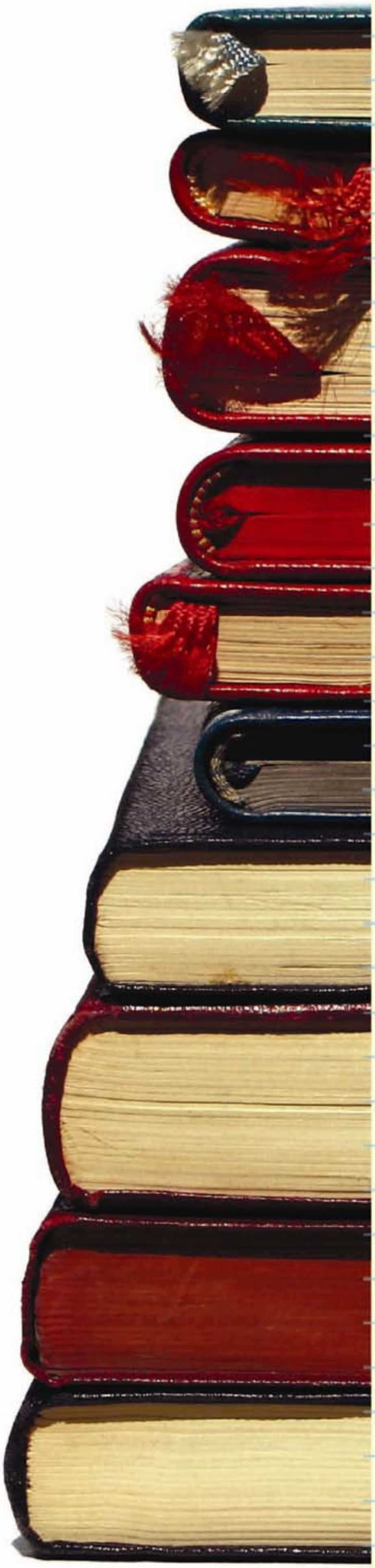

Comprehensive Curriculum

Revised 2008

Advanced Mathematics Pre-Calculus

Louisiana Department of
EDUCATION

Paul G. Pastorek, State Superintendent of Education

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Pre-test Functions

Name___________________________________

1. Given $f(x) = x^2 - x$

a) Find f(3) __________ b) Find $f(x + 1)$___ _________________

2. Find the domain and range for each of the following:

a) $f(x) = \sqrt{x}$ _____________________ Domain _________________ Range

b) $g(x) = x - 3$ _________________ Domain __________________ Range

c) $h(x) = \dfrac{1}{x}$ _________________ Domain ________________ Range

3. Write a linear equation in standard form if

a) the slope of the line is – ½ and the line passes through (4, -2)

____

b) the line passes through the points (5, 4) and (6, 3). ____________________

4. Given $f(x) = 2x - 5$ and $g(x) = \dfrac{3}{x-4}$

a) Find $f(x) + g(x)$. Write your answer in simplest form.

____

b) Find $f(x) \div g(x)$

____

5. Given $f(x) = x + 3$ find $f^{-1}(x)$

____

6. Find the zeroes of each of the following functions:

a) $f(x) = x^2 - x - 2$

____

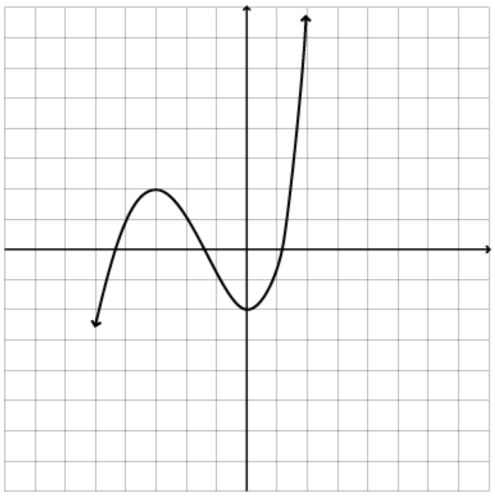
b) $f(x) = \sqrt{x-3}$ __________________

7. Solve: $\frac{3x}{5} + \frac{3}{2} = \frac{7x}{10}$ __________________________

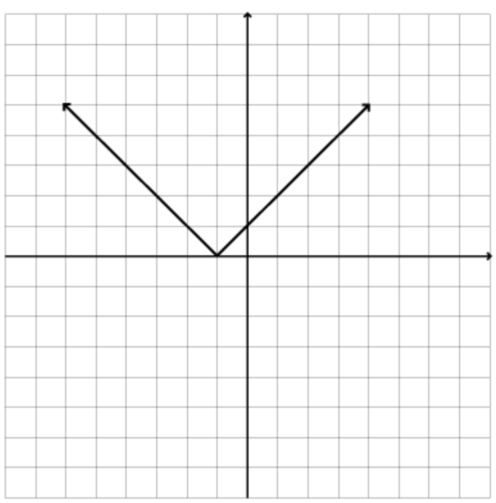
8. Solve $|x-2| \le 3$ Write your answer in interval notation. __________________

9. Given the graph of $y = f(x)$ below. Over what interval/s is the graph

increasing?____________________________ decreasing?_________________________



10. Given the graph of $y = f(x)$ below. Using the same coordinate system sketch
$y = f(x - 1) + 2$



Unit 1, Pre-test Functions with Answers

Name_____*Key*_____________

1. Given $f(x) = x^2 - x$

a) find $f(3)$ ___*6*_______ b) find $f(x + 1)$___x^2+x________________

2. Find the domain and range for each of the following:

a) $f(x) = \sqrt{x}$ _____*{x:x≥0}*____ Domain _*{y: y ≥ 0}*_____ Range

b) $g(x) = x - 3$ ___*D: all reals*_______ Domain *R: all reals*__ Range

c) $h(x) = \dfrac{1}{x}$ __*{x: x ≠ 0}*_______ Domain __*{y: y ≠ 0 }*______ Range

3. Write a linear equation in standard form if

a) the slope of the line is – ½ and the line passes through (4, -2)

______*x + 2y = 0*

b) the line passes through the points (5, 4) and (6, 3).

______$x + y = 9$

4. Given $f(x) = 2x - 5$ and $g(x) = \dfrac{3}{x-4}$

a) Find $f(x)$ + g(x). Write your answer in simplest form.

$$\frac{2x^2 - 13x + 23}{x - 4}$$

b) Find $f(x) \div g(x)$

$$\frac{2x^2 - 13x + 20}{3}$$

Unit 1, Pre-test Functions with Answers

5. Given $f(x) = x + 3$ find $f^{-1}(x)$

$\underline{f^{-1}(x) = x - 3}$

6. Find the zeroes of each of the following functions:

a) $f(x) = x^2 - x - 2$ $\underline{\{2, -1\}}$

b) $f(x) = \sqrt{x-3}$ $\underline{\{3\}}$

7. Solve: $\frac{3x}{5} + \frac{3}{2} = \frac{7x}{10}$ $\underline{\{15\}}$

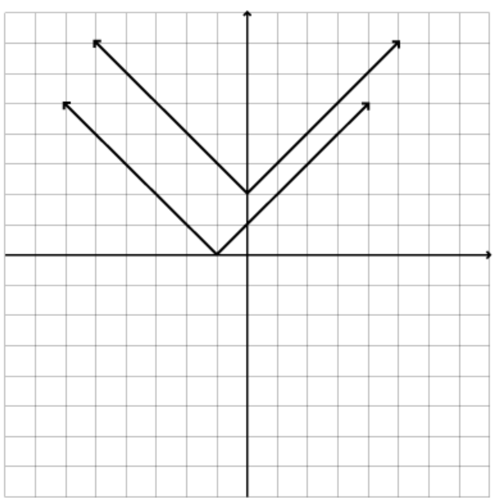
8. Solve $|x-2| \leq 3$ Write your answer in interval notation.

$\underline{[-1, 5]}$

9. Given the graph of $y = f(x)$ below. Over what interval/s is the graph

increasing? $\underline{(-\infty, -3) \text{ and } (0, \infty)}$ decreasing? $\underline{(-3, 0)}$

10. Given the graph of $y = f(x)$ below. Using the same coordinate system sketch $y = f(x - 1) + 2$



Unit 1, What Do You Know about Functions?

Word	**+**	***?***	**-**	***What do I know about this topic?***
function				
domain				
range				
independent variable				
dependent variable				
open intervals				
closed intervals				
function notation				
vertical line test				
implied domain				
increasing intervals				

decreasing intervals				
relative maximum				
relative minimum				
local extrema				
even function				
odd function				
translations				
zeros				
reflections				
dilations				
one-to-one				

composition				
inverse function				
horizontal line test				
piecewise defined function				
continuous function				
function with discontinuities				

Unit 1, Activity 1, Finding Functions in Situations

Name________________________________

Date________________________________

In each of the following identify the independent variable, the dependent variable, and sketch a possible graph of each.

1. The distance required to stop a car depends on how fast it is going when the brakes are applied.

2. The height of a punted football and the number of seconds since it was kicked.

3. The volume of a sphere is a function of its radius.

4. The amount of daylight in Shreveport depends on the time of year.

5. If you blow up a balloon, its diameter and the number of breaths blown into it are related.

1. The distance required to stop a car depends on how fast it is going when the brakes are applied.

independent variable is speed; dependent variable is distance

There is a formula $d = \frac{s^2}{20} + s$ *where s > 0. Students should realize that the graph will be found in the first quadrant and should be a curve that is concave up since it takes more distance to sto, the faster one is going. More information can be found at http://www.hintsandthings.co.uk/garage/stopmph.htm*

2.The height of a punted football and the number of seconds since it was kicked.

The independent variable is time in seconds and the dependent variable is height in feet. The graph should be a parabola opening down and found in the first quadrant.

3. The volume of a sphere is a function of its radius.

The independent variable is the radius r and the dependent variable is the volume V. This again is a curve (cubic polynomial) found in the first quadrant only, corresponding to the formula $V = \frac{4}{3}\pi r^3$*, r > 0.*

4. The amount of daylight in Shreveport depends on the time of year.

The independent variable is time and the dependent variable is the number of hours of daylight. This is a periodic function and will be studied in Unit 5. Students could use the following points to sketch a graph:
For the northern hemisphere-

- *vernal equinox in March has 12 hours of daylight and 12 hours of dark*
- *summer solstice in June has the largest amount of daylight; the amount depends on the latitude*
- *autumnal equinox in September has 12 hours of daylight and 12 hours of dark*
- *winter solstice in December has the smallest amount of daylight; the amount depends on the latitude*

5. If you blow up a balloon, its diameter and the number of breaths blown into it are related.

The independent variable is the number of breaths and the dependent variable is the diameter of the balloon. The graph should be linear. This makes an excellent activity. Let the students collect data by giving each group a balloon, having one person blow into the balloon and measuring the diameter after every 4 to 5 blows.

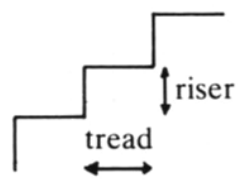
Directions: Use the procedure below to solve the following problems.

Procedure for Developing a Mathematical Model

1. Set up a table using data that appear to be related.
2. Set up a coordinate system, label axes, and choose appropriate scales.
3. Plot data points as ordered pairs on the coordinate system.
4. Sketch a curve that passes through the points.
5. Describe the functional relationship (or an approximation of it) with a symbolic formula.
6. Use the curve and the equation to predict other outcomes.
7. Consider the reasonableness of your results.
8. Consider any limitations of the model.
9. Consider the appropriateness of the scales.

Problems:

1. Stairs are designed according to the following principle:
   The normal pace length is 60 cm. This must be decreased by 2 cm for every 1 cm that a person's foot is raised when climbing stairs. According to this design, how should the "tread length" (see diagram) depend upon the height of each "riser"?
   a) Set up a table to show the relationship.
   b) Graph the points.
   c) What equation models this relationship?



2. Amy needs to buy a new gas water heater and has narrowed her choice down to two different models. One model has a purchase price of $278.00 and will cost $17.00 per month to operate. The initial cost of the second model is $413.00, but because of the higher energy factor rating, it will cost an average of $11.00 a month to operate.
   a. Set up a table to compare the two heaters.
   b. Write an equation for each of the two models.
   c. Construct a graph for each using the same coordinate system.
   d. Explain which model would be a better buy.

3. A craftsman making decorative bird houses has invested $350.00 in materials. He plans to sell his houses at craft and garden shows for $14.99.
   a. Set up a model, draw a graph, and determine an equation to calculate his profit.
   b. Use the graph to estimate how many bird houses he would have to sell to break even.
   c. Use the equation to determine how many bird houses he would have to sell to break even.
   d. How many birdhouses would he have to sell in order to make a profit of $250.00?

4. Suppose a dump truck, purchased new for $150,000 by the ABC construction company, has an expected useful life of 10 years and has an expected salvage value of

\$30,000 at the end of that time. IRS allows the loss in value of the truck as a deductible expense of doing business. Although there are a number of methods for determining the annual amount of depreciation, the simplest is the straight-line method. Using this model, the company assumes that the value V, at any time, t, can be represented by a straight line that contains the two ordered pairs, (0, 150,000) and (10, 30,000).

a. Describe how to interpret the two ordered pairs.

b. Determine the equation of the line that passes through the two ordered pairs and draw the graph.

c. Interpret the slope within the context of the problem. What does the y-intercept stand for?

d. What are the domain and range of the linear function in (b)? How does this differ from the domain and range within context of problem?

5. Writing exercise: In problem #3 you obtained the equation of a line to model the profit from selling bird houses. What is the difference between the domain of the linear function obtained and the domain within the context of the problem?

Unit 1, Activity 1, Solving Problems Using Mathematical Modeling with Answers

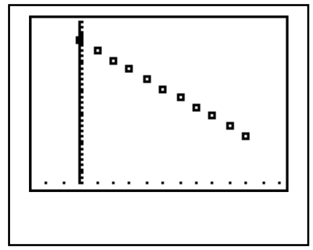
1. Stairs are designed according to the following principle:

   The normal pace length is 60 cm. This must be decreased by 2 cm for every 1 cm that a person's foot is raised when climbing stairs. According to this design, how should the "tread length" (see diagram) depend upon the height of each "riser"?

Students should recognize that the table shows a constant rate of change so the equation will be linear in nature. A carefully constructed graph will show a linear <u>set of points</u>.

Riser (R)	*Tread (T)*
0	*60*
1	*58*
2	*56*
3	*54*

equation: $T = -2R + 60$



2. Amy needs to buy a new gas water heater and has narrowed her choice down to two different models. One model has a purchase price of \$278.00 and will cost \$17.00 per month to operate. The initial cost of the second model is \$413.00 but because of the higher energy factor rating, it will cost an average of \$11.00 a month to operate.

   a. Set up a table to compare the two heaters.

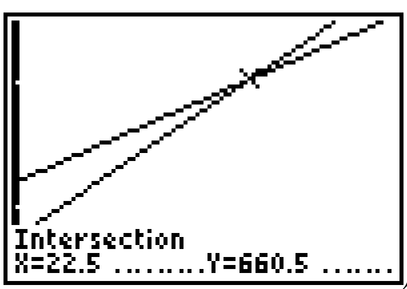
Month	*Heater 1*	*Heater 2*
0	*\$278.*	*\$413.*
1	*\$295.*	*\$424.*
2	*\$312.*	*\$435.*
3	*\$329.*	*\$446.*

   b) Use the table to write equations for the two models.

   Heater 1has the equation $c = 17m + 278$ and Heater 2 has the equation

   $c = 11m + 413$, where c is the cost per month and m is the number of months

   c. Graph the equations. What do you see?



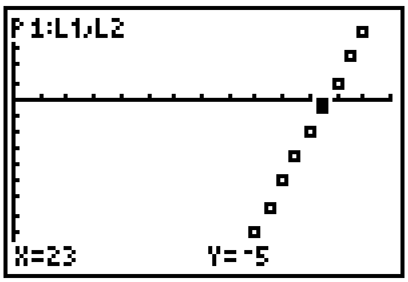
)

d. Explain which model would be a better buy.
A graph of the two lines on the same coordinate system show that by the 23rd month heater 1 becomes the more expensive buy. Since water heaters last for quite a few years, the second heater with its higher energy efficiency is the better buy.

3 A craftsman making decorative bird houses has invested $350.00 in materials. He plans to sell his houses at craft and garden shows for $14.99.

a. Set up a model, determine the equation, and draw a graph that can be used to calculate his profit.
The model needed is Profit = Revenue – Cost
$P = 14.99n - 350$

b. Graph that equation and use it to estimate how many bird houses he would have to sell to break even.



On the graph the break even point is the x-intercept ≈ 24 bird houses. The graph is really a series of points with the domain the whole numbers.

c. Use the equation to determine how many bird houses he would have to sell to break even.
Setting the equation = to 0 and rounding up gives 24 bird houses.

d. How many birdhouses would he have to sell in order to make a profit of $250.00?
He would have to sell 41 bird houses to make a profit of $250.00

4. Suppose a dump truck, purchased new for $150,000 by the ABC construction company, has an expected useful life of 10 years and has an expected salvage value of $30,000 at the end of that time. IRS allows the loss in value of the truck as a deductible expense of doing business. Although there are a number of methods for determining the annual amount of depreciation, the simplest is the straight-line method. Using this model, the company assumes that the value *V*, at any time, *t*, can be represented by a straight line that contains the two ordered pairs, (0, 150,000) and (10, 30,000).

a. Describe how to interpret the two ordered pairs.
(0, 150,000) represents the initial cost of the dump truck and (10, 30,000) represents what the truck is worth at the end of 10 years

b. Determine the equation of the line that passes through the two ordered pairs and draw the graph.
V = -12,000n + 150,000, where V is the value of the truck at n years

c. Interpret the slope within the context of the problem.
$12,000 is depreciation per year

d. What does the y-intercept stand for?
The y-intercept is the initial cost of the truck.

e. What are the domain and range of the linear function in (b)? How does this differ from the domain and range within context of problem?
The domain and range of any linear function is the set of reals. In this case the domain is a set of whole numbers $0 \le n \le 10$ and the range $30{,}000 \le c \le 150{,}000$.

5. Writing exercise: In problem #3 you obtained the equation of a line to model the profit from selling bird houses. What is the difference between the domain of the linear function obtained and the domain within the context of the problem?

   The domain of a linear function is the set of real numbers. However, within the context of this problem, the domain is a subset of whole numbers with the largest value of n depending on the number of bird houses that can be built from the $350.00 worth of materials.

Name______________________________

Date_______________________________

1. Find the domain and range of each of the following functions. Support your answer with a graphing utility. Show a sketch of the graph beside each answer.

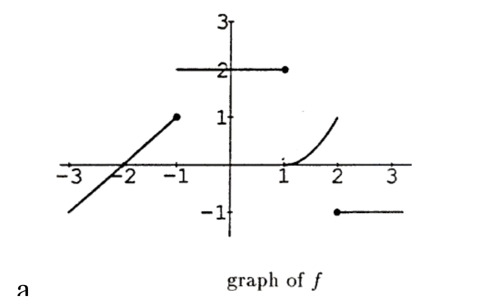
a. $f(x) = 2 + \sqrt{x-1}$

b. $g(x) = \sqrt{x-3} - 4$

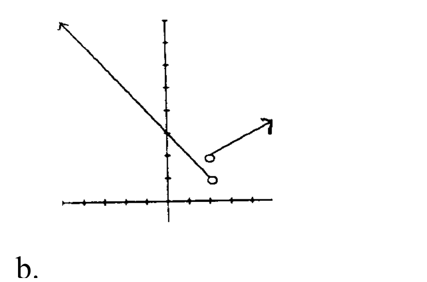
c. $h(x) = \sqrt{x^2 - 4}$

d. $f(x) = \dfrac{2}{x-3}$

2. Find the domain and range of each of the graphs below.



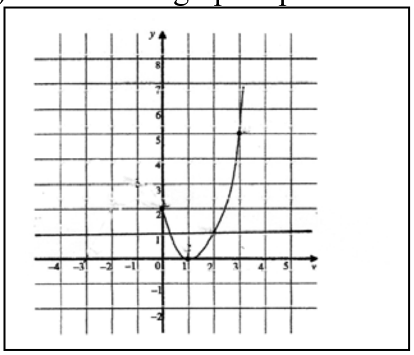
a


b.

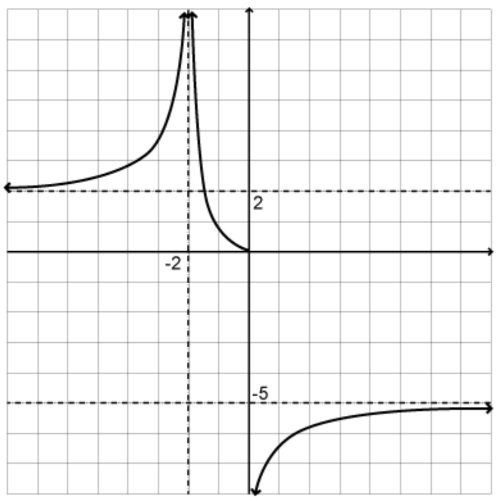
Domain:___________________ Domain:________________________

Range: ___________________ Range: ________________________

3. Using graph paper complete the graph below so that
   a) the finished graph represents an even function and
   b) the finished graph represents an odd function.



4. Below is the graph of $y = f(x)$



   a) What is the domain of $f(x)$?

   b) What is the range?

   c) On which intervals is $f(x)$ increasing?

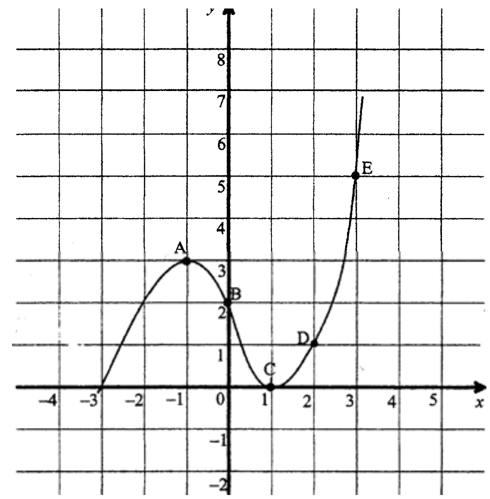
   d) On which intervals is $f(x)$ decreasing?

   e) On which intervals is $f(x)$ negative?

f) On which intervals is $f(x)$ positive?

g) Describe the end-behavior of $f(x)$.

5.The sketch below shows part of the graph of $y = f(x)$ which passes through the points A(-1,3), B(0, 2), C(1,0), D(2, 1), and E(3, 5).



a) On which intervals is $f(x)$ increasing?

b) On which intervals is $f(x)$ decreasing?

c) What are the zeros?

d) Identify the location of the relative maximum. What is its value?

e) Identify the location of the relative minimum. What is its value?

f) A second function is defined by
$g(x) = f(x - 1) + 2$.
i) Calculate $g(0)$, and $g(3)$.
ii) On the same set of axes sketch a graph of the function $g(x)$.

6. Writing activity: Compare the domain and ranges of the functions defined by $y = \sqrt{x^2}$ and $y = \left(\sqrt{x}\right)^2$. Explain any differences you might see

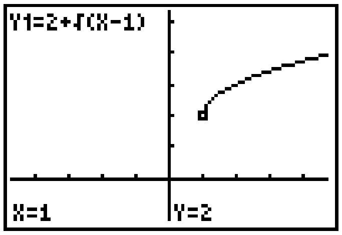
Unit 1, Activity 2, Functions and their Graphs with Answers

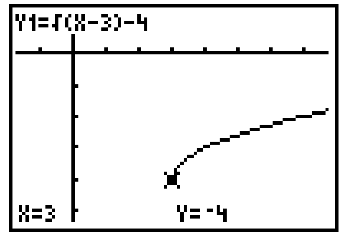
1. Find the domain and range of each of the following functions. Support your answer with a graphing utility. Show a sketch of the graph beside each answer.

   a. $f(x) = 2 + \sqrt{x-1}$ *b.* $g(x) = \sqrt{x-3} - 4$

a) domain {x: x ≥ 1} range {y: y≥ 2}
The endpoint is (1,2)

b) domain {x : x ≥ 3} range {y : y ≥ -4}
The endpoint is (3, -4)

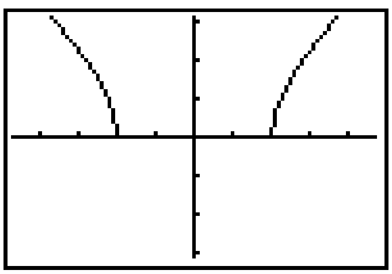


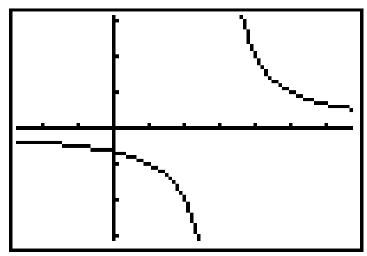


c. $h(x) = \sqrt{x^2 - 4}$ d. $f(x) = \dfrac{2}{x-3}$

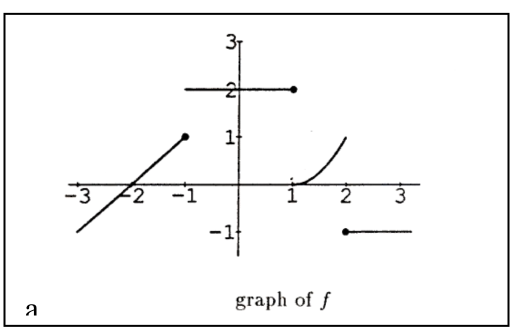
c) domain {x: x ≤ -2 or x ≥ 2} range {y: y ≥ 0}

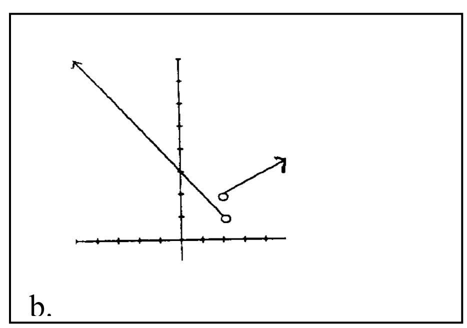
d) domain {x : all reals except 3}
range {y: all reals except 0}





2. Find the domain and range of each of the graphs below.

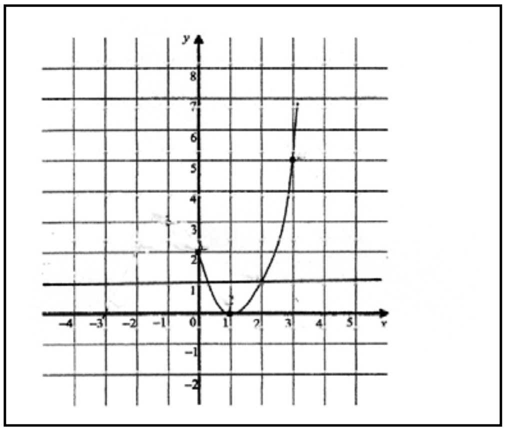




2. a) domain {x: -3 ≤ x ≤ 3} and range {y: -1 ≤ y ≤ 1, y = 2}
b) domain {x: reals except 2}and range {y: y > 1}

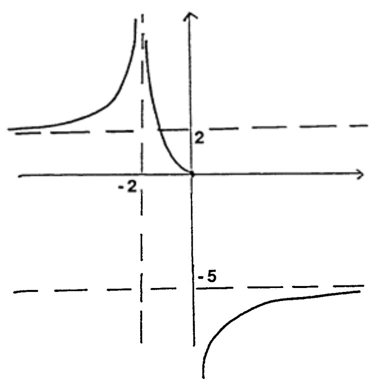
Unit 1, Activity 2, Functions and their Graphs with Answers

3. Using graph paper complete the graph below so that
   a) the finished graph represents an even function and
   b) the finished graph represents an odd function



3. a) For an even function reflect the graph over the y-axis.
b) For an odd function reflect the graph around the origin

4.

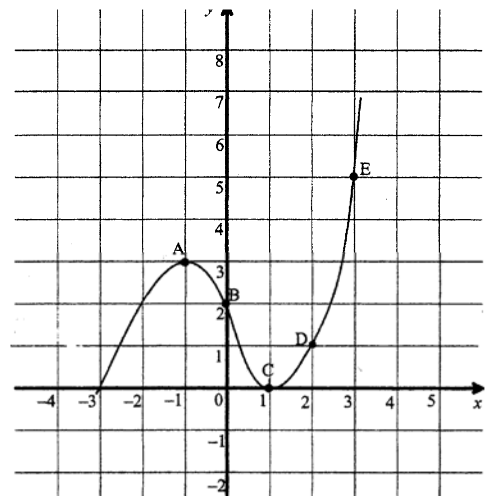


a) What is the domain of *f (x)*?
domain {x: x reals except -2}
b) What is the range?
{y: y < -5 or y ≥ 0}
c) On which intervals is *f (x)* increasing?
increasing for x < -2 or x > 0
d) On which intervals is *f (x)* decreasing?
decreasing for -2 < x ≤ 0
e) On which intervals is *f (x)* negative?
Negative for x > 0
f) On which intervals is *f (x)* positive?
Positive for x < 0
g) Describe the end-behavior of *f (x)*.
When $x \to -\infty$ f(x) $\to 2$ *and when* $x \to \infty\, f(x) \to -5$

5. The sketch below shows part of the graph of $y = f(x)$ which passes through the points A(-1,3), B(0, 2), C(1,0), D(2, 1), and E(3, 5).

   a) On which intervals is $f(x)$ increasing?

   increasing $-3 < x < -1$ *and* $1 < x < 3$



   b) On which intervals is $f(x)$ decreasing?

   decreasing $-1 < x < 1$

   c) What are the zeros?

   $f(-3) = 0$ *and* $f(1) = 0$

   d) Identify the location of the relative maximum. What is its value?

   Relative maximum is located at x = -1. Its value is 3.

   e) Identify the location of the relative minimum. What is its value?

   The relative minimum is located at x = 1. Its value is 0.

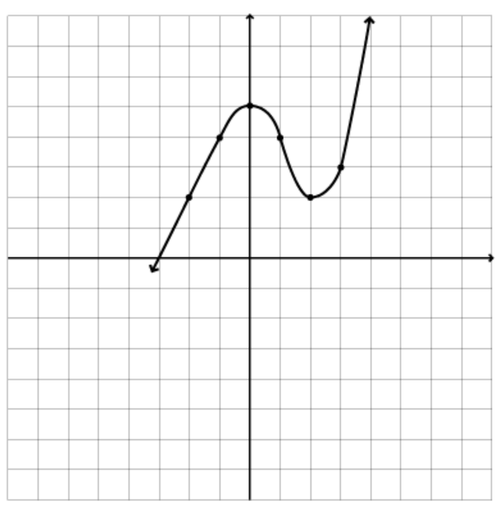
   f) A second function is defined by $g(x) = f(x - 1) + 2$.

   i) Calculate $g(0)$, and $g(3)$.

   $g(0) = 5$ *and* $g(3) = 3$

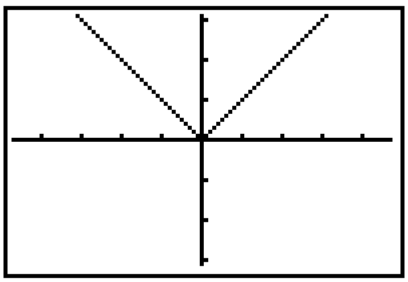
   ii) On the same set of axes sketch a graph of the function $g(x)$.

   (ii) The new graph should pass through the points (0, 5), (1, 4), (2, 2) (3, 3) and (4, 7) as shown below.

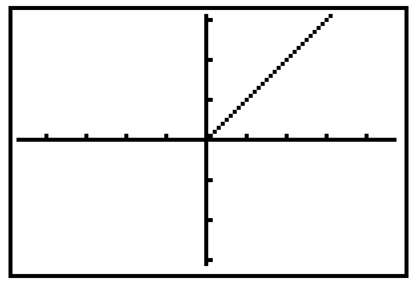


6. Writing activity: Compare the domain and ranges of the functions defined by $y=\sqrt{x^2}$ and $y=\left(\sqrt{\text{x}}\right)^2$. Explain any difference you might see.

Students should graph both y_1 and y_2 using their graphing calculators. Both have the same range but the domain of y_1 is the set of reals and the domain of y_2 is $\{x: x \geq 0\}$. In y_1 the input is squared before the square root is taken so the number is always positive. In y_2 the square root is taken before the result is squared. This problem can also be used with composition of functions.



$y_1 = \sqrt{x^2}$



$y_2 = \left(\sqrt{x}\right)^2$

Unit 1, Activity 3, Operations on Functions

Name____________________________

Date_____________________________

1. Given the following functions: $f(x) = x + 2,\ g(x) = \dfrac{1}{x}$, and $h(x) = x^2 - x - 2$.

   Find and simplify your answers:

   $a. (f + g)(x)$

   $b. \left(\dfrac{f}{g}\right)(x)$

   $c. (g \cdot h)(x)$

   $d. (f \circ h)(x)$

2. Each of the following is a composition of functions *f(g(x))*. Fill in the grid below.

f(g(x))	**f(x)**	**g(x)**	**Domain of f(g(x))**	**Domain of *f***	**Domain of *g***
$a.\ \sqrt{3x+2}$					
$b.\ \left(\dfrac{x+1}{x-2}\right)^3$					
$c.\ (\lvert x \rvert)^3$					
d. $\left(\dfrac{1}{\sqrt{9-x^2}}\right)^2$					
e. $\sqrt{\dfrac{x+1}{x-5}}$					

3. Given the table:

x	0	1	2	3	4	5
$f(x)$	10	6	3	4	7	11
$g(x)$	-3	-1	0	1	3	5

Find:

a) $3f(x)$

b) $2 - f(x)$

c) $f(x) - g(x)$

4. In each of the problems below the order in which the transformations are to be applied to the graph is specified. In each case, sketch the graph and write an equation for the transformed graph.

a) $y = x^2$, vertical stretch by a factor of 3, then a shift up by 4

b) $y = |x|$, shift left 3, vertical shrink by ½ , shift down 4

c) $y = \sqrt{x}$, vertical stretch by 2, reflect through x-axis, shift left 5, shift down 2

5. Suppose a store sells calculators by marking up the price 20%. The price, then, of one calculator costing c dollars is $p(c) = c + 0.2c$. The cost of manufacturing n calculators is $50n + 200$ dollars. Thus the cost of each calculator is $c(n) = \dfrac{50n + 200}{n}$
   a) Find the price for one calculator if only one calculator is manufactured.

   b) Find the price for one calculator if 1000 calculators are manufactured.

   c) Express the price as a function of the number of calculators produced by finding $p(c(n))$.

   d) Sketch a graph of the resulting function.

6. Writing activity: What can be said about the composition of an even function with an odd function? Using several even and odd functions, investigate their composition both algebraically and graphically. Show your work and write a paragraph summarizing what you found.

Unit 1, Activity 3, Operations on Functions with Answers

1. Given the following functions: $f(x) = x + 2,\ g(x) = \frac{1}{x}$, and $h(x) = x^2 - x - 2$.

Find and simplify your answers:

$a. (f + g)(x)$	$a.\ \frac{x^2 + 2x + 1}{x}$
$b. \left(\frac{f}{g}\right)(x)$	$b.\ x^2 + 2x$
$c. (g \cdot h)(x)$	$c.\ \frac{x^2 - x - 2}{x}$
$d. (f \circ h)(x)$	$d.\ x^2 - x$

2. Key for composition of functions is provided below.

f(g(x))	***f(x)***	***g(x)***	**Domain of *f(g(x))***	**Domain of *f***	**Domain of *g***
$a.\ \sqrt{3x+2}$	$\sqrt{x}$	$3x + 2$	$\{x{:}x \geq -2/3\}$	$x \geq 0$	Reals
$b.\ \left(\frac{x+1}{x-2}\right)^{1/3}$	$x^{1/3}$	$\frac{x+1}{x-2}$	$\{x{:}x \neq 2\}$	Reals	$\{x{:}x \neq 2\}$
$c.\ (\lvert x\rvert)^3$	x^3	$\lvert x\rvert$	Reals	Reals	Reals
d. * $\left(\frac{1}{\sqrt{9-x^2}}\right)^2$	x^2	$\frac{1}{\sqrt{9-x^2}}$	$-3 < x < 3$	Reals	$-3 < x < 3$
e. $\sqrt{\frac{x+1}{x-5}}$	$\sqrt{x}$	$\frac{x+1}{x-5}$	$x \leq -1$ or $x > 5$	$x \geq 0$	$x : x \neq 5$

** This is one of several answers for this problem.*

3. Given the table:

x	0	1	2	3	4	5
$f(x)$	10	6	3	4	7	11
$g(x)$	-3	-1	0	1	3	5

Find:

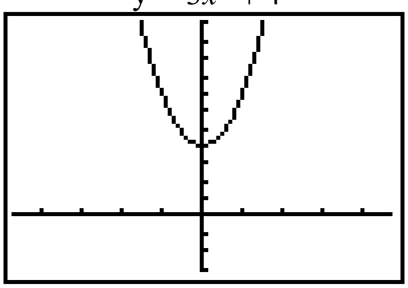
a) $3f(x)$
b) $2 - f(x)$
c) $f(x) - g(x$

x	*0*	*1*	*2*	*3*	*4*	*5*
$3f(x)$	*30*	*18*	*9*	*12*	*21*	*33*
$2\text{-}f(x)$	*-8*	*-4*	*-1*	*-2*	*-5*	*-9*
$f(x)\text{-}g(x)$	*13*	*7*	*3*	*3*	*4*	*6*

4. In each of the problems below the order in which the transformations are to be applied to the graph is specified. In each case, sketch the graph and write an equation for the transformed graph.

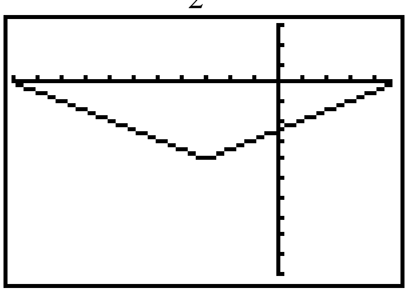
a) $y = x^2$, vertical stretch by a factor of 3 and a shift up by 4

$y = 3x^2 + 4$



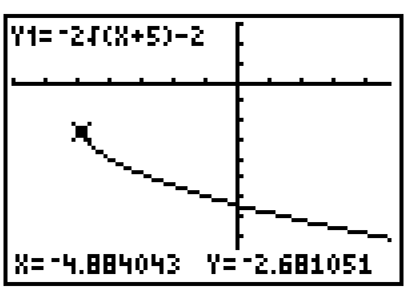
b) $y = |\,x\,|$, shift left 3, vertical shrink by ½ , shift down 4

$y = \frac{1}{2}|x+3| - 4$



c) $y = \sqrt{x}$, vertical stretch by 2, reflect through x-axis, shift left 5, shift down 2

$y = -2\sqrt{x+5} - 2$



5. Suppose a store sells calculators by marking up the price 20%. The price, then, of one calculator costing *c* dollars is *p(c)* = *c* + 0.2*c*. The cost of manufacturing *n* calculators is 50*n* + 200 dollars. Thus the cost of each calculator is $c(n) = \dfrac{50n + 200}{n}$

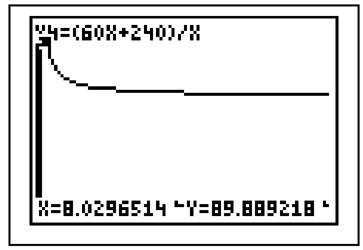
a) Find the price for one calculator if only one calculator is manufactured.
p(1) = $300. if 1 calculator is manufactured

b) Find the price for one calculator if 1000 calculators are manufactured.
p(1) = $60.24 if 1000 calculators are manufactured

c) Express the price as a function of the number of calculators produced by finding *p(c(n))*.

$$p = \frac{60n + 240}{n}$$

d) Sketch a graph of the resulting function.



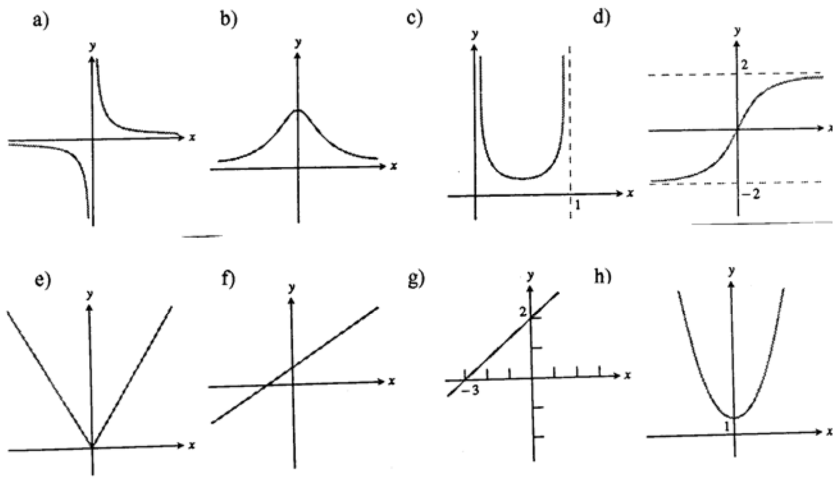
6. Writing activity: What can be said about the composition of an even function with an odd function? Using several even and odd functions, investigate their composition both algebraically and graphically. Show your work and write a paragraph summarizing what you found.
In each case, the composition of an even function with an odd function will give an even function.

Unit 1, Activity 4, Inverse Functions

Name______________________________

Date_______________________________

1. Which of the functions graphed below have an inverse that is also a function? Explain why or why not using the horizontal line test.



2. Use a graphing utility to graph the functions given below and apply the horizontal line test to determine if the functions are one-to-one. Show a sketch of each graph.

a) $y = \dfrac{3}{x-2} - 1$

b) $y = x^3 - 4x + 6$

c) $y = x^2 + 5x$

d) $y = 2^{3-x}$

e) $y = \log x^2$

3. Look at each of the tables given below. Decide which have an inverse that is also a function and give the numerical representation of that inverse function.

a)

x	-1	0	1	3	5
$f(x)$	5	3	2	1	-3

b)

x	-3	-2	-1	0	1
$f(x)$	5	4	3	5	1

c)

x	-2	0	2	4	6
$f(x)$	-6	-4	-2	0	2

4. Functions f and g are inverse functions if $f(g(x)) = g(f(x)) = x$. Use this to determine whether or not the following are inverse functions.

