Neutron stars: from first principles to cutting edge

Joonas Nättilä

Acknowledgements

I thank everybody.

List of publications

The work on this thesis is based on the following publications:

Theory

Radiation from rapidly rotating oblate neutron stars

Models of neutron star atmospheres enriched with nuclear burning ashes

Observations

The effect on accretion on the measurement of neutron star mass and radius in the low-mass X-ray binary 4U 1608−52

The influence of accretion geometry on the spectral evolution during the thermonuclear (type I) X-ray bursts

Equation of state constraints for the cold dense matter inside neutron stars using the cooling tail method

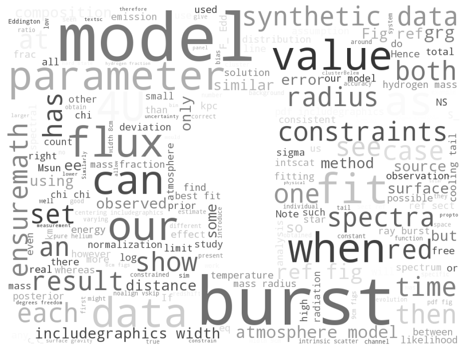
Detection of burning ashed from thermonuclear X-ray bursts

The direct cooling tail method for X-ray burst analysis to constrain neutron star masses and radii

Abstract

Neutron stars are the best.

Contents



1 Introduction

Neutrons stars are curious objects encompassing many still unsolved problems of modern physics and astrophysics. Their unique nature makes them ideal laboratories for many of the most energetic phenomena in space. Born from the ashes of a supernova, they begin their life only when some other normal star fades away and dies in a spectacular supernova explosion.[1] From there on, they continue their life by devouring the surrounding interstellar matter or an unlucky companion star floating next to them. It is not the impressive $\sim 10^{30}$ kilos they weight but the mere $\sim 10\,\mathrm{km}$ in radius sphere that they encapsulate this material into that is then able to bend the spacetime itself. Such an impressive feat rewards them with a categorization into to a group called *compact objects*.

In the next few sections, we will start by building our intuition of these peculiar objects habiting the space around us.

1.1 From first principles to a neutrons star

Let us first see, what can we learn from neutron stars using simple estimates and conservation laws.

Suppose a neutron star is, like any normal star, a blob of gas held together by the inwards pulling gravity. Gravity does not prefer any direction more than some other and so a stable end-result is an isotropic configuration. A pure inward pulling force is, of course, not enough so we also need a countering outward-facing force to resist the compression of the material. As an first approximation, there is no need to assume that this force would have any preferred direction either. Hence, our expected outcome is a sphere held together by the gravity originating from the mass M of the matter itself. Let us, for a while forget the exact origin and nature of the compression-resisting force and see what can we learn solely from the current information only.

Neutrons star are born from the death of a normal star. Most familiar such a star is our Sun 1.496×10^{13} cm from us.[2] With a mass of $M_\odot = 1.99 \times 10^{33}\,\mathrm{g}$ and radius of $R_\odot = 6.96 \times 10^{10}\,\mathrm{cm}$, our Sun then introduces us some typical stellar dimensions. Curiously, this means that the mean density of the Sun is $\rho_\odot \approx 1.41\,\mathrm{g\,cm^{-3}}$, a mere $1.4\times$ the density of the water. It turns out that Sun is also not as stable as one would think: With a rotation period of about 25.5 days it then takes Sun about a month to revolve around itself. Similar to a bicycle dynamo hub, this rotation also gives rise to a detectable surface magnetic field of $B_\odot \approx 1\,\mathrm{Gauss}$.[3]

Orders of magnitude

A typical neutron star, on the other hand, weights about $M \sim 1.5\,M_\odot$ but extends only up to $R \sim 10\,\mathrm{km}$. Such dimensions give us an impressive mean density of $\rho \sim 7 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. In comparison, a typical nucleon (such as a neutron) weights about 1.67×10^{-24} g and has a radius of about 1.25×10^{-13} cm, yielding

[1] For the pedantic ones, we specify that a supernova is actually an implosion followed by a subsequent explosion.

[2] Throughout this thesis we will typically present our quantities only up to some fixed precision instead of the full litany of numbers. We will also adopt the centimeter-gram-second (cgs) unit system instead of the (maybe) more common SI-system. Such a selection is sure to disappoint some, but try to endure.

[3] A typical refrigerator magnet is about $50\times$ stronger with a magnetic field of 50 Gauss.

us a nuclear density of $\rho_{\mathrm{n}} \approx 2 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. Hence, even the mean density inside the star is already on the same order of magnitude as the internal density inside nuclei. This suggests us that the composition inside a neutron star is not our typical every-day matter.

