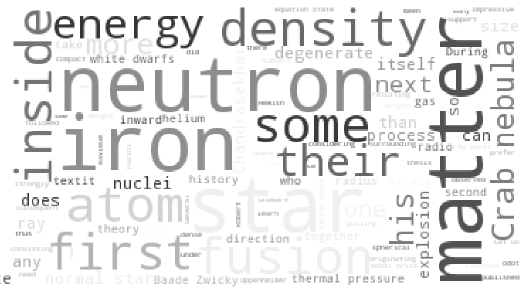


1 Introduction

Neutrons stars are curious objects encompassing many still unsolved problems of modern physics and astrophysics. Their unique nature makes them ideal laboratories for many of the most energetic phenomena in space. Born from the ashes of a supernova, they begin their life only when some other normal star fades away and dies in a spectacular supernova explosion.[1] From there on, they continue their life by slowly collecting the surrounding interstellar matter or by devouring an unlucky companion star floating next to them. It is not the impressive $\sim 10^{30}$ kilograms they weight but the mere $\sim 10\,\mathrm{km}$ in radius sphere that they encapsulate this material into that is then able to bend the spacetime itself. Such an impressive feat rewards them with a categorization into a special stellar group called *compact objects*. Along with white dwarfs and black holes, these strange compact objects have been under a scientific scrutiny for almost nine decades now. Still, some of the most fundamental questions remain: What are neutron stars made of? How big are they? How do we see them?

In the next few sections, we will discuss these peculiar objects habiting the Space around us in detail. First, a short history of their discovery is given, followed by simple physical arguments why we actually expect them to exists in a first place. This will hopefully help us in building some intuition of physics that are possible near and inside these stars. Next, some actual observable phenomena is reviewed. These are also closely connected to the surroundings and environments of the neutron stars, and hence those will be discussed and described also.

1.1 Short history

1.1.1 From imagination into a reality

In 1908, Lev Landau was born in Baku, Azerbaijan. Lev turned out to become a brilliant Soviet scientist who already at an age of 14 matriculated from the Baku State University studying in both, Department of Physics and Mathematics, and the Department of Chemistry simultaneously. Soon after that he became disinterested in chemistry but continued to be fascinated by physics through his whole career. In 1931, at an age of 23, Landau published an exceptional paper where he speculated on the existence of stars containing matter on nuclear densities:[2]

> *...the density of matter becomes so great that atomic nuclei come in close contact, forming one gigantic nucleus.*

What made this paper exceptional was that it was done before the existence of neutrons were confirmed, hence proposing that quantum mechanics might be violated as the atoms consisting of protons and electrons only would live together. Nevertheless, it marked the first theoretical speculation on the existence of what we now know as neutron stars.

[1] For the pedantic ones, we specify that a supernova is actually an implosion followed by a subsequent explosion.

[2] [1] L. D. Landau. *Phys. Z. Sowjetunion*. (1932).

Lev needed not to wait for long. It was already next year in 1932, that James Chadwick then confirmed that neutrons really were a fundamental part of our nuclear physics with his works dubbed as *Possible Existence of a Neutron*[3] and a later follow-up of *Existence of a Neutron*[4]. His findings then became to confirm the theoretical predictions of his supervisor Ernest Rutherford made already in 1920[5]. Later on Chadwick was awarded the Nobel Prize in physics (1935) of his findings. Chadwick himself continued his career as part of the Manhattan project, as it was basically his work that inspired the U.S. government to begin a serious atomic bomb research.

Now that the existence of the neutron was confirmed, it did not take long for others, independent of Landau, to propose similar stars. One year after the discovery, in December 1933 at the Meeting of the American Physical Society at Standford, Wilhelm Baade and Frank Zwicky made their famous propose that Supernovae should be considered as a new category of astronomical objects.[6] At the same time they also stated:

> *...we advance the view that a super-nova represents the transition of an ordinary star into a neutron star, consisting mainly of neutrons. Such a star may possess a very small radius and an extremely high density.*

Such a statements were, however, deemed as a work of imagination from a bunch of weird theorists. Zwicky, on the other hand, kept on insisting that neutron stars are really out there. A.G.W. Cameron, a former post-doc during the 1959−1969 at Caltech where Zwicky was also situated, recalls:

> *For years Fritz [Zwicky] had been pushing his ideas about neutron stars to anyone who would listen and had been universally ignored. I believe that the part of the problem was his personality, which implied strongly that people were idiots if they did not believe in neutron stars.* (Cameron, 1999)

Progress on the theoretical understanding of neutron stars was also tightly connected to understanding the interiors of white dwarfs. Unlike the mysterious nuclear forces related to neutrons, white dwarfs physics was more related to understanding the behavior of electrons. A breakthrough came in 1925 when a young Paul Dirac formulated the quantum wave equations for the motion of the electrons[7]. What soon followed, was a description of a pressure of degenerate electron gas by Ralph Fowler, Dirac's supervisor[8]. Implications of this were severely against the previously known physics: Even in zero temperature, there would be a degeneracy pressure preventing matter from collapsing caused by the exclusion principle of quantum mechanics.

Using a simplified uniform density approximation, Edmund Stoner was then able to show that this implied a maximum mass limit for white dwarfs.[9] Thus, a surprising result was obtained: when the density of a white dwarf approaches infinity, the mass reaches a maximum value. It was later on realized by German-Estonian Wilhelm Anderson that the electrons in this problem must actually be treated relativistically[10], something omitted by Stoner. Anderson also tried to correct the crude mistake by deriving the equation of state of relativistic degenerate electron gas but ended up making severe mistakes. It was then Stoner who corrected his equations, based on the communications with Anderson, and re-derived his maximum mass

[3] [2] J. Chadwick. *Nature*. (1932).

[4] [3] J. Chadwick. *Proceedings of the Royal Society of London Series A*. (1932).

[5] [4] E. Rutherford. *Proceedings of the Royal Society of London Series A*. (1920).

[6] [5] W. Baade and F. Zwicky. *Proceedings of the National Academy of Science*. (1934); [6] W. Baade and F. Zwicky. *Physical Review*. (1934).

[7] [7] P. A. M. Dirac. *Proceedings of the Royal Society of London Series A*. (1925).

[8] [8] R. H. Fowler. *MNRAS*. (1926).

[9] [9] E. C. Stoner. *Philos. Mag.* (1930).

[10] [10] W. Anderson. *Zeitschrift für Physik*. (1929).

limit. Regardless of Stoner's efforts, the maximum mass limit was later named Chandrasekhar's mass limit for its importance in astrophysics.

This was to honor Subrahmanyan Chandrasekhar, a young prolific Indian physicist and astrophysicist who was working on the same topic after reading Fowler's paper on the degenerate electron gas. Unlike Stoner's limit computed using the uniform density approximation, Chandrasekhar realized that a polytropic density profile is a more physical, albeit mathematically more challenging, formulation. 19 years old Chandrasekhar, already known for his mathematical skills, was still able to numerically integrate the equations by hand and obtained a similar limiting mass[11]. Later on, it has been, however, found that Chandrasekhar was not even the second person to derive the mass limit, but third:[12] A soviet physicist Yakov Frenkel published a similar derivation, independently and unknowingly of the progress in the west, where he applied the relativistic degenerate electron gas results to white dwarfs, and concluded that a similar upper limit on the mass exists[13]. It was, however, due to the slow publishing speed in the Soviet journals, that his work never became available to his western peers.