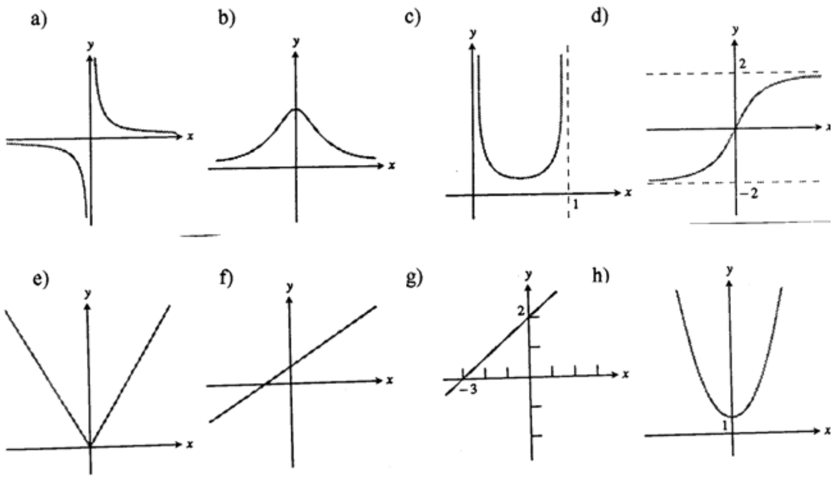
a) $f(x) = 3x - 5$ and $g(x) = \dfrac{x+5}{3}$

b) $f(x) = \sqrt[3]{x+2}$ and $g(x) = x^3 - 2$

c) $f(x) = \dfrac{1}{x-2}$ and $g(x) = \dfrac{1+2x}{x}$

Unit 1, Activity 4, Inverse Functions with Answers

1. Which of the functions graphed below have an inverse that is also a function? Explain why or why not using the horizontal line test.



a, d, f, g are one-to-one functions that will have an inverse. All are either strictly increasing or strictly decreasing functions and pass the horizontal line test. They are one-to-one functions.

2. Use a graphing utility to graph the functions given below and apply the horizontal line test to determine if the functions are one-to-one.
   a) $y = \dfrac{3}{x-2} - 1$
   b) $y = x^3 - 4x + 6$
   c) $y = x^2 + 5x$
   d) $y = 2^{3-x}$
   e) $y = \log x^2$

Equations a and d have an inverse that is a function. Students should show the graph of each one and determine if they are one-to-one functions using the horizontal line test.

3. Look at each of the tables given below. Decide which have an inverse that is also a function and give the numerical representation of that inverse function.
a) *yes* *original* *inverse*

x	-1	0	1	3	5		*x*	5	3	2	1	-3
f(x)	5	3	2	1	-3		*f(x)*	-1	0	1	3	5

b) no

c) *yes* *original* *inverse*

x	-2	0	2	4	6		*x*	-6	-4	-2	0	2
f(x)	-6	-4	-2	0	2		*f(x)*	-2	0	2	4	6

4. Functions *f* and *g* are inverse functions if *f(g(x)) = g(f(x)) = x*. Use this to determine whether or not the following are inverse functions.

a) $f(x) = 3x - 5$ and $g(x) = \dfrac{x+5}{3}$

b) $f(x) = \sqrt[3]{x+2}$ and $g(x) = x^3 - 2$

c) $f(x) = \dfrac{1}{x-2}$ and $g(x) = \dfrac{1+2x}{x}$

All of the functions are inverses. Therefore, f(g(x)) = g(f(x)) = x.

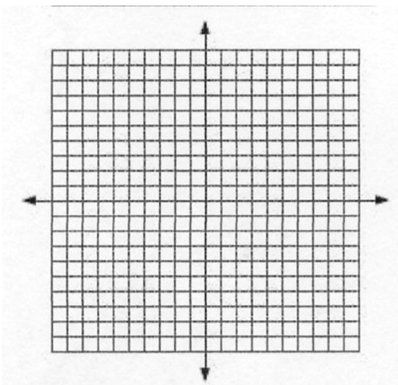
Unit 1, Activity 5, Piecewise Defined Functions

Name________________________

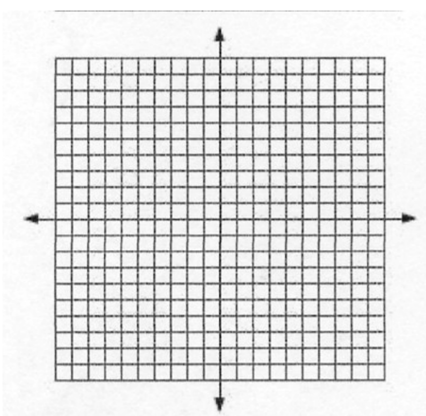
Date_________________________

Part I. Given *f*(*x*). Graph each of the following. Is the graph continuous or discontinuous? How do you know?

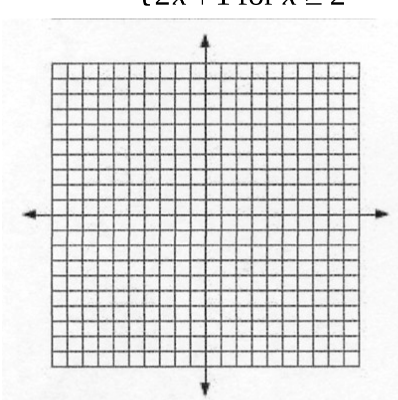
1) $f(x) = \begin{cases} 1 \text{ for } x = 1 \\ x - 2 \text{ for } x \neq 1 \end{cases}$



2) $f(x) = \begin{cases} x \quad \text{if x} \leq \text{-1} \\ x + 3 \text{ if x} > \text{-1} \end{cases}$

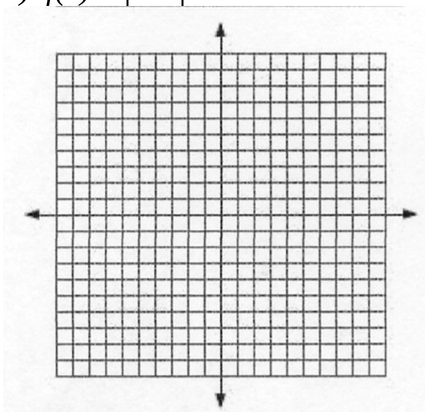


3) $f(x) = \begin{cases} 3 - x \text{ for } x < 2 \\ 2x + 1 \text{ for } x \geq 2 \end{cases}$

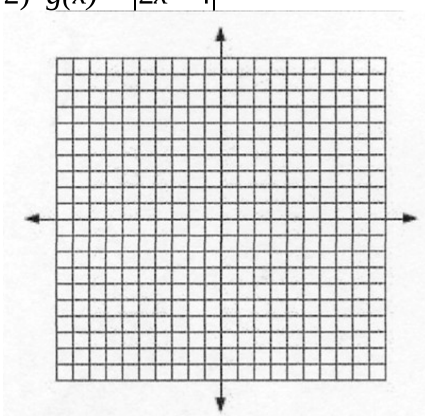


Part II. Rewrite each of the absolute value functions as a piecewise defined function and then sketch the graph. Is the graph continuous or discontinuous? How do you know?

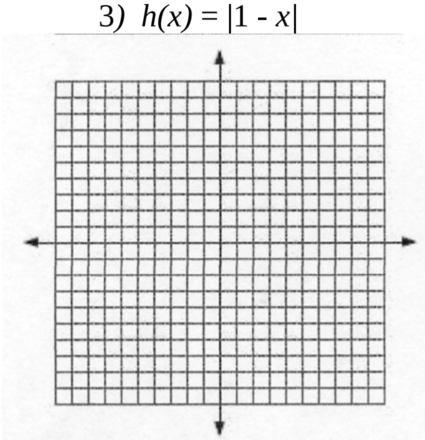
1) $f(x) = |x+1|$



2) $g(x) = |2x - 4|$



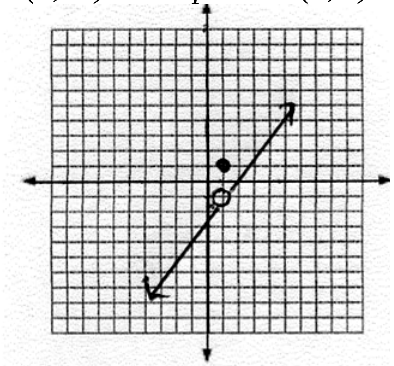
3) $h(x) = |1 - x|$



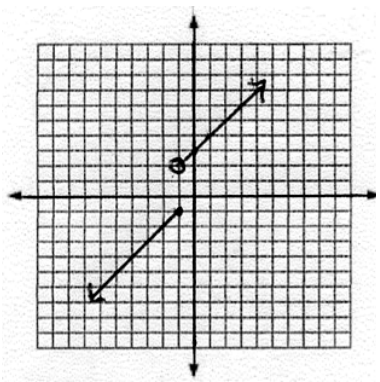
Unit 1, Activity 5, Piecewise Defined Functions with Answers

Part I.

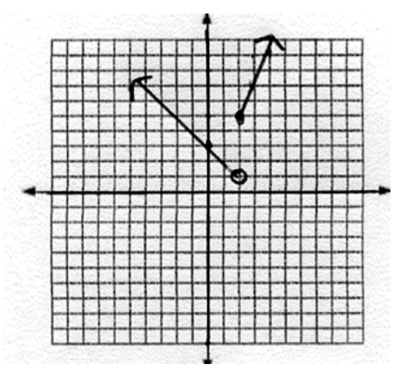
1) $f(x)=\begin{cases}1 \text{ for } x=1\\ x-2 \text{ for } x\neq 1\end{cases}$ *This is a discontinuous function. There is a hole in the graph at (1, -1) and a point at (1, 1).*



2) $f(x)=\begin{cases}x \quad \text{if x} \leq \text{-1}\\ x+3 \text{ if x} > \text{-1}\end{cases}$ *This is a discontinuous function. There is a jump at x = -1.*



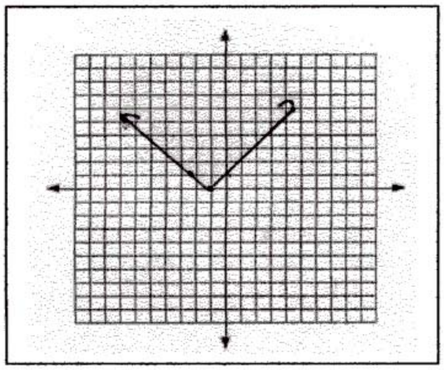
3. $f(x)=\begin{cases}3-x \text{ for } x<2\\ 2x+1 \text{ for } x\geq 2\end{cases}$ *This is a discontinuous function. There is a jump at x = 2.*



Unit 1, Activity 5, Piecewise Defined Functions with Answers

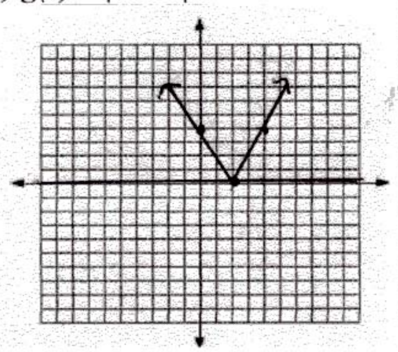
Part II

1) $f(x) = |x+1|$



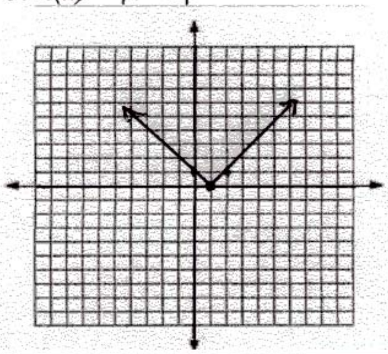
$$f(x) = \begin{cases} x + 1, & x \geq -1 \\ -x - 1, & x < -1 \end{cases}$$

2) $g(x) = |2x - 4|$



$$g(x) = \begin{cases} 2x - 4, & x \geq 2 \\ 4 - 2x, & x < 2 \end{cases}$$

3. $h(x) = |1 - x|$



$$h(x) = \begin{cases} x - 1, & x \geq 1 \\ 1 - x, & x < 1 \end{cases}$$

Unit 1, Activity 6, Library of Functions – Linear Functions

1. You are to create an entry for your Library of Functions. Introduce the parent function f(x) = x and include a table and graph of the function. Give a general description of the function written in paragraph form. Your description should include
   (a) the domain and range
   (b) local and global characteristics of the function – look at your glossary list and choose the words that best describe this function
2. Give some examples of family members using translation, reflection and dilation in the coordinate plane. Show these examples symbolically, graphically and numerically. Explain what has been done to the parent function to find each of the examples you give.
3. What are the common characteristics of a linear function?
4. Find a real-life example of how this function family can be used. Be sure to show a representative equation, table and graph. Does the domain and range differ from that of the parent function? If so, why? Describe what the slope, y-intercept, and zero mean within the context of your example.
5. Be sure that
   - ✓ your paragraph contains complete sentences with proper grammar and punctuation
   - ✓ your graphs are properly labeled, scaled, and drawn
   - ✓ you have used the correct math symbols and language in describing this function

Unit 1, General Assessments Spiral

1. Write an equation in standard form for

   a) the line containing the points (1, 1) and (-1, 3).

   b) a slope of $\frac{4}{3}$ and passing through the point (-1, 6).

2. Write an equation of a line parallel to $2x - 3y = 4$ and passing through (1, 3).

3. Write an equation of a line perpendicular to $x = 4$ and passing through (4, 6).

4. Given $f(x) = 3x^2 + 2x - 4$ find:

   a) $f(-1)$

   b) $f(2x)$

5. Solve the following system of equations:

$$3x + y = 13$$
$$2x - y = 2$$

6. Determine whether each of the following is a function. Explain.

   a) {(-2, 3), (-1, 5), (3, 7), (4, 3)}

   b) $y^2 = 4 - \underline{x}^2$

7. Solve:

   a) $x^2 - 8x - 9 = 0$

   b) $\sqrt{2x+4} = 4$

   c) $|x - 6| = 8$

Unit 1, General Assessments Spiral with Answers

1. Write an equation in standard form for

a) the line containing the points (1, 1) and (-1, 3).

$x + y = 2$

b) a slope of $\frac{4}{3}$ and passing through the point (-1, 6).

$4x - 3y = -22$

2. Write an equation of a line parallel to $2x - 3y = 4$ and passing through (1, 3).

$2x - 3y = -7$

3. Write an equation of a line perpendicular to x = 4 and passing through (4, 6).

$y = 6$

4. Given $f(x) = 3x^2 + 2x - 4$ find:

a) $f(-1) = -3$

b) $f(2x) = 12x^2 + 4x - 4$

5. Solve the following system of equations:

$3x + y = 13$
$2x - y = 2$ *(3, 4)*

6. Determine whether each of the following is a function. Explain.

a) {(-2, 3), (-1, 5), (3, 7), (4, 3)} *Yes, each value of the domain {-2, -1, 3, 4} is used exactly one time*

b) $y^2 = 4 - x^2$ *No, a distinct x, the domain, corresponds to two different values of y, the range. For example if x is 0, y is 2 or -2.*

7. Solve:

a) $x^2 - 8x - 9 = 0$ *solution: 9 or -1*

b) $\sqrt{2x + 4} = 4$ *solution: 6*

c) $|x - 6| = 8$ *solution: 14 or -2*

Unit 2, Pretest on Polynomials and Rational Functions

Name________________________

Date_________________________

1. Factor to find the roots in each of the problems below.

   a. $x^2 - 5x - 14 = 0$ Factors___________________

   Roots___________

   b. $2x^2 - 13x + 6 = 0$ Factors___________________

   Roots________________

2. Use the quadratic formula to solve $2x^2 - 3x - 4 = 0$

3. Find the discriminant and use it to describe the nature of the roots in each of the following equations.

   a. $2x^2 + 10x + 11 = 0$ _______________________

   b. $3x^2 + 4x = -2$ _______________________

   c. $4x^2 + 20x + 25 = 0$ _______________________

4. Given $P(x) = x^3 - 4x^2 + x + 6$

   a) Find $P(-2)$ using synthetic division. ________________

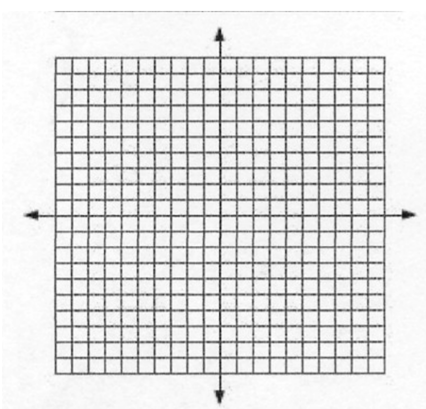
   b) $x - 2$ is a factor of $x^3 - 4x^2 + x + 6$. Use this fact to find all of the roots of $x^3 - 4x^2 + x + 6 = 0$.

   ____

5. Solve $(x - 2)(x + 4) \geq 0$.

____

6. Graph $y = x^2 - 6x + 8$. For what values is $x^2 - 6x + 8 < 0$?



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7. Simplify $\dfrac{x^2 - 3x - 18}{x + 3}$. _________________

Unit 2, Pretest on Polynomials and Rational Functions with Answers

1. Factor to find the roots in each of the problems below.

a. $x^2 - 5x - 14 = 0$ Factors _*(x – 7)(x + 2)*_

Roots *7 and -2*

b. $2x^2 - 13x + 6 = 0$ Factors _*(2x – 1)(x – 6)*_

Roots *1/2 and 6*

2. Use the quadratic formula to solve $2x^2 - 3x - 4 = 0$

$$x = \frac{3+\sqrt{41}}{4} \text{ or } \frac{3-\sqrt{41}}{4}$$

3. Find the discriminant and use it to describe the nature of the roots in each of the following equations.

a. $2x^2 + 10x + 11 = 0$ *100 – 88 > 0 2 real roots*

b. $3x^2 + 4x = -2$ *16 – 24 < 0 no real roots*

c. $4x^2 + 20x + 25 = 0$ *400 – 400 = 0 double root*

4. Given $P(x) = x^3 - 4x^2 + x + 6$

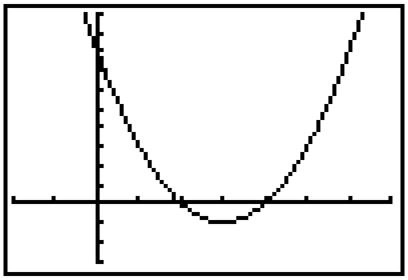
a) Find P(-2) using synthetic division. *P(-2) = -36*

b) $x - 2$ is a factor of $x^3 - 4x^2 + x + 6$. Use this fact to find all of the roots of $x^3 - 4x^2 + x + 6 = 0$.

roots are 2, 3, and -1

5. Solve $(x - 2)(x + 4) \geq 0$ $x \geq 2$ *or* $x \leq -4$

6. Graph $y = x^2 - 6x + 8$. For what values is $x^2 - 6x + 8 < 0$?



-4 < x < 2

7. Simplify $\dfrac{x^2 - 3x - 18}{x + 3}$ ___*x – 6*_____

Unit 2, What Do You Know about Polynomials and Rational Functions?

Word	+	?	-	What do I know about this topic?
polynomial				
terms				
factors				
leading coefficient				
zeros of a function				
multiplicity of zeros				
The connection between roots of an equation, zeros of a function and *x*-intercepts on a graph				
parabola				
axis of symmetry				
vertex				
continuous				

concavity				
leading coefficient				
intermediate value theorem				
synthetic division				
Remainder Theorem				
Factor Theorem				
Rational Root Theorem				
rational functions				
asymptotic discontinuity				
point discontinuity				
vertical asymptote				

Unit 2, Activity1, A Review of the Quadratic Function

Name______________________

Date ______________________

1. In each of the quadratic functions below determine the nature of the zeros of the function.

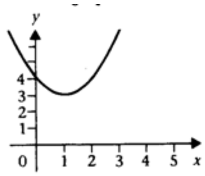
a) $f(x) = x^2 - 2x + 4$

b) $f(x) = x^2 - 4x + 4$

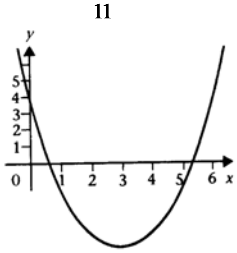
c) $f(x) = x^2 - 6x + 4$

2. The graph of each of the functions whose equations appear in #1 is shown below. Use the information about the zeros that you found in #1 to match each function to its graph.

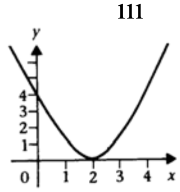
i



ii

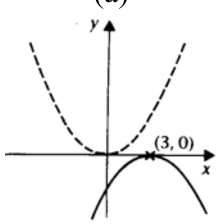


iii

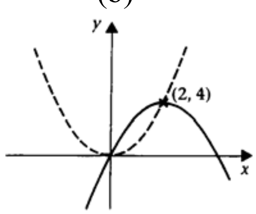


3. Each of the following is a translation of the graph of the function f(x) = x^2. Write the equation for each in standard form. Use a graphing utility to graph the equation and verify your result.

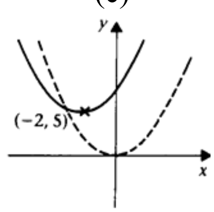
(a)



(b)



(c)



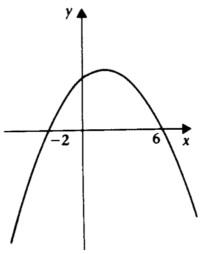
(a) ____________________________

(b)____________________________

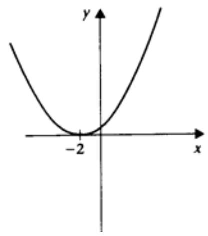
(c)____________________________

4. For each of the graphs below write a) an equation in factored form and (b) a possible equation for the function in the form $y = ax^2 + bx + c$

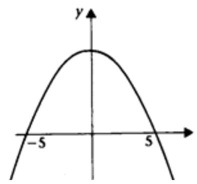
a)



b)



c)



5. If a ball is thrown upward from a building 20 meters tall, then its approximate height above the ground t seconds later is given by $h(t) = 20 + 25t - 4.9t^2$.

a. After how many seconds does the ball hit the ground?
b. What is the domain of this function?
c. How high does the ball go?
d. What is the range of this function?

6. The path around a square flower bed is 2 feet wide. If the area of the flower bed is equal to the area of the path, find the dimensions of the flower bed.

7. A rectangular lawn which is three times as wide as it is long has a 3 foot path around three sides only. The area of the path is equal to the area of the lawn. Find the dimensions of the lawn.

8. Writing exercise: Discuss the connection between the zeros of a function, the roots of an equation, and the *x*-intercepts of a graph. Give examples of your finding.

9. The quadratic function is to be added to the Library of Functions. You should consider

- ✓ the function in each of the 4 representations.
- ✓ domain and range
- ✓ the vertex as the local maximum or minimum of the function, concavity, symmetry, increasing/decreasing intervals, and zeros
- ✓ examples of translation in the coordinate plane
- ✓ whether or not an inverse exists
- ✓ a real-life example of how the function can be used

This is to go in your student portfolio.

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Unit 2, Activity 1, A Review of the Quadratic Function with Answers

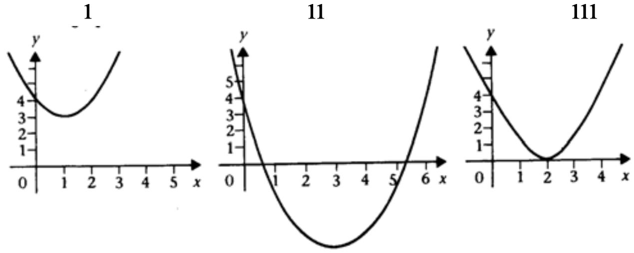
1. In each of the quadratic functions below determine the nature of the zeros of the function.

a) $f(x) = x^2 - 2x + 4$
b) $f(x) = x^2 - 4x + 4$
c) $f(x) = x^2 - 6x + 4$

a) $b^2 - 4ac < 0$*; no real roots*
b) $b^2 - 4ac = 0$*; 1 real root, a double root*
c) $b^2 - 4ac > 0$*; 2 real roots*

2. The graph of each of the functions whose equations appear in #1 is shown below. Use the information about the zeros that you found in #1 to match each function to its graph.

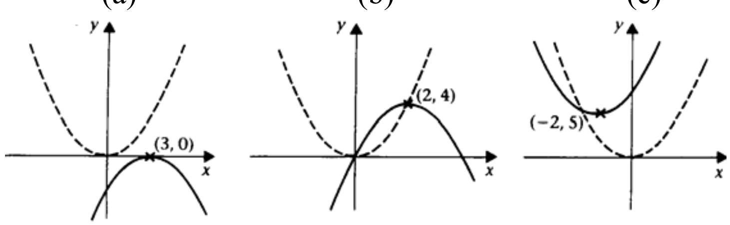
i ii iii



(a) matches with i; (b) matches with iii; (c) matches with ii

3. Each of the following is a translation of the graph of the function f(x) = x^2. Write the equation for each in standard form. Use a graphing utility to graph the equation and verify your result.

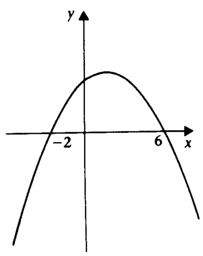
(a) (b) (c)



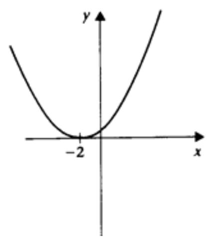
a) $f(x) = -(x-3)^2$
b) $f(x) = -(x-2)^2 + 4$
c) $f(x) = (x+2)^2 + 5$

4. For each of the graphs below write a) an equation in factored form and (b) a possible equation for the function in the form $y = ax^2 + bx + c$.

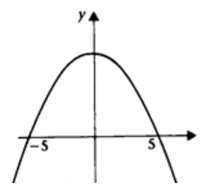
a)



b)



c)



a) y = (x – 6)(x + 2)in factored form and a possible equation $y = x^2 - 4x - 12$
b) y = (x + 2)(x + 2) in factored form and a possible equation $y = x^2 + 4x + 4$
c) y = -(x + 5)(x – 5) in factored form and a possible equation $y = 25 - x^2$

5. If a ball is thrown upward from a building 20 meters tall, then its approximate height above the ground t seconds later is given by $h(t) = 20 + 25t - 4.9t^2$.

a. After how many seconds does the ball hit the ground?
$20 + 25t - 4.9t^2 = 0$ at $t \approx 5.8$ seconds

b. What is the domain of this function?
domain $\{t : 0 < t < 5.8\}$

c. How high does the ball go?
maximum height is ≈ 52 meters

d. What is the range of this function?
range$\{h : 0 \leq h \leq 52\}$

6. The path around a square flower bed is 2 feet wide. If the area of the flower bed is equal to the area of the path, find the dimensions of the flower bed.

$x^2 = 8x + 16$ so the dimensions are approximately 9.66 by 9.66 feet

7. A rectangular lawn which is three times as wide as it is long has a 3 foot path around three sides only. The area of the path is equal to the area of the lawn. Find the dimensions of the lawn.

There are two answers depending on what three sides are chosen.
$3x^2 = 15x + 18$; so the dimensions are 6 by 18 feet or
$3x^2 = 21x + 18$; so the dimensions are approximately 7.8 by 23.3 feet

8. Writing exercise: Discuss the connection between the zeros of a function, the roots of an equation, and the *x*-intercepts of a graph. Give examples of your finding.
Students should see that the terms are all describing the same thing. Whether you refer to the zeros of a given function, solve an equation to find it roots, or use the x-intercepts in a graph, you are referring to the same set of points.

9. Because of its importance the quadratic function should be added to the Library of Functions. Students should consider

- ✓ the function in each of the 4 representations.
- ✓ domain and range
- ✓ the vertex as the local maximum or minimum of the function, symmetry, increasing/decreasing intervals, and zeros
- ✓ examples of translation in the coordinate plane
- ✓ whether or not an inverse exists
- ✓ a real-life example of how the function can be used

Unit 2, Activity 2, Discovery using Technology

1. Plot on the same screen the following graphs: $y = x^2$, $y = x^3$, y = x^4, y = x^5

    a) What points do all of the graphs have in common?

    b) Which function increases most rapidly, and which increases least rapidly, as x becomes large?

    c) What are the main differences between the graphs of the even powers of x and the odd powers of x?

2. Plot on the same screen the following graphs: $y = -x^2$, $y = -x^3$, y =$-x^4$, y = $-x^5$

    a) What characteristics do these graphs share with the graphs in #1?

    b) How do these graphs differ from the graphs in #1?

3. Plot on the same screen the following graphs: $y = x^2$, $y = 4x$, $y = x^2 + 4x$

    a) What do you notice about the graphs of $y = x^2$ and $y = x^2 + 4x$ when x is a large positive or negative number?

    b) What do you notice about the graphs of $y = 4x$ and the graph of $y = x^2 + 4x$ when x is a small positive or negative number?

4. Plot on the same screen the following graphs: $y = x^3$, $y = -4x^2$, $y = x^3 - 4x^2$

    a) What do you notice about the graphs of $y = -4x^2$ and $y = x^3 - 4x^2$ when x is a small positive or negative number?

    b) What do you notice about the graphs of $y = x^3$ and $y = x^3 - 4x^2$ when x is a large positive or negative number?

5. Plot on the same screen the following graphs: $y = x^3 + 2x^2 - 4x - 6$, $y = x^3$, and $y = -4x - 6$

    a) Compare the three graphs for large positive and negative values of x.

    b) Compare the three graphs for small positive and negative values of x.

    c) Suggest a reason for ignoring the terms x^2, $-4x$, and -6 when considering the shape of the graph in part (a) for large values of x.

    d) Suggest a reason for ignoring the terms x^3 and x^2 when considering the shape of the graph in part (a) for very small values of x.

Unit 2, Activity 2, Discovery using Technology with Answers

1. Plot on the same screen the following graphs: $y = x^2$, $y = x^3, y = x^4, y = x^5$

   a) What points do all of the graphs have in common?
   All of the graphs pass through (0, 0) and (1, 1)

   b) Which function increases most rapidly, and which increases least rapidly, as x becomes large?
   $y = x^5$ increases most rapidly and $y = x^2$ increases least rapidly

   c) What are the main differences between the graphs of the even powers of x and the odd powers of x?
   The graphs of the even powers of x are even functions; that is, they are symmetrical over the y-axis. The graphs of the odd powers of x are odd functions; that is, they are symmetrical around the origin. The end-behavior of the even powers is the same as $x \to \infty, y \to \infty$ and as $x \to -\infty, y \to \infty$. *The same can be said for the end-behavior of the odd powers: as* $x \to \infty, y \to \infty$ and as $x \to -\infty, y \to -\infty$.

2. Plot on the same screen the following graphs: $y = -x^2$, $y = -x^3, y = -x^4, y = -x^5$

   a) What characteristics do these graphs share with the graphs in #1? *The end-behavior of the even powers is the same as* $x \to \infty, y \to -\infty$ and as $x \to -\infty, y \to -\infty$. *The same can be said for the end-behavior of the odd powers: as* $x \to \infty, y \to -\infty$ and as $x \to -\infty, y \to \infty$.

   b) How do these graphs differ from the graphs in #1?
   The graphs of #1 have been reflected over the x-axis. The graphs now pass through (0, 0) and (1,-1). The symmetry remains the same but now $y = - x^5$ decreases most rapidly and $y = x^2$ decreases least rapidly.

3. Plot on the same screen the following graphs: $y = x^2$, $y = 4x$, $y = x^2 + 4x$

   a) What do you notice about the graphs of $y = x^2$ and $y = x^2 + 4x$ when x is a large positive or negative number?
   When x is a large positive or negative number the graphs of $y = x^2$ and $y = x^2 + 4x$ are very similar. They increase at the same rate. This is because x^2 is dominant for very large values of x.

   b) What do you notice about the graphs of $y = 4x$ and the graph of $y = x^2 + 4x$ when x is a small positive or negative number?
   When x is very small the graphs of $y = 4x$ and $y = x^2 + 4x$ are very close together. Now the 4x term is dominant.

4. Plot on the same screen the following graphs: $y = x^3$, $y = -4x^2$, $y = x^3 - 4x^2$

   a) What do you notice about the graphs of $y = -4x^2$ and $y = x^3 - 4x^2$ when x is a small positive or negative number?

The same effect is noted with this problem as we saw in problem 3. When x is very small -4x^2 is dominant.

b) What do you notice about the graphs of $y = x^3$ and $y = x^3 - 4x^2$ when x is a large positive or negative number?
The same effect is noted with this problem as we saw in problem 3. When x is very large x^3 is the dominant term.

5. Plot on the same screen the following graphs: $y = x^3 + 2x^2 - 4x - 6$, $y = x^3$, and $y = -4x - 6$

a) Compare the three graphs for large positive and negative values of x.
The graph of $y = x^3 + 2x^2 - 4x - 6$ *is similar to that of* $y = x^3$ *for large values of x.*

b) Compare the three graphs for small positive and negative values of x.
The graph of $y = x^3 + 2x^2 - 4x - 6$ *is similar to that of the graph of* $y = -4x - 6$ *for small values of x.*

c) Suggest a reason for ignoring the terms x^2, -4x, and -6 when considering the shape of the graph in part (a) for large values of x.
For the graph of a polynomial function, the term of largest degree is dominant when x is a very large positive or negative number. Let x take on the values of 100, 1000 etc. to illustrate this statement.

d) Suggest a reason for ignoring the terms x^3 and x^2 when considering the shape of the graph in part (a) for very small values of x.
When x is a small positive or negative number, the terms of smallest degree are the dominant factors. Let x take on the values ¼ and ½ to illustrate this statement.

Unit 2, Activity 3, The Zeros of Polynomials

Name_____________________________

Date ______________________________

This is a non-calculator exercise.

1. If $P(x) = x^3 - 2x^2 - 11x + 12$,

   a) Show that x + 3 is a factor of $P(x)$

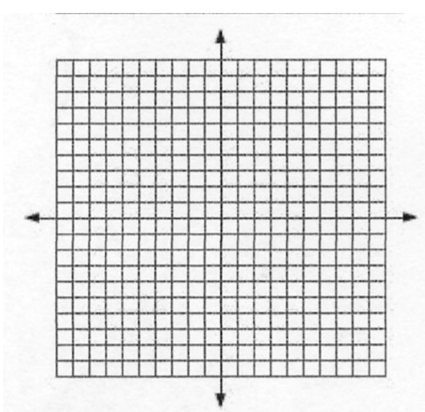
   b) Find $Q(x)$, if $P(x) = (x + 3)Q(x)$.

2. If $P(x) = x^3 - 5x^2 + 2x + 8$ and $P(-1) = 0$,

   a) Factor $P(x)$ completely.

   b) Write the solutions of $P(x) = 0$.

   c) Sketch the graph of $P(x)$.

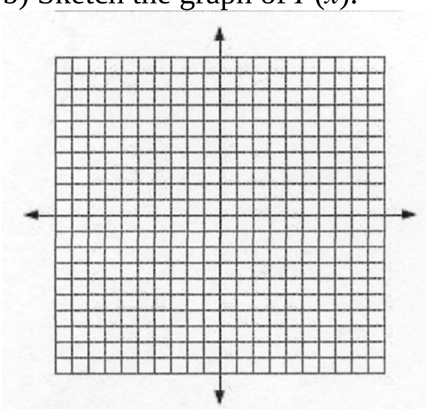


3. If $P(x) = x^3 - 3x^2 + x + 2$ and $(x – 2)$ is a factor of *P(x),*
   a) Find $Q(x)$, if $P(x) = (x – 2)Q(x)$.

   b) Find the zeros of $P(x)$. Exact values please!

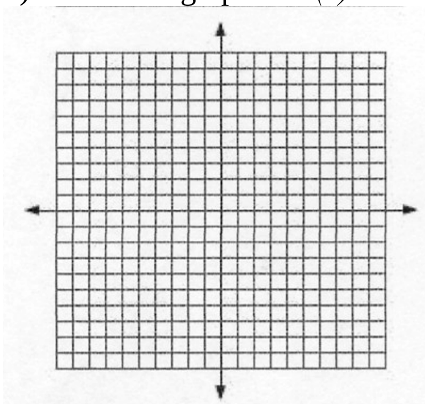
4. If $P(x) = 2x^3 + 3x^2 - 23x - 12$ and P(-4) = 0,
   a) Find all of the zeros of P(x).

   b) Sketch the graph of $P(x)$.



5. If $P(x) = x^3 - x^2 - 7x - 2$,
   a) Find all zeros of *P(x).* Exact values please!

b) Sketch the graph of $P(x)$



Unit 2, Activity 3, The Zeros of Polynomials with Answers

This is a non-calculator exercise.

1. If $P(x) = x^3 - 2x^2 - 11x + 12$,

    a) Show that $x + 3$ is a factor of $P(x)$,
    Using synthetic division P(-3)=0 so by the factor theorem (x + 3) is a factor

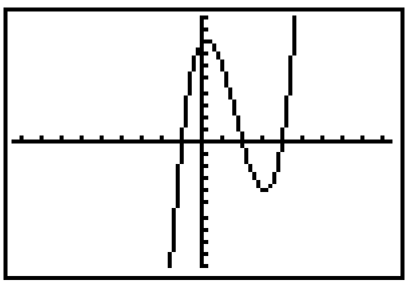
    b) Find $Q(x)$, if $P(x) = (x + 3)Q(x)$,
    The depressed equation left from part (a) is Q(x). $Q(x) = x^2 - 5x + 4$

2. If $P(x) = x^3 - 5x^2 + 2x + 8$ and $P(-1) = 0$,

    a) Factor $P(x)$ completely.
    (x + 1)(x – 2)(x – 4)

    b) Write the solutions of $P(x) = 0$.
    P(x) = 0 for x = -1, 2, and 4

c) Sketch the graph of $P(x)$



3. If $P(x) = x^3 - 3x^2 + x + 2$ and (x – 2) is a factor of *P(x),*

    a) Find $Q(x)$, if $P(x) = (x - 2)Q(x)$ and $Q(x) = x^2 - x - 1$

b) Find the zeros of P(x). Exact values please!

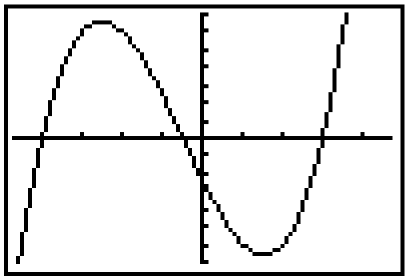
P(x) = 0 for x = $2, \frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

4. If $P(x) = 2x^3 + 3x^2 - 23x - 12$ and $P(-4) = 0$,

   a) Find all of the zeros of P(x).

   P(x) =0 for x = - ½ , -4, 3

   b) Sketch the graph of P(x).

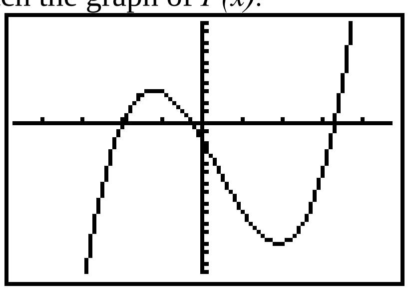


5. If $P(x) = x^3 - x^2 - 7x - 2$,

a) Find all zeros of *P(x.)* Exact values please!

P(x)=0 for x = -2, $\dfrac{3+\sqrt{13}}{2}, \dfrac{3-\sqrt{13}}{2}$

b) Sketch the graph of *P(x).*



Unit 2, Activity 4, Analyzing Polynomials

Given each of the polynomials below

(a) Find the real zeros.

(b) Identify the multiplicity of zeros and tell whether the graph will cross or be tangent to the *x*-axis.

(c) What polynomial does it resemble for large values of *x*?

(d) Find the location and value for each of the relative maxima and minima.

(e) Over what intervals is the polynomial increasing, decreasing?

(f) Using graph paper sketch a graph labeling the relative maxima and minima and the zeros.

1. $f(x) = (x-3)^2(x+2)$
2. $f(x) = 2x^2(x-3)(x^2+1)$
3. $f(x) = (x^2-4)(x+2)$
4. $f(x) = -x^2(x^2-1)$
5. $f(x) = -x^3(x-4)(x^2-4)$

Unit 2, Activity 4, Analyzing Polynomials with Answers

The answers for the relative extrema were obtained with a TI-83 and are written to the nearest thousandth.

#	zeros	root characteristic	end behavior	relative extrema	increasing and decreasing intervals
1	-2, 3	crosses at -2 double root at 3 tangent to x-axis	resembles $y=x^3$	min is 0 at x = 3 max is 18.519 at x = -.333	increasing (-∞, -.333) U (3,∞) decreasing (-.333, 3)
2	0,3	0 is double root tangent; crosses at 3	resembles $y=2x^5$	min is -46.835 at x = 2.361 and max is 0 at 0	increasing (-∞, 0) U (2.361, ∞) decreasing (0, 2.361)
3	-2, 2	-2 is double root tangent; crosses at 2	resembles $y=x^3$	min is -9.482 at x = .667 and max is 0 at x = -2	increasing (-∞, -2) U (0.667, ∞) decreasing (-2, 0.667)
4	-1, 0, 1	0 is a double root tangent; crosses at x = -1, 1	resembles $y = -x^4$	min is 0 at x = 0 and max is .25 at x = -.707 and x = .707	increasing (-∞, -.707) U (0, .707) decreasing(-.707, 0) U (0.707, ∞)
5	-2, 0, 2, 4	0 is a triple root, crosses; -2, 2, and 4 single roots cross	resembles $y = -x^6$	min is -14.783 at x = 1.479 2 maxima: one is 33.094 at x = -1.577 and one is 178.605 at x = 3.431	increasing (-∞, -1.577) U (1.479, ∞) decreasing(-1.577, 1.479) U (3.413, ∞)

Unit 2, Activity 5, Applications of Polynomial Functions

1. When the sunlight shines on auto exhausts, certain pollutants are produced that can be unsafe to breathe. In a study done of runners of many ages and levels of training, it has been shown that a level of 0.3 parts per million (ppm) of pollutants in the air can cause adverse effects. Though the level of pollution varies from day to day and even hour to hour, it has been shown that the level during the summer can be approximated by the model $L = 0.04t^2 - 0.16t + 0.25$, where L measures the level of pollution and t measures time with t = 0 corresponding to 6:00 a.m. When will the level of pollution be at a minimum? What time of day should runners exercise in order to be at a safe level?

