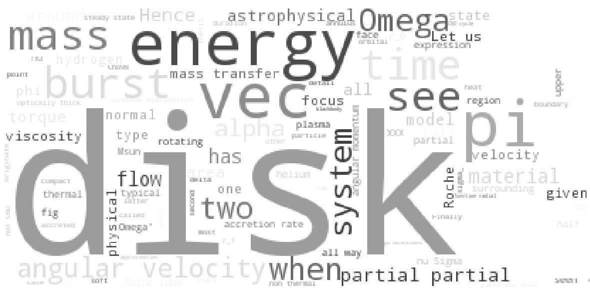
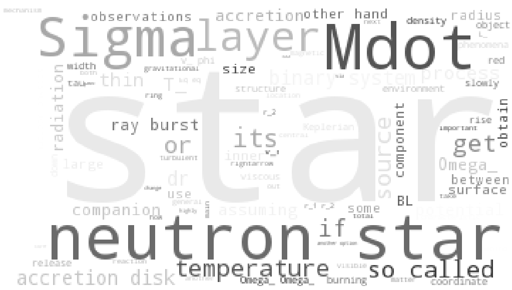
Contents

CONTENTS





1 Astrophysics around neutron stars

Let us next discuss the violent environments around neutron stars and the related astrophysics therein. These surroundings also play an important role when we try to decipher various different kinds of observations of neutron stars. In the heart of this whole problem is an astrophysical process called accretion. This is a physical process where matter is transferred from one source to another because of the gravitational forces. Most importantly, the infalling material has to somehow loose its angular momentum, before it is able to travel all the way to the neutron star surface. Nature's mechanism to do this is called an accretion disk. Sometimes the disk also continues all the way to the surface of the neutron star. Instead of smoothly joining the star, this process is often violent and leads to another source of radiation as this so-called boundary layer heats up when the rapidly rotating inner disk tries to slow down to the velocities similar to that of the more gently rotating star.

We begin by discussing the basics of the accretion process and present some order-of-magnitude estimates relevant for the accretion disks and boundary layers. Most importantly, we try to characterize the strength of the radiation originating from them. Lastly, we will also discuss the unstable thermonuclear runaways, X-ray bursts, that can be seen as the end-results of this accretion.

1.1 Accretion

Accretion is an astrophysical process that has its roots in the gravitational potential energy. It can be a source of enormous amounts of energy if the central object is compact, because the depth of a gravitational well is directly proportional to the compactness of the central source. Hence, it is an important, and often dominating, process for neutron stars.*

Gravitational potential energy release for a mass m that is accreted onto a compact

*For an introduction, see e.g., [1] J. Frank, A. King, and D. J. Raine. 2002.

object of radius R and mass M is

$$\Delta E_{\mathrm{acc}} = m\frac{GM}{R} \sim 10^{20}\left(\frac{m}{\mathrm{g}}\right)\left(\frac{10\,\mathrm{km}}{R}\right)\left(\frac{M}{\mathrm{M}_\odot}\right)\,\mathrm{erg}, \tag{1.1}$$

where in the latter expression typical dimensions of neutron star are used.

This energy, 10^{20} erg per each gram that is accreted, is usually released as radiation. The rate of this energy release is simply related to the mass accreted per time, i.e., accretion rate $\dot{M}$,

$$L_{\mathrm{acc}} = \dot{M}\frac{GM}{R} \approx 1.3\times10^{36}\left(\frac{\dot{M}}{10^{16}\,\mathrm{g\,s^{-1}}}\right)\left(\frac{10\,\mathrm{km}}{R}\right)\left(\frac{M}{\mathrm{M}_\odot}\right)\,\mathrm{erg\,s^{-1}}, \tag{1.2}$$

where a typical value of $\dot{M} \sim 10^{16}\,\mathrm{g\,s^{-1}} \approx 1.5\times10^{-10}\,\mathrm{M_\odot\,yr^{-1}}$ is taken for the accretion rate. Hence, depending on the accretion rate, this value can be about the same as the Eddington luminosity (Eq. (**??**)) of a neutron star.

