4} = \frac{x+1}{x+10}$

10.  $\frac{3}{k} - \frac{4}{3k} > 0$

11.  $2 - \frac{3}{v} < \frac{5}{v}$

12.  $n + \frac{3}{n} < \frac{12}{n}$

13.  $\frac{1}{2m} - \frac{3}{m} < -\frac{5}{2}$

14.  $\frac{1}{2x} < \frac{2}{x} - 1$

15.  $\frac{15}{x} + \frac{9x-7}{x+2} = 9$

16.  $\frac{3b-2}{b+1} = 4 - \frac{b+2}{b-1}$

17.  $2 = \frac{5}{2q} + \frac{2q}{q+1}$

18.  $8 - \frac{4}{z} = \frac{8z-8}{z+2}$

19.  $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$

20.  $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$

21.  $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

22.  $\frac{12s+19}{s^2+7s+12} - \frac{3}{s+3} = \frac{5}{s+4}$

23.  $\frac{2e}{e^2-4} + \frac{1}{e-2} = \frac{2}{e+2}$

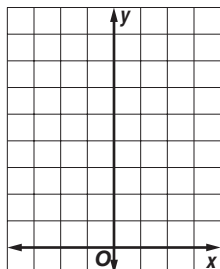
24.  $\frac{8}{t^2-9} + \frac{4}{t+3} = \frac{2}{t-3}$



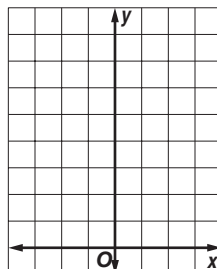
**9-1 Skills Practice****Exponential Functions**

Sketch the graph of each function. Then state the function's domain and range.

1.  $y = 3(2)^x$



2.  $y = 2\left(\frac{1}{2}\right)^x$



Determine whether each function represents exponential *growth* or *decay*.

3.  $y = 3(6)^x$

4.  $y = 2\left(\frac{9}{10}\right)^x$

5.  $y = 10^{-x}$

6.  $y = 2(2.5)^x$

Write an exponential function whose graph passes through the given points.

7. (0, 1) and (−1, 3)

8. (0, 4) and (1, 12)

9. (0, 3) and (−1, 6)

10. (0, 5) and (1, 15)

11. (0, 0.1) and (1, 0.5)

12. (0, 0.2) and (1, 1.6)

Simplify each expression.

13.  $(3^{\sqrt{3}})^{\sqrt{3}}$

14.  $(x^{\sqrt{2}})^{\sqrt{7}}$

15.  $5^{2\sqrt{3}} \cdot 5^{4\sqrt{3}}$

16.  $x^{3\pi} \div x^{\pi}$

Solve each equation or inequality. Check your solution.

17.  $3^x > 9$

18.  $2^{2x+3} = 32$

19.  $49^x \leq \frac{1}{7}$

20.  $4^{3x-2} = 16$

21.  $3^{2x+5} = 27^x$

22.  $27^x = 3^{2x+3}$

**9-2 Skills Practice*****Logarithms and Logarithmic Functions*****Write each equation in logarithmic form.**

1.  $2^3 = 8$

2.  $3^2 = 9$

3.  $8^{-2} = \frac{1}{64}$

4.  $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

**Write each equation in exponential form.**

5.  $\log_3 243 = 5$

6.  $\log_4 64 = 3$

7.  $\log_9 3 = \frac{1}{2}$

8.  $\log_5 \frac{1}{25} = -2$

**Evaluate each expression.**

9.  $\log_5 25$

10.  $\log_9 3$

11.  $\log_{10} 1000$

12.  $\log_{125} 5$

13.  $\log_4 \frac{1}{64}$

14.  $\log_5 \frac{1}{625}$

15.  $\log_8 8^3$

16.  $\log_{27} \frac{1}{3}$

**Solve each equation or inequality. Check your solutions.**

17.  $\log_3 x = 5$

18.  $\log_2 x = 3$

19.  $\log_4 y < 0$

20.  $\log_{\frac{1}{4}} x = 3$

21.  $\log_2 n > -2$

22.  $\log_b 3 = \frac{1}{2}$

23.  $\log_6 (4x + 12) = 2$

24.  $\log_2 (4x - 4) > 5$

25.  $\log_3 (x + 2) = \log_3 (3x)$

26.  $\log_6 (3y - 5) \geq \log_6 (2y + 3)$

**9-3 Skills Practice*****Properties of Logarithms***

Use  $\log_2 3 \approx 1.5850$  and  $\log_2 5 \approx 2.3219$  to approximate the value of each expression.

1.  $\log_2 25$

2.  $\log_2 27$

3.  $\log_2 \frac{3}{5}$

4.  $\log_2 \frac{5}{3}$

5.  $\log_2 15$

6.  $\log_2 45$

7.  $\log_2 75$

8.  $\log_2 0.6$

9.  $\log_2 \frac{1}{3}$

10.  $\log_2 \frac{9}{5}$

Solve each equation. Check your solutions.

11.  $\log_{10} 27 = 3 \log_{10} x$

12.  $3 \log_7 4 = 2 \log_7 b$

13.  $\log_4 5 + \log_4 x = \log_4 60$

14.  $\log_6 2c + \log_6 8 = \log_6 80$

15.  $\log_5 y - \log_5 8 = \log_5 1$

16.  $\log_2 q - \log_2 3 = \log_2 7$

17.  $\log_9 4 + 2 \log_9 5 = \log_9 w$

18.  $3 \log_8 2 - \log_8 4 = \log_8 b$

19.  $\log_{10} x + \log_{10} (3x - 5) = \log_{10} 2$

20.  $\log_4 x + \log_4 (2x - 3) = \log_4 2$

21.  $\log_3 d + \log_3 3 = 3$

22.  $\log_{10} y - \log_{10} (2 - y) = 0$

23.  $\log_2 s + 2 \log_2 5 = 0$

24.  $\log_2 (x + 4) - \log_2 (x - 3) = 3$

25.  $\log_4 (n + 1) - \log_4 (n - 2) = 1$

26.  $\log_5 10 + \log_5 12 = 3 \log_5 2 + \log_5 a$

**9-4 Skills Practice****Common Logarithms**

Use a calculator to evaluate each expression to four decimal places.

1.  $\log 6$

2.  $\log 15$

3.  $\log 1.1$

4.  $\log 0.3$

Use the formula  $\text{pH} = -\log[H^+]$  to find the pH of each substance given its concentration of hydrogen ions.

5. gastric juices:  $[H^+] = 1.0 \times 10^{-1}$  mole per liter

6. tomato juice:  $[H^+] = 7.94 \times 10^{-5}$  mole per liter

7. blood:  $[H^+] = 3.98 \times 10^{-8}$  mole per liter

8. toothpaste:  $[H^+] = 1.26 \times 10^{-10}$  mole per liter

Solve each equation or inequality. Round to four decimal places.

9.  $3^x > 243$

10.  $16^v \leq \frac{1}{4}$

11.  $8^p = 50$

12.  $7^y = 15$

13.  $5^{3b} = 106$

14.  $4^{5k} = 37$

15.  $12^{7p} = 120$

16.  $9^{2m} = 27$

17.  $3^{r-5} = 4.1$

18.  $8^{y+4} > 15$

19.  $7.6^{d+3} = 57.2$

20.  $0.5^{t-8} = 16.3$

21.  $42^{x^2} = 84$

22.  $5^{x^2+1} = 10$

Express each logarithm in terms of common logarithms. Then approximate its value to four decimal places.

23.  $\log_3 7$

24.  $\log_5 66$

25.  $\log_2 35$

26.  $\log_6 10$

**9-5 Skills Practice*****Base e and Natural Logarithms***

Use a calculator to evaluate each expression to four decimal places.

1.  $e^3$

2.  $e^{-2}$

3.  $\ln 2$

4.  $\ln 0.09$

Write an equivalent exponential or logarithmic equation.

5.  $e^x = 3$

6.  $e^4 = 8x$

7.  $\ln 15 = x$

8.  $\ln x \approx 0.6931$

Evaluate each expression.

9.  $e^{\ln 3}$

10.  $e^{\ln 2x}$

11.  $\ln e^{-2.5}$

12.  $\ln e^y$

Solve each equation or inequality.

13.  $e^x \geq 5$

14.  $e^x < 3.2$

15.  $2e^x - 1 = 11$

16.  $5e^x + 3 = 18$

17.  $e^{3x} = 30$

18.  $e^{-4x} > 10$

19.  $e^{5x} + 4 > 34$

20.  $1 - 2e^{2x} = -19$

21.  $\ln 3x = 2$

22.  $\ln 8x = 3$

23.  $\ln (x - 2) = 2$

24.  $\ln (x + 3) = 1$

25.  $\ln (x + 3) = 4$

26.  $\ln x + \ln 2x = 2$

**9-6 Skills Practice*****Exponential Growth and Decay***

- 1. FISHING** In an over-fished area, the catch of a certain fish is decreasing at an average rate of 8% per year. If this decline persists, how long will it take for the catch to reach half of the amount before the decline?
- 2. INVESTING** Alex invests \$2000 in an account that has a 6% annual rate of growth. To the nearest year, when will the investment be worth \$3600?
- 3. POPULATION** A current census shows that the population of a city is 3.5 million. Using the formula  $P = ae^{rt}$ , find the expected population of the city in 30 years if the growth rate  $r$  of the population is 1.5% per year,  $a$  represents the current population in millions, and  $t$  represents the time in years.
- 4. POPULATION** The population  $P$  in thousands of a city can be modeled by the equation  $P = 80e^{0.015t}$ , where  $t$  is the time in years. In how many years will the population of the city be 120,000?
- 5. BACTERIA** How many days will it take a culture of bacteria to increase from 2000 to 50,000 if the growth rate per day is 93.2%?
- 6. NUCLEAR POWER** The element plutonium-239 is highly radioactive. Nuclear reactors can produce and also use this element. The heat that plutonium-239 emits has helped to power equipment on the moon. If the half-life of plutonium-239 is 24,360 years, what is the value of  $k$  for this element?
- 7. DEPRECIATION** A Global Positioning Satellite (GPS) system uses satellite information to locate ground position. Abu's surveying firm bought a GPS system for \$12,500. The GPS depreciated by a fixed rate of 6% and is now worth \$8600. How long ago did Abu buy the GPS system?
- 8. BIOLOGY** In a laboratory, an organism grows from 100 to 250 in 8 hours. What is the hourly growth rate in the growth formula  $y = a(1 + r)^t$ ?