Nevertheless, a maximum mass for a white dwarf was laid out, and in the end after all the relevant physical inclusions, it turned out to be 1.44 $M_\odot$, or 1.44 times the mass of our Sun. What makes this limit even more important is that a maximum mass for a white dwarf is just an minimum mass for a neutron star. An important connection first made by George Gamow in 1939[14]. Idea behind this is simple: If it is the degenerate electron gas pressure, quantum mechanical in nature, that keeps the white dwarfs from collapsing, what happens when the maximum mass is attained and even this pressure is note able to resist the forces of gravity? At a conference in Paris in 1939, Chandrasekhar then laid out the answer:

> *If the degenerate core attain sufficiently high densities, the protons and electrons will combine to form neutrons. This would cause a sudden diminution of pressure resulting in the collapse of the star to a neutron core.*

A neutron star should thus have a mass close to the Chandrasekhar limit, i.e. $M \sim 1.44\, M_\odot$, and consists of neutron only. Exactly like proposed by Landau eight years ago without knowing anything about neutrons yet, or later on by Baade and Zwicky when they presented their theory of supernovae!

It was then before the second World War, that a solid basis for a theory of neutron stars was laid out. This was, however, just the beginning. Next question would be the critical one that we are still trying to find the answer to: If they exists, how big are they? The problem was that because of the extremely dense nature of these objects, the standard stellar equilibrium equations were not valid anymore, and because of this, it was not possible to estimate even the size of the star. The problem was unwieldy because it was general relativistic in nature: The immense mass of the neutron star was bending the spacetime itself, and more compact it was, the more it could bend it. On the other hand, the more curved the spacetime was, the more the star would gain weight.

It was already during the same year in 1939 as Gamow's remark, that a theoretical framework for studying this was published. This was done independently by Richard Tolman[15], and Robert Oppenheimer together with his student George Volkoff[16]. Both papers were even submitted on the same day, 3rd of January, to Physical Review and were published on the same February issue. More importantly, they both described a hydrostatic equilibrium for a spherically symmetric object in general relativity, exactly what is needed to study neutron stars. Because of the utter importance, the solution is now known as Tolman-Oppenheimer-Volkoff equation. In addition, Oppenheimer and Volkoff applied their equation and numerically calculated

[11] [11] S. Chandrasekhar. *MNRAS*. (1931).
[12] [12] D. G. Yakovlev. *Physics Uspekhi*. (1994).
[13] [13] J. Frenkel. *Zeitschrift für Physik*. (1928).
[14] [14] G. Gamow. *Physical Review*. (1939).
[15] [15] R. C. Tolman. *Physical Review*. (1939).
[16] [16] J. R. Oppenheimer and G. M. Volkoff. *Physical Review*. (1939).

a structure of a neutron star consisting of non-interacting strongly degenerate neutron gas. This marked the first try in characterizing neutron stars. Similar to white dwarfs, they also obtained an upper limit for the mass. As a disappointment for everybody, it was, however, calculated to be of around 0.7 $M_{\odot}$, i.e. less than the Chandrasekhar limit of 1.44 $M_{\odot}$ for white dwarfs, indicating that neutron stars could not exist in nature. It took almost two decades then to show that it was actually the assumption of no interaction between the neutrons that was causing this hiccup.

More over, it was not actually Tolman, or Oppenheimer and Volkoff who first discovered general relativistic hydrostatic equation. It was now Chandrasekhar's turn to avoid having an important result credited to him: Together with John Von Neumann, Chandrasekhar extended his work on white dwarfs to cover also neutron stars and in the process derived exactly the same equilibrium equation in 1934, i.e., five years before the groundbreaking publication of Tolman, Oppenheimer and Volkoff.[17] It is, however, worth mentioning that later on, in 1983, Chandrasekhar received a Noble prize in physics for his work on "theoretical studies of the physical processes of importance to the structure and evolution of the stars". So he certainly received at least some credit from his important work.

Around the same time, in 1937, Gamow and Landau also independently proposed that a accretion of matter onto a dense neutron star core could be the missing source of energy for stars. This increased the interest towards neutron stars and the field flourished on the 1930s. Soon it was, however, shown that stars are powered not by accretion but by thermonuclear reactions (as suggested in the 1920s by Eddington and others). The interest in neutron stars then faded away and the research focused on weaponing the nuclear forces.

Next big breakthrough came almost 20 years later in the 1950s, when John Wheeler and his collaborators constructed the first realistic equation of state of dense matter[18]. For the outer layers, known as the crust, they applied a semi-empirical mass formula together with the equation of state of degenerate electrons. For the dense core, they assumed a mixture of three ideal Fermi gases compose of neutrons, protons, and electrons. This marked the first consistent formulation for the neutron star structure. It was followed by Cameron who applied the Skyrme equation of state for the high-density matter. This had important conclusions, as he was then able to show that nuclear forces stiffen the matter considerably in comparison to the non-interacting neutrons. Similar to Tolman and Volkoff, he then went on and calculated the maximum possible mass of approximately 2 $M_{\odot}$. This marked an important theoretical breakthrough as it implied that neutron stars can, after all, exists. A new wave of interest towards neutron stars was thus launched as everybody wanted to observe them.

1.1.2 Many observational faces of neutron stars

After Wheeler and Cameron laying the modern foundation for the neutron star structure studies, everybody was eager to find these strange objects from the night sky. It did not take long, when researchers realized that as neutron stars are born in the supernova explosions, we expect them to be hot. Most of the theoretical effort in the 60s was then focused on developing models for the cooling of neutron stars[19] It was the possible thermal radiation from this cooling that could then be used to detect them, as was first shown by Hong-Yee Chiu[20]. First calculations predicted surface temperatures of $T \sim 10^6$ K for a neutron star of around 1000 years old. This had important implications for the observers as it means that neutron stars would radiate mainly in the X-rays. Atmosphere of Earth, on the other hand, was impenetrable in the X-ray wavelengths. Luckily, 60s also marked the beginning of golden era for spaceborn observatories.

17 [17] G. Baym. 1982.

18 [18] J. A. Wheeler. *ARA&A*. (1966).

19 [19] R. C. Stabler. 1960; [20] H.-Y. Chiu. *Annals of Physics*. (1964); [21] D. C. Morton. *Nature*. (1964); [22] H.-Y. Chiu and E. E. Salpeter. *Physical Review Letters*. (1964); [23] J. N. Bahcall and R. A. Wolf. *Physical Review*. (1965); [24] J. N. Bahcall and R. A. Wolf. *ApJ*. (1965); [25] S. Tsuruta and A. G. W. Cameron. *Canadian Journal of Physics*. (1966).

20 [20] H.-Y. Chiu. *Annals of Physics*. (1964).

When X-rays did not reach the earth, humankind went to space. In the late 1950s and early 1960s it was the pioneering experiments of italian astrophysicist Riccardo Giacconi that opened this new window the Universe. Giacconi started first with rocket-borne experiments and later on continued by leading the development of the first orbiting X-ray satellite Uhuru, "freedom" in Swahili.[21] After the first X-ray satellite, Giacconi then continued with the Einstein Observatory, the first fully imaging X-ray satellite, and later with Chandra X-ray observatory. For all of his efforts, he received the Noble Prize in Physics in 2002 "for pioneering contributions to astrophysics, which have led to the discovery of cosmic X-ray sources".