2. Postal regulations limit the size of packages that can be mailed. The regulations state that "the girth plus the length cannot exceed 108 inches." We plan to mail a package with a square bottom.

   a) Write the volume V in terms of x.
   b) Considering the physical limitations, what is the domain of this function?
   c) What should the dimensions of the package be to comply with the postal regulations and at the same time maximize the space within the box?

3. One hundred feet of fencing is used to enclose three sides of a rectangular pasture. The side of a barn closes off the fourth side. Let x be one of the sides perpendicular to the barn.

   a) The farmer wants the area to be between 800 square feet and 1000 square feet. Determine the possible dimensions to the nearest foot needed to give that square footage.
   b) Suppose he wants the maximum area possible. What dimensions would he need? What is the maximum area?

4. Equal squares of side length x are removed from each corner of a 20- inch by 25-inch piece of cardboard. The sides are turned up to form a box with no top.

   a) Write the volume V of the box as a function of x.
   b) Draw a complete graph of the function $y = V(x)$.
   c) What values of x make sense in this problem situation? How does this compare with the domain of the function $V(x)$?
   d) What value of x will give the maximum possible volume? What is the maximum possible volume?

1. When the sunlight shines on auto exhausts, certain pollutants are produced that can be unsafe to breathe. In a study done of runners of many ages and levels of training, it has been shown that a level of 0.3 parts per million (ppm) of pollutants in the air can cause adverse effects. Though the level of pollution varies from day to day and even hour to hour, it has been shown that the level during the summer can be approximated by the model $L = 0.04t^2 - 0.16t + 0.25$, where L measures the level of pollution and t measures time with $t = 0$ corresponding to 6:00 a.m. When will the level of pollution be at a minimum? What time of day should runners exercise in order to be at a safe level?

1) The level of pollution will be at a minimum at 8 a.m. The pollution levels will be below 0.3 ppm between 6 a.m. and 10:17 a.m.

2. Postal regulations limit the size of packages that can be mailed. The regulations state that "the girth plus the length cannot exceed 108 inches." We plan to mail a package with a square bottom.

a) Write the volume V in terms of x.

$V(x) = x^2(108-4x)$

b) Considering the physical limitations, what is the domain of this function?

$\{x: 0 < x < 27\}$

c) What should the dimensions of the package be to comply with the postal regulations and at the same time maximize the space within the box?

18" by 18" by 36"

3. One hundred feet of fencing is used to enclose three sides of a rectangular pasture. The side of a barn closes off the fourth side. Let x be one of the sides perpendicular to the barn.

a) The farmer wants the area to be between 800 square feet and 1000 square feet. Determine the possible dimensions to the nearest foot needed to give that square footage.

The length of the perpendicular side should be between 10' and 14'.

b) Suppose he wants the maximum area possible. What dimensions would he need? What is the maximum area?

The maximum area would be 1250 square feet with dimensions of 25' by 50'.

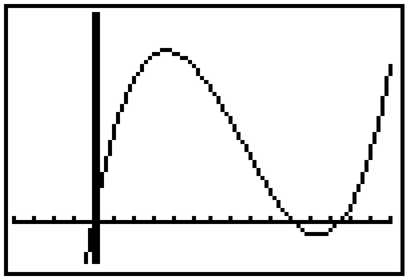
4. Equal squares of side length x are removed from each corner of a 20- inch by 25-inch piece of cardboard. The sides are turned up to form a box with no top.

a) Write the volume V of the box as a function of x.

$V(x) = x(25 - 2x)(20 - 2x)$

b) Draw a complete graph of the function $y = V(x)$.

From the TI-83 with x-min -4, x-max 15, y-min -100, and y-max 1000



c) What values of x make sense in this problem situation? How does this compare with the domain of the function V(x)?

For this problem x must lie between 0 and 10. The domain is the set of real numbers (-∞, ∞)

d) What value of *x* will give the maximum possible volume? What is the maximum possible volume?

x≈3.7" for a volume of approximately 820.5 cubic inches

Unit 2, Activity 6, Rational Functions and their Graphs

PART I

For each of the following problems

a) Find the domain. Set the denominator equal to zero and solve $D(x) = 0$. The solutions to that equation are the discontinuities of the function and not in the domain.

b) Find all zeros of the function. Set the numerator equal to zero and solve $N(x) = 0$. If one or more of the solutions are also solutions to $D(x) = 0$, then that value of x represents the location of a hole in the graph. Solutions that are unique to $N(x) = 0$ represent the x-intercepts of the graph. Plot those intercepts.

c) Identify any vertical and horizontal asymptotes. Draw them on the graph with a dashed line.

d) Find the y-intercept (if any) by evaluating $f(0)$. Plot that point.

e) Find and plot one or two points prior to and beyond each of the vertical asymptotes.

e) Graph the function

Check your answer graphically using a graphing utility, and numerically, by creating a table of values using the table function.

1. $f(x) = \dfrac{x^2 - 12}{x^2 - 16}$ 2. $f(x) = \dfrac{x^2}{x^2 - 9}$ 3. $g(x) = \dfrac{2x - 4}{x + 4}$

4. $g(x) = \dfrac{x + 1}{x^2 + x - 6}$ 5. $h(x) = \dfrac{3}{x^2 - 6x + 8}$

PART II

1. What symmetry do you see in the graphs drawn in Part I? (Think in terms of even and odd functions)

2. The double zero in #2 caused the tangency to the x-axis. What can we do to #5 to make its “parabola” tangent to the x-axis? How would this change your answers to this question?

3. None of these graphs had “holes” because of discontinuities. Suppose I want the discontinuity in #3 to be a hole rather than asymptotic. How should I change the equation? How would this change the graph?

4. Which of the problems above have a range that is the set of all reals? How can you tell from the graph?

5. Write an equation for a rational function that has (a) at least one zero, (b) two vertical asymptotes and one “hole”, and c) a horizontal asymptote other than 0. Hand it to another student to solve.

Unit 2, Activity 6, Rational Functions and their Graphs with Answers

PART I

For each of the following problems

a) Find the domain. Set the denominator equal to zero and solve *D(x)* = 0. The solutions to that equation are the discontinuities of the function and not in the domain.

b) Find all zeros of the function. Set the numerator equal to zero and solve *N(x)* = 0. If one or more of the solutions are also solutions to *D(x)* = 0, then that value of x represents the location of a hole in the graph. Solutions that are unique to *N(x)* = 0 represent the *x*-intercepts of the graph. Plot those intercepts.

c) Identify any vertical and horizontal asymptotes. Draw them on the graph with a dashed line.

d) Find the *y*-intercept (if any) by evaluating *f*(0). Plot that point.

e) Find and plot one or two points prior to and beyond each of the vertical asymptotes.

e) Graph the function

Check your answer graphically using a graphing utility, and numerically, by creating a table of values.

1. $f(x) = \dfrac{x^2 - 12}{x^2 - 16}$

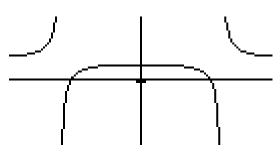
a*) domain = {x:x≠-4, 4}*

b) The zeros are at $\sqrt{12}, -\sqrt{12}$

c) The vertical asymptotes are at x = 4 and x = -4 and the horizontal asymptote is y = 1.

d) The y-intercept is ¾ .

The graph :



2. $f(x) = \dfrac{x^2}{x^2 - 9}$

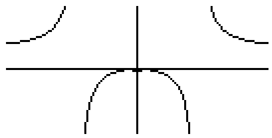
a) Domain = {x: x ≠-3, 3}

b) There is one zero at x = 0. However, it is a double zero so the graph will be tangent to the x-axis at that point.

c) There are two vertical asymptotes, at x = 3 and x = -3. The horizontal asymptote is y = 1.

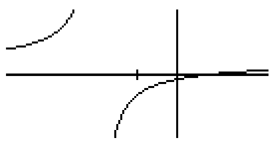
d) The y-intercept is 0.

The graph:



3. $g(x) = \dfrac{2x-4}{x+4}$

   a) Domain = {x≠ -4}
   b) There is one zero at x = 2.
   c) There is one vertical asymptote at x = -4 and one horizontal asymptote at y =2.
   d) The y-intercept is -1.
   The graph

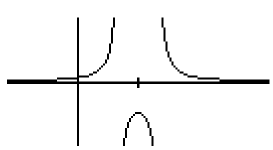


4. $g(x) = \dfrac{x+1}{x^2+x-6}$

   a) Domain = {x:x≠ -3, 2}
   b) There is one zero at x = -1.
   c) There are two vertical asymptotes x = -3 and x = 2. The horizontal asymptote is y = 0.
   d) The y-intercept is $-\dfrac{1}{6}$.
   The graph:

5. $h(x) = \dfrac{3}{x^2-6x+8}$

   a) Domain = {x:x ≠2, 4}
   b) There are no zeros in this function.
   c) The vertical asymptotes are at x = 4 and x = 2. The horizontal asymptote is y = 0.
   d) The y-intercept is $\dfrac{3}{8}$.
   The graph:



PART II

1. What symmetry do you see in the graphs drawn in Part I? (Think in terms of even and odd functions)

#1 and #2 are even functions

2. The double zero in #2 caused the tangency to the *x*-axis. What can we do to #5 to make its "parabola" tangent to the *x*-axis? How would this change your answers to this question?

Rewrite the function to be $f(x)=\dfrac{3x^2}{x^2-6x+8}$. *The horizontal asymptote will now be y = 3.*

3. None of these graphs had "holes" because of discontinuities. Suppose I want the discontinuity in #3 to be a hole rather than asymptotic. How should I change the equation? How would this change the graph?

Add (x + 4) as a factor in the numerator. The function is now $f(x)=\dfrac{(2x+4)(x+4)}{x+4}$ *and the graph would be the line y = 2x + 4 with a hole at (-4, -4).*

4. Which of the problems above have a range that is the set of all reals? How can you tell from the graph?

. Problem # 4 has a range that is the set of all reals. For $x\to -3^+, f(x)\to +\infty$ *and for* $x\to 2^-, f(x)\to -\infty$

5. Write an equation for a rational function that has (a) at least one zero, (b) two vertical asymptotes and one "hole", and c) a horizontal asymptote other than 0. Hand it to another student to solve.

One example would be $f(x)=\dfrac{x^3+4x^2}{x^3+4x^2-x-4}$

Unit 2, Activity 8, Library of Functions – Quadratic Functions, Polynomial Functions and Rational Functions

There are three functions that should be added to your Library of Functions.

- The quadratic function
- Polynomial functions
- Rational functions

Each function should have a separate entry.

1. Introduce the parent function and include a table and graph of the function. Give a general description of the function written in paragraph form. Your description should include:

   (a) the domain and range

   (b) local and global characteristics of the function – look at your glossary list from units 1 and 2 and choose the words that best describe this function.

2. Give some examples of family members using translation, reflection and dilation in the coordinate plane. Show these examples symbolically, graphically, and numerically. Explain what has been done to the parent function to find each of the examples you give.

3. What are the common characteristics of the function?

4. Find a real-life example of how this function family can be used. Be sure to show a representative equation, table and graph. Does the domain and range differ from that of the parent function? If so, why? Describe what the local and global characteristics mean within the context of your example.

5. Be sure that

   ✓ your paragraph contains complete sentences with proper grammar and punctuation.

   ✓ your graphs are properly labeled, scaled, and drawn using graph paper.

   ✓ you have used the correct math symbols and language in describing this function.

Unit 2, General Assessments Spiral

1. Let $f(x) = x^2 - 6x - 7$ and $g(x) = |x|$

   a. Find $f(g(x))$ and sketch the graph.

   b. Find $g(f(x))$ and sketch the graph.

   c. Which, if any, are even functions? How can you tell?

2. Let $f(x) = x^2 + 7x + 10$ and $g(x) = x + 2$

   a. Find $\dfrac{f(x)}{g(x)}$

   b. Find $f \cdot g(x)$

3. Given $f(x) = x^2 - 4$

   a. Find the inverse of this function.

   b. Graph both the function and its inverse on the same set of axes.

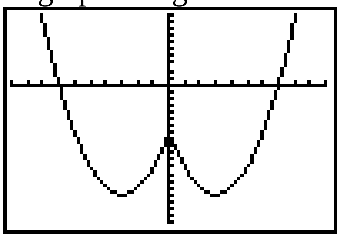
   c. Is the inverse of f a function? How do you know?

1. Let $f(x) = x^2 - 6x - 7$ and $g(x) = |x|$

   a. Find $f(g(x))$ and sketch the graph.

   $f(g(x)) = |x|^2 - 6|x| - 7$

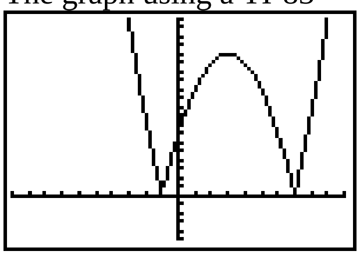
   The graph using a TI-83



   b. Find $g(f(x))$ and sketch the graph.

   $g(f(x)) = |x^2 - 6x - 7|$

   The graph using a TI-83



   c. Which, if any, are even functions? How can you tell?

   f(g(x)) is an even function. The graph is symmetrical over the y-axis. In the equation f(x) = f(-x)

2. Let $f(x) = x^2 + 7x + 10$ and $g(x) = x + 2$

   a. Find $\dfrac{f(x)}{g(x)}$

   $$\frac{x^2+7x+10}{x+2} = \frac{(x+5)(x+2)}{x+2}$$

   $$= x+5$$

   b) Find $f \cdot g(x)$

   $$(x+2)(x^2+7x+10) =$$

   $$x^3+9x^2+24x+20$$

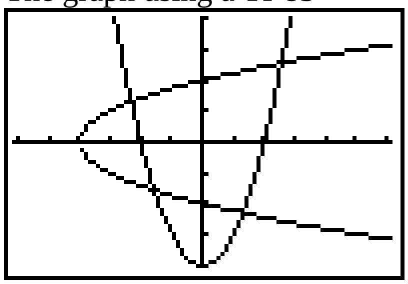
3. Given $f(x) = x^2 - 4$

a. Find the inverse of this function.

$y = \pm\sqrt{x+4}$

b. Graph both the function and its inverse on the same set of axes.

The graph using a TI-83



c. Is the inverse of *f* a function? How do you know?

The inverse is not a function. Each element in the domain is matched to more than one element in the range. Looking at the graphs the inverse fails the vertical line test. The function fails the horizontal line test.

Unit 3, Pretest

Name___________________________

Date ___________________________

1. Simplify each of the following. Write your answer with positive exponents:

a. $(x^4)(x^6)$ a. _____________

b. $(x^2)^3$ b. _____________

c. $\dfrac{(2x)^2}{4x}$ c. _____________

d. $\dfrac{x^2 y^3}{x^4 y^0}$ d. ______________

e. $\dfrac{15x^{-2}y^{14}}{5xy^{-3}}$ e. ______________

2. Simplify:

a. $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ 2a. ______________

b. $\dfrac{5^{3\sqrt{2}}}{5^{\sqrt{2}}}$ 2b.______________

c. $\left(3^{\sqrt{2}}\right)\left(3^{4\sqrt{2}}\right)$ 2c. ______________

Unit 3, Pretest

3. Rewrite using rational exponents:

$\sqrt[3]{a^2b^3c^4}$ 3. ______________

4. Solve each equation:

a. $3^{2x+3} = 3^{x-4}$ 4a. ______________

b. $7 - 2e^x = 5$ 4b. ______________

5. Rewrite in exponential form:

$\log_3 81 = 4$ 5. ________________

6. Use the laws of logarithms to rewrite the expression as a single logarithm:

a. $\log_3 x + \log_3 (x-1)$ 6a.________________

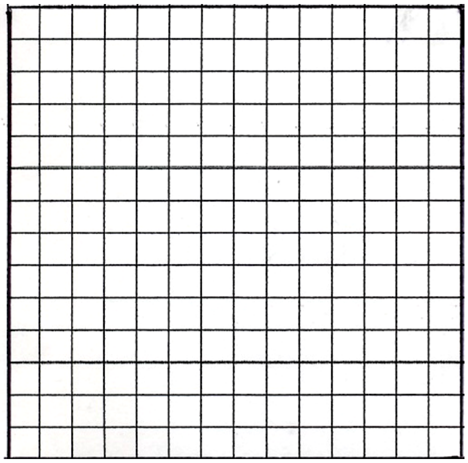
b. $2\log_5 (x-1) - \log_5 (x-1)$ 6b.________________

7. Solve for *x*:

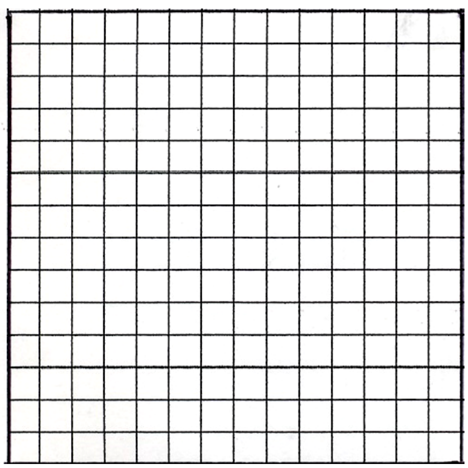
a. $\log_5 x = 2$ 7a. ________________

b. $\log_2 \frac{1}{16} = x$ 7b. ________________

8. Use the grid below to graph $f(x) = 2^x$. Give the domain and range of this function.



9. Use the grid below to graph $f(x) = \log_2 x$. Give the domain and range of this function.



Unit 3, Pretest with Answers

1. Simplify each of the following. Write your answer with positive exponents:

. 1a. $(x^4)(x^6)$ 1a. x^{10}

b. $(x^2)^3$ 1 b. x^6

c. $\dfrac{(2x)^2}{4x}$ 1c. x

d. $\dfrac{x^2y^3}{x^4y^0}$ 1d. $\dfrac{y^3}{x^2}$

e. $\dfrac{15x^{-2}y^{14}}{5xy^{-3}}$ 1e. $\dfrac{3y^{17}}{x^3}$

2. Simplify Give the exact values.

a. $\left(\frac{1}{4}\right)^{\frac{1}{2}}$ 2a. ½

b. $\dfrac{5^{3\sqrt{2}}}{5^{\sqrt{2}}}$ 2b. $5^{2\sqrt{2}}$

c. $\left(3^{\sqrt{2}}\right)\left(3^{4\sqrt{2}}\right)$ 2c. $3^{5\sqrt{2}}$

3. Rewrite using rational exponents:

$\sqrt[3]{a^2b^3c^4}$ 3. $a^{2/3}b^1c^{4/3}$

4. Solve each equation:

a. $3^{2x+3} = 3^{x-4}$ 4a. x = -7

b. $7 - 2e^x = 5$ 4b. $x = 0$

5. Rewrite in exponential form:

$\log_3 81 = 4$ 5. $3^4 = 81$

6. Use the laws of logarithms to rewrite the expression as a single logarithm:

a. $\log_3 x + \log_3(x-1)$ 6a. $\log_3 x(x-1)$

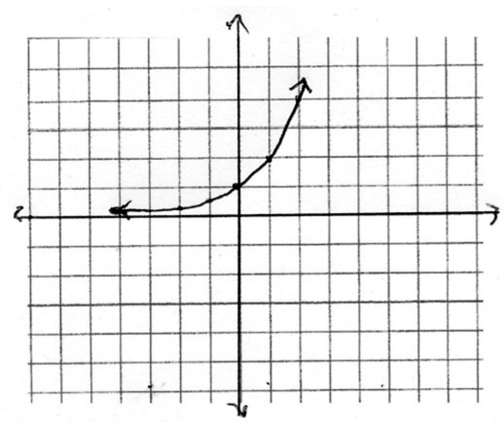
b. $2\log_5(x-1) - \log_5(x-1)$ 6b. $\log_5(x-1)$

7. Solve for x:

a. $\log_5 x = 2$ 7a. *x = 25*

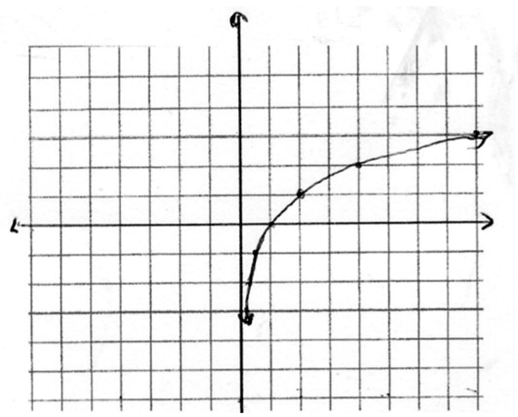
b. $\log_2 \frac{1}{16} = x$ 7b.*x = -4*

8. Use the grid below to graph f(x) = 2^x. Give the domain and range of this function.



Domain = *{x: x is the set of reals} and the* Range = *{y: y> 0}*

9. Use the grid below to graph $f(x) = \log_2 x$. Give the domain and range of this function.



Domain = {x: x > 0} and Range = {y: y is the set of all reals}

Unit 3, What do You Know about the Exponential and Logarithmic Functions?

Word	+	?	-	What do you know about the exponential and Logarithmic functions?
algebraic functions				
transcendental functions				
exponential functions				
logarithmic functions				
growth rate				
growth factor				
exponential growth model				
exponential decay model				
natural base e				

Unit 3, What do You Know about the Exponential and Logarithmic Functions?

symbolic form of the natural exponential function				
logarithmic function to base a				
common log function				
natural log function				
properties of exponents				
laws of logarithms				
definition of zero, negative, and fractional exponents				
change of base formula				

Unit 3, Activity 1, The Four Representations of Exponential Functions

1. Growth is called exponential when there is a constant, called the growth factor, such that during each unit time interval the amount present is multiplied by this factor. Use this fact to decide if each of the situations below is represented by an exponential function.
a) To attract new customers a construction company published its pre-tax profit figures for the previous ten years.

Year	Profit before Tax (millions of dollars)
1992	27.0
1993	32.4
1994	38.9
1995	46.7
1996	56.0
1997	67.2
1998	80.6
1999	96.7
2000	116.1
2001	139.3

Was the growth of profits exponential? How do you know? If exponential, what is the growth factor?

b) A second company also published its pre-tax profit figures for the previous ten years. Its results are shown in the table below.

Year	Profit before Tax (millions of dollars)
1992	12.6
1993	13.1
1994	14.1
1995	16.2
1996	20.0
1997	29.6
1998	42.7
1999	55.2
2000	71.5
2001	90.4

Was the growth of profits for this company exponential? How do you know? If exponential what is the growth factor?

c) Determine whether each of the following table of values could correspond to a linear function, an exponential function, or neither.

i)

x	$f(x)$
0	10.5
1	12.7
2	18.9
3	36.7

ii)

x	$f(x)$
0	27
2	24
4	21
6	18

iii)

x	$f(x)$
-1	50.2
0	30.12
1	18.072
2	10.8432

2. Carbon dating is a technique for discovering the age of an ancient object by measuring the amount of Carbon 14 that it contains. All plants and animals contain Carbon 14. While they are living the amount is constant, but when they die the amount begins to decrease. This is referred to as radioactive decay and is given by the formula $A = A_o(.886)^t$ A_o represents the initial amount. The quantity "A" is the amount remaining after t thousands of years. Let $A_0 = 15.3$ cpm/g.

a) Fill in the table below:

Age of object (1000's of years)	0	1	2	3	4	5	6	7	8	9	10	12	15	17
Amount of C14 (cpm/g)														

b) Sketch a graph using graph paper.

c) There are two samples of wood. One was taken from a fresh tree and the other from Stonehenge and is 4000 years old. How much Carbon 14 does each sample contain? (Answer in cpms)

Use the graph and the table above to answer the questions below. Check your answer by using the given equation.

d) How long does it take for the amounts of Carbon 14 in each sample to be halved?
e) Charcoal from the famous Lascaux Cave in France gives a count of 2.34 cpm. Estimate the date of formation of the charcoal and give a date to the paintings found in the cave.

3. Each of the following functions gives the amount of a substance present at time t. In each case

- give the amount present initially
- state the growth/decay factor
- state whether or not the function represents an exponential growth model or exponential decay model

$a) A = 100(1.04)^t$

$b) A = 150(.89)^t$

$c) A = 1200(1.12)^t$

4. For each of the following functions state (a) whether exponential growth or decay is represented and (b) give the percent growth or decay rate.

a) $A = 22.3(1.07)^t$
b) $A = 10(.91)^t$
c) $A = 1000(.85)^t$

5. In a recent newspaper article the 2003-04 jobs report for the Baton Rouge metro area was given. Job changes by industry sector (since August 2003) were as follows:

- Leisure & hospitality up 4.5%
- Education & health services down 2%
- Construction down 6%
- Financial up 3.6%

For each what is the growth/decay factor and what is the growth/decay rate?

6. Find a possible formula for the function represented by the data below and explain what each of the values in the formula mean:

x	0	1	2	3
f(x)	4.30	6.02	8.43	11.80

Unit 3, Activity 1, The Four Representations of Exponential Functions with Answers

1. Growth is called exponential when there is a constant, called the growth factor, such that during each unit time interval the amount present is multiplied by this factor. Students should use this fact to decide if each of the situations below is represented by an exponential function.

a) To attract new customers a construction company published its pre-tax profit figures for the previous ten years.

Year	Profit before Tax (millions of dollars)
1992	27.0
1993	32.4
1994	38.9
1995	46.7
1996	56.0
1997	67.2
1998	80.6
1999	96.7
2000	116.1
2001	139.3

Was the growth of profits exponential? How do you know? If exponential what is the growth factor?

yes; $\frac{32.4}{27} = \frac{38.9}{32.4} = 1.2$ *the growth factor is 1.2*

b) A second company also published its pre-tax profit figures for the previous ten years. Its results are shown in the table below.

Year	Profit before Tax (millions of dollars)
1992	12.6
1993	13.1
1994	14.1
1995	16.2
1996	20.0
1997	29.6
1998	42.7
1999	55.2
2000	71.5
2001	90.4

Was the growth of profits for this company exponential? How do you know? If exponential what is the growth factor?

The growth of the second company is not exponential. There is not a constant growth factor.

c) Determine whether each of the following table of values could correspond to a linear function, an exponential function, or neither.

i)

x	f(x)
0	10.5
1	12.7
2	18.9
3	36.7

ii)

x	f(x)
0	27
2	24
4	21
6	18

iii)

x	f(x)
-1	50.2
0	30.12
1	18.072
2	10.8432

i) neither *ii) linear* *iii) exponential*

2. Carbon dating is a technique for discovering the age of an ancient object by measuring the amount of Carbon 14 that it contains. All plants and animals contain Carbon 14. While they are living the amount is constant, but when they die the amount begins to decrease. This is referred to as radioactive decay and is given by the formula $A = A_o(.886)^t$ A_o represents the initial amount. The quantity "A" is the amount remaining after t thousands of years. Let $A_0 = 15.3$ cpm/g.

a) Fill in the table below:

Age of object (1000's of years)	0	1	2	3	4	5	6	7	8	9	10	12	15	17
Amount of C14 (cpm/g)	*15.3*	*13.56*	*12.01*	*10.64*	*9.43*	*8.35*	*7.4*	*6.56*	*5.81*	*5.15*	*4.56*	*3.58*	*2.49*	1.95

b) Sketch a graph using graph paper.

c) There are two samples of wood. One was taken from a fresh tree and the other from Stonehenge and is 4000 years old. How much Carbon 14 does each sample contain? (answer in cpms)

Reading from the table: The fresh wood will contain 15.3 cpm's of Carbon 14 while the wood from Stonehenge will contain 9.43 cpm's of Carbon 14.

Use the graph and the table above to answer the questions below. Check your answer by using the given equation.

d) How long does it take for the amounts of Carbon 14 in each sample to be halved?
Fresh wood: the amount of C14 to be halved is 7.65 cpm/g. Both the table and the graph would put the age between 5000 and 6000 years.
Wood from Stonehenge: there would be 4.715 cpm/g present. That would put the age between 9000 and 10,000 years. Since this wood is already 4000 years old, the half-life is the same as the fresh wood. It is the same in each case. Using the equation solve $\frac{1}{2} = .886^t$ *which is ≈ 5700 years*

e) Charcoal from the famous Lascaux Cave in France gives a count of 2.34 cpm. Estimate the date of formation of the charcoal and give a date to the paintings found in the cave.

The charcoal from the caves is about 15,500 years old, so the paintings date back to about 13,500 BC.

3. Each of the following functions gives the amount of a substance present at time t. In each case
- give the amount present initially
- state the growth/decay factor
- state whether or not the function represents an exponential growth model or exponential decay model

a) $A = 100(1.04)^t$
100 is initial substance, 1.04 is growth factor, exponential growth

b) $A = 150(.89)^t$
150 is initial substance, .89 is the growth factor, exponential decay

c) $A = 1200(1.12)^t$
1000 is initial substance, 1.12 is growth factor, exponential growth

4. For each of the following functions state (a) whether exponential growth or decay is represented and (b) give the percent growth or decay rate.

a) $A = 22.3(1.07)^t$
growth at 7%

b) $A = 10(.91)^t$
decay at 9%

c) $A = 1000(.85)^t$
decay at 15%

5. In a recent newspaper article the 2003-04 jobs report for the Baton Rouge metro area was given. Job changes by industry sector (since August 2003) were as follows:

- Leisure & hospitality up 4.5%
- Education & health services down 2%
- Construction down 6%
- Financial up 3.6%

For each, what is the growth/decay factor and what is the growth/decay rate?

- *Leisure has a growth factor of 1.045 and a growth rate of 4.5%.*
- *Education has a decay factor of .98 with a decay rate of 2%.*
- *Construction has a decay factor of .94 with a decay rate of 6%.*
- *Financial has a growth factor of 1.036 and a growth rate of 3.6%.*

6. Find a possible formula for the function represented by the data below and explain what each of the values in the formula mean:

x	0	1	2	3
$f(x)$	4.30	6.02	8.43	11.80

$f(x) = 4.3(1.4)^x$

4.3 is the initial amount A_o; 1.4 is the growth factor with the growth rate being 40%.

Unit 3, Activity 2, Continuous Growth and the Number e

1. In each of the following equations tell whether or not there is growth or decay and give the continuous rate of growth or decay.

a) $A = 1000e^{0.08t}$

b) $A = 200e^{-.2t}$

c) $A = 2.4e^{-o.oo4t}$

2. Write the following exponential functions in the form $P = a^t$.

a) P = e.25t

b) P = e-.4t

3. The population of a city is 50,000 and it is growing at the rate of 3.5% per year. Find a formula for the population of the city *t* years from now if the rate of 3.5% is:
a) an annual rate and b) a continuous annual rate. In each case find the population at the end of 10 years.

4. Air pressure, *P*, decreases exponentially with the height above the surface of the earth, h: $P = P_o e^{-0.00012h}$ where P_o is the air pressure at sea level and *h* is in meters.

a) Crested Butte ski area in Colorado is 2774 meters (about 9100 feet) high. What is the air pressure there as a percent of the air pressure at sea level?

b) The maximum cruising altitude of commercial airplanes is 12,000 meters (around 29,000 feet). At that height what is the air pressure as a percent of the air pressure at sea level?

Unit 3, Activity 2, Continuous Growth and the Number e with Answers

1. In each of the following equations tell whether or not there is growth or decay and give the continuous rate of growth or decay.

a) $A = 1000e^{0.08t}$
growth of 8%

b) $A = 200e^{-.2t}$
decay of 20%

c) $A = 2.4e^{-o.oo4t}$
decay of .4%

2. Write the following exponential functions in the form $P = a^t$

a) $P = e^{.25t}$ $b = e^{.25}$ *so* $P = (1.28)^t$

b) $P = e^{-.4t}$ $b = e^{-0.4}$ *so* $P = (.67)^t$

3. The population of a city is 50,000 and it is growing at the rate of 3.5% per year. Find a formula for the population of the city at time t years from now if the rate of 3.5% is: a) an annual rate and b) a continuous annual rate. In each case find the population at the end of 10 years.

a) $A= 50000(1.035)^t$ *when* $t = 10$ $A = 70{,}530$

b) $A = 50000e^{.0344t}$ *when* $t = 10$ $A=70{,}529$

4. Air pressure, P, decreases exponentially with the height above the surface of the earth, h: $P = P_o e^{-0.00012h}$ where P_o is the air pressure at sea level and h is in meters.

a) Crested Butte ski area in Colorado is 2774 meters (about 9100 feet) high. What is the air pressure there as a percent of the sea level air pressure?

a) 72%

b) The maximum cruising altitude of commercial airplanes is 12,000 meters (around 29,000 feet). At that height what is the air pressure as a percent of the sea level air pressure?

b) 24%

Unit 3, Activity 3, Saving for Retirement

Two friends, Jack and Bill, both begin their careers at 21. By age 23 Jack begins saving for retirement. He is able to put $6,000 away each year in a fund that earns on the average 7% per year. He does this for 10 years, then at age 33 he stops putting money in the retirement account. The amount he has at that point continues to grow for the next 32 years, still at the average of 7% per year. Bill on the other hand doesn't start saving for retirement until he is 33. For the next 32 years, he puts $6000 into his retirement account that also earns on the average 7% per year. At 65 ,Jack has the greatest amount of money in his retirement fund.

Unit 3, Activity 3, Saving for Retirement with Answers

Jack's Retirement Fund

Use the Future Value Formula: $F = P\left[\frac{(1+i)^n - 1}{i}\right]$ with P = \$6000. I = 7% and n = 10

$$F = 6000\left[\frac{(1+.07)^{10} - 1}{.07}\right]$$

F = \$82,898.69

Then, since Jack does not add to his account, use the exponential growth formula compounded annually.

$P = 82898.69(1.07)^{32}$
P = \$722,484.51

Bill's Retirement Fund

Bill will save \$6000 per year for 32 years. Using the Future Value Formula:

$$F = 6000\left[\frac{(1+.07^{32} - 1)}{.07}\right]$$

F = \$661,308.93

Jack put \$60,000 of his own money in the account. Bill put \$192,000 in his account.

(1) What if the interest rate averages 10% instead of 7%, how much will each have then?

- *For Jack: He will have \$95,624 in the account at age 33. At retirement he will have \$ 2,018,983 in the account.*

- *For Bill: He will have \$1,206,826 at retirement.*

(2) Suppose Jack retires at 62. How does his retirement fund compare to Bill's who will retire at 65?

At age 33 Jack will have \$82,898.69 to invest. This will be worth \$589,763. when he is 62. Below is the table from age 61 to 67.

X	Y1	
28	551180	
29	589763	
30	631046	
31	675219	
32	722485	
33	773058	
34	827173	

X=28

Unit 3, Activity 4, The Local and Global Behavior of Ln x

Part I

1. Using graph paper graph the function $f(x) = \ln x$. Use a window with $-1 \le x \le 10$ and $-10 \le y \le 5$. Sketch the graph. What is the domain of $f(x)$? For what values of x is $\ln x < 0$? $\ln x = 0$? $\ln x > 0$? Run the trace feature and find the farthest point to the left on the graph. What is it?

2. Reset your window to $0 \le x \le 0.01$ and $-10 \le y \le -6$. Sketch this graph. Run the trace feature and find the furthest point to the left on the graph. What is it?

3. To get a feel for how rapidly the natural log of x is falling as the x values are getting closer to zero, fill in the table below.

x	**ln *x***
.01	
.001	
.0001	
.00001	
.000001	

Why do you think these points are not evident on the graph of $f(x) = \ln x$?

4. Look at the end-behavior of the function.

a) Set your window $0 \le x \le 100$ and adjust the y-values so that the graph exits at the right. Is the graph increasing, decreasing, or constant? Describe its concavity.

b) Increase the window to $-1 \le x \le 1000$ and if necessary adjust the y-values. What do you see?

5. Based on this information how would you describe the global behavior of this function? What is its range?

Unit 3, Activity 4, The Local and Global Behavior of Ln x with Answers

Part I

1. Graph the function $f(x) = \ln x$. Use a window with $-1 \le x \le 10$ and $-10 \le y \le 5$. Sketch the graph. What is the domain of $f(x)$? For what values of x is $\ln x < 0$? $\ln x = 0$? $\ln x > 0$? Run the trace feature and find the farthest point to the left on the graph. What is it?

The domain is {x: x>0}. lnx = 0 at x = 1 so lnx < 0 when x < 1 and lnx > 0 when x > 1. The farthest point to the left is (.053, -2.93)

2. Reset your window to $0 \le x \le 0.01$ and $-10 \le y \le -6$. Sketch this graph. Run the trace feature and find the farthest point to the left on the graph. What is it?

(0.000106, -9.148)

3. To get a feel for how rapidly the natural log of x is falling as the x values are getting closer to zero, fill in the table below.

x	**ln *x***
.01	-4.605
.001	-6.907
.0001	-9.210
.00001	-11.513
.000001	-13.816

Why do you think these points are not evident on the graph of $f(x) = \ln x$?
The y-axis is acting as a vertical asymptote. The calculator is unable to graph the complete function due to the limitations of the viewing screen. The TABLE function will give a much more precise answer. Set TblStart to 0 and ΔTbl to increasingly small increments up to $1 \times 1O^{99}$.

4. Look at the end-behavior of the function.

a) Set your window $0 \le x \le 100$ and adjust the y-values so that the graph exits at the right. Is the graph increasing, decreasing, or constant? Describe its concavity.
The graph is increasing. The graph is concave down.

b) Increase the window to $-1 \le x \le 1000$ and if necessary adjust the y-values. What do you see?
The graph continues to increase no matter how large x becomes. The graph continues to be concave down.

5. Based on this information how would you describe the global behavior of this function? What is its range?
The range is the set of all real numbers. As $x \to 0, y \to -\infty$ and as $x \to \infty, y \to \infty$.

Unit 3, Activity 4, Translations, Dilations, and Reflections of Ln x

Place a √ in the column marked My Opinion if you agree with the statement. Place an X if you disagree with the statement. If you have disagreed, explain why.

My Opinion	Statement	If you disagree, why?	Calculator Findings
	1. The graph of $f(x) = \ln(x + 3)$ is translated 3 units to the right. The zero is at (4, 0).		
	2. The graph of $f(x) = \ln(-x)$ is reflected over the y-axis. The domain is $(-\infty, 0)$.		
	3. The graph of $f(x) = 3\ln(x)$ has a zero at $x = 3$.		
	4. The graph of $f(x) = \ln(4 - x)$ has been reflected over $x = 4$. Its domain is $\{x: x < 4\}$.		
	5. The graph of $f(x) = \ln(x) + 2$ has a zero at e^{-2}.		
	6. The graph of $f(x) = \ln(x - 4)$ has a vertical asymptote at $x = 4$ and a zero at $x = 5$.		

Unit 3, Activity 4, Translations, Dilations, and Reflections of Ln x

Part II.

Sketch the graph of each function. Give the domain, the zero, the vertical asymptote, and the y-intercept.

1. $f(x) = \ln(x + 3)$

2. $f(x) = \ln(x - 2) + 1$

3. $f(x) = 2\ln(x - 2)$

4. $f(x) = -\ln(x)$

Unit 3, Activity 4, Translations, Dilations, and Reflections of Ln x with Answers

My Opinion	Statement	If you disagree why?	Calculator Findings
	1. The graph of $f(x) = \ln(x + 3)$ is translated 3 units to the right. The zero is at (4, 0).		*This graph is translated 3 units to the left. The zero will be at (-2, 0).*
	2. The graph of $f(x) = \ln(-x)$ is reflected over the y-axis. The domain is $(-\infty, 0)$.		*The statement is correct.*
	3. The graph of $f(x) = 3\ln(x)$ has a zero at $x = 3$.		*The value 3 causes a dilation. The zero stays at x = 1.*
	4. The graph of $f(x) = \ln(4 - x)$ has been reflected over $x = 4$. Its domain is $\{x: x < 4\}$		*The statement is correct.*
	5. The graph of $f(x) = \ln(x) + 2$ has a zero at e^{-2}.		*The graph of f(x) is translated 2 units up. The zero is* $e^{-2} \approx .1353...$ *Let f(x) = 0 then ln x = -2 so* $e^{-2} = x$
	6. The graph of $f(x) = \ln(x - 4)$ has a vertical asymptote at $x = 4$ and a zero at $x = 5$.		*The statement is correct.*

Unit 3, Activity 4, Translations, Dilations, and Reflections of Ln x with Answers

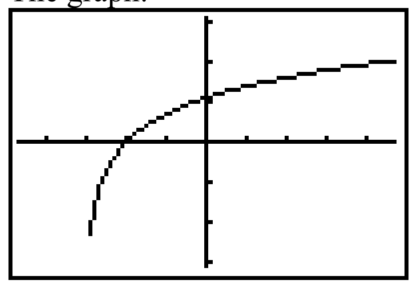
Part II.

Sketch the graph of each function. Give the domain, the zero, the vertical asymptote, and the *y*-intercept.

1. $f(x) = \ln(x + 3)$

The domain is $(-3,\infty)$. *The zero is -2. The vertical asymptote is x = -3. The y-intercept is* $\ln 3 \approx 1.098\ldots$

The graph:

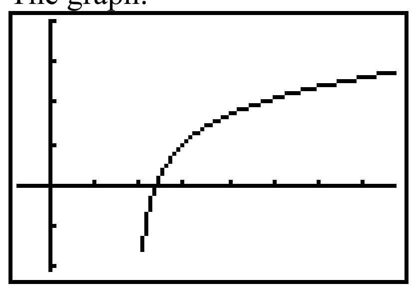


2. $f(x) = \ln(x - 2) + 1$
The domain is {x: x > 2}. The line x = 2 is a vertical asymptote. The zero is $e^{-1} + 2 \approx 2.367\ldots$.

$$\ln(x-2)+1=0$$
$$\ln(x-2)=-1$$
$$e^{-1}=x-2$$
$$e^{-1}+2=x$$

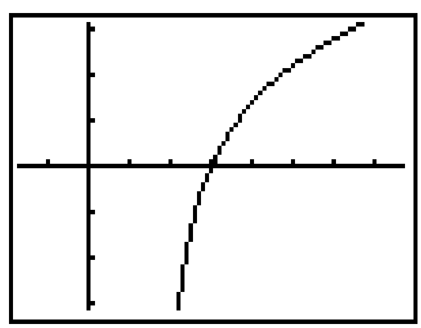
There is no y-intercept.

