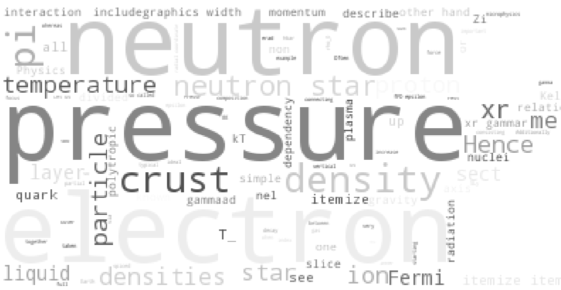

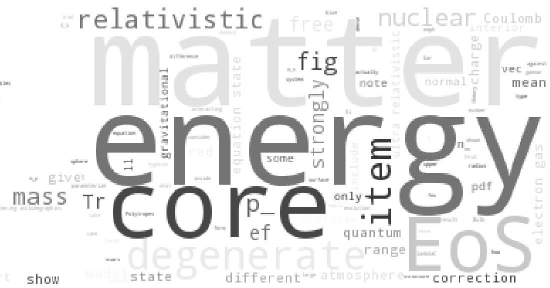

1 Physics of neutron star interiors

In this chapter, we will study the neutron star interiors in more detail. In practice, this means describing the behavior of the matter from a densities of $\sim 10^{-2}\,\mathrm{g\,cm^{-3}}$ to $\sim 10^{16}\,\mathrm{g\,cm^{-3}}$, an impressive 18 orders of magnitude range starting from a hot and rarefied electron corona to an ultra-dense neutron liquid.

The structure of the star can be roughly divided into three distinct sections: atmosphere, crust, and core. Neutron star atmosphere holds a negligible amount of matter in comparison to the whole star, but it is important for the radiative processes. It is the radiation from the atmosphere that we actually observe. The crust, like the name implies, can be understood as a solidified layer surrounding the liquid core. Physics describing the crust is relatively well known and same type of matter consisting of ions, protons, and electrons can be found inside white dwarf stars. Bulk of the mass, on the other hand, is located on the liquid neutron core. Detailed microphysics of such matter are still unknown and this is reflected in a large uncertainty in the actual size of the star that is still unconstrained.

We begin by giving an overview of the characteristics of each of the different layers. By combining this information, we can then build different models for the neutron stars and describe some more global aspects of them such as mass and radius. For this we need to solve the relativistic equations of hydrostatic equilibrium, that is also discussed. In the end, this enables us to build a mapping between the (un)known microphysics of the dense matter and the astrophysical observables.

1.1 Equation of state

In thermodynamics, we speak of *state variables* that describe a current state of the matter under a given physical conditions. These include, for example, the density ρ, pressure P, and temperature T of the matter. Equation of state is a thermodynamical equation connecting these states variables together. Often, when focusing on neutron stars, what we mean by EoS is a function connecting the pressure and the density of the matter only, $P(\rho)$.

Degenerate matter

The dependency on the rest of the variables such as temperature can be often forgotten because the matter is *degenerate*. In contrast to the "normal" matter where a statistical moments such as temperature can be used to describe a large ensemble of particles, the degenerate matter is dominated by quantum mechanical effects of single particles. Because of the immense densities, a free particle in a degenerate matter is actually bounded into a finite volume. Inside this small volume, the energy levels of the particle are restricted to take only a discrete set of values called quantum states, because of the underlying wave-nature of the quantum mechanical description. Hence, a notion of temperature, for example, does not make much sense.