During the starting boom, several extra-terrestrial X-ray sources were discovered. Like is common in science, first discovery actually came by accident. A team led by Giacconi launched an Aerobee 150 rocket in June 1962 to the skies with a payload of highly sensitive soft X-ray detector meant to observe the X-rays from the moon. Due to a slight change (or a mistake) in the planned trajectory it ended up observing towards the constellation of Scorpius and caught a glimpse of what is now known as an X-ray source Sco X−1. Little did they know that this was actually the first neutron star radiating to us. Five years later, in 1967 Iosif Shklovsky was first to propose that Scorpius X-1 is a neutron star[22], but his work attracted little to no attention.

First intended observations to find neutron stars were aimed towards the Crab nebula, a well-known candidate to hosting a neutron star. The Crab Nebula, already known from 1920s and 1930s to be a supernova remnant, is known to be exploded exactly in 4th of July, 1054.[23] In the contemporary Chinese, Japanese, and Arab history writings, a "guest star" is described to appear in the constellation of Taurus, and to persist even in the day-sky for 23 consecutive days. Even after that, it remained visible in the night sky for two years. For astronomers, this was a clear sign of a nearby supernova going off.

But it was not only the spectacular supernova, but what was left behind that eluded astronomers. Already in 1942, our old friend Baade and Rudolf Minkowski correctly found that the center of the Crab nebula contained an unusual star.[24] In the following years, the mystery deepened when radio emission was also detected.[25] This gathered lot of interest from the theorists as they were trying to explain the origin of the energy powering the nebula. In 1953, Shklovsky was on the right track again, when he predicted that the emission is due to synchrotron radiation from relativistic electrons spiraling along magnetic field lines. Next piece of the puzzle came in 1964, when Lodewijk Woltjer, who did his PhD on the Crab nebula and was supervised by Jan Oort, argued based on conservation of magnetic flux, that neutron stars should have a strong magnetic field.[26] Similar result was independently obtained in the East by Vitaly Ginzburg.[27]

Early X-ray telescopes of the time, had a very poor angular resolution so imaging the Crab nebula only to get an answer to the puzzle was hard. The first observation in 1964 by S. Bowyer et al. used a clever method of partial lunar occultation to cover unwanted part of the sky with the Moon, and what followed was the first X-ray observation of the neutron star candidate everybody was waiting for.[28] It was, however, followed by a disappointment when a follow-up observation measured the source size as about 1 light-year in size (10^{13} km) in comparison to the 11 light-years for the whole nebula.[29] The result was much larger than what was expected for a neutron star that should be of mere $\sim 10\,\mathrm{km}$ in radius. Ironically, what they did not know, was that this just as expected: For young neutron stars like the one in the Crab nebula, a pulsar wind (consisting of charged particles similar to Solar wind) is expected. This wind will then create a surrounding shell, much bigger in size, around the neutron star called plerion, that is the source of the X-rays. Hence, the

[21] [26] R. Giacconi *et al*. *Physical Review Letters*. (1962).
[22] [27] I. S. Shklovsky. *ApJ*. (1967).
[23] [28] J. H. Oort. 1997; [29] K. Lundmark. *PASP*. (1921); [30] N. U. Mayall. *Leaflet of the Astronomical Society of the Pacific*. (1939); [31] D. A. Green and F. R. Stephenson. 2003.
[24] [32] W. Baade. *ApJ*. (1942); [33] R. Minkowski. *ApJ*. (1942).
[25] [34] J. G. Bolton, G. J. Stanley, and O. B. Slee. *Nature*. (1949).
[26] [35] L. Woltjer. *ApJ*. (1964).
[27] [36] V. L. Ginzburg. *Soviet Physics Doklady*. (1964).
[28] [37] S. Bowyer *et al*. *Nature*. (1964).
[29] [38] S. Bowyer *et al*. *Science*. (1964).

mystery remained even though Nikolai Kardashev in the East and Franco Pacini in the West, gave plausible pioneering explanations for the formation of the wind in 1964 and 1967, respectively.[30]

Despite all the efforts (and partly due to bad luck) no concrete observations supporting the existence of neutron stars still existed. This all changed in the July 1967, in the farmlands near Cambridge. There, a pasture was filled with primitive antenna consisting of wires that were hanging from stakes. The idea was to use this newly build radio telescope to study interplanetary scintillation that could help in resolving quasars, another form of compact objects powered by black holes from extended sources in the sky. Among several other students who were working for Anthony Hewish, was a young post-doc Jocelyn Bell. In addition to the signal from the scintillation, she discovered a deviation on her chart-recorded papers: an extremely regular signal of 1.3373012 seconds caught Bell's attention. Originally this was dubbed (partially as a joke) as Little Green Men 1 (LGM-1). In reality, what they were seeing, and what Bell quickly realized, was the first pulsar observed, a rapidly rotating neutron star whose beam sometimes points towards us like a distant lighthouse. More Little Green Man quickly followed and by the end of 1968 a dozens of LGMs were known. The finding was later published in the Nature 1968 by Hewish.[31]

Hewish's announcement was quickly followed by a more than 100 papers on pulsars, speculating the possible origin of the signal. Winning argument came from Timothy Gold where he showed that pulsars are strongly magnetized rapidly rotating neutron stars.[32] However, one should not forget the similar seminal theoretical paper already in 1967, before the discovery, from Pacini.[33] More proof came when our old friend Crab nebula was shown to host a pulsar rotating at a period of merely 33 milliseconds.[34] Anything but a neutron star would be destroyed by the centrifugal forces from such a rotation.

The finding of Bell and Hewish was sensational and marked the first detection of a neutron stars almost 40 years after the theoretical speculation by Landau. Later on, Hewish was awarded the Noble prize in Physics in 1974 "for the discovery of pulsars", a somewhat unfair recognition taken into account that it was Bell who found them in practice. Hence, despite all the efforts in X-ray astronomy, the concluding evidence finally came from the radio wavelengths.

One should not, however, feel sorry for the X-ray astronomers as they got their fair share of neutron star-related revelations during the next decade. Important discoveries especially to study the nature of accretion, how matter infalls to a compact objects, came from the first long-duration observations done with the dutch astronomy satellite ANS. As a direct competitor the Europeans had the mighty machinery of U.S. funded Los Alamos. Their Vela-satellites were send to space mostly to monitor the compliance to the 1963 Partial Test Ban Treaty of nuclear weapons but they were used for science, too. In 1975 the ANS satellite was commisioned to study possible black holes in the center of globular cluster but happened to stumble upon somthing completely different: A short, ~ 60 second long X-ray flares were detected from a globular cluster NGC6624 Grindlay and Heise.[35] The competing Los Alamos group found similar energetic bursts, but due to the poor angular resolution (pointing to Earth for X-rays was easy and hence no effort was put for a good spatial accuracy) they could not pin point the exact location of the sources.[36] Later on, Clark et al went through the existing SAS-3 data from May 1975 and found a series of ten similar bursts from the same location, NGC6624.[37] Even more retrospect, it turned out that these strange flares had been observed already in 1969 from Cen X-4[38] with another Vela-satellite and in 1971 with the Soviet Kosmos 428 X-ray detector[39]. Their nature, however, remained elusive.

