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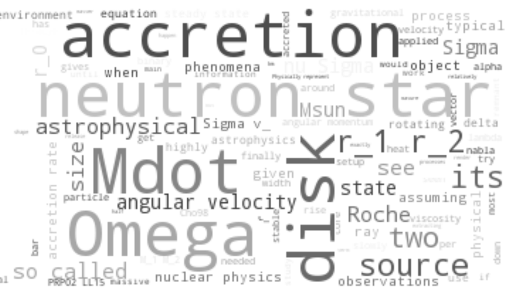

1 Constraining the equation of state with astrophysics

The main motive for this thesis was to set constraints for the ultra-dense equation of state. Instead of starting from the nuclear physics that works on the smallest scales, we use astrophysical observations to study large-scale “global” aspects of neutron stars. It is then possible to make a step back to the nuclear physics because the size of a compact star is strongly coupled to the composition of its core.

Looking from the astrophysical point of view, it is the size of the neutron star that will define many of its observable features. One of the most important characteristics is the compactness of the object that will then define the exact shape of the spacetime surrounding it. The strongly curved spacetime, in turn, influences many of the phenomena occurring in the close vicinity of the star and will also leave its distinct imprints on the observations.

The physical phenomena behind the observable features on the other hand, are often highly energetic, otherwise they would not be seen by distant observers, such as us. It is these highly energetic physical processes that will then render the neutron stars visible to us, and that at the same time carry a plethora of information from the surroundings of where they originated from. This gives birth to a beautiful cosmic connection where the delicate and unattainable nuclear physics of the ultra-dense matter is coupled to vigorous astrophysical phenomena that we can observe. The caveat here is that the astrophysical processes are often messy and poorly understood. Hence, a thorough understanding of both, the nature of the observed phenomena and how it exactly couples to the nuclear physics, is needed.

In this thesis, we will focus on extracting the information from the so-called X-ray bursts that ignite in the upper layers of neutron stars. These bursts originate from the unstable nuclear fusion runaways in the neutron star ocean that produce excessive heat that is then radiated away as photons. These photons will then emerge through the atmosphere of the star, travel astronomical distances towards Earth until they will land on one of our scientific instruments, and be recorded by us as X-ray events. In theory, this method of using the X-ray bursts to probe the neutron star interiors is robust as we can theoretically model the characteristics of the emerging radiation and these models can be applied to describe the data that we see. In practice, however, caution is needed when applying the models as the environment near the neutron star plays a huge role.

In this final chapter, we will shortly discuss the complex astrophysical environment surrounding the bursting neutron star. We will also review the relevant physics behind the X-ray bursts. Finally we will lay out the basics of how observing the burst cooling can set constraints on the size of the emitting area, and in the end, the radius of the neutron star.

1.1 Astrophysics around neutron stars

Let us begin by discussing the violent environments around the neutron stars as the these surroundings play an important role when we try to decipher the real observations of these stars. Typically, the neutron stars can be found (or rather seen) either in binary systems where they are accompanied by another star, or as a lonely remnant left behind from a supernova explosion. In the latter case it is the neutron star itself that is the source of the energy that renders it visible as it will slowly cool down and radiate away all the left-over heat from the explosion. In some cases, the rotating magnetic field of the star can also create radiation when it propels in the medium that is left behind. This gives rise to a particle acceleration as the charged plasma is dragged along by the magnetic field producing radiation as the particles try to resist this motion.

In the binary systems, on the other hand, the energy originates not from the neutron star itself but from the companion. In the heart of this whole problem is an astrophysical process called accretion. This is a physical process where matter is transferred from one source to another because of the gravitational forces. In this thesis and in the following discussion we will focus on these binary systems and on the so-called accretion powered phenomena. We, however, note that it is possible to use the observations of the single neutron star remnants too, to constrain the mass and radius.[*]

1.1.1 Accretion

Accretion is an astrophysical process that taps into the gravitational potential energy of particles. It can be a source of enormous amounts of energy if the central object is compact, because the depth of a gravitational well is directly proportional to the compactness of the source. Hence, it is an important, and often dominating, process for neutron stars.[†]

Gravitational potential energy release for a mass m that is accreted onto a compact

[*] see, e.g., [1] D. Page and S. Reddy. *Annual Review of Nuclear and Particle Science*. (2006).