1.1.1 Roche lobes and mass transfer in binary systems

In order to use the accretion as an energy source, mass transfer needs to take place in the system. For the mass transfer to keep on operating, a source of fresh material is needed. In binary systems with two stars, the companions star is the obvious fuel resource. Here we will focus on the so-called Low Mass X-ray Binary (LMXB) systems where the companion, like the name implies, is a relatively low-weight star.* Typically, it is a normal or a late-type star with a mass $M \lesssim 1\,\mathrm{M}_\odot$. Such a setup leads to a mass-transfer quite naturally as the heavier neutron star will just rip out the outer layers of its poor companion and slowly devours it, until nothing is left. As another option, the system could be a so-called High Mass X-ray Binary (HMXB) system, where the companion of the neutron star has $M \sim 10\,\mathrm{M}_\odot$, and the accretion happens, for example, via a neutron star traveling through the outer envelope of the other stars. Here, we will, however, only focus on the LMXB systems, as they provide a relatively stable mass-transfer mechanism.

How exactly is the material transferred form the companion to the primary star is an interesting problem. We can begin to understand the physical setup by considering a general hydrodynamical system of two objects in a rotating frame. Here we select the frame such that it co-rotates with the binary system. The subsequent flow of gas between the two stars can then be described by the Euler equation with additional Corriolis and centrifugal terms.† In practice the Euler equation describes the time evolution of the velocity $\boldsymbol{v}$ of the gas that has a pressure P and density ρ. In a reference frame rotating together with the

* [2] T. M. Tauris and E. P. J. van den Heuvel. 2006.

†see, e.g, [3] A. R. Choudhuri. 1998, for a good introduction.

binary system with angular velocity ω the Euler equation takes the form

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla\Phi_{\mathrm{R}} - 2\boldsymbol{\omega} \times \boldsymbol{v} - \frac{1}{\rho}\nabla P, \tag{1.3}$$

where the angular velocity of the binary is

$$\boldsymbol{\omega} = \left(\frac{GM}{a^3}\right)^{1/2} \boldsymbol{e}, \tag{1.4}$$

as given with the unit vector $\boldsymbol{e}$ normal to the orbital plane. Here M is the total mass of the system, i.e., $M = M_1 + M_2$, where M_1 and M_2 are the individual masses of the two stars in the system, respectively, and a is their orbital separation.

The effects originating from the gravitation and from the centrifugal forces are encapsulated in the so-called Roche potential, given as a function of radial vector $\boldsymbol{r}$ as*

$$\Phi_{\mathrm{R}}(\boldsymbol{r}) = -\frac{GM_1}{|\boldsymbol{r} - \boldsymbol{r_1}|} - \frac{GM_2}{|\boldsymbol{r} - \boldsymbol{r_2}|} - \frac{1}{2}\,(\boldsymbol{\omega} \times \boldsymbol{v}\,)^2, \tag{1.5}$$

where the location of the stars are given with $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$. By studying the shape of the potential, we see that in between the two stars, in the so-called L_1 point there exists a location where the countering gravitational forces from the two stars are balanced. This can be though of as a physical nozzle in the system from which the less massive star will leak into the more massive star. Such a mass transfer, also known as a Roche lobe overflow, will then occur if the companion star's radius exceeds the size of its own individual Roche lobe visualized in Fig. 1.1. Typically such a thing can happen when the star evolves and expands at the end of its life cycle.

1.1.2 Accretion disks

When the mass transfer has started via the Roche lobe overflow, and we have a stable source of material that is being transferred from the companion to the more massive neutron star, we can next focus on the region where gravitational forces of the neutron star dominate. Originally, we can think of each individual incoming particle having their own circular orbit around the central object. The mass flow, i.e., the stream of particles, is confined into the orbital plane of the binary system, hence, the problem of describing the physics of the flow is two dimensional as a first approximation. This is known as the so-called thin-disk approximation. The nested circular orbits of the plasma can be naturally described in cylindrical coordinates, hence the word "disk". Secondly, because we neglect

*see, e.g., [4] P. Podsiadlowski, S. Rappaport, and E. D. Pfahl. *ApJ*. (2002); [5] D. A. Leahy and J. C. Leahy. *Computational Astrophysics and Cosmology*. (2015).