1.2 More advanced considerations...

1.2.1 Why neutrons then?

Let us first consider ideal gas of degenerate electron-proton-neutron plasma. In a degenerate plasma all the quantum states are filled up all the way to the Fermi energy. It is the Pauli exclusion principle that then prevents occupying all of these already taken quantum states. Normal beta-decay mode for the neutrons, on the other hand, is $n \to p + e^- + \bar{\nu}_e$, that describes the possible path of how a neutron n will decay into a proton p, electron e^-, and electron neutrino $\bar{\nu}_e$. Such a decay is, however, blocked because there is no room for an emission of an extra electron e^- or a proton p.[4]

Let us then only focus on the decay of the most energetic neutrons with an energy equal to the Fermi energy $\epsilon_{\mathrm{F}}(n)$. Co-existence of neutrons, protons, and electrons is then guaranteed (at zero temperature) if

$$\epsilon_{\mathrm{F}}(n) = \epsilon_{\mathrm{F}}(p) + \epsilon_{\mathrm{F}}(e^-). \tag{1.1}$$

Fermi momentum of a particle is related to its concentration via

$$p_{\mathrm{F}} = \left(\frac{3n}{8\pi}\right)^{1/3} h, \tag{1.2}$$

where n is the number density, and h the Planck constant. Massive neutrons and protons are to a good approximation non-relativistic up to a densities of ρ_{n}, and hence energy is simply a sum of their rest mass energy and kinetic energy

$$\epsilon_{\mathrm{F}}(n) \approx m_n c^2 + \frac{p_{\mathrm{F}}(n)^2}{2m_n}, \tag{1.3}$$

and

$$\epsilon_{\mathrm{F}}(p) \approx m_p c^2 + \frac{p_{\mathrm{F}}(p)^2}{2m_p}. \tag{1.4}$$

Electrons, on the other hand, are already ultra-relativistic, and so

$$\epsilon_{\mathrm{F}}(e^-) \approx p_{\mathrm{F}}(e^-) c^2. \tag{1.5}$$

Also note that $n_p = n_e$, as the star is electrically neutral. From this we find relation of the $n_n/n_p \sim 1/200$ by taking into account the rest mass difference $m_p - m_n = 2.6\mathrm{MeV}\,c^2$ at $\rho \sim \rho_{\mathrm{n}}$. Thus, we conclude that the matter inside is neutron rich.

1.2.2 Tolman-Volkoff-Oppenheimer equations

Newtonian pressure gradient needed to oppose the gravity is

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}. \tag{1.6}$$

Taking into account the general relativistic corrections we get

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \times \frac{(1 + P/\rho c^2)(1 + 4\pi r^3 P/mc^2)}{1 - 2Gm/rc^2}. \tag{1.7}$$

[4] see e.g. [1] A. C. Phillips. 1994.

Difference originates from the source of gravity: in the Newtonian case it is the mass m, whereas in the General relativity it is the energy momentum tensor that depend both on the energy density and the pressure. As a result, energy and pressure give rise to a gravitational fields.

It has an important consequence to the stability of neutron stars: Successive increase in the pressure to counter the gravity is ultimately self-defeating.

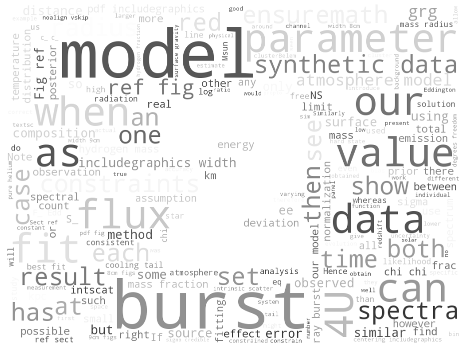
Solution for a constant density ρ_0 gives

$$P(r) = G\frac{2\pi}{3}\rho_0^2(R^2 - r^2) \tag{1.8}$$

whereas the GR gives

$$P(r) = \rho_0^2 c^2 \left[\frac{(1-u\left(\frac{r}{R}\right)^2)^{1/2} - (1-u)^{1/2}}{3(1-u)^{1/2} - (1-u\left(\frac{r}{R}\right)^2)^{1/2}}\right], \tag{1.9}$$

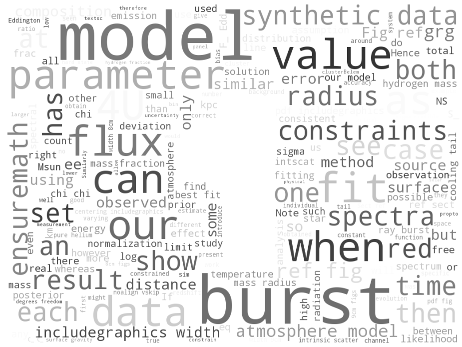
where $u = 2GM/Rc^2$.



2 First principle physics

Blaa blaa here we ref to[1].

1 [2] J. Nättilä *et al*. *A&A*. (2016).



3 Summary of the original publications

3.1 The author’s contribution to the publications

3.2 Radiation from rapidly rotating oblate neutron stars

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4 Bibliography

[1] A. C. Phillips. *The physics of stars*. 1994.

[2] J. Nättilä, A. W. Steiner, J. J. E. Kajava, V. F. Suleimanov, and J. Poutanen. *A&A*. 591, A25. (2016) . "Equation of state constraints for the cold dense matter inside neutron stars using the cooling tail method" DOI: 10.1051/0004-6361/201527416.