30 [39] N. S. Kardashev. *AZh*. (1964); [40] F. Pacini. *Nature*. (1967).

31 [41] A. Hewish *et al*. *Nature*. (1968).

32 [42] T. Gold. *Nature*. (1968).

33 [40] F. Pacini. *Nature*. (1967).

34 [43] J. M. Comella *et al*. *Nature*. (1969).

35 [44] J. Grindlay *et al*. *ApJ*. (1976).

36 [45] R. D. Belian, J. P. Conner, and W. D. Evans. *ApJ*. (1976).

37 [46] G. W. Clark *et al*. *ApJ*. (1976).

38 [47] R. D. Belian, J. P. Conner, and W. D. Evans. *ApJ*. (1972).

39 [48] O. P. Babushkina *et al*. *Soviet Astronomy Letters*. (1975).

Pioneering theoretical work on thermonuclear instabilities on the surface layers of accreting neutron stars was initiated by Hansen and van Horn in 1975.[40] They constructed stationary burning shells to lay on top of neutron stars but in contrast found out that most of them were actually unstable. Unstable here might not give the full meaning to the physical issue though: Such a layer on top of neutron star burning uncontrollably meant a spectacular firework. Shortly after the Los Alamos results came in, an italian astrophysicist Laura Maraschi, while visiting MIT in February 1976 was able to connect the dots and speculated that these recently observed X-ray bursts were due to thermonuclear flashes on the surface of accreting neutron stars.[41] Similar conclusion was done by Woosley and Taam in their 1976 paper titled "Gamma-ray bursts from thermonuclear explosions on neutron stars". Observational evidence soon followed when van Parajids et al and Thorstensen et al were independently able to optically resolve the companions of two known bursting sources Cen X-4[42] and Aql X-1[43]. Not only did these observations confirm that there is a companion star close by, but that it must be such a close to the neutron star that accretion, i.e., a constant flow of new fuel for the explosions, must exist.

All of the aforementioned discoveries were, however, nothing but a prelude to what was discovered on the years to follow. We will end this short historical review by listing some of the most important more modern findings. A big revelation came in 1979 when a very intense burst of gamma-rays was detected by two Soviet satellites Venera 11 and Venera 12.[44] Later dubbed as Soft Gamma Repeaters (SGRs), their energy source remained mysterious for decades. Theoretical breakthrough came in 1992 when Robert Duncan, Chistropher Thompson, and Bohdan Paczynski showed that the bursts could originate from a neutron star with a magnetic field 100 to 1000 times more powerful then what was previously known.[45] Today these neutron stars are more commonly known as magnetars, a subclass of young neutron stars where the initial magnetic field has been amplified by some delicate dynamo processes during the supernova explosion. Another surprise came in 1982, when a team lead by Backer changed on how we look at pulsars when they found, using the world's largest radiotelescope in Arecibo, a pulsar spinning 641 times per second.[46] This new neutron star was dubbed as a millisecond pulsar and unlike its predesessors, we now know that instead of slowly decreasing in spin, it belongs to a class of old pulsars that have been spun up by the accretion. In 2000, our understanding of the thermonuclear X-ray bursts was also changed when Cornelisse observed a very long, not minutes but hours long, burst from a neutron normally exhibiting regular bursts.[47] These were then dubbed as superbursts, in contrast to the normal burst. The reason in the difference is, we think, because of the burning material: normal bursts use hydrogen and helium as their fuel but superbursts can devour a more rare carbon shell in a matter of hours if the conditions are just right.

1.2 From first principles to a neutrons star

1.2.1 Background: Sun and stars

Let us first see, what can we learn from neutron stars using simple estimates and conservation laws. Neutrons star are born from a death of a normal star. Most familiar such a star is our Sun 1.496×10^{13} cm from us.[48] With a mass of $M_\odot = 1.99 \times 10^{33}$ g and radius of $R_\odot = 6.96 \times 10^{10}$ cm, our Sun gives us an idea of the typical stellar size scales. Curiously, these numbers also means that the mean density of the Sun is $\rho_\odot \approx 1.41\,\mathrm{g\,cm^{-3}}$, a mere 1.4× the density of the water. Stellar orders of magnitude

Like all stars, our Sun is held together by the inward pulling gravity. Gravity does not prefer any direction

40 [49] C. J. Hansen and H. M. van Horn. *ApJ*. (1975).
41 [50] L. Maraschi and A. Cavaliere. 1977; [51] W. H. G. Lewin, J. van Paradijs, and R. E. Taam. *SSRv*. (1993).
42 [52] J. van Paradijs *et al*. *ApJ*. (1980).
43 [53] J. Thorstensen, P. Charles, and S. Bowyer. *ApJ*. (1978).
44 [54] E. P. Mazets *et al*. *Nature*. (1979).
45 [55] R. C. Duncan and C. Thompson. *ApJ*. (1992).
46 [56] D. C. Backer *et al*. *Nature*. (1982).
47 [57] R. Cornelisse *et al*. *A&A*. (2000).
48 Throughout this thesis we will typically present our quantities only up to some fixed precision instead of the full litany of numbers. We will also adopt the centimeter-gram-second (cgs) unit system instead of the (maybe) more common SI-system. Such a selection is sure to disappoint some, but try to endure.

more than some other and, just like we easily observe, a spherical object is expected to form. In addition to the inward-facing force, an outward-facing force is needed to balance the system. For normal stars this force is originating from the thermal pressure.

We observe stars in the night sky because they shine on us. This radiation, and also the origin of the thermal pressure, is from the thermonuclear fusion burning inside the star. *Thermo* here refers to the temperature and heat, *nuclear* to the atomic nuclei, and *fusion* to the process where elements are fused together. During the thermonuclear fusion process, the star's core fuses light elements such as protons into heavier ones like helium. Mass of two protons is less than a mass of one helium atom. This mass difference between the start and end results is then transferred into energy in accordance to the Einstein's famous formula $E = mc^2$. A whole sequence of such a fusions takes place inside the star where lighter elements are merged together building heavier and heavier elements. This energy release from the mass-to-energy conversion will then give the star a sufficient thermal pressure support to keep it from not collapsing under the relentless gravity.

Thermonuclear fusion process

The fusion of elements does not continue forever. In the beginning, two protons collide two form a helium. In the next stage three helium nuclei collide to form a carbon, and so on, until iron is created. Production of iron marks the end of the possible fusion chain because fusion of two iron cores does not release energy anymore, it requires it to take place.[49] This iron produced, will then sink to the center of star forming a dead core without any energy input.

Like all big furnaces, at some point the stars will run out of fuel to burn. What is then left behind is a inner core of iron with a subsequent onion-like layers of lighter and lighter elements. Crucial question to ask next is: What is supporting this iron core now that the thermal pressure from the fusion processes are lost? To answer this, we need to look inside the iron atom: Iron nucleus, consisting of 26 protons and 32 neutrons, and surrounded by 26 electrons, repulses its neighbors because of the negatively charged electron cloud does not want to get in touch with its neighbors flying next to it in the iron-atom lattice. [50] It is this antisocial behavior originating from the electric charge repulsion that also gives internal support to our everyday materials and prevents us from not falling through the floor. Electrons and protons inside the atom itself, on the other hand, are repulsing each other because of the quantum effects. explain quantum repulsion

1.2.2 Neutron stars at last

From this setup it only takes a short step into realizing the existence of neutron stars. What if at some point even these quantum effects are not enough to support the core? Let us consider the consequences of this thought play. More detailed calculations show that the resulting iron-core sitting at the center of the star is weighting at a maximum of around $\sim M_\odot$. Hence, there are $M_{\mathrm{core}}/m_{\mathrm{atom}} \sim M_\odot/m_{\mathrm{p}} \approx 1.99 \times 10^{33}\,\mathrm{g}/1.67 \times 10^{-24}\,\mathrm{g} \sim 10^{57}$ atoms trapped inside the core. [51] Here we are already considering not iron atoms but pure hydrogen atoms only to simplify the presentation. Working backwards from these numbers, we can estimate the size of the core.

