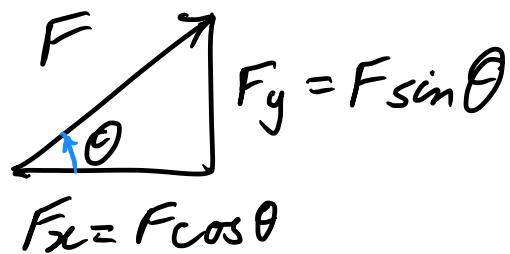


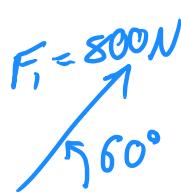
Tutorial Week 4 - Equilibrium of structures

state structure - not moving (all forces must equal 0) (balanced)



Force Components Example

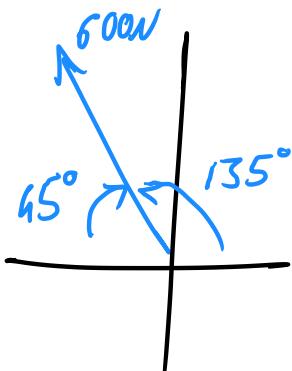
For F_1 :



$$F_x = 800 \cos 60^\circ = 400N \quad \checkmark$$

$$\begin{aligned} F_y &= 800 \sin 60^\circ \approx 692.820 N \\ &\approx 692.8N \quad \checkmark \end{aligned}$$

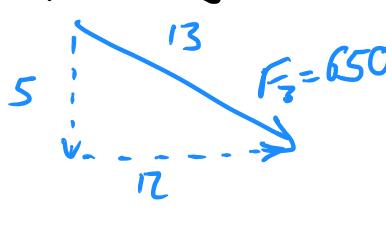
For F_2 :



$$F_x = 600 \cos 135^\circ = -424.3 N \quad \checkmark$$

$$F_y = 600 \sin 135^\circ = 424.3 N \quad \checkmark$$

For F_3 :



$$650 \div 13 = 50$$

$$\therefore F_x = 12 \times 50 = 600N \quad \checkmark$$

$$F_y = -5 \times 50 = -250N \quad \checkmark$$

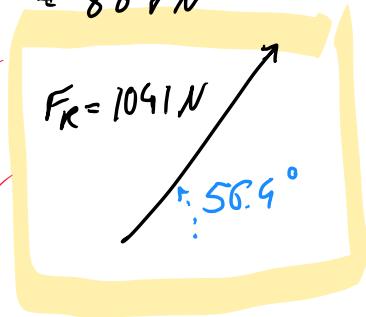
Now add the forces to find the resultant force:

$$F_{Rx} = \sum F_x = F_{1x} + F_{2x} + F_{3x} \\ = 400 - 924 + 600 \\ = 576 N$$

$$F_{Ry} = \sum F_y = F_{1y} + F_{2y} + F_{3y} \\ = 693 + 424 - 250 \\ = 867 N$$

$$|F_R| = \sqrt{867^2 + 576^2} \approx 1041 N \quad \checkmark$$

$$\theta = \tan^{-1} \left(\frac{y}{x} \right) = \tan^{-1} \left(\frac{867}{576} \right) \approx 56.9^\circ \quad \checkmark$$



Moments:

$$M = E \times J$$

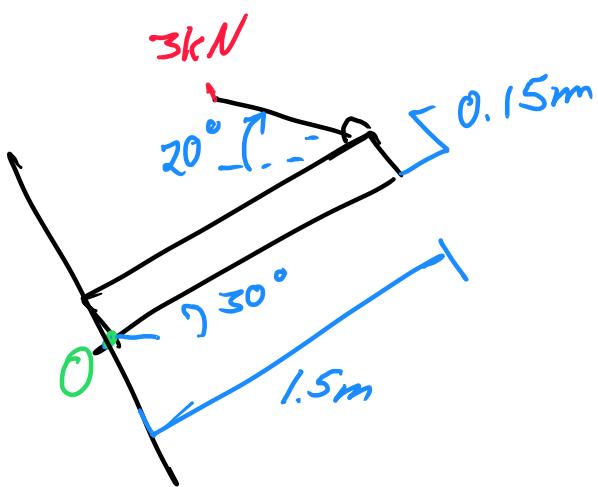
$$M = F \times d$$

where d = perpendicular force

Units: Newton Metre (N. m)

- This is in accordance with the unit circle

Moments Example 1



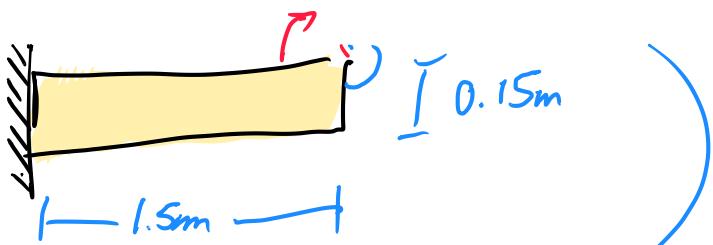
Determine moment of force about point O.

$$M = F \times d$$

$$F_x = -3 \cos 50^\circ$$

$$F_y = 3 \sin 50^\circ$$

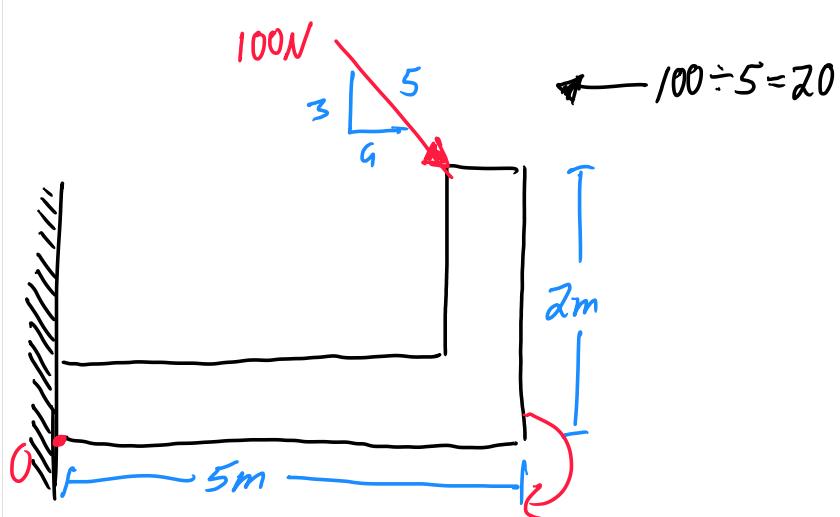




$$M = 3 \sin 50^\circ \times 1.5 + 3 \cos 50^\circ \times 0.15 = +3.79 \text{ kNm}$$

Moments Example 2

- Calculate the moment about 0 created by the horizontal component of the force
- Calculate the moment about 0 created by the vertical component of the force
- Calculate the total moment about 0 created by the 100N force



1. by horizontal:

1. by horizontal:

$$F_x = 9 \times 20 = 80 \quad \therefore M_x = F \times d = 80 \times 2 = 160 \text{ Nm}$$

2. by vertical:

$$F_y = 3 \times 20 = 60$$

$$\therefore M_y = F \times d = 60 \times 5 = -300 \text{ Nm}$$

because
clockwise
direction

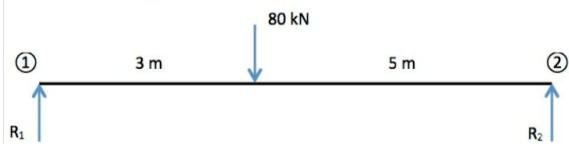
3. Total moment:

$$M = M_x + M_y = 160 + 300 = -460 \text{ Nm}$$

Support reaction Ex. I

Support Reaction Example 1

- Lecture Example 3
- Calculate support reaction forces



[Ans: $R_1 = 50 \text{ kN}$; $R_2 = 30 \text{ kN}$]

$$\sum M_i = 0 \quad R_2 \times 8 - 80 \times 3 = 0$$

$$\therefore R_2 = \frac{80 \times 3}{8} = 30 \text{ kN}$$

$$\sum F_y = 0 \quad R_1 + R_2 - 80 = 0$$

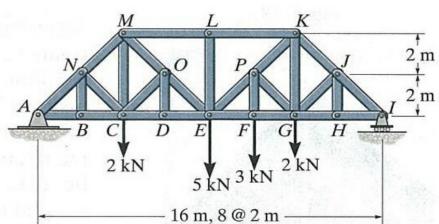
$$\therefore R_1 = 80 - R_2$$

$$= 80 - 30$$

$$= 50 \text{ kN}$$

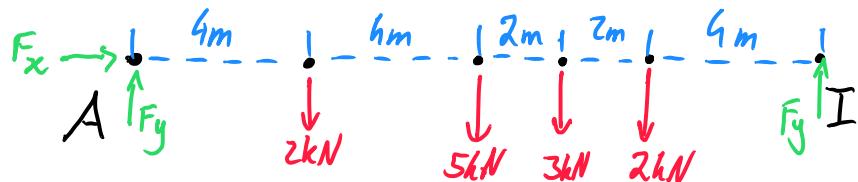
Support Reaction Example 2

- Calculate the support reactions at A and I



[Ans: $A_x = 0$; $A_y = 5.625 \text{ kN}$; $I_y = 6.375 \text{ kN}$]

$$2+5+3+2 = 12 \text{ kN total}$$



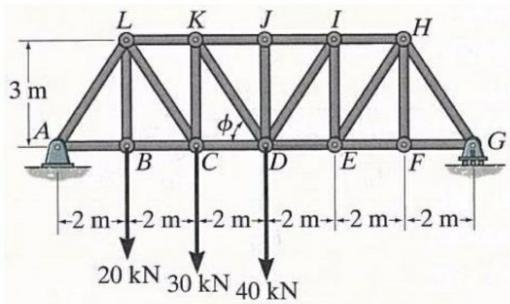
$$\begin{aligned} \sum M_A = 0 & \quad \therefore 0 = (I_y \times 16) + (-2 \times 12) + (-3 \times 10) + (-5 \times 8) + (-2 \times 4) \\ & \quad \therefore I_y = 6.375 \text{ kN} \end{aligned}$$

$$\sum F_y = 0 \quad A_y + I_y$$

- do Support Reaction Ex. 3 at Home

Support Reaction Example 3

- Calculate the vertical support reaction force at G
 - Calculate the vertical support reaction force at A