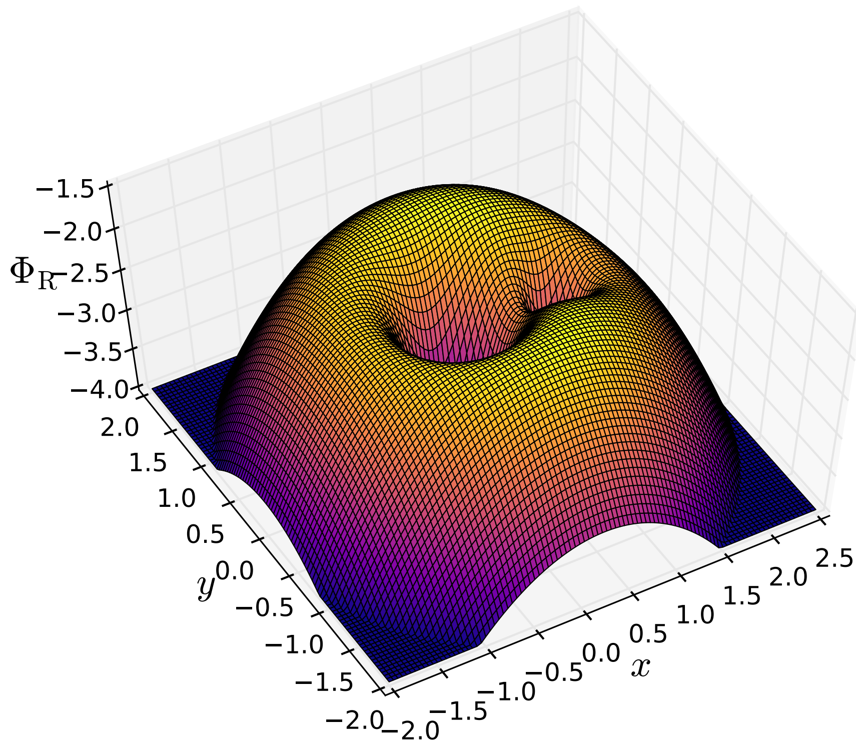


Figure 1.1: Two-dimensional Roche potential $\Phi_R(x, y)$ visualized for a binary systems with $M_1/M_2 = 0.25$ and $a = 1$. The nozzle (L_1 point) is visible as a valley (or more specifically, a saddle point) between the gravitational wells of the two stars.

all the structure that the accreted layer might have in the vertical direction, we say that the disk is infinitely thin.

Radial structure of the disk can then be obtained from the Keplerian rotation law as

$$\Omega_K(r) = \left(\frac{GM}{R^3}\right)^{1/2}, \tag{1.6}$$

describing an angular velocity as a function of radial coordinate from the disk center. There is an important detail hidden here in the Kepler's law: Keplerian angular velocity implies differential rotation, i.e., varying angular velocities as a function of the distance from the center of the disk. Such a shearing between two adjacent annulus will then lead to viscous stresses when the inner annulus rotates faster than the outer one.

The disk's structure can be obtained from the hydrodynamical equations in cylindrical coordinates. Instead of the density ρ, let us use the surface density $\Sigma = \rho H$ to describe the mass, where H is the height of the disk. Conservation of mass and angular momentum can then be written as*

$$r\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r}(r\Sigma v_r) = 0, \tag{1.7}$$

and

$$r\frac{\partial}{\partial t}(\Sigma r^2 \Omega) + \frac{\partial}{\partial r}(r\Sigma v_r r^2 \Omega) = \frac{1}{2\pi}\frac{\partial \tau}{\partial r}, \tag{1.8}$$

where v_r is the velocity in r direction and τ is the viscous torque of the differentially rotating disk.

Torque, in general, is the net outward angular momentum flux given as

$$\tau_i = \epsilon_{ijk} r_j f_k, \tag{1.9}$$

where ϵ_{ijk} is the Levi-Civita symbol, and f_k is the force density given as

$$f_k = \sigma_{kh} n_h. \tag{1.10}$$

Here σ_{kh} is the kh-component of a shear tensor σ, and n_h is some surface normal of the area where the shearing takes place. In our case, we can compute the shear in cylindrical coordinate system focusing on r and ϕ coordinates only as

$$\sigma_{r\phi} = \rho\nu\left(r\frac{\partial}{dr}\left\{\frac{v_\phi}{r}\right\} + \frac{1}{r}\frac{dv_r}{d\phi}\right), \tag{1.11}$$

which simplifies to

$$\sigma_{r\phi} = \rho\nu r\frac{d\Omega}{dr}, \tag{1.12}$$

when we remember that $v_\phi = r\Omega$ and assuming the flow to be symmetric on ϕ-direction ($\partial v_r/\partial\phi = 0$). The total torque exerted by the $2\pi r H$ area corresponding to the disk rim at location r is then

$$\tau(r,t) = 2\pi r\nu\Sigma r^2\Omega', \tag{1.13}$$

where $\Omega' = d\Omega/dr$.

*see e.g., [3] A. R. Choudhuri. 1998; [1] J. Frank, A. King, and D. J. Raine. 2002, for full derivation.