1.2.3 Neutron star dimensions

It turns out that Sun is also not as stable as one would think: With a rotation period of about 25.5 days it then takes Sun about a month to revolve around itself. Similar to a bicycle dynamo hub, this rotation also gives rise to a detectable surface magnetic field of $B_\odot \approx 1$ Gauss.[52]

[49] This opens up another possibility of creating energy by splitting heavy elements. Such a process is called fission and is familiarly taken advantage of in earthly nuclear power plants.

[50] More precise consideration shows that nickel ($^{56}_{28}$Ni) is actually thermodynamically more favored in the core because of the lack of neutrons needed to synthesize iron-58 ($^{58}_{26}$Fe).

[51] The mass of the atom, $m_{\mathrm{atom}} = m_{\mathrm{p}} + m_{\mathrm{e}}$, is also approximated (to an excellent accuracy) by just considering the central nuclei alone as the electron mass $m_{\mathrm{e}} \approx 9.11 \times 10^{-28}$ g is negligible in comparison to the proton mass.

[52] A typical refrigerator magnet is about 50× stronger with a magnetic field of 50 Gauss.

A typical neutron star, on the other hand, weights about $M \sim 1.5\,M_\odot$ but extends only up to $R \sim 10\,\mathrm{km}$. Such dimensions give us an impressive mean density of $\rho \sim 7 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. In comparison, a typical nucleon (such as a neutron) weights about 1.67×10^{-24} g and has a radius of about 1.25×10^{-13} cm, yielding us a nuclear density of $\rho_\mathrm{n} \approx 2 \times 10^{14}\,\mathrm{g\,cm^{-3}}$. Hence, even the mean density inside the star is already on the same order of magnitude as the internal density inside nuclei. This suggests us that the composition inside a neutron star is not our typical every-day matter.

Suppose a neutron star is, like any normal star, a blob of gas held together by the inwards pulling gravity. Gravity does not prefer any direction more than some other and so a stable end-result is an isotropic configuration. A pure inward pulling force is, of course, not enough so we also need a countering outward-facing force to resist the compression of the material. As an first approximation, there is no need to assume that this force would have any preferred direction either. Hence, our expected outcome is a sphere held together by the gravity originating from the mass M of the matter itself. Let us, for a while forget the exact origin and nature of the compression-resisting force and see what can we learn solely from the current information only.

1.3 What do we actually know?

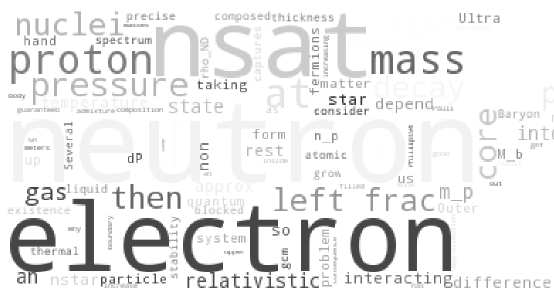
magnetic fields in pulsars $10^{11} - 10^{13}$ G[53] in binaries accretion screens b field[54]

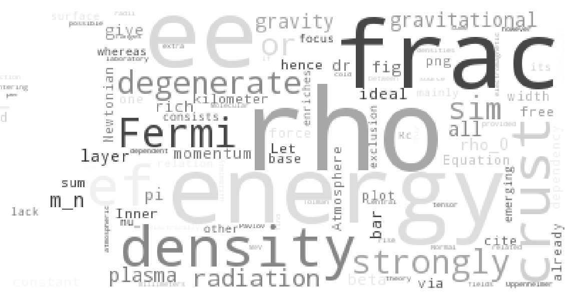
Maximum from virial theorem $B \sim 10^{18} - 10^{19}$ Gauss. After that magnetic energy $R^3 B^2/6$ is grater than graviational binding energy $3GM^2/5R$.[55]

1.4 Connection to this thesis

In this thesis we study many aspects of the neutron stars, their environment and the internal processes that will in the end help us in constraining the behavior of the ultra-dense matter inside the star.

53 [58] R. N. Manchester *et al*. *AJ*. (2005).
54 [59] A. Cumming, E. Zweibel, and L. Bildsten. *ApJ*. (2001).
55 [60] S. Chandrasekhar and E. Fermi. *ApJ*. (1953); [61] S. L. Shapiro and S. A. Teukolsky. 1983; [62] D. Lai and S. L. Shapiro. *ApJ*. (1991).





2 Physics of neutron star interiors

In this chapter, we will study the neutron star interiors in more detail. In practice, this means describing the behavior of the matter from a densities of $\sim 10^{-2}\,\mathrm{g\,cm^{-3}}$ to $\sim 10^{16}\,\mathrm{g\,cm^{-3}}$, an impressive 18 orders of magnitude range starting from a hot and rarefied electron corona to an ultra-dense neutron liquid.

The structure of the star can be roughly divided into three distinct sections: atmosphere, crust, and core. Neutron star atmosphere holds a negligible amount of matter in comparison to the whole star, but it is important for the radiative processes. It is the radiation from the atmosphere that we actually observe. The crust, like the name implies, can be understood as a solidified layer surrounding the liquid core. Physics describing the crust is relatively well known and same type of matter consisting of ions, protons, and electrons can be found inside white dwarf stars. Bulk of the mass, on the other hand, is located on the liquid neutron core. Detailed microphysics of such matter are still unknown and this is reflected in a large uncertainty in the actual size of the star that is still unconstrained.

We begin by giving an overview of the characteristics of each of the different layers. By combining this information, we can then build different models for the neutron stars and describe some more global aspects of them such as mass and radius. For this we need to solve the relativistic equations of hydrostatic equilibrium, that is also discussed. In the end, this enables us to build a mapping between the (un)known microphysics of the dense matter and the astrophysical observables.

2.1 Equation of state

In thermodynamics, we speak of *state variables* that describe a current state of the matter under a given physical conditions. These include, for example, the density ρ, pressure P, and temperature T of the matter. Equation of state is a thermodynamical equation connecting these states variables together. Often, when focusing on neutron stars, what we mean by EoS is a function connecting the pressure and the density of the matter only, $P(\rho)$.

The dependency on the rest of the variables such as temperature can be often forgotten because the matter is *degenerate*. In contrast to the "normal" matter where a statistical moments such as temperature can be used to describe a large ensemble of particles, the degenerate matter is dominated by quantum mechanical effects of single particles. Because of the immense densities, a free particle in a degenerate matter is actually bounded into a finite volume. Inside this small volume, the energy levels of the particle are restricted to take only a discrete set of values called quantum states, because of the underlying wave-nature of the quantum mechanical description. Hence, a notion of temperature, for example, does not make much sense.