Overview of the EoS for the full range of densities relevant to neutron stars is shown in Fig. 1.1. Another perspective is shown in Fig. 1.2 that visualizes the dependency of the pressure and cumulative mass of the star against the radial coordinate as measured starting from the core. From here it is easy to see that temperature only plays a role in the very uppermost ~ 10 meters of the atmosphere. Behavior of the matter is also quite well known all the way up to the crust-core interface, after which we start to see larger deviations because of the different EoS models. In the Earthly laboratories we can probe the matter somewhere close to

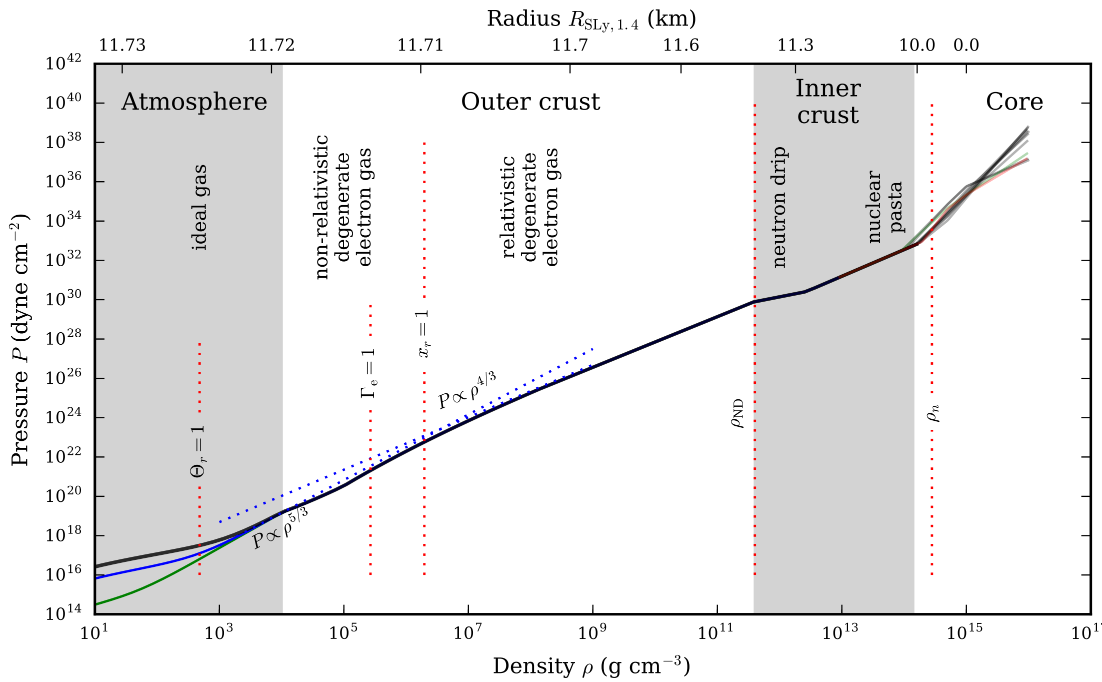

Figure 1.1: Overview of the pressure versus density relation for the full range of densities relevant for neutron stars. Here the evolution of the pressure is shown against the densities depicted in the bottom vertical axis. Green solid line shows the EoS for matter at $T = 10^6$ K, whereas blue line is for $T = 5 \times 10^6$ K, and black for $T = 10^7$ K. Additionally, the upper vertical axis shows the evolution of the radial coordinate computed for one particular EoS (SLy, see Sect. 1.4) and neutron star configuration (mass of $1.4\,M_\odot$). Different shaded vertical regions show the corresponding interior structures of the star. Additionally, some interesting densities are highlighted with dashed red lines and text labels (see Sects. 1.2–1.4).

10^{14} g cm^{-3}, after which the densities becomes too great for us to handle in.* On the other hand, it is exactly starting from this density range that the bulk of the neutron star just starts. Another curious quirk of Nature is how all of the complicated microphysics gets reduced to simple line segments in the logarithmic scales, also known as polytropic pressure relations. In the following sections, we will focus on deriving these simple relations as it helps us in understanding the underlying physics.

1.2 Atmosphere

†

radiation espaces to the surrounding space without considerable losses.

Gaseous uppermost layers. For strong magnetic field strenghts and low temperatures, the hydrogen can condensate into a liquid or solid surface. This is, however, quite rare and usually the hydrogen remains

*Maximum densities reached in the Earth are usually obtained by colliding heavy nucleons together, momentarily creating a core of even denser matter. The densest naturally occurring element found on top of planet Earth is osmium that has a density of "just" $\rho \approx 2.2 \times 10^1$ g cm^{-3}.