Combining the latter expressions and assuming Keplerian rotation $\Omega(r) = \Omega_{\mathrm{K}}(r)$ we can then solve the system to obtain expressions for the surface density and the radial velocity as

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\left[r^{1/2}\frac{\partial}{\partial r}(\nu\Sigma r^{1/2})\right], \tag{1.14}$$

and

$$v_r = -\frac{3}{\Sigma r^{1/2}}\frac{\partial}{\partial r}(\nu\Sigma r^{1/2}). \tag{1.15}$$

In order to continue further, we would need a description for the viscosity ν. This could have its roots on the normal molecular viscosity* or like the current theories imply, be magnetohydrodynamic in nature. The latter is known as the magneto-rotational instability (MRI) where magnetic stresses of turbulent field lines cause viscosity for the plasma† Velikhov59 As another approach, we should also mention the widely successful α-parameterization by Shakura & Sunyaev, known as the "standard disk" or α-disk.‡ This method is mostly mathematical as we just reparameterize our ignorance of the viscosity into a new parameter called α, that is taken to be proportional to the local pressure in the disk. Even though mathematical in nature, this formulation has turned out to be extremely successful in helping to explain the basic functionality of accretion disks.

To get some idea of the disk dynamics, we can, as our zeroth order approximation assume ν = constant. Then, the time-dependent disk structure (Eqs. (1.7)-1.8) can be solved, for example, by assuming as an initial condition a ring of mass m at $r = r_0$,

$$\Sigma(r, t = 0) = \frac{m}{2\pi r_0}\delta(r - r_0), \tag{1.16}$$

where δ is the Dirac delta function. As a solution, we then obtain a mass distribution that slowly diffuses due to viscosity as

$$\Sigma(\tilde{x}, \tilde{\tau}) = \frac{m}{\pi r^2}\frac{1}{\tilde{\tau}\tilde{x}^{1/4}}\exp\left[-\frac{(1+\tilde{x}^2)}{\tilde{\tau}}\right] I_{1/4}\left(\frac{2\tilde{x}}{\tilde{\tau}}\right), \tag{1.17}$$

where $I_{1/4}(x)$ is the modified Bessel function and radial coordinate r is non-dimensionalized as $\tilde{x} = r/r_0$, and time as $\tilde{\tau} = 12\nu t/r_0^2$. Even from this simplified treatment we can understand the basic physics of how the accretion disks operate: The viscous shear stresses help the rotating plasma loose its angular momentum, which will then lead to a mass flow inwards, towards the center.

* [6] S. Chapman and T.G. Cowling. 1970.

† [7] S. Chandrasekhar. *Proceedings of the National Academy of Science*. (1960); [8] S. A. Balbus and J. F. Hawley. *ApJ*. (1991).

‡ [9] N. I. Shakura and R. A. Sunyaev. *A&A*. (1973).

Let us next study a steady-state disk solution by setting $\partial/\partial t \to 0$. From angular momentum conservation (1.8) we obtain

$$r\Sigma v_r r^2\Omega = \frac{\tau}{2\pi} + \frac{C}{2\pi}, \tag{1.18}$$

with a constant C that physically represent a torque term from the coupling of the inner disk and the star at $r = R$. In short, it is given by

$$C = -\dot{M}(GMR)^{1/2}, \tag{1.19}$$

by considering the expression for the mass accretion rate

$$\dot{M} = -\frac{2\pi r\Sigma dr}{dt} = -2\pi r\Sigma v_r, \tag{1.20}$$

and assuming a thin layer for the zone where the inner disk angular velocity is slowed down to the angular velocity of the star. Substituting this, and assuming Keplerian angular velocity profile, we obtain

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R}{r}\right)^{1/2}\right]. \tag{1.21}$$

Physically this represents a steady-state solution of a disk with central torque applied to it. Viscous dissipation rate per unit area in the disk is then*

$$D(r) = \frac{\tau\Omega'}{4\pi r} = \frac{1}{2}\nu\Sigma(r\Omega')^2 = \frac{3GM\dot{M}}{8\pi r^3}\left[1 - \left(\frac{R}{r}\right)^{1/2}\right]. \tag{1.22}$$