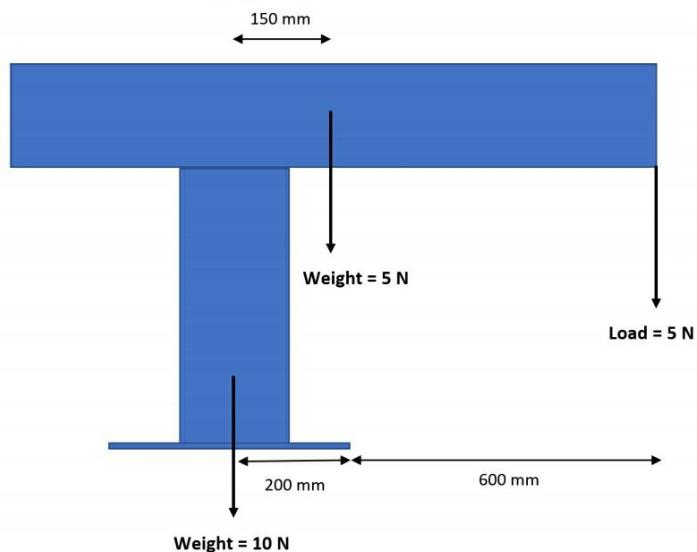
[Ans: $A_x = 0$; $A_y = 56.67 \text{ kN}$; $G_y = 33.33 \text{ kN}$]

Balance Calculations on Freestanding Structures

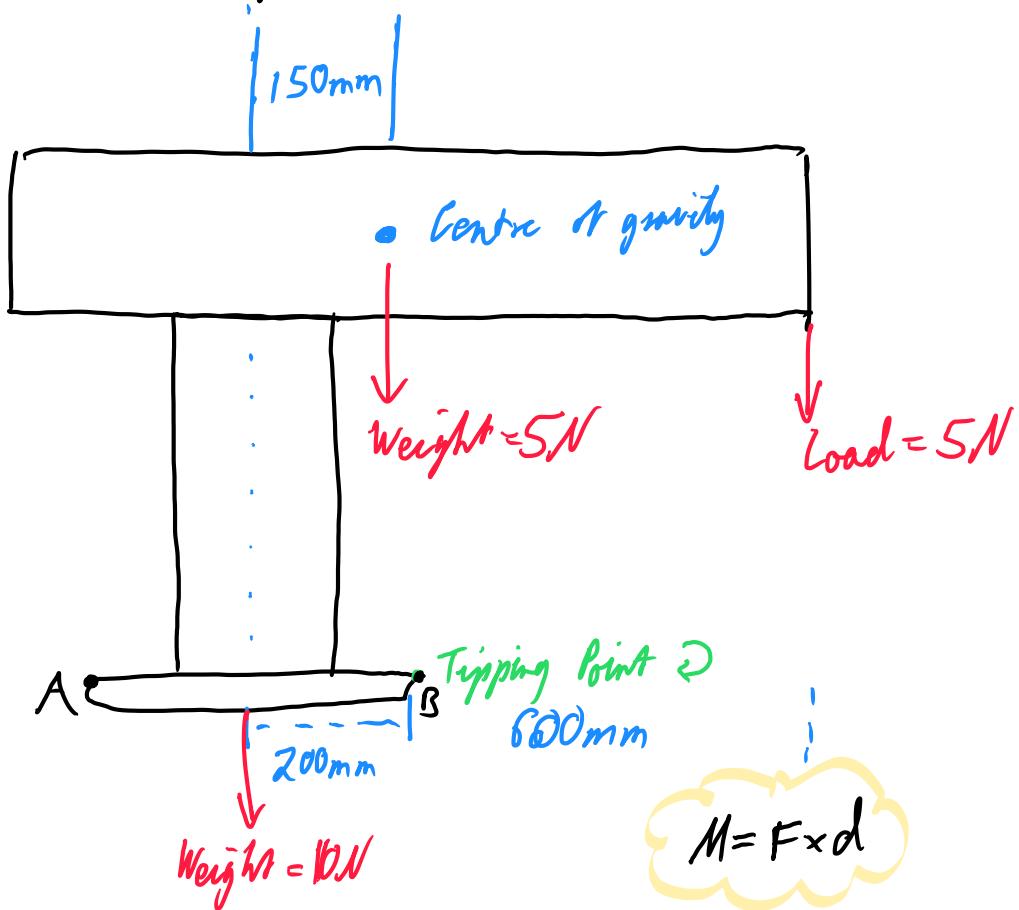
- If a structure is freestanding, we can use the theory learnt in today's class to test whether it will be balanced, or whether it will topple over.
- For the structure to be balanced, all reaction forces must point up. If any point down, then it will topple.

Balance Example 1

- Will this structure stay upright or fall over?



Will it fall over?

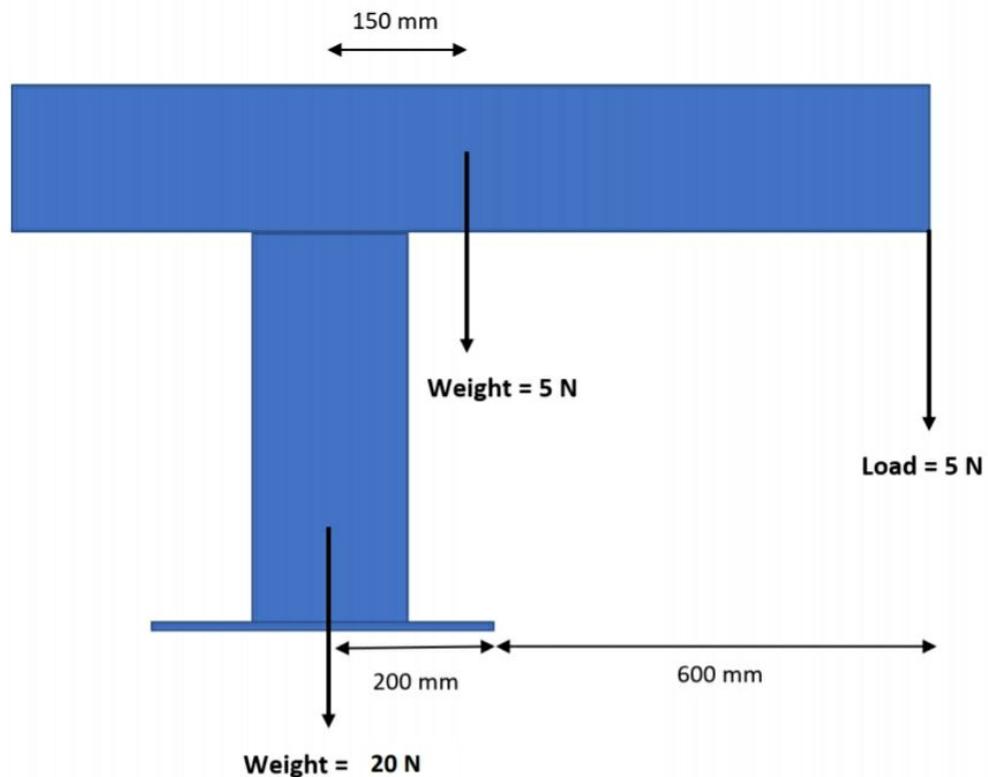


$$\sum M_B : 10 \times 200\text{mm} + 5 \times 50\text{mm} - 5 \times 600\text{mm} \\ = -0.75\text{Nm (clockwise)}$$

\therefore given $M_B \neq 0$, the structure will tip.

Balance Example 2

- Will this structure stay upright or fall over?



Tutorial Week 5 - Truss Analysis

Truss Analysis Assumptions

- all joints are frictionless pins (pin joints)
- all entered loads are applied at joints
- two members are axial loading (comp. or tens.)
- truss must be determinate to be able to calculate forces

Truss Determinancy

Truss is determinate when condition met:

$$J = M + R$$

J = no. joints (nodes)

M = no. members

R = no. support reactions

Method of joints overview

- Apply $\sum F_x = 0$ and $\sum F_y = 0$ at each joint to determine up to 2 unknown forces

Method of joints - Step 1

- draw free body diagram

Step 2:

- Solve for reaction forces:

$$\sum M_o = 0 \quad \sum F_x = 0 \quad \sum F_y = 0$$

Step 3:

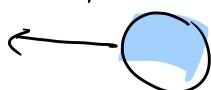
- Find joint w/ ≤ 2 unknowns + draw free body diagram of the joint
 → + force = tension
 - force = compression

Member in Tension



Joint Opposing Member

in Tension



Step 4:

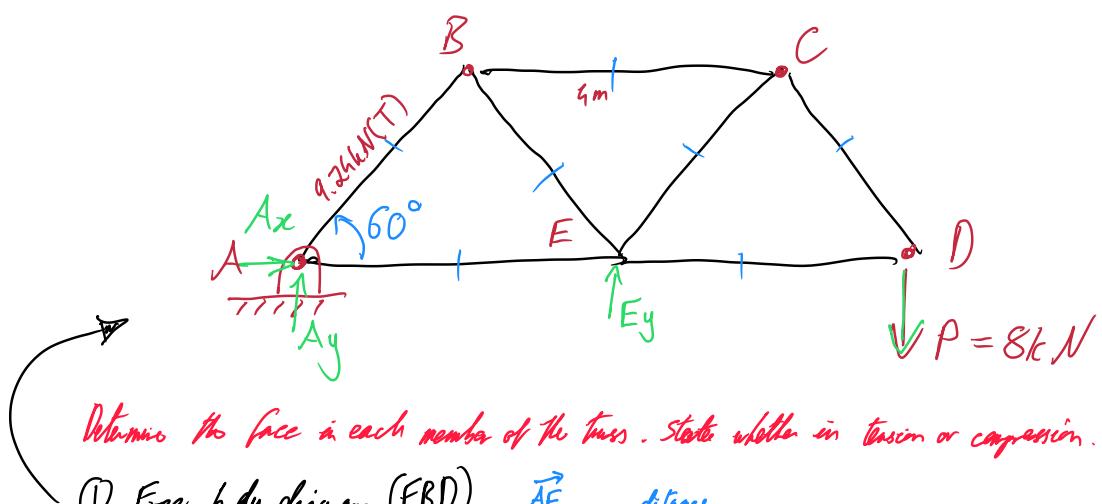
- Apply $F_x = 0$ and $F_y = 0$ to solve the 2 unknown forces

- Apply $F_x=0$ and $F_y=0$ to solve the 2 unknown forces
→ Always put T and C next to answer to indicate tens.
or comp.

Step 5:

- repeat Steps 3+4

Example 1:



Determine the force in each member of the truss. State whether in tension or compression.

