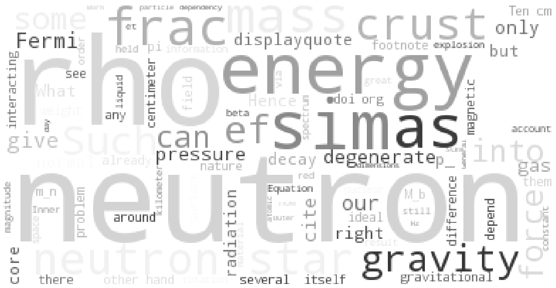

1 Introduction

Neutrons stars are curious objects encompassing many still unsolved problems of modern physics and astrophysics. Their unique nature makes them ideal laboratories for many of the most energetic phenomena in space. Born from the ashes of a supernova, they begin their life only when some other normal star fades away and dies in a spectacular supernova explosion.[1] From there on, they continue their life by slowly collecting the surrounding interstellar matter or by devouring an unlucky companion star floating next to them. It is not the impressive $\sim 10^{30}$ kilograms they weight but the mere $\sim 10\,\mathrm{km}$ in radius sphere that they encapsulate this material into that is then able to bend the spacetime itself. Such an impressive feat rewards them with a categorization into a special stellar group called *compact objects*. Along with white dwarfs and black holes, these strange compact objects have been under a scientific scrutiny for almost nine decades now. Still, some of the most fundamental questions remain: What are neutron stars made of? How big are they? How do we see them?

In the next few sections, we will start our journey by building our intuition of these peculiar objects habiting the Space around us. A short history of their discovery is given, followed by simple physical arguments why we actually expect them to exists in a first place. Next, some actual observable phenomena is reviewed. These are also closely connected to the surroundings and environments of the neutron stars, and hence those will be discussed and described also.

1.1 Short history: from imagination into a reality

Anticipation by theorist before the second World War.[2]

> ...the density of matter becomes so great that atomic nuclei come in close contact, forming one gigantic nucleus. (Landau)

Blaablaa[3]

> With all reserve we advance the view that a super-nova represents the transition of an ordinary star into a neutron star, consisting mainly of neutrons. Such a star may possess a very small radius and an extremely high density. (Baade och Zwicky)

Work of imagination from weird theorists. On July 1967 everything changed because of Jocelyn Bell, a graduate student of Anthony Hewish in Mullard Radio Astronomy Observatory. Primitive antenna of wires strung on stakes in a pasture.

She discovered that the signal was pulsing with great regularity, at a rate of about one pulse per second. (1.3s) Temporarily dubbed "Little Green Man 1" (LGM-1) the source (now known as PSR B1919+21) was identified after several years as a rapidly rotating neutron star. Dupped as pulsar, short of pulsating radio source. Later published in Nature 1968 by Hewish.[4]

[1] For the pedantic ones, we specify that a supernova is actually an implosion followed by a subsequent explosion.

[2] [1] L. D. Landau. *Phys. Z. Sowjetunion*. (1932).

[3] [2] W. Baade and F. Zwicky. *Proceedings of the National Academy of Science*. (1934).

[4] [3] A. Hewish *et al*. *Nature*. (1968).

1.1.1 History of degenerate electron gas and Chandrasekhar limit

electrons as fermions[5] electron deg. pressure[6] elecs must be relativistic[7] equation of state for uniform density[8] Similar independent study by Landau[9] full equation of state of degenerate electron gas unaware of Fowler[10][11]

Finally put together by Chandrasekhar[12] work done 5 years before Stoner.

1.1.2 Ideas about neutron stars

1.2 From first principles to a neutrons star

1.2.1 Background: Sun and stars

Stellar orders of magnitude

Let us first see, what can we learn from neutron stars using simple estimates and conservation laws. Neutrons star are born from a death of a normal star. Most familiar such a star is our Sun 1.496×10^{13} cm from us.[13] With a mass of $M_\odot = 1.99 \times 10^{33}\,\mathrm{g}$ and radius of $R_\odot = 6.96 \times 10^{10}\,\mathrm{cm}$, our Sun gives us an idea of the typical stellar size scales. Curiously, these numbers also means that the mean density of the Sun is $\rho_\odot \approx 1.41\,\mathrm{g\,cm^{-3}}$, a mere $1.4\times$ the density of the water.