The graph:



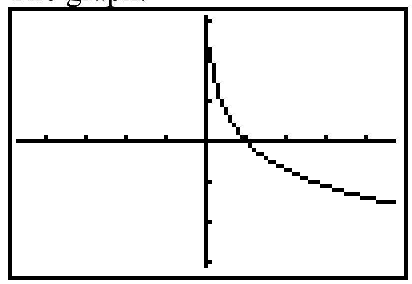
3. $f(x) = 2\ln(x - 2)$
domain is $(2,\infty)$. *The zero is 3. The vertical asymptote is x = 2. There is no y-intercept.*

The graph:



4. $f(x) = -\ln(x)$
The domain is $(0,\infty)$. The zero is 1. The vertical asymptote is $x = 0$. There is no y-intercept.

The graph:



Unit 3, Activity 5, Working with the Laws of Logarithms

Name____________________

Date_____________________

1. Write each expression as a rational number or as a logarithm of a single quantity.

a) $\log 8 - \log 5 - \log 3$

b) $\ln 10 - \ln 5 - \frac{1}{3}\ln 8$

c) $4\log M - 3\log N$

d) $\frac{1}{2}(3\log M + \log N)$

e) $1/3(2\log_a M - \log_a N - \log_a P)$

2. Express y in terms of x

a) $\log y = 2\log x$

b) $\ln y - \ln x = 2\ln 7$

c)) $\log y = 3 - .2x$

d) $\ln y = \frac{1}{4}(\ln 3 + \ln x)$

3. Solve the following logarithmic equations:

a) $\log(x+1)-\log(x)=\log\left(\frac{4}{3}\right)$

b) $\log(x + 3) + \log(x – 2) = \log(x + 10)$

c) $\log_{\frac{1}{2}}(x)+\log_{\frac{1}{2}}(x-2)=-3$

d) $\log_4 x + \log_4(x - 3) = 1$

Unit 3, Activity 5, Working with the Laws of Logarithms with Answers

1. Write each expression as a rational number or as a single logarithm.

a) $\log 8 - \log 5 - \log 3$

answer: $\log\left(\frac{8}{15}\right)$

b) $\ln 10 - \ln 5 - \frac{1}{3}\ln 8$

answer: 0

c) $4\log M - 3\log N$

answer: $\log\left(\frac{M^4}{N^3}\right)$

d) $\frac{1}{2}(3\log M + \log N)$

answer: $\log(M^3 N)^{1/2}$

e) $1/3\,(2\log_a M - \log_a N - \log_a P)$

answer: $\log\left(\frac{M^2}{NP}\right)^{1/3}$

2. Express y in terms of x

a) $\log y = 2\log x$

$y = x^2$

b) $\ln y - \ln x = 2\ln 7$

$y = 49x$

c) $\log y = 3 - .2x$

$y = 10^{3-.2x}$

d) $\ln y = \frac{1}{4}(\ln 3 + \ln x)$

$y = \sqrt[4]{3x}$ *or* $(3x)^{1/4}$

Unit 3, Activity 5, Working with the Laws of Logarithms with Answers

3. Solve the following logarithmic equations:

a) $\log(x+1)-\log(x)=\log\left(\frac{4}{3}\right)$

x = 3

b) $\log(x + 3) + \log(x - 2) = \log(x + 10)$

x = 4 *(-4 is not in the domain)*

c) $\log_{\frac{1}{2}}(x)+\log_{\frac{1}{2}}(x-2)=-3$

x = 4 *(-2 is not in the domain)*

d) $\log_4 x + \log_4(x - 3) = 1$

x = 4 *(-1 is not in the domain)*

Unit 3, Activity 6, Working with Exponential and Logarithmic Functions

1. Start with the graph of $y = 3^x$. Write an equation for each of the conditions below and sketch the graph labeling all asymptotes and intercepts. Verify your answer with a graphing utility.

a) Reflect the graph through the x-axis.
b) Reflect the graph through the y-axis.
c) Shift the graph up 3 units and translate the graph 4 units to the left.
d) Reflect the graph over the x-axis, then shift it 2 units to the right.

2. Graph on the same set of axes: $f(x) = 2^x$ and $g(x) = \log_2 x$. What is the relationship of $f(x)$ and $g(x)$?

3. Fill in the table below:

f(g(x)) =	***f(x)***	***g(x)***	**♦Domain of *f(g(x))***	**Domain of *f***	**Domain of *g***
1. $\ln(x^2-4)$					
2. $e^{\|x\|}$					
3. $(1 - \ln x)^2$					
4. $2^{1/x}$					

4. Which of the composite functions in the table above are even, odd, or neither? How do you know?

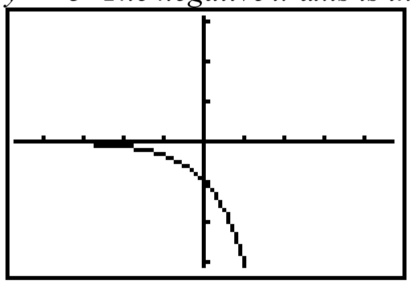
5. Which of the functions in the table have an inverse that is a function? Justify your answer.

6. For those function/s that have an inverse find $f^{-1}(x)$.

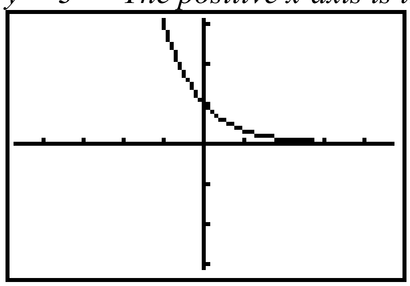
Unit 3, Activity 6, Working with Exponential and Logarithmic Functions with Answers

1. Start with the graph of $y = 3^x$. Write an equation for each of the conditions below and sketch the graph labeling all asymptotes and intercepts. Verify your answer with a graphing utility.

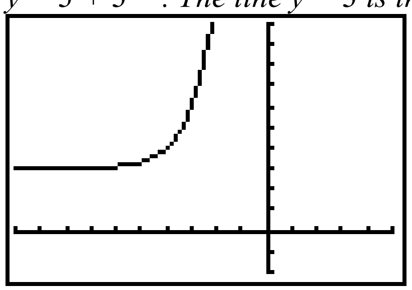
a) Reflect the graph through the x-axis.
$y = -3^x$ The negative x-axis is the horizontal asymptote. The y-intercept is (0,-1).



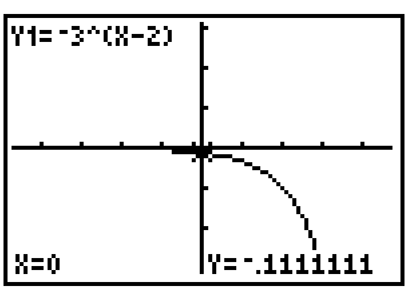
b) Reflect the graph through the y-axis.
$y = 3^{-x}$ The positive x-axis is the horizontal asymptote. The y-intercept is (0, 1).



c) Shift the graph up 3 units and translate the graph 4 units to the left.
$y = 3 + 3^{x+4}$. The line y = 3 is the horizontal asymptote. The y-intercept is (0, 84).

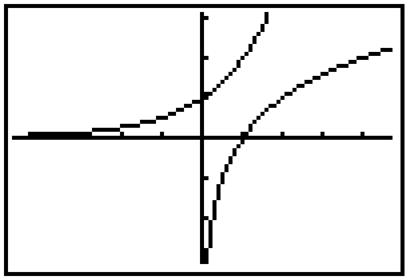


d) Reflect the graph over the x-axis, then shift it 2 units to the right.
$y = -3^{x-2}$ *The negative x-axis is the horizontal asymptote. The y-intercept is (0, - $\frac{1}{9}$)*



Unit 3, Activity 6, Working with Exponential and Logarithmic Functions with Answers

2. Graph on the same set of axes: $f(x) = 2^x$ and $g(x) = \log_2 x$. What is the relationship of f(x) and g(x)? *Each is the inverse of the other. If* $f(x) = 2^x$ then $f^{-1}(x) = \log_2 x$



3. Fill in the table below:

f(g(x)) =	***f(x)***	***g(x)***	**Domain of *f(g(x))***	**Domain of *f***	**Domain of *g***
1. $\ln(x^2-4)$	*Ln(x)*	x^2-4	*{x: x < -2 or x > 2}*	*x > 0*	*Reals*
2. $e^{\|x\|}$	e^x	$\|x\|$	*Reals*	*Reals*	*Reals*
3. $(1 - \ln x)^2$	x^2	$1 - \ln(x)$	*x > 0*	*Reals*	*x > 0*
4. $2^{1/x}$	2^x	$\frac{1}{x}$	*{x:x≠0}*	*Reals*	*{x:x≠0}*

4. Which of the composite functions in the table above are even, odd, or neither? How do you know? *1 and 2 are even because they are symmetric over the y-axis. Numbers 3 and 4 are neither symmetric over the y-axis nor around the origin.*

5. Which of the functions in the table have an inverse that is a function? Justify your answer. *Numbers 1, 2, and 3 do not have an inverse that is a function. Number 4 has an inverse that is a function.*
Justification:

- *Numbers 1, 2, and 3 are not-one-to-one functions or*
- *because 1 and 2 are even functions they will not have an inverse that is a function, and number 3 decreases into a minimum then increases.*
- *Looking at the graph or at the numerical tables of #4, the function is strictly decreasing, but y = 1 is a horizontal asymptote so the values of the range are never repeated. When x < 0, y < 1 and when x > 0, y > 1.*

6. For those function/s that have an inverse find $f^{-1}(x)$.

The composite function in #4 has an inverse: $f^{-1}(x) = (\log_2 x)^{-1}$

Unit 3, Activity 7, Solving Exponential Equations

Part A

Solve for *x* and give the exact answer.

1. $\left(\frac{1}{3}\right)^x = 9^{x+2}$

2. $16^x = 8^{x-1}$

3. $2^{x-2} = 3$

4. $2^{x-1} = 3^{x+1}$

Part B

Solve for *x*. Use your calculator to obtain the answer. Write the answer to the nearest thousandth.

1. $1400 = 350e^{-.2x}$

2. $14.53(1.09)^{2x} = 2013.$

3. $2500 = 5100(.79)^x$

4. $8^{2-x} = 45$

5. $200 = 40(1.12)^{3t}$

Part C. Use a graphing calculator to solve the equation $e^x = 5\text{-}2x$.

Unit 3, Activity 7, Solving Exponential Equations with Answers

Part A
Solve for x and give the exact answer.

1. $\left(\frac{1}{3}\right)^x = 9^{x+2}$

   $x = -4/3$

2. $16^x = 8^{x-1}$

   $x = -3$

3. $2^{x-2} = 3$

   $x = \frac{\ln 3}{\ln 2} + 2 \; or \; \frac{\ln 12}{\ln 2} \; which \; is \approx 3.585$

4. $2^{x-1} = 3^{x+1}$

   $\frac{\ln 6}{\ln \frac{2}{3}} \; which \; is \approx -4.419$

Part B
Solve for x. Use your calculator to obtain the answer. Write the answer to the nearest thousandth.

1. $1400 = 350e^{-.2x}$

   -6.93

2. $14.53(1.09)^{2x} = 2013$

   15.24

3. $2500 = 5100(.79)^x$

   3.02

4. $8^{2-x} = 45$

   $= \frac{\log\left(\frac{45}{64}\right)}{\log 8} \approx -.169$

5. $200 = 40(1.12)^{3t}$

   $t = \frac{\ln 5}{3 \ln 1.12} \approx 4.734$

Part C. Use a graphing utility to solve the equation $e^x = 5-2x$

The two graphs intersect at $x \approx 1.06$.

1. The number of radioactive atoms N of a particular material present at time t years may be written in the form $N = 5000e^{-kt}$, where 5000 is the number of atoms present when $t = 0$, and k is a positive constant. It is found that $N = 2500$ when $t = 5$ years.

    a) Determine the value of k.
    b) At what value of t will $N = 50$?

2. A cup of coffee contains about 100 mg of caffeine. The half-life of caffeine in the body is about 4 hours, which means that the level of caffeine in the body is decaying at the rate of about 16% per hour.

    a) Write a formula for the level of caffeine in the body as a function of the number of hours since the coffee was drunk.

    b) How long will it take until the level of caffeine reaches 20 mg?

3. A radioactive substance has a half-life of 8 years. If 200 grams are present initially,

    a) How much will remain at the end of 12 years?

    b) How long will it be until only 10% of the original amount remains?

4. The Angus Company has a manufacturing process that produces a radioactive waste byproduct with a half-life of twenty years.

    a) How long must the waste be stored safely to allow it to decay to one-quarter of its original mass?

    b) How long will it take to decay to10% of its original mass?

    c) How long will it take to decay to 1% of its original mass?

5. A bacteria population triples every 5 days. The population is P_o bacteria.

    a) Write an equation that reflects this statement.

    b) If the initial population is 120, what is the population
        i) after 5 days?
        ii) after 2 weeks?

Unit 3, Activity 8, Applications Involving Exponential Growth and Decay with Answers

1. The number of radioactive atoms N of a particular material present at time t years may be written in the form $N = 5000e^{-kt}$, where 5000 is the number of atoms present when $t = 0$, and k is a positive constant. It is found that $N = 2500$ when $t = 5$ years.

a) Determine the value of k.

$k = .1386$

b) At what value of t will $N = 50$?

$t \approx 33.3$ years

2. A cup of coffee contains about 100 mg of caffeine. The half-life of caffeine in the body is about 4 hours which means that the level of caffeine in the body is decaying at the rate of about 16% per hour.

a) Write a formula for the level of caffeine in the body as a function of the number of hours since the coffee was drunk.

$A = 100(.84)^t$

b) How long will it take until the level of caffeine reaches 20 mg?

$t \approx 9.2$ hours

3. A radioactive substance has a half-life of 8 years.

a) If 200 grams are present initially, how much will remain at the end of 12 years?

$A \approx 70.7$grams

b) How long will it be until only 10% of the original amount remains?

b) $t \approx 26.6$ years

4. The Angus Company has a manufacturing process that produces a radioactive waste byproduct with a half-life of twenty years.

a) How long must the waste be stored safely to allow it to decay to one-quarter of its original mass?

40 years

b) How long will it take to decay to 10% of its original mass?

66.4 years

c) How long will it take to decay to 1% of its original mass?

132.9 years

5. A bacteria population triples every 5 days. The population is P_o bacteria.

a) Write an equation that reflects this statement. $P = P_o(3)^{t/5}$

b) If the initial population is 120, what is the population

i) after 5 days *360*

ii) after 2 weeks *2600*

Unit 3, Activity 9, Library of Functions – The Exponential Function and Logarithmic Function

1. You are to create an entry for your Library of Functions. Two functions are to be added for this unit – the exponential function, $f(x) = a^x$ and the logarithmic function, $f(x) = \log_b x$. Write a separate entry for each function. Follow the same outline that you did for the previous entries. Introduce the parent function, include a table and graph for each function. Give a general description of the function written in paragraph form. Your description should include:

   (a) the domain and range

   (b) local and global characteristics of the function – look at your glossary list and choose the words that best describe this function.

2. Give some examples of family members using translation, reflection and dilation in the coordinate plane. Show these examples symbolically, graphically, and numerically. Explain what has been done to the parent function to find each of the examples you give.

3. What are the common characteristics of each function?

4. Find a real-life example of how this function family can be used. Be sure to show a representative equation, table and graph. Does the domain and range differ from that of the parent function? If so, why? Describe what the special characteristics mean within the context of your example.

5. Be sure that:
   - ✓ your paragraph contains complete sentences with proper grammar and punctuation
   - ✓ your graphs are properly labeled, scaled and drawn
   - ✓ you have used the correct math symbols and language in describing this function

Unit 3, General Assessment, Spiral

Unit 3, Spiral

Simplify:

1. $4^{3/2}$

2. $\left(8^{-1/6}\right)^{-2}$

3. $\dfrac{x^{1/3}}{2x^{-2/3}}$

4. $2x^{-2/3}\left(x^{8/3} - 3x^{5/3}\right)$

5. $\dfrac{4ab^{-1/2} - 2ab^{1/2}}{\left(a^2b\right)^{-1/2}}$

6. $\dfrac{x^{-1/3} - 3x^{2/3}}{x^{-4/3}}$

Solve:

7. $9^x = 3^5$

8. $x^{-1/2} = 8$

9. $8^x = 2^7 \cdot 4^9$

10. $(2x)^{-2} = 16$

Unit 3, General Assessment, Spiral with Answers

Simplify:

1. $4^{3/2} = \underline{8}$

2. $\left(8^{-1/6}\right)^{-2} = \underline{2}$

3. $\dfrac{x^{1/3}}{2x^{-2/3}} = \underline{x/2}$

4. $2x^{-2/3}\left(x^{8/3} - 3x^{5/3}\right) = \underline{2x^2 - 6x}$

5. $\dfrac{4ab^{-1/2} - 2ab^{1/2}}{\left(a^2b\right)^{-1/2}} = \underline{4a^2 - 2a^2b}$

6. $\dfrac{x^{-1/3} - 3x^{2/3}}{x^{-4/3}} = \underline{x - 3x^2}$

Solve:

7. $9^x = 3^5$

$x = 5/3$

8. $x^{-1/2} = 8$

$x = 1/64$

9. $8^x = 2^7 \cdot 4^9$

$x = 25/3$

10. $(2x)^{-2} = 16$

$x = 1/8$

Unit 4, Pretest Triangle Trigonometry

Name______________________

Date_______________________

1. Match each element in row A with an element in row B.

A. sine cosine tangent

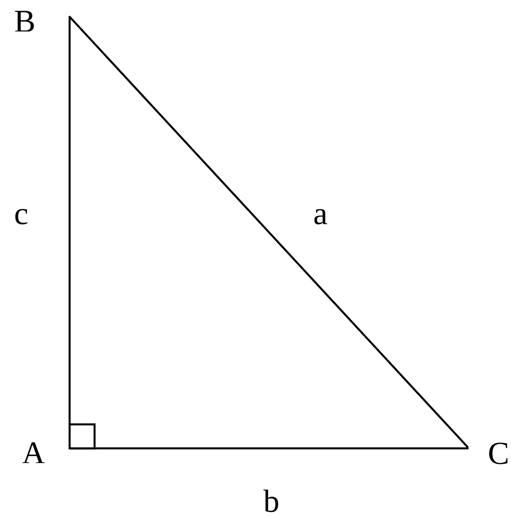
B. $\frac{\text{adjacent}}{\text{hypotenuse}}$ $\frac{\text{opposite}}{\text{adjacent}}$ $\frac{\text{opposite}}{\text{hypotenuse}}$

2. Express the sine, cosine, and tangent of angle A in terms of sides a, b, and c.

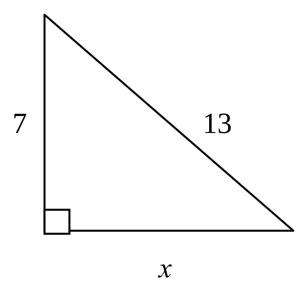
sin B________________

cos B________________

tan B________________



3. State an equation that can be used to solve for *x*.



4. In △DEF ∠D $=90^{o}$, ∠E $= 64^{o}$, and side $e = 9$. Find the length of side d.

5. Sketch △ABC with ∠$C = 90^{o}$. What is the relationship between cos A and sin B?

6. Given isosceles right triangle RST, with ∠S $= 90^{o}$ and sides RS and ST each 1 unit long, find the exact value of each of the following. Write your answer in simplest radical form.

a) side RT

b) cos R

c) sin R

7. Given a $30^o - 60^o - 90^o$ triangle. The hypotenuse is 2 units long.
   a) find the length of the legs in simplest radical form

   b) Find the exact value of

   cos 30^o ______________ sin 30^o______________ tan 30^o____________________

Unit 4, Pretest Triangle Trigonometry with Answers

1. Match each element in row A with an element in row B.

A. sine   cosine   tangent

B. $\dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\dfrac{\text{opposite}}{\text{adjacent}}$ $\dfrac{\text{opposite}}{\text{hypotenuse}}$

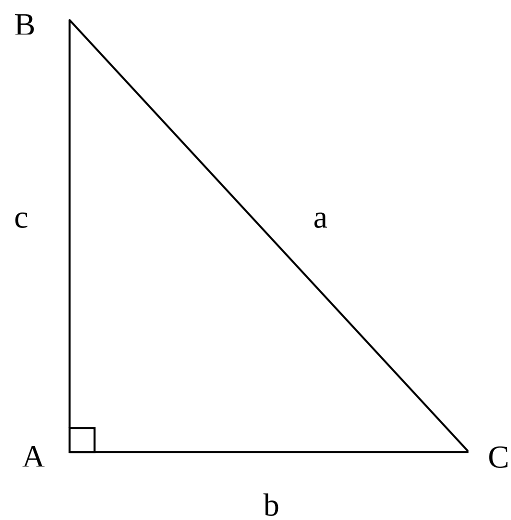
The sine is $\dfrac{opposite}{hypotenuse}$*, the cosine is* $\dfrac{adjacent}{hypotenuse}$*, the tangent is* $\dfrac{opposite}{adjacent}$

2. Express the sine, cosine, and tangent of angle A in terms of sides *a, b,* and *c.*

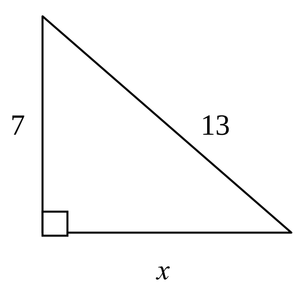
sin B $= {}^{b}\!/_{a}$

cos B $= {}^{c}\!/_{a}$

tan B $= {}^{b}\!/_{c}$



3. State an equation that can be used to solve for x.



$7^2 + x^2 = 13^2$

4. In $\triangle$DEF $\angle$D $=90^o$, $\angle$E $= 64^o$, and side e = 9. Find the length of side *d.*

$9\sin 64^o \approx 8.09$

5. Sketch $\triangle ABC$ with $\angle C = 90^o$. What is the relationship between cos A and sin B?

cos A and sin B are equal.

6. Given isosceles right triangle *RST,* with $\angle S = 90^o$ and sides *RS* and *ST* each 1 unit long, find the exact value of each of the following. Write your answer in simplest radical form.

a) side $RT = \sqrt{2}$

b) cos *R* $1/\sqrt{2} = \sqrt{2}/2$

c) sin *R* $1/\sqrt{2} = \sqrt{2}/2$

7. Given a $30^o - 60^o - 90^o$ triangle. The hypotenuse is 2 units long.

a) find the length of the legs in simplest radical form

The side opposite the 30º angle is ½ and the side opposite the 60º angle is $\sqrt{3}/2$.

b) Find the exact value of

$\cos 30^o = \sqrt{3}/2$ $\quad$ $\sin 30^o = 1/2$ $\quad$ $\tan 30^o = \sqrt{3}/2$

Unit 4, What Do You Know about Triangle Trig and Vectors?

Word	+	?	-	What do I know about triangle trig and vectors?
sine				
cosine				
tangent				
secant				
cosecant				
cotangent				
exact value				
angles of elevation				
angles of depression				
line of sight				
oblique triangles				

degree, minute, second used as angle measurement				
Law of Sines				
Law of Cosines				
vector				
initial point				
terminal point				
vector in standard position				
unit vectors				
zero vector				
equal vectors				
magnitude of a vector				

Unit 4, What Do You Know about Triangle Trig and Vectors?

scalar				
horizontal and vertical components of a vector				
resultant				
bearing				
heading				
ground speed				
air speed				
true course				

Name_____________________________

1. One function of acute angle A is given. Find the other five trigonometric functions of A. Leave answers in simplest radical form.

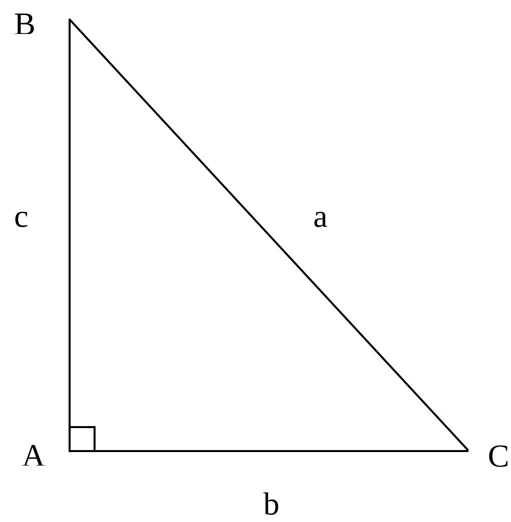
a)

sin A	cos A	tan A	sec A	csc A	cot A
	$\frac{4}{5}$				

b)

sin A	cos A	tan A	sec A	csc A	cot A
		$\frac{4\sqrt{2}}{7}$			

2. The triangle at the right establishes the notation used in the two problems below. Find the lengths and angle measures that are not given.



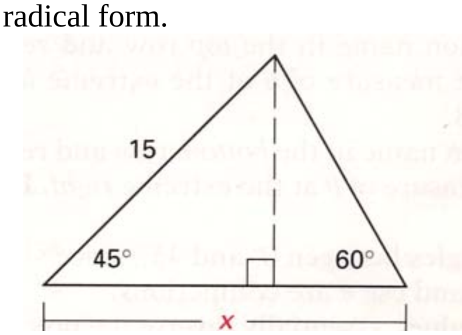
a) $\angle C = 60^{o}$, $a = 12$

b) $\angle B = 45^{o}$, b = 4

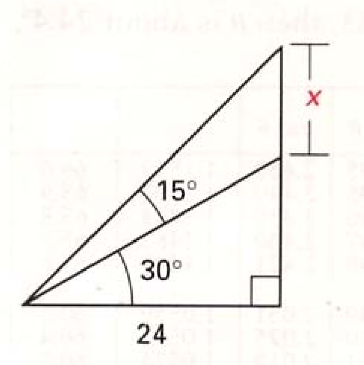
3. Draw $\triangle ABC$ with sin A = $^{5}\!/_{13}$. Find

cos A______________ tan A_________________

4. Find the length x. Leave the answer in simplest radical form.



5. Find the length x. Leave the answer in simplest radical form.



6. Express 51.724° in degrees, minutes, and seconds.

7. Use the fundamental identities to transform one side of the equation into the other.

a) $(1 + \sin A)(1 - \sin A) = \cos^2 A$

b) $\csc A \cdot \cos A = \cot A$

c) $\tan A \cdot \csc A = \sec A$

8. Writing Assignment: What does it mean to solve a right triangle?

Unit 4, Activity 1, Solving Right Triangles with Answers

1. One function of acute angle A is given. Find the other five trigonometric functions of A. Leave answers in simplest radical form.

a)

sin A	cos A	tan A	sec A	csc A	cot A
$3/5$	$4/5$	$3/4$	$5/4$	$5/3$	$4/3$

b)

sin A	cos A	tan A	sec A	csc A	cot A
$4\sqrt{2}/9$	$7/9$	$4\sqrt{2}/7$	$9/7$	$9\sqrt{2}/8$	$7\sqrt{2}/8$

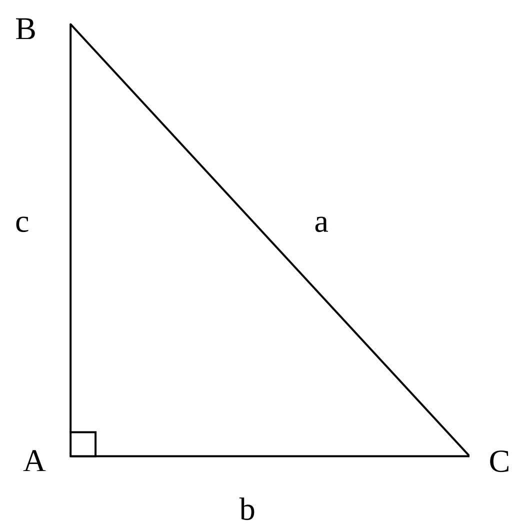
2. The triangle at the right establishes the notation used in the two problems below. Find the lengths and angle measures that are not given.

a) $\angle C = 60^o$, $a = 12$

$\angle B = 30^o,\ b = 6,\ c = 6\sqrt{3}$

b) $\angle$ B = 45^o, b = 4

$\angle C = 45^o,\ c = 4,\ a = 4\sqrt{2}$

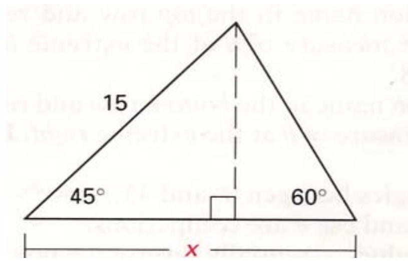


3. Draw $\triangle ABC$ with sin A = $5/13$. Find

cos A = $12/13$ tan A = $5/12$

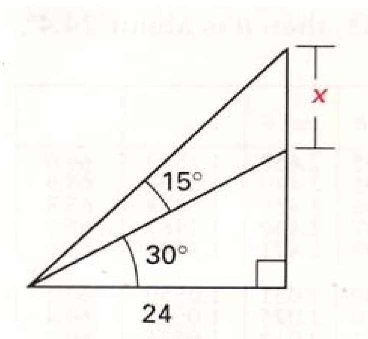
4. Find the length x. Leave the answer in simplest radical form.

$$x = \frac{15\sqrt{2}}{2} + \frac{15\sqrt{6}}{6}$$
$$= \frac{15\sqrt{2} + 5\sqrt{6}}{2}$$



5. Find the length x. Leave the answer in simplest radical form.

$$x = 24 - 8\sqrt{3}$$



6. Express 51.724^o in degrees, minutes, and seconds. *51^{o}43'26"*

7. Use the fundamental identities to transform one side of the equation into the other.

a) $(1 + \sin A)(1 - \sin A) = \cos^2 A$

$1 - \sin^2 A = \cos^2 A$

$\cos^2 A = \cos^2 A$

b) $\csc A \cdot \cos A = \cot A$

$$\frac{1}{\sin A} \cdot \cos A = \cot A$$
$$\frac{\cos A}{\sin A} = \cot A$$
$$\cot A = \cot A$$

c) $\tan A \cdot \csc A = \sec A$

$$\frac{\sin A}{\cos A} \cdot \frac{1}{\sin A} =$$
$$\frac{1}{\cos A} = \sec A$$

8. Writing Assignment: What does it mean to solve a right triangle?

Possible answer:
Each triangle has three sides and three angles. To solve a triangle means to find the unknown sides and angles using the given sides and angles.

Unit 4, Activity 2, Right Triangles in the Real World

For each of the problems below

- Draw a picture and identify the known quantities
- Set up the problem using the desired trigonometric ratio
- Give the answer in degrees, minutes, and seconds using significant digits.

1. A ramp 8.30 meters in length rises to a loading platform that is 1.25 meters off of the ground. What is the angle of elevation of the ramp?

2. A television camera in a blimp is focused on a football field with an angle of depression $25^{o}48'$. A range finder on the blimp determines that the field is 925.0 meters away. How high up is the blimp?

3. A 35 meter line is used to tether a pilot balloon. Because of a breeze the balloon makes a 75^{o} angle with the ground. How high is the balloon?

4. The levels in a parking garage are 12.5 feet a part, and a ramp from one level to the next level is 135 feet long. What angle does the ramp make with horizontal?

5. From a point on the North Rim of the Grand Canyon, a surveyor measures an angle of depression of $1^{o}\ 02'$. The horizontal distance between the two points is estimated to be 11 miles. How many feet is the South Rim below the North Rim?

6. In one minute a plane descending at a constant angle of depression of $12^{o}24'$ travels 1600 meters along its line of flight. How much altitude has it lost?

Writing activity:
Compare an angle of elevation to an angle of depression.

Unit 4, Activity 2, Right Triangles in the Real World with Answers

1. A ramp 8.30 meters in length rises to a loading platform that is 1.25 meters off of the ground. What is the angle of elevation of the ramp? *$8^{o}40'$*

2. A television camera in a blimp is focused on a football field with an angle of depression $25^{o}48'$. A range finder on the blimp determines that the field is 925.0 meters away. How high up is the blimp? *402.6 meters*

3. A 35 meter line is used to tether a pilot balloon. Because of a breeze the balloon makes a 75^{o} angle with the ground. How high is the balloon? *34 meters*

4. 4. The levels in a parking garage are 12.5 feet a part, and a ramp from one level to the next level is 135 feet long. What angle does the ramp make with horizontal? *5.3 degrres*

5. From a point on the North Rim of the Grand Canyon, a surveyor measures an angle of depression of $1^{o}\ 02'$. The horizontal distance between the two points is estimated to be 11 miles. How many feet is the South Rim below the North Rim? *1048 feet*

6. In one minute a plane descending at a constant angle of depression of $12^{o}24'$ travels 1600 meters along its line of flight. How much altitude has it lost? *343.6 meters*

Writing activity:
Compare an angle of elevation to an angle of depression.

> *Possible answer should include the fact that both angles are made with the horizontal. The angle of elevation is the angle between the horizontal and the line of sight when looking up at the object. The angle of depression is the angle between the horizontal and the line of sight when looking down on an object.*

Unit 4, Activity 3, Solving Oblique Triangles

Use the geometry postulates and the laws of sines or cosines to fill in the following table.

Problem #	Which geometric postulate applies? SAS, SSS, AAS, ASA, SSA	Which law should be used?	Solution
1. $a = 30$ $b = 60$ $\angle C = 23^{o}50'$			
2. $b = 23.5$ $\angle B = 15.0^{o}$ $\angle C = 18.0^{o}$			
3. $a = 8.302$ $b = 10.40$ $c = 14.40$			
4. $a = 8.0$ $b = 9.0$ $\angle A = 75.0^{o}$			
5. $a = 6.0$ $\angle A = 64^{o}30'$ $\angle C = 56^{o}20'$			
6. $a = 3.0$ $b = 9.0$ $c = 4.0$			

Writing activity: You are given two sides and an angle. What procedures do you use to determine if the given information will form one unique triangle, two triangles, or no triangle?

Unit 4, Activity 3, Solving Oblique Triangles with Answers

Problem #	Which geometric postulate applies? SAS, SSS, AAS, ASA, SSA	Which law should be used ?	Solution
1. $a = 30.00$ $b = 60.00$ $\angle C = 23^{o}50'$	*SAS*	*law of cosines*	$\angle B = 135^{o}43'$ $\angle A = 20^{o}27'$ $c = 34.74$
2. $b = 23.5$ $\angle B = 15.0^{o}$ $\angle C = 18.0^{o}$	*AAS*	*law of sines*	$\angle A = 147^{o}$ $a = 49.5$ $c = 28.1$
3. $a = 8.302$ $b = 10.40$ $c = 14.40$	*SSS*	*law of cosines*	$\angle A = 34^{o}35'$ $\angle B = 45^{o}19'$ $\angle C = 100^{o}06'$
4. $a = 8.00$ $b = 9.00$ $\angle A = 75.0^{o}$	*SSA*	*If* $b\sin A < a < b$ *there are 2 triangles, therefore 2 solutions. use law of sines*	*Solution1:* $\angle B = 51^{o}20'$ $\angle C = 86^{o}40'$ $c = 44.8$ *or* *Solution 2:* $\angle B = 128^{o}40'$ $\angle C = 9^{o}20'$ $c = 7.3$
5. $a = 6.0$ $\angle A = 64^{o}30'$ $\angle C = 56^{o}20'$	*AAS*	*law of sines*	$b = 5.9$ $c = 6.2$ $\angle B = 64^{o}30'$
6. $a = 3.0$ $b = 9.0$ $c = 4.0$	*SSS*	$a + c < b$ *Therefore there is no triangle.*	

Unit 4, Activity 3, Solving Oblique Triangles with Answers

Writing activity:

A possible answer would be:

When you are given two sides and a non- included angle, you can use the Law of Cosines with the given information letting the side opposite the given angle be x. This will give a quadratic equation. The quadratic formula can be used to solve for x. Pay special attention to the discriminant. If it is negative, then there is no triangle. If it is a perfect square, then there is just one side and therefore one triangle. Otherwise, there are two real values for x, therefore two triangles.

Unit 4, Activity 4, Real-Life Problems Involving Oblique Triangles

For each problem

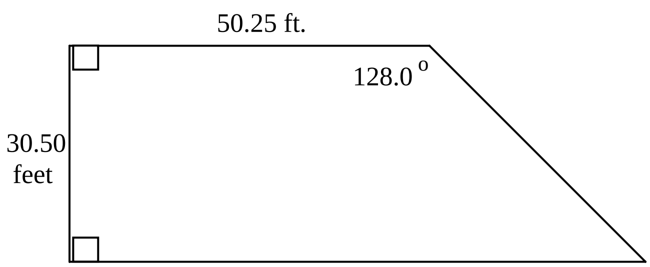
- Draw a picture and identify the known quantities.
- Solve using the appropriate law.
- Use significant digits.

1. A twelve meter long loading ramp that makes a 25^o angle with the horizontal is to be replaced by a ramp whose angle of inclination is only 10^o. How long will the new ramp be?

2. A pilot approaching a 3000 meter runway finds that the angles of depression of the ends of the runway are 14.00^o and 20.00^o. How far is the plane from the nearer end of the runway?

3. At a certain point, the angle of elevation of the top of a tower which stands on level ground is 30.0^o. At a point 100 meters nearer the tower, the angle of elevation is 58.0^o. How high is the tower?

4. The Johnsons plan to fence in their back lot shown below. If fencing costs $4.50 a foot, how much will it cost?



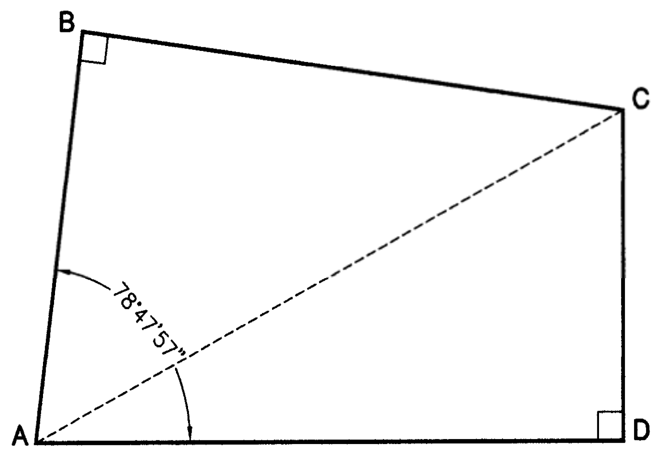
5. To find the distance between two points A and B, on opposite sides of a swamp, a surveyor laid off a base line AC 25.0 meters long and found that $\angle BAC = 82^o$ and $\angle BCA = 69^o$. Find AB

6. A surveyor needs to find the area of a plot of land. The directions are as follows: From an iron post proceed 500 meters northeast to the edge of Bayou Cocodrie. Then move 300 meters east along the bayou to the fence surrounding Brown's camp. Then continue 200 meters S15°E to Wiggners Road. Finally, go along Wiggners road back to the iron post. Sketch the plot of land and find its area.

7. The following problem is taken from the National Society of Professional Surveyors Annual Trig-Star Contest.

TRIG-STAR PROBLEM LOCAL CONTEST

REQUIRED ANSWER FORMAT
DISTANCES: NEAREST HUNDREDTH
ANGLES: DEGREES-MINUTES-SECONDS TO THE NEAREST SECOND



KNOWN: DISTANCE BC = 95.73 DISTANCE CD = 50.15
∡BAD = 78°47'57"

FIND: DISTANCE AB = ____________ (10 POINTS)

DISTANCE AD = ____________ (10 POINTS)

DISTANCE AC = ____________ (10 POINTS)

Unit 4, Activity 4, Real-Life Problems Involving Oblique Triangles

Writing assignment: How do you determine which technique to use in solving real-life problems involving oblique triangles?

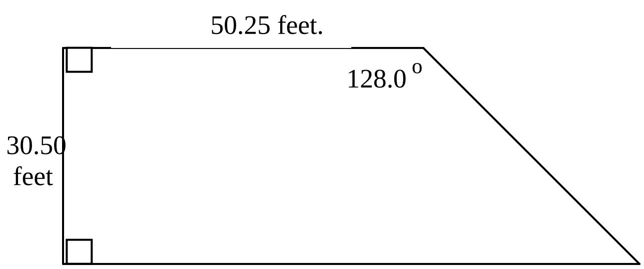
Unit 4, Activity 4, Real-Life Problems Involving Oblique Triangles with Answers

1. A twelve meter long loading ramp that makes a 25^{o} angle with the horizontal is to be replaced by a ramp whose angle of inclination is only 10^{o}. How long will the new ramp be? *29 meters*

2. A pilot approaching a 3000 meter runway finds that the angles of depression of the ends of the runway are 14.00^{o} and 20.00^{o}. How far is the plane from the nearer end of the runway? *6943 meters*

3. At a certain point, the angle of elevation of the top of a tower which stands on level ground is 30.0^{o}. At a point 100 meters nearer the tower, the angle of elevation is 58.0^{o}. How high is the tower? *90.3 feet*

4. The Johnson's plan to fence in their back lot shown below. If fencing costs $4.50 a foot, how much will it cost?



193.6 feet costing $871.20

5. To find the distance between two points A and B, on opposite sides of a swamp, a surveyor laid off a base line AC 25.0 meters long and found that $\angle BAC = 82.0^{o}$ and $\angle BCA = 69.0^{o}$. Find AB. *48.1 meters*

6. A surveyor needs to find the area of a plot of land. The directions are as follows: From an iron post proceed 500 meters northeast to the edge of Bayou Cocodrie. Then move 300 meters east along the bayou to the fence surrounding Brown's camp. Then continue 200 meters S15^{o}E to Wiggners Road. Finally, go along Wiggners road back to the iron post. Sketch the plot of land and find its area.*125,000 m^2*

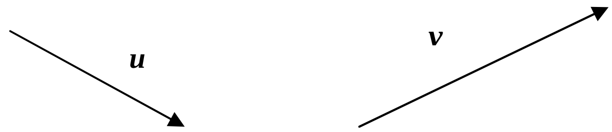
7. Trig-Star problem
Distance AB = 86.48
Distance AD = 139.04
Distance AC = 155.16

Writing activity: How do you determine which technique to use in solving real-life problems involving oblique triangles?
Possible answer: Set up the problem by drawing a diagram and filling in the values that are known then labeling what needs to be found. Determine whether we are given SSS, ASA, AAS, or SAS. Use the appropriate law to solve.