① Free body diagram (FBD)

$$\frac{\vec{AE}}{AE} \downarrow \text{distance} \downarrow$$

$$\textcircled{2} \quad \sum M_A = 0 \rightarrow 0 = +E_y(4) - 8(8)$$

$$E_y = \frac{64}{4} = 16 \text{ kN}$$

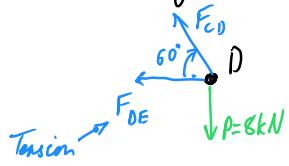
$$\sum F_y = 0 \rightarrow 0 = +A_y + E_y - 8 \stackrel{(c)}{=} A_y + 16 - 8$$

$$A_y = -8 \text{ kN}$$

$$\sum F_x = 0 \rightarrow A_x = 0$$

$$\sum F_x = 0 \rightarrow A_x = 0$$

③ For joint D:



$$\sum F_y = 0$$

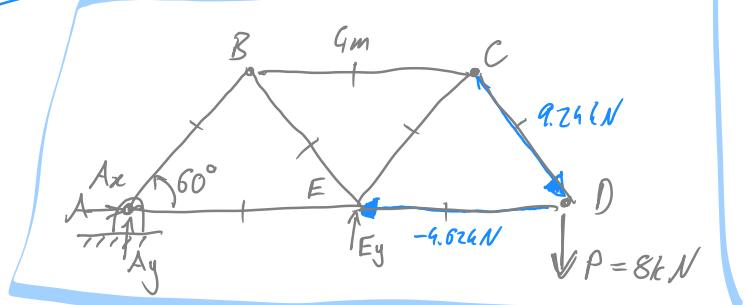
$$+ F_{CD} \sin 60^\circ - 8 = 0$$

$$\therefore F_{CD} = \frac{8}{\sin 60^\circ} = 9.24 \text{ kN (T)}$$

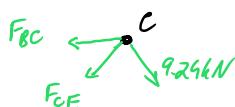
$$\sum F_x = 0$$

$$-F_{DE} - F_{CD} \cos 60^\circ = 0$$

$$\therefore F_{DE} = -9.62 \text{ kN (C)}$$



For joint C:



$$\sum F_y = 0$$

$$-F_{CE} \sin 60^\circ - F_{CD} \sin 60^\circ = 0$$

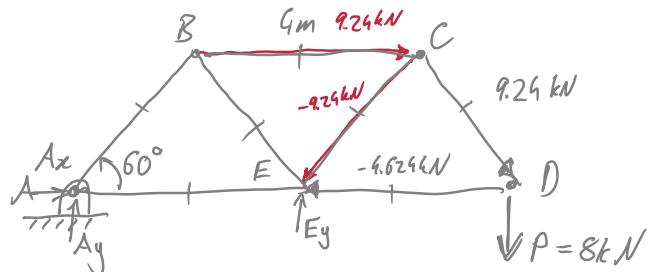
$$\therefore F_{CE} = \frac{-9.24 \sin 60^\circ}{\sin 60^\circ} = -9.24 \text{ kN (C)}$$

$$\sum F_x = 0$$

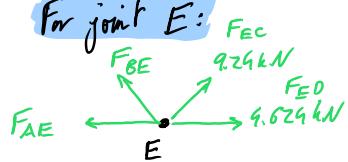
$$-F_{BC} - F_{CE} \cos 60^\circ + F_{CD} \cos 60^\circ = 0$$

$$F_{BC} = +9.24 \cos 60^\circ + 9.24 \cos 60^\circ$$

$$F_{BC} = 9.24 \text{ kN (T)}$$



For joint E:



$$\sum F_y = 0$$

$$0 = +F_{BE} \sin 60^\circ + F_{EC} \sin 60^\circ$$

$$0 = F_{BE} \sin 60^\circ + 9.24 \sin 60^\circ$$

$$\therefore F_{BE} = -9.24 \text{ kN (C)}$$

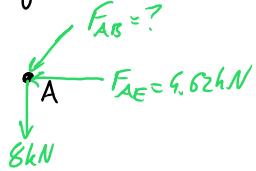
$$\sum F_x = 0$$

$$0 = +F_{AE} + F_{BE} \cos 60^\circ - F_{EC} \cos 60^\circ - F_{ED}$$

$$0 = +F_{AE} - 9.24 \cos 60^\circ - 9.24 \cos 60^\circ - 9.62kN$$

$$F_{AE} = -4.64 \text{ kN (C)}$$

For joint A:

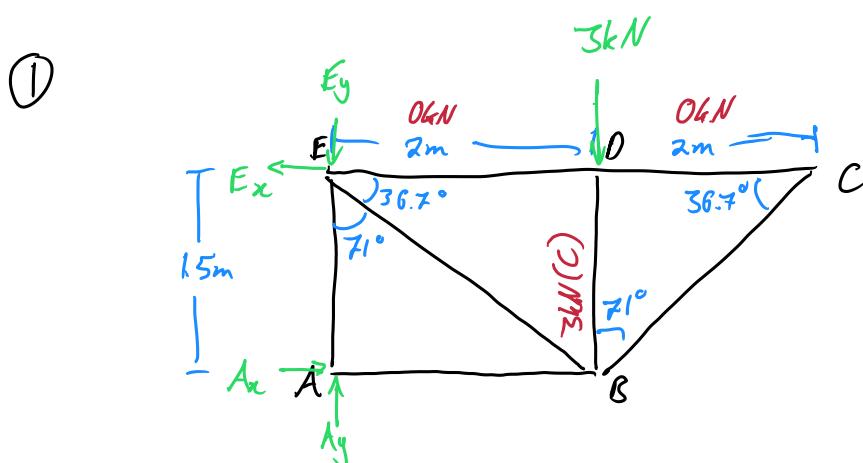


$$\sum F_y = 0$$

$$F_{AB} \sin 60 - 8 = 0$$

$$\therefore F_{AB} = 9.24kN (T)$$

Example 2



$$\textcircled{2} \quad \sum M_o = 0 \quad \sum F_x = 0 \quad \sum F_y = 0$$

For Reactions:

$$\sum M_E = 0 \quad \therefore +A_x(1.5) - 3 \times 2 = 0$$

$$A_x = 9kN$$

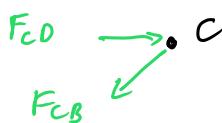
$$\sum F_y = 0 \quad E_y - 3 = 0 \quad \therefore E_y = 3kN$$

$$\sum F_x = 0 \quad +A_x + E_x = 0$$

$$4 + E_x = 0 \quad \therefore E_x = -4kN$$

(3)

For joint C:



$$\sum F_x = 0$$

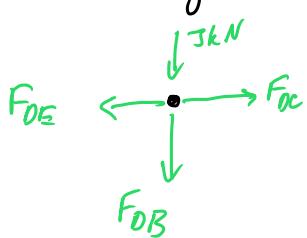
$$\therefore F_{C0} = 0$$

$$\sum F_y = 0$$

$$0 = F_{Cb} \sin 36.7^\circ$$

$$\therefore F_{Cb} = 0$$

For joint D:



$$\sum F_y = 0$$

$$0 = 3kN + F_{D0}$$

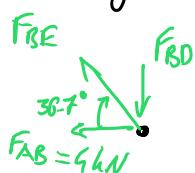
$$\therefore F_{D0} = -3kN \quad (C)$$

$$\sum F_x = 0$$

$$0 = F_{Dc} - F_{Bc}$$

$$\therefore F_{Dc} = 0$$

For joint B:



$$\sum F_y = 0$$

$$0 = F_{BD} + F_{BE} \sin 36.7^\circ$$

$$0 = -3kN + F_{BE} \sin 36.7^\circ$$

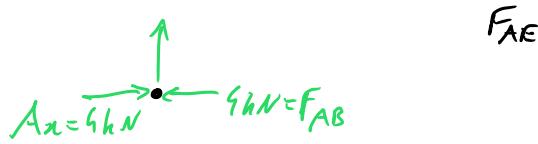
$$F_{BE} = \frac{3}{\sin 36.7^\circ} = 5kN \quad (T)$$

For joint A:



$$\sum F_y = 0$$

$$\therefore F_{AE} = 0$$



Zero Force Members

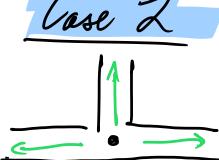
- Members w/ no force
- can be ignored for purpose of analysis

Case 1



- only 2 members
- no external load applied
- both are zero force members

Case 2

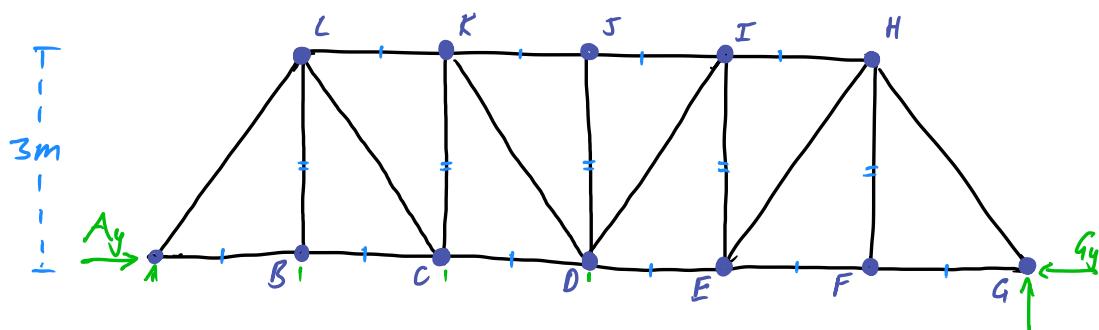


- 3 members at a joint
→ 2 collinear
- no external load applied
- 3rd member is zero force member

Truss Analysis for Design

- Interested in **worst case** tension/compression failure
- No failure under 0.5kg or 1kg statue load; should fail under 1.5kg statue applied
- compare to materials testing data

Example 3





Tutorial Week 6 - Mechanical Systems Analysis 1

Today's aim is to learn basic mechanical systems analysis, incl:

- Linear dynamics
- Rotational dynamics
- Power transfer
- Efficiency
- Friction

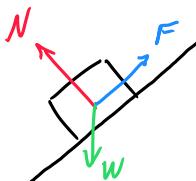
Linear Kinematics

Displacement (m): $s = x_f - x_i$

$$v = \frac{dx}{dt} = \frac{s}{t} \text{ (ms}^{-1}\text{)} \quad a = \frac{dv}{dt} = \frac{v}{t} \text{ (ms}^{-2}\text{)}$$

Linear mechanical power can be defined:

$$\sum F = ma \longrightarrow P_{\text{linear}} = Fv \text{ (in Watts)}$$



1 Example Problem P. A

A 10kg mass is lifted 50cm in 20 seconds. Determine:

a) Velocity required to lift mass assuming it's constant

$$v = \frac{s}{t} \quad \therefore v = \frac{0.5}{20} = 0.025 \text{ ms}^{-1}$$
$$s = 0.5 \text{ m}, t = 20$$

b) The tension in the cable at constant velocity

$$F = mg = 98 \text{ N} \quad \therefore \text{Tension} = 98 \text{ N}$$

c) The lifting power.

$$\text{assume } \sum F = 0$$

$$P = Fv = 98 \times 0.025 = 2.45 \text{ Watts}$$

d) The tension in the cable if the mass initially accelerates at 20 cm/s^2

$$a = 9.8 + 0.2 = 10 \text{ ms}^{-2}$$

$$\therefore F = ma$$

$$= 10 \times 10 = 100 \text{ N}$$

$$\begin{aligned} F &= ma \\ T - W &= ma \\ T - 98 &= 10 \times 0.2 \\ \therefore T &= 100 \text{ N} \end{aligned}$$

Rotational Kinematics

Angular displacement (radians): $\theta = \theta_f - \theta_i$

Angular velocity (rad/s): $\omega = \frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t}$

Angular acceleration (rad/s²): $\alpha = \frac{d\omega}{dt} = \frac{\Delta\omega}{\Delta t}$