† [1] A. Y. Potekhin. *Physics Uspekhi.* (2014).

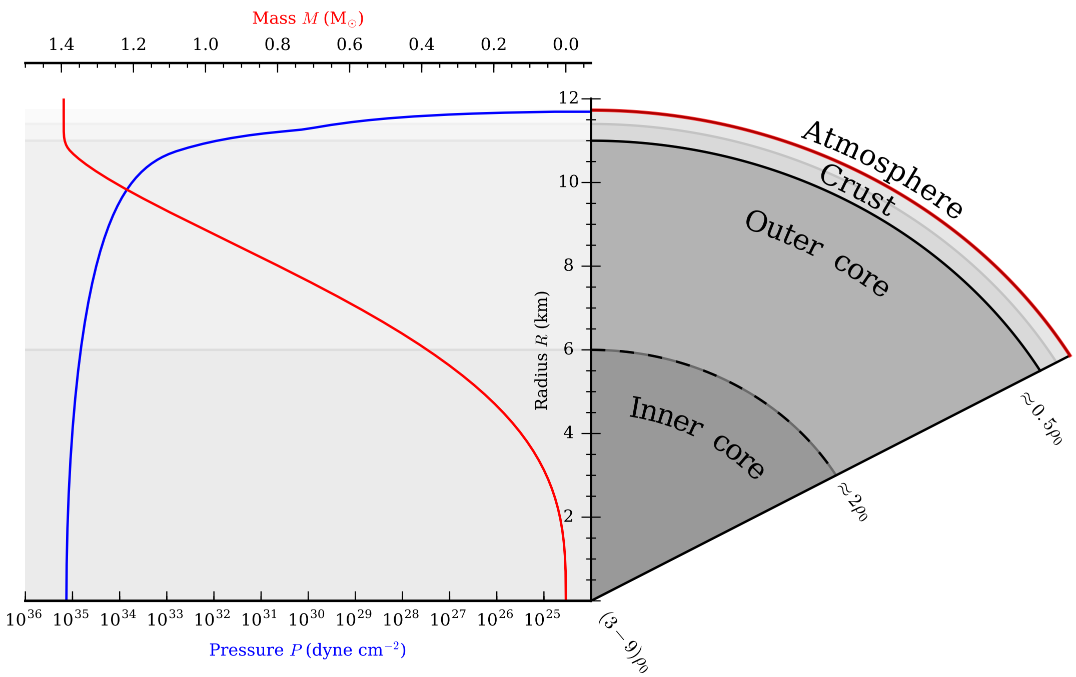

Figure 1.2: Overview of the neutron star structure. Right side of the figure shows a schematic presentation of the star's interiors against the radial coordinate, whereas the left side shows the pressure (blue; bottom axis) and cumulative mass (red; top axis) evolution from the core to the surface.

gaseous forming an atmosphere. Properties of emergent radiation strongly depend on the chemical composition of the atmosphere. Difference to normal stars is that we only expect the composition to consist of the lightest element as heavier ones sink to the bottom.* Consequence of gravitational force. Enormous surface gravitational acceleration of $g \sim 10^1 4$ to $10^{15}\,\mathrm{cm\,s^{-2}}$. Strong gravitational field also bends the photon trajectories.† First low-field models by pioneering work of Romani.‡

Compactness parameter

$$u = \frac{R_\mathrm{S}}{R} \tag{1.1}$$

and

$$R_\mathrm{S} = \frac{2GM}{c^2} \approx 2.95 \frac{M}{M_\odot}\ \mathrm{km} \tag{1.2}$$

Gravitational acceleration under general relativistic theory is then

$$g = \frac{GM}{R^2} \frac{1}{\sqrt{1-u}} = 1.38 \times 10^{14}\ \frac{1}{\sqrt{1-u}} \left(\frac{M}{M_\odot}\right) \left(\frac{R}{10\,\mathrm{km}}\right)^2\ \mathrm{cm\,s^{-2}} \tag{1.3}$$

Atmosphere of a star is the first and uppermost layer responsible for the emergent radiation. It consists of a thin layer of plasma and ranges from a few millimeters to couple of centimeters in height.