**10-1 Skills Practice*****Midpoint and Distance Formulas***

**Find the midpoint of each line segment with endpoints at the given coordinates.**

**1.**  $(4, -1), (-4, 1)$

**2.**  $(-1, 4), (5, 2)$

**3.**  $(3, 4), (5, 4)$

**4.**  $(6, 2), (2, -1)$

**5.**  $(3, 9), (-2, -3)$

**6.**  $(-3, 5), (-3, -8)$

**7.**  $(3, 2), (-5, 0)$

**8.**  $(3, -4), (5, 2)$

**9.**  $(-5, -9), (5, 4)$

**10.**  $(-11, 14), (0, 4)$

**11.**  $(3, -6), (-8, -3)$

**12.**  $(0, 10), (-2, -5)$

**Find the distance between each pair of points with the given coordinates.**

**13.**  $(4, 12), (-1, 0)$

**14.**  $(7, 7), (-5, -2)$

**15.**  $(-1, 4), (1, 4)$

**16.**  $(11, 11), (8, 15)$

**17.**  $(1, -6), (7, 2)$

**18.**  $(3, -5), (3, 4)$

**19.**  $(2, 3), (3, 5)$

**20.**  $(-4, 3), (-1, 7)$

**21.**  $(-5, -5), (3, 10)$

**22.**  $(3, 9), (-2, -3)$

**23.**  $(6, -2), (-1, 3)$

**24.**  $(-4, 1), (2, -4)$

**25.**  $(0, -3), (4, 1)$

**26.**  $(-5, -6), (2, 0)$

**10-2 Skills Practice****Parabolas****Write each equation in standard form.**

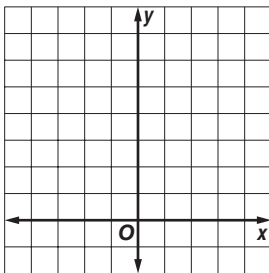
1.  $y = x^2 + 2x + 2$

2.  $y = x^2 - 2x + 4$

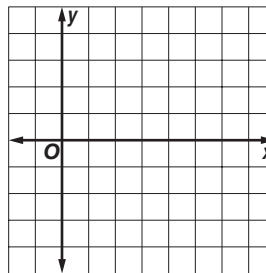
3.  $y = x^2 + 4x + 1$

**Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola.**

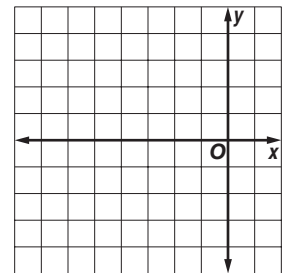
4.  $y = (x - 2)^2$



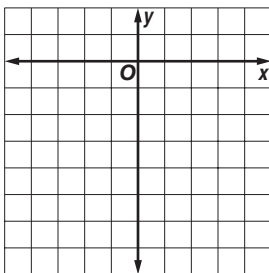
5.  $x = (y - 2)^2 + 3$



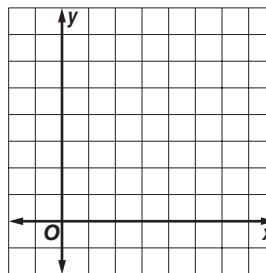
6.  $y = -(x + 3)^2 + 4$

**Write an equation for each parabola described below. Then draw the graph.**

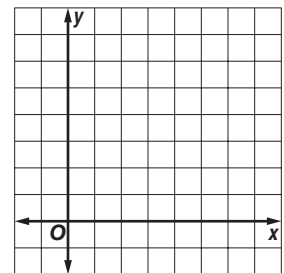
7. vertex  $(0, 0)$ ,  
focus  $\left(0, -\frac{1}{12}\right)$



8. vertex  $(5, 1)$ ,  
focus  $\left(5, \frac{5}{4}\right)$



9. vertex  $(1, 3)$ ,  
directrix  $x = \frac{7}{8}$





**10-3 Skills Practice****Circles****Write an equation for the circle that satisfies each set of conditions.**

1. center  $(0, 5)$ , radius 1 unit

2. center  $(5, 12)$ , radius 8 units

3. center  $(4, 0)$ , radius 2 units

4. center  $(2, 2)$ , radius 3 units

5. center  $(4, -4)$ , radius 4 units

6. center  $(-6, 4)$ , radius 5 units

7. endpoints of a diameter at  $(-12, 0)$  and  $(12, 0)$

8. endpoints of a diameter at  $(-4, 0)$  and  $(-4, -6)$

9. center at  $(7, -3)$ , passes through the origin

10. center at  $(-4, 4)$ , passes through  $(-4, 1)$

11. center at  $(-6, -5)$ , tangent to  $y$ -axis

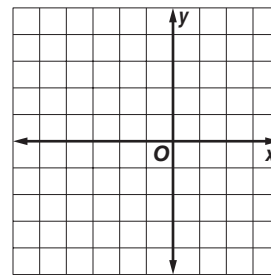
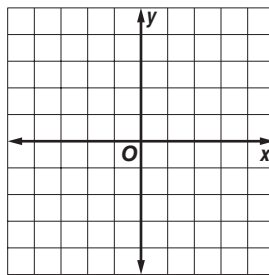
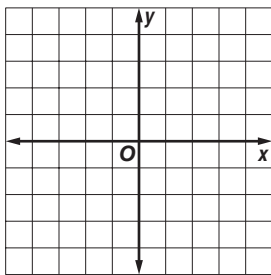
12. center at  $(5, 1)$ , tangent to  $x$ -axis

**Find the center and radius of the circle with the given equation. Then graph the circle.**

13.  $x^2 + y^2 = 9$

14.  $(x - 1)^2 + (y - 2)^2 = 4$

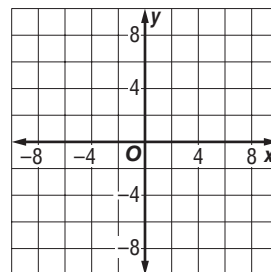
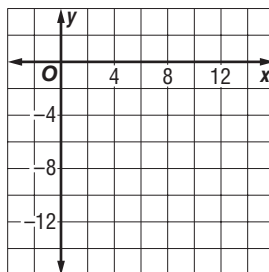
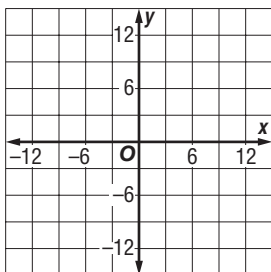
15.  $(x + 1)^2 + y^2 = 16$

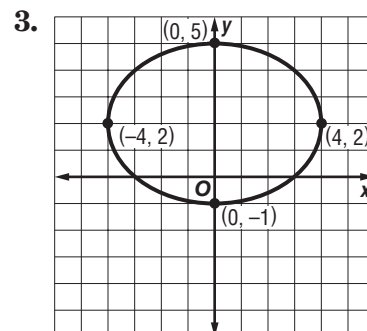
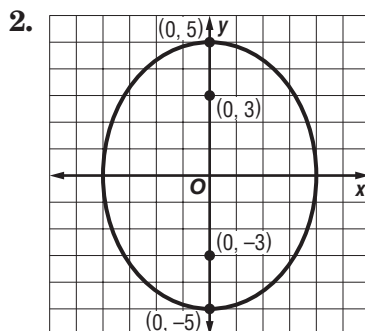
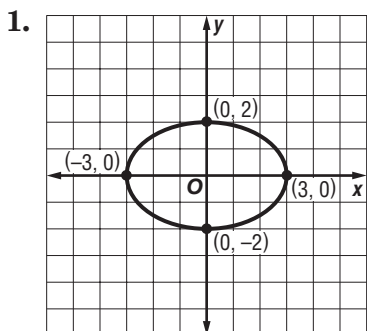


16.  $x^2 + (y + 3)^2 = 81$

17.  $(x - 5)^2 + (y + 8)^2 = 49$

18.  $x^2 + y^2 - 4y - 32 = 0$



**10-4 Skills Practice****Ellipses****Write an equation for each ellipse.****Write an equation for the ellipse that satisfies each set of conditions.**

4. endpoints of major axis at  $(0, 6)$  and  $(0, -6)$ , endpoints of minor axis at  $(-3, 0)$  and  $(3, 0)$

5. endpoints of major axis at  $(2, 6)$  and  $(8, 6)$ , endpoints of minor axis at  $(5, 4)$  and  $(5, 8)$

6. endpoints of major axis at  $(7, 3)$  and  $(7, 9)$ , endpoints of minor axis at  $(5, 6)$  and  $(9, 6)$

7. major axis 12 units long and parallel to  $x$ -axis, minor axis 4 units long, center at  $(0, 0)$

8. endpoints of major axis at  $(-6, 0)$  and  $(6, 0)$ , foci at  $(-\sqrt{32}, 0)$  and  $(\sqrt{32}, 0)$

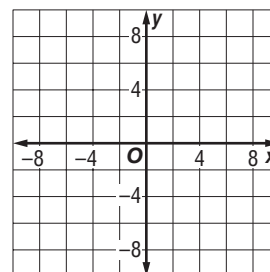
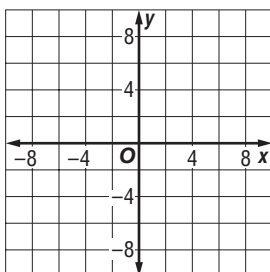
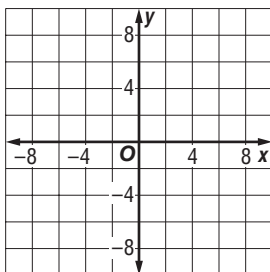
9. endpoints of major axis at  $(0, 12)$  and  $(0, -12)$ , foci at  $(0, \sqrt{23})$  and  $(0, -\sqrt{23})$

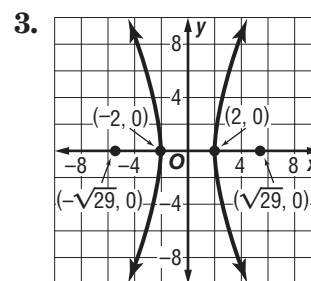
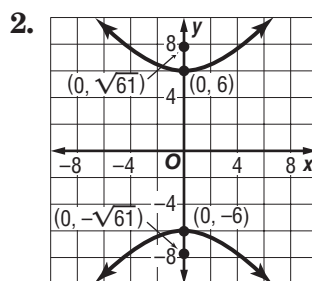
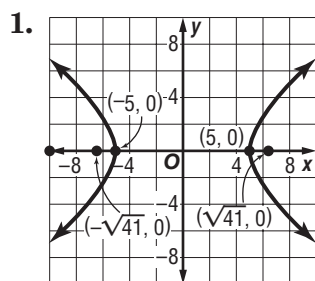
**Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.**

10.  $\frac{y^2}{100} + \frac{x^2}{81} = 1$

11.  $\frac{x^2}{81} + \frac{y^2}{9} = 1$

12.  $\frac{y^2}{49} + \frac{x^2}{25} = 1$



**10-5 Skills Practice****Hyperbolas****Write an equation for each hyperbola.****Write an equation for the hyperbola that satisfies each set of conditions.**