Rotational Dynamics can be defined:

$$\sum F = ma \rightarrow \sum \tau = I \alpha$$

→ To calculate inertia (I):
 $I_{\text{rot, point mass}} = mr^2$ e.g. static

we won't really be using this too

$\begin{cases} \tau = \text{Torque (Nm)} \\ I = \text{Moment of Inertia (kg m}^2\text{)} \\ \alpha = \text{Angular acceleration} \end{cases}$

really be using this too
much for the project

$$I_{\text{rot, cylinder}} = \frac{1}{2}mr^2 \quad \text{e.g. rod seat to structure (motor)}$$

different types of objects/masses have different types of inertia

Rotational Chemical Power can be defined: (search inertia formula on internet)

τ = torque (Nm)

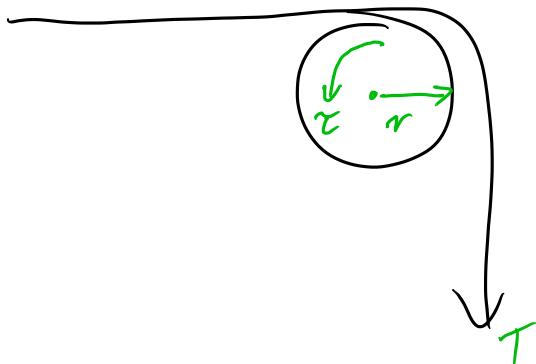
ω = rotational velocity (rad/s)

known as omega

Converting Between Linear + Rotational Properties

Force - Torque Conversion:

$$\tau = F_r = \text{Force} \times \text{radius}$$



Displacement: $\theta = \frac{s}{r}$

Velocity: $\omega = \frac{v}{r}$

Acceleration: $\alpha = \frac{a}{r}$

1 Example Problem Part B

A motor-drum combination is used to lift the mass from the previous example.

Given a drum radius of 25cm and drum mass of 1kg, and assuming no losses, determine:

From Part A

From Part A: $v = 2.5 \text{ cm/s}$, $T_a = 98 \text{ N}$,
 $T_d = 100 \text{ N}$

- a) The torque, angular velocity and power of the motor-drum (given constant velocity)

$$r = 25 \text{ cm} = 0.25 \text{ m} \quad \omega = \frac{v}{r} = \frac{0.025}{0.25} = 0.1 \text{ rad/s}$$

$$m = 1 \text{ kg}$$

$$\begin{aligned} \tau &= F \times r \\ &= 98 \times 0.25 \\ &= 24.5 \text{ Nm} \end{aligned} \quad \begin{aligned} P_{\text{rotational}} &= \tau \times \omega \\ &= 24.5 \times 0.1 \\ &= 2.45 \text{ W} \end{aligned}$$

- b) What effect doubling the drum radius have on torque, angular velocity and power?

$$r = 0.5 \text{ m} \quad \omega = \frac{v}{r} = \frac{0.025}{0.5} = 0.05 \text{ rad/s}$$

$$\begin{aligned} \tau &= F \times r \\ &= 98 \times 0.5 \\ &= 49 \text{ Nm} \end{aligned} \quad \begin{aligned} P &= \tau \times \omega = 49 \times 0.05 = 2.45 \text{ W} \end{aligned} \quad \text{constant}$$

- c) Find the angular acceleration and torque if the initial acceleration of mass is 20 cm/s^2 .

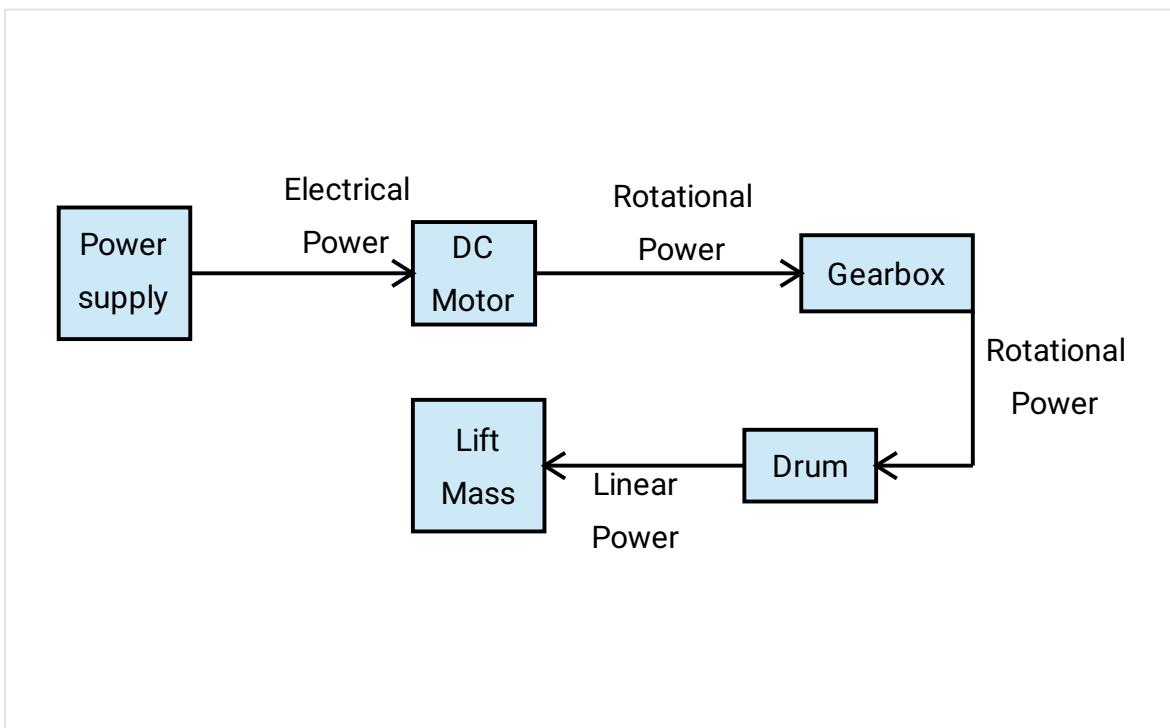
$$\alpha = \frac{\omega}{t} \quad \sum \tau = I \alpha$$

$$\tau_{\text{applied}} = T \times r = \left(\frac{1}{2} m r^2\right) \alpha \quad \begin{array}{l} \text{100N} \\ \text{0.25m} \end{array} \quad \begin{array}{l} \text{1kg} \\ \text{0.25} \end{array} \quad \alpha = \frac{\alpha}{r} = \frac{0.2}{0.25} = 0.8 \text{ rad/s}^2$$

$$\tau_{\text{app}} = 25.025 \text{ Nm}$$

Power Transfer

- P.T. systems convert the power in a system into different forms



- There will always be losses in power transfer, so efficiency of components is very important

Efficiency

efficiency →
$$\eta = \frac{P_{out}}{P_{in}}$$
power out
power in

- Losses to power transferred in a system only effects load-type properties:
 - Current
 - Torque
 - Force
- doesn't effect speed type properties:
 - Voltage
 - Angular velocity
 - Velocity

1 Example Problem Part C:

The drum from prev. example is rotated through the combination of a power supply, DC motor and gearbox. Given the efficiencies below determine the following:

$$\eta_{motor} = 40\%$$

$$\eta_{gearbox} = 80\%$$

$$\eta_{drum} = 95\%$$

a) The torque and angular velocity required by the drum

$$\omega = 0.1 \text{ rad/s} \quad \text{speed isn't affected by losses}$$

$$T_{drum,in} = \frac{\tau_{out}}{\eta_{drum}} = \frac{24.5}{0.95} = 25.79 \text{ Nm}$$

Recall from Part B:

$$P_{out} = 2.45 \text{ W}$$

$$\tau = 24.5 \text{ Nm}$$

$$\omega = 0.1 \text{ rad/s}$$

b) The power required by the power supply

$$\begin{aligned} \eta_{total} &= \eta_{motor} \times \eta_{GB} \times \eta_{drum} \\ &= 0.4 \times 0.8 \times 0.95 \\ \eta &= 30.9\% \end{aligned} \quad \begin{aligned} \therefore \eta_{total} &= \frac{P_{out}}{P_{in}} = \frac{P_{linear}}{P_{power}} \\ \therefore 30.9\% &= \frac{2.45 \text{ W}}{P_{power}} \\ \therefore P_{power} &= 8.06 \text{ W} \end{aligned}$$

Friction

- 3 types: **Static, Kinetic + Rolling**

Frictional Force: always opposes direction of motion

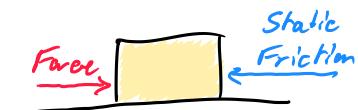
$$F_f = \mu N$$

μ = coefficient of friction - depends on materials in contact

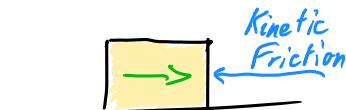
$$\mu_s > \mu_k > \mu_r$$

N = normal force

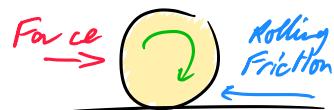
Static Friction



Kinetic Friction



Rolling Friction



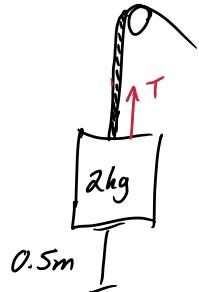
Practice Problem

A lifting device is being designed in order to move a mass of 2kg to a platform 50cm higher than the starting point. A gear motor-drum system is proposed for the lifting of the mass (details below). For Q.1-4, assume steady state motion i.e. constant

velocity.

System Details: $\gamma_{GM} = 2 \text{ Nm}$, $\omega_{GM} = 3 \text{ rad/s}$, $\eta_{drum} = 95\%$

- a) Use a free-body diagram to determine tension in the cable.



$$\begin{aligned} F &= ma \\ F &= 2 \times 9.8 \\ &= 19.6 \text{ N} \\ \therefore T &= 19.6 \text{ N} \checkmark \end{aligned}$$

- b) Determine the lifting power the gear motor can provide at the output of the drum.

$$\begin{aligned} P &= \gamma_{GM} \times \omega_{GM} \\ P_{GM} &= 2 \times 3 = 6 \text{ Watts} \quad \left| \begin{array}{l} \eta = \frac{P_{out}}{P_{in}} \\ 95\% = \frac{P_{out}}{6} \\ P_{GM\ out} = 6 \times 0.95 = 5.7 \text{ W} \checkmark \end{array} \right. \end{aligned}$$

c) Determine the max. allowable drum radius so the motor can lift the 2kg mass.

$$P_{\text{motor}} = T_{\text{Gmax}} \times \omega$$

$$T = \frac{\alpha}{\omega} = \frac{5.1}{3} = 1.9 \text{ Nm}$$

$$\begin{aligned} T &= F_r \\ r &= \frac{T}{F} \\ &= \frac{1.9}{10 \cdot g} = 0.097 \text{ m} \quad \checkmark \end{aligned}$$

d) For this drum radius, determine how long it takes to complete the lift.

$$\begin{aligned} \omega &= \frac{V}{r} \\ V &= \omega r \\ &= 3 \times 0.097 \\ V &= 0.291 \text{ ms}^{-1} \end{aligned} \quad \left| \begin{array}{l} d = s \times t \\ t = \frac{d}{s} = \frac{0.5}{0.291} \\ t = 1.718 \text{ seconds} \quad \checkmark \end{array} \right.$$

e) If the start-up angular acceleration of the motor is 3 rad/s², determine the tension in the cable.

$$\begin{aligned} \alpha &= \frac{\alpha}{r} \\ \alpha &= \alpha \cdot r = 3 \times 0.097 \\ &= 0.291 \text{ ms}^{-2} \end{aligned} \quad \left| \begin{array}{l} a = 9.8 + 0.291 = 10.09 \text{ ms}^{-2} \\ F = ma \\ = 2 \times 10.09 \\ = 20.182 \text{ N} \quad \checkmark \end{array} \right.$$

Tutorial Week 7- Machine Elements and Actuators

Learn about:

- Electric Motors
- Gearboxes
- Bearings
- Pulleys

From W 6 Practical, we could observe:

- As voltage increases:
 - speed < es
 - current unaffected
 - efficiency changes
- as torque (load) increases:
 - speed > es
 - current increases
 - efficiency changes

But how are these quantified?