[†] For an introduction, see e.g., [2] J. Frank, A. King, and D. J. Raine. 2002.

object of radius R and mass M is

$$\Delta E_{\mathrm{acc}} = m\frac{GM}{R} \sim 10^{20}\left(\frac{m}{\mathrm{g}}\right)\left(\frac{10\,\mathrm{km}}{R}\right)\left(\frac{M}{\mathrm{M}_\odot}\right)\mathrm{erg}, \tag{1.1}$$

where in the latter expression typical dimensions of neutron star are inserted to the formula.

This energy, 10^{20} erg per each gram that is accreted, is usually released as radiation. The rate of this energy release is simply related to the mass accreted per time, i.e., accretion rate $\dot{M}$,

$$L_{\mathrm{acc}} = \dot{M}\frac{GM}{R} \approx 1.3\times10^{36}\left(\frac{\dot{M}}{10^{16}\,\mathrm{g\,s^{-1}}}\right)\left(\frac{10\,\mathrm{km}}{R}\right)\left(\frac{M}{\mathrm{M}_\odot}\right)\mathrm{erg\,s^{-1}}, \tag{1.2}$$

where a typical value of $\dot{M} \sim 10^{16}\,\mathrm{g\,s^{-1}} \approx 1.5\times10^{-10}\,\mathrm{M}_\odot\,\mathrm{yr}^{-1}$ is taken for the accretion rate. Hence, depending on the accretion rate, this value can be about the same as the Eddington luminosity Eq. XXX of a neutron star.

X-rays from blackbody T

1.1.2 Roche lobes and mass transfer in binary systems

In order to use the accretion as an energy source, we need mass transfer to occur. For the mass transfer to keep on operating, a source of fresh material is needed. In binary systems, the companions star is the obvious fuel resource. Here we will focus on the so-called Low Mass X-ray Binary (LMXB) systems where the companion, like the name implies, is a relatively low-weight star.* Typically, it is a normal or late-type star with a mass $M \lesssim 1\,\mathrm{M}_\odot$. Such a setup leads to a mass-transfer quite naturally as the more heavy-weight neutron star will just rip out the outer layers of its poor companion and slowly devours it, until nothing is left. As another option, the system could be a so-called High Mass X-ray Binary (HMXB) system, where the neutron star companion is $M \sim 10\,\mathrm{M}_\odot$, and the accretion happens, for example, via a neutron star traveling through the other stars extended outer envelope. Here, we will, however, only focus on the LMXB systems, as they provide a relatively stable mass-transfer mechanism.

How exactly is the material transferred form the companion to the primary star is an interesting problem. We can begin to understand the physical setup by considering a general hydrodynamical system of two objects in a rotating frame. Here we select the frame such that it co-rotates with the binary system. The subsequent flow of gas between the two stars can then be described by the Euler equation with additional Corriolis and XXX terms.† In practice the Euler equation describes the time evolution of the velocity $\boldsymbol{v}$ of the gas that has a pressure P and density ρ. In a reference frame rotating together with

* [3] T. M. Tauris and E. P. J. van den Heuvel. 2006.

† see, e.g, [4] A. R. Choudhuri. 1998, for a good introduction.

the binary system with angular velocity ω the Euler equation takes the form

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla \Phi_{\mathrm{R}} - 2\boldsymbol{\omega} \times \boldsymbol{v} - \frac{1}{\rho}\nabla P, \tag{1.3}$$

where the angular velocity of the binary is

$$\boldsymbol{\omega} = \left(\frac{GM}{a^3}\right)^{1/2} \boldsymbol{e}, \tag{1.4}$$

as given with the unit vector $\boldsymbol{e}$ normal to the orbital plane. Here M is the total mass of the system, i.e., $M = M_1 + M_2$, where M_1 and M_2 are the individual masses of the two stars in the system, respectively, and a is their orbital separation.