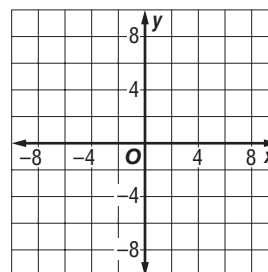
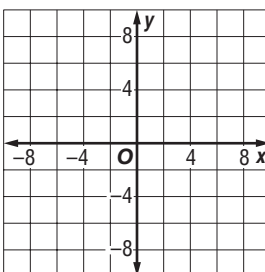
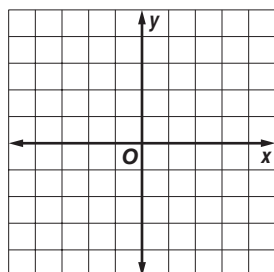
4. vertices  $(-4, 0)$  and  $(4, 0)$ , conjugate axis of length 8
5. vertices  $(0, 6)$  and  $(0, -6)$ , conjugate axis of length 14
6. vertices  $(0, 3)$  and  $(0, -3)$ , conjugate axis of length 10
7. vertices  $(-2, 0)$  and  $(2, 0)$ , conjugate axis of length 4
8. vertices  $(-3, 0)$  and  $(3, 0)$ , foci  $(\pm 5, 0)$
9. vertices  $(0, 2)$  and  $(0, -2)$ , foci  $(0, \pm 3)$
10. vertices  $(0, -2)$  and  $(6, -2)$ , foci  $(3 \pm \sqrt{13}, -2)$

**Find the coordinates of the vertices and foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.**

11.  $\frac{x^2}{9} - \frac{y^2}{36} = 1$

12.  $\frac{y^2}{49} - \frac{x^2}{9} = 1$

13.  $\frac{x^2}{16} - \frac{y^2}{1} = 1$



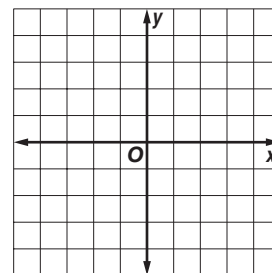
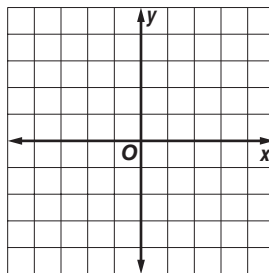
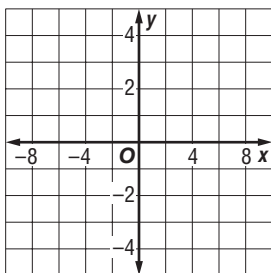
**10-6 Skills Practice****Conic Sections**

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

1.  $x^2 - 25y^2 = 25$

2.  $9x^2 + 4y^2 = 36$

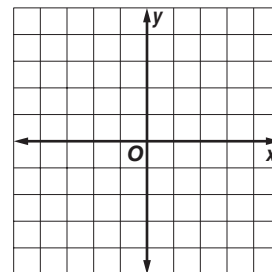
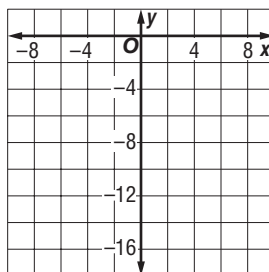
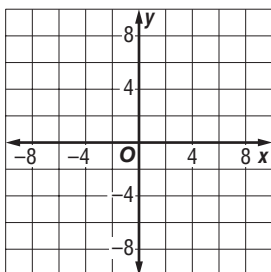
3.  $x^2 + y^2 - 16 = 0$



4.  $x^2 + 8x + y^2 = 9$

5.  $x^2 + 2x - 15 = y$

6.  $100x^2 + 25y^2 = 400$



Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

7.  $9x^2 + 4y^2 = 36$

8.  $x^2 + y^2 = 25$

9.  $y = x^2 + 2x$

10.  $y = 2x^2 - 4x - 4$

11.  $4y^2 - 25x^2 = 100$

12.  $16x^2 + y^2 = 16$

13.  $16x^2 - 4y^2 = 64$

14.  $5x^2 + 5y^2 = 25$

15.  $25y^2 + 9x^2 = 225$

16.  $36y^2 - 4x^2 = 144$

17.  $y = 4x^2 - 36x - 144$

18.  $x^2 + y^2 - 144 = 0$

19.  $(x + 3)^2 + (y - 1)^2 = 4$

20.  $25y^2 - 50y + 4x^2 = 75$

21.  $x^2 - 6y^2 + 9 = 0$

22.  $x = y^2 + 5y - 6$

23.  $(x + 5)^2 + y^2 = 10$

24.  $25x^2 + 10y^2 - 250 = 0$

**10-7 Skills Practice****Solving Quadratic Systems**

Find the exact solution(s) of each system of equations.

1.  $y = x - 2$   
 $y = x^2 - 2$

2.  $y = x + 3$   
 $y = 2x^2$

3.  $y = 3x$   
 $x = y^2$

4.  $y = x$   
 $x^2 + y^2 = 4$

5.  $x = -5$   
 $x^2 + y^2 = 25$

6.  $y = 7$   
 $x^2 + y^2 = 9$

7.  $y = -2x + 2$   
 $y^2 = 2x$

8.  $x - y + 1 = 0$   
 $y^2 = 4x$

9.  $y = 2 - x$   
 $y = x^2 - 4x + 2$

10.  $y = x - 1$   
 $y = x^2$

11.  $y = 3x^2$   
 $y = -3x^2$

12.  $y = x^2 + 1$   
 $y = -x^2 + 3$

13.  $y = 4x$   
 $4x^2 + y^2 = 20$

14.  $y = -1$   
 $4x^2 + y^2 = 1$

15.  $4x^2 + 9y^2 = 36$   
 $x^2 - 9y^2 = 9$

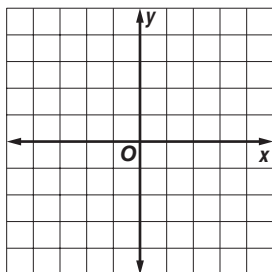
16.  $3(y + 2)^2 - 4(x - 3)^2 = 12$   
 $y = -2x + 2$

17.  $x^2 - 4y^2 = 4$   
 $x^2 + y^2 = 4$

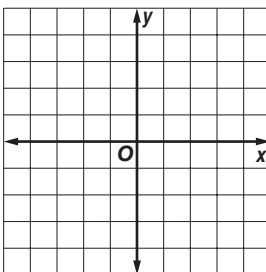
18.  $y^2 - 4x^2 = 4$   
 $y = 2x$

Solve each system of inequalities by graphing.

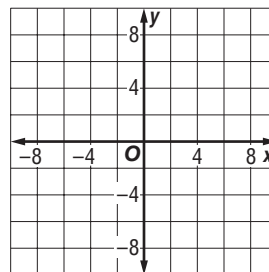
19.  $y \leq 3x - 2$   
 $x^2 + y^2 < 16$



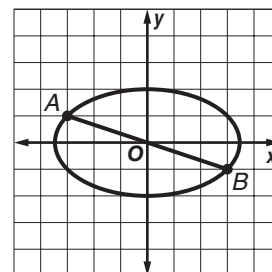
20.  $y \leq x$   
 $y \geq -2x^2 + 4$



21.  $4y^2 + 9x^2 < 144$   
 $x^2 + 8y^2 < 16$



- 22. GARDENING** An elliptical garden bed has a path from point A to point B. If the bed can be modeled by the equation  $x^2 + 3y^2 = 12$  and the path can be modeled by the line  $y = -\frac{1}{3}x$ , what are the coordinates of points A and B?



# 11-1 Skills Practice

## Arithmetic Sequences

Find the next four terms of each arithmetic sequence.

1. 7, 11, 15, ...

2. -10, -5, 0, ...

3. 101, 202, 303, ...

4. 15, 7, -1, ...

5. -67, -60, -53, ...

6. -12, -15, -18, ...

Find the first five terms of each arithmetic sequence described.

7.  $a_1 = 6, d = 9$

8.  $a_1 = 27, d = 4$

9.  $a_1 = -12, d = 5$

10.  $a_1 = 93, d = -15$

11.  $a_1 = -64, d = 11$

12.  $a_1 = -47, d = -20$

Find the indicated term of each arithmetic sequence.

13.  $a_1 = 2, d = 6, n = 12$

14.  $a_1 = 18, d = 2, n = 8$

15.  $a_1 = 23, d = 5, n = 23$

16.  $a_1 = 15, d = -1, n = 25$

17.  $a_{31}$  for 34, 38, 42, ...

18.  $a_{42}$  for 27, 30, 33, ...

Complete the statement for each arithmetic sequence.

19. 55 is the ?th term of 4, 7, 10, ...

20. 163 is the ?th term of -5, 2, 9, ...

Write an equation for the  $n$ th term of each arithmetic sequence.

21. 4, 7, 10, 13, ...

22. -1, 1, 3, 5, ...

23. -1, 3, 7, 11, ...

24. 7, 2, -3, -8, ...

Find the arithmetic means in each sequence.

25. 6, ?, ?, ?, 38

26. 63, ?, ?, ?, 147

# 11-2 Skills Practice

## Arithmetic Series

Find  $S_n$  for each arithmetic series described.

1.  $a_1 = 1, a_n = 19, n = 10$

2.  $a_1 = -5, a_n = 13, n = 7$

3.  $a_1 = 12, a_n = -23, n = 8$

4.  $a_1 = 7, n = 11, a_n = 67$

5.  $a_1 = 5, n = 10, a_n = 32$

6.  $a_1 = -4, n = 10, a_n = -22$

7.  $a_1 = -8, d = -5, n = 12$

8.  $a_1 = 1, d = 3, n = 15$

9.  $a_1 = 100, d = -7, a_n = 37$

10.  $a_1 = -9, d = 4, a_n = 27$

11.  $d = 2, n = 26, a_n = 42$

12.  $d = -12, n = 11, a_n = -52$

Find the sum of each arithmetic series.

13.  $1 + 4 + 7 + 10 + \dots + 43$

14.  $5 + 8 + 11 + 14 + \dots + 32$

15.  $3 + 5 + 7 + 9 + \dots + 19$

16.  $-2 + (-5) + (-8) + \dots + (-20)$

17.  $\sum_{n=1}^5 (2n - 3)$

18.  $\sum_{n=1}^{18} (10 + 3n)$

19.  $\sum_{n=2}^{10} (4n + 1)$

20.  $\sum_{n=5}^{12} (4 - 3n)$

Find the first three terms of each arithmetic series described.

21.  $a_1 = 4, a_n = 31, S_n = 175$

22.  $a_1 = -3, a_n = 41, S_n = 228$

23.  $n = 10, a_n = 41, S_n = 230$

24.  $n = 19, a_n = 85, S_n = 760$

# 11-3 Skills Practice

## Geometric Sequences

Find the next two terms of each geometric sequence.

1.  $-1, -2, -4, \dots$

2.  $6, 3, \frac{3}{2}, \dots$

3.  $-5, -15, -45, \dots$

4.  $729, -243, 81, \dots$

5.  $1536, 384, 96, \dots$

6.  $64, 160, 400, \dots$

Find the first five terms of each geometric sequence described.

