

1.2 From first principles to a neutrons star

Let us first see, what can we learn from neutron stars using simple estimates and conservation laws.

Suppose a neutron star is, like any normal star, a blob of gas held together by the inwards pulling gravity. Gravity does not prefer any direction more than some other and so a stable end-result is an isotropic configuration. A pure inward pulling force is, of course, not enough so we also need a countering outward-facing force to resist the compression of the material. As an first approximation, there is no need to assume that this force would have any preferred direction either. Hence, our expected outcome is a sphere held together by the gravity originating from the mass M of the matter itself. Let us, for a while forget the exact origin and nature of the compression-resisting force and see what can we learn solely from the current information only.

Orders of magnitude Neutrons star are born from the death of a normal star. Most familiar such a star is our Sun 1.496×10^{13} cm from us.⁵ With a mass of $M_{\odot} = 1.99 \times 10^{33}$ g and radius of $R_{\odot} = 6.96 \times 10^{10}$ cm, our Sun then introduces us some typical stellar dimensions. Curiously, this means that the mean density of the Sun is $\rho_{\odot} \approx 1.41 \text{ g cm}^{-3}$, a mere 1.4× the density of the water. It turns out that Sun is also not as stable as one would think: With a rotation period of about 25.5 days it then takes Sun about a month to revolve around itself. Similar to a bicycle dynamo hub, this rotation also gives rise to a detectable surface magnetic field of $B_{\odot} \approx 1 \text{ Gauss}$.⁶

A typical neutron star, on the other hand, weights about $M \sim 1.5 M_{\odot}$ but extends only up to $R \sim 10$ km. Such dimensions give us an impressive mean density of $\rho \sim 7 \times 10^{14} \text{ g cm}^{-3}$. In comparison, a typical nucleon (such as a neutron) weights about 1.67×10^{-24} g and has a radius of about 1.25×10^{-13} cm, yielding us a nuclear density of $\rho_n \approx 2 \times 10^{14} \text{ g cm}^{-3}$. Hence, even the mean density inside the star is already on the same order of magnitude as the internal density inside nuclei. This suggests us that the composition inside a neutron star is not our typical every-day matter.

1.3 What do we actually know?

⁵Throughout this thesis we will typically present our quantities only up to some fixed precision instead of the full litany of numbers. We will also adopt the centimeter-gram-second (cgs) unit system instead of the (maybe) more common SI-system. Such a

selection is sure to disappoint some, but try to endure.

⁶A typical refrigerator magnet is about 50× stronger with a magnetic field of 50 Gauss.

2.0.2 Tolman-Volkoff-Oppenheimer equations

Newtonian pressure gradient needed to oppose the gravity is

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}. \quad (2.6)$$

Taking into account the general relativistic corrections we get

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \times \frac{(1 + P/\rho c^2)(1 + 4\pi r^3 P/mc^2)}{1 - 2Gm/rc^2}. \quad (2.7)$$

Difference originates from the source of gravity: in the Newtonian case it is the mass m , whereas in the General relativity it is the energy momentum tensor that depend both on the energy density and the pressure. As a result, energy and pressure give rise to a gravitational fields.

It has an important consequence to the stability of neutron stars: Successive increase in the pressure to counter the gravity is ultimately self-defeating.

Solution for a constant density ρ_0 gives

$$P(r) = G \frac{2\pi}{3} \rho_0^2 (R^2 - r^2) \quad (2.8)$$

whereas the GR gives

$$P(r) = \rho_0^2 c^2 \left[\frac{(1 - u(\frac{r}{R})^2)^{1/2} - (1 - u)^{1/2}}{3(1 - u)^{1/2} - (1 - u(\frac{r}{R})^2)^{1/2}} \right], \quad (2.9)$$

where $u = 2GM/Rc^2$.

2.1 Equation of state

Often means dependency between P and ρ . Or sometimes the associated energy density $\epsilon = \rho c^2$. Also depends on T but composed mainly on strongly degenerate fermions so so temperature dependency is negligible.

Bulk property of the sea of fermions.

Eos for $\rho > \rho_n$ can not be produced in laboratory. Can not be calculated because of the lack of precise many-body theory of strongly interacting particles.

Baryon mass M_b that is sum of baryon masses. Gravitational mass M that is M_b from where the gravitational binding energy is subtracted.²

2.2 Atmosphere

Thin layer of plasma From centimeters in hot to millimeters in cold Zavlin & Pavlov 2002

Where spectrum or thermal electromagnetic radiation is formed. Spectrum, beaming and polarization of emerging radiation can be determined from radiation transfer problem in atmospheric layers.

Contains information on the parameters of the surface: effective temperature surface gravity chemical composition geometry of the system mass and radii.

Eddington limit of where radiation force exceeds the gravitational one.

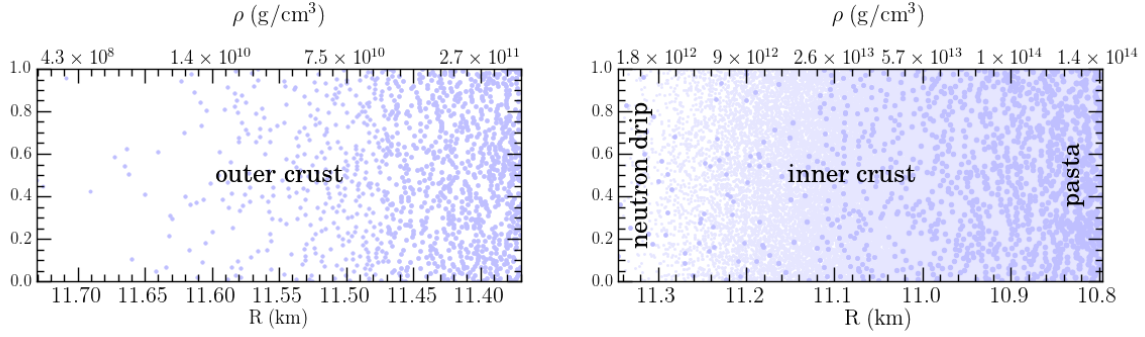


Figure 2.1: Molecular simulation of the crust. Figure adapted from <https://github.com/awsteiner/nstar-plot>.

2.3 Crust

Outer crust

From atmosphere to $\rho_N D \sim 4 \times 10^{11} \text{ g cm}^{-3}$. In thickness some hundred meters. Non degenerate electron gas Ultra-relativistic electron gas $\rho > 10^6 \text{ g cm}^{-3}$. Pressure provided by electrons here.

In deeper layers ions form a strongly coupled Coulomb system (liquid or solid). Hence, crust. Fermi energy grows with increasing ρ . Induces β captures and enriches nuclei with neutrons. At the base neutrons start to drip out from nuclei.

Inner crust About one kilometer thick. Density from $\rho \sim \rho_{ND}$ (at upper boundary) to $\sim 0.5 \rho_n$ at the base. Matter consists of electrons, free neutrons n and neutron-rich atomic nuclei. Fraction of free n grow with ρ .

Finally, nuclei disappear at the crust-core interface.

2.4 Core

Outer core. Density ranges $0.5 \rho_n < \rho < 2 \rho_n$. Several kilometers. Neutrons with several per cent admixture of protons p and electrons e^- . Strongly degenerate. Electrons form almost ideal Fermi gas. Neutrons and protons, interacting via nuclear forces, constitute a strongly interacting Fermi liquid.

Inner core. Where $\rho > 2 \rho_n$. Central density can be around $(10 - 15) \rho_n$. Very model dependent. Main problem.

² [5] F. Zwicky. *ApJ*. (1938).

3 Bibliography

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