Degenerate matter

Overview of the EoS for the full range of densities relevant to neutron stars is shown in Fig. 2.1. From here it is easy to see that temperature only plays a role in the very uppermost ~ 100 meters of the atmosphere. Behavior of the matter is also quite well known all the way up to the crust-core interface, after which we start to see any deviation from different EoS models. In the Earthly laboratories we can probe the matter

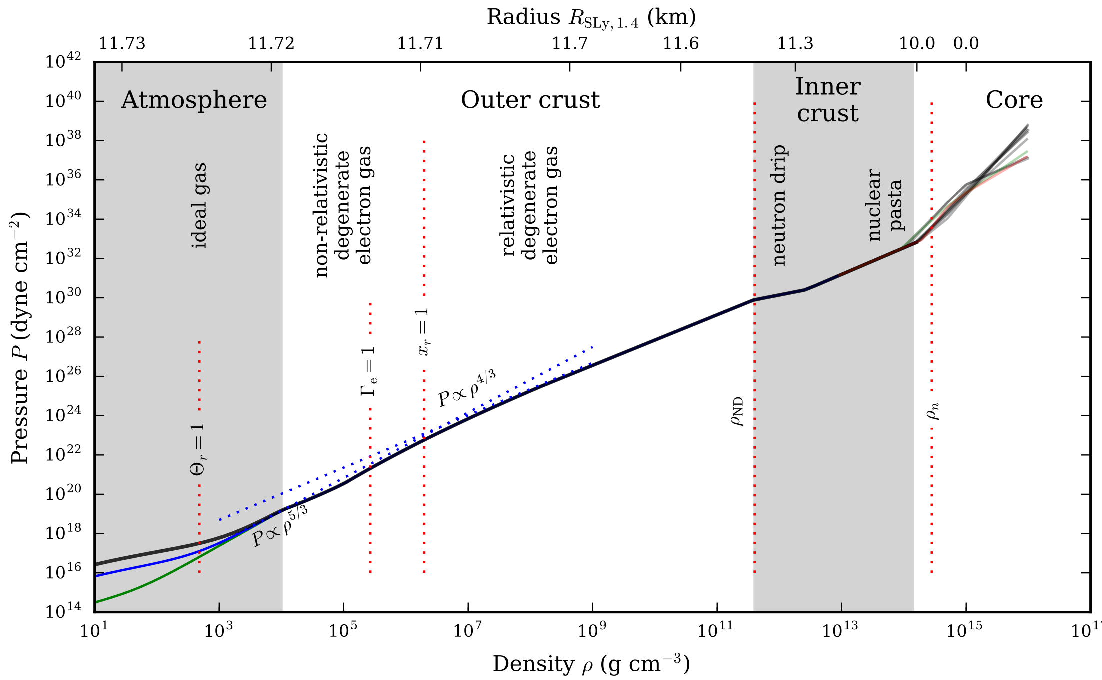


Figure 2.1: Overview of the pressure versus density relation for the full range of densities relevant for neutron stars. Here the evolution of the pressure is shown against the densities depicted in the bottom vertical axis. Green solid line shows the EoS for matter at $T = 10^6$ K, whereas blue line is for $T = 5 \times 10^6$ K, and black for $T = 10^7$ K. Additionally, the upper vertical axis shows the evolution of the radial coordinate computed for one particular EoS (SLy, see Sect. 2.4) and neutron star configuration (mass of $1.4\,M_\odot$). Different shaded vertical regions show the corresponding interior structures of the star. Additionally, some interesting densities are highlighted with dashed red lines and text labels (see Sects. 2.2–2.4).

somewhere close to 10^{14} g cm^{-3}, after which the densities becomes too great for us to handle in Earth.[1] On the other hand, it is exactly starting from this density range that the bulk of the neutron star just starts. Other curious quirk of Nature is how all of the complicated microphysics gets reduced to simple line segments in the logarithmic scales, also known as polytropic pressure relations. In the following sections, we will focus on deriving these simple relations as it helps us in understanding the underlying physics.

2.2 Atmosphere

Thin layer of plasma From centimeters in hot to millimeters in cold Zavlin & Pavlov 2002

Where spectrum or thermal electromagnetic radiation is formed. Spectrum, beaming and polarization of emerging radiation can be determined from radiation transfer problem in atmospheric layers.

Contains information on the parameters of the surface: effective temperature surface gravity chemical

[1] Maximum densities reached in the Earth are usually obtained by colliding heavy nucleons together, momentarily creating a core of even denser matter. The densest naturally occurring element found on top of planet Earth is osmium that has a density of “just” $\rho \approx 2.2 \times 10^1$ g cm^{-3}.

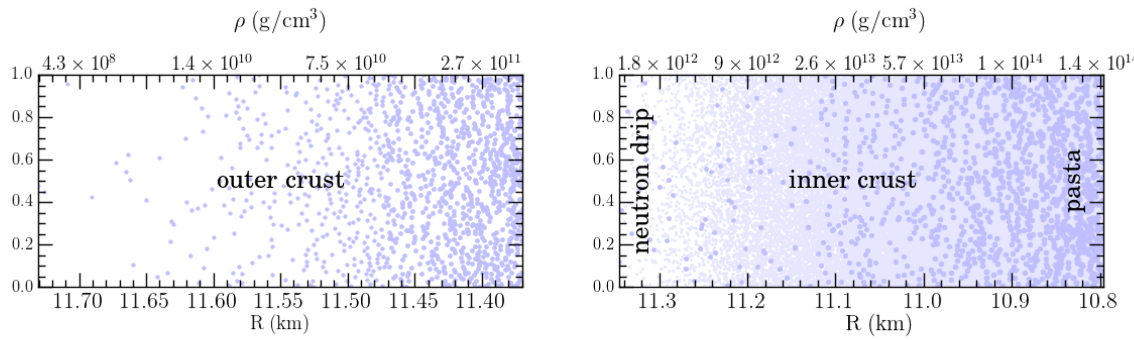


Figure 2.2: Molecular simulation of the crust. Figure adapted from `https://github.com/awsteiner/nstar-plot`.

composition geometry of the system mass and radii.

Eddington limit of where radiation force exceeds the gravitational one.

2.3 Crust

Outer crust

From atmosphere to $\rho_N D \sim 4\times10^{11}\,\mathrm{g\,cm^{-3}}$. In thickness some hundred meters. Non degenerate electron gas Ultra-relativistic electron gas $\rho > 10^6\,\mathrm{g\,cm^{-3}}$. Pressure provided by electrons here.

Here the equation of state is decribed by the relativistic degenerate electron gas. Physics behind this are quite simple and we repeat the calculations here to give the reader ideas about what are the most important physical processes. The result also bears some historical value.