Electric Motors

Converts electrical power to rotational power

$$\begin{aligned}P_{in} &= VI \\P_{out} &= \tau \omega \\&\therefore \eta = \frac{P_{out}}{P_{in}} = \frac{\tau \omega}{VI}\end{aligned}$$

↑
torque
↓
rotational velocity
voltage → current

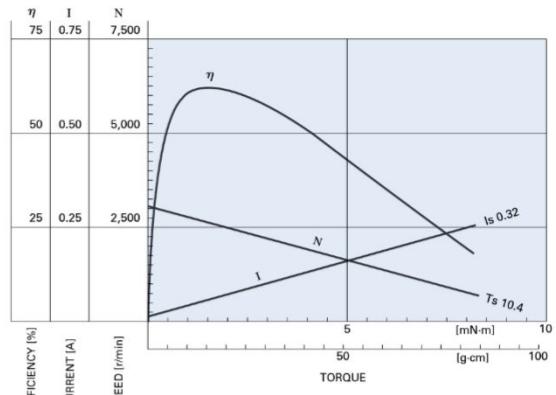
- Nominal Voltage: the voltage at which motors run most efficiently

Motor Curves

- to describe behaviour of nominal voltage, motor curves are used
 - as torque (load) increases:
 - speed > es
 - current increases
 - efficiency changes
- electric motors: high speed, low torque
- units not usually in SI form: must convert

Motor Curve Example

RF-370CB-11670 12.0V



Common units + conversions

Torque:

$$1 \text{ cm} = 1 \times 10^{-2} \text{ m}$$

$$1 \text{ g} = 1 \times 10^{-3} \text{ kg}$$

$$1 \text{ N} = 1 \text{ kg} \times 9.8 \text{ ms}^{-2}$$

torque in Nm
 \downarrow
 $T_{(Nm)} = T_{(gcm)} \times 9.8 \times 10^{-5}$
 \uparrow
 in gcm units

Speed (rpm):

$$N_{(\text{rad/s})} = N_{(\text{rpm})} \times \frac{2\pi}{60}$$

Electric Motor Data

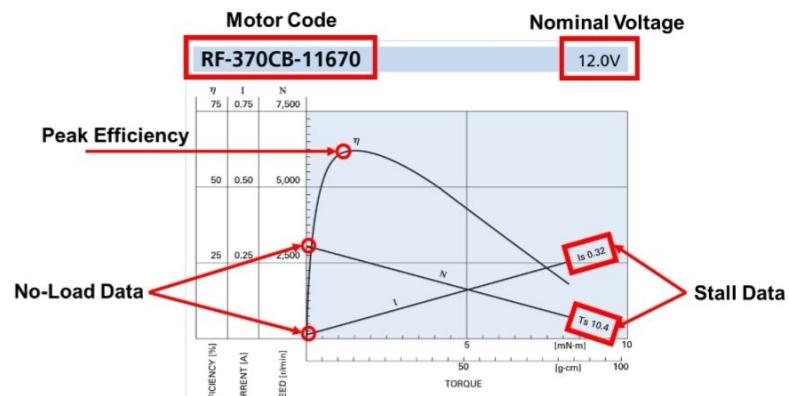
- Motor curves don't usually come w/ cheap motors, but can be generated provided data.
- Nominal Voltage:
 - the voltage that motor most efficiently runs at. If too far away from N.V., will be inefficient
- No load data:
 - motor data when spinning freely (when $T=0$ Nm)
- Stall data:
 - motor data when stalling ($w= 0$ rad/s)
- Max. efficiency data:
 - where a motor should be operating in a well-designed system

Motor Data Example

MOTOR	
Motor Nominal Voltage	12V
Motor Type	High Power
Maximum Efficiency RPM	10850RPM
No load current	0.67A
Maximum Efficiency Torque	252N m
Maximum Efficiency Current	4.5A
Stall Torque	2500N m
No Load RPM	14600RPM
Stall current	18A
Maximum Efficiency	55.1%
Maximum Efficiency Power	26.5W
shaft diameter	3.175mm
shaft length	9.5mm

Obviously should be gcm units

Motor Data Example on a Curve



Drawing Motor Curves

- draw them by plotting the no-load + stall conditions, and connecting w/a straight line
- or substitute gradient and y-int for a straight line: $y = mx + c$

Example 1

Nominal voltage = 12V	No Load	Maximum Efficiency	Stall
Speed (rpm)	13000	10489	-
Torque (g.cm)	-	246	1200
Current (A)	0.6	4	17.2

a) Determine the input power, output power + efficiency of the motor at max efficiency. Would this motor be good to use for your project?

$$\therefore \eta = \frac{P_{out}}{P_{in}} = \frac{\tau \omega}{VI}$$

P_{out} = rotational power

$$= \tau \omega$$

$$\omega = 10989 \text{ rpm} \times \frac{2\pi}{60}$$

$$\omega = 1098.4055 \text{ rad/s}$$

$$\tau = 246 \text{ gcm} \times 9.8 \times 10^{-5} \approx 0.0246 \text{ Nm}$$

$$\therefore P_{out} = \tau \omega$$

$$= 1098.4055 \times 0.024611$$

$$= 26.48256 \text{ W}$$

P_{in} = electrical power

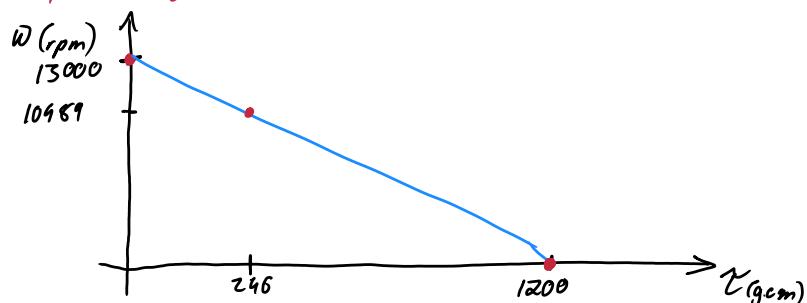
$$= VI$$

$$\therefore \eta = \frac{P_{out}}{P_{in}} = \frac{26.48256}{48} \approx 55\%$$

$$= 12 \times 4 = 48 \text{ W}$$

- b) Draw the motor curves for speed + current and find the speed and current for a torque of 50 gcm.

$$\tau = \frac{r \times i}{r_m}$$



Normal condition: $\tau = 0$

$$\tau = \frac{r \times i}{r_m} \omega + I_0 \quad \leftarrow \omega = m \tau + c$$

$$\tau = -\frac{13000}{1200} \omega + 1300$$

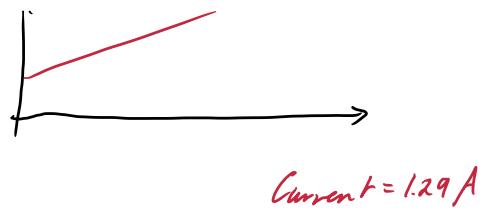
$$\text{when } \tau = 50 \text{ gcm: } 50 = -\frac{13000}{1200} \omega + 1300$$

$$\omega = (50 - 1300) \times \left(-\frac{1200}{13000}\right)$$

$$\omega = 115.3846 \times 12458 \text{ rpm}$$

1

—



Operating away from Nominal Voltage

- Motor curve valid @ nominal... how can we predict motor behaviour if voltage is unchanged?
- Voltage is proportional to speed near nominal voltage. This means $Voltage \times 1.2$

- would increase speed by factor 1.2 aswell.
- Hint: don't operate too far away from nominal voltage; can cause motor to operate inefficiently or burn out

Motor selection + design tips

- select an appropriate motor considering power requirements and design system to get at max. efficiency
- make sure motor has data
- if operating motor outside of max efficiency conditions, use moles curves to predict behaviour
- test mob once you receive them
- avoid high voltages and testing near stall conditions; will cause motor burnout

Motor Troubleshooting

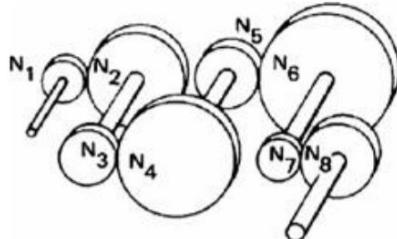
- it motor needs to move faster:
 - < voltage
 - 2 drum size (if far away from still conditions)
 - select more appropriate motor
- it motor can't lift load:
 - 2 current limit
 - > drum size
 - select more appropriate meter
 - **use gearbox**

Gearboxes

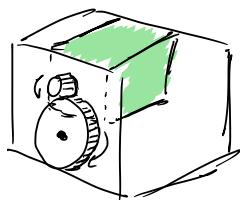
- they use mechanical advantage to < torque capability at expense of >ing speed

Gearboxes

- Gearboxes use mechanical advantage to increase torque capability at the expense of decreasing speed
- **Gear Ratio** = $\frac{N_2}{N_1} \times \frac{N_4}{N_3} \dots$
 - N = No. of Teeth
- $GR = \frac{\omega_{IN}}{\omega_{OUT}} = \frac{\tau_{OUT}}{\tau_{IN} * \eta}$