Finally, from here we can compute the luminosity of the disk faces due to energy lost by viscous dissipation

$$\begin{aligned} L(r_1, r_2) &= 2\int_{r_1}^{r_2} D(r)2\pi r dr = \frac{3GM\dot{M}}{2}\int_{r_1}^{r_2}\left[1 - \left(\frac{R}{r}\right)^{1/2}\right]\frac{dr}{r^2} \\ &= \frac{3GM\dot{M}}{2}\left\{\frac{1}{r_1}\left[1 - \frac{2}{3}\left(\frac{R}{r_1}\right)^{1/2}\right] - \frac{1}{r_2}\left[1 - \frac{2}{3}\left(\frac{R}{r_2}\right)^{1/2}\right]\right\}, \end{aligned} \tag{1.23}$$

and by then setting $r_1 \to R$ and $r_2 \to \infty$, we get

$$L_{\rm disk} = \frac{GM\dot{M}}{2R} = \frac{1}{2}L_{\rm acc}. \tag{1.24}$$

*Viscous dissipation rate in ring of width dr is $\tau\Omega' dr$ and the total area of the ring, taking into account both the lower and upper faces, is $4\pi r dr$. Hence, we obtain Eq. (1.22) as the ratio of these.

Hence, half of the potential energy will be lost by the viscous shearing and is radiated away by the upper and lower faces of the accretion disk. Importantly, the other remaining half will be transferred all the way to the star.

Temperature of the hottest inner disk can be estimated by assuming a blackbody radiator and using the dissipation rate D as our energy. This innermost region of the disk is the brightest as here the gravity is the strongest and the dissipation area the smallest. Estimate for the disk temperature can then be obtained from

$$\sigma_{\mathrm{SB}} T_{\mathrm{disk}}^4 \sim D(R), \tag{1.25}$$

where optically thick disk and blackbody radiator are assumed. From here we get

$$T_{\mathrm{disk}} = T_* \left[1 - \left(\frac{R}{r} \right)^{1/2} \right]^{1/4}, \tag{1.26}$$

where

$$T_* = \left(\frac{3GM\dot{M}}{8\pi R^3 \sigma_{\mathrm{SB}}} \right)^{1/4} \approx 2.3 \times 10^7 \left(\frac{\dot{M}}{10^{16}\,\mathrm{g\,s^{-1}}} \right)^{1/4} \left(\frac{M}{\mathrm{M}_\odot} \right)^{1/4} \left(\frac{R}{10\,\mathrm{km}} \right)^{-3/4} \mathrm{K}, \tag{1.27}$$

is a typical temperature in the innermost regions of the disk.

Observationally we see that the disks are, however, not as simple as discussed here.* The standard α-disk model assumes steady-state, whereas in reality the disk structure evolves in time. Most importantly, the mass accretion rate is seen to vary over long timescales. From observations we also know that the disks alternates between two states called hard and soft state.†

The soft (also known as "high" or "thermal-dominant") state is characterized by a strong soft component in the observed spectra.‡ There is, however, also a complex non-thermal tail usually present.§ Here the soft component could be interpreted as a thermal radiation from an optically thick disk but the non-thermal tail implies that this picture is not complete. The hard (also known as "low") state is even more complicated as the observational spectra is dominated by a strong hard X-ray part but some signs of low-temperature thermal disk component is also sometimes visible.¶

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*see [10] C. Done, M. Gierliński, and A. Kubota. *A&A Rev.* (2007), ,for a review.

† [11] K. Mitsuda *et al. PASJ*. (1989); [12] G. Hasinger and M. van der Klis. *A&A*. (1989); [13] M. Gierliński and C. Done. *MNRAS*. (2002); [14] T. J. Maccarone and P. S. Coppi. *MNRAS*. (2003); [15] T. Muñoz-Darias *et al. MNRAS*. (2014); [10] C. Done, M. Gierliński, and A. Kubota. *A&A Rev.* (2007).

‡ [16] M. Gierliński *et al. MNRAS*. (1999).

§ [17] M. L. McConnell *et al. ApJ*. (2000).

¶see, e.g., [18] A. A. Zdziarski and M. Gierliński. *Progress of Theoretical Physics Supplement*. (2004).

The current physical interpretation of these two states can be satisfactorily explained by a truncated disk model with a hot inner flow. Here the disk, somewhat well-described by the Shakura & Sunyaev α-disk is truncated, i.e., does not always reach the central object. The cool and optically thick, geometrically thin, disk is then responsible of the low-energy thermal radiation. In the innermost parts, the disk turns into hot, optically thin flow, that is also responsible for the non-thermal radiation component. Depending on the mass accretion rate, the disk truncation radius varies and so the strength of the thermal disk and non-thermal hot inner flow components can vary.