Contains information on the parameters of the surface: effective temperature surface gravity chemical composition geometry of the system mass and radii.

Because the thickness of the atmosphere is much smaller than the radius of the star, the atmosphere can be considered in plane-parallel approximation. Rather high densities, on the other hand, allow to consider the plasma of the atmosphere in local thermodynamical equilibrium.

The standard approach in describing the atmosphere structure includes solving three main equations.

Firstly, radiative transfer equation for the specific spectral intensity I_ν is

$$\mu \frac{dI_\nu}{dy} = k_\nu (I_\nu - S_\nu) \tag{1.4}$$

where μ is the cosine of the angle θ between the propagated radiation and the surface normal, y is the column density (mass per area) defined via $dy = \rho dz$, where z is the horizontal distance from the surface, $k_\nu = \alpha_\nu + \sigma_\nu$ is the total radiative opacity including contributions from the “true” opacity α_ν and from the scattering opacity σ_ν. In addition we need the source function

$$S_\nu = (\sigma_\nu J_\nu + \alpha B_\nu) k_\nu^{-1} \tag{1.5}$$

, that can be described using the mean spectral intensity

$$J_\nu = \frac{1}{2} \int_{-1}^{+1} I_\nu d\mu \tag{1.6}$$

, and

$$B_\nu = XX \tag{1.7}$$

is the thermal Planck function. As an boundary conditions for the RTE we can use $I_\nu = 0$ for $\mu < 0$ at $y = 0$.

* [2] C. Alcock and A. Illarionov. *ApJ*. (1980).

† [3] K. R. Pechenick, C. Ftaclas, and J. M. Cohen. *ApJ*. (1983).

‡ [4] R. W. Romani. *ApJ*. (1987).

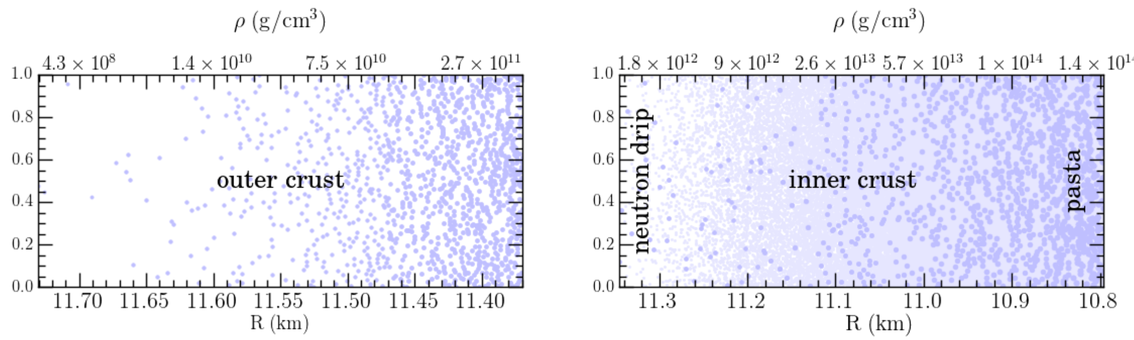

Figure 1.3: Molecular simulation of the crust. Figure adapted from https://github.com/awsteiner/nstar-plot.

The atmospheres are also usually considered to be in radiative and hydrostatic equilibirum. First requirement can be formulated as

$$\int_0^\infty d\nu \int_{-1}^{+1} \mu I_\nu d\mu = \sigma_{\mathrm{SB}} T_{\mathrm{eff}}, \tag{1.8}$$

where σ_{SB} is the Stefan-Boltzmann constant and T_{eff} is the effective temperature of the atmosphere. The second, hydrostatic equilibrium, demands that

$$P = gy \tag{1.9}$$

if the radiative pressure forces are neglected.