The effects originating from the gravitation and from the centrifugal forces are encapsulated in the so-called Roche potential, given as a function of radial vector $\boldsymbol{r}$ as*

$$\Phi_{\mathrm{R}}(\boldsymbol{r}) = -\frac{GM_1}{|\boldsymbol{r} - \boldsymbol{r_1}|} - \frac{GM_2}{|\boldsymbol{r} - \boldsymbol{r_2}|} - \frac{1}{2}\left(\boldsymbol{\omega} \times \boldsymbol{v}\right)^2, \tag{1.5}$$

where the location of the stars are given with $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$. By studying the shape of the potential, we see that in between the stars, in the so-called L_1 point there exists a location where the countering gravitational forces from the two stars are balanced. This can be though of as a physical nozzle in the system from which the less massive star will leak into the more massive star. Such a mass transfer, also known as a Roche lobe overflow, will then occur if the companion star's radius exceeds the size of its own individual Roche lobe visualized in Fig. 1.1. Typically such a thing can happen when the star evolves and expands at the end of its life cycle.

1.1.3 Accretion disks

When the mass transfer has started via the Roche lobe overflow, and we have stable source of material transferred from the companion, we can next focus on the region where gravitational forces of the neutron star dominate. Most importantly, the infalling material has to somehow loose its angular momentum, before it is able to travel all the way to the neutron star surface. Nature's mechanism to do this is called an accretion disk.

accretion disk: machine for slowly lowering material in the graviational potential and extracting energy.

orbital kinetic energy to heat

Confined to the orbital plane, hence thin-disk approximation is sufficient.

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*see, e.g., [5] P. Podsiadlowski, S. Rappaport, and E. D. Pfahl. *ApJ*. (2002); [6] D. A. Leahy and J. C. Leahy. *Computational Astrophysics and Cosmology*. (2015).

Radial disk structure from Keplerian rotation law

$$\Omega_{\mathrm{K}}(R) = \left(\frac{GM}{R^3}\right)^{1/2} \tag{1.6}$$

implies differential rotation.

Viscous stress from shear viscosicty.

Disk luminosity:

Conservation of mass and angular momentum

$$r\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r}(r\Sigma v_{\mathrm{R}}) = 0 \tag{1.7}$$

$$r\frac{\partial}{\partial t}(\Sigma r^2 \Omega) + \frac{\partial}{\partial r}(r\Sigma v_{\mathrm{r}} r^2 \Omega) = \frac{1}{2\pi}\frac{\partial G}{\partial r}, \tag{1.8}$$

where G is the viscous torque of the differentially rotating disk.

Torque, in general, is the net outward angular momentum flux given as

$$\tau_i = \epsilon_{ijk} r_j f_k, \tag{1.9}$$

where ϵ_{ijk} is the Levi-Civita symbol, and f_k is the force density given as

$$f_k = \sigma_{kh} n_h. \tag{1.10}$$

Here σ_{kh} is the kh-component of the shear tensor σ and n_h is some surface normal. In our case, we can compute the shear in cylindrical coordinate system focusing on r and ϕ coordinates only as

$$\sigma_{r\phi} = \rho\nu \left(r\frac{\partial}{dr}\left\{\frac{v_\phi}{r}\right\} + \frac{1}{r}\,\frac{dv_r}{d\phi} \right), \tag{1.11}$$

which simplifies to

$$\sigma_{r\phi} = \rho\nu r\frac{d\Omega}{dr}, \tag{1.12}$$

when we remember that $v_\phi = r\Omega$ and assuming the flow to be symmetric on ϕ-direction ($\partial v_r/\partial\phi = 0$). Blaablaa* The total torque exerted by the $2\pi r H$ area corresponding to the disk rim is then

$$G(r,t) = 2\pi r\nu\Sigma r^2 \Omega', \tag{1.13}$$

where $\Omega' = d\Omega/dr$.

*see, e.g., [7] S. Chapman and T.G. Cowling. 1970.

Combining these and assuming Keplerian rotation $\Omega = \Omega_{\rm K}$ we get

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\left[r^{1/2}\frac{\partial}{\partial r}(\nu\Sigma r^{1/2})\right] \tag{1.14}$$

$$v_{\rm r} = -\frac{3}{\Sigma r^{1/2}}\frac{\partial}{\partial r}(\nu\Sigma r^{1/2}). \tag{1.15}$$

In order to continue, we need a description for the viscosity ν.