7.  $a_1 = 6, r = 2$

8.  $a_1 = -27, r = 3$

9.  $a_1 = -15, r = -1$

10.  $a_1 = 3, r = 4$

11.  $a_1 = 1, r = \frac{1}{2}$

12.  $a_1 = 216, r = -\frac{1}{3}$

Find the indicated term of each geometric sequence.

13.  $a_1 = 5, r = 2, n = 6$

14.  $a_1 = 18, r = 3, n = 6$

15.  $a_1 = -3, r = -2, n = 5$

16.  $a_1 = -20, r = -2, n = 9$

17.  $a_8$  for  $-12, -6, -3, \dots$

18.  $a_7$  for  $80, \frac{80}{3}, \frac{80}{9}, \dots$

Write an equation for the  $n$ th term of each geometric sequence.

19.  $3, 9, 27, \dots$

20.  $-1, -3, -9, \dots$

21.  $2, -6, 18, \dots$

22.  $5, 10, 20, \dots$

Find the geometric means in each sequence.

23.  $4, \underline{\quad}, \underline{\quad}, \underline{\quad}, 64$

24.  $1, \underline{\quad}, \underline{\quad}, \underline{\quad}, 81$



# 11-4 Skills Practice

## Geometric Series

Find  $S_n$  for each geometric series described.

1.  $a_1 = 2, a_5 = 162, r = 3$

2.  $a_1 = 4, a_6 = 12,500, r = 5$

3.  $a_1 = 1, a_8 = -1, r = -1$

4.  $a_1 = 4, a_n = 256, r = -2$

5.  $a_1 = 1, a_n = 729, r = -3$

6.  $a_1 = 2, r = -4, n = 5$

7.  $a_1 = -8, r = 2, n = 4$

8.  $a_1 = 3, r = -2, n = 12$

9.  $a_1 = 8, r = 3, n = 5$

10.  $a_1 = 6, a_n = \frac{3}{8}, r = \frac{1}{2}$

11.  $a_1 = 8, r = \frac{1}{2}, n = 7$

12.  $a_1 = 2, r = -\frac{1}{2}, n = 6$

Find the sum of each geometric series.

13.  $4 + 8 + 16 + \dots$  to 5 terms

14.  $-1 - 3 - 9 - \dots$  to 6 terms

15.  $3 + 6 + 12 + \dots$  to 5 terms

16.  $-15 + 30 - 60 + \dots$  to 7 terms

17.  $\sum_{n=1}^4 3^{n-1}$

18.  $\sum_{n=1}^5 (-2)^{n-1}$

19.  $\sum_{n=1}^4 \left(\frac{1}{3}\right)^{n-1}$

20.  $\sum_{n=1}^9 2(-3)^{n-1}$

Find the indicated term for each geometric series described.

21.  $S_n = 1275, a_n = 640, r = 2; a_1$

22.  $S_n = -40, a_n = -54, r = -3; a_1$

23.  $S_n = 99, n = 5, r = -\frac{1}{2}; a_1$

24.  $S_n = 39,360, n = 8, r = 3; a_1$

**11-5 Skills Practice*****Infinite Geometric Series*****Find the sum of each infinite geometric series, if it exists.**

1.  $a_1 = 1, r = \frac{1}{2}$

2.  $a_1 = 5, r = -\frac{2}{5}$

3.  $a_1 = 8, r = 2$

4.  $a_1 = 6, r = \frac{1}{2}$

5.  $4 + 2 + 1 + \frac{1}{2} + \dots$

6.  $540 - 180 + 60 - 20 + \dots$

7.  $5 + 10 + 20 + \dots$

8.  $-336 + 84 - 21 + \dots$

9.  $125 + 25 + 5 + \dots$

10.  $9 - 1 + \frac{1}{9} - \dots$

11.  $\frac{3}{4} + \frac{9}{4} + \frac{27}{4} + \dots$

12.  $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

13.  $5 + 2 + 0.8 + \dots$

14.  $9 + 6 + 4 + \dots$

15.  $\sum_{n=1}^{\infty} 10\left(\frac{1}{2}\right)^{n-1}$

16.  $\sum_{n=1}^{\infty} 6\left(-\frac{1}{3}\right)^{n-1}$

17.  $\sum_{n=1}^{\infty} 15\left(\frac{2}{5}\right)^{n-1}$

18.  $\sum_{n=1}^{\infty} \left(-\frac{4}{3}\right)\left(\frac{1}{3}\right)^{n-1}$

**Write each repeating decimal as a fraction.**

19.  $0.\overline{4}$

20.  $0.\overline{8}$

21.  $0.\overline{27}$

22.  $0.\overline{67}$

23.  $0.\overline{54}$

24.  $0.\overline{375}$

25.  $0.\overline{641}$

26.  $0.\overline{171}$

**11-6 Skills Practice*****Recursion and Special Sequences*****Find the first five terms of each sequence.**

1.  $a_1 = 4, a_{n+1} = a_n + 7$

2.  $a_1 = -2, a_{n+1} = a_n + 3$

3.  $a_1 = 5, a_{n+1} = 2a_n$

4.  $a_1 = -4, a_{n+1} = 6 - a_n$

5.  $a_1 = 1, a_{n+1} = a_n + n$

6.  $a_1 = -1, a_{n+1} = n - a_n$

7.  $a_1 = -6, a_{n+1} = a_n + n + 1$

8.  $a_1 = 8, a_{n+1} = a_n - n - 2$

9.  $a_1 = -3, a_{n+1} = 2a_n + 7$

10.  $a_1 = 4, a_{n+1} = -2a_n - 5$

11.  $a_1 = 0, a_2 = 1, a_{n+1} = a_n + a_{n-1}$

12.  $a_1 = -1, a_2 = -1, a_{n+1} = a_n - a_{n-1}$

13.  $a_1 = 3, a_2 = -5, a_{n+1} = -4a_n + a_{n-1}$

14.  $a_1 = -3, a_2 = 2, a_{n+1} = a_{n-1} - a_n$

**Find the first three iterates of each function for the given initial value.**

15.  $f(x) = 2x - 1, x_0 = 3$

16.  $f(x) = 5x - 3, x_0 = 2$

17.  $f(x) = 3x + 4, x_0 = -1$

18.  $f(x) = 4x + 7, x_0 = -5$

19.  $f(x) = -x - 3, x_0 = 10$

20.  $f(x) = -3x + 6, x_0 = 6$

21.  $f(x) = -3x + 4, x_0 = 2$

22.  $f(x) = 6x - 5, x_0 = 1$

23.  $f(x) = 7x + 1, x_0 = -4$

24.  $f(x) = x^2 - 3x, x_0 = 5$

**11-7 Skills Practice*****The Binomial Theorem*****Evaluate each expression.**

1.  $8!$

2.  $10!$

3.  $12!$

4.  $\frac{15!}{13!}$

5.  $\frac{6!}{3!}$

6.  $\frac{10!}{2!8!}$

7.  $\frac{9!}{3!6!}$

8.  $\frac{20!}{15!5!}$

**Expand each power.**

9.  $(x - y)^3$

10.  $(a + b)^5$

11.  $(g - h)^4$

12.  $(m + 1)^4$

13.  $(r + 4)^3$

14.  $(a - 5)^4$

15.  $(y - 7)^3$

16.  $(d + 2)^5$

17.  $(x - 1)^4$

18.  $(2a + b)^4$

19.  $(c - 4d)^3$

20.  $(2a + 3)^3$

**Find the indicated term of each expansion.**

21. fourth term of  $(m + n)^{10}$

22. seventh term of  $(x - y)^8$

23. third term of  $(b + 6)^5$

24. sixth term of  $(s - 2)^9$

25. fifth term of  $(2a + 3)^6$

26. second term of  $(3x - y)^7$

**11-8 Skills Practice*****Proof and Mathematical Induction***

**Prove that each statement is true for all positive integers.**

1.  $1 + 3 + 5 + \dots + (2n - 1) = n^2$

2.  $2 + 4 + 6 + \dots + 2n = n^2 + n$

3.  $6^n - 1$  is divisible by 5.

**Find a counterexample for each statement.**

4.  $3^n + 3n$  is divisible by 6.

5.  $1 + 4 + 8 + \dots + 2^n = \frac{n(n+1)(2n+1)}{6}$

**12-1 Skills Practice*****The Counting Principle***

State whether the events are *independent* or *dependent*.

1. finishing in first, second, or third place in a ten-person race
2. choosing a pizza size and a topping for the pizza
3. Seventy-five raffle tickets are placed in a jar. Three tickets are then selected, one after the other, without replacing a ticket after it is chosen.
4. The 232 members of the freshman class all vote by secret ballot for the class representative to the Student Senate.

Solve each problem.

5. A surveying firm plans to buy a color printer for printing its maps. It has narrowed its choice to one of three models. Each of the models is available with either 32 megabytes of random access memory (RAM), 64 megabytes of RAM, or 128 megabytes of RAM. From how many combinations of models and RAM does the firm have to choose?
6. How many arrangements of three letters can be formed from the letters of the word *MATH* if any letter will not be used more than once?
7. Allan is playing the role of Oliver in his school's production of *Oliver Twist*. The wardrobe crew has presented Allan with 5 pairs of pants and 4 shirts that he can wear. From how many possible costumes consisting of a pair of pants and a shirt does Allan have to choose?
8. The 10-member steering committee that is preparing a study of the public transportation needs of its town will select a chairperson, vice-chairperson, and secretary from the committee. No person can serve in more than one position. In how many ways can the three positions be filled?
9. Jeanine has decided to buy a pickup truck. Her choices include either a V-6 engine or a V-8 engine, a standard cab or an extended cab, and 2-wheel drive or 4-wheel drive. How many possible models does she have to choose from?
10. A mail-order company that sells gardening tools offers rakes in two different lengths. Customers can also choose either a wooden, plastic, or fiberglass handle for the rake. How many different kinds of rakes can a customer buy?
11. A Mexican restaurant offers chicken, beef, or vegetarian fajitas wrapped with either corn or flour tortillas and topped with either mild, medium, or hot salsa. How many different choices of fajitas does a customer have?

**12-2 Skills Practice*****Permutations and Combinations*****Evaluate each expression.**

- |               |               |               |
|---------------|---------------|---------------|
| 1. $P(6, 3)$  | 2. $P(8, 2)$  | 3. $P(2, 1)$  |
| 4. $P(3, 2)$  | 5. $P(10, 4)$ | 6. $P(5, 5)$  |
| 7. $C(2, 2)$  | 8. $C(5, 3)$  | 9. $C(4, 1)$  |
| 10. $C(8, 7)$ | 11. $C(3, 2)$ | 12. $C(7, 4)$ |

**Determine whether each situation involves a *permutation* or a *combination*. Then find the number of possibilities.**

13. seating 8 students in 8 seats in the front row of the school auditorium
14. introducing the 5 starting players on the Woodsville High School basketball team at the beginning of the next basketball game
15. checking out 3 library books from a list of 8 books for a research paper
16. choosing 2 movies to rent from 5 movies
17. the first-, second-, and third-place finishers in a race with 10 contestants
18. electing 4 candidates to a municipal planning board from a field of 7 candidates
19. choosing 2 vegetables from a menu that offers 6 vegetable choices
20. an arrangement of the letters in the word *rhombus*
21. selecting 2 of 8 choices of orange juice at a store
22. placing a red rose bush, a yellow rose bush, a white rose bush, and a pink rose bush in a row in a planter
23. selecting 2 of 9 kittens at an animal rescue shelter
24. an arrangement of the letters in the word *isosceles*

**12-3 Skills Practice*****Probability***

Ahmed is posting 2 photographs on his website. He has narrowed his choices to 4 landscape photographs and 3 portraits. If he chooses the two photographs at random, find the probability of each selection.