We have reached densities where the EoS is dominated mainly by the electrons, hence it is characterized by the electron number density n_e and temperature T_e (hereafter just T in this section). Moving on from an ideal plasma, we can start by introducing corrections produced by the closely packed charges. In practice we can use the so-called ion-sphere model to describe our Coulomb liquid of ions. We now assume that our ions are emerged into a sea of rigid electron background that takes care of the charge neutrality. Let us begin by defining a so-called electron sphere radius

$$r_\mathrm{e} = \left(\frac{4\pi n_\mathrm{e}}{3}\right)^{-1/3} \tag{2.1}$$

We can also parameterize the strength between Coulomb (charge) interactions by considering a ratio of potential energy to the kinetic energy with

$$\Gamma_\mathrm{e} = \frac{e^2}{r_\mathrm{e} kT}. \tag{2.2}$$

Similarly, for ion with a charge number of Z_i, we can define the ion-sphere radius

$$r_\mathrm{i} = r_\mathrm{e} Z_\mathrm{i}^{1/3} \tag{2.3}$$

that encapsulates enough area to be charge neutral, when considering a static electron-induced background from n_e. Ion Coulomb coupling factor is similarly

$$\Gamma_\mathrm{i} = \Gamma_\mathrm{e} Z_\mathrm{i}^{5/3} = \frac{(Z_\mathrm{i} e)^2}{r_\mathrm{i} kT} \tag{2.4}$$

In the weak-coupling limit ($\Gamma_\mathrm{i} \ll 1$) Debye-Hückel results for the free energy are valid[2]

$$\frac{F_\mathrm{ex}}{V} = \frac{1}{\sqrt{3}} n_\mathrm{i} k T \Gamma^{3/2} \tag{2.5}$$

Hence, the pressure correction due to the Coulomb interactions is[3]

$$P_\mathrm{ii} \approx -0.3 n_\mathrm{i} \frac{Z^2 e^2}{r_\mathrm{i}}. \tag{2.6}$$

For a degenerate system it also makes sense to present the Fermi quantities: momentum

$$p_\mathrm{F} = \hbar (3\pi^2 n_\mathrm{e})^{1/3} \tag{2.7}$$

energy

$$\epsilon_\mathrm{F} = c^2 \sqrt{(m_\mathrm{e} c)^2 + p_\mathrm{F}^2}, \tag{2.8}$$

and temperature

$$T_\mathrm{F} = \frac{m_\mathrm{e} c^2}{k} \left(\sqrt{1 + \left(\frac{p_\mathrm{F}}{m_\mathrm{e} c} \right)^2} - 1 \right) = T_\mathrm{r} (\gamma_\mathrm{r} - 1), \tag{2.9}$$

where we have defined a typical temperature

$$T_\mathrm{r} \equiv \frac{m_\mathrm{e} c^2}{k} \sim 6 \times 10^9\ \mathrm{K} \tag{2.10}$$

relativistic scaling factor

$$\gamma_\mathrm{r} \equiv \sqrt{1 + x_\mathrm{r}^2}, \tag{2.11}$$

and typical (dimensionless) momentum

$$x_\mathrm{r} \equiv \frac{p_\mathrm{F}}{m_\mathrm{e} c}. \tag{2.12}$$

Using these definitions, it is easy to characterize our electron gas into regions of

- non-relativistic, for which $T \ll T_\mathrm{r}$ and $x_\mathrm{r} \ll 1$,
- mildly-relativistic, $T \sim T_\mathrm{r}$ and $x_\mathrm{r} \sim 1$,
- ultra-relativistic, $T \gg T_\mathrm{r}$ and $x_\mathrm{r} \gg 1$,
- non-degenerate, $T \gg T_\mathrm{F}$,
- mildly degenerate, $T \sim T_\mathrm{F}$,
- and strongly degenerate $T \ll T_\mathrm{F}$.

[2] [63] L. D. Landau and E. M. Lifshitz. 1980.

[3] [64] D. R. Dewitt *et al*. *Phys. Rev. A*. (1996).

Free energy

$$F = (\mu - m_{\rm e}c^2)n_{\rm e} - \frac{2}{(2\pi\hbar)^3}\int d\vec{p}\frac{1}{3}\vec{p}\cdot\vec{v}f_{\rm F-D}(\epsilon) \tag{2.13}$$

where

$$n_{\rm e} = \frac{2}{(2\pi\hbar)^3}\int d\vec{p}f_{\rm F-D}(\epsilon), \tag{2.14}$$

and

$$f_{\rm F-D}(\epsilon, T) = \frac{1}{\exp\left(\frac{\epsilon-\mu}{kT}\right) - 1} \tag{2.15}$$

and

$$\epsilon = \sqrt{m_{\rm e}^2c^4 + p^2c^2} \tag{2.16}$$

Pressure

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,\{N_j\}} \tag{2.17}$$

Sommerfield expand free energy in powers of $T/T_{\rm F}$

$$\frac{F}{V} = \frac{m_{\rm e}c^2}{\lambda_{\rm C}^3}\frac{1}{8\pi^2}\left(x_{\rm r}(1+2x_{\rm r}^2)\gamma_{\rm r} - \ln(x_{\rm r}+\gamma_{\rm r}) + \frac{4\pi^2}{9}t_{\rm r}^2x_{\rm r}(\gamma_{\rm r}+\gamma_{\rm r}^{-1})\right) + O(t_{\rm r}^4) \tag{2.18}$$

and obtain pressure

$$P_{\rm id}^{(\rm e)} = \frac{P_{\rm r}}{8\pi^2}\left(x_{\rm r}(1+2x_{\rm r}^2)\gamma_{\rm r} - \ln(x_{\rm r}+\gamma_{\rm r})\right) \tag{2.19}$$

where again typical pressure

$$P_{\rm r} = \frac{m_{\rm e}c^2}{\lambda_{\rm C}} \sim 1.4\times10^{25}\,{\rm dyn\,cm^{-2}} \tag{2.20}$$

Hence, it takes the simple polytropic form

$$P_{\rm id}^{(\rm e)} \approx \frac{P_{\rm r}}{9\pi^2\gamma_{\rm AD}}x_{\rm r}^{3\gamma_{\rm AD}} \tag{2.21}$$

where the polytropic index $\gamma_{\rm AD} = \frac{5}{3}$ for the non-relativistic $x_{\rm r} \ll 1$ and $\gamma_{\rm AD} = \frac{4}{3}$ for the ultra-relativistic case (recall also that $x_{\rm r} \propto n_{\rm e}^{1/3}$).

Degenerate electron gas pressure accompanied with the ion Coulomb correction will then actually give us a rather good approximation for the equation of state

$$P(x_{\rm r}) = P_{\rm id}^{(\rm e)} + P_{\rm ii} \tag{2.22}$$

this is valid in a large density range of $10^4 < \rho < 10^{10}\,{\rm g\,cm^{-3}}$.

In deeper layers ions form a strongly coupled Coulomb system (liquid or solid). Hence, crust. Fermi energy grows with increasing ρ. Induces β captures and enriches nuclei with neutrons. At the base neutrons start to drip out from nuclei.

2.3.1 Inner crust

About one kilometer thick. Density from $\rho \sim \rho_{ND}$ (at upper boundary) to $\sim 0.5\rho_{\rm n}$ at the base. Matter consists of electrons, free neutrons n and neutron-rich atomic nuclei. Fraction of free n grow with ρ.

nuclei are immersed in a neutron gas and hence nuclear interactions play a crucial role in defining the matter.

Finally, nuclei disappear at the crust-core interface.