Unit 4, Activity 5, Practice with Vectors

Name________________________________

1. Use vectors u and v shown below. Measure each then draw each of the following:



a) 2***u*** + ***v*** b) ***u*** – ***v*** c) ***u*** + ½ ***v***

2. Work the following:

a) Illustrate the following situation using a ruler and protractor. Label all known parts. A ship sails due west for 300 kilometers then changes its heading to 200^{o} and sails 100 km to point C.

b) How far is the ship from its starting point ,and what is the bearing from the ship to the starting point?

3. For each of the following problems:

i. Find the resultant of two given displacements. Express the answer as a distance and a bearing (clockwise from the north) from the starting point to the ending point.
ii. Tell the bearing from the end point back to the starting point.
iii. Draw the vectors on graph paper, using ruler and protractor. Show that your answers are correct to 0.1 units of length and 1 degree of angle. Show your work for i and ii below.

a) 9 units north followed by 6 units along a bearing of 75 degrees

b) 8 units east followed by 10 units along a bearing of 170 degrees

c) 6 units south followed by 12 units along a bearing of 300 degrees

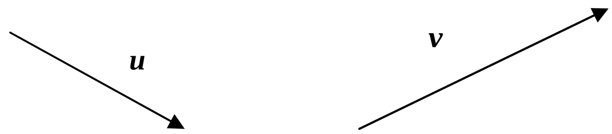
4. An object moves 90 meters due south (bearing 180 degrees) turns and moves 50 more meters along a bearing of 240 degrees.

a) Find the resultant of these two displacement vectors.
b) What is the bearing from the end point back to the starting point?

Writing activity: How does a vector differ from a line segment?

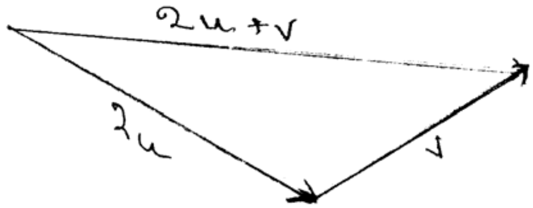
Unit 4, Activity 5, Practice with Vectors with Answers

1. 1. Use vectors u and v shown below. Measure each then draw each of the following:

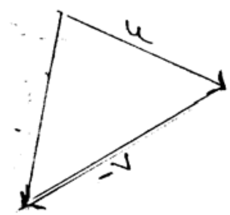


a) 2***u*** + ***v*** b) ***u*** – ***v*** c) ***u*** + ½ ***v***

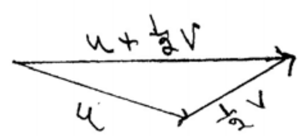
a)



b)

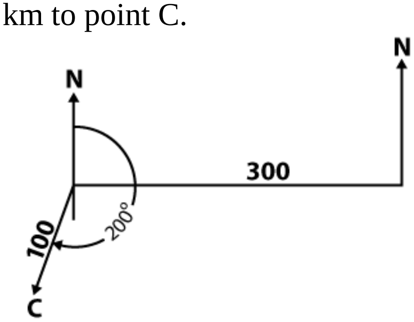


c)



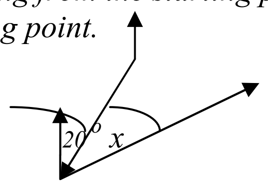
2. Work the following:

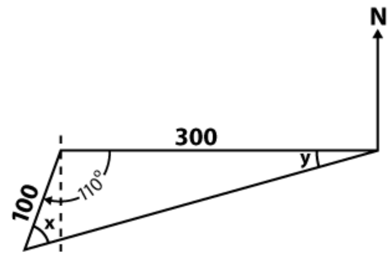
a) A ship sails due west for 300 kilometers then changes its heading to 200^o and sails 100 km to point C.



b) How far is the ship from its starting point, and what is the bearing from the ship to the starting point?

> *Use the law of cosines to find the resultant: 347 km*
>
> *Find $\angle x$ using the law of sines: 54.3^o*
>
> *Add 20^o to 54.3^o giving 74.3^o for the bearing from the starting point to the ending point.*





3. For each of the following problems:

   i. Find the resultant of two given displacements. Express the answer as a distance and a bearing (clockwise from the north) from the starting point to the ending point.
   ii.Tell the bearing from the end point back to the starting point.
   iii. Draw the vectors on graph paper, using ruler and protractor. Show that your answers are correct to 0.1 units of length and 1 degree of angle.

   a) 9 units north followed by 6 units along a bearing of 75 degrees
   resultant 12.0; bearing 245^o

   b) 8 units east followed by 10 units along a bearing of 170 degrees
   resultant 4.2; bearing 346^o

   c) 6 units south followed by 12 units along a bearing of 300 degrees
   resultant 10.4; bearing 90^o

4. An object moves 90 meters due south (bearing 180 degrees) turns and moves 50 more meters along a bearing of 240 degrees.
   a) Find the resultant of these two displacement vectors. *122.9*
   b) What is the bearing from the end point back to the starting point? *200°*

Writing activity: How does a vector differ from a line segment?
A possible answer would be "A vector has both magnitude and direction while a line segment has only magnitude."

Unit 4, Activity 6, Vectors and Navigation

Directions: Your group is going to write a problem, find its solution, and then challenge the other groups in the class to solve your problem. Each of you have been given a sheet to record the problem. Use one of them for your group solution and use the other three to pass out for the challenge.

Student #1: Begin the story by picking the starting and ending cities and using a map or the latitude and longitude for each determine the direction in which the plane will have to fly. This will give you the heading. Choose an airplane from the list found below to determine the air speed of the plane. Choose a wind speed and its bearing. Fill in the information on the story page. Put your vectors on the solutions page labeling the vectors with the airspeed and wind speed.

Student #2: Draw in the vector representing the ground speed of the airplane on the solutions page, find the angle formed by ***u***(the velocity of the plane) and ***w***(the wind velocity), then find the ground speed. Show all of your work on the solutions page.

Student # 3: Find the true course of the airplane and put your work on the solutions page.

Student #4: Use what has been found and draw a vector diagram on the solutions page showing θ, the needed course correction. Determine the heading needed so that the plane will fly on the needed course to reach the correct destination. Show all of your work.

Name	**Range (in miles)**	**Cruising Speed (mph)**
Airbus A310	5200	557
Boeing 737-800	4880	577
Lockheed L1011	5405	615
Beechcraft C99	1048	286
Embraer 170	2100	343
Lockheed Electra	2200	390
Saab-Fairchild 340	920	300

Unit 4, Activity 6, Vectors and Navigation

Challenge problem:

Names: ________________________________

____

____

____

A _________________________ with an air speed of ___________ is flying from
(type of plane) (speed)

________________________ to _____________________. The heading is __________.
(starting city) (ending city)

A _____________ wind is blowing with a bearing of ________________.
(speed of wind)

a) Find the ground speed and the true course of the plane.
b) What heading should the pilot use so that the true course will be the correct heading?

Draw the vector diagrams and show all of the needed work. Circle the answers.

Unit 4, Activity 7, Algebraic Representation of Vectors

1. Plot points A and B. Give the component form of $\vec{AB}$ and find $\left|\vec{AB}\right|$. Give the answer as an exact value.

a. A(2, -4) and B(1, 5)   b. A(-3, -3) and B(4, 3)

2. Use the vectors, $\boldsymbol{v}$ = (2, 7) and $\boldsymbol{w}$ = (-3, 5), to perform the given operations:
a. $-\boldsymbol{v}$ b. $2\boldsymbol{v} + 4\boldsymbol{w}$ c. $\boldsymbol{w} - \boldsymbol{v}$

3. Vector $\vec{v}$ has a magnitude of 4 and direction 150^o from the horizontal. Resolve $\vec{v}$ into horizontal and vertical components.

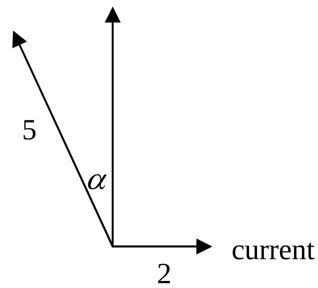
4. If $\boldsymbol{v} = (3\cos 40^o, 3\sin 40^o)$ find $2\boldsymbol{v}$ and $-\boldsymbol{v}$ in component form.

5. Vector $\boldsymbol{v}$ is 5 at 60^o and vector w is 4 at 100^o. Find the resultant r, as
a. the sum of two components
b. a magnitude and direction.

6. A baseball is thrown with an initial velocity of 75 feet per second at an angle of 40^o. Find the horizontal and vertical components of velocity.

7. A force $\boldsymbol{f}$ of 10 newtons acts at an angle of 130^o with the positive x-axis. Find $\boldsymbol{f}$ in component form.

8. A boy can paddle a canoe at 5 mph. Suppose he wants to cross a river whose current is moving at 2 mph. At what angle α to the perpendicular from one bank to the other should he direct his canoe?



Writing exercise: Below are four very common applications of vectors and their peculiar method of indicating direction. Describe each vector quantity and sketch a picture.

1. A force $\boldsymbol{f}$ of 20 lb at 60^{o}
2. A velocity $\boldsymbol{v}$ of 500 mph at 100^{o}
3. Wind velocity $\boldsymbol{w}$ of 10 knots from 200^{o}
4. A displacement $\boldsymbol{OA}$ from O to A of 25 km N30^{o}W

Unit 4, Activity 7, Algebraic Representation of Vectors with Answers

1. Plot points A and B. Give the component form of $\vec{AB}$ and find $\left|\vec{AB}\right|$. Give the answer as an exact value.

a. A(2, -4) and B(1, 5)   b. A(-3, -3) and B(4, 3)

a. $\vec{AB} = (-1, 9)$ $\left|\vec{AB}\right| = \sqrt{82}$ *b.* $\vec{AB} = (7, 6)$, $|\vec{AB}| = \sqrt{85}$

2. Use the vectors $\boldsymbol{v} = (2, 7)$ and $\boldsymbol{w} = (-3, 5)$ to perform the given operations:

a. $-\boldsymbol{v}$ b. $2\boldsymbol{v} + 4\boldsymbol{w}$ c. $\boldsymbol{w} - \boldsymbol{v}$

a) (-2, -7)
b) (4, 14) + (-12, 20) = (-8, 34)
c) (2, 7) – (-3, 5) = (5, 2)

3. Vector $\vec{v}$ has a magnitude of 4 and direction 150^o from the horizontal. Resolve $\vec{v}$ into horizontal and vertical components.

(4cos150, 4sin150) = (3.5, 2)

4. If $\boldsymbol{v} = (3\cos 40^o, 3\sin 40^o)$ find $2\boldsymbol{v}$ and $-\boldsymbol{v}$ in component form.

$2\boldsymbol{v} = (6\cos 40^o, 6\sin 40^o)$ *and* $-\boldsymbol{v} = (-3\cos 40^o, -3\sin 40^o)$
$= (4.6, 3.9)$ $= (-2.3, -1.9)$

5. Vector $\boldsymbol{v}$ is 5 at 60^o and vector w is 4 at 100^o. Find the resultant r, as

a. The sum of two components
b. A magnitude and direction.

a) $(5\cos 60^o)\vec{i} + (5\sin 60^o)\vec{j} + (4\cos 100^o)\vec{i} + (4\sin 100^o)\vec{j}$
$= (2.5 + -.6945\ldots)\vec{i} + (4.330 + 3.939\ldots)\vec{j}$
$= 1.81\vec{i} + 8.27\vec{j}$

b) *magnitude is* ≈ 8.5 *and direction is* $\tan^{-1}\left(\frac{8.27}{1.81}\right) \approx 78^o$

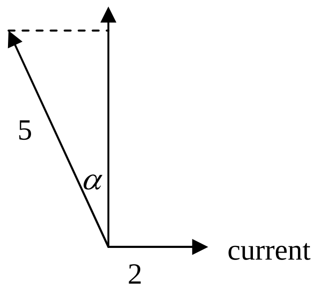
6. A baseball is thrown with an initial velocity of 75 feet per second at an angle of 40^o. Find the horizontal and vertical components of velocity.

horizontal 58 ft/sec, vertical 48 ft./sec

7. A force $\boldsymbol{f}$ of 10 newtons acts at an angle of 130^o with the positive x-axis. Find $\boldsymbol{f}$ in component form.

$10\sin 130^o = 7.66$ *and* $10\cos 130^o = -6.43$ *so the component form is (-6.43, 7.66)*

8. A boy can paddle a canoe at 5 mph. Suppose he wants to cross a river whose current is moving at 2 mph. At what angle α to the perpendicular from one bank to the other should he direct his canoe?



Slide the vector representing the current up so that its initial point is at the terminal point of the vector representing the velocity of the canoe. Then $\alpha = \sin^{-1}\left(\frac{2}{5}\right)$. $\alpha = 24^o$.

Writing activity: Below are four very common applications of vectors and their peculiar method of indicating direction. Describe each vector quantity and sketch a picture.

1. A force ***f*** of 20 lb at 60^o *The angle is measured counterclockwise from the positive x-axis to the vector. The magnitude of the vector is 20.*

2. A velocity ***v*** of 500 mph at 100^o. *The angle, called a heading, bearing, or course is measured clockwise from the north to the velocity vector.*

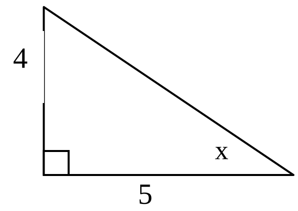
3. Wind velocity ***w*** of 10 knots from 200^o *The angle is measured clockwise from north to the direction from which the wind is blowing.*

4. A displacement ***OA*** from *O* to *A* of 25 km N30^oW *The angle is measured from north (or south) toward west (or east).*

.

For those problems where the answers are the measure of angles or the length of the sides, give the angle measure in degrees, minutes, and seconds, and the length of the sides to the nearest hundredth.

1. Find the measure of angle x in the triangle below.

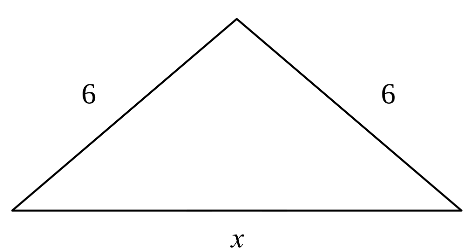


2. In triangle ABC $\angle$ A = 90^o, b = 11, and $\angle$ C = 56^o. Solve the triangle.

3. An isosceles rhombus has sides with lengths 5, 5, 7, and10 cm. Find the angles of the rhombus.

4. Find the measures of the acute angles of a right triangle whose legs are 8 cm. and 15 cm.

5. Find x in the triangle below.



Unit 4, General Assessments, Spiral

Simplify the following trigonometric expressions.

6. $\sec x - \sin x \tan x$
7. $(\csc x - 1)(\csc x + 1)$
8. $\dfrac{\sin x \cos x}{\left(1 - \cos^2 x\right)}$

Prove the given identities:

9. $\dfrac{1 - \sin^2 x}{1 + \cot^2 x} = \sin^2 x \cos^2 x$

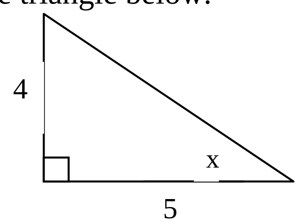
10. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

Unit 4, General Assessments, Spiral with Answers

For those problems where the answers are the measure of angles or the length of the sides give the angle measure in degrees, minutes, and seconds and the length of the sides in the nearest hundredth.

1. Find the measure of angle x in the triangle below.

$x = 38^o39'35''$



2. In triangle ABC $\angle$ A = 90^o, b = 11 and $\angle$ C = 56^o. Solve the triangle.

$a = 19.67$, $\angle B = 34^o$, and $c = 16.31$

3. An isosceles rhombus has sides with lengths 5, 5, 7, and10 cm. Find the angles of the rhombus.

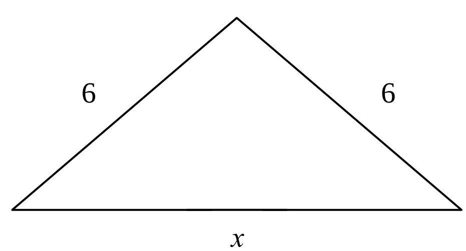
Two angles measure $107^o27'27''$ and the other two measure $72^o32'33''$.

4. Find the measures of the acute angles of a right triangle whose legs are 8 cm. and 15 cm.

$28^o\ 4'\ 21''$ *and* $61^o55'39''$

5. Find x in the triangle below.

$x = 9.19$



Simplify the following trigonometric expressions.

6. $\sec x - \sin x \tan x = \cos x$
7. $(\csc x - 1)(\csc x + 1) = \cot^2 x$
8. $\dfrac{\sin x \cos x}{\left(1 - \cos^2 x\right)} = \cot x$

Prove the given identities:

9. $\dfrac{1-\sin^2 x}{1+\cot^2 x} = \sin^2 x \cos^2 x$

$\dfrac{\cos^2 x}{\csc^2 x}$

$\cos^2 x \sin^2 x = \sin^2 x \cos^2 x$

10. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

$\dfrac{\sin^2 x}{\cos^2 x} - \sin^2 x$

$\dfrac{\sin^2 x - \sin^2 x \cos^2 x}{\cos^2 x}$

$\dfrac{\sin^2 x\left(1-\cos^2 x\right)}{\cos^2 x}$

$\dfrac{\sin^2 x \cdot \sin^2 x}{\cos^2 x}$

$\sin^2 x \cdot \tan^2 x = \tan^2 x \cdot \sin^2 x$

Unit 5, What Do You Know about Trigonometric Functions?

Word or Concept	**+**	**?**	**-**	**What do You know about trigonometric functions?**
angles in trigonometry • **size** • **movement** • **measure**				
radians				
method of converting degree measure to radian measure and vice versa				
coterminal angles				
quadrantal angles				
reference angles				
central angles				
arcs of circles				
linear speed				

Unit 5, What Do You Know about Trigonometric Functions?

angular speed				
unit circle				
periodic functions				
fundamental period				
amplitude				
phase shift				
sinusoidal axis				

Name________________________________

Fill in the following chart:

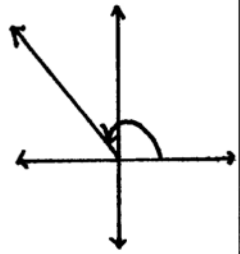
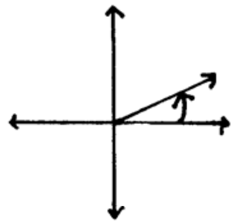
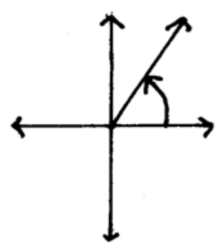
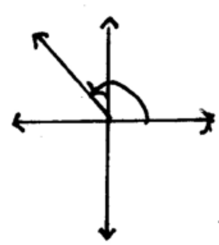
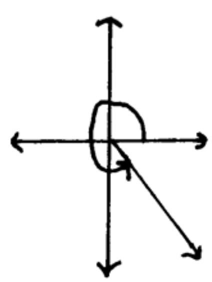
Angle	**Quadrant**	**Sketch in Standard Position**	**Degree Measure**	**A Positive Coterminal Angle**	**A Negative Coterminal Angle**
1. $2\pi/3$					
2. $\pi/6$					
3. $\pi/3$					
4. $3\pi/4$					
5. $7\pi/4$					

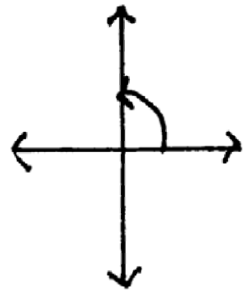
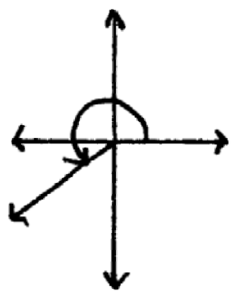
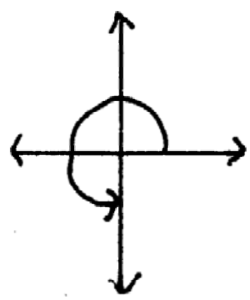
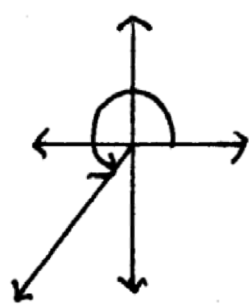
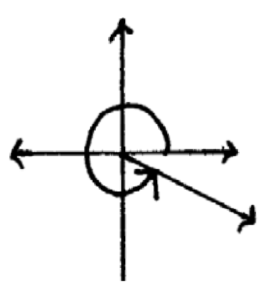
Unit 5, Activity 1, Angles and Their Measure

Angle	Quadrant	Sketch in Standard Position	Degree Measure	A Positive Coterminal Angle	A Negative Coterminal Angle
6. $\pi/2$					
7. $7\pi/6$					
8. $3\pi/2$					
9. $5\pi/4$					
10. $11\pi/6$					

Unit 5, Activity 1, Angles and Their Measure with Answers

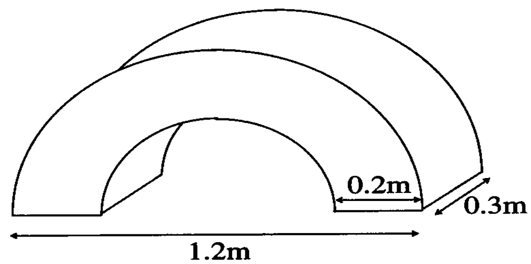
Fill in the following chart:

Angle	Quadrant	Sketch in Standard Position	Degree Measure	Positive Coterminal Angle	Negative Coterminal Angle
1. $\frac{2\pi}{3}$	II		120°	$\frac{8\pi}{3}$ or 480°	$-\frac{4\pi}{3}$ or -240°
2. $\frac{\pi}{6}$	I		30°	$\frac{13\pi}{6}$ or 390°	$-\frac{11\pi}{6}$ or -330°
3. $\frac{\pi}{3}$	I		60°	$\frac{7\pi}{3}$ or 420°	$-\frac{5\pi}{3}$ or -300°
4. $\frac{3\pi}{4}$	II		135°	$\frac{11\pi}{4}$ or 495°	$-\frac{5\pi}{4}$ or -225°
5. $\frac{7\pi}{4}$	IV		315°	$\frac{15\pi}{4}$ or 675°	$-\frac{\pi}{4}$ or -45°

6. $\pi/2$	none		90°	$5\pi/2$ or 450°	$-3\pi/2$ or -270°
7. $7\pi/6$	III		210°	$19\pi/6$ or 570°	$-5\pi/6$ or -150°
8. $3\pi/2$	none		270°	$7\pi/2$ or 630°	$-\pi/2$ or -90°
9. $5\pi/4$	III		225°	$13\pi/4$ or 585°	$-3\pi/4$ or -135°
10. $11\pi/6$	IV		330°	$23\pi/6$ or 690°	$-\pi/6$ or -30°

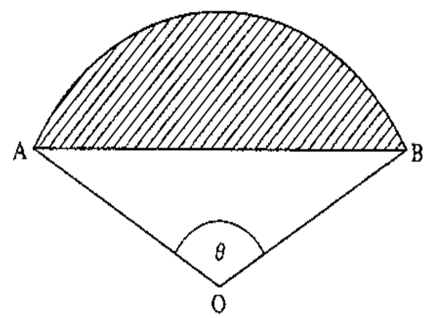
Name_______________________

1. The diagram to the right shows part of a Norman arch. Find the volume of stone in the arch.



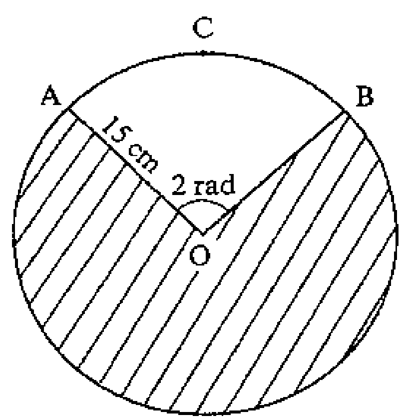
2. The diagram to the right shows a sector AOB of a circle with a 15 cm radius and a center at O. The angle θ at the center of the circle is 115°.

a) Calculate the area of the sector AOB.
b) Calculate the area of the shaded region.

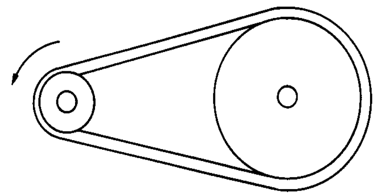


3. The diagram to the right shows a circle with center O and a radius of 15 cm. The arc ACB subtends a central angle of 2 radians.

a. Find the length of the arc ACB.
b. Find the area of the shaded region.



4. The small pulley in the diagram to the right has a radius of 3 cm and is turning at 120 rpm. It is connected by a belt to the larger pulley that has a radius of 8 cm.



a. Find the angular speed of the small pulley in radians per second.

b. Find the linear speed of the rim of the small pulley.

c. How many rpm is the larger pulley turning?

d. Find the angular speed of the larger pulley in radians per second.

e. What is the linear speed of the rim of the large pulley?

Unit 5, Activity 2, Arcs, Sectors, Linear and Angular Speed with Answers

1. Find the volume of stone in the arch.. *≈0.094 m^3*

2. a) Calculate the area of the sector AOB. ≈ *225.8 cm^2*
   b) Calculate the area of the shaded region. ≈ *123.8 cm^2*

3. a) Find the length of the arc ACB. *30 cm*
   b) Find the area of the shaded region ≈ *481.86 cm^2*

4. a) Find the angular speed of the small pulley in radians per second.
   The pulley is turning 120 rpm which would be 2 revolutions per second.
   $2 \cdot 2\pi =$ *4π radians/sec*

   b) Find the linear speed of the rim of the small pulley.
   ≈ 37.70 cm/sec

   c) How many rpm is the larger pulley turning?
   The small pulley makes 8 revolutions to the larger pulleys 3 revolutions so the larger pulley is turning at 45 rpm.

   d) Find the angular speed of the larger pulley in radians per second.
   The angular velocity is 1.5π radians/sec

   e) What is the linear speed of the rim of the large pulley?
   37.70 cm/sec

Unit 5, Activity 3, A Completed Unit Circle

The unit circle is a circle with a radius of 1 unit.

You have been given a large circle superimposed on a coordinate system. The radius is one unit.

1. Label the coordinates (1, 0), (0, 1), (-1, 0) and (0, -1) where the circle intersects the coordinate system.

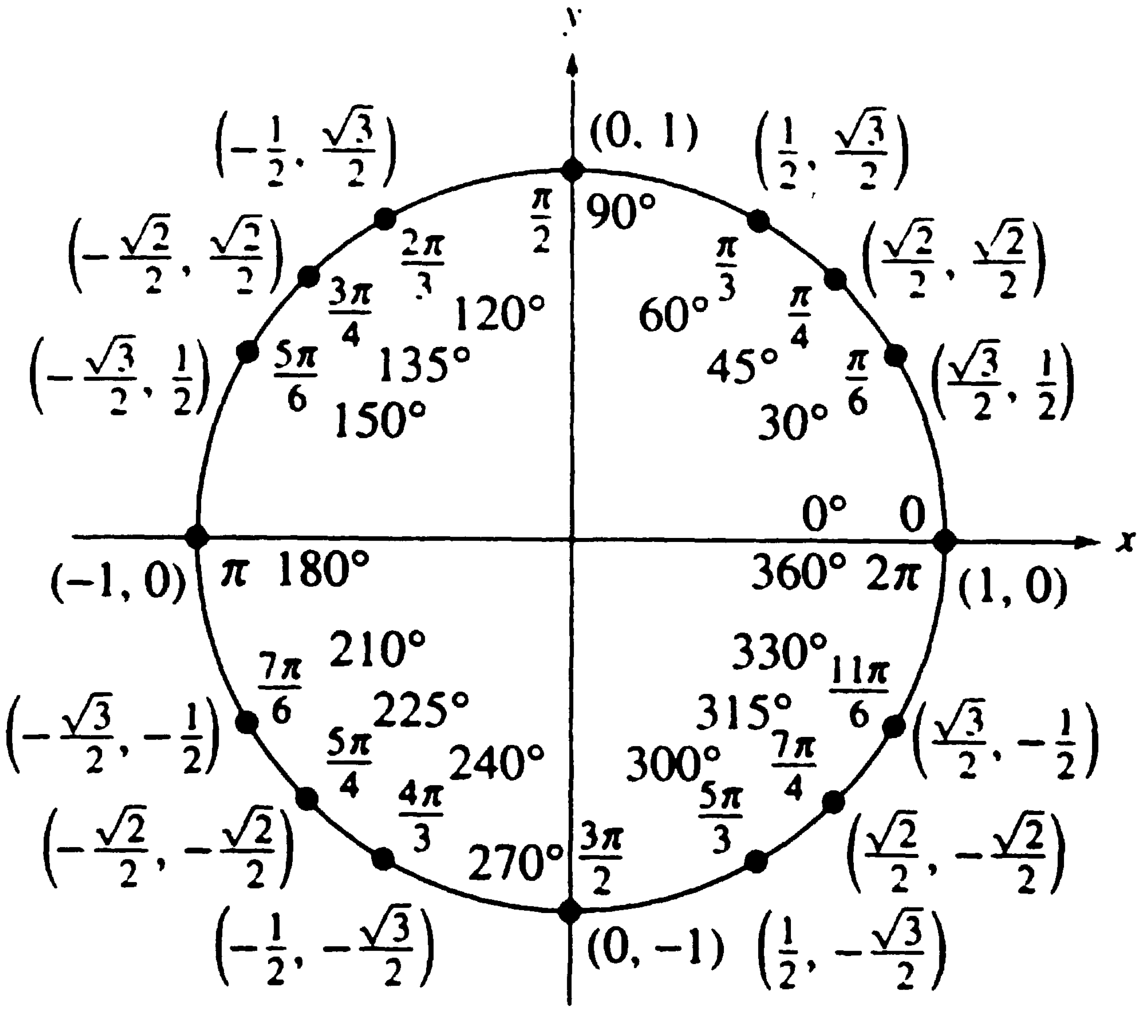
2. Divide the circle into 8 equal arcs. Mark each with the length of the arc, corresponding to values of $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \ldots$ and $0^\circ, 45^\circ, 90^\circ, 120^\circ \ldots$

3. Find the coordinates of each of those endpoints using the 45^o-45^o-90^o right triangle. Exact values should be put on the circle.

4. Divide the circle into 12 equal arcs corresponding to values of $0, \frac{\pi}{6}, \frac{\pi}{3}, \ldots$ and 0^o, 30^o, $60^o \ldots$, 360^o

5. Find the coordinates of each of those endpoints using the 30^o-60^o-90^o right triangle. Exact values should be put on the circle.

Unit 5, Activity 3, A Completed Unit Circle



Unit 5, Activity 5, Computing the Values of Trigonometric Functions of General Angles

Name_______________________________________

1. Fill in the following chart:

Angle θ	**Coterminal Angle 0 - 360°**	**Reference Angle**	**sin** θ	**cos** θ	**tan** θ
-225°					
570°					
-840°					
675°					
-390°					
780°					

2. Fill in the following chart:

Angle θ	**Coterminal 0 - 2π**	**Reference Angle**	**sin θ**	**cos θ**	**tan θ**
$\frac{7\pi}{2}$					
$-\frac{11\pi}{4}$					
$\frac{23\pi}{6}$					
$\frac{14\pi}{3}$					
$-\frac{23\pi}{3}$					
15π					

3. In the problems below, a point on the terminal side of an angle θ is given. Find the exact value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

a) (4, 3) b) (-3, -3) c) (-1, 0)

4. In the problems below, name the quadrant in which the angle θ lies.

a) $\sin \theta < 0$, $\tan \theta > 0$ ___________________

b) $\cos \theta > 0$, $\csc \theta < 0$___________________

c) $\cos \theta < 0$, $\cot \theta < 0$___________________

d) $\sec \theta > 0$, $\sin \theta < 0$___________________

5. Find the exact value of each of the remaining trigonometric functions of θ

a) $\sin\theta = \frac{12}{13}, \theta$ in quadrant II

$\cos\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$

b) $\cos\theta = -\frac{4}{5}, \theta$ in quadrant III

$\sin\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$

Unit 5, Activity 5, Computing the Values of Trigonometric Functions of General Angles with Answers

1. Fill in the following chart:

Angle θ	Coterminal Angle 0 - 360^o	Reference Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
-225^o	135^o	45^o	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
570^o	210^o	30^o	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
-840^o	240^o	60^o	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$
675^o	315^o	45^o	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1
-390^o	330^o	30^o	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
780^o	60^o	60^o	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Unit 5, Activity 5, Computing the Values of Trigonometric Functions of General Angles with Answers

2. Fill in the following chart:

Angle θ	**Coterminal 0 - 2π**	**Reference Angle**	**sin θ**	**cos θ**	**tan θ**
$\frac{7\pi}{2}$	$\frac{3\pi}{2}$	none	-1	0	undefined
$-\frac{11\pi}{4}$	$\frac{5\pi}{4}$	$\frac{\pi}{4}$ $\frac{\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
$\frac{23\pi}{6}$	$\frac{11\pi}{6}$	$\frac{\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
$\frac{14\pi}{3}$	$\frac{2\pi}{3}$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
$-\frac{23\pi}{3}$	$\frac{\pi}{3}$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
15π	π	none	0	-1	0

3. In the problems below, a point on the terminal side of an angle θ is given. Find the exact value of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

a) (4, 3) b) (-3, -3) c) (-1, 0)

Problem	$\sin\theta$	$\cos\theta$	$\tan\theta$
3(a) (4, 3)	$3/5$	$4/5$	$3/4$
3(b) (-3,-3)	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1
3(c) (-1, 0)	0	-1	0

Unit 5, Activity 5, Computing the Values of Trigonometric Functions of General Angles with Answers

4. In the problems below, name the quadrant in which the angle θ lies.

a) sin θ < 0, tan θ > 0 ______third quadrant____

b) cos θ > 0, csc θ < 0______fourth quadrant___

c) cos θ < 0, cot θ < 0______second quadrant___

d) sec θ > 0, sin θ < 0______fourth quadrant___

5. Find the exact value of each of the remaining trigonometric functions of θ

a) $\sin\theta = \frac{12}{13}, \theta$ in quadrant II

$\cos\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
$-\frac{5}{13}$	$-\frac{12}{5}$	$-\frac{13}{5}$	$\frac{13}{12}$	$-\frac{5}{12}$

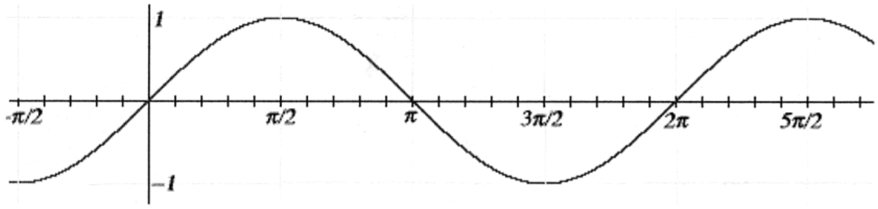
b) $\cos\theta = -\frac{4}{5}, \theta$ in quadrant III

$\sin\theta$	$\tan\theta$	$\sec\theta$	$\csc\theta$	$\cot\theta$
$-\frac{3}{5}$	$\frac{3}{4}$	$-\frac{5}{4}$	$-\frac{5}{3}$	$\frac{4}{3}$

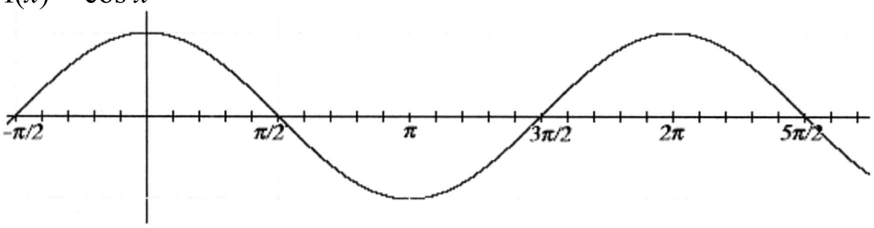
Unit 5, Activity 6, The Family of Functions - Sine and Cosine Part I

Part I: Below are the graphs of the parent functions

f(x) = sin x



f(x) = cos x



Today we are going to study the families of each of these two functions:
f(x) = Asin$(Bx + C) + D$ and f(x) = Acos$(Bx + C) + D$. Begin by answering the questions below:

1. What will the value "A" in $f(x) = A\sin x$ do to change the graph of the sine function? the cosine function? What do you know mathematically that will give credence to your answer?

____

____

____

2. What will the value "B" in $f(x) = \sin(Bx)$ do to change the graph of the sine function? the cosine function? What do you know mathematically that will give credence to your answer?

____

____

____

3. What will the value "C" in $f(x) = \sin(x + C)$ do to change the graph of the sine function? the cosine function? What do you know mathematically that will give credence to your answer?

____

____

____

4. What will the value "D" in $f(x) = \sin x + D$ do to change the graph of the sine function? the cosine function? What do you know mathematically that will give credence to your answer?

____

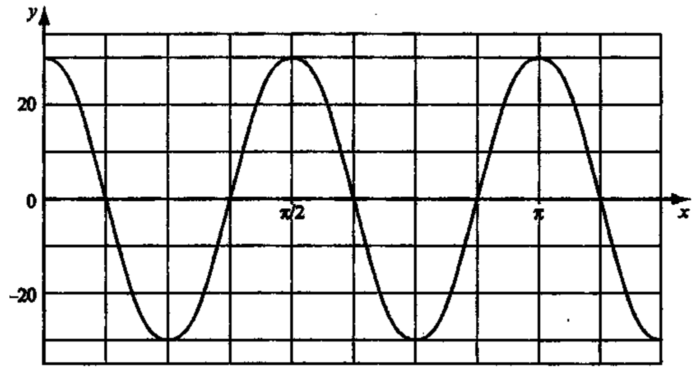
____

____

Unit 5, Activity 6, The Family of Functions - Sine and Cosine Part II

Part II. Use what you have learned about the families of sine and cosine functions to work the following problems.

1. The graph of a function of the form $f(x) = A\cos Bx$ is shown below. Find the values of A and B.



2. Write an equation and sketch the graph of a sine function with amplitude $\frac{1}{2}$, period 3π, phase shift $\frac{\pi}{4}$ units to the right.

3. The fundamental period of a cosine function is $\frac{\pi}{2}$. Its maximum value is 5 and its minimum value is 3. Sketch a graph of the function and write a rule for your graph.

4. Write an equation and sketch the graph of a cosine function if the amplitude is 3, the fundamental period is π and there is a phase shift of $\pi/2$ to the left.

5. As the moon circles the earth, its gravitational force causes tides. The height of the tide can be modeled by a sine or cosine function. Assume there is an interval of 12 hours between successive high tides.
a) Sketch the graph of the height if it is 2.6 meters at low tide and 9.6 meters at high tide.

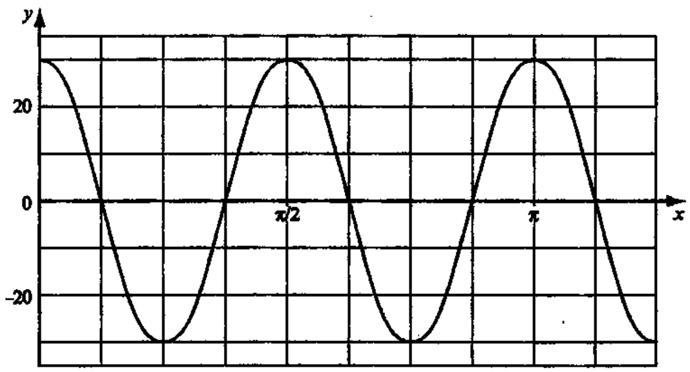
b) Use the graph to help express the height of the tide h meters, as a function of the time t hours after high tide.

Unit 5, Activity 6, The Family of Functions - Sine and Cosine Parts I and II with Answers

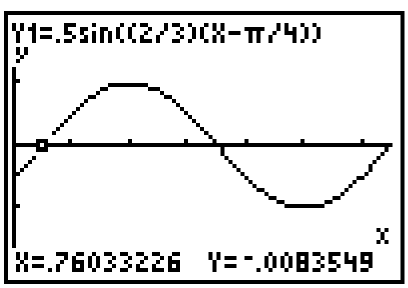
Part I. *Students should recall the function of the values A, B, C, and D from using them in Algebra II and Units 1, 2, and 3 in this course. Mathematically y = Af(x) causes a dilation, a vertical stretch or shrink in the graph of y = f(x). B also causes a dilation. y =f(Bx) stretches or shrinks the graph of y = f(x) horizontally. The graph of y = f(x) is translated C units left if C < 0 and C units right if C > 0.*

Part II

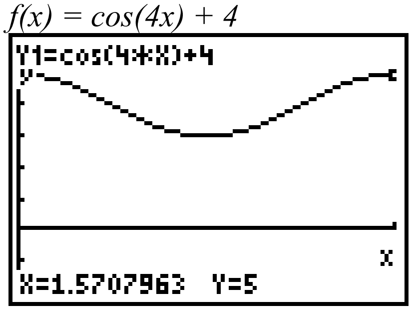
1. The graph of a function of the form $f(x) = A\cos Bx$ is shown below. Find the values of A and B. *A = 30, B = 4*



2. Write an equation and sketch the graph of a sine function with amplitude ½, period 3π, phase shift $\frac{\pi}{4}$ units to the right. *equation is* $f(x) = \frac{1}{2}\sin\frac{2}{3}\left(\theta - \frac{\pi}{4}\right)$

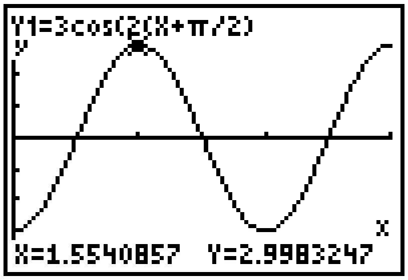


3. The fundamental period of a cosine function is $\frac{\pi}{2}$. Its maximum value is 5 and its minimum value is 3. Sketch a graph of the function and write a rule for your graph.
f(x) = cos(4x) + 4



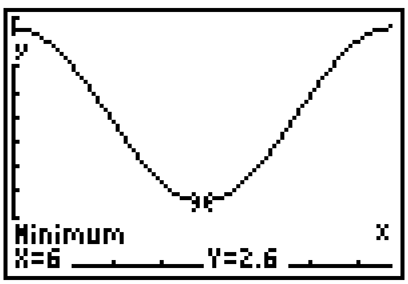
4. Write an equation and sketch the graph of a cosine function if the amplitude is 3, the fundamental period is π and there is a phase shift of $\pi/2$ to the left.