1.  $P(2 \text{ portrait})$                       2.  $P(2 \text{ landscape})$                       3.  $P(1 \text{ of each})$

The Carubas have a collection of 28 video movies, including 12 westerns and 16 science fiction. Elise selects 3 of the movies at random to bring to a sleep-over at her friend's house. Find the probability of each selection.

4.  $P(3 \text{ westerns})$                       5.  $P(3 \text{ science fiction})$   
 6.  $P(1 \text{ western and } 2 \text{ science fiction})$                       7.  $P(2 \text{ westerns and } 1 \text{ science fiction})$   
 8.  $P(3 \text{ comedy})$                       9.  $P(2 \text{ science fiction and } 2 \text{ westerns})$

For Exercises 10–13, use the chart that shows the class and gender statistics for the students taking an Algebra 1 or Algebra 2 class at La Mesa High School.

If a student taking Algebra 1 or Algebra 2 is selected at random, find each probability. Express as decimals rounded to the nearest thousandth.

Class/Gender	Number
Freshman/Male	95
Freshman/Female	101
Sophomore/Male	154
Sophomore/Female	145
Junior/Male	100
Junior/Female	102

10.  $P(\text{sophomore/female})$   
 11.  $P(\text{junior/male})$   
 12.  $P(\text{freshman/male})$   
 13.  $P(\text{freshman/female})$

Find the odds of an event occurring, given the probability of the event.

14.  $\frac{5}{8}$                       15.  $\frac{2}{7}$                       16.  $\frac{3}{5}$   
 17.  $\frac{1}{10}$                       18.  $\frac{5}{6}$                       19.  $\frac{5}{12}$

Find the probability of an event occurring, given the odds of the event.

20. 2:1                      21. 8:9                      22. 4:1  
 23. 1:9                      24. 2:7                      25. 5:9



**12-4 Skills Practice*****Multiplying Probabilities*****A die is rolled twice. Find each probability.**

1.  $P(5, \text{ then } 6)$
2.  $P(\text{no } 2\text{s})$
3.  $P(\text{two } 1\text{s})$
4.  $P(\text{any number, then not } 5)$
5.  $P(4, \text{ then not } 6)$
6.  $P(\text{not } 1, \text{ then not } 2)$

**A board game uses a set of 6 different cards. Each card displays one of the following figures: a star, a square, a circle, a diamond, a rectangle, or a pentagon. The cards are placed face down, and a player chooses two cards. Find each probability.**

7.  $P(\text{circle, then star}), \text{ if no replacement occurs}$
8.  $P(\text{diamond, then square}), \text{ if replacement occurs}$
9.  $P(2 \text{ polygons}), \text{ if replacement occurs}$
10.  $P(2 \text{ polygons}), \text{ if no replacement occurs}$
11.  $P(\text{circle, then hexagon}), \text{ if no replacement occurs}$

**Determine whether the events are *independent* or *dependent*. Then find each probability.**

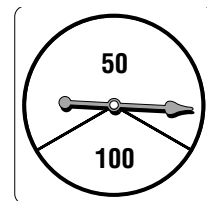
12. A mixed box of herbal teas contains 2 lemon teas, 3 orange-mango teas, 3 chamomile teas, and 1 apricot-ginger tea. Kevin chooses 2 teas at random to bring to work with him. What is the probability that he first chooses a lemon tea and then a chamomile tea?

13. The chart shows the selection of olive oils that Hasha finds in a specialty foods catalog. If she randomly selects one type of oil, then randomly selects another, different oil, what is the probability that both selections are domestic, first cold pressed oils?

Type of Oil	Domestic	Imported
Pure	2	5
Cold Pressed	4	8
First Cold Pressed	7	15

**For Exercises 14 and 15, two thirds of the area of the spinner earns you 50 points. Suppose you spin the spinner twice.**

14. Sketch a tree diagram showing all of the possibilities. Use it to find the probability of spinning 50 points, then 100 points.



15. What is the probability that you get 100 points on each spin?

**12-5 Skills Practice*****Adding Probabilities***

Eli has 10 baseball cards of 10 different players in his pocket. Three players are pitchers, 5 are outfielders, and 2 are catchers. If Eli randomly selects a card to trade, find each probability.

1.  $P(\text{pitcher or outfielder})$       2.  $P(\text{pitcher or catcher})$       3.  $P(\text{outfielder or catcher})$

A die is rolled. Find each probability.

4.  $P(5 \text{ or } 6)$       5.  $P(\text{at least a } 3)$       6.  $P(\text{less than } 4)$

Determine whether the events are *mutually exclusive* or *inclusive*. Then find the probability.

7. A die is rolled. What is the probability of rolling a 3 or a 4?
8. A die is rolled. What is the probability of rolling an even number or a 4?
9. A card is drawn from a standard deck of cards. What is the probability of drawing a king or a queen?
10. A card is drawn from a standard deck of cards. What is the probability of drawing a jack or a heart?
11. The sophomore class is selling Mother's Day plants to raise money. Susan's prize for being the top seller of plants is a choice of a book, a CD, or a video. She can choose from 6 books, 3 CDs, and 5 videos. What is the probability that Susan selects a book or a CD?

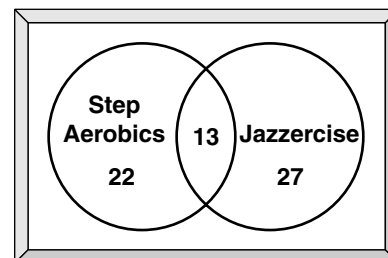
A spinner numbered 1–10 is spun. Find each probability.

12.  $P(\text{less than } 5 \text{ or even})$       13.  $P(\text{even or odd})$       14.  $P(\text{prime or even})$

Two cards are drawn from a standard deck of cards. Find each probability.

15.  $P(\text{both red or both black})$       16.  $P(\text{both aces or both red})$   
 17.  $P(\text{both 2s or both less than } 5)$       18.  $P(\text{both black or both less than } 5)$

For Exercises 19 and 20, use the Venn diagram that shows the number of participants in two different kinds of aerobic exercise classes that are offered at a health club. Determine each probability if a person is selected at random from the participants.



19.  $P(\text{step aerobics or jazzercise, but not both})$   
 20.  $P(\text{step aerobics and jazzercise})$

# 12-6 Skills Practice

## Statistical Measures

Find the variance and standard deviation of each set of data to the nearest tenth.

- {32, 41, 35, 35, 46, 42}
- {13, 62, 77, 24, 38, 19, 88}
- {89, 99, 42, 16, 42, 71, 16}
- {450, 400, 625, 225, 300, 750, 650, 625}
- {17, 23, 65, 94, 33, 33, 33, 8, 57, 75, 44, 12, 11, 68, 39}
- {7.2, 3.1, 3.8, 9.5, 8.3, 8.4}
- {1.5, 2.5, 3.5, 4.5, 4.5, 5.5, 6.5, 7.5}

For Exercises 8 and 9, use the table that shows the profit in billions of dollars reported by U.S. manufacturers for the first quarter of the years from 1997 through 2001.

Year	1997	1998	1999	2000	2001
Seasonally-Adjusted Profit (\$ billions)	\$61.4	\$75.6	\$60.9	\$78.5	\$45.3

Source: U. S. Census Bureau

- Find the mean and median of the data to the nearest tenth.
- Which measure of central tendency best represents the data? Explain.

For Exercises 10 and 11, use the table that shows the percent of fourth grade students reading at or above the proficiency level in a nationally-administered reading assessment.

Year	1992	1994	1998	2000
Percent at or above proficiency level	29%	30%	31%	32%

Source: National Center for Education Statistics

- Find the mean, median, and standard deviation of the data to the nearest tenth.
- What do the statistics from Exercise 11 tell you about the data?

**12-7 Skills Practice*****The Normal Distribution***

Determine whether the data in each table appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

1.

Miles Run	Track Team Members
0–4	3
5–9	4
10–14	7
15–19	5
20–23	2

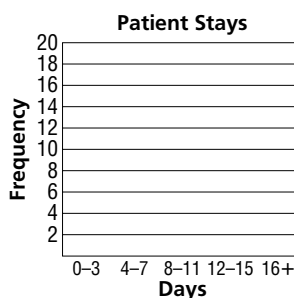
2.

Speeches Given	Political Candidates
0–5	1
6–11	2
12–17	3
18–23	8
24–29	8

For Exercises 3 and 4, use the frequency table that shows the average number of days patients spent on the surgical ward of a hospital last year.

3. Make a histogram of the data.

4. Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.



Days	Number of Patients
0–3	5
4–7	18
8–11	11
12–15	9
16+	6

**DELIVERY** For Exercises 5–7, use the following information.

The time it takes a bicycle courier to deliver a parcel to his farthest customer is normally distributed with a mean of 40 minutes and a standard deviation of 4 minutes.

5. About what percent of the courier's trips to this customer take between 36 and 44 minutes?

6. About what percent of the courier's trips to this customer take between 40 and 48 minutes?

7. About what percent of the courier's trips to this customer take less than 32 minutes?

**TESTING** For Exercises 8–10, use the following information.

The average time it takes sophomores to complete a math test is normally distributed with a mean of 63.3 minutes and a standard deviation of 12.3 minutes.

8. About what percent of the sophomores take more than 75.6 minutes to complete the test?

9. About what percent of the sophomores take between 51 and 63.3 minutes?

10. About what percent of the sophomores take less than 63.3 minutes to complete the test?

**12-8 Skills Practice****Exponential and Binomial Distribution**

For Exercises 1–4, use the following information. The exponential distribution shown at the right has a mean of 3. Use the graph or function equation to find each probability. Round to the nearest tenth.

1.  $x > 7$
2.  $x < 8$
3.  $x < 5$
4.  $x > 2$

For Exercises 5–6, use the following information. A binomial distribution has a 80% rate of success. There are 65 trials.

5. What is the expected number of successes?
6. What is the expected number of failures?

For Exercises 7–11, use the following information. A binomial distribution has a 40% rate of success. There are 23 trials.

7. What is the probability that there will be 13 successes?
8. What is the probability that there will be 15 failures?
9. What is the probability that there will be at least 4 successes?
10. What is the probability that there will be 20 or more failures?
11. What is the expected number of successes?