Finally, we need to supplement these equations with an equation connecting the pressure and density, the EoS. For the rarefied atmosphere, the ideal gas law is an excellent approximation

$$P = nkT. \tag{1.10}$$

Eddington limit of where radiation force exceeds the gravitational one.

$$L_{\mathrm{Edd}} = \frac{4\pi G M m_{\mathrm{p}}}{\sigma_{\mathrm{T}}} \approx 1.3 \times 10^{38} \left(\frac{M}{M_\odot}\right) \mathrm{erg\,s^{-1}} \tag{1.11}$$

1.3 Crust

Outer crust

From atmosphere to $\rho_N D \sim 4 \times 10^{11}$ g cm^{-3}. In thickness some hundred meters. Non degenerate electron gas Ultra-relativistic electron gas $\rho > 10^6$ g cm^{-3}. Pressure provided by electrons here.

Here the equation of state is decribed by the relativistic degenerate electron gas. Physics behind this are quite simple and we repeat the calculations here to give the reader ideas about what are the most important physical processes. The result also bears some historical value.

We have reached densities where the EoS is dominated mainly by the electrons, hence it is characterized by the electron number density n_{e} and temperature T_{e} (hereafter just T in this section). Moving on from an ideal plasma, we can start by introducing corrections produced by the closely packed charges. In practice we can use the so-called ion-sphere model to describe our Coulomb liquid of ions. We now assume that our ions are emerged into a sea of rigid electron background that takes care of the charge neutrality. Let us begin by defining a so-called electron sphere radius

$$r_e = \left(\frac{4\pi n_e}{3}\right)^{-1/3} \tag{1.12}$$

We can also parameterize the strength between Coulomb (charge) interactions by considering a ratio of potential energy to the kinetic energy with

$$\Gamma_e = \frac{e^2}{r_e kT}. \tag{1.13}$$

Similarly, for ion with a charge number of Z_i, we can define the ion-sphere radius

$$r_i = r_e Z_i^{1/3} \tag{1.14}$$

that encapsulates enough area to be charge neutral, when considering a static electron-induced background from n_e. Ion Coulomb coupling factor is similarly

$$\Gamma_i = \Gamma_e Z_i^{5/3} = \frac{(Z_i e)^2}{r_i kT} \tag{1.15}$$

In the weak-coupling limit ($\Gamma_i \ll 1$) Debye-Hückel results for the free energy are valid*

$$\frac{F_{ex}}{V} = \frac{1}{\sqrt{3}} n_i kT \Gamma^{3/2} \tag{1.16}$$

Hence, the pressure correction due to the Coulomb interactions is†

$$P_{ii} \approx -0.3 n_i \frac{Z^2 e^2}{r_i}. \tag{1.17}$$

For a degenerate system it also makes sense to present the Fermi quantities: momentum

$$p_F = \hbar(3\pi^2 n_e)^{1/3} \tag{1.18}$$

energy

$$\epsilon_F = c^2 \sqrt{(m_e c)^2 + p_F^2}, \tag{1.19}$$

and temperature

$$T_F = \frac{m_e c^2}{k}\left(\sqrt{1 + \left(\frac{p_F}{m_e c}\right)^2} - 1\right) = T_r(\gamma_r - 1), \tag{1.20}$$

where we have defined a typical temperature

$$T_r \equiv \frac{m_e c^2}{k} \sim 6 \times 10^9\ \mathrm{K} \tag{1.21}$$

relativistic scaling factor

$$\gamma_r \equiv \sqrt{1 + x_r^2}, \tag{1.22}$$

and typical (dimensionless) momentum

$$x_r \equiv \frac{p_F}{m_e c}. \tag{1.23}$$

Using these definitions, it is easy to characterize our electron gas into regions of

* [5] L. D. Landau and E. M. Lifshitz. 1980.

† [6] D. R. Dewitt *et al*. *Phys. Rev. A*. (1996).