Like all stars, our Sun is held together by the inward pulling gravity. Gravity does not prefer any direction more than some other and, just like we easily observe, a spherical object is expected to form. In addition to the inward-facing force, an outward-facing force is needed to balance the system. For normal stars this force is originating from the thermal pressure.

hermonuclear fusion process

We observe stars in the night sky because they shine on us. This radiation, and also the origin of the thermal pressure, is from the thermonuclear fusion burning inside the star. *Thermo* here refers to the temperature and heat, *nuclear* to the atomic nuclei, and *fusion* to the process where elements are fused together. During the thermonuclear fusion process, the star's core fuses light elements such as protons into heavier ones like helium. Mass of two protons is less than a mass of one helium atom. This mass difference between the start and end results is then transferred into energy in accordance to the Einstein's famous formula $E = mc^2$. A whole sequence of such a fusions takes place inside the star where lighter elements are merged together building heavier and heavier elements. This energy release from the mass-to-energy conversion will then give the star a sufficient thermal pressure support to keep it from not collapsing under the relentless gravity.

The fusion of elements does not continue forever. In the beginning, two protons collide two form a helium. In the next stage three helium nuclei collide to form a carbon, and so on, until iron is created. Production of iron marks the end of the possible fusion chain because fusion of two iron cores does not release energy anymore, it requires it to take place.[14] This iron produced, will then sink to the center of star forming a dead core without any energy input.

Like all big furnaces, at some point the stars will run out of fuel to burn. What is then left behind is a inner core of iron with a subsequent onion-like layers of lighter and lighter elements. Crucial question to ask next is: What is supporting this iron core now that the thermal pressure from the fusion processes are lost? To answer this, we need to look inside the iron atom: Iron nucleus, consisting of 26 protons and 32

5 [4] P. A. M. Dirac. *Proceedings of the Royal Society of London Series A*. (1925).
6 [5] R. H. Fowler. *MNRAS*. (1926).
7 [6] Wilhelm Anderson. *Zeitschrift für Physik*. (1929).
8 [7] E. C. Stoner. *Philos. Mag.* (1930).
9 [1] L. D. Landau. *Phys. Z. Sowjetunion*. (1932).
10 [8] J. Frenkel. *Zeitschrift für Physik*. (1928).
11 [9] D. G. Yakovlev. *Physics Uspekhi*. (1994).
12 [10] S. Chandrasekhar. *MNRAS*. (1931).

13 Throughout this thesis we will typically present our quantities only up to some fixed precision instead of the full litany of numbers. We will also adopt the centimeter-gram-second (cgs) unit system instead of the (maybe) more common SI-system. Such a selection is sure to disappoint some, but try to endure.

14 This opens up another possibility of creating energy by splitting heavy elements. Such a process is called fission and is familiarly taken advantage of in earthly nuclear power plants.

neutrons, and surrounded by 26 electrons, repulses its neighbors because of the negatively charged electron cloud does not want to get in touch with its neighbors flying next to it in the iron-atom lattice. [15] It is this antisocial behavior originating from the electric charge repulsion that also gives internal support to our everyday materials and prevents us from not falling through the floor. Electrons and protons inside the atom itself, on the other hand, are repulsing each other because of the quantum effects. explain quantum repulsion

1.2.2 Neutron stars at last

From this setup it only takes a short step into realizing the existence of neutron stars. What if at some point even these quantum effects are not enough to support the core? Let us consider the consequences of this thought play. More detailed calculations show that the resulting iron-core sitting at the center of the star is weighting at a maximum of around $\sim M_\odot$. Hence, there are $M_{\mathrm{core}}/m_{\mathrm{atom}} \sim M_\odot/m_{\mathrm{p}} \approx 1.99 \times 10^{33}\,\mathrm{g}/1.67 \times 10^{-24}\,\mathrm{g} \sim 10^{57}$ atoms trapped inside the core. [16] Here we are already considering not iron atoms but pure hydrogen atoms only to simplify the presentation. Working backwards from these numbers, we can estimate the size of the core.

1.2.3 Neutron star dimensions

It turns out that Sun is also not as stable as one would think: With a rotation period of about 25.5 days it then takes Sun about a month to revolve around itself. Similar to a bicycle dynamo hub, this rotation also gives rise to a detectable surface magnetic field of $B_\odot \approx 1\,\mathrm{Gauss}$.[17]

A typical neutron star, on the other hand, weights about $M \sim 1.5\,M_\odot$ but extends only up to $R \sim 10\,\mathrm{km}$. Such dimensions give us an impressive mean density of $\rho \sim 7 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. In comparison, a typical nucleon (such as a neutron) weights about $1.67 \times 10^{-24}\,\mathrm{g}$ and has a radius of about $1.25 \times 10^{-13}\,\mathrm{cm}$, yielding us a nuclear density of $\rho_{\mathrm{n}} \approx 2 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. Hence, even the mean density inside the star is already on the same order of magnitude as the internal density inside nuclei. This suggests us that the composition inside a neutron star is not our typical every-day matter.