**MAIL** For Exercises 12–14, use the following information.

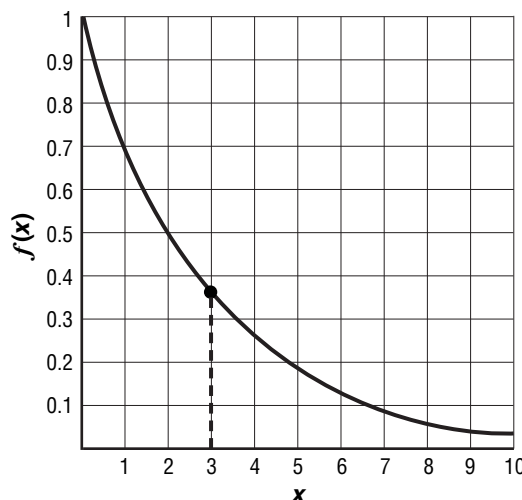
The amount of time it takes a parcel of mail to travel first class from San Francisco to New York follows an exponential distribution. The mean travel time is 5 days.

12. What is the probability that a randomly selected package will take longer than 7 days?
13. What is the probability that a randomly selected package will take less than 6 days?
14. What is the probability that a randomly selected package will take longer than 3 days?

**SOCKS** For Exercises 15–17, use the following information.

You grab 6 pairs of socks out of your drawer for a trip without looking. The socks are 75% white.

15. What is the probability that 5 pairs of the socks are white?
16. What is the probability that at least 3 pairs of the socks are white?
17. What is the probability that all of the socks are white?



## 12-9 Skills Practice

### *Binomial Experiments*

Find each probability if a coin is tossed 4 times.

1.  $P(4 \text{ heads})$
2.  $P(0 \text{ heads})$
3.  $P(\text{exactly } 3 \text{ heads})$
4.  $P(\text{exactly } 2 \text{ heads})$
5.  $P(\text{exactly } 1 \text{ head})$
6.  $P(\text{at least } 3 \text{ heads})$

Find each probability if a die is rolled 3 times.

7.  $P(\text{exactly one } 2)$
8.  $P(\text{exactly two } 2\text{s})$
9.  $P(\text{exactly three } 2\text{s})$
10.  $P(\text{at most one } 2)$

A town that presents a fireworks display during its July 4 celebration found the probability that a family with two or more children will watch the fireworks is  $\frac{3}{5}$ . If 5 of these families are selected at random, find each probability.

11.  $P(\text{exactly } 3 \text{ families watch the fireworks})$
12.  $P(\text{exactly } 2 \text{ families watch the fireworks})$
13.  $P(\text{exactly } 5 \text{ families watch the fireworks})$
14.  $P(\text{no families watch the fireworks})$
15.  $P(\text{at least } 4 \text{ families watch the fireworks})$
16.  $P(\text{at most } 1 \text{ family watches the fireworks})$

One section of a standardized English language test has 10 true/false questions. Find each probability when a student guesses at all ten questions.

17.  $P(\text{exactly } 8 \text{ correct})$
18.  $P(\text{exactly } 2 \text{ correct})$
19.  $P(\text{exactly half correct})$
20.  $P(\text{all } 10 \text{ correct})$
21.  $P(0 \text{ correct})$
22.  $P(\text{at least } 8 \text{ correct})$

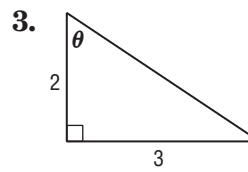
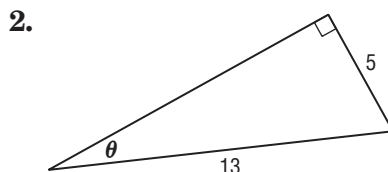
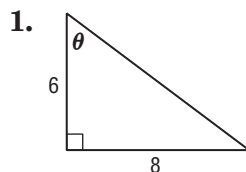
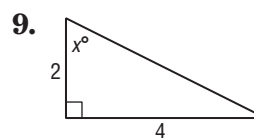
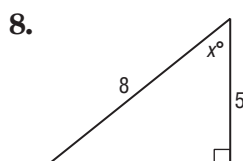
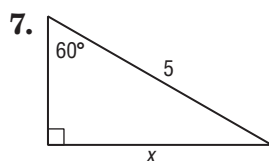
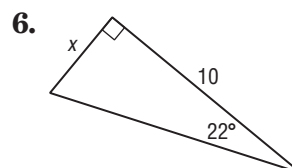
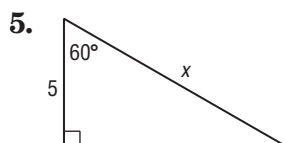
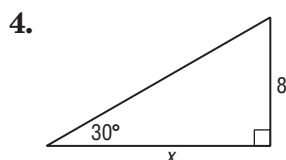
**12-10 Skills Practice*****Sampling and Error***

**Determine whether each situation would produce a random sample. Write *yes* or *no* and explain your answer.**

1. calling households at 3:30 P.M. on Tuesday to determine a political candidate's support
2. polling customers as they exit a sporting goods store about their attitudes about exercise
3. recording the number of sit-ups performed by 15-year old girls in the high schools of a large school district to determine the fitness of all high-school girls in the district
4. selecting two of a city's 20 apartment buildings for a survey to determine the desire of apartment dwellers in the city to own a home
5. In a large school district, the superintendent of schools interviews two teachers at random from each school to determine whether teachers in the district think students are assigned too much or too little homework.
6. For seven consecutive days, one hour each in the morning, afternoon, and evening, every tenth customer who enters a mall is asked to choose her or his favorite store.

**Find the margin of sampling error to the nearest percent.**

- |                             |                             |
|-----------------------------|-----------------------------|
| 7. $p = 85\%$ , $n = 100$   | 8. $p = 78\%$ , $n = 100$   |
| 9. $p = 15\%$ , $n = 100$   | 10. $p = 37\%$ , $n = 500$  |
| 11. $p = 12\%$ , $n = 500$  | 12. $p = 93\%$ , $n = 500$  |
| 13. $p = 23\%$ , $n = 1000$ | 14. $p = 56\%$ , $n = 1000$ |
15. **HEALTH** In a recent poll of cigarette smokers, 67% of those surveyed said they had tried to quit smoking within the last year. The margin of error was 3%. About how many people were surveyed?

**13-1 Skills Practice*****Right Triangle Trigonometry***Find the values of the six trigonometric functions for angle  $\theta$ .Write an equation involving  $\sin$ ,  $\cos$ , or  $\tan$  that can be used to find  $x$ . Then solve the equation. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.Solve  $\triangle ABC$  by using the given measurements. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.

10.  $A = 72^\circ, c = 10$

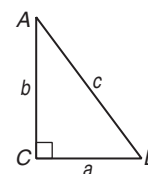
11.  $B = 20^\circ, b = 15$

12.  $A = 80^\circ, a = 9$

13.  $A = 58^\circ, b = 12$

14.  $b = 4, c = 9$

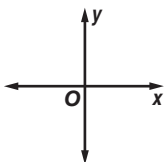
15.  $a = 7, b = 5$



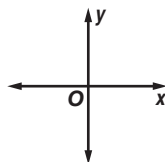


**13-2 Skills Practice*****Angles and Angle Measure*****Draw an angle with the given measure in standard position.**

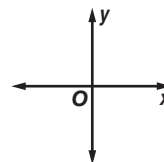
1.  $185^\circ$



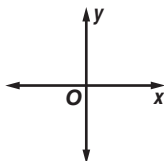
2.  $810^\circ$



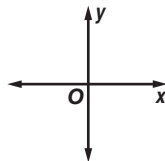
3.  $390^\circ$



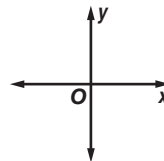
4.  $495^\circ$



5.  $-50^\circ$



6.  $-420^\circ$

**Rewrite each degree measure in radians and each radian measure in degrees.**

7.  $130^\circ$

8.  $720^\circ$

9.  $210^\circ$

10.  $90^\circ$

11.  $-30^\circ$

12.  $-270^\circ$

13.  $\frac{\pi}{3}$

14.  $\frac{5\pi}{6}$

15.  $\frac{2\pi}{3}$

16.  $\frac{5\pi}{4}$

17.  $-\frac{3\pi}{4}$

18.  $-\frac{7\pi}{6}$

**Find one angle with positive measure and one angle with negative measure coterminal with each angle.**

19.  $45^\circ$

20.  $60^\circ$

21.  $370^\circ$

22.  $-90^\circ$

23.  $\frac{2\pi}{3}$

24.  $\frac{5\pi}{2}$

25.  $\frac{\pi}{6}$

26.  $-\frac{3\pi}{4}$

**13-3 Skills Practice*****Trigonometric Functions of General Angles***

Find the exact values of the six trigonometric functions of  $\theta$  if the terminal side of  $\theta$  in standard position contains the given point.

1. (5, 12)

2. (3, 4)

3. (8, -15)

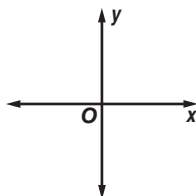
4. (-4, 3)

5. (-9, -40)

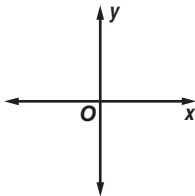
6. (1, 2)

Sketch each angle. Then find its reference angle.

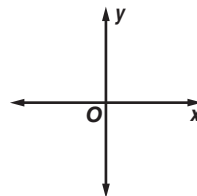
7.  $135^\circ$



8.  $200^\circ$



9.  $\frac{5\pi}{3}$



Find the exact value of each trigonometric function.

10.  $\sin 150^\circ$

11.  $\cos 270^\circ$

12.  $\cot 135^\circ$

13.  $\tan (-30^\circ)$

14.  $\tan \frac{\pi}{4}$

15.  $\cos \frac{4\pi}{3}$

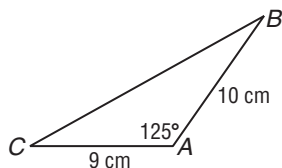
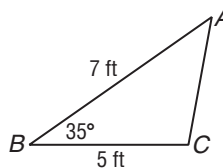
16.  $\cot (-\pi)$

17.  $\sin \left(-\frac{3\pi}{4}\right)$

Suppose  $\theta$  is an angle in standard position whose terminal side is in the given quadrant. For each function, find the exact values of the remaining five trigonometric functions of  $\theta$ .