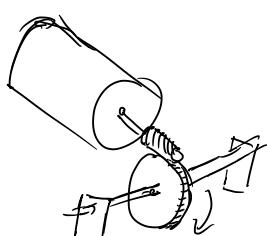


Gearbox examples



Standard gearbox

$$GR = \frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} \dots$$



Worm Drive gearbox

$GR = \text{no. teeth on larger gear}$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5}$$

larger gear

Example Gearbox Question

The motor from prev. example is used at max efficiency ($\tau = 0.029 \text{ Nm}$, $\omega = 1098 \text{ rad/s}$). The output of the gearbox is required to provide a torque of 5 Nm . Determine the following, assuming the gearbox has an efficiency of 80%:

a) The gear ratio required by the gearbox

$$GR = \frac{\tau_{out}}{\tau_{in} \times \eta} = \frac{5}{0.029 \times 0.8} \approx 260.4167$$

b) The output speed from the gearbox

$$GR = \frac{\omega_{in}}{\omega_{out}} \quad \therefore \omega_{out} = \frac{\omega_{in}}{GR}$$

$$= \frac{1098}{260.4167}$$

$$\approx 4.2163 \text{ rad/s}$$

c) The gearbox size you'd select if gear ratios were only available in multiples of 50

gearbox size you'll select if gear ratios were only available in multiples of 50

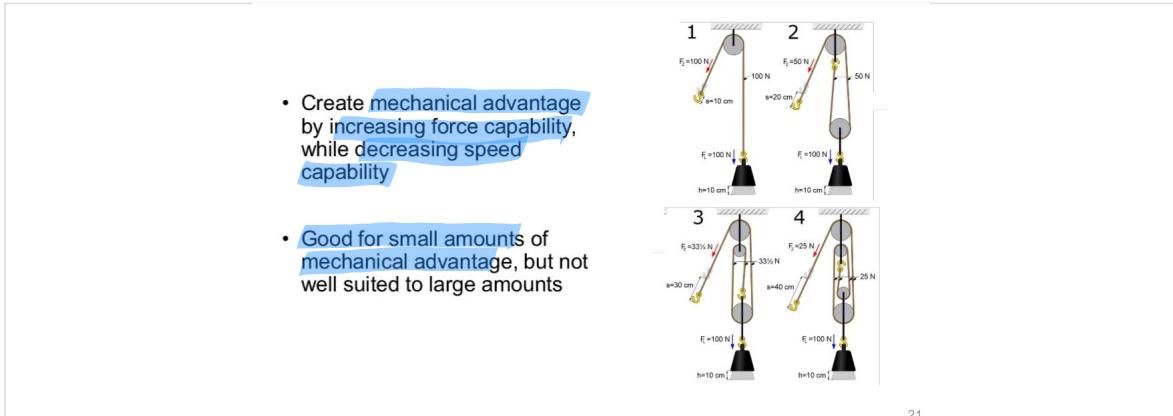
$$GR = 300$$

Bearings

- Bearings are a machine component that restrain motion to a particular direction and **reduce friction** in that direction.
- **Highly recommended** for designs that **require rotating members**. Bearings must be **strong enough to support the moment** created by the 0.5 kg mass as well as the self weight of the crane.



Pulleys



Mechanical Design Tips

- Design system for load capacity, then check speed capability
- don't make speed too fast-will make difficult to control
- select best gearbox for application
- use bearings for rotating meat.
- remember to include efficiencies in calculations
- test all components before assembling device
- be careful of thin plaster gears under high torque loads
- mites w/gearboxes already attached will hemere efficient

Project Application

- consider 1 meter
- mechanical advantage required-motors can only provide small torques
- must operate > 10W, moves object in under 1.5min or 45 seconds

How do it get these components?

There are many places you can purchase electromechanical components if you wish to:

- Hobby stores
 - Electronic stores – e.g. Jaycar
 - The internet – Little Bird Electronics and Pololu (parts may take 1-2 weeks to be delivered)
-
- You can quite often build many components using resources available at the university

Practice Problem

This problem will continue the analysis of the lifting device from last week.

Last week a drum was designed to lift a 2kg mass with an input of $\tau = 2 \text{ Nm}$, $\omega = 3 \text{ rad/s}$. For this tutorial, a motor-gearbox system needs to be designed to meet these input requirements.

A motor has been selected for the design with details provided below.

Nominal Voltage = 6V	No Load	Max efficiency	Stall
Speed (rpm)	13500	9679	-
Torque (g.cm)	-	60	191
Current (A)	0.5	2	5.25
Output Power (W)	-	5.96	-

25

a) At max efficiency, determine motor speed in rad/s and the torque in Nm.

$$\begin{aligned} \gamma &= 60 \text{ gcm} \times 9.8 \times 10^{-5} & \omega &= 967.9 \text{ rpm} \times \frac{\pi}{60} \\ &= 0.00588 \text{ Nm} & & \approx 1014 \text{ rad/s} \end{aligned}$$

b) Determine gear ratio required to provide a torque of 2Nm at max efficiency, assuming $\eta_{gear} = 90\%$.

$$\begin{aligned} GR &= \frac{T_{out}}{T_{in} \times \eta} \\ &= \frac{2}{0.00588 \times 0.9} \end{aligned}$$

$$GR \approx 378$$

c) The angular velocity out of gearbox for the gear ratio.

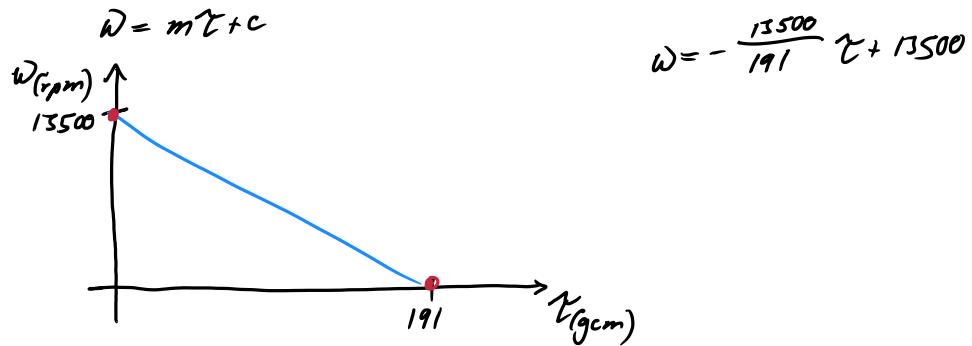
$$GR = \frac{\omega_{in}}{\omega_{out}}$$

$$\begin{aligned} \omega_{out} &= \frac{\omega_{in}}{GR} \\ &= \frac{1014}{378} = 2.68 \text{ rad/s} \end{aligned}$$

d) How can the system be sped up to 3rad/s without replacing any of the components? by increasing the voltage

e) Draw the torque-speed curve for the motor at nominal voltage. The

a) Draw the torque-speed curve for the motor at nominal voltage. The x-intercept, y-intercept and equation of the line must all be included. Use gcm and rpm for units.



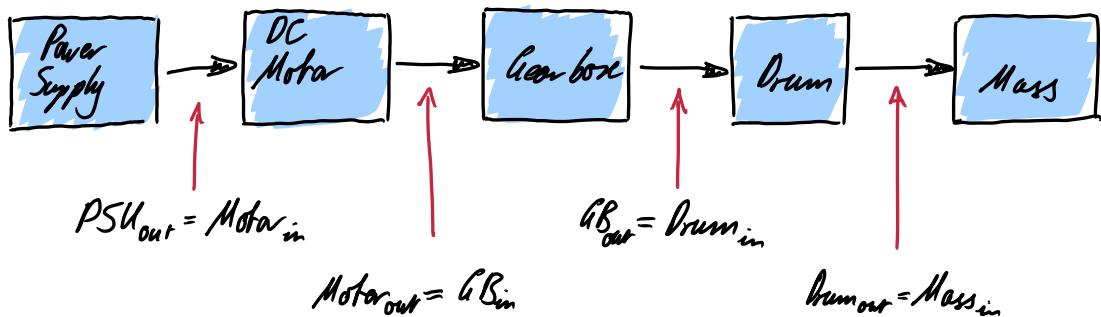
Tutorial Week 8 - Mechanical Systems Design

Today we will use concepts covered in previous mechanical tutorials to analyse and design a mechanical system

- Physics for mechanical systems
- Machine elements and actuators

Representing Electro-Mechanical Systems

- clearly analyse electro-mechanical systems via a block/system diagram:



- each block-component in system
- each arrow can be viewed at output of prev. block or input of following block

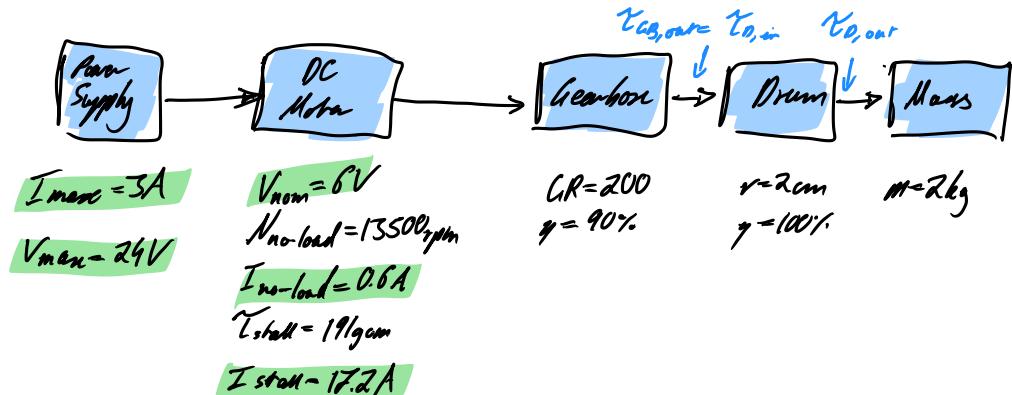
Analysing Electro-Mechanical Systems

- below in example we can analyse torque, speed + power

Torque Analysis

- load/torque is critical in many systems, so we generally start analysis here
- based on mass required to move, we can solve all torques in system moving from right to left through systems diagram

For example:



Complete Torque Analysis.

$$T_{0,out} = R \times r^{lo} \times r^{cm} = 2000^9 \times 2^{cm} = 4000 \text{ gcm}$$

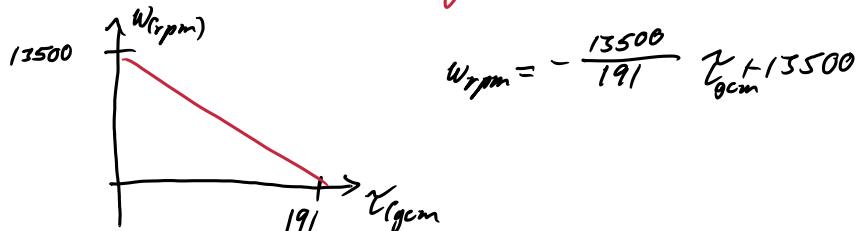
$$T_{0,in} = 4000 \text{ gcm}$$

$$T_{GB,in} = \frac{T_{GB,out}}{G.R. \times GB} = \frac{4000}{0.9 \times 200} = 22.2 \text{ gcm} \leftarrow = T_{M,out}$$

$$T_{M,out} = 22.2 \text{ gcm} \leftarrow T_{M,out} < T_{M,stall} \text{ which is good}$$

this means the Motor won't stall with the load

Complete Speed Analysis of System.