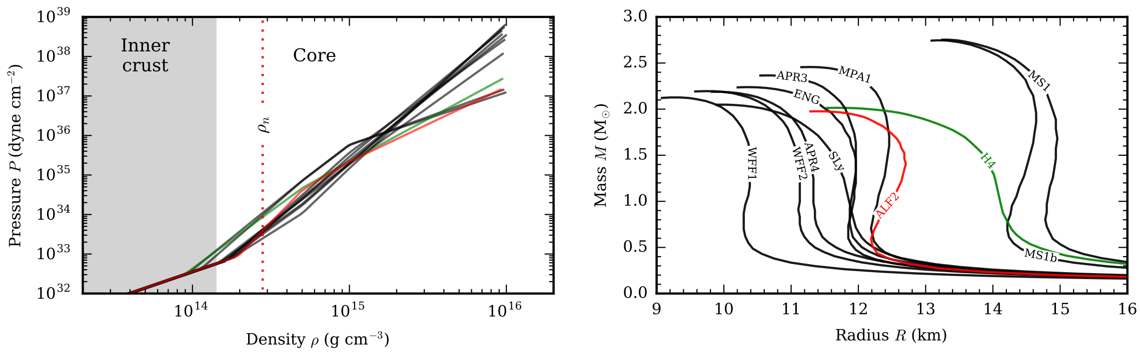


Figure 2.3: Core EoS.

2.4 Liquid core

Outer core. Density ranges $0.5\,\rho_{\rm n} < \rho < 2\,\rho_{\rm n}$. Several kilometers. Neutrons with several per cent admixture of protons p and electrons e^-. Strongly degenerate. Electrons form almost ideal Fermi gas. Neutrons and protons, interacting via nuclear forces, constitute a strongly interacting Fermi liquid.

Inner core. Where $\rho > 2\,\rho_{\rm n}$. Appearance of muon. Central density can be around $(10-15)\,\rho_{\rm n}$. Very model dependent. Main problem.

Here we consider few (relatively) modern descriptions for the dense matter equation of state. The EoSs can be divided into few different classes based on the particles that they consist of. Here they are divided into matter consisting of

- plain $(npe\mu)$nuclear matter,
- normal nuclear matter spiced up with hyperons (H),
- normal nuclear matter together with more exotic particles like pion and kaon condensates (πK),
- and matter consisting of (or normal matter spiced up with) quarks (q).

For $(npe\mu)$we include models computed with

- potential method using SLy effective nuclear interaction that is of Skyrme-type,[4],
- four variational method EoSs, APR3/4[5] and WFF1/2[6],
- two relativistic Brueckner-Hartree-Fock calculations, ENG[7] and MPA1[8],
- two relativistic mean-field theory, MS1 and MS1b (same as MS1 but with lower symmetry energy)[9].

For hyperon models (H)we include

[4] [65] F. Douchin and P. Haensel. *A&A*. (2001).
[5] [66] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall. *Phys. Rev. C*. (1998).
[6] [67] R. B. Wiringa, V. Fiks, and A. Fabrocini. *Phys. Rev. C*. (1988).
[7] [68] L. Engvik *et al*. *ApJ*. (1996).
[8] [69] H. Müther, M. Prakash, and T. L. Ainsworth. *Physics Letters B*. (1987).
[9] [70] H. Müller and B. D. Serot. *Nuclear Physics A*. (1996).

- one variant of relativistic mean-field theory EoS H4[10].

EoSs where mesons, like pion and kaon condensates (πK), are taken into account end up not being stiff enough.

Finally, for the hybrid nuclear matter and quark matter compositions (q)we include

- mixed APR nuclear matter and color-flavor-locked quark matter EoS ALF2.[11]

2.4.1 Why neutrons then?

Let us first consider ideal gas of degenerate electron-proton-neutron plasma. In a degenerate plasma all the quantum states are filled up all the way to the Fermi energy. It is the Pauli exclusion principle that then prevents occupying all of these already taken quantum states. Normal beta-decay mode for the neutrons, on the other hand, is $n \to p + e^- + \bar{\nu}_e$, that describes the possible path of how a neutron n will decay into a proton p, electron e^-, and electron neutrino $\bar{\nu}_e$. Such a decay is, however, blocked because there is no room for an emission of an extra electron e^- or a proton p.[12]

Let us then only focus on the decay of the most energetic neutrons with an energy equal to the Fermi energy $\epsilon_\mathrm{F}(n)$. Co-existence of neutrons, protons, and electrons is then guaranteed (at zero temperature) if

$$\epsilon_\mathrm{F}(n) = \epsilon_\mathrm{F}(p) \ + \epsilon_\mathrm{F}(e^-). \tag{2.23}$$

Fermi momentum of a particle is related to its concentration via

$$p_\mathrm{F} = \left(\frac{3n}{8\pi}\right)^{1/3} h, \tag{2.24}$$

where n is the number density, and h the Planck constant. Massive neutrons and protons are to a good approximation non-relativistic up to a densities of ρ_n, and hence energy is simply a sum of their rest mass energy and kinetic energy

$$\epsilon_\mathrm{F}(n) \ \approx m_n c^2 + \frac{p_\mathrm{F}(n)^2}{2m_n}, \tag{2.25}$$

and

$$\epsilon_\mathrm{F}(p) \ \approx m_p c^2 + \frac{p_\mathrm{F}(p)^2}{2m_p}. \tag{2.26}$$

Electrons, on the other hand, are already ultra-relativistic, and so

$$\epsilon_\mathrm{F}(e^-) \approx p_\mathrm{F}(e^-)c^2. \tag{2.27}$$

Also note that $n_p = n_e$, as the star is electrically neutral. From this we find relation of the $n_n/n_p \sim 1/200$ by taking into account the rest mass difference $m_p - m_n = 2.6\mathrm{MeV}\, c^2$ at $\rho \sim \rho_\mathrm{n}$. Thus, we conclude that the matter inside is neutron rich.

2.4.2 Polytropes

Parameterize everything with polytropes.

[10] [71] B. D. Lackey, M. Nayyar, and B. J. Owen. *Phys. Rev. D.* (2006).

[11] [72] M. Alford *et al. ApJ.* (2005).

[12] see e.g. [73] A. C. Phillips. 1994.

2.5 Tolman-Volkoff-Oppenheimer equations

Newtonian pressure gradient needed to oppose the gravity is

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}. \tag{2.28}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{2.29}$$

Taking into account the general relativistic corrections we get

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \times \frac{(1 + P/\rho c^2)(1 + 4\pi r^3 P/mc^2)}{1 - 2Gm/rc^2}. \tag{2.30}$$

Difference originates from the source of gravity: in the Newtonian case it is the mass m, whereas in the General relativity it is the energy momentum tensor that depend both on the energy density and the pressure. As a result, energy and pressure give rise to a gravitational fields.

Severness of the GR corrections can be estimated from the so called compactness parameter

$$x = \frac{GM}{Rc^2} \approx 2.95\ M/\ M_\odot\ \mathrm{km} \tag{2.31}$$

It has an important consequence to the stability of neutron stars: Successive increase in the pressure to counter the gravity is ultimately self-defeating.

Solution for a constant density ρ_0 gives

$$P(r) = G\frac{2\pi}{3}\rho_0^2(R^2 - r^2) \tag{2.32}$$

whereas the GR gives

$$P(r) = \rho_0^2 c^2 \left[\frac{(1 - u\left(\frac{r}{R}\right)^2)^{1/2} - (1 - u)^{1/2}}{3(1 - u)^{1/2} - (1 - u\left(\frac{r}{R}\right)^2)^{1/2}} \right], \tag{2.33}$$

where $u = 2GM/Rc^2$.

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