18.  $\sin \theta = \frac{4}{5}$ , Quadrant II

19.  $\tan \theta = -\frac{12}{5}$ , Quadrant IV

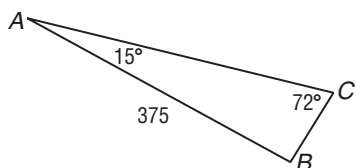
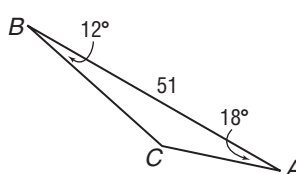
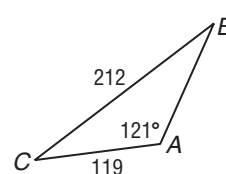
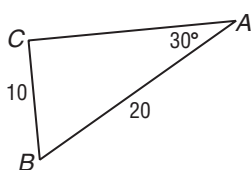
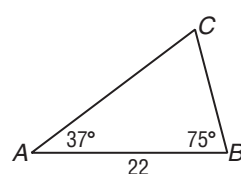
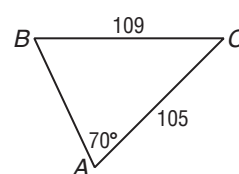
**13-4 Skills Practice****Law of Sines****Find the area of  $\triangle ABC$  to the nearest tenth.****1.****2.**

**3.**  $A = 35^\circ$ ,  $b = 3$  ft,  $c = 7$  ft

**4.**  $C = 148^\circ$ ,  $a = 10$  cm,  $b = 7$  cm

**5.**  $C = 22^\circ$ ,  $a = 14$  m,  $b = 8$  m

**6.**  $B = 93^\circ$ ,  $c = 18$  mi,  $a = 42$  mi

**Solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.****7.****8.****9.****10.****11.****12.****Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.**

**13.**  $A = 30^\circ$ ,  $a = 1$ ,  $b = 4$

**14.**  $A = 30^\circ$ ,  $a = 2$ ,  $b = 4$

**15.**  $A = 30^\circ$ ,  $a = 3$ ,  $b = 4$

**16.**  $A = 38^\circ$ ,  $a = 10$ ,  $b = 9$

**17.**  $A = 78^\circ$ ,  $a = 8$ ,  $b = 5$

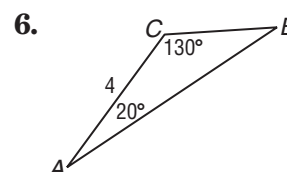
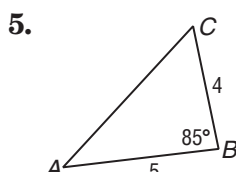
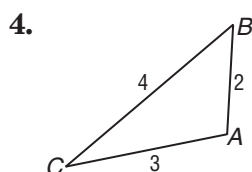
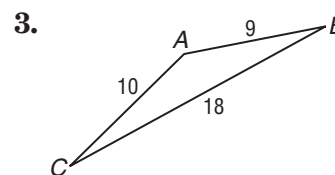
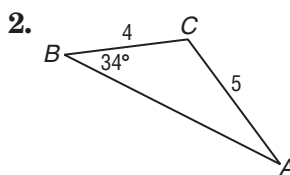
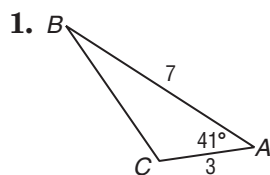
**18.**  $A = 133^\circ$ ,  $a = 9$ ,  $b = 7$

**19.**  $A = 127^\circ$ ,  $a = 2$ ,  $b = 6$

**20.**  $A = 109^\circ$ ,  $a = 24$ ,  $b = 13$

**13-5 Skills Practice*****Law of Cosines***

Determine whether each triangle should be solved by beginning with the Law of Sines or Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measures of angles to the nearest degree.



7.  $C = 71^\circ, a = 3, b = 4$

8.  $A = 11^\circ, C = 27^\circ, c = 50$

9.  $C = 35^\circ, a = 5, b = 8$

10.  $B = 47^\circ, a = 20, c = 24$

11.  $A = 71^\circ, C = 62^\circ, a = 20$

12.  $a = 5, b = 12, c = 13$

13.  $A = 51^\circ, b = 7, c = 10$

14.  $a = 13, A = 41^\circ, B = 75^\circ$

15.  $B = 125^\circ, a = 8, b = 14$

16.  $a = 5, b = 6, c = 7$

# 13-6 Study Guide and Intervention

## Circular Functions

The given point  $P$  is located on the unit circle. Find  $\sin \theta$  and  $\cos \theta$ .

1.  $P\left(\frac{3}{5}, \frac{4}{5}\right)$

2.  $P\left(\frac{5}{13}, -\frac{12}{13}\right)$

3.  $P\left(-\frac{9}{41}, -\frac{40}{41}\right)$

4.  $P(0, 1)$

5.  $P(-1, 0)$

6.  $P\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Find the exact value of each function.

7.  $\cos 45^\circ$

8.  $\sin 210^\circ$

9.  $\sin 330^\circ$

10.  $\cos 330^\circ$

11.  $\cos (-60^\circ)$

12.  $\sin (-390^\circ)$

13.  $\sin 5\pi$

14.  $\cos 3\pi$

15.  $\sin \frac{5\pi}{2}$

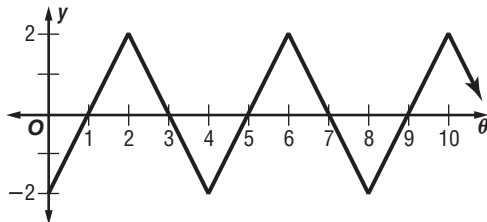
16.  $\sin \frac{7\pi}{3}$

17.  $\cos \left(-\frac{7\pi}{3}\right)$

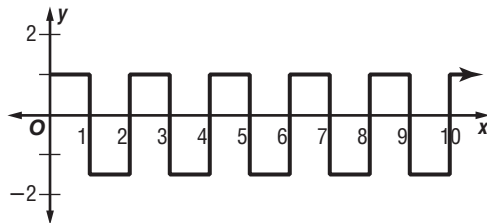
18.  $\cos \left(-\frac{5\pi}{6}\right)$

Determine the period of each function.

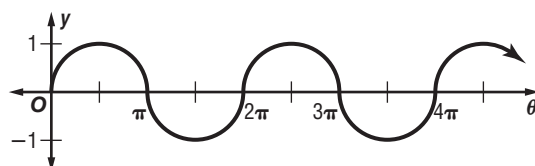
19.



20.



21.



**13-7 Skills Practice*****Inverse Trigonometric Functions*****Write each equation in the form of an inverse function.**

1.  $\alpha = \cos \beta$

2.  $\sin b = a$

3.  $y = \tan x$

4.  $\cos 45^\circ = \frac{\sqrt{2}}{2}$

5.  $b = \sin 150^\circ$

6.  $\tan y = \frac{4}{5}$

**Solve each equation by finding the value of  $x$  to the nearest degree.**

7.  $x = \cos^{-1}(-1)$

8.  $\sin^{-1}(-1) = x$

9.  $\tan^{-1} 1 = x$

10.  $x = \arcsin\left(-\frac{\sqrt{3}}{2}\right)$

11.  $x = \arctan 0$

12.  $x = \arccos \frac{1}{2}$

**Find each value. Write angle measures in radians. Round to the nearest hundredth.**

13.  $\sin^{-1} \frac{\sqrt{2}}{2}$

14.  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

15.  $\tan^{-1} \sqrt{3}$

16.  $\arctan\left(-\frac{\sqrt{3}}{3}\right)$

17.  $\arccos\left(-\frac{\sqrt{2}}{2}\right)$

18.  $\arcsin 1$

19.  $\sin(\cos^{-1} 1)$

20.  $\sin\left(\sin^{-1} \frac{1}{2}\right)$

21.  $\tan\left(\arcsin \frac{\sqrt{3}}{2}\right)$

22.  $\cos(\tan^{-1} 3)$

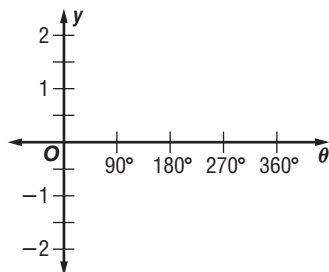
23.  $\sin[\arctan(-1)]$

24.  $\sin\left[\arccos\left(-\frac{\sqrt{2}}{2}\right)\right]$

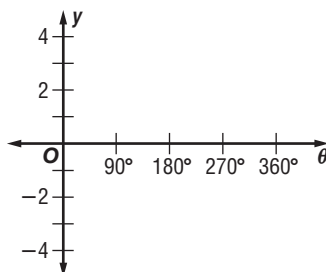
**14-1 Skills Practice****Graphing Trigonometric Functions**

Find the amplitude, if it exists, and period of each function.  
Then graph each function.

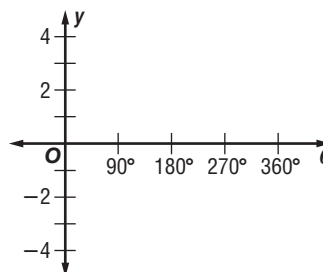
1.  $y = 2 \cos \theta$



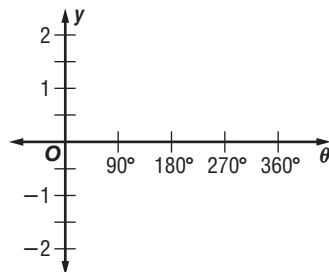
2.  $y = 4 \sin \theta$



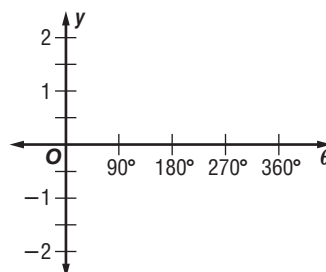
3.  $y = 2 \sec \theta$



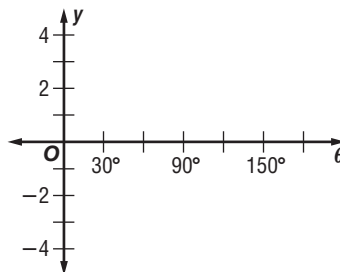
4.  $y = \frac{1}{2} \tan \theta$



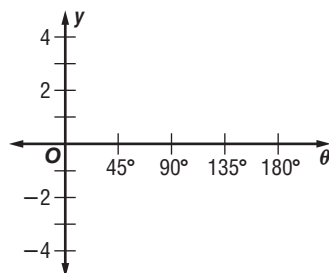
5.  $y = \sin 3\theta$



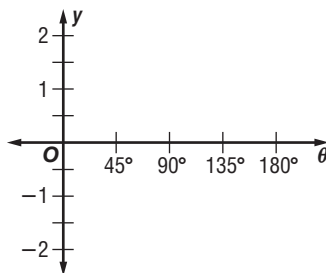
6.  $y = \csc 3\theta$



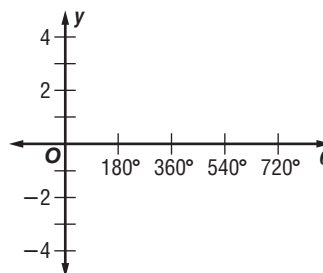
7.  $y = \tan 2\theta$



8.  $y = \cos 2\theta$



9.  $y = 4 \sin \frac{1}{2}\theta$



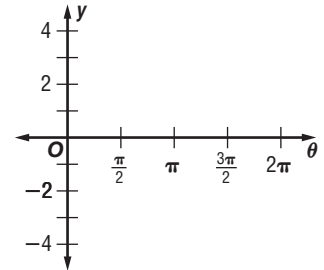
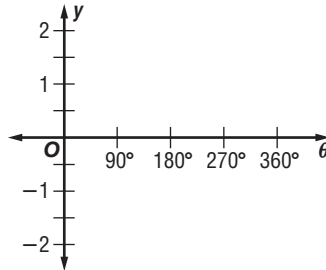
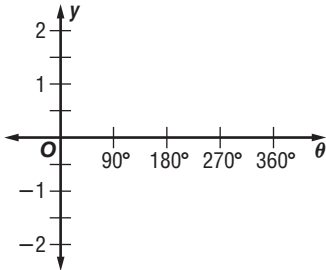
**14-2 Skills Practice****Translations of Trigonometric Graphs**

State the amplitude, period, and phase shift for each function. Then graph the function.