1.1.3 Boundary layers

Our simple analysis of accretion disk physics has shown that viscous dissipation can get rid of up to half of the potential energy of the incoming matter. Where the other half goes, we shall look next.

Imagine the accretion disk extending all the way down to the central star. The angular frequency of this disk edge can be taken to be of around Keplerian velocity, $\Omega_{\mathrm{K}}(R)/2\pi \sim$ 2000 Hz. The star, on the other hand, usually rotates anywhere from 100 to 600 revolutions per second.* Hence, we expect a thin interface between the star and the disk where angular velocity changes by a factor of $\sim$ 3 to 20. This region we call the boundary layer.

Let us assume a thin layer of width b so that $\Omega(R+b) \approx \Omega_{\mathrm{K}}(R+b)$. Within this layer, the angular velocity should decrease to Ω_* as we move from radial location $R+b$ to R towards the star's surface. The work done by the viscous torque to spin up the star is then

$$\tau_* = \dot{M}R^2(\Omega_{\mathrm{K}} - \Omega_*). \tag{1.28}$$

Rate of kinetic energy change is

$$\dot{E} = \frac{1}{2}\,\dot{M}R^2(\Omega_{\mathrm{K}}^2 - \Omega_*^2) = \frac{1}{2}\dot{M}\frac{GM}{R}\left[1 - \left(\frac{\Omega_*}{\Omega_{\mathrm{K}}}\right)^2\right] \tag{1.29}$$

For the expression of the total rate of energy change we need to subtract the constant work done by the viscous torque to get

$$L_{\mathrm{BL}} = \frac{1}{2}\,\dot{M}R^2(\Omega_{\mathrm{K}}^2 - \Omega_*^2) - \dot{M}R^2(\Omega_* - \Omega_{\mathrm{K}}) = \frac{1}{2}\,\frac{GM\dot{M}}{R}\left(1 - \frac{\Omega_*}{\Omega_{\mathrm{K}}}\right)^2 \tag{1.30}$$

In the limit $\Omega_* \ll \Omega_{\mathrm{K}}$ we obtain

$$L_{\mathrm{BL}} = \frac{1}{2}\,\frac{GM\dot{M}}{R} = \frac{1}{2}L_{\mathrm{acc}}. \tag{1.31}$$

* [19] A. L. Watts. *ARA&A*. (2012); [20] A. Papitto *et al. A&A*. (2014).

Let us finally estimate the temperature of the this layer. By assuming an optically thick region around the star we get a characteristic blackbody temperature from

$$A_{\rm BL}\sigma_{\rm SB}T_{\rm BL}^4 \sim L_{\rm BL}, \tag{1.32}$$

where $A_{\rm BL}$ is the area of the emitting region. As a reasonable first guess we can use $b \sim H$ so an annulus around the star has an area of $A_{\rm BL} = 2 \times 2\pi RH$ taking into account both top and bottom face. This corresponds to a temperature of

$$T_{\rm BL} \sim \left(\frac{GM\dot{M}}{8\pi\sigma_{\rm SB}R^2H}\right)^{1/4} \sim T_* \left(\frac{R}{H}\right)^{1/4} \tag{1.33}$$

As another option we can consider a so-called spreading layer.* Instead of assuming that the energy is dissipated in a thin equatorial ring, we can assume that it will spread to cover the whole star. In this case, $A_{\rm SL} \sim 4\pi R^2$, and using Eq. (1.32), we get

$$T_{\rm SL} \sim T_*, \tag{1.34}$$

i.e., a smaller temperature ($T_{\rm SL} < T_{\rm BL}$) that is comparable to the temperature of the disk.

Finally, one should note that the physical processes discussed here assume Newtonian gravity. In a general relativistic treatment the energy release in the boundary layer can be almost two times that of the energy released in the disk.†

1.2 X-ray bursts and unstable thermonuclear burning

Accretion can be a powerful energy source but this is the not end of the material that slowly spirals down to the star. After it has traveled all the way to the surface of the neutron star, it will slowly sink and mix with the material in the star's ocean. Here this new material — fresh fuel — will get compressed and heats up. Eventually the kinetic energy of the nuclei is large enough so that the protons will collide, fuse together, and release their rest-mass energy. This heat injection will then start an unstoppable chain reaction that creates a burning front that eventually covers the whole star. This thermonuclear fusion reaction will then last until all the available fuel on the star's upper layers is exhausted and consumed.