$f(x) = 3\cos 2\left(x + \dfrac{\pi}{2}\right)$ *or* $f(x) = 3\cos(2x + \pi)$



5.As the moon circles the earth, its gravitational force causes tides. The height of the tide can be modeled by a sine or cosine function. Assume there is an interval of 12 hours between successive high tides.

a) Sketch the graph of the height if it is 2.6 meters at low tide and 9.6 meters at high tide.



b) Use the graph to help express the height of the tide *h* meters, as a function of the time *t* hours after high tide. $h(t) = 3.5\cos\left(\dfrac{\pi}{6}t\right)$

Unit 5, Activity 7, Finding Daylight

Instructions

How much daylight do we have each day? If you are reading this during the winter you would say that sunset comes awfully early. However, if it's May where you live ,you will have noticed that it stays light well into the evening.

This activity will have you explore daylight hours for various locations throughout the United States. You will record the hours of daylight for 12 different times during a year (the 21^{st} of each month) and fit these data points to a cosine curve.

Each member of your group will be recording and graphing the daylight hours for a different location. You will compare certain aspects of each graph and make some conjectures about the amount of daylight in different locations throughout the United States. Finally, you will write an equation for your data and test it to see if it will give the correct amount of daylight for particular times of the year.

There are many websites that have the times of sunrise and sunset. Try this site

http://aa.usno.navy.mil/AA/data/docs/RS_OneYear.html#forma

The "Sun or Moon Rise/Set Table for One Year" will appear. Enter the city and state following the site directions. A daylight table will appear for that city. The latitude and longitude will appear in the upper left hand corner.

Each group in the class will be assigned one of the city groups listed below. Each member of the group needs to select one of the city choices in the group they are assigned. No member of the group can select the same city.

Group A	**Group B**	**Group C**	**Group D**
Seattle, Washington	Portland, Oregon	Minneapolis, MN	Boston, Mass.
Denver, Colorado	Philadelphia, PA	Chicago, IL	Richmond, VA
Memphis, Tenn.	Oklahoma City, OK	San Diego, CA	Phoenix, AZ
Miami, FL	New Orleans, LA	Atlanta, GA	San Antonio, TX

The sunrise/sunset times will be listed as 24 hour time (military time). In other words, 3 p.m. will be shown as 15:00. Suppose your time reads sunrise: 5:15 and sunset: 18:45. This means 5:15 a.m. and 6:45 p.m. The easy and quick way to figure the amount of daylight is to type in (18 + 45/60) – (5 + 15/60). The amount of daylight is 13.5 hours.

Name___________________________

Date____________________________

Complete the table below with the information from your city.

Your City ____

Latitude__________________ **Longitude**___________________

Day #	Hours of Daylight	Day #	Hours of Daylight
21		202	
52		233	
80		264	
111		294	
141		325	
172		355	

1. Enter the data into two lists, L_1 and L_2.

2. Check to make sure that you do not have any functions turned on in the Y= editor.

3. To plot the points, turn on the Statplots for the two lists. Make sure that you have chosen scatterplot and that L_1 and L_2 are your Xlist and Ylist.

4. Choose ZoomStat to view the graph. TRACE along your plot and make sure that the points have been entered correctly.

5. What is the maximum daylight time for your city?________________

6. Which day number is the longest day?_______________

   What month and day is this? __________________

7. What is the minimum daylight time for your city?________________

8. What day number is the shortest day?_______________

   What month and day is this?____________________

9. Compare your information with the rest of the group. Which city in your group has the

   longest day?______________________, the shortest day?______________________

10. Explain the relationship between the location's latitude and the amount of daylight hours it has?

11. Obtain an equation for your scatterplot. Choose STAT, CALC, and C:SinReg. Press ENTER twice. Write the equation you obtain below.

____

12. Transfer the equation into Y= and press graph. What do you see?

____

____

13. Using graph paper hand sketch a scatter plot of your data. The horizontal axis represents the number of days, the vertical axis the amount of hours.

14. Find the amplitude___________________, the period__________________,

    the phase shift____________________, and the sinusoidal axis _______________.

.

15. How do these values compare to values of the other cities in your group? What is different and what is the same?

____

____

16. Write a sinusoidal equation that models your data.

____

17. Enter your equation into the calculator, then graph it with your data points and calculator equation. How does it compare?

____

____

18. Use the rule you obtained from the calculator as well as your rule to predict the amount of daylight on May 2, 2007. Use the website to check the answers. How do they compare?

Unit 5, Activity 8, Library of Functions - The Sine Function and The Cosine Function

1. You are to create two entries for your Library of Functions. Introduce the parent functions $f(x) = \sin x$ and $f(x) = \cos x$. Include a table and graph of each of the functions. Give a general description of each function written in paragraph form. Your description should include
   (a) the domain and range
   (b) local and global characteristics of the function – look at your glossary list and choose the words that best describe this function.
   Include a paragraph that compares and contrasts the two functions.

2. Give some examples of family members using translation, reflection, and dilation in the coordinate plane. Show these examples symbolically, graphically, and numerically. Explain what has been done to the parent function to find each of the examples you give.

3. What are the common characteristics of two functions? How do they differ?

4. Find a real-life example of how these function families can be used. Can either function be used to describe the real-life example you have chosen? Would one be better than the other? Why? Be sure to show a representative equation, table, and graph. Does the domain and range differ from that of the parent function? If so, why? Describe what the maximum, minimum, y-intercept, and zero or zeroes mean within the context of your example.

5. Be sure that
   - ✓ your paragraph contains complete sentences with proper grammar and punctuation
   - ✓ your graphs are properly labeled, scaled and drawn
   - ✓ you have used the correct math symbols and language in describing these functions

Unit 5, General Assessment, Spiral

1. Simplify: $\dfrac{2x^2+5x-3}{x+3}$

2. Find the inverse of $f(x) = x^2 - 1$. Is the inverse a function? Why or why not?

3. Given:

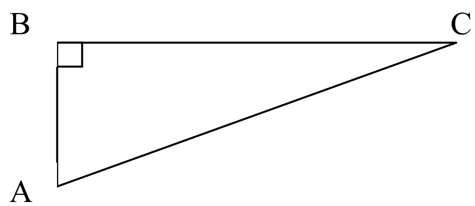
$$kx + 2y = 6$$
$$8x + ky = 8$$

Find the value of k that makes the lines parallel.

4. $x^2 - kx - 15 = 0$ One root of this quadratic is 5. Find the value of k.

5. If one leg of a right triangle is 4 and the other is 6, find the hypotenuse.

6. In the triangle below find BC in simplest form.



$\angle$ C = 30°
AB = 10
Find BC

7. Given the parent function $f(x) = x^2$. Describe the transformations that have taken place in the related graph of $y = -2(x + 1)^2$.

8. Given the parent function $f(x) = \dfrac{1}{x^2}$. Describe the transformations that have taken place with $y = \dfrac{1}{(x+1)^2} - 2$.

9. In triangle ABC $\angle A$ and $\angle B$ are acute angles. $\cos A = 5/8$. Find $\sin A$.

10. Find the area of triangle ABC if $\angle A = 16^\circ$, $\angle B = 31^\circ 45'$ and $c = 3.2$.

1. Simplify: $\dfrac{2x^2+5x-3}{x+3}$

   2x - 1

2. Find the inverse of $f(x) = x^2 - 1$. Is the inverse a function? Why or why not?
The inverse is $y = \pm\sqrt{x+1}$. *It is not a function because y is matched to two values.*

3. Given:

   $kx + 2y = 6$
   $8x + ky = 8$

   Find the value of k that makes the lines parallel.

   k = 4

4. $x^2 - kx - 15 = 0$ One root of this quadratic is 5. Find the value of k.

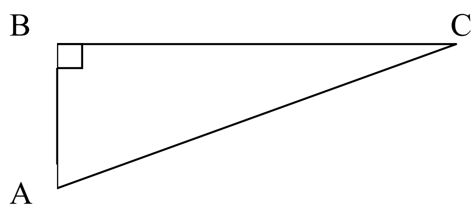
   k = 2

5. If one leg of a right triangle is 4 and the other is 6, find the hypotenuse.

   $2\sqrt{13}$

6. In the triangle below find BC in simplest form.

   $BC = 10\sqrt{3}$



$\angle C = 30o$
$AB = 10$
Find BC

7. Given the parent function $f(x) = x^2$. Describe the transformations that have taken place in the related graph of $y = -2(x + 1)^2$. *The graph is reflected over the x-axis, moved one unit to the left and has a vertical stretch by a factor of 2.*

8. Given the parent function $f(x) = \dfrac{1}{x^2}$. Describe the transformations that have taken place with $y = \dfrac{1}{(x+1)^2} - 2$. *The graph has shifted horizontally one unit to the left and vertically down 2 units.*

9. In triangle ABC $\angle A$ and $\angle B$ are acute angles. $\cos A = \frac{5}{8}$. Find $\sin A$.

$\sin^2 A + \left(\frac{5}{8}\right)^2 = 1,\ \sin A = \frac{\sqrt{39}}{8}$

10. Find the area of triangle ABC if $\angle A = 16^o$, $\angle B = 31^o45'$ and $c = 3.2$.
area of triangle is 1.0

Unit 6, What Do You Know About these Topics in Trigonometry?

Word or Concept	**+**	**?**	**-**	**What do you know about these topics in trigonometry ?**
inverse sine function				
sin^{-1}*x*				
inverse cosine function				
cos^{-1}*x*				
inverse tangent function				
tan^{-1}*x*				
co-functions				
even/odd identities				
sum and difference identities				
double angle identities				

Unit 6, What Do You Know About these Topics in Trigonometry?

polar coordinate system				
pole				
polar axis				
polar coordinates				

Unit 6, Activity 1, Graphs of the Four Remaining Functions

Use graph paper to sketch the graphs of the functions below over two periods. Be sure the axes are scaled properly and carefully labeled. For each graph find

1) the period
2) the phase shift
3) the asymptotes
4) location and value of the maximum points if any
5) location and value of the minimum points if any
6) any x- and y-intercepts? If so, where?
7) verify your sketch with a graphing calculator

1. $y = \csc 2x$

2. $y = \tan\left(x + \frac{\pi}{2}\right)$

3. $y = 2\sec x - 1$

4. $y = -\cot\left(x - \frac{\pi}{4}\right)$

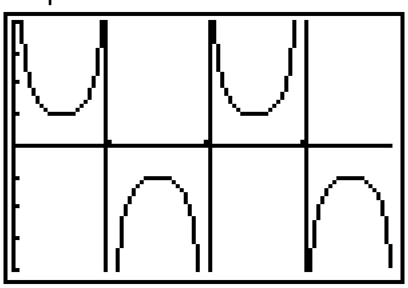
5. $y = \frac{1}{2}\csc 3(x-1)$

Unit 6, Activity 1, Graphs of the Four Remaining Functions with Answers

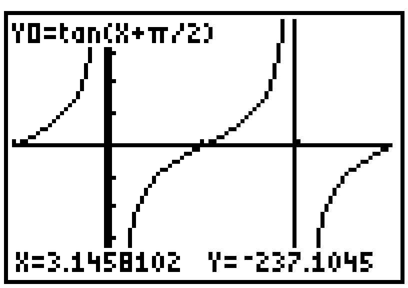
Use graph paper to sketch the graphs of the functions below over two periods. Be sure the axes are scaled properly and carefully labeled. For each graph find

1. the period
2. the phase shift
3. the asymptotes
4. location and value of the maximum points if any
5. location and value of the minimum points if any
6. any x- and y-intercepts? If so, where?
7. verify your sketch with a graphing calculator

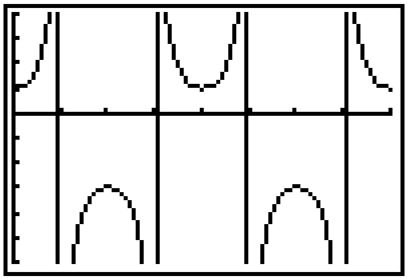
1. $y = \csc 2x$ *The period is π. There is no phase shift. The vertical asymptotes are x = 0,* $x = \pi/2$ *in the first period, and x = π and* $x = \frac{3\pi}{2}$ *in the second period. The minimums are located at* $x = \frac{\pi}{4}$, $x = \frac{5\pi}{4}$. *The value is 1. The maximums are located at* $x = \frac{3\pi}{4}$, $\frac{7\pi}{4}$. *The value -1.*

Graph: 

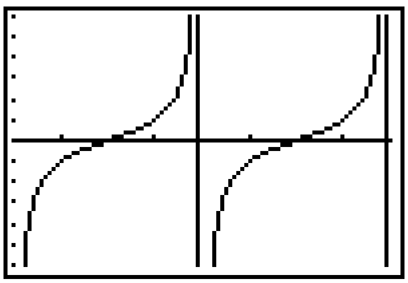
2. $y = \tan\left(x + \frac{\pi}{2}\right)$ *The period is π. The phase shift is* $\pi/2$ *to the left. The asymptotes are found at x = 0 and x = π. There are x-intercepts at* $-\frac{\pi}{2}$, $\frac{\pi}{2}$.

The graph: 

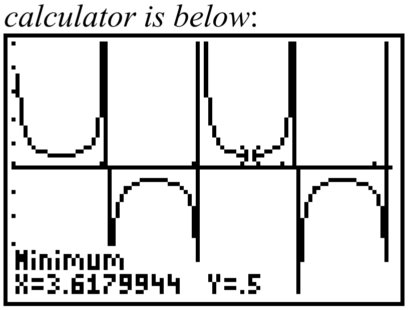
3. y = 2secx – 1 The period is 2π. There is no phase shift. The graph has a vertical translation of -1. This means that y = -1 is the sinusoidal axis. Minimum points are located at 0 and 2π with a value of 1. The maximum points are located at π and 3π with a value of -3. Asymptotes are located at $x = \frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{5\pi}{2}$, $\frac{7\pi}{2}$.

The graph: 

4. $y = -\cot\left(x - \frac{\pi}{4}\right)$ *The graph is reflected over the x-axis. The period is π. The phase shift is* $\pi/4$ *units to the right. Asymptotes are located at* $x = \frac{3\pi}{4}, \frac{7\pi}{4}$

The graph: 

5. $y = \frac{1}{2}\csc 3(x-1)$ *The period is* $\frac{2\pi}{3}$. *The phase shift is 1 unit to the right. The asymptotes are found at: x = 1, x =* $1+\pi/3$, *and* $x = 1 + 2\pi/3$. *The minimum points are located at* $1+\frac{\pi}{6} = 1.52$ and $1+\frac{5\pi}{6} = 3.62$. *The value is 1/2. The maximum points are located at* $1+\frac{\pi}{2} = 2.57$ and $1+\frac{7\pi}{6} = 4.67$. *The value is -1/2. The graph as shown on the calculator is below*:



Unit 6, Activity 2, Working with Inverse Trigonometric Functions

Name________________________________

I. Evaluate. Give the exact value in radians.

1. $\text{Cos}^{-1}(0)$ ____________________

2. $\text{Tan}^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ ______________________

3. $\text{Sin}^{-1}(-1)$ _______________________

4. $\text{Tan}^{-1}(1)$ _______________________

5. $\text{Sin}^{-1}\left(\frac{1}{2}\right)$ _______________________

6. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ _________________________

Part II. Give the value to the nearest degree.

1. $\text{Sin}^{-1}(.3574)$ ________________________

2. $\text{Cos}^{-1}(-.7321)$ ______________________

3. $\text{Csc}^{-1}(1.5163)$ _______________________

4. $\text{Tan}^{-1}(4.8621)$ ________________________

5. $\text{Sec}^{-1}(-3.462)$ ________________________

6. $\text{Cot}^{-1}(-.2111)$ ________________________

Part III. Find the exact values:

1. $\cos\left(Sin^{-1}\left(-\tfrac{5}{13}\right)\right)$ __________

2. $\sin\left(Sin^{-1}(1)\right)$ _______________

3. $\sin\left(Tan^{-1}(2)\right)$ ______________

4. $\cos\left(Sin^{-1}\left(\tfrac{1}{4}\right)\right)$ _____________

Write each of the following as an algebraic expression in x. Use your graphing calculator to graph the problem and your answer, in the same window, to verify the answer is correct.

5. $\cos\left(Tan^{-1}(2x)\right)$ _______________

6. $\csc\left(Cos^{-1}(x)\right)$ ______________

Unit 6, Activity 2, Working with Inverse Trigonometric Functions with Answers

I. Evaluate. Give the exact value in radians.

1. $\text{Cos}^{-1}(0) = \pi/2$
2. $\text{Tan}^{-1}\left(-\dfrac{\sqrt{3}}{3}\right) = -\pi/6$
3. $\text{Sin}^{-1}(-1) = -\pi/2$
4. $\text{Tan}^{-1}(1) = \pi/4$
5. $\text{Sin}^{-1}\left(1/2\right) = \pi/6$
6. $\text{Cos}^{-1}\left(-\sqrt{3}/2\right) = 5\pi/6$

Part II. Give the value to the nearest degree.

1. $\text{Sin}^{-1}(.3574) = 21^o$
2. $\text{Cos}^{-1}(-.7321) = 137^o$
3. $\text{Csc}^{-1}(1.5163) = 41^o$
4. $\text{Tan}^{-1}(4.8621) = 78^o$
5. $\text{Sec}^{-1}(-3.462) = 107^o$
6. $\text{Cot}^{-1}(-.2111) = -78^o$

Part III. Find the exact values:

1. $\cos\left(Sin^{-1}\left(-5/13\right)\right) = -12/13$
2. $\sin\left(Sin^{-1}(1)\right) = 1$
3. $\sin\left(Tan^{-1}(2)\right) = \dfrac{2\sqrt{5}}{5}$
4. $\cos\left(Sin^{-1}\left(1/4\right)\right) = \dfrac{\sqrt{15}}{4}$
5. $\cos\left(Tan^{-1}(2x)\right) = \dfrac{1}{\sqrt{4x^2+1}}$
6. $\csc\left(Cos^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$

Unit 6, Activity 3, Solving Trigonometric Equations

Solve algebraically the following equations for all values of x over the interval $[0, 360^{o})$. Work must be shown. Use a graphing calculator to verify your answers.

1. $5\sin(2x-3) = 4$

2. $2\cos(4x) + 2 = 1$

3. $3\tan(3x) = 2$

4. $5\sin\left(\frac{1}{2}x\right) = 4$

5. $\cos^2 x + 2\cos x + 1 = 0$

6. $2\sin^2 x = \sin x$

7. Suppose that the height of the tide, *h* meters, at the harbor entrance is modeled by the function

$$h = 2.5\sin 30t^{o} + 5$$

where *t* is the number of hours after midnight.

   a) When is the height of the tide 6 meters?

   b) If a boat can only enter and leave the harbor when the depth of the water exceeds 6 meters, for how long each day is this possible?

.

8. How can you determine, prior to solving a trigonometric equation, how many possible answers you should have?

Unit 6, Activity 3, Solving Trigonometric Equations with Answers

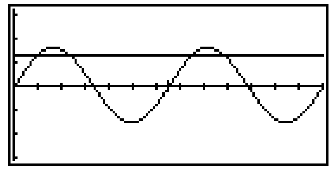
Solve the following equations for all values of *x* over the interval [0, 360^{o}). Verify your answers with a graphing calculator.

1. 5sin(2*x*-3) = 4

   5sin(2x-3) = 4

   sin^{-1}(.8) = 2x - 3

   28^{o}4', 151^{o} 56', 208^{o} 4', 331^{o} 56'

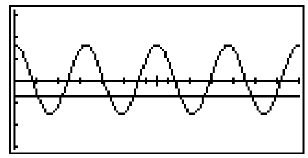


2. 2cos(4*x*) +2 = 1

   cos(4x) = -.5

   cos^{-1}(-.5) = 4x

   x = 30^{o}, 60^{o}, 120^{o}, 150^{o}, 210^{o}, 240^{o}, 300^{o}, 330^{o}



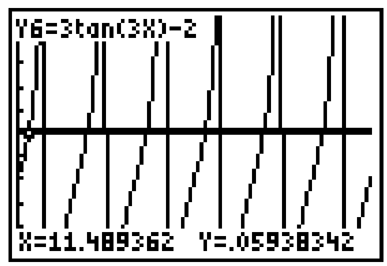
3. 3 tan(3x) = 2

$$\tan(3x) = \frac{2}{3}$$

$$3x = \tan^{-1}\left(\frac{2}{3}\right)$$

$$3x = 33.7^{o}$$

$$x = 11.2^{o}, 71.2^{o}, 131.2^{\circ}, 191.2^{o}, 251.2^{o} 311.2^{o}$$

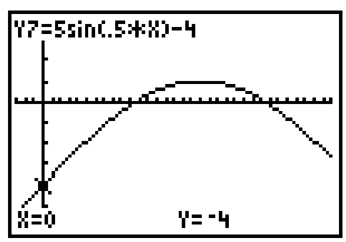


4. $5\sin\left(\frac{1}{2}x\right) = 4$

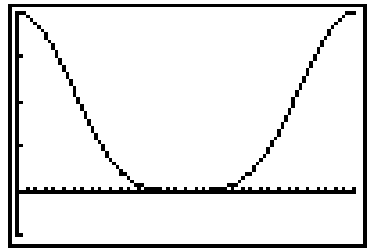
$$\sin\left(\frac{1}{2}x\right) = .8$$

$$\sin^{-1}(.8) = \frac{1}{2}x$$

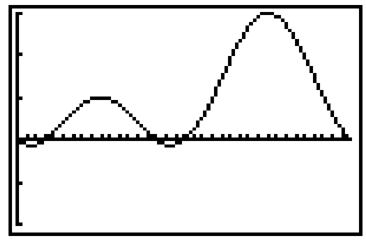
$$x = 105.26^{o}, 253.7^{o}$$



5. $\cos^2 x + 2\cos x + 1 = 0$
$(\cos x + 1)(2\cos x + 1) = 0$
$\cos x = -1$
$x = 180^{\circ}$



6. *$2\sin^2 x = \sin x$*
$2\sin^2 x - \sin x = 0$
$\sin x(2\sin x - 1) = 0$
$\sin x = 0$ or $\sin x = ½$
$x = 0, 180^{o}, 30^{o}, 150^{o}$

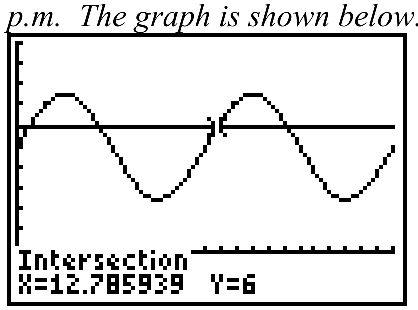


7. Suppose that the height of the tide, *h* meters, at the harbor entrance is modeled by the function

$$h = 2.5\sin 30t^{o} + 5$$

where *t* is the number of hours after midnight.

a) When is the height of the tide 6 meters?
If $h = 6$, $6 = 2.5\sin 30t + 5$
$\sin 30t = 0.4$
$30t = 23.58$ or 156.42
$t = 0.786$ hours or 5.214 hours
The height is 6 meters at 12:47 a.m. and 7:13 a.m. Since the period is 12 (12 hours) there is also 12 + 0.786 and 12 + 5.214 in the 24 hour period. This gives 12:47 p.m. and 5:13 p.m. The graph is shown below. The line represents the 6 meter tide.



b) If a boat can only enter and leave the harbor when the depth of the water exceeds 6 meters, for how long each day is this possible? *Twice a day for 4 hours and 26 minutes.*

8. How can you determine, prior to solving a trigonometric equation, how many possible answers you should have? *Determine how many periods are in the interval in which the solutions lie and how many solutions are found within one period. This will give the total number of solutions.*

Unit 6, Activity 4, Using the Fundamental Identities to Solve Trigonometric Equations

Name________________________________

Solve the following equations for all values of *x* over the interval $[0, 360^o)$. Show the work you did to obtain the answer. Verify your answers with a graphing calculator.

1. $2\cos^2 x + 7\sin x = 5$

2. $2\sin x = \sin x \tan x$

3. $3\sin x = \cos x$

4. $2\csc x = \sec x$

5. $\cos^2 x + \sin x + 1 = 0$

6. csc2 x – 2cot x -1 = 0

7. 2sin x – csc x = 0

8. sin x(tan^2 x – 1) = 0

Unit 6, Activity 4, Using the Fundamental Identities to Solve Trigonometric Equations with Answers

Solve the following equations for all values of x over the interval [0, 360°). Show the work you did to obtain the answer. Verify your answers with a graphing calculator.

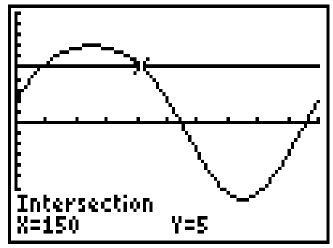
1. $2\cos^2 x + 7\sin x = 5$

   $2(1 - \sin^2 x) + 7\sin x - 5 = 0$

   $2\sin^2 x - 7\sin x + 3 = 0$

   $x = 30^o, 150^o$

Y1: $2\cos^2 x + 7\sin x$
Y2 : 5



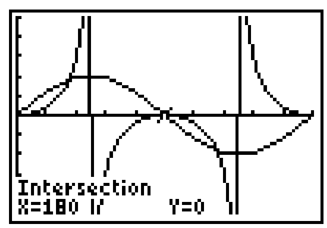
2. $2\sin x = \sin x \tan x$

$\sin x(2 - \tan x) = 0$

$\sin x = 0$ or $2 - \tan x = 0$

$x = 0^o, 180^o, 63.4^o, 243.4^o$

Y1: 2sin x   Y2: sin x tan x



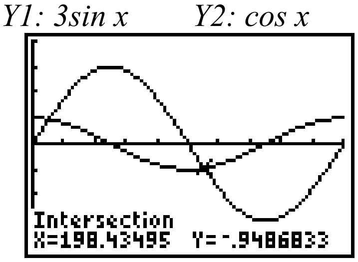
3. $3\sin x = \cos x$

$$\frac{1}{3} = \frac{\sin x}{\cos x}$$

$$\tan^{-1}\left(\frac{1}{3}\right)$$

$x = 18.4^o, 198.4^o$

Y1: 3sin x   Y2: cos x



4. $2\csc x = \sec x$

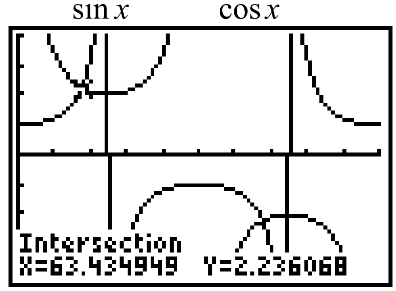
$$\frac{2}{\sin x} = \frac{1}{\cos x}$$

$$2 = \frac{\sin x}{\cos x}$$

$\tan^{-1}(2) = x$

$x = 63.43^o, 243.4^o$

Y1: $\dfrac{2}{\sin x}$ Y2: $\dfrac{1}{\cos x}$



Unit 6, Activity 4, Using the Fundamental Identities to Solve Trigonometric Equations with Answers

5. $\cos^2 x + \sin x + 1 = 0$

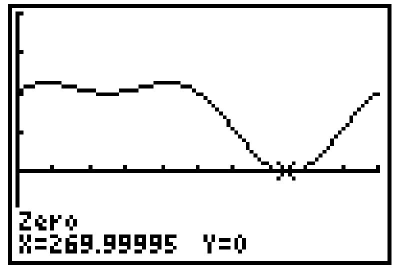
   $1 - \sin^2 x + \sin x + 1 = 0$

   $\sin^2 x - \sin x - 2 = 0$

   $(\sin x - 2)(\sin x + 1) = 0$

   $x = 270^o$

Y1: $\cos^2 x + \sin x + 1 = 0$



6. csc2 x – 2cot x -1 = 0

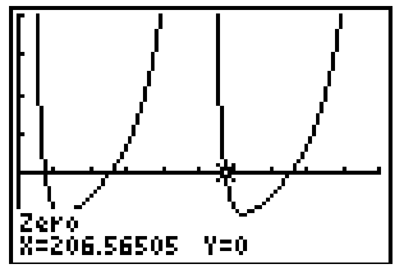
   $1 + \cot^2 x - 2\cot x - 1 = 0$

   $\cot x(\cot x - 2) = 0$

   cot x = 0 or cot x = 2

   x = 90°, 270 °, 26.6 °, 206.6 °

Y1: $\dfrac{1}{\sin^2 x} - 2\dfrac{\cos x}{\sin x} - 1$



7. $2\sin x - \csc x = 0$

   $2\sin x - \dfrac{1}{\sin x} = 0$

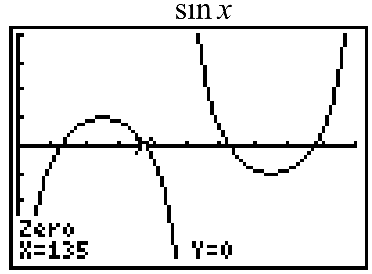
   $2\sin^2 x - 1 = 0$

   $\sin^2 x = ½$

   $\sin x = \pm\dfrac{\sqrt{2}}{2}$

   $x = 45^o, 135^o, 225^o, 315^o$

Y1: $2\sin x - \dfrac{1}{\sin x}$



8. $\sin(\tan^2 x - 1) = 0$

   $\sin x(\sec^2 x - 1 - 1) = 0$

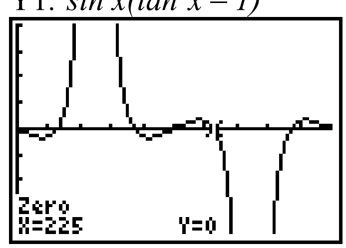
   $\sin x(\sec^2 x - 2) = 0$

   $\sin x = 0$ or $\sec^2 x = 2$

   $\sec x = \pm\sqrt{2}$

   $x = 0^o, 180^o, 45^o, 135^o, 225^o, 315^o$

Y1: $\sin x(\tan^2 x - 1)$



Unit 6, Activity 5, Working with the Properties and Formulas for the Trigonometric Functions

1. Which of the following equations represents the same function? Use the sum and difference formulas, the co-function and the even/odd properties to find your answer.

a) $y = \sin x$ b) $y = \cos x$ c) $y = \sin(-x)$

d) $y = \cos(-x)$ e) $y = -\sin x$ f) $y = -\cos x$

g) $y = \sin\left(x + \frac{\pi}{2}\right)$ h) $y = \sin\left(x - \frac{\pi}{2}\right)$ i) $y = \cos\left(x + \frac{\pi}{2}\right)$

j) $y = \cos\left(x - \frac{\pi}{2}\right)$ k) $y = \cos(x + \pi)$ l) $y = \cos(x - \pi)$

m) $y = \sin(x + \pi)$ n) $y = \sin(x - \pi)$ o) $y = \cos\left(\frac{\pi}{2} - x\right)$

2. Solve each of the following equations for $0 \le x < 2\pi$ using the sum and difference formulas and verifying your answer with a graph. Show the work you did to obtain the answer.

a) $\sin\left(x + \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{6}\right) = 1$

b) $\cos\left(x + \frac{\pi}{3}\right) - \cos\left(x - \frac{\pi}{3}\right) = 1$

c) $2\sin(x + \pi) + \tan(x + \pi) = 0$

d) $\sin 2x \cos 3x - \cos 2x \sin 3x = 1$

e) $\dfrac{\tan 4x - \tan x}{1 + \tan 4x \tan x} = \sqrt{3}$

3. Solve each of the following equations for $0 \le x < 2\pi$ using the double angle formulas.

a) $\sin 2x - \sin x = 0$

b) $\sin(2x)\sin x = \cos x$

c) $\cos(2x) + \sin x = 0$

Unit 6, Activity 5, Working with the Properties and Formulas for the Trigonometric Functions

4. Find the exact values without use of the calculator:

a) $\sin\left(Tan^{-1}1 - Tan^{-1}\frac{4}{3}\right)$

b) $\sin\left(2Sin^{-1}\frac{1}{3}\right)$

c) $\cos\left(Cos^{-1}\frac{1}{4} + Sin^{-1}\left(-\frac{1}{4}\right)\right)$

d) $\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)$

e) $\tan\left(2Tan^{-1}2\right)$

5. Verify the given identities:

a) $\cos\left(2Cos^{-1}x\right) = 2x^2 - 1$

b) $\tan\left(2Tan^{-1}x\right) = \dfrac{2x}{1-x^2}$

Unit 6, Activity 5, Working with the Properties and Formulas for the Trigonometric Functions with Answers

1. Which of the following equations represent the same function? Use the sum and difference formulas, the co-function and the even/odd properties to find your answer.

a) y = sin x	b) y = cos x	c) y = sin (-x)
d) y = cos(-x)	e) y = -sin x	f) y = -cos x
g) $y = \sin\left(x + \frac{\pi}{2}\right)$	h) $y = \sin\left(x - \frac{\pi}{2}\right)$	i) $y = \cos\left(x + \frac{\pi}{2}\right)$
j) $y = \cos\left(x - \frac{\pi}{2}\right)$	k) $y = \cos(x + \pi)$	l) $y = \cos(x - \pi)$
m) $y = \sin(x + \pi)$	n) $y = \sin(x - \pi)$	o) $y = \cos\left(\frac{\pi}{2} - x\right)$

1. a, j , and o represent the same function; b, d, and g represent the same function; c, e, i, m, and n represent the same function, and f, h, k, and l represent the same function.

2. Solve each of the following equations for $0 \le x < 2\pi$, using the sum and difference formulas and verifying your answer with a graph. Give exact answers where possible.

a) $\sin\left(x + \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{6}\right) = 1$

$$\sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} + \sin x \cos\frac{\pi}{6} - \cos x \sin\frac{\pi}{6} = 1$$

$$\sin x\left(\frac{\sqrt{3}}{2}\right) + \cos x\left(\frac{1}{2}\right) + \sin x\left(\frac{\sqrt{3}}{2}\right) - \cos x\left(\frac{1}{2}\right) = 1$$

$$2\sin x\left(\frac{\sqrt{3}}{2}\right) = 1$$

$$\sin x = \frac{1}{\sqrt{3}}$$

$x = 3.76$ or 5.67

b) $\cos\left(x + \frac{\pi}{3}\right) - \cos\left(x - \frac{\pi}{3}\right) = 1$

$$\cos x \cos\frac{\pi}{3} - \sin x \sin\frac{\pi}{3} - \left(\cos x \cos\frac{\pi}{3} + \sin x \sin\frac{\pi}{3}\right) = 1$$

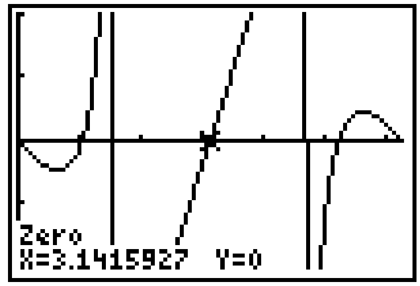
$$\cos x \cdot \frac{1}{2} - \sin x \cdot \frac{\sqrt{3}}{2} - \cos x \cdot \frac{1}{2} - \sin x \cdot \frac{\sqrt{3}}{2} = 1$$

$$-2\sin x = 1\left(\frac{2}{\sqrt{3}}\right)$$

$$\sin x = -\frac{1}{\sqrt{3}}$$

$x = 3.76$ or 5.67

c) $2\sin(x+\pi)+\tan(x+\pi)=0$



$$2(\sin x\cos\pi+\cos x\sin\pi)+\frac{\tan x+\tan\pi}{1-\tan x\tan\pi}=0$$

$$-2\sin x+\frac{\tan x+0}{1-\tan x(0)}=0$$

$$-2\sin x+\tan x=0$$

$$-2\sin x+\frac{\sin x}{\cos x}=0$$

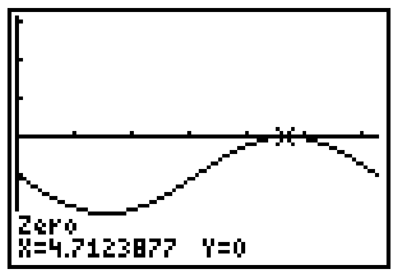
$$\sin x\left(-2+\frac{1}{\cos x}\right)=0$$

$$\sin x(-2+\sec x)=0$$

$$\sin x=0 \text{ or } \sec x=2$$

$$x=0,\ \pi,\ \frac{\pi}{3},\ \frac{5\pi}{3}$$

d) $\sin 2x\cos 3x-\cos 2x\sin 3x=1$



$$\sin(2x-3x)=1$$

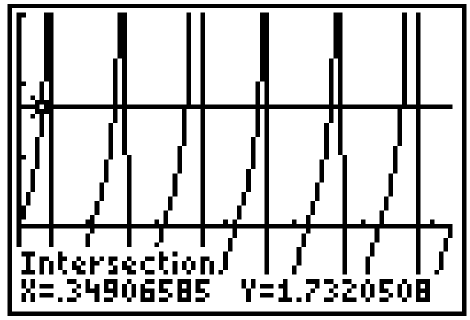
$$\sin(-x)=1$$

$$\sin^{-1}1=-x$$

$$\frac{\pi}{2}=-x$$

$$x=\frac{3\pi}{2}$$

e) $\dfrac{\tan 4x - \tan x}{1 + \tan 4x \tan x} = \sqrt{3}$



$\tan(4x - x) = \sqrt{3}$

$\tan 3x = \sqrt{3}$

$3x = \tan^{-1}\left(\sqrt{3}\right)$

$3x = \dfrac{\pi}{3}$

$x = \dfrac{\pi}{9}$

The period is $\dfrac{\pi}{3}$. There are six periods in the interval. The other five solutions are:
$\dfrac{4\pi}{9}, \dfrac{7\pi}{9}, \dfrac{10\pi}{9}. \dfrac{13\pi}{9}, \dfrac{16\pi}{9}$

3. Solve each of the following equations for $0 \le x < 2\pi$ using the double angle formulas.

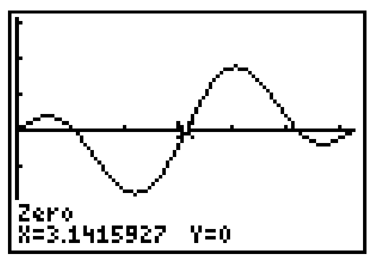
a) $\sin 2x - \sin x = 0$

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

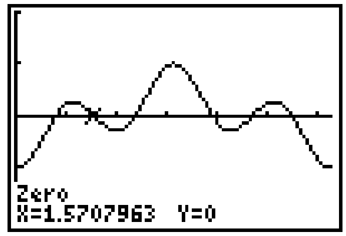
$\sin x = 0 \text{ or } \cos x = \dfrac{1}{2}$

$0,\ \pi,\ \dfrac{\pi}{3},\ \dfrac{5\pi}{3}$



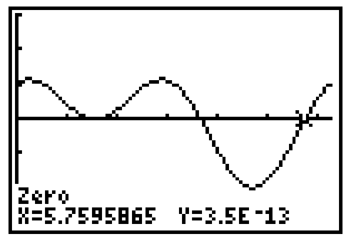
b) $\sin(2x)\sin x = \cos x$

$$2\sin x\cos x(\sin x) = \cos x$$
$$2\sin^2 x\cos x - \cos x = 0$$
$$\cos x\left(2\sin^2 x - 1\right) = 0$$
$$\cos x = 0 \text{ or } \sin^2 x = \frac{1}{2}$$
$$\cos x = 0 \text{ or } \sin x = \pm\frac{\sqrt{2}}{2}$$
$$x = \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{\pi}{4},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{7\pi}{4}$$



c) $\cos(2x) + \sin x = 0$

$$1 - 2\sin^2 x + \sin x = 0$$
$$2\sin^2 x - \sin x - 1 = 0$$
$$(2\sin x + 1)(\sin x - 1) = 0$$
$$\sin x = -\frac{1}{2} \text{ or } \sin x = 1$$
$$x = \frac{7\pi}{6},\ \frac{11\pi}{6},\ \frac{\pi}{2}$$



4. Find the exact values without use of the calculator:

a) $\sin\left(Tan^{-1}1 - Tan^{-1}\frac{4}{3}\right)$

Let $\text{Tan}^{-1}1 = A$ and $\text{Tan}^{-1}\frac{4}{3} = B$

$$\sin\left(A + B\right) = \sin A\cos B + \cos A\sin B$$

Set up right triangles for angles A and B. Tan A = 1 so $\sin A = \frac{1}{\sqrt{2}}$ and $\cos A = \frac{1}{\sqrt{2}}$.

$\tan B = \frac{4}{3}$ so $\sin B = \frac{4}{5}$ and $\cos B = \frac{3}{5}$.