- non-relativistic, for which $T \ll T_\mathrm{r}$ and $x_\mathrm{r} \ll 1$,
- mildly-relativistic, $T \sim T_\mathrm{r}$ and $x_\mathrm{r} \sim 1$,
- ultra-relativistic, $T \gg T_\mathrm{r}$ and $x_\mathrm{r} \gg 1$,
- non-degenerate, $T \gg T_\mathrm{F}$,
- mildly degenerate, $T \sim T_\mathrm{F}$,
- and strongly degenerate $T \ll T_\mathrm{F}$.

Free energy

$$F = (\mu - m_\mathrm{e}c^2)n_\mathrm{e} - \frac{2}{(2\pi\hbar)^3}\int d\vec{p}\frac{1}{3}\vec{p}\cdot\vec{v}f_{\mathrm{F-D}}(\epsilon) \tag{1.24}$$

where

$$n_\mathrm{e} = \frac{2}{(2\pi\hbar)^3}\int d\vec{p}f_{\mathrm{F-D}}(\epsilon), \tag{1.25}$$

and

$$f_{\mathrm{F-D}}(\epsilon, T) = \frac{1}{\exp\left(\frac{\epsilon-\mu}{kT}\right) - 1} \tag{1.26}$$

and

$$\epsilon = \sqrt{m_\mathrm{e}^2c^4 + p^2c^2} \tag{1.27}$$

Pressure

$$P = -\left(\frac{\partial F}{\partial V}\right)_{T,\{N_j\}} \tag{1.28}$$

Sommerfield expand free energy in powers of T/T_F

$$\frac{F}{V} = \frac{m_\mathrm{e}c^2}{\bar{\lambda}_\mathrm{C}^3}\frac{1}{8\pi^2}\left(x_\mathrm{r}(1+2x_\mathrm{r}^2)\gamma_\mathrm{r} - \ln(x_\mathrm{r}+\gamma_\mathrm{r}) + \frac{4\pi^2}{9}t_\mathrm{r}^2x_\mathrm{r}(\gamma_\mathrm{r}+\gamma_\mathrm{r}^{-1})\right) + O(t_\mathrm{r}^4) \tag{1.29}$$

and obtain pressure

$$P_\mathrm{id}^{(\mathrm{e})} = \frac{P_\mathrm{r}}{8\pi^2}\left(x_\mathrm{r}(1+2x_\mathrm{r}^2)\gamma_\mathrm{r} - \ln(x_\mathrm{r}+\gamma_\mathrm{r})\right) \tag{1.30}$$

where again typical pressure

$$P_\mathrm{r} = \frac{m_\mathrm{e}c^2}{\lambda_\mathrm{C}} \sim 1.4\times 10^{25}\,\mathrm{dyn\,cm^{-2}} \tag{1.31}$$

Hence, it takes the simple polytropic form

$$P_\mathrm{id}^{(\mathrm{e})} \approx \frac{P_\mathrm{r}}{9\pi^2\gamma_\mathrm{AD}}x_\mathrm{r}^{3\gamma_\mathrm{AD}} \tag{1.32}$$

where the polytropic index $\gamma_\mathrm{AD} = \frac{5}{3}$ for the non-relativistic $x_\mathrm{r} \ll 1$ and $\gamma_\mathrm{AD} = \frac{4}{3}$ for the ultra-relativistic case (recall also that $x_\mathrm{r} \propto n_\mathrm{e}^{1/3}$).