Observationally these bursts are classified as type-I X-ray bursts.‡ As another option, we might see flaring also from instabilities in the incoming mass flow. These events are

* [21] N. A. Inogamov and R. A. Sunyaev. *Astronomy Letters*. (1999); [22] V. Suleimanov and J. Poutanen. *MNRAS*. (2006).

† [23] R. A. Syunyaev and N. I. Shakura. *Soviet Astronomy Letters*. (1986); [24] N. R. Sibgatullin and R. A. Sunyaev. *Astronomy Letters*. (2000).

‡ see [25] W. H. G. Lewin, J. van Paradijs, and R. E. Taam. *SSRv*. (1993); [26] T. Strohmayer and L. Bildsten. 2010, for a review.

classified as type-II bursts, and we do not focus on them, as here we are interested in probing the neutron star itself. cite type-II.

A typical X-ray burst has a rise time of 0.1 to 10 seconds and a duration from 10 to 100 seconds. During this time, it releases $10^{39} - 10^{40}$ erg of energy. Temperature in the upper layers is of around $T \sim 10^7$ K and the main ingredient of the composition is either hydrogen, helium, or both.

The driving engine for a X-ray burst is the unstable thermonuclear fusion process.* The burning of hydrogen plasma is dominated by the CNO-cycle if temperature is around 10^7 K. For a slightly hotter plasma, $T \gtrsim 8 \times 10^7$ K, we have to modify the reaction a bit into a so-called hot CNO-cycle.† Helium plasma, on the other hand, burns via the triple-α process (active when $T \gtrsim 2 \times 10^8$ K). In addition to these two main reactions, the αp-process can operate when $T \gtrsim 5 \times 10^8$ K, creating heavier elements like Ne, Na, and Mg. The rp-process to synthesize even heavier elements can take place if $T \sim 10^9$ K. The end result is anyway the same, the accreted matter is fused together into heavier elements, and at the same time, energy is released into the neutron star ocean.

If the accreted material does not have any hydrogen, or if the hot CNO-cycle has enough time to burn all the available hydrogen into helium, the ignition starts in the helium shell. On the other hand, if the hot CNO burning of hydrogen is not continuous, it can trigger the runaway in the hydrogen shell, after which the helium shell will also ignite. These minor details have observational importance, as we sometimes see bursts with very short rise times, and other times it might take seconds for the burst to really get going.‡ This discrepancy is believed to originate from this changing ignition mechanism.

In addition to the normal type-I bursts, we have also recently detected more rare long-duration bursts, now commonly dubbed as "superbursts".§ These are thought to be powered by carbon burning¶ When looking at the duration, there are also a third class of bursts in between the superbursts and normal bursts, named "intermediate bursts".‖

All in all, the energy production of bursts appears to be a complex mechanisms that we do not fully understand yet. This is also reflected in the large variety of burst durations, energetics, rise times, etc. that we observationally see.**

* [27] M. Y. Fujimoto, T. Hanawa, and S. Miyaji. *ApJ*. (1981); [28] R. K. Wallace and S. E. Woosley. *ApJS*. (1981); [29] J. L. Fisker, H. Schatz, and F.-K. Thielemann. *ApJS*. (2008).

† [30] W. A. Fowler and F. Hoyle. 1965.

‡ [26] T. Strohmayer and L. Bildsten. 2010.

§ [31] R. Cornelisse *et al*. *A&A*. (2000); [32] E. Kuulkers *et al*. *A&A*. (2002); [33] T. E. Strohmayer and E. F. Brown. *ApJ*. (2002).

¶ [34] A. Cumming and L. Bildsten. *ApJ*. (2001).

‖ [35] A. Cumming *et al*. *ApJ*. (2006).

** [36] D. K. Galloway *et al*. *ApJS*. (2008).

2 Probing the ultra-dense matter

The main motive for this thesis was to set constraints for the ultra-dense equation of state. Instead of starting from the nuclear physics that works on the smallest scales, we use astrophysical observations to study large-scale "global" aspects of neutron stars. It is then possible to make a step back to the nuclear physics because the size of a compact star is strongly coupled to the composition of its core.

Looking from the astrophysical point of view, it is the size of the neutron star that will define many of its observable features. One of the most important characteristics is the compactness of the object that will then define the exact shape of the spacetime surrounding it. The strongly curved spacetime, in turn, influences many of the phenomena occurring in the close vicinity of the star and will also leave its distinct imprints on the observations.