To get some idea of the disk dynamics, we can, as our zeroth order approximation assume $\nu =$ const.. Then, the time-dependent disk structure can be solved for example by assuming as an initial condition a ring of mass m at $r = r_0$,

$$\Sigma(r, t=0) = \frac{m}{2\pi r_0}\delta(r - r_0), \tag{1.16}$$

where δ is the Dirac delta function. We then obtain a time-dependent solution that diffuses due to viscosity as

$$\Sigma(x, \tau) = \frac{m}{2\pi r^2}\frac{1}{\tau x^{1/4}}\exp\left[-\frac{(1+x^2)}{\tau}\right] I_{1/4}\left(\frac{2x}{\tau}\right), \tag{1.17}$$

where $I_{1/4}$ is the modified Bessel function and radial coordinate r is non-dimensionalized as $x = r/r_0$, and time as $\tau = 12\nu t/r_0^2$.

Let us next study a steady-state disk solution by setting $\partial/\partial t \to 0$. From angular momentum conservation we then obtain

$$r\Sigma v_{\rm r} r^2 \Omega = \frac{G}{2\pi} + \frac{C}{2\pi}, \tag{1.18}$$

with a constant C that physically represent a torque term from the coupling of the inner disk and the star. In short, it is given by

$$C = -\dot{M}(GMR)^{1/2}, \tag{1.19}$$

by considering the expression for the mass accretion rate

$$\dot{M} = -\frac{2\pi r\Sigma dr}{dt} = -2\pi r\Sigma v_{\rm r}, \tag{1.20}$$

and assuming a thin layer for the zone where inner disk angular velocity is slowed down to the angular velocity of the star. Substituting this, and assuming Keplerian velocity again, we obtain

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R}{r}^{1/2}\right)\right]. \tag{1.21}$$

Physically this represents a steady-state solution of a disk with central torque applied to it.

Viscous dissipation rate per unit area is*

$$D(r) = \frac{G\Omega'}{4\pi r} = \frac{1}{2}\nu\Sigma r^2\Omega' = \frac{3GM\dot{M}}{8\pi r^3}\left[1 - \left(\frac{R}{r}^{1/2}\right)\right]. \tag{1.22}$$

Finally, from here we can compute the luminosity of the disk faces due to energy lost by viscous dissipation

$$\begin{aligned} L(r_1, r_2) &= 2\int_{r_1}^{r_2} D(r)2\pi r dr = \frac{3GM\dot{M}}{2}\int_{r_1}^{r_2}\left[1 - \left(\frac{R}{r}\right)^{1/2}\right]\frac{dr}{r^2} \\ &= \frac{3GM\dot{M}}{2}\left\{\frac{1}{r_1}\left[1 - \frac{2}{3}\left(\frac{R}{r_1}\right)^{1/2}\right] - \frac{1}{r_2}\left[1 - \frac{2}{3}\left(\frac{R}{r_2}\right)^{1/2}\right]\right\}, \end{aligned} \tag{1.23}$$

and by then setting $r_1 \to R$ and $r_2 \to \infty$, we get

$$L_{\text{disk}} = \frac{GM\dot{M}}{2R} = \frac{1}{2}L_{\text{acc}}. \tag{1.24}$$

Hence, half of the potential energy will be lost by the viscous dissipation and is radiated away by the upper and lower faces of the accretion disk. Importantly, the other remaining half will be transferred all the way to the star.

Hard and soft state† Alternates between these two states‡§

1.1.4 Boundary layers

Our simple analysis of accretion disk physics has shown that viscous dissipation can get rid of up to half of the potential energy of the incoming matter. Where the other half goes, we shall look next.