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A + B) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{3}{5}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{4}{5}\right)$$
$$= \frac{3\sqrt{2} - 4\sqrt{2}}{10}$$
$$= -\frac{\sqrt{2}}{10}$$

b) $\sin\left(2Sin^{-1}\dfrac{1}{3}\right)$

Let A = $\sin^{-1}\dfrac{1}{3}$ then sin 2A = 2sin A cos A. Set up a right triangle with side opposite A = 1 and the hypotenuse = 3. Find cos A

$$\sin 2A = 2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$$

$$= \frac{4\sqrt{2}}{9}$$

c) $\cos\left(Cos^{-1}\dfrac{1}{4} + Sin^{-1}\left(-\dfrac{1}{4}\right)\right)$

Follow the same procedure as in 4(a). Use the sum formula
cos(A + B) = cos A cos B-sin A sin B

$$\cos(A+B) = \left(\frac{1}{4}\right)\left(\frac{\sqrt{15}}{4}\right) - \left(\frac{\sqrt{15}}{4}\right)\left(\frac{1}{4}\right)$$

$$= 0$$

d) $\cos\left(2\sin^{-1}\left(\dfrac{1}{2}\right)\right)$

Let $A = \sin^{-1}\dfrac{1}{2}$ and use cos 2A = 1 – 2sin^{2}A

$$\cos 2A = 1 - 2\left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2}$$

e) $\tan\left(2Tan^{-1}2\right)$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2(2)}{1-(2)^2}$$

$$= -\frac{4}{3}$$

5. Verify the given identities:

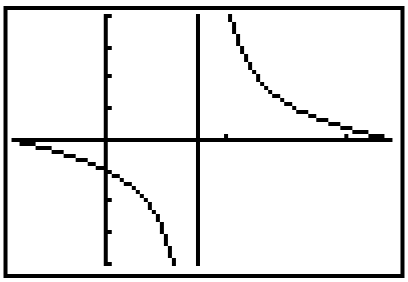
a) $\cos\left(2Cos^{-1}x\right) = 2x^2 - 1$

b) $\tan\left(2Tan^{-1}x\right) = \dfrac{2x}{1-x^2}$

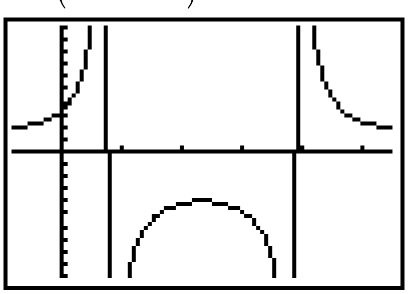
Unit 6, Activity 6, Answers to the problem set for Playing Mr. Professor

Problems for the graphing portion of the game

1. $y = -\tan\left(x + \frac{\pi}{4}\right)$

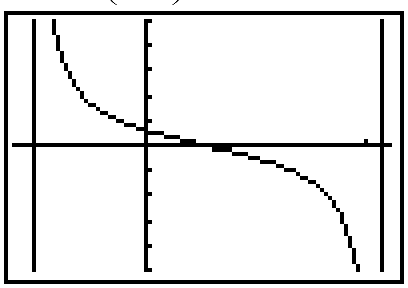
- period is π
- the phase shift is $\frac{\pi}{4}$ units to the left
- asymptote is x = $\frac{\pi}{4}$
- there are no relative maximum or minimum values
- the x-intercept is $\frac{3\pi}{4}$, the y-intercept is (0, -1)
- the graph 

2. $y = 3\sec\left(x + \frac{\pi}{4}\right) - 1$

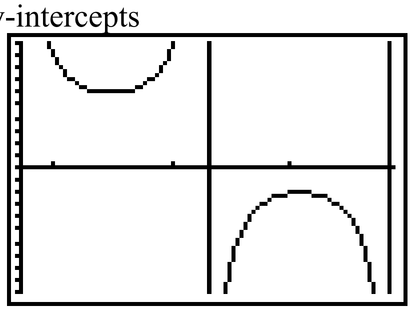
- period is 2π
- phase shift is $\frac{\pi}{4}$ units to the left
- asymptotes are x = $\frac{\pi}{4}$ and x = $\frac{5\pi}{4}$
- relative minimum value is 2 located at x = -$\frac{\pi}{4}$ and x = $\frac{7\pi}{4}$
- relative maximum value is -4 located at x = $\frac{3\pi}{4}$
- y-intercept is $\left(0, 3\sqrt{2} - 1\right)$
- the graph 

Unit 6, Activity 6, Answers to the problem set for Playing Mr. Professor

3. $y = \cot\left(2x + \frac{\pi}{3}\right)$

- period is $\frac{\pi}{2}$
- phase shift is $\frac{\pi}{6}$ units to the left
- asymptotes are x = $-\frac{\pi}{6}$ and x = $\frac{5\pi}{6}$
- there are no relative maximum or minimum values
- the y-intercept is $(0, \frac{\sqrt{3}}{3})$
- the x-intercept is $\left(\frac{\pi}{3}, 0\right)$
- the graph 

4. $y = 4\csc(2x - 3\pi) + 2$

- period is π
- phase shift is $\frac{3\pi}{2}$ units to the right
- asymptotes are x = $\frac{3\pi}{2}$, x = 2π, x = $\frac{5\pi}{2}$
- relative maximum value is -2 located at $\frac{9\pi}{4}$
- relative minimum value is 6 located at $\frac{7\pi}{4}$
- no x- or y-intercepts
- the graph 

Unit 6, Activity 6, Answers to the problem set for Playing Mr. Professor

Problems for solving equations Solve over $0 \le x < 360^o$.

$$1 + \cos x = 4\sin^2 x$$

$$1 + \cos x = 4\left(1 - \cos^2 x\right)$$

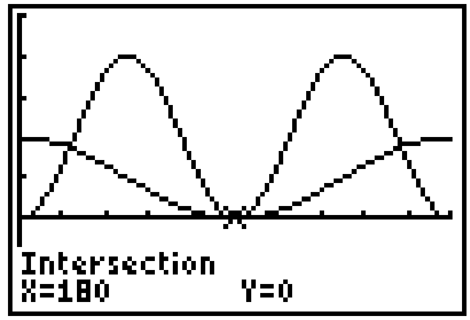
$$4\cos^2 x + \cos x - 3 = 0$$

$$(4\cos x - 3)(\cos x + 1) = 0$$

$$x = \cos^{-1}\left(3/4\right) \text{ or } x = \cos^{-1}(-1)$$

$$41.4^o, 318.6^o, 180^o$$

Y1:1 + cos x and Y2: $4\sin^2 x$



Set window: xmin: 0; xmax:360, ymin:-2, ymax:5
Use CALC5: intersect feature to obtain answers

2. $3\sin x + 2 = \cos 2x$

$$3\sin x + 2 = 1 - 2\sin^2 x$$

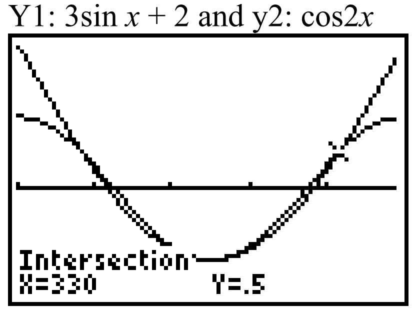
$$2\sin^2 x + 3\sin x + 1 = 0$$

$$(2\sin x + 1)(\sin x + 1) = 0$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = x \text{ or } \sin^{-1}(-1) = x$$

$$x = 210^o, 330^o, 270^o$$

Y1: 3sin x + 2 and y2: cos2x



The points of intersection are hard to see when graphed over 0 to 360. Try using zoom box or reset the window to 180 to 360 to find the three points of intersection.

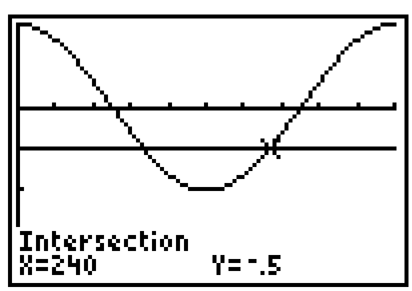
3. $\cos 2x\cos x + \sin 2x\sin x = -\frac{1}{2}$

$$\cos(2x - x) = -\frac{1}{2}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = x$$

$$120^o, 240^o$$

Y1: $\cos 2x\cos x + \sin 2x\sin x$ and Y2: $-\frac{1}{2}$



Set window: xmin: 0; xmax:360, ymin:-2, ymax:5
Use CALC5: intersect feature to obtain answers

Unit 6, Activity 6, Answers to the problem set for Playing Mr. Professor

4. $\dfrac{\tan 3x - \tan x}{1 + \tan 3x \tan x} = \sqrt{3}$

$\tan(3x - x) = \sqrt{3}$

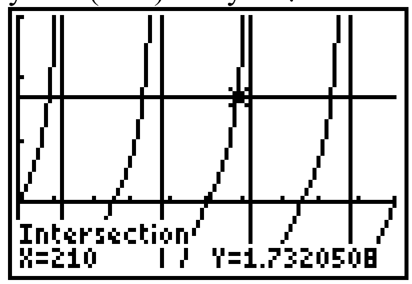
$\tan 2x = \sqrt{3}$

$2x = Tan^{-1}\sqrt{3}$

$x = \dfrac{60^o}{2}$

x = 30^o, 120^o, 210^o, 300^o

y1:tan(3x-x) and y2: $\sqrt{3}$



Set window: xmin: 0; xmax:360, ymin:-2, ymax:5
Use CALC5: intersect feature to obtain answers

<u>Problems for simplifying expressions</u>

1. $\dfrac{\cos x}{1 + \sin x} + \tan x$

$= \dfrac{\cos x}{1 + \sin x} + \dfrac{\sin x}{\cos x}$

$= \dfrac{\cos^2 x + \sin x + \sin^2 x}{(1 + \sin x)\cos x}$

$= \dfrac{1 + \sin x}{(1 + \sin x)\cos x}$

$= \sec x$

2. $\dfrac{\sec x + \csc x}{1 + \tan x}$

$= \dfrac{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}{1 + \dfrac{\sin x}{\cos x}}$

$= \dfrac{\sin x + \cos x}{\cos x \sin x} \div \dfrac{\cos x + \sin x}{\cos x}$

$= \dfrac{\sin x + \cos x}{\cos x \sin x} \cdot \dfrac{\cos x}{\cos x + \sin x}$

$= \csc x$

3. $\sin x(\cos x + \sin x \tan x)$

$= \sin x(\cos x + \sin x \tan x)$

$= \sin x\left(\cos x + \sin x\left(\dfrac{\sin x}{\cos x}\right)\right)$

$= \sin x\left(\left(\dfrac{\cos^2 x + \sin^2 x}{\cos x}\right)\right)$

$= \sin x\left(\dfrac{1}{\cos x}\right)$

$= \tan x$

4. $\dfrac{\sec x - \cos x}{\sin^2 x \sec^2 x}$

$= \dfrac{\dfrac{1}{\cos x} - \cos x}{\sin^2 x \dfrac{1}{\cos^2 x}}$

$= \dfrac{1 - \cos^2 x}{\cos x} \div \dfrac{\sin^2 x}{\cos^2 x}$

$= \dfrac{\sin^2 x}{\cos x} \cdot \dfrac{\cos^2 x}{\sin^2 x}$

$= \cos x$

<u>Problems for finding the exact values</u>

1. $\sin\left(\tan^{-1} 3 - \cos^{-1} \dfrac{1}{3}\right)$

Use:

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

and $A = \tan^{-1} 3,\ B = \cos^{-1}\left(-\dfrac{1}{3}\right)$, B is in quadrant II

$\sin A \cos B - \cos A \sin B$

$= \dfrac{3}{\sqrt{10}} \cdot -\dfrac{1}{3} - \dfrac{1}{\sqrt{10}} \cdot \dfrac{\sqrt{8}}{3}$

$= \dfrac{-3 - \sqrt{8}}{3\sqrt{10}}$

2. $\cos\left(\cos^{-1} \dfrac{1}{4} + \cos^{-1}\left(-\dfrac{1}{3}\right)\right)$

Use $\cos(A + B) = \cos A \cos B - \sin A \sin B$

with $A = \cos^{-1}\dfrac{1}{4}$ and $B = \cos^{-1} -\dfrac{1}{3}$, B in quadrant II

$$\cos A \cos B - \sin A \sin B$$
$$= \frac{1}{4} \cdot -\frac{1}{3} - \frac{\sqrt{15}}{4} \cdot \frac{\sqrt{8}}{3}$$
$$= -\frac{1}{12} - \frac{2\sqrt{30}}{12}$$
$$= \frac{-1 - 2\sqrt{30}}{12}$$

3. $\sin\left(\cos^{-1}\dfrac{1}{3} + \tan^{-1}(-2)\right)$

Use $\sin A \cos B + \cos A \sin B$

with $A = \cos^{-1}\dfrac{1}{3}$ and $B = \tan^{-1}(-2)$, B is in quadrant IV

$$\sin A \cos B + \cos A \sin B$$
$$= \frac{\sqrt{8}}{3} \cdot \frac{1}{\sqrt{5}} + \frac{1}{3} \cdot \left(\frac{-2}{\sqrt{5}}\right)$$
$$= \frac{\sqrt{8} - 2}{3\sqrt{5}}$$

4. $\tan 2\left(Sin^{-1} x\right)$

Use $\dfrac{2\tan A}{1 - \tan^2 A}$ with $A = \sin^{-1} x$ and $\tan A = \dfrac{x}{\sqrt{1 - x^2}}$

$$\frac{2\tan A}{1 - \tan^2 A}$$
$$\frac{2\left(\frac{x}{\sqrt{1 - x^2}}\right)}{1 - \left(1 - x^2\right)}$$
$$= \frac{\frac{2x}{\sqrt{1 - x^2}}}{x^2}$$
$$= \frac{2x}{\sqrt{1 - x^2}} \cdot \frac{1}{x^2}$$
$$= \frac{2}{x\sqrt{1 - x^2}}$$

Unit 6, Activity 7, Polar Representation of Complex Numbers

Name________________________________

1. Given the complex number $a + bi$ rewrite in polar form. Give θ in degrees and minutes.

a) -5 + 3i ________________________________

b) $1 - i$ ________________________________

c) $-1 - i\sqrt{3}$ ________________________________

2. Rewrite each of the numbers in polar form then perform the indicated operation.

a) $(3 + 4i)\left(\sqrt{3} - i\right)$

b) $3i(2 - i)$

c) $\dfrac{\sqrt{3} + i}{\sqrt{3} - i}$

d) $\frac{5}{1+i}$

e) Find the reciprocal of $6 - 3i$.

3. Perform the operations and then write the answer in rectangular form.

a) $(10cis35^o)(2cis100^o)$

b) $(2cis120^o)(3cis180^o)$

c) $\frac{5cis29^o}{3cis4^o}$

4. Use your graph paper to plot the complex numbers and then express each in rectangular form.

$a)$ $5cis135^o$

$b)$ $3cis300^o$

Unit 6, Activity 7, Polar Representations of Complex Numbers with Answers

1. Given the complex number a + bi rewrite in polar form. Give θ in degrees and

a) -5 + 3i $\underline{\sqrt{34} cis 149^o 02'}$

b) $1 - i$ $\underline{\sqrt{2} cis 270^o}$

c) $-1 - i\sqrt{3}$ $\underline{2 cis 240^o}$

2. Rewrite each of the numbers in polar form then perform the indicated operation.

a) $(3+4i)(\sqrt{3}-i)$

$(5cis53^o 13')(2cis330^o)$

$= 10cis383^o 13'$ or $10\text{cis}23^\circ 13'$

b) $3i(2-i)$

$(3cis90^o)(\sqrt{5}cis333^o 24')$

$= 3\sqrt{5}cis423^o 24'$ or $3\sqrt{5}cis63^o 24'$

c) $\dfrac{\sqrt{3}+i}{\sqrt{3}-i}$

$\dfrac{2cis30^o}{2cis330^o}$

$cis(-300^o)$ or $cis60^o$

d) $\dfrac{5}{1+i}$

$\dfrac{5cis0^o}{\sqrt{2}cis45^o}$

$= \dfrac{5}{\sqrt{2}}cis(-45^o)$ or $\dfrac{5\sqrt{2}}{2}cis315^o$

e) Find the reciprocal of 6 – 3i

$3\sqrt{5}cis333.4^o$ in polar form. The reciprocal is $\dfrac{1}{3\sqrt{5}}cis(-333.4^o)$ or $\dfrac{\sqrt{5}}{15}cis26^o 24'$.

3. Perform the operations and then write the answer in rectangular form.

a) $\left(10cis35^{o}\right)\left(2cis100^{o}\right)$ *=20cis135°*

In a + bi form $-10\sqrt{2}+10i\sqrt{2}$

b) $\left(2cis120^{o}\right)\left(3cis180^{o}\right)$ *=6cis300°*

In a + bi form: $3+3i\sqrt{3}$

c) $\dfrac{5cis29^{o}}{3cis4^{o}}=\dfrac{5}{3}cis25^{o}$

In a + bi form $\approx 1.5+.7i$

4. Use your graph paper to plot the complex numbers and then express each in *a + bi* form.

a) $5cis135^{o} = -\dfrac{5\sqrt{2}}{2}+\dfrac{5}{2}i$

b) $3cis300^{o} = \dfrac{3}{2}-\dfrac{3\sqrt{3}}{2}i$

Unit 6, Activity 8, The Graphs of Polar Functions

1. Graph each of the following polar equations. Fill out the table below. To answer some of the questions it would be helpful to either trace the graph or look at the table values. Try setting the ΔTbl to $\frac{\pi}{24}$. This is the same as the θ step in the window. Scroll through the table to find the answers.

The Equation $r = a\sin(n\theta)$ $r = a\cos(n\theta)$	**Number of Petals**	**Domain**	**Number of Zeros**	**Symmetry**	**Maximum r-values**
$r= \sin 2\theta$					
$r = \sin 3\theta$					
$r = \sin 4\theta$					
$r = \sin 5\theta$					
$r = \cos 2\theta$					
$r = \cos 3\theta$					
$r = \cos 4\theta$					
$r = \cos 5\theta$					

II. Writing exercise

1. In each of the problems above a = 1. Let *a* take on other values. What changes as far as the table is concerned?

2. What in general can you say about the value of *n*?

3. What do you see with the symmetry of each graph? Is there a pattern?

4. What is the least domain needed for a complete graph? Is it the same for all of the rose curves?

5. Is there a pattern with the zeros?

Unit 6, Activity 8, The Graphs of Polar Functions with Answers

I.

The Equation *r=asin(nθ)* *r=acos(nθ)*	*Number of Petals*	*Domain*	*Number of Zeros*	*Symmetry*	*Maximum r-values*
r= sin2θ	*4*	*[0, 2π]*	*5*	*with respect to the pole*	*1*
r = sin3θ	*3*	*[0, π]*	*4*	*with respect to the pole*	*1*
r = sin4θ	*8*	*[0, 2π]*	*9*	*with respect to the pole*	*1*
r = sin5θ	*5*	*[0, π]*	*6*	*with respect to the pole*	*1*
r = cos2θ	*4*	*[0, 2π]*	*5*	*with respect to the polar axis*	*1*
r = cos3θ	*3*	*[0, π]*	*4*	*with respect to the polar axis*	*1*
r = cos4θ	*8*	*[0, 2π]*	*9*	*with respect to the polar axis*	*1*
r = cos5θ	*5*	*[0, π]*	*6*	*with respect to the polar axis*	*1*

II.
Writing exercise

1. In each of the problems above $a = 1$. Let a take on other values. What changes are there as far as the table is concerned? *The maximum r-values change and reflect the value of a.*

2. What in general can you say about the value of n? *If n is odd then the number of petals is equal to n. If n is even the number of petals is equal to 2n.*

3. What do you see with the symmetry of each graph? Is there a pattern? *All of the sine graphs have the same symmetry as do the cosine graphs.*

4. What is the least domain needed for a complete graph? Is it the same for all of the rose curves? *When n is odd the graph can be completed in [0, π]. When n is even then the graph is completed in [0, 2π]*

5. Is there a pattern with the zeros? *The number of zeros is 1 more than the number of petals.*

Unit 6, General Assessments, Spiral

Reduce each of the following expressions to a single trigonometric function.

1. $\csc\theta - \cot\theta\cos\theta$

2. $\dfrac{1+\cos\theta}{1+\sec\theta}$

3. $\sec\theta - \sin\theta\tan\theta$

4. $(\tan\theta + \cot\theta)\sin\theta$

5. $\dfrac{\sec\theta - \cos\theta}{\tan\theta}$

6. $\cos\theta(\cot\theta + \tan\theta)$

7. $(\cos^2\theta - 1)(\tan^2\theta + 1)$

8. $\dfrac{1+\sec\theta}{\tan\theta + \sin\theta}$

9. $\csc\theta\cos\theta$

10. $\dfrac{\tan\theta\sin\theta}{\sec^2\theta - 1}$

Reduce each of the following expressions to a single trigonometric function.

1. $\csc\theta - \cot\theta\cos\theta$

$$\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\cdot\cos\theta$$

$$\frac{1-\cos^2\theta}{\sin\theta}$$

$$\sin\theta$$

2. $\dfrac{1+\cos\theta}{1+\sec\theta}$

$$\frac{1+\cos\theta}{1+\dfrac{1}{\cos\theta}}$$

$$\frac{1+\cos\theta}{\dfrac{\cos\theta+1}{\cos\theta}}$$

$$=\cos\theta$$

3. $\sec\theta - \sin\theta\tan\theta$

$$\frac{1}{\cos\theta} - \sin\theta\cdot\frac{\sin\theta}{\cos\theta}$$

$$\frac{1-\sin^2\theta}{\cos\theta}$$

$$\frac{\cos^2\theta}{\cos\theta}$$

$$\cos\theta$$

4. $(\tan\theta + \cot\theta)\sin\theta$

$$\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\cdot\sin\theta$$

$$\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)\cdot\sin\theta$$

$$\left(\frac{1}{\cos\theta}\right)$$

$$=\sec\theta$$

5. $\dfrac{\sec\theta - \cos\theta}{\tan\theta}$

$$\frac{\dfrac{1}{\cos\theta} - \cos\theta}{\dfrac{\sin\theta}{\cos\theta}}$$

$$\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)$$

$$\frac{1-\cos^2\theta}{\sin\theta}$$

$$= \sin\theta$$

6. $\cos\theta(\cot\theta + \tan\theta)$

$$\cos\theta\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$\cos\theta\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\right)$$

$$\frac{1}{\sin\theta}$$

$$= \csc\theta$$

7. $(\cos^2\theta - 1)(\tan^2\theta + 1)$

$$(-\sin^2\theta)(\sec^2\theta)$$

$$-\sin^2\theta \cdot \frac{1}{\cos^2\theta}$$

$$= -\tan^2\theta$$

8. $\dfrac{1+\sec\theta}{\tan\theta+\sin\theta}$

$$\frac{1+\dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}+\sin\theta}$$

$$\frac{\dfrac{\cos\theta+1}{\cos\theta}}{\dfrac{\sin\theta+\sin\theta\cos\theta}{\cos\theta}}$$

$$\frac{\cos\theta+1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$\frac{1}{\sin\theta}$$

$$=\csc\theta$$

9. $\csc\theta\cos\theta$

$$\frac{1}{\sin\theta}\cdot\cos\theta$$

$$\frac{\cos\theta}{\sin\theta}$$

$$=\cot\theta$$

10. $\dfrac{\tan\theta\sin\theta}{\sec^2\theta-1}$

$$\frac{\tan\theta\sin\theta}{\tan^2\theta+1-1}$$

$$\frac{\tan\theta\sin\theta}{\tan^2\theta}$$

$$\frac{\sin\theta}{\tan\theta}$$

$$\sin\theta\cdot\frac{\cos\theta}{\sin\theta}$$

$$=\cos\theta$$

Unit 7, What Do You Know about Sequences and Series?

Word	+	*?*	-	*What do I know about sequences and series?*
finite sequence				
terms of a sequence				
infinite sequence				
recursion formula				
explicit or nth term formula				
arithmetic sequence				
common difference				
geometric sequence				
common ratio				
finite series				
n^{th} partial sum				

infinite series				
***n*th term of a series**				
convergence				
divergence				
summation (sigma) notation				
index of summation				
lower limit of summation				
upper limit of summation				

Unit 7, Activity 1, Arithmetic and Geometric Sequences

1. Find the first 4 terms of the given sequence and tell whether the sequence is arithmetic, geometric, or neither.

a) $t_n = 3(2)^n$

b) $t_n = 3 - 7n$

c) $t_n = n + \dfrac{1}{n}$

2. Find the formula for t_n. Using graph paper sketch the graph of each arithmetic or geometric sequence.

a) 8, 6, 4, 2,…

b) 8, 4, 2, 1, …

c) 24, -12, 6, -3,…

d) $\dfrac{1}{6}, -\dfrac{1}{6}, -\dfrac{1}{2}, -\dfrac{5}{6}$

3. A field house has a section in which the seating can be arranged so that the first row has 11 seats, the second row has 15 seats, the third row has 19 seats, and so on. If there is sufficient space for 20 rows in the section, how many seats are in the last row?

4. A company began doing business four years ago. Its profits for the last 4 years have been \$11 million, \$15 million, \$19 million and \$23 million. If the pattern continues, what is the expected profit in 26 years?

5. The school buys a new copy machine for \$15,500. It depreciates at a rate of 20% per year. How much has it depreciated at the end of the first year? Find the depreciated value after 5 full years.

6. Explain how you use the first two terms of a geometric sequence to find the explicit formula.

1. Find the first 4 terms of the given sequence and tell whether the sequence is arithmetic, geometric, or neither.

   a) $t_n = 3(2)^n$ *6, 12, 24, 48 geometric*

   b) $t_n = 3 - 7n$ *-4, -11, -18, -25 arithmetic*

   c) $t_n = n + \frac{1}{n}, 2, \frac{5}{2}, \frac{10}{3}, \frac{17}{4}$ *neither*

2. Find the formula for t_n, and sketch the graph of each arithmetic or geometric sequence.

   a) 8, 6, 4, 2,… $t_n = 10 - 2n$

   b) 8, 4, 2, 1, … $t_n = 16\left(\frac{1}{2}\right)^n$

   c) 24, -12, 6, -3,… $t_n = -48\left(-\frac{1}{2}\right)^n$

   d) $\frac{1}{6}, -\frac{1}{6}, -\frac{1}{2}, -\frac{5}{6} \ldots$ $t_n = \frac{1}{2} - \frac{1}{3}n$

3. A field house has a section in which the seating can be arranged so that the first row has 11 seats, the second row has 15 seats, the third row has 19 seats, and so on. If there is sufficient space for 20 rows in the section, how many seats are in the last row? *86 seats*

4. A company began doing business four years ago. Its profits for the last 4 years have been $11 million, $15 million, $19 million and $23 million. If the pattern continues, what is the expected profit in 26 years? *127 million*

5. The school buys a new copy machine for $15,500. It depreciates at a rate of 20% per year. How much has it depreciated at the end of the first year? Find the depreciated value after 5 full years. *$ 3100 at the end of the first year and, $6348.80 after five years*

6. Explain how you use the first two terms of a geometric sequence to find the explicit formula.

   Since a geometric sequence is an exponential function with a domain that is the set of natural numbers, you find the growth/decay factor $\frac{t_2}{t_1}$ *just as you did with exponential functions. This value is r in the geometric sequence formula* $t_n = t_1 r^{n-1}$

Unit 7, Activity 2, Using the Recursion Formula

Name______________________________________

1. Find the second, third, fourth, and fifth term of the sequence, then write the nth term formula for the sequence.

a) $t_1 = 6$, $t_n = t_{n-1} + 4$

b) $t_1 = 1$, $t_n = 3t_{n-1}$

c) $t_1 = 9$, $t_n = \frac{1}{3}t_{n-1}$

2. Give a recursive definition for each sequence.

a) 81, -27, 9, -3, …

b) 1, 3, 6, 10, 15, 21…

c) 8, 12, 16, 20,…

3 A pond currently has 2000 catfish in it. A fish hatchery decides to add an additional 20 catfish each month. In addition, it is known that the catfish population is growing at a rate of 3% per month. The size of the population after *n* months is given by the recursively defined sequence $p_1 = 2000$, $p_n = 1.03p_{n-1} + 20$.

How many catfish are in the pond at the end of the second month?

____

the third month? ____________________

Use a graphing utility to determine how long it will be before the catfish population reaches 5000. ____________________________________

4. Your group is to write a math *story chain* problem using recursion formulas and an appropriate real-life situation. The first student initiates the story. The next student adds a sentence and passes it to the third student to do the same. If one of you disagrees with any of the previous sentences, you should discuss the work that has already been done. Then either revise the problem or move on as it is written. Once the problem has been written at least three questions should be generated. Work out a key then challenge another group to solve your problem.

Unit 7, Activity 2, Using the Recursion Formula with Answers

1. Find the third, fourth, and fifth term of the sequence then write the nth term formula for the sequence.

a) $t_1 = 6$, $t_n = t_{n-1} + 4$ *10, 14, 18, 22* $t_1 = 6$, $t_n = 4n + 2$

b) $t_1 = 1$, $t_n = 3t_{n-1}$ *3, 9, 27, 81* $t_1 = 1$, $t_n = 3^{n-1}$ or $\frac{1}{3}(3)^n$

c) $t_1 = 9$, $t_n = \frac{1}{3}t_{n-1}$ *3,* $1, \frac{1}{3}, \frac{1}{9}$ $t_1 = 9, t_n = 9\left(\frac{1}{3}\right)^{n-1}$ *or* $27\left(\frac{1}{3}\right)^n$

2. Give a recursive definition for each sequence.

a) 81, -27, 9, -3 … $t_1 = 81,\ t_n = \left(-\frac{1}{3}\right)t_{n-1}$

b) 1, 3, 6, 10, 15, 21… $t_1 = 1,\ t_n = t_{n-1} + n$

c) 8, 12, 16, 20,… $t_1 = 8,\ t_n = t_{n-1} + 4$

3. A pond currently has 2000 catfish in it. A fish hatchery decides to add an additional 20 catfish each month. In addition, it is known that the catfish population is growing at a rate of 3% per month. The size of the population after *n* months is given by the recursively defined sequence $p_1 = 2000,\ p_n = 1.03p_{n-1} + 20$.

How many trout are in the pond at the end of the second month? *2162 trout*

the third month? *2246 trout*

Use a graphing utility to determine how long it will be before the trout population reaches 5000 *28 months*

4. *story chain* problem

Unit 7, Activity 3, Series and Partial Sums

Name________________________________

1. Write the formula for t_n, the nth term of the series, then find the sum of each of the following using the algebraic formula.

a) The first 10 terms of $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\ldots$

b) $1^4+2^4+3^4+\ldots+10^4$

c) 2 + 6 + 10 +14 +……+ 30

2. The chain letter reads:

Dear Friend,

Copy this letter 6 times and send it to 6 of your friends. In twenty days you will have good luck. If you break this chain you will have bad luck!

Assume that every person who receives the letter sends it on and does not break the chain.

a) Fill in the table below to show the number of letters sent in each mailing:

1st mailing	2nd mailing	3rd mailing	4th mailing	…	10th mailing

b) After the 10th mailing, how many letters have been sent?

c) The population of the United States is approximately 294,400,000. Would the 11th mailing exceed this population? How do you know?

3. A company began doing business four years ago. Its profits for the last 4 years have been \$32 million, \$38 million, \$42 million ,and \$48 million. If the pattern continues, what is the expected total profit in the first ten years?

4. A production line is improving its efficiency through training and experience. The number of items produced in the first four days of a month is 13, 15, 17, and 19, respectively. Project the total number of items produced by the end of a 30 day month if the pattern continues.

5. The Internal Revenue Service assumes that the value of an item which can wear out decreases by a constant number of dollars each year. For instance, a house depreciates by $\frac{1}{40}$ of its value each year.

   a. If a rental house is worth \$125,000 originally, by how many dollars does it depreciate each year?
   b. What is the house worth after 1, 2, or 3 years?
   c. Do these values form an arithmetic or a geometric sequence?
   d. Calculate the value of the house at the end of 30 years.
   e. According to this model is the house ever worth nothing? Explain.

6. A deposit of \$300 is made at the beginning of each quarter into an account that pays 5% compounded quarterly. The balance *A* in the account at the end of 25 years is

$A = 300\left(1+\frac{0.05}{4}\right)^1 + 300\left(1+\frac{0.05}{4}\right)^2 + \ldots + 300\left(1+\frac{0.05}{4}\right)^{100}$. Find *A*.

1. Write the formula for t_n, the nth term of the sequence, then find the sum of each of the following using the algebraic formula. Verify your answers with the calculator.

a) The first 10 terms of $1-\frac{1}{3}+\frac{1}{9}-\frac{1}{27}+\ldots$

The formula is $t_n = \left(-\frac{1}{3}\right)^{n-1}$. *The sum is -1.4999…*

b) $1^4 + 2^4 + 3^4 + \ldots + 10^4$

The formula is $t_n = n^4$. *The sum is 25,333.*

c) 2 + 6 + 10 +14 +……+ 30

The formula is $t_n = 4n-2$. *The sum is 128.*

2. The chain letter reads:

Dear Friend,

Copy this letter 6 times and send it to 6 of your friends. In twenty days you will have good luck. If you break this chain you will have bad luck!

Assume that every person who receives the letter sends it on and does not break the chain.

a) Fill in the table below to show the number of letters sent in each mailing:

1st mailing	*2nd mailing*	*3rd mailing*	*4th mailing*	…	*10th mailing*
6	6^2	6^3	6^4		6^{10}

b) After the 10th mailing, how many letters have been sent? *60,166,176*

c) The population of the United States is approximately 294,400,000. Would the 11th mailing exceed this population? How do you know?

Yes, because the 11th mailing would be 362,797,056 letters

3. A company began doing business four years ago. Its profits for the last 4 years have been \$32 million, \$38 million, \$42 million and \$48 million. If the pattern continues, what is the expected total profit in the first ten years? *590 million*

4. A production line is improving its efficiency through training and experience. The number of items produced in the first four days of a month is 13, 15, 17, and 19, respectively. Project the total number of items produced by the end of a 30 day month if the pattern continues. *1,260 items*

5. The Internal Revenue Service assumes that the value of an item which can wear out decreases by a constant number of dollars each year. For instance, a house depreciates by $\frac{1}{40}$ of its value each year.

a) If a rental house is worth \$125,000 originally, by how many dollars does it depreciate each year? *\$3125*

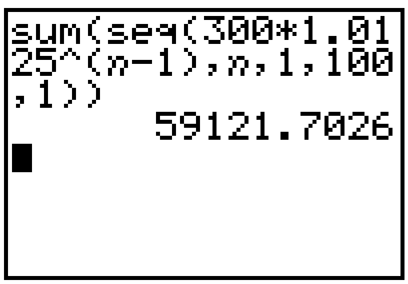
b) What is the house worth after 1, 2, or 3 years? *\$121,875, \$118,750, \$115,625*

c) Do these values form an arithmetic or a geometric sequence? *arithmetic sequence*

d) Calculate the value of the house at the end of 30 years. *\$34,375*

e) According to this model is the house ever worth nothing? Explain. *Yes, the model used is linear and will equal 0 at 41 years. However, the model does not take into account the fact that property values usually appreciate over time.*

6. A deposit of \$300 is made at the beginning of each quarter into an account that pays 5% compounded quarterly. The balance A in the account at the end of 25 years is $A = 300\left(1+\frac{0.05}{4}\right)^{1} + 300\left(1+\frac{0.05}{4}\right)^{2} + \ldots + 300\left(1+\frac{0.05}{4}\right)^{100}$. Find A.



\$59,121.70

Unit 7, Activity 4, Infinite Sequences and Convergence Part I

Place a ✓ in the column marked My Opinion if you agree with the statement. Place an X if you disagree with the statement. If you have disagreed, explain why. Use the calculator to help answer lessons learned.

My Opinion	**Statement**	**If you disagree, why?**	**Lessons Learned**
	1. Sequences whose nth term formula is geometric: $t_n = ar^{n-1}$ are convergent.		
	2. Arithmetic sequences are always divergent.		
	3. All sequences whose nth term uses a rational function formula are convergent.		
	4. All sequences whose nth term formula is a composition f(g(x)) will converge only if both f and g are convergent.		
	5. All sequences whose nth term is a periodic function are divergent.		

Unit 7, Activity 4, Infinite Sequences and Convergence Part I with Answers

My Opinion			
My Opinion	Statement	If you disagree why?	Lessons Learned
	1. Sequences whose nth term formula is geometric: $t_n = ar^{n-1}$ are convergent.		It depends on the value of *r*. If -1 < r < 1 then the sequence is convergent. If r > 1 or r < -1 then the sequence is divergent.
	2. Arithmetic sequences are always divergent.		Arithmetic sequences have nth term formulas that are linear. The end-behavior for a linear function is $\pm\infty$.
	3. All sequences whose nth term uses a rational function formula are convergent.		Only those sequences whose nth term formula has a horizontal asymptote *y* = k, k a real number, will converge.
	4. . All sequences whose nth term formula is a composition f(g(x)) will converge only if both f and g are convergent.		Not necessarily. If $f(g(x)) \to k$, k a real number, as *n* increases without bound, then the sequence is convergent.
	5. All sequences whose nth term is a periodic function are divergent.		Not necessarily. See the answer above. If the periodic function is *f* of *f*(*g*(*x*)) *and* *g*(*x*) converges to a real number, then the sequence is convergent.

Unit 7, Activity 4, Infinite Sequences and Convergence Part II

Graph each of the following using graph paper. Determine whether or not they converge.

1) $f(n) = 3n - 5$

2) $f(n) = \left(\frac{3}{4}\right)^n$

3) $f(n) = \frac{1}{n-1}$

4) $f(n) = (-2)^{n-1}$

5. $f(n) = -\sin\left(\frac{n}{2}\right)$

6. $f(n) = \frac{\cos n\pi}{n}$

7. $f(n) = \frac{n^3}{n^2 - n - 1}$

8. $f(n) = 4\left(-\frac{9}{10}\right)^n$

Unit 7, Activity 4, Infinite Sequences and Convergence Part II with Answers

Graph each of the following using graph paper. Determine whether or not they converge.

1) $f(n) = 3n - 5$ *divergent; an arithmetic sequence that is linear*

2) $f(n) = \left(\frac{3}{4}\right)^n$ *convergent; the values get closer and closer to 0*

3) $f(n) = \frac{1}{n-1}$ *convergent; the values get closer and closer to 0*

4) $f(n) = (-2)^{n-1}$ *divergent; the sequence is both oscillatory and divergent*

5. $f(n) = -\sin\left(\frac{n}{2}\right)$ *Divergent; the sequence is periodic*

6. $f(n) = \frac{\cos n\pi}{n}$ *Convergent and oscillatory (It helps to go into FORMAT and turn off the axes to see it clearly.)*

7. $f(n) = \frac{n^3}{n^2 - n - 1}$ *Divergent; the values get larger and larger*

8. $f(n) = 4\left(-\frac{9}{10}\right)^n$ *Convergent and oscillatory*

Name________________________________

Evaluate each limit or state that the limit does not exist.

1) $\lim\limits_{n \to \infty} e^{-n}$

2) $\lim\limits_{n \to \infty} \dfrac{2n-1}{n+10}$

3) $\lim\limits_{n \to \infty} \dfrac{3n+12}{n-5}$

4) $\lim\limits_{n \to \infty} \dfrac{n^2-3n-4}{n-4}$

5) $\lim\limits_{n \to \infty} \dfrac{3-4n}{n^2-4}$

6) $\lim\limits_{n \to \infty} \dfrac{8n^2-3n}{5n^2-4}$

Unit 7, Activity 4, Finding Limits of Infinite Sequences with Answers

Evaluate each limit or state that the limit does not exist.

1) $\lim_{n \to \infty} e^{-n} = 0$

2) $\lim_{n \to \infty} \frac{2n-1}{n+10} = 2$

3) $\lim_{n \to \infty} \frac{3n+12}{n-5} = 3$

4) $\lim_{n \to \infty} \frac{n^2-3n-4}{n-4}$ *does not exist - divergent*

5) $\lim_{n \to \infty} \frac{3-4n}{n^2-4} = 0$

6) $\lim_{n \to \infty} \frac{8n^2-3n}{5n^2-4} = \frac{8}{5}$

Unit 7, Activity 5, Working with Summation Notation

Name______________________________

Express the given series using summation notation, then find the sum.

1. $5 + 9 + 13 + \ldots + 101$

2. $48 + 24 + 12 \ldots$

3. $1 + 4 + 9 + \ldots + 144$

4. $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+...-\frac{1}{512}$

Expand each of the following, then find the sum:

5. $\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^{k}$

6. $\sum_{i=1}^{10} 2i$

7. $\sum_{j=1}^{15} 6$

8. $\sum_{1}^{8} 3^{k} - 1$

9. $\sum_{i=2}^{7} 5 - 2i$

10. $\sum_{k=1}^{6} \left(-1^{k}\right)\left(2^{k}\right)$

Unit 7, Activity 5, Working with Summation Notation with Answers

Express the given series using summation notation, then find the sum.

1) 5 + 9 + 13 + …+ 101

$$\sum_{i=1}^{25} 4i + 1 = 1325$$

2) 48 + 24 + 12 …

$$\sum_{i=1}^{\infty} 48\left(\frac{1}{2}\right)^{i-1} = 96$$

3) 1 + 4 + 9 +…+ 144

$$\sum_{i=1}^{12} i^2 = 650$$

4. $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots-\frac{1}{512}$

$$\sum_{i=1}^{9}\left(-\frac{1}{2}\right)^{i-1} = \frac{171}{256}$$

Expand each of the following, then find the sum:

5. $\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^{k}$

$$1+\frac{1}{3}+\frac{1}{9}+\ldots=\frac{3}{2}$$

6. $\sum_{i=1}^{10} 2i$

2 + 4 + 6+ …+ 20 = 110

7. $\sum_{j=1}^{15} 6$

6 + 6 + …. =6 = 90   6(15) = 90

8. $\sum_{1}^{8} 3^k - 1$

2 + 8 + 26 + … 6560 = 9832

9. $\sum_{i=2}^{7} 5-2i$

1 – 1 – 3 - ...-9 = 24

10. $\sum_{k=1}^{6} \left(-1^{k}\right)\left(2^{k}\right)$

-2 + 4 – 8 + 16 – 32 + 64 = 42

Unit 7, General Assessment, Spiral

1. Find the vertical and horizontal asymptotes for $f(x) = \dfrac{2x^2 - 4}{x^2 - 3x - 4}$.

2. Describe the end-behavior of each of the following functions:

a) $g(x) = x - 3$

b) $f(x) = 4 - x^3$

c) $h(x) = \dfrac{3x - 4}{2x + 1}$

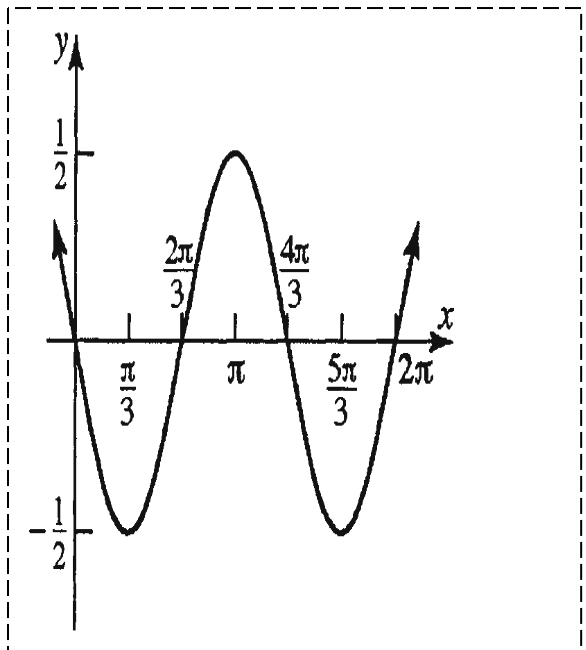
d) $g(x) = 5(3)^{-x}$

3.

a) Over what intervals is the graph to the right increasing?

b) Identify the location of the relative maximum. What is its value?

c) The graph belongs to a sinusoidal function (sine or cosine). What is its period?
What is the amplitude?