Degenerate electron gas pressure accompanied with the ion Coulomb correction will then actually give us a rather good approximation for the equation of state

$$P(x_\mathrm{r}) = P_\mathrm{id}^{(\mathrm{e})} + P_\mathrm{ii} \tag{1.33}$$

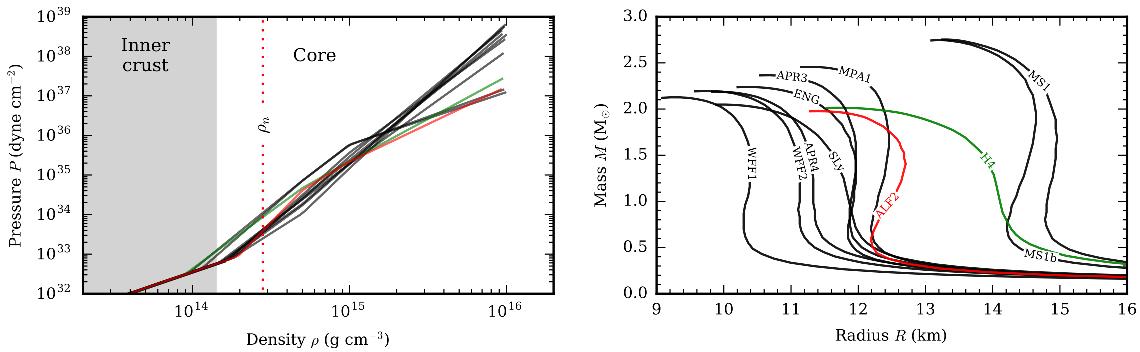

Figure 1.4: Core EoS.

this is valid in a large density range of $10^4 < \rho < 10^{10}$ g cm^{-3}.

In deeper layers ions form a strongly coupled Coulomb system (liquid or solid). Hence, crust. Fermi energy grows with increasing ρ. Induces β captures and enriches nuclei with neutrons. At the base neutrons start to drip out from nuclei.

1.3.1 Inner crust

About one kilometer thick. Density from $\rho \sim \rho_{ND}$ (at upper boundary) to $\sim 0.5\rho_{\rm n}$ at the base. Matter consists of electrons, free neutrons n and neutron-rich atomic nuclei. Fraction of free n grow with ρ.

nuclei are immersed in a neutron gas and hence nuclear interactions play a crucial role in defining the matter.

Finally, nuclei disappear at the crust-core interface.

1.4 Liquid core

Outer core. Density ranges $0.5\rho_{\rm n} < \rho < 2\rho_{\rm n}$. Several kilometers. Neutrons with several per cent admixture of protons p and electrons e^-. Strongly degenerate. Electrons form almost ideal Fermi gas. Neutrons and protons, interacting via nuclear forces, constitute a strongly interacting Fermi liquid.

Inner core. Where $\rho > 2\rho_{\rm n}$. Appearance of muon. Central density can be around $(10-15)\rho_{\rm n}$. Very model dependent. Main problem.

Here we consider few (relatively) modern descriptions for the dense matter equation of state. The EoSs can be divided into few different classes based on the particles that they consist of. Here they are divided into matter consisting of

- plain $(npe\mu)$nuclear matter,
- normal nuclear matter spiced up with hyperons (H),
- normal nuclear matter together with more exotic particles like pion and kaon condensates (πK),
- and matter consisting of (or normal matter spiced up with) quarks (q).

For $(npe\mu)$we include models computed with

- potential method using SLy effective nuclear interaction that is of Skyrme-type,*,

* [7] F. Douchin and P. Haensel. *A&A*. (2001).

- four variational method EoSs, APR3/4* and WFF1/2†,
- two relativistic Brueckner-Hartree-Fock calculations, ENG‡ and MPA1§,
- two relativistic mean-field theory, MS1 and MS1b (same as MS1 but with lower symmetry energy)¶.

For hyperon models (H)we include

- one variant of relativistic mean-field theory EoS H4‖.

EoSs where mesons, like pion and kaon condensates (πK), are taken into account end up not being stiff enough.