Suppose a neutron star is, like any normal star, a blob of gas held together by the inwards pulling gravity. Gravity does not prefer any direction more than some other and so a stable end-result is an isotropic configuration. A pure inward pulling force is, of course, not enough so we also need a countering outward-facing force to resist the compression of the material. As an first approximation, there is no need to assume that this force would have any preferred direction either. Hence, our expected outcome is a sphere held together by the gravity originating from the mass M of the matter itself. Let us, for a while forget the exact origin and nature of the compression-resisting force and see what can we learn solely from the current information only.

1.3 What do we actually know?

1.4 Connection to this thesis

In this thesis we study many aspects of the neutron stars, their environment and the internal processes that will in the end help us in constraining the behavior of the ultra-dense matter inside the star.

[15] More precise consideration shows that nickel ($^{56}_{28}\mathrm{Ni}$) is actually thermodynamically more favored in the core because of the lack of neutrons needed to synthesize iron-58 ($^{58}_{26}\mathrm{Fe}$).

[16] The mass of the atom, $m_{\mathrm{atom}} = m_{\mathrm{p}} + m_{\mathrm{e}}$, is also approximated (to an excellent accuracy) by just considering the central nuclei alone as the electron mass $m_{\mathrm{e}} \approx 9.11 \times 10^{-28}\,\mathrm{g}$ is negligible in comparison to the proton mass.

[17] A typical refrigerator magnet is about 50× stronger with a magnetic field of 50 Gauss.

2 Physics of neutron stars

2.0.1 Why neutrons then?

Let us first consider ideal gas of degenerate electron-proton-neutron plasma. In a degenerate plasma all the quantum states are filled up all the way to the Fermi energy. It is the Pauli exclusion principle that then prevents occupying all of these already taken quantum states. Normal beta-decay mode for the neutrons, on the other hand, is $n \rightarrow p + e^- + \bar{\nu}_e$, that describes the possible path of how a neutron n will decay into a proton p, electron e^-, and electron neutrino $\bar{\nu}_e$. Such a decay is, however, blocked because there is no room for an emission of an extra electron e^- or a proton p.[1]

Let us then only focus on the decay of the most energetic neutrons with an energy equal to the Fermi energy $\epsilon_{\mathrm{F}}(n)$. Co-existence of neutrons, protons, and electrons is then guaranteed (at zero temperature) if

$$\epsilon_{\mathrm{F}}(n) = \epsilon_{\mathrm{F}}(p) \ + \epsilon_{\mathrm{F}}(e^-). \tag{2.1}$$

Fermi momentum of a particle is related to its concentration via

$$p_{\mathrm{F}} = \left(\frac{3n}{8\pi}\right)^{1/3} h, \tag{2.2}$$

where n is the number density, and h the Planck constant. Massive neutrons and protons are to a good approximation non-relativistic up to a densities of ρ_{n}, and hence energy is simply a sum of their rest mass energy and kinetic energy

$$\epsilon_{\mathrm{F}}(n) \ \approx m_n c^2 + \frac{p_{\mathrm{F}}(n)^2}{2m_n}, \tag{2.3}$$

and

$$\epsilon_{\mathrm{F}}(p) \ \approx m_p c^2 + \frac{p_{\mathrm{F}}(p)^2}{2m_p}. \tag{2.4}$$

Electrons, on the other hand, are already ultra-relativistic, and so

$$\epsilon_{\mathrm{F}}(e^-) \approx p_{\mathrm{F}}(e^-)c^2. \tag{2.5}$$

Also note that $n_p = n_e$, as the star is electrically neutral. From this we find relation of the $n_n/n_p \sim 1/200$ by taking into account the rest mass difference $m_p - m_n = 2.6\mathrm{MeV}\ c^2$ at $\rho \sim \rho_{\mathrm{n}}$. Thus, we conclude that the matter inside is neutron rich.

[1] see e.g. [11] A. C. Phillips. 1994.

2.0.2 Tolman-Volkoff-Oppenheimer equations

Newtonian pressure gradient needed to oppose the gravity is

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}. \tag{2.6}$$

Taking into account the general relativistic corrections we get

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \times \frac{(1 + P/\rho c^2)(1 + 4\pi r^3 P/mc^2)}{1 - 2Gm/rc^2}. \tag{2.7}$$

Difference originates from the source of gravity: in the Newtonian case it is the mass m, whereas in the General relativity it is the energy momentum tensor that depend both on the energy density and the pressure. As a result, energy and pressure give rise to a gravitational fields.