$$\text{when } T_{GB,in} = 22.2 \text{ gcm} \rightarrow \omega_{GB,in} \approx 11929 \text{ rpm}$$

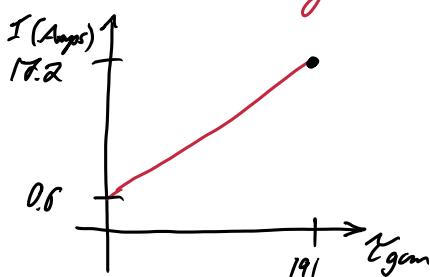
$$\omega_{m,out} = \frac{\omega_{GB,in}}{G.R.} = \frac{11929}{200} = 59.6 \text{ rpm}$$

$$\omega_{D, \text{out}} (\text{in rad/s}) = \frac{\pi R}{60} \times 59.6 \text{ rpm} = 6.25 \text{ rad/s}$$

$$V = \omega_{D, \text{out}} \times r \text{ cm} = 6.25 \times 2 = 12.5 \text{ cm/s}$$

do decrease speed, increase gear ratio

Power Analysis



$$m = \frac{\Delta y}{\Delta x} = \frac{17.2 - 0.6}{191} = \frac{16.6}{191}$$

$$\therefore I = \frac{16.6}{191} z + 0.6$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$I \cdot z_{M, \text{out}} = 22.2 \text{ rad/s} \cdot 17.2 \text{ cm} = I = 2.53 \text{ A}$$

Input Power:

$$P_{\text{in}} = V \times I = 6 \times 2.53 = 15.18 \text{ W}$$

Output Power:

$$\begin{aligned} P_{\text{out}} &= F \times v = (2 \times 9.81) \times 12.5 \text{ cm/s} \\ &= (2 \times 9.81) \times 0.125 \text{ m/s} = 2.45 \text{ W} \end{aligned}$$

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = 16.13\% \leftarrow \text{as long as } \eta > 10\%, \text{ it is alright}$$

Speed Analysis

- w/ torque analysis completed we now can do speed analysis
- use torque/speed curve to predict speed of motor, then move left-right through system to solve velocity the mass can be lifted at

Power Analysis

- w/ torque + speed determined, we can do power analysis (input power, output power + efficiency of system)
- torque-current curve used to determine current and therefore input power of system

Milestone 2

- mechanical systems analysis required (system block diagram, torque analysis,

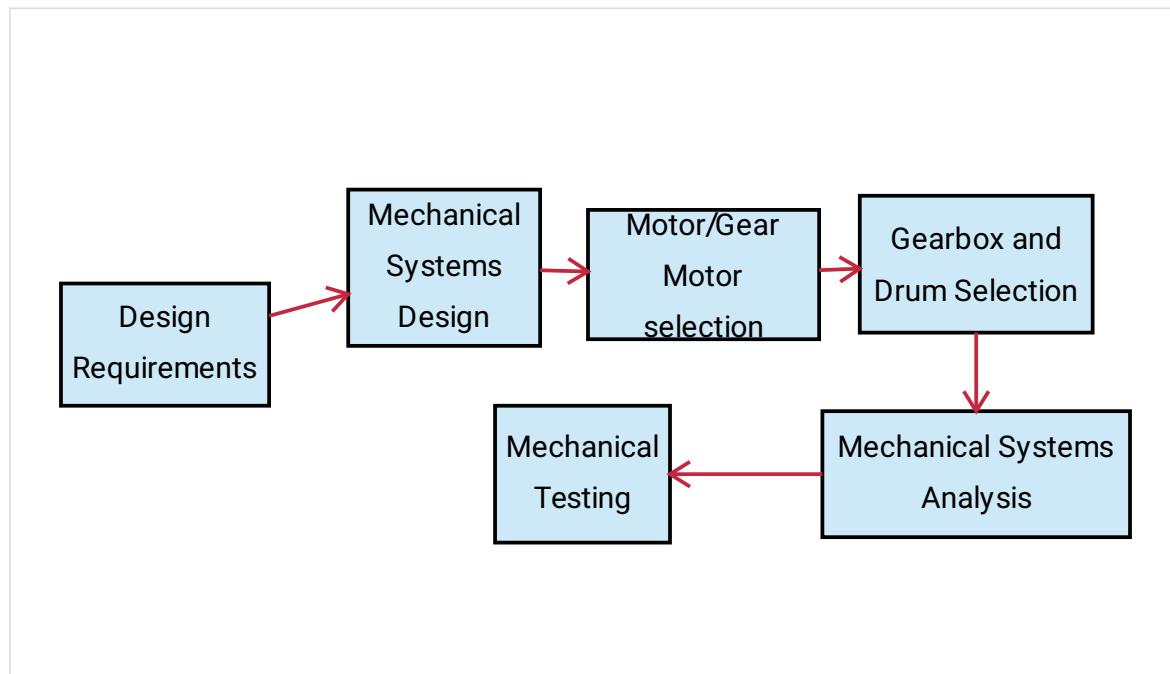
(speed analysis + power analysis)

- to be completed on one mechanical system (e.g. lifting)

Mechanical Design

- mechanical design is more challenging; multiple design requirements to be met
 - some requirements conflict
 - components unknown, like motor, drum size, gear ratio etc.
- no 'correct' way of designing mechanical system, but some are more efficient than others
- programming the calculations can be useful, e.g. Excel
- Mechanical design isn't only calculations, it's also testing of the motor and components.

Mechanical Design Process

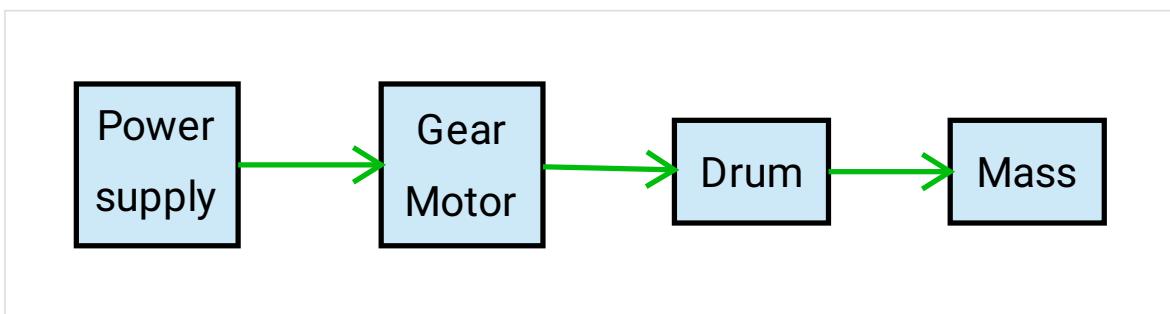


Step 1: Design Requirements

- identify requirements of mechanical design
 - load requirement
 - speed requirement
 - power requirement

Step 2: System Design

- understand systematically what your design will look like



- info we know like limitations and load requirements can be added
- add data for components that we might need to assume data for

Step 3: Motor (or generator) selection

- data likely to be given:
 - nominal voltage
 - no load speed
 - no load current
 - stall torque
 - stall current

Ideal Operating Range for a Motor:

Peak Power occurs at $\frac{1}{2} \omega_{n1}$

↳ find torque at this speed

*↳ you might not want to operate at peak power:
leaves no room for initial transients*

Aim for operating at 75% of peak power (0.25 T_s)

Motor Selection

- If planning to use motor + gearbox as 2 separate components, main

consideration when selecting motor is power

- determine lifting power required based on system requirements

If we're aiming to lift 1m in 30 seconds (6kg):

$$\text{Using SI units: } P_{lift} = Fv = 6 \times 9.8 \times \frac{1}{30} = 1.31W$$

$$\text{kg, cm, s: } P_{lift} = Fv = 6 \times \frac{100}{30} = 13.3 \text{ kg cm/s}$$

 Mass $m = 6\text{kg}$
 $P_{lift} = 1.31W$

- we can determine the max output power of motor using motor specs:

$$P_{max} = \frac{T_{stall} \times \omega_{no\ load}}{9}$$

- Typically, we use lifting power 19-75% of max output of motor
∴ we want max power of motor:

$$1.75W < P_{max} < 6.89W$$

$$17.73 \text{ kg.cm/s} < P_{max} < 70 \text{ kg.cm/s}$$

- a gear meter will need to consider torque
- if lifting 4kg, 1- 5cm is reasonable, then gearmotor will operate @ torque between:

$$\tau_1 = 4 \times 1 = 4 \text{ kg.cm}$$

$$\tau_2 = 4 \times 5 = 20 \text{ kg.cm}$$

- **NOTE:** operating torque preferably around 5- 25% of stall torque

- we want stall torque of our motor to be :

$$16 \text{ kg.cm} < \tau_{\text{stall}} < 400 \text{ kg.cm}$$

- research gearmetas online (Pololu and Jaycar) good places to start
- Find a motor that:

- meets power requirement
- meets torque requirement

<https://www.pololu.com/product/3484>

- this is a gear motor that meets requirements

Checking gear motor against criteria

Torque criteria :

$$\text{e.g. } \tau_{\text{stall}} = 21 \text{ kg.cm}$$

$$16 \text{ kg.cm} < \tau_{\text{stall}} < 400 \text{ kg.cm}$$

∴ GM meets torque requirement

Power Criteria :

$$P_{\max} = \frac{\tau_{\text{stall}} \times \omega_{\text{no load}}}{G \cdot b_g} = \frac{21 \times 36 \times 2 \times \frac{\pi}{60}}{4}$$

$$= 39.58 \text{ kg.cm/s}$$

$$17.73 \text{ kgcm/s} < P_{\max} < 70 \text{ kgcm/s}$$

∴ GM meets power requirement

Step 4: Drum Radius

- once Gear motor chosen, we can enter details into motion curve spreadsheet on Blackboard
 - then find max. efficiency torque for Gearbox and select a drum radius such that we can operate motor at/near max efficiency

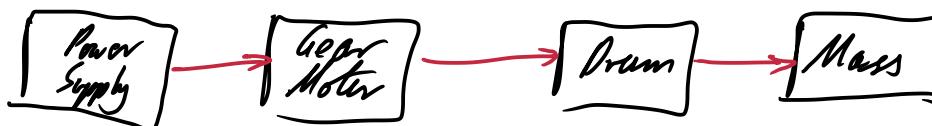
remember : $T = F_r$

- in this case, a drum radius of 1cm achieves near max. efficiency conditions

Step 5: Mechanical Systems Analysis

- mechanical components have been selected: now do a full mechanical system analysis
 - incl. torque, speed + power analysis, as done above

Complete a torque, speed and power analysis for the proposed system. Verify whether load, speed + power requirements have been met.



$$\begin{array}{lll} V_{nom} = 12V & r = 1\text{ cm} & m = 4\text{ kg} \\ N_{NL} = 36\text{ rpm} & \eta = 100\% & P_{lift} = 1.31\text{ W} \\ I_{NL} = 0.09A & & \\ T_{stall} = 21\text{ kg.cm} & & \\ I_{stall} = 1.6A & & \end{array}$$

Step 6: Mechanical Testing

- now its'confirmed that the design works: now, order the parts.
- Appropriate testing includes:
 - Test components of mechanical system
 - then test mechanical system once assembled
 - then test entire system once mechanical system is attached to structure

Mechanical Systems for lateral Motion

- Friction + inertial loads difficult to predict
- Ideally, you can construct design + test force/torque requirement of system
- If you can experimentally determine load requirement, then mechanical design calculations can be performed
- instead of experimentally testing force/torque...
 - decide on a speed requirement for the system
 - assume friction is small, therefore motor operates close to no load speed
 - choose a motor for similar power to lifting motor), and determine gear ratio for required speed reduction.
 - construct and test the system.
- this involves more assumptions than our earlier calculations, so expect modifications to be needed after building and testing.