The physical phenomena behind the observable features on the other hand, are often highly energetic, otherwise they would not be seen by distant observers, such as us. It is these highly energetic physical processes that will then render the neutron stars visible to us, and that at the same time carry a plethora of information from the surroundings of where they originated from. This gives birth to a beautiful cosmic connection where the delicate and unattainable nuclear physics of the ultra-dense matter is coupled to vigorous astrophysical phenomena that we can observe. The caveat here is that the astrophysical processes are often messy and poorly understood. Hence, a thorough understanding of both, the nature of the observed phenomena and how it exactly couples to the nuclear physics, is needed.

In this thesis, we will focus on extracting the information from the X-ray bursts. In theory, this method of using the X-ray bursts to probe the neutron star interiors is robust as we can theoretically model the characteristics of the emerging radiation and these models can be applied to describe the data that we see. In practice, however, caution is needed when applying the models as the environment and the astrophysics near the neutron star play a huge role.

In this final chapter, we will lay out the basics of how by observing the burst cooling it is possible to set constraints on the size of the emitting area, and in the end, the radius of the neutron star. We will also summarize the content of the articles in this thesis, and discuss discuss our work where we try to understand not only the complex role of the astrophysical surroundings but in the end, the composition of the core.

2.1 Measuring the size of the bursting source

Even though the bursts characteristics change from one burst to another as we saw in Sect. XXX, the cooling appears to obey some common trends. This means that as long as we have some kind of an energy injection deep below the neutron star's atmosphere, the energy will radiate out and the uppermost layers of the star will then shape into a similar cooling curve, independent of the actual details of the injection. If we are then able to model the atmosphere and the processes therein, we can then use the bursts as probes for the neutron star interiors.

The big caveat here are the surroundings. We have seen that the astrophysical environment of neutron stars can be very active and lively. In the general picture, we have the accretion as an energy source and the disk to dissipate this energy. The disk is, however, not a simple geometrically thin steady layer but can have complex inner flow. On the other hand, if the disk does extend all the way down to the star, an additional complication originates from the boundary or spreading layer that can not only cover the star but also radiate on its own. These are some of the complications that we face when trying to analyze our neutron star observations, as after all when trying to constrain the mass and radius of the star we must make sure that it is actually the star that we are looking.

2.2 Scientific summary of the results

2.2.1 Modeling of neutron star atmospheres and emergent radiation

2.2.2 Understanding the astrophysical environments of X-ray bursts

2.2.3 Constraining the mass and radius of neutron stars

2.3 The author's contribution to the publications

Paper I.

The author of the thesis made contributions to the manuscript, reduced and analyzed the observational X-ray data, and contributed to the scientific discussions related to the manuscript.

Paper II.

The author participated in the reduction and analysis of the observational data, made significant contributions to the development of the data reduction software, and helped in the preparation of the manuscript.

Paper III.

The author contributed to the main idea of the paper, independently redesigned the neutron star atmosphere code used for the calculations, and implemented new physical processes to this numerical framework. The author also prepared most of the manuscript.

Paper IV.

The author independently designed the Bayesian fitting framework for the cooling tail method, reduced and analyzed the X-ray observations, and finally led the equation of state modeling from the observations. The author also prepared the manuscript.

Paper V.

The author contributed to the main idea of this research and was responsible of the atmosphere modeling of the observations. The Bayesian atmosphere spectral model fitting framework was also independently designed by the author. Author also made significant contributions to the manuscript.

Paper VI.

Author helped in designing the fitting framework, based on his own previous results, and contributed to the scientific and statistical discussions of the paper. The author also contributed to the manuscript.

Paper VII.

In this paper, the author proposed the usage of the dynamic power-law method and co-supervised the project which was originally based on the Master's thesis of J. Kuuttila. Author also made significant contributions to the manuscript.

Paper VIII.

The author of the thesis took part in the discussion of the theoretical explanation for the obtained observational results and contributed significantly to the statistical analysis of data. The author also contributed to the manuscript.

Paper IX.

The author independently proposed the idea of applying the split-Hamilton method to the ray tracing problem of photons, derived the theoretical framework and all the related formulae, designed the numerical code, and prepared most of the manuscript.

Paper X.

The author independently designed the hierarchical Bayesian fitting framework, implemented it into a code together with M.C. Miller and A.W. Steiner, analyzed the data, and, finally, prepared most of the manuscript together with M.C. Miller.

Paper XI.

In this paper, the author took part in the scientific discussion of the results, helped in the statistical analysis and contributed to the manuscript.

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