4. a) Graph: $f(x) = \begin{cases} 1 + x, x < 0 \\ x^2, x \geq 0 \end{cases}$

b) Describe the discontinuities, if any.

1. Find the vertical and horizontal asymptotes for $f(x) = \dfrac{2x^2-4}{x^2-3x-4}$.

The vertical asymptotes are x = 4 and x = -1. The horizontal asymptote is y = 2.

2. Describe the end-behavior of each of the following functions"

a) $g(x) = x-3$, as $x \to +\infty, y \to +\infty$ and as $x \to -\infty, y \to -\infty$

b) $f(x) = 4 - x^3$, as $x \to +\infty, y \to -\infty$ and as $x \to -\infty, y \to +\infty$

c) $h(x) = \dfrac{3x-4}{2x+1}$, *there is a horizontal asymptote* $y = \dfrac{3}{2}$

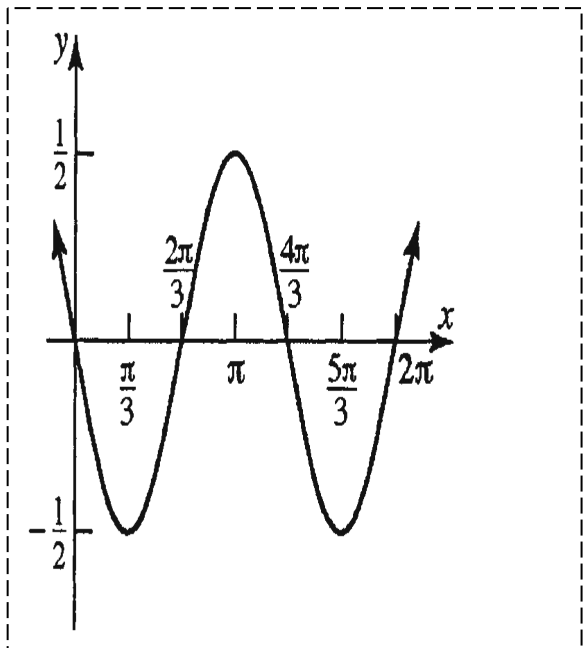
d) $g(x) = 5(3)^{-x}$ *This exponential function is decreasing and uses the positive x-axis as a horizontal asymptote so* $x \to +\infty, y \to 0$

3.

a) Over what intervals is the graph to the right increasing?
$\dfrac{\pi}{3} < x < \pi$ and $x > \dfrac{5\pi}{3}$

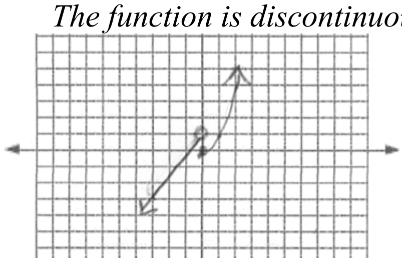
b) Identify the location of the relative maximum. What is its value? *It is located at x = π and has a value of ½ .*

c) The graph belongs to a sinusoidal function (sine or cosine).
What is its period? $\dfrac{4\pi}{3}$
What is the amplitude? ½



4. a) Graph: $f(x) = \begin{cases} 1+x, x<0 \\ x^2, x \ge 0 \end{cases}$

b) Describe the discontinuities, if any.
The function is discontinuous at x = 0. It has a jump discontinuity.



Unit 8, Pre-test Conic Sections

Name_________________________________

1. Determine which of the following equations is a circle, ellipse, parabola, or hyperbola.

a) $x^2 + 4x + y + 3 = 0$

1a)__________________

b) $2x^2 + y^2 - 8x + 4y + 2 = 0$

1b)__________________

c) $2y^2 - x^2 + x - y = 0$

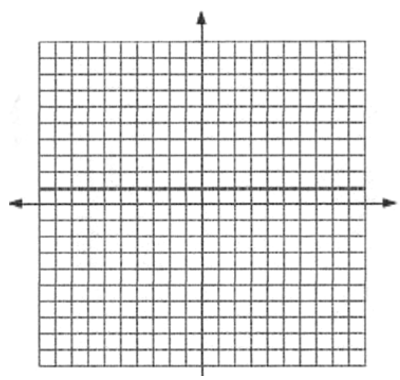
1c)__________________

d) $2x^2 + 2y^2 - 8x + 8y = 0$

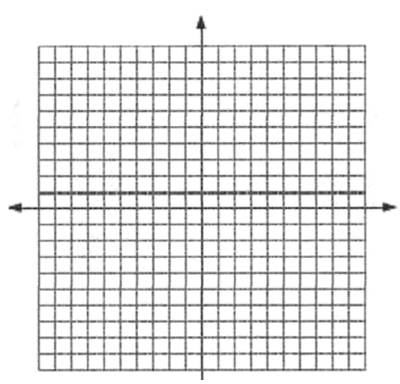
1d)__________________

2. Sketch the graph of each of the following:

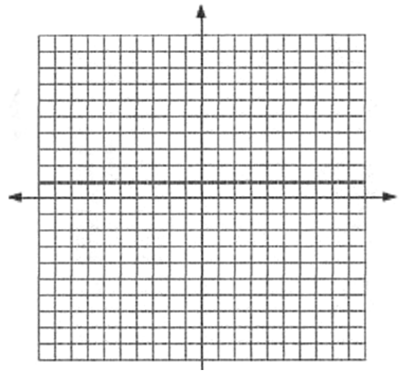
a) $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$



b) $x^2 + y^2 - 2x + 4y - 20 = 0$

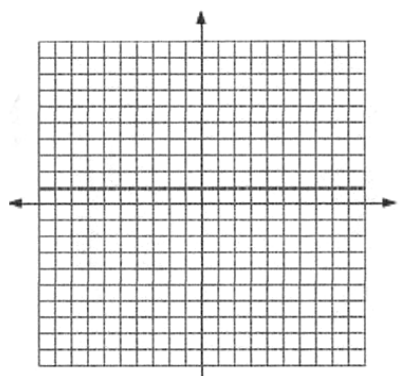


c) A hyperbola with
1) center at (4, -1)
2) focus at (7, -1)
3) vertex at (6, -1)



What are the asymptotes for (c)?

d) $y^2 - 4y - 4x = 0$
What are the vertex, focus, and directrix of the parabola?



Unit 8, Pre-test Conic Sections with Answers

1. Determine which of the following equations is a circle, ellipse, parabola, or hyperbola.

a) $x^2 + 4x + y + 3 = 0$

1a)_*parabola*

b) $2x^2 + y^2 - 8x + 4y + 2 = 0$

1b)_*ellipse*

c) $2y^2 - x^2 + x - y = 0$

1c)_*hyperbola*

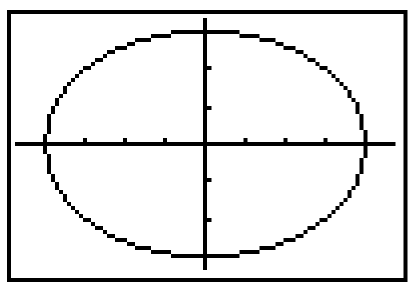
d) $2x^2 + 2y^2 - 8x + 8y = 0$

1d)_*circle*__

2. Sketch the graph of each of the following:

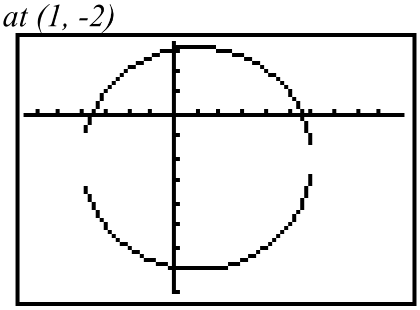
a) $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

major axis is horizontal with length 8 and vertices (-4, 0) and (4, 0); minor axis length 6, vertices at (0,-3) and (0,3)

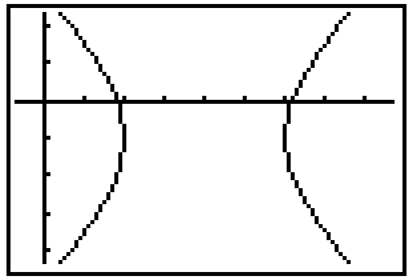


b) $x^2 + y^2 - 2x + 4y - 20 = 0$

a circle with radius of 5 and a center at (1, -2)

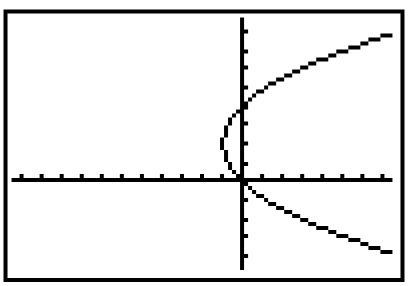


c) A hyperbola with
   4) center at (4, -1)
   5) focus at (7, -1)
   6) vertex at (6, -1)



The asymptotes are $y = \frac{\sqrt{5}}{2}(x-4)-1$ and $y = -\frac{\sqrt{5}}{2}(x-4)-1$

d) $y^2 - 4y - 4x = 0$
What are the vertex, focus, and directrix of the parabola?
The vertex is (-1, 2), the focus is (0, 2) and the directrix is x = -2



Unit 8, What Do You Know about Conic Sections and Parametric Equations?

Term or Concept	**+**	**?**	**-**	**What do you know about these terms or concepts in conic sections and parametric equations?**
double napped cone				
conic section				
locus				
general equation of a conic section				
standard form of the equation of a circle				
parabola as a conic section				
standard form of the equation of a parabola				
directrix of a parabola				
focus of a parabola				
ellipse				
foci of the ellipse				
major axis of the				

ellipse minor axis of the ellipse				
general form for the equation of an ellipse				
standard (graphing) form for the equation of an ellipse				
vertices of an ellipse				
foci of an ellipse				
eccentricity				
hyperbola				
general form for the equation of a hyperbola				
standard (graphing) form for the equation of a hyperbola				

transverse axis of a hyperbola				
asymptotes of a hyperbola				
foci of a hyperbola				
conjugate axis of a hyperbola				
parametric equations				
plane curve				
parameter				

Unit 8, Activity 1, Working with Circles

Name_______________________________

Show the work needed to find the answers. Graphs needed in #2 should be done by hand using graph paper.

1. Write both the standard and general forms of an equation of the circle given the following information.

a) the center at (2, -3) and radius is equal to 4

b) the center at (4, 2) and the circle passes through (1, 7)

c) the endpoints of the diameter of the circle are (1, 5) and (-3, -7)

d) the center is at (-2, 5) and the circle is tangent to the line $x = 7$.

2. Each of the problems below are equations of semi-circles. For each one find
   i. the center
   ii. the radius
   iii. the domain and range
   iv. sketch the graph

a) $f(x) = \sqrt{9 - x^2}$

b) $f(x) = -\sqrt{4 - (x+4)^2}$

c) $f(x) = \sqrt{4x - 3 - x^2}$

d) $f(x) = \sqrt{16 + 6x - x^2}$

3. Given the equation of the semi-circle $y = \sqrt{1 - x^2}$ .Write an equation of the semi-circle if

a) the center is moved to (-3, 0)

b) the center is moved to (0, -2)

c) the center is moved to (3, -5)

d) the semi-circle is reflected over the x-axis

Unit 8, Activity 1, Working with Circles with Answers

1. Write both the standard and general form of an equation of the circle given the following information.

   a) the center at (2, -3) and radius is equal to 4
   $(x-2)^2+(y+3)^2=16$ and $x^2+y^2-4x+6y-3=0$

   b) the center at (4, 2) and the circle passing through (1, 7)
   $(x-4)^2+(y-2)^2=34$ and $x^2+y^2-4x+6y-3=0$

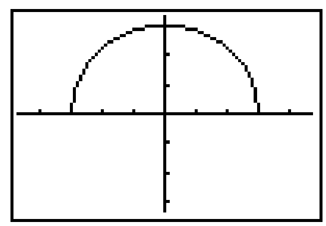
   c) the endpoints of the diameter of the circle are (1, 5) and (-3, -7)
   $(x+1)^2+(y+1)^2=40$ and $x^2+y^2+2x+2y-38=0$

   d) the center is at (-2, 5) and the circle is tangent to the line x = 7.
   $(x+2)^2+(y-5)^2=81$ and $x^2+y^2-8x-4y-14=0$

2. Each of the problems below are equations of semi-circles. For each one find
   i. the center
   ii. the radius
   iii. the domain and range
   iv. sketch the graph

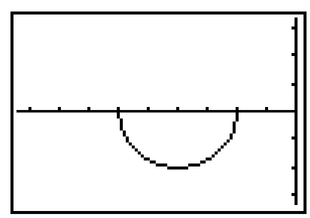
a) $f(x)=\sqrt{9-x^2}$

i. *the center is (0, 0)*
ii. *the radius is 3*
iii. *the domain is* $\{x: -3 \le x \le 3\}$ *and range is* $\{y: 0 \le y \le 3\}$
iv. graph



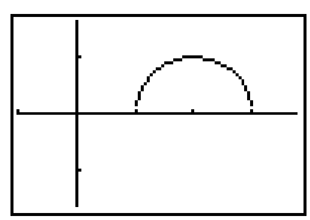
b) $f(x)=-\sqrt{4-(x+4)^2}$

i. *the center is (-4, 0)*
ii. *the radius is 2*
iii. *the domain is* $\{x: -4 \le x \le 0\}$ *and the range is* $\{y: -2 \le y \le 0\}$
iv. *the graph*



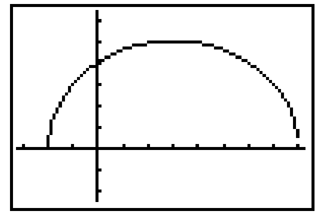
c) $f(x)=\sqrt{4x-3-x^2}$

i. *the center is (2, 0)*
ii. *the radius is 1*
iii. *the domain is* $\{1 \le x \le 3\}$ *and the range is* $\{y: 0 \le y \le 1\}$
iv. *the graph*



d) $f(x) = \sqrt{16 + 6x - x^2}$

- *i.* *the center is (3, 0)*
- *ii.* *the radius is 5*
- *iii.* *the domain is {x: -2 ≤ x ≤ 8} and the range is {y: 0 ≤ y ≤ 5}*
- *iv.* *the graph*



3. Given the equation of the semi-circle $y = \sqrt{1 - x^2}$ .Write an equation of the semi-circle if

a) the center is moved to (-3, 0)

$y = \sqrt{1 - (x + 3)^2}$

b) the center is moved to (0, -2)

$y = \sqrt{1 - x^2} - 2$

c) the center is moved to (3, -5)

$y = \sqrt{1 - (x - 3)^2} - 5$

d) the semi-circle is reflected over the x-axis

$y = -\sqrt{1 - x^2}$

Unit 8, Activity 2, Parabolas as Conic Sections

1. For each of the parabolas, find the coordinates of the focus and vertex, an equation of the directrix, sketch the graph putting in the directrix and focus, then label the vertex and the axis of symmetry. Use graph paper.

a) $y^2 = -8x$

b) $(x+2)^2 = -4(y+1)$

c) $2y^2 + 9x = 0$

d) $y = x^2 - 4x - 4$

2. Find the standard form of the equation of each parabola using the given information. Sketch the graph.

a) Focus: (-1, 0); directrix the line x = 1

b) Vertex (5, 2); Focus (3, 2)

c) Focus at (0, 3) and vertex at (0, -1)

3. For each of the following equations find

- the zeros
- the domain and range
- sketch the graph

Verify your answers with the graphing calculator.

a) $x = \sqrt{y}$

b) $y = -\sqrt{x}$

c) $y = -\sqrt{x-3}$

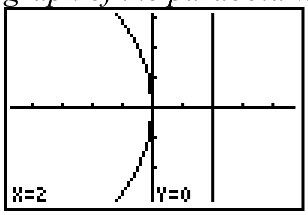
d) $x = -\sqrt{y+3} + 2$

e) $y = \sqrt{x-2} - 1$

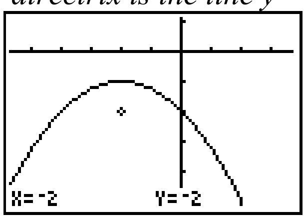
Unit 8, Activity 2, Parabolas as Conic Sections with Answers

1. For each of the parabolas, find the coordinates of the focus and vertex, an equation of the directrix, sketch the graph putting in the directrix and focus, then label the vertex and the axis of symmetry. Use graph paper.

a) $y^2 = -8x$ *vertex at (0, 0) focus at (-2, 0) directrix is the line x = 2; graph of the parabola with directrix shown below*

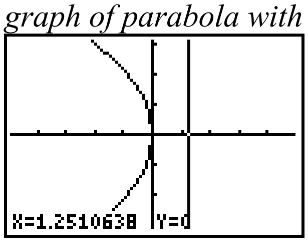


b) $(x+2)^2 = -4(y+1)$ *vertex at (-2, -1), focus at (-2, -2), directrix is the line y = 0; graph of parabola with focus shown below*

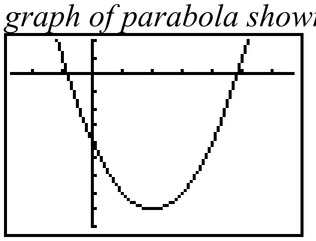


c) $2y^2 + 9x = 0$ *vertex at (0, 0), focus at* $\left(-\frac{9}{8}, 0\right)$*, directrix is the line* $x = \frac{9}{8}$*;*

graph of parabola with directrix shown below



d) $y = x^2 - 4x - 4$ *vertex at (2, -8), focus at (2, 7.75) and directrix at y = -8.25; graph of parabola shown below*



2. Find the standard form of the equation of each parabola using the given information. Sketch the graph.

a) Focus: (-1, 0); directrix the line x = 1
The equation is $y^2 = 4x$

b) Vertex (5, 2); Focus (3, 2)
The equation is $(y-2)^2 = -8(x-5)$

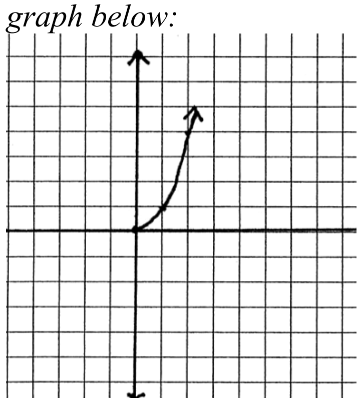
c) Focus at (0, 3) and vertex at (0, -1)
The equation is $(x-0)^2 = 16(y+1)$

3. For each of the following equations find

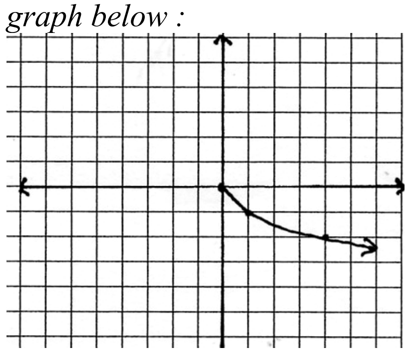
- o the zeros
- o the domain and range
- o sketch the graph

Verify your answers with the graphing calculator.

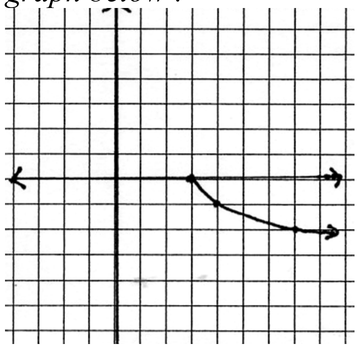
a) $x = \sqrt{y}$ *. domain: {x: x≥ 0} range {y :y≥ 0} zeros: (0, 0)*
graph below:



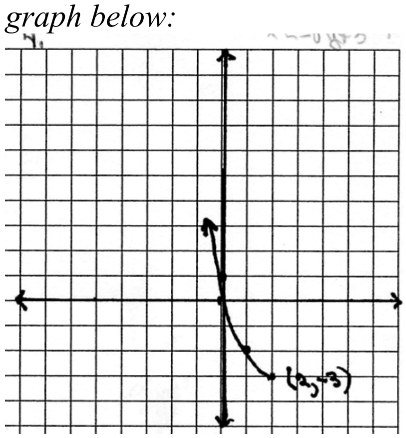
b) $y = -\sqrt{x}$ *domain: {x: x≥ 0}, range : {y: y ≤ 0}, zeros: (0, 0)*
graph below :



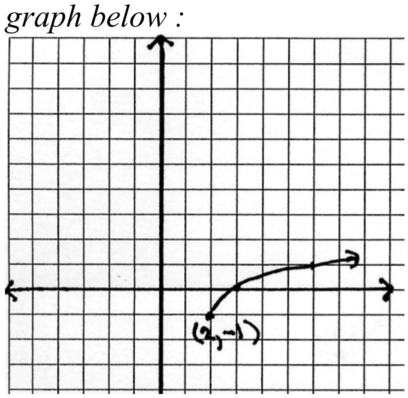
c) $y = -\sqrt{x-3}$ *domain: {x: x ≥3}, range : {y: y ≤ 0}, zeros: (3, 0)*
graph below :



d) $x = -\sqrt{y+3} + 2$ *domain: {x: x ≤ 2}, range: {y ≥ -3}, zeros:***(** $2-\sqrt{3}$ *)*
graph below:



e) $y = \sqrt{x-2} - 1$ *domain: {x: x ≥ 2}, range: {y: y ≥ -1}, zeros: (3, 0)*
graph below :



Unit 8, Activity 3, Using Eccentricity to Write Equations and Graph Conics

1. Identify the conic and find its equation having the given properties:

a) Focus at (2, 0); directrix: x = -4; $e = \frac{1}{2}$

b) Focus at (-3, 2); directrix x = 1; $e = 3$

c) Center at the origin; foci on the x-axis ; $e = 2$; containing the point (2, 3)

d) Center at (4, -2); one vertex at (9, -2), and one focus at (0, -2)

e) One focus at $(-3-3\sqrt{13},1)$, asymptotes intersecting at (-3, 1), and one asymptote passing through the point (1, 7)

2. Find the eccentricity, center, foci, and vertices of the given ellipse and draw a sketch of the graph:

a) $(x+1)^2 + 9(y-6)^2 = 9$

b) $\dfrac{x^2}{100} + \dfrac{(y-3)^2}{64} = 1$

3. Find the eccentricity, center, foci, vertices, and equations of the asymptotes of the given hyperbolas and draw a sketch of the graph.

a) $(x-5)^2 - \dfrac{(y-7)^2}{3} = 1$

b) $\dfrac{(y-4)^2}{4} - \dfrac{(y+1)^2}{9} = 1$

Unit 8, Activity 3, Using Eccentricity to Write Equations and Graph Conics with Answers

1. Identify the conic and find its equation having the given properties:

a) Focus at (2, 0); directrix: x = -4; *e* = ½

$3x^2 - 24x + 4y^2 = 0$ *ellipse*

b) Focus at (-3, 2); directrix x = 1; *e* = 3

$8x^2 - 24x - y^2 + 4y - 4 = 0$ *hyperbola*

c) Center at the origin ; foci on the x-axis ; *e* = 2 ; containing the point (2, 3)

$3x^2 - y^2 + 4y^2 = 0$ *hyperbola*

d) Center at (4, -2); one vertex at (9, -2), and one focus at (0, -2)

$$\frac{(x-4)^2}{25} + \frac{(y+2)^2}{9} = 0 \text{ } ellipse$$

e) One focus at $(-3-3\sqrt{13},1)$, asymptotes intersecting at (-3, 1), and one asymptote passing through the point (1, 7)

$$\frac{(x+3)^2}{36} - \frac{(y-1)^2}{81} = 1 \text{ } hyperbola$$

2. Find the eccentricity, center, foci, and vertices of the given ellipse and draw a sketch of the graph.

a) $(x+1)^2 + 9(y-6)^2 = 9$

eccentricity is $\frac{\sqrt{2}}{3} \approx .47$*, center is (-1, 6), foci are*

$\left(-1+2\sqrt{2},6\right)$ and $\left(-1-2\sqrt{2},6\right)$*, and vertices are (5, 6) and (-7, 6)*

b) $\frac{x^2}{100} + \frac{(y-3)^2}{64} = 1$

eccentricity is .6, center at (0, 3), foci at (6, 3) and (-6, 3), and vertices at (10, 3) and (-10, 3)

3. Find the eccentricity, center, foci, vertex, and equations of the asymptotes of the given hyperbolas.

a) $(x-5)^2 - \frac{(y-7)^2}{3} = 1$

eccentricity is 2, center at (5, 7), foci at (3, 7) and (7, 7), vertices are (4, 7) and (6, 7), equations are $y - 7 = \sqrt{3}(x-5)$ and $y - 7 = -\sqrt{3}(x-5)$

b) $\frac{(y+4)^2}{4} - \frac{(x+3)^2}{12} = 1$

eccentricity is 2 , center at (-3, -4) , foci at (-3, 0) and (-3, -8),vertices are (-3, -2) and (-3, -6,) equations are $y + 4 = -\frac{\sqrt{3}}{3}(x+3)$ and $- y + 4 = -\frac{\sqrt{3}}{3}(x+3)$

Unit 8, Activity 4, Polar Equations of Conics

The equations below are those of conics having a focus at the pole. In each problem (a) find the eccentricity; (b) identify the conic; (c) describe the position of the conic and (d) write an equation of the directrix which corresponds to the focus at the pole. Graph the conic. Verify your answers by graphing the polar conic and the directrix on the same screen.

1. $r = \dfrac{2}{1-\cos\theta}$

2. $r = \dfrac{6}{3-2\cos\theta}$

3. $r = \dfrac{5}{2+\sin\theta}$

4. $r = \dfrac{9}{5-6\sin\theta}$

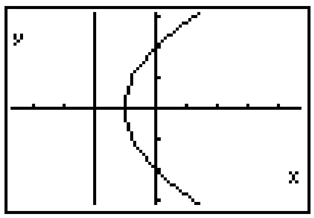
5. Given the eccentricity, $e = \dfrac{4}{3}$ and the directrix y = -1.5, find the polar equation for this conic section. Verify the answer by graphing the polar conic and directrix on the same screen using the graphing calculator.

6. Given the eccentricity, $e = \dfrac{2}{3}$ and a directrix y = 3, find the polar equation for this conic section. Verify the answer by graphing the polar conic and directrix on the same screen using the graphing calculator.

The equations below are those of conics having a focus at the pole. In each problem (a) find the eccentricity; (b) identify the conic; (c) describe the position of the conic; and (d) write an equation of the directrix which corresponds to the focus at the pole. Graph the conic. Verify your answers by graphing the polar conic and the directrix on the same screen.

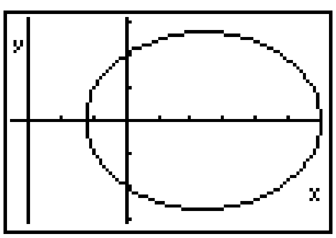
1. $r = \dfrac{2}{1-\cos\theta}$

   a) 1 b) parabola c) the focus is at the pole and the directrix is perpendicular to the polar axis and 2 units to the left of the pole (d) rcosθ = -2

   graph: 

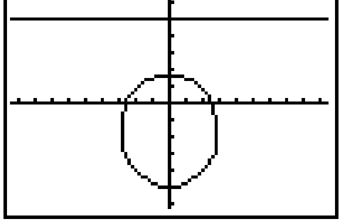
2. $r = \dfrac{6}{3-2\cos\theta}$

   a) 2/3 b) ellipse c) one of the foci is at the pole and the directrix is perpendicular to the polar axis a distance of 3 units to the left of the pole, the major axis is along the polar axis (d) rcosθ = -3

   graph: 

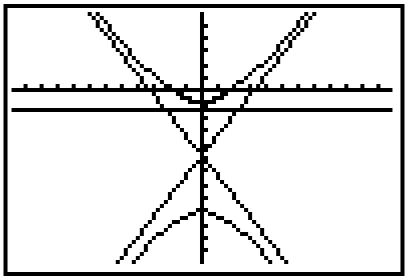
3. $r = \dfrac{5}{2+\sin\theta}$

   a) ½ b) ellipse c) one of the foci is at the pole and the directrix is parallel to the polar axis a distance of 5 units above the polar axis, the major axis is along the pole (d) rsinθ = - 5

   graph: 

4. $r = \dfrac{9}{5 - 6\sin\theta}$

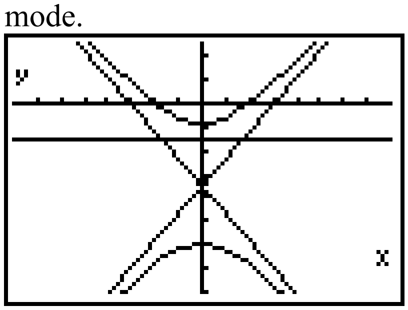
a) 6/5 b) hyperbola c) one of the foci is at the pole and the directrix is parallel to the polar axis a distance of 1.5 units below the polar axis, the major axis is along the pole (d) rsinθ = -1.5

graph: 

The asymptotes are drawn because the calculator is in connected mode.

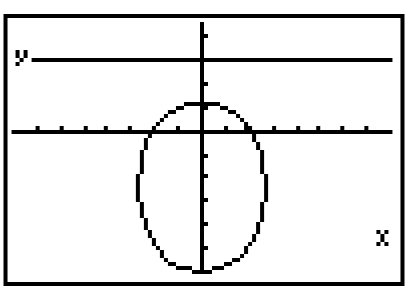
5. Given the eccentricity, $e = \dfrac{4}{3}$ and the directrix y = -1.5, find the polar equation for this conic section. Verify the answer by graphing the polar conic and directrix on the same screen using the graphing calculator.

$r = \dfrac{6}{3 - 4\sin\theta}$, the asymptotes are drawn because the calculator is in connected mode.



6. Given the eccentricity, $e = \dfrac{2}{3}$ and a directrix y = 3, find the polar equation for this conic section. Verify the answer by graphing the polar conic and directrix on the same screen using the graphing calculator.

$r = \dfrac{6}{3 + 2\sin\theta}$



Unit 8, Activity 5, Plane Curves and Parametric Equations

For each of the problems, set up a table such as the one below:

t									
x									
y									

Part I. Fill in the table and sketch the curve given by the following parametric equations. Describe the orientation of the curve.

1. Given the parametric equations: $x = 1 - t$ and $y = \sqrt{t}$ for $0 \le t \le 10$.
   a) Complete the table.
   b) Plot the points (x, y) from the table labeling each point with the parameter value t.
   c) Describe the orientation of the curve.

2. Given the parametric equations $x = 6 - t^3$ and $y = 3\ln t$ $0 < t \le 5$.
   a) Complete the table.
   b) Plot the points (x, y) from the table, labeling each point with the parameter value t.
   c) Describe the orientation of the curve.

Part II.
a) Graph using a graphing calculator.
b) Eliminate the parameter and write the equation with rectangular coordinates.
c) Answer the following questions.

i. For which curves is y a function of x?
ii. What if any restrictions are needed for the two graphs to match?

1. $x = 4\cos t$, $y = 4\sin t$ for $0 \le t < 2\pi$

2. $x = \cos t$, $y = \sin 2t$ for $0 \le t < 2\pi$

3. $x = -3\sqrt{t}$, $y = e^t$ for $0 \le t \le 10$

4. $x = \dfrac{t}{2}$, $y = \dfrac{2}{t-3}$ for $0 \le t \le 10$

Part III. In the rectangular coordinate system, the intersection of two curves can be found either graphically or algebraically. With parametric equations, we can distinguish between an intersection point (the values of *t* at that point are different for the two curves) and a collision point (the values of *t* are the same).

1. Consider two objects in motion over the time interval $0 \le t \le 2\pi$. The position of the first object is described by the parametric equations $x_1 = 2\cos t$ and $y_1 = 3\sin t$. The position of the second object is described by the parametric equations $x_2 = 1 + \sin t$ and $\cos t - 3$. At what times do they collide?

2. Find all intersection points for the pair of curves.
$x_1 = t^3 - 2t^2 + t;\ y_1 = t$ and $x_2 = 5t;\ y_2 = t^3$. Indicate which intersection points are true collision points. Use the interval $-5 \le t \le 5$.

Unit 8, Activity 6, Modeling Motion using Parametric Equations

Part I. Fill in the table and sketch the curve given by the following parametric equations. Describe the orientation of the curve.

1. Given the parametric equation: $x = 1 - t$ and $y = \sqrt{t}$ for $0 \le t \le 10$

   a) Complete the table.

t	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
x	*1*	*0*	*-1*	*-2*	*-3*	*-4*	*-5*	*-6*	*-7*	*-8*	*-9*
y	*0*	*1*	*1.4*	*1.7*	*2*	*2.24*	*2.25*	*2.65*	*2.83*	*3*	*3.16*

   b) Plot the points (x, y) from the table labeling each point with the parameter value t.

   c) Describe the orientation of the curve. *The orientation is from right to left.*

2. Given the parametric equations $x = 6 - t^3$ and $y = 3\ln t$ $0 < t \le 5$.

   a) Complete the table

t	*1*	*2*	*3*	*4*	*5*
x	*5*	*-2*	*-21*	*-58*	*-119*
y	*0*	*2.08*	*3.30*	*4.16*	*4.83*

   b) Plot the points (x, y) from the table labeling each point with the parameter value t.

   c) Describe the orientation of the curve. *The orientation is from right to left.*

Part II.

a) Graph using a graphing calculator.

b) Eliminate the parameter and write the equation with rectangular coordinates.

c) Answer the following questions.

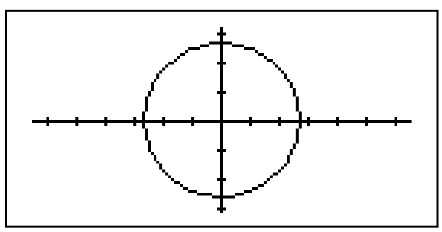
   i. For which curves is y a function of x?

   ii. What if any restrictions are needed for the two graphs to match?

1. $x = 4\cos t$, $y = 4\sin t$ for $0 \le t < 2\pi$

   $x^2 + y^2 = 16$

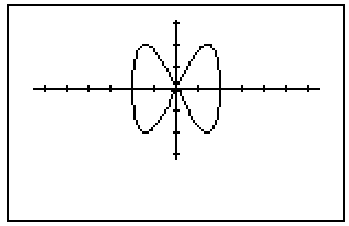
   not a function, no restrictions needed



2. $x = \cos t$, $y = \sin 2t$ for $0 \le t < 2\pi$

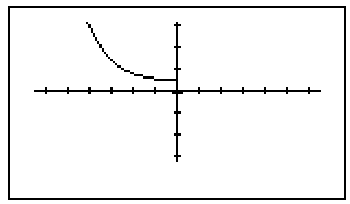
$y^2 = 4x^2(1 - x^2)$

not a function
no restrictions needed



3. $x = -3\sqrt{t}$, $y = e^t$ for $0 \le t \le 10$

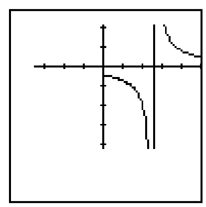
$y = e^{x^2/9}$ $0 \le x \le -9.49$
$1 \le y \le 22{,}026.47$



4. $x = \dfrac{t}{2}$, $y = \dfrac{2}{t-3}$ for $0 \le t \le 10$

$y = \dfrac{2}{2x-3}$ $0 \le x < 1.5$ or $1.5 < x \le 5$

$y < -\dfrac{2}{3}$ or $y > \dfrac{2}{7}$



Part III.

1. Consider two objects in motion over the time interval $0 \le t \le 2\pi$. The position of the first object is described by the parametric equations $x_1 = 2\cos t$ and $y_1 = 3\sin t$. The position of the second object is described by the parametric equations $x_2 = 1 + \sin t$ and $\cos t - 3$. At what times do they collide?

Set $x_1 = x_2$ and $y_1 = y_2$ and solve the resulting system of equations. The objects collide when t = $\dfrac{3\pi}{2}$. This occurs at the point (0, -3). Graphing the two parametric equations shows another point of intersection but it is not a point of collision.

2. Find all intersection points for the pair of curves.
$x_1 = t^3 - 2t^2 + t;\ y_1 = t$ and $x_2 = 5t;\ y_2 = t^3$. Indicate which intersection points are true collision points. Use the interval $-5 \le t \le 5$.

Point of collision is (0, 0). Other points of intersection: ≈ (-5.22, -1.14) and ≈ (6.89, 2.62)

Unit 8, Activity 6, Modeling Motion using Parametric Equations

1. A skateboarder goes off a ramp at a speed of 15.6 meters per second. The angle of elevation of the ramp is 13.5°, and the ramp's height above the ground is 1.57 meters.

    a) Give the set of parametric equations for the skater's jump.

    b) Find the horizontal distance along the ground from the ramp to the point he lands.

2. A baseball player hits a fastball at 146.67 ft/sec (100 mph) from shoulder height (5 feet) at an angle of inclination 15° to the horizontal.

    a) Write parametric equations to model the path of the project.

    b) A fence 10 feet high is 400 feet away. Does the ball clear the fence?

    c) To the nearest tenth of a second, when does the ball hit the ground? Where does it hit?

    d) What angle of inclination should the ball be hit to land precisely at the base of the fence?

    e) At what angle of inclination should the ball be hit to clear the fence?

3. A bullet is shot at a ten foot square target 330 feet away. If the bullet is shot at the height of 4 feet with the initial velocity of 200 ft/sec and an angle of inclination of 8°, does the bullet reach the target? If so, when does it reach the target and what will be its height when it hits?

4. A toy rocket is launched with a velocity of 90 ft/sec at an angle of 75° with the horizontal.

    a) Write the parametric equations that model the path of the toy rocket.

    b) Find the horizontal and vertical distance of the rocket at $t = 2$ seconds and $t = 3$ seconds.

    c) Approximately when does the rocket hit the ground? Give your answer to the nearest tenth of a second.

1. A skateboarder goes off a ramp at a speed of 15.6 meters per second. The angle of elevation of the ramp is 13.5°, and the ramp's height above the ground is 1.57 meters.
a) Give the set of parametric equations for the skater's jump.

a) $x = (15.6\cos 13.5^o)t$ and $y = -\frac{1}{2}(9.8)t^2 + (15.6\sin 13.5^o)t + 1.57$

$x = 15.2t$ and $y = -4.9t^2 + 3.64t + 1.57$

b) Find the horizontal distance along the ground from the ramp to the point he lands.
He will land when y = 0. y = 0 when t = 0.954
then x = 15.2(0.954) = 14.5 meters

2. A baseball player hits a fastball at 146.67 ft/sec (100 mph) from shoulder height (5 feet) at an angle of inclination 15° to the horizontal.

a) Write parametric equations to model the path of the project.

$x = (146.67\cos 15^o)t$

$y = -16t^2 \_(146.67\sin 15^o)t + 5$

b) A fence 10 feet high is 400 feet away. Does the ball clear the fence?
No

c) To the nearest tenth of a second when does the ball hit the ground? Where does it hit?
2.5 seconds; 354.2 feet

d) What angle of inclination should the ball be hit to land precisely at the base of the fence?
17.4° (Note: Have the students set the xmax to 400) Change the angle in increments and trace to the point where the graph touches the x-axis.)

e) At what angle of inclination should the ball be hit to clear the fence?
≈ 19.2° (Change the angle in increments until the graph intersects the point (400, 10.1))

3. A bullet is shot at a ten foot square target 330 feet away. If the bullet is shot at the height of 4 feet with the initial velocity of 200 ft/sec and an angle of inclination of 8°, does the bullet reach the target? If so, when does it reach the target and what will be its height when it hits?
5 feet 10 inches at ≈1.67 seconds

4. A toy rocket is launched with a velocity of 90 ft/sec at an angle of 75^{o} with the horizontal.

a) Write the parametric equations that model the path of the toy rocket.

$$x = 90\cos 75^{o}\, t$$

$$y = 90\sin 75^{o}\, t - 16t^{2}$$

b) Find the horizontal and vertical distances of the rocket at t = 2 seconds and t = 3 seconds.

At t = 2 seconds x = 46.59 feet and y = 109.87

At t = 3 seconds x = 69.88 feet and y = 116.8 feet

c) Approximately when does the rocket hit the ground? Give your answer to the nearest tenth of a second.

5.4 seconds

Unit 8, General Assessments, Spiral

Work each of the following. Show all work and any formulas used.

1. The polar coordinates of a point are given. Find the rectangular coordinates of that point.

   a) $\left(4, \frac{3\pi}{4}\right)$

   b) $\left(-3, -\frac{\pi}{3}\right)$

2. The rectangular coordinates of a point are given. Find polar coordinates of the point.

   a) (-3, -3)

   b) $\left(-2, -2\sqrt{3}\right)$

3. Below are equations written in rectangular coordinates. Rewrite the equations using polar coordinates.

   a) $y^2 = 2x$

   b) $2xy = 1$

   c) $2x^2 + 2y^2 = 3$

4. Below are equations written in polar form. Rewrite the equations using rectangular coordinates.

   a) r = cos θ

   b) $r = \frac{4}{1 - \cos\theta}$

   c) $2r^2 \sin 2\theta = 9$

Unit 8, General Assessments, Spiral with Answers

1. The polar coordinates of a point are given. Find the rectangular coordinates of that point.

a) $\left(4, \frac{3\pi}{4}\right)$ $\quad\left(-2\sqrt{2}, 2\sqrt{2}\right)$

b) $\left(-3, -\frac{\pi}{3}\right)$ $\quad\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$

2. The rectangular coordinates of a point are given. Find polar coordinates of the point that lie in the interval [0, 2π).

a) (-3, -3) $\quad\left(3\sqrt{2}, \frac{5\pi}{4}\right)$

b) $\left(-2, -2\sqrt{3}\right)$ $\quad\left(4, \frac{4\pi}{3}\right)$

3. Below are equations written in rectangular coordinates. Rewrite the equations using polar coordinates.

a) $y^2 = 2x$ $\quad r^2 \sin^2\theta = 2r\cos\theta$

b) $2xy = 1$ $\quad r^2 \sin 2\theta = 1$

c) $2x^2 + 2y^2 = 3$ $\quad 2r^2 = 3 \text{ or } r^2 = \frac{3}{2}$

4. Below are equations written in polar form. Rewrite the equations using rectangular coordinates.

a) $r = \cos\theta$ $\quad x^2 + y^2 = x$

b) $r = \dfrac{4}{1-\cos\theta}$ $\quad y^2 = 8(x + 2)$

c) $2r^2 \sin 2\theta = 9$ $\quad 4xy = 9$