Imagine the accretion disk extending all the way down to the central star. The angular velocity of this disk rim can be taken to be of around Keplerian velocity $\Omega_{\text{K}}(R) \sim xxx$. The star, on the other hand, usually rotates anywhere from 100 to 600 revolutions per second. Hence, we expect a thin layer in the disk-star interface where angular velocity goes down from XXX to maybe half of that. This region we call the boundary layer.

$$\Omega(R) \approx \Omega_{\text{K}}(R) = \left(\frac{GM}{R^3}\right)^{1/2} \tag{1.25}$$

*Viscous dissipation rate in ring of width dr is $G\Omega' dr$ and the total area of the ring, taking into account both the lower and upper faces, is $4\pi r dr$. Hence, we obtain Eq. (1.22) as the ratio of these.

† [8] G. Hasinger and M. van der Klis. *A&A.* (1989).

‡ [9] T. Muñoz-Darias *et al. MNRAS.* (2014).

§ [10] C. Done, M. Gierliński, and A. Kubota. *A&A Rev.* (2007).

Layer of thickness b equals $\Omega(R+b) \approx \Omega_{\mathrm{K}}(R+b)$ that must slow down to Ω_*.
Energy difference

$$\dot{E} = \frac{1}{2}\,\dot{M}R^2(\Omega_{\mathrm{K}}^2 - \Omega_*^2) = \frac{1}{2}\dot{M}\frac{GM}{R}\left[1 - \left(\frac{\Omega_*}{\Omega_{\mathrm{K}}}\right)^2\right] \tag{1.26}$$

Viscous torque $G_{\mathrm{T}} = \dot{M}R^2(\Omega_{\mathrm{K}} - \Omega_*)$ Hence,

$$\dot{E} = \frac{1}{2}\,\frac{GM\dot{M}}{R}\left(1 - \frac{\Omega_*}{\Omega_{\mathrm{K}}}\right)^2 \tag{1.27}$$

1.2 X-ray bursts

1.2.1 Unstable thermonuclear burning on top of neutron stars

1.2.2 Constraining the size of bursting source

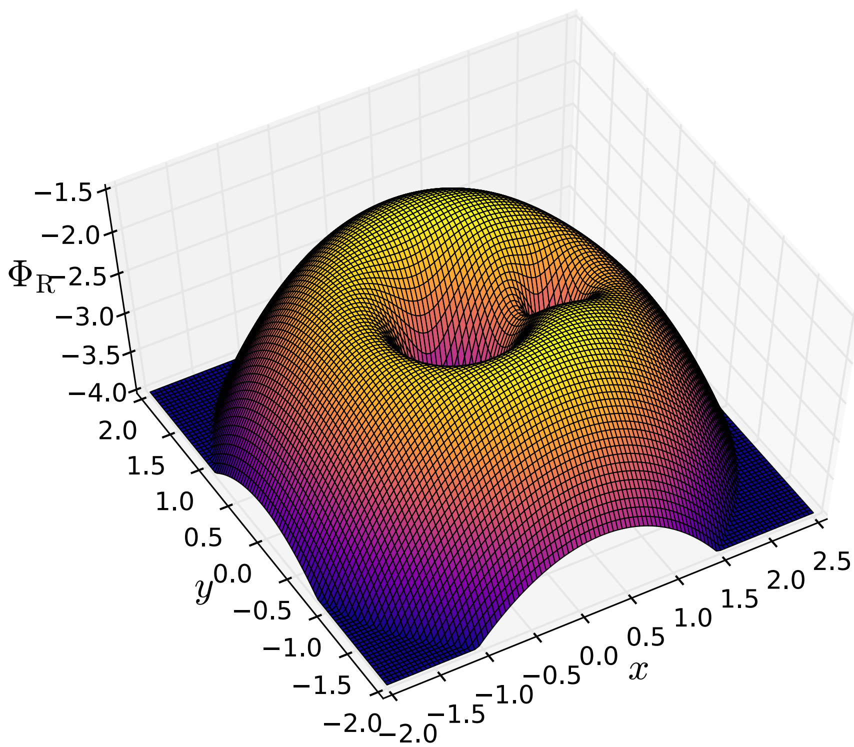

Figure 1.1: Two-dimensional Roche potential $\Phi_R(x, y)$ visualized for a binary systems with $M_1/M_2 =$ and $a =$.The nozzle (L_1 point) is visible as a valley (or more specifically, a saddle point) between the gravitational wells of the two stars.

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