It has an important consequence to the stability of neutron stars: Successive increase in the pressure to counter the gravity is ultimately self-defeating.

Solution for a constant density ρ_0 gives

$$P(r) = G\frac{2\pi}{3}\rho_0^2(R^2 - r^2) \tag{2.8}$$

whereas the GR gives

$$P(r) = \rho_0^2 c^2 \left[\frac{(1 - u\left(\frac{r}{R}\right)^2)^{1/2} - (1 - u)^{1/2}}{3(1 - u)^{1/2} - (1 - u\left(\frac{r}{R}\right)^2)^{1/2}} \right], \tag{2.9}$$

where $u = 2GM/Rc^2$.

2.1 Equation of state

Often means dependency between P and ρ. Or sometimes the associated energy density $\epsilon = \rho c^2$. Also depends on T but composed mainly on strongly degenerate fermions so so temperature dependency is negligible.

Bulk property of the sea of fermions.

Eos for $\rho > \rho_{\mathrm{n}}$ can not be produced in laboratory. Can not be calculated because of the lack of precise many-body theory of strongly interacting particles.

Baryon mass M_b that is sum of baryon masses. Gravitational mass M that is M_b from where the gravitational binding energy is subtracted.[2]

2.2 Atmosphere

Thin layer of plasma From centimeters in hot to millimeters in cold Zavlin & Pavlov 2002

Where spectrum or thermal electromagnetic radiation is formed. Spectrum, beaming and polarization of emerging radiation can be determined from radiation transfer problem in atmospheric layers.

Contains information on the parameters of the surface: effective temperature surface gravity chemical composition geometry of the system mass and radii.

Eddington limit of where radiation force exceeds the gravitational one.

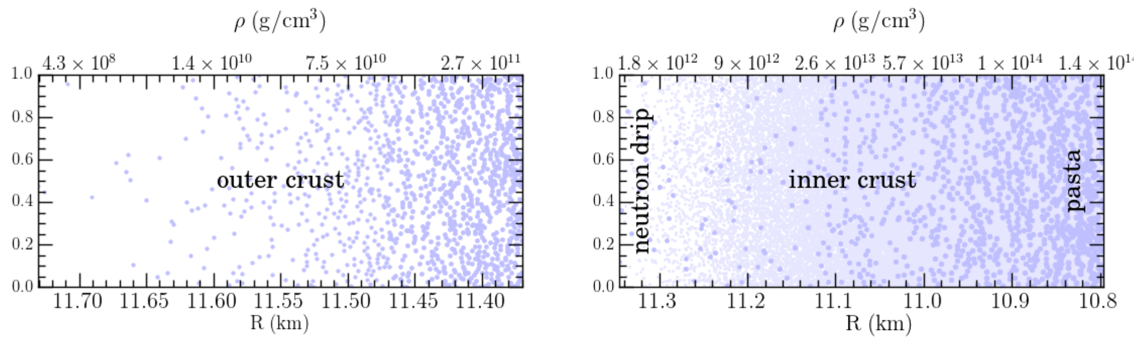

Figure 2.1: Molecular simulation of the crust. Figure adapted from `https://github.com/awsteiner/nstar-plot`.

2.3 Crust

Outer crust

From atmosphere to $\rho_N D \sim 4 \times 10^{11}\,\mathrm{g\,cm^{-3}}$. In thickness some hundred meters. Non degenerate electron gas Ultra-relativistic electron gas $\rho > 10^6\,\mathrm{g\,cm^{-3}}$. Pressure provided by electrons here.

In deeper layers ions form a strongly coupled Coulomb system (liquid or solid). Hence, crust. Fermi energy grows with increasing ρ. Induces β captures and enriches nuclei with neutrons. At the base neutrons start to drip out from nuclei.

Inner crust About one kilometer thick. Density from $\rho \sim \rho_{ND}$ (at upper boundary) to $\sim 0.5\,\rho_\mathrm{n}$ at the base. Matter consists of electrons, free neutrons n and neutron-rich atomic nuclei. Fraction of free n grow with ρ.

Finally, nuclei disappear at the crust-core interface.

2.4 Core

Outer core. Density ranges $0.5\,\rho_\mathrm{n} < \rho < 2\,\rho_\mathrm{n}$. Several kilometers. Neutrons with several per cent admixture of protons p and electrons e^-. Strongly degenerate. Electrons form almost ideal Fermi gas. Neutrons and protons, interacting via nuclear forces, constitute a strongly interacting Fermi liquid.

Inner core. Where $\rho > 2\,\rho_\mathrm{n}$. Central density can be around $(10-15)\,\rho_\mathrm{n}$. Very model dependent. Main problem.

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