Finally, for the hybrid nuclear matter and quark matter compositions (q)we include

- mixed APR nuclear matter and color-flavor-locked quark matter EoS ALF2.**

1.4.1 Why neutrons then?

Let us first consider ideal gas of degenerate electron-proton-neutron plasma. In a degenerate plasma all the quantum states are filled up all the way to the Fermi energy. It is the Pauli exclusion principle that then prevents occupying all of these already taken quantum states. Normal beta-decay mode for the neutrons, on the other hand, is $n \rightarrow p + e^- + \bar{\nu}_e$, that describes the possible path of how a neutron n will decay into a proton p, electron e^-, and electron neutrino $\bar{\nu}_e$. Such a decay is, however, blocked because there is no room for an emission of an extra electron e^- or a proton p.††

Let us then only focus on the decay of the most energetic neutrons with an energy equal to the Fermi energy $\epsilon_{\mathrm{F}}(n)$. Co-existence of neutrons, protons, and electrons is then guaranteed (at zero temperature) if

$$\epsilon_{\mathrm{F}}(n) = \epsilon_{\mathrm{F}}(p) \ + \epsilon_{\mathrm{F}}(e^-). \tag{1.34}$$

Fermi momentum of a particle is related to its concentration via

$$p_{\mathrm{F}} = \left(\frac{3n}{8\pi}\right)^{1/3} h, \tag{1.35}$$

where n is the number density, and h the Planck constant. Massive neutrons and protons are to a good approximation non-relativistic up to a densities of ρ_{n}, and hence energy is simply a sum of their rest mass energy and kinetic energy

$$\epsilon_{\mathrm{F}}(n) \ \approx m_n c^2 + \frac{p_{\mathrm{F}}(n)^2}{2m_n}, \tag{1.36}$$

and

$$\epsilon_{\mathrm{F}}(p) \ \approx m_p c^2 + \frac{p_{\mathrm{F}}(p)^2}{2m_p}. \tag{1.37}$$

Electrons, on the other hand, are already ultra-relativistic, and so

$$\epsilon_{\mathrm{F}}(e^-) \approx p_{\mathrm{F}}(e^-) c^2. \tag{1.38}$$

Also note that $n_p = n_e$, as the star is electrically neutral. From this we find relation of the $n_n/n_p \sim 1/200$ by taking into account the rest mass difference $m_p - m_n = 2.6\mathrm{MeV}\, c^2$ at $\rho \sim \rho_{\mathrm{n}}$. Thus, we conclude that the matter inside is neutron rich.

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** [14] M. Alford *et al. ApJ*. (2005).
†† see e.g. [15] A. C. Phillips. 1994.

1.4.2 Polytropes

Parameterize everything with polytropes.

$P(\rho) = K\rho^{\gamma}$, γ is a the polytropic index and is a measure of *stiffness* of the EoS. how strongly the matter responds to an increase in density with an increase in pressure

1.5 Tolman-Volkoff-Oppenheimer equations

Newtonian pressure gradient needed to oppose the gravity is

$$\frac{dP}{dr} = -\frac{Gm\rho}{r^2}. \tag{1.39}$$

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{1.40}$$

Taking into account the general relativistic corrections we get

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} \times \frac{(1 + P/\rho c^2)(1 + 4\pi r^3 P/mc^2)}{1 - 2Gm/rc^2}. \tag{1.41}$$

Difference originates from the source of gravity: in the Newtonian case it is the mass m, whereas in the General relativity it is the energy momentum tensor that depend both on the energy density and the pressure. As a result, energy and pressure give rise to a gravitational fields.

Severness of the GR corrections can be estimated from the so called compactness parameter

$$x = \frac{GM}{Rc^2} \approx 2.95\ M/\ M_{\odot}\,\mathrm{km} \tag{1.42}$$

It has an important consequence to the stability of neutron stars: Successive increase in the pressure to counter the gravity is ultimately self-defeating.

Solution for a constant density ρ_0 gives

$$P(r) = G\frac{2\pi}{3}\rho_0^2(R^2 - r^2) \tag{1.43}$$

whereas the GR gives

$$P(r) = \rho_0^2 c^2 \left[\frac{(1 - u\left(\frac{r}{R}\right)^2)^{1/2} - (1 - u)^{1/2}}{3(1 - u)^{1/2} - (1 - u\left(\frac{r}{R}\right)^2)^{1/2}} \right], \tag{1.44}$$

where $u = 2GM/Rc^2$.

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