1.  $y = \sin(\theta + 90^\circ)$

2.  $y = \cos(\theta - 45^\circ)$

3.  $y = \tan\left(\theta - \frac{\pi}{2}\right)$

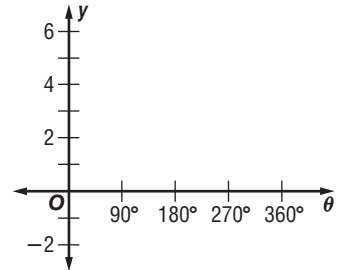
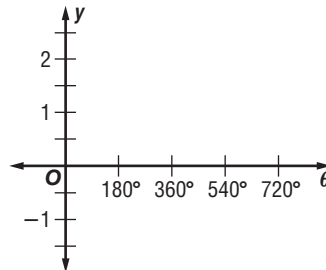
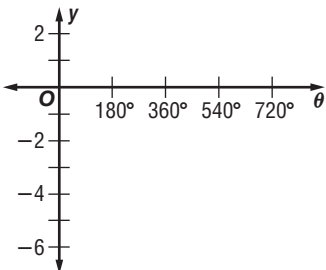


State the vertical shift, equation of the midline, amplitude, and period for each function. Then graph the function.

4.  $y = \csc \theta - 2$

5.  $y = \cos \theta + 1$

6.  $y = \sec \theta + 3$

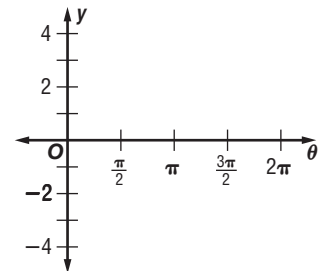
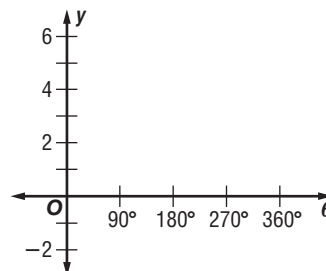
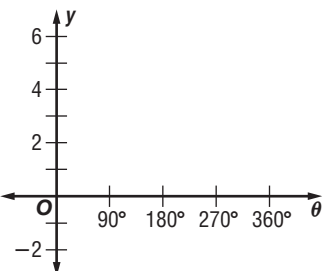


State the vertical shift, amplitude, period, and phase shift of each function. Then graph the function.

7.  $y = 2 \cos [3(\theta + 45^\circ)] + 2$

8.  $y = 3 \sin [2(\theta - 90^\circ)] + 2$

9.  $y = 4 \cot \left[ \frac{4}{3} \left( \theta + \frac{\pi}{4} \right) \right] - 2$





**14-3 Skills Practice*****Trigonometric Identities*****Find the value of each expression.**

1.  $\sin \theta$ , if  $\cos \theta = -\frac{4}{5}$  and  $90^\circ < \theta < 180^\circ$

2.  $\cos \theta$ , if  $\tan \theta = 1$  and  $180^\circ < \theta < 270^\circ$

3.  $\sec \theta$ , if  $\tan \theta = 1$  and  $0^\circ \leq \theta < 90^\circ$

4.  $\cos \theta$ , if  $\tan \theta = \frac{1}{2}$  and  $0^\circ \leq \theta < 90^\circ$

5.  $\tan \theta$ , if  $\sin \theta = -\frac{\sqrt{2}}{2}$  and  $180^\circ < \theta < 270^\circ$

6.  $\cos \theta$ , if  $\sec \theta = 2$  and  $270^\circ < \theta < 360^\circ$

7.  $\cos \theta$ , if  $\csc \theta = -2$  and  $180^\circ < \theta < 270^\circ$

8.  $\tan \theta$ , if  $\cos \theta = -\frac{2\sqrt{5}}{5}$  and  $180^\circ < \theta < 270^\circ$

9.  $\cos \theta$ , if  $\cot \theta = -\frac{3}{2}$  and  $90^\circ < \theta < 180^\circ$

10.  $\csc \theta$ , if  $\cos \theta = \frac{8}{17}$  and  $0^\circ < \theta < 90^\circ$

11.  $\cot \theta$ , if  $\csc \theta = -2$  and  $180^\circ < \theta < 270^\circ$

12.  $\tan \theta$ , if  $\sin \theta = -\frac{5}{13}$  and  $180^\circ < \theta < 270^\circ$

**Simplify each expression.**

13.  $\sin \theta \sec \theta$

14.  $\csc \theta \sin \theta$

15.  $\cot \theta \sec \theta$

16.  $\frac{\cos \theta}{\sec \theta}$

17.  $\tan \theta + \cot \theta$

18.  $\csc \theta \tan \theta - \tan \theta \sin \theta$

19.  $\frac{1 - \sin^2 \theta}{\sin \theta + 1}$

20.  $\csc \theta + \cot \theta$

21.  $\frac{\sin^2 \theta + \cos^2 \theta}{1 - \cos^2 \theta}$

22.  $1 + \frac{\tan^2 \theta}{1 + \sec \theta}$

**14-4 Skills Practice*****Verifying Trigonometric Identities***

Verify that each of the following is an identity.

1.  $\tan \theta \cos \theta = \sin \theta$

2.  $\cot \theta \tan \theta = 1$

3.  $\csc \theta \cos \theta = \cot \theta$

4.  $\frac{1 - \sin^2 \theta}{\cos \theta} = \cos \theta$

5.  $(\tan \theta)(1 - \sin^2 \theta) = \sin \theta \cos \theta$

6.  $\frac{\csc \theta}{\sec \theta} = \cot \theta$

7.  $\frac{\sin^2 \theta}{1 - \sin^2 \theta} = \tan^2 \theta$

8.  $\frac{\cos^2 \theta}{1 - \sin \theta} = 1 + \sin \theta$

**14-5 Skills Practice*****Sum and Difference of Angles Formulas*****Find the exact value of each expression.**

1.  $\sin 330^\circ$

2.  $\cos (-165^\circ)$

3.  $\sin (-225^\circ)$

4.  $\cos 135^\circ$

5.  $\sin (-45^\circ)$

6.  $\cos 210^\circ$

7.  $\cos (-135^\circ)$

8.  $\sin 75^\circ$

9.  $\sin (-195^\circ)$

**Verify that each of the following is an identity.**

10.  $\sin (90^\circ + \theta) = \cos \theta$

11.  $\sin (180^\circ + \theta) = -\sin \theta$

12.  $\cos (270^\circ - \theta) = -\sin \theta$

13.  $\cos (\theta - 90^\circ) = \sin \theta$

14.  $\sin \left( \theta - \frac{\pi}{2} \right) = -\cos \theta$

15.  $\cos (\pi + \theta) = -\cos \theta$

**14-6 Skills Practice*****Double-Angle and Half-Angle Formulas***

Find the exact values of  $\sin 2\theta$ ,  $\cos 2\theta$ ,  $\sin \frac{\theta}{2}$ , and  $\cos \frac{\theta}{2}$  for each of the following.

1.  $\cos \theta = \frac{7}{25}$ ,  $0^\circ < \theta < 90^\circ$

2.  $\sin \theta = -\frac{4}{5}$ ,  $180^\circ < \theta < 270^\circ$

3.  $\sin \theta = \frac{40}{41}$ ,  $90^\circ < \theta < 180^\circ$

4.  $\cos \theta = \frac{3}{7}$ ,  $270^\circ < \theta < 360^\circ$

5.  $\cos \theta = -\frac{3}{5}$ ,  $90^\circ < \theta < 180^\circ$

6.  $\sin \theta = \frac{5}{13}$ ,  $0^\circ < \theta < 90^\circ$

Find the exact value of each expression by using the half-angle formulas.

7.  $\cos 22\frac{1}{2}^\circ$

8.  $\sin 165^\circ$

9.  $\cos 105^\circ$

10.  $\sin \frac{\pi}{8}$

11.  $\sin \frac{15\pi}{8}$

12.  $\cos 75^\circ$

Verify that each of the following is an

13.  $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

14.  $\tan \theta + \cot \theta = 2 \csc 2\theta$

**14-7 Skills Practice*****Solving Trigonometric Equations*****Find all solutions of each equation for the given interval.**

1.  $\sin \theta = \frac{\sqrt{2}}{2}, 0^\circ \leq \theta < 360^\circ$

2.  $2 \cos \theta = -\sqrt{3}, 90^\circ < \theta < 180^\circ$

3.  $\tan^2 \theta = 1, 180^\circ < \theta < 360^\circ$

4.  $2 \sin \theta = 1, 0 \leq \theta < \pi$

5.  $\sin^2 \theta + \sin \theta = 0, \pi \leq \theta < 2\pi$

6.  $2 \cos^2 \theta + \cos \theta = 0, 0 \leq \theta < \pi$

**Solve each equation for all values of  $\theta$  if  $\theta$  is measured in radians.**

7.  $2 \cos^2 \theta - \cos \theta = 1$

8.  $\sin^2 \theta - 2 \sin \theta + 1 = 0$

9.  $\sin \theta + \sin \theta \cos \theta = 0$

10.  $\sin^2 \theta = 1$

11.  $4 \cos \theta = -1 + 2 \cos \theta$

12.  $\tan \theta \cos \theta = \frac{1}{2}$

**Solve each equation for all values of  $\theta$  if  $\theta$  is measured in degrees.**

13.  $2 \sin \theta + 1 = 0$

14.  $2 \cos \theta + \sqrt{3} = 0$

15.  $\sqrt{2} \sin \theta + 1 = 0$

16.  $2 \cos^2 \theta = 1$

17.  $4 \sin^2 \theta = 3$

18.  $\cos 2\theta = -1$

**Solve each equation for all values of  $\theta$ .**

19.  $3 \cos^2 \theta - \sin^2 \theta = 0$

20.  $\sin \theta + \sin 2\theta = 0$

21.  $2 \sin^2 \theta = \sin \theta + 1$

22.  $\cos \theta + \sec \theta = 2$