Tutorial Week 9- Electrical Systems Analysis

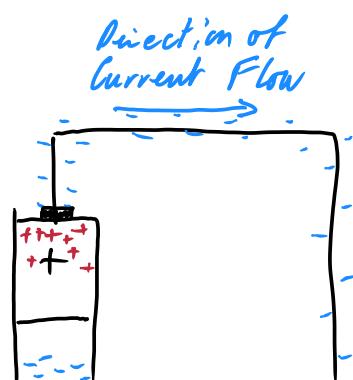
Today's Tutorial

- Electricity
- Ohms Law
- Voltage, Current + Resistance
- Circuits + Power
- Series + Parallel Circuits

Electricity

- always moves from higher to lower energy potential

- Here, current will flow from + terminal to - terminal





Ohm's law

Voltage

- electric potential, referring to the energy in an amount of charge
 - measured in volts V (symbol is V)

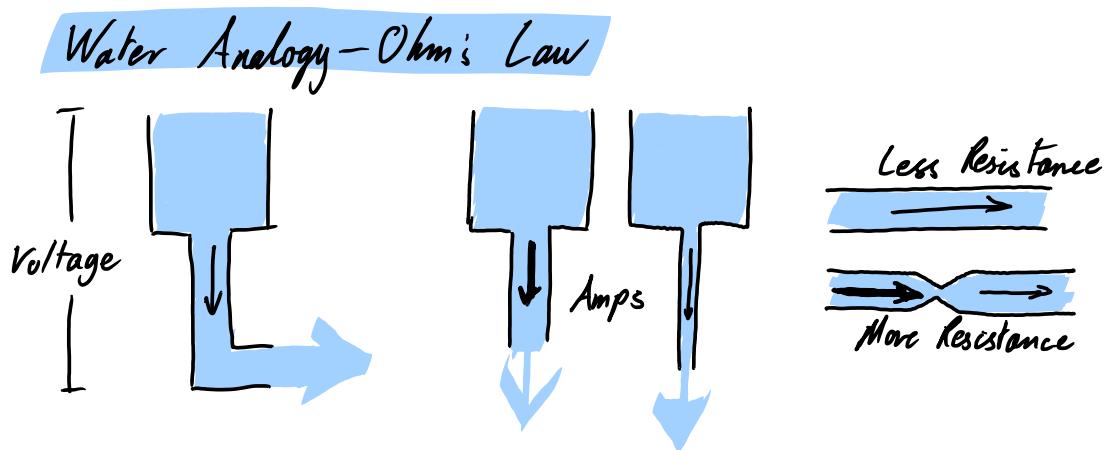
Current

- rate of flow of charge
 - measured in Amps (A), symbol of i

Resistance

- refers to any element that limits the flow of current
 - measured in Ohms, symbol R

$$\text{Voltage} = \text{Current} \times \text{Resistance}$$



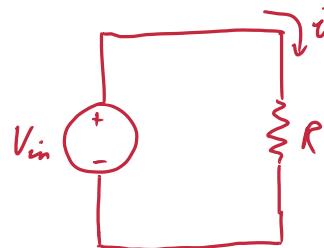
Ohms law Example

a) If $V_{in} = 12V$, $R = 200\Omega$, what is i ?

$$V = iR \rightarrow i = \frac{V}{R} = \frac{12}{200} = 0.06A$$

b) If $V_{in} = 12V$, $i = 0.08A$, what is R ?

$$V = iR \rightarrow R = \frac{V}{i} = \frac{12}{0.08} = 150\Omega$$



c) If $i = 0.06A$, $R = 400\Omega$, what is V_{in} ?

$$V = iR = 0.06 \times 400 = 24V$$

d) Would there be a difference between V_{in} and V_R ?

No, as $V_{in} = V_{out}$ and the only element for V_{out} is V_R

$$\therefore V_{in} = V_R$$

Ohm's law - Simplified

- increased voltage = more current
- increased resistance = less current

Power Example

For the following 3 examples find the power dissipated by the resistor:

a) If $V_{in} = 12V$, $R = 200\Omega$ and $i = 0.06A$

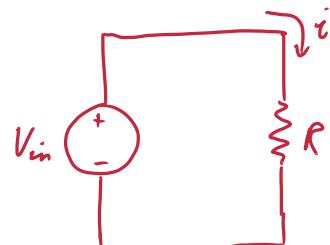
$$P = VI = iR \times i = i^2 R = 0.72W$$

b) If $V_{in} = 12V$, $R = 150\Omega$ and $i = 0.08A$

$$P = i^2 R = 0.08^2 \times 150 = 0.96W$$

c) If $V_{in} = 12V$, $R = 400\Omega$ and $i = 0.06A$

$$P = i^2 R = 0.06^2 \times 400 = 1.44W$$

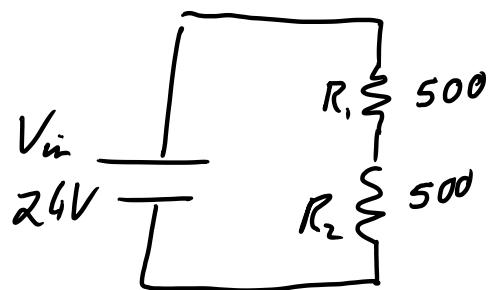


$$\begin{aligned} P &= VI \\ P &= i^2 R \\ P &= \frac{V^2}{R} \end{aligned}$$

- **Power Resistors:** designed to have a higher power rating so they have a higher voltage/current

Resistance in Series

- resistors- considered to be in series it same current goes through all of the resistors in order
- all voltage drops add to equal total voltage
- in a series, every component will have the same current

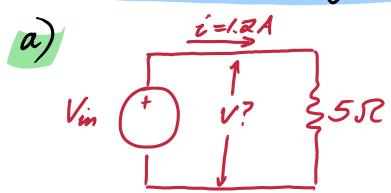


Resistance in Parallel

- two resistors are considered in parallel if both terminals are respectively connected together at 2 common nodes
- Branch currents add to equal current: $i = i_1 + i_2$
- in parallel will have exact same voltage for each component

Ohms Law and Power Example

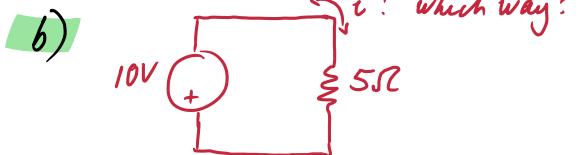
Determine unknown parameter of each circuit shown below, and calculate the power dissipated by the resistors.



$$V_{out} = iR = 1.2 \times 5$$

$$V = 6V$$

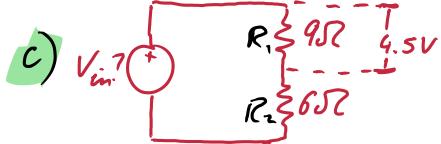
$$P_{out} = i^2 R = 5 \times 1.2^2 = 7.2W$$



$$V = iR \rightarrow i = \frac{V}{R}$$

$$i = \frac{10}{5} = 2A$$

$$P = i^2 R = 5 \times 2^2 = 20W$$

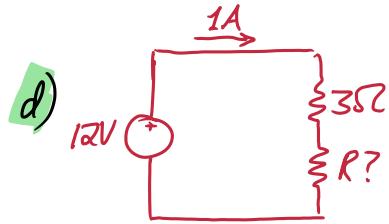


$$\text{For } R_1: i = \frac{V}{R} = 0.5A$$

$$\text{For } R_2: V = iR = 0.5 \times 6 = 3V$$

$$\therefore V_{in} = V_{out} = 3 + 4.5 = 7.5V$$

$$P = VI = 7.5 \times 0.5 = 3.75W$$



$$V_{in} = V_{out}$$

$$12V = i(R_1 + R_2)$$

$$12 = 3 + R_2$$

$$\therefore R_2 = 12 - 3 = 9\Omega$$

$$P = VI = 12 \times 1 = 12W$$

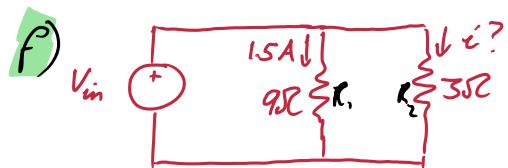


$$V = iR = 4 \times 3 = 12V$$

$$\therefore R = \frac{V}{i} = \frac{12}{4} = 3\Omega$$

$$i = i_1 + i_2 = 4 + 2 = 6A$$

$$P = VI = 12 \times 6 = 72W$$



$$V = iR = 1.5 \times 9 = 13.5V$$

$$i = \frac{V}{R_2} = \frac{13.5}{3} = 4.5A$$

$$i = 1.5 + 4.5 = 6A$$

$$\therefore P = iV = 6 \times 13.5 = 81W$$

Open Circuit (Series DC Circuit)

- where no current flows because there is a gap in the circuit that does not allow current to flow
- an open Series DC circuit will result in no power being consumed by any of the loads

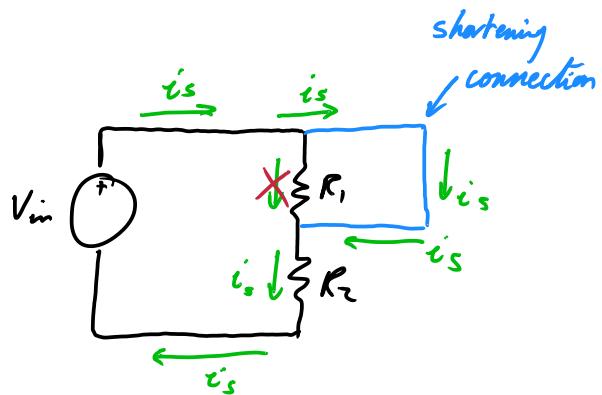
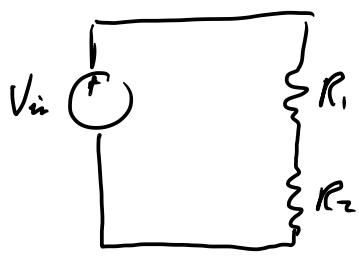
Open Circuit (Parallel DC Circuit)

- where no current flows because there's a gap in the circuit that doesn't allow current to flow
 - the effect of an open in parallel circuit is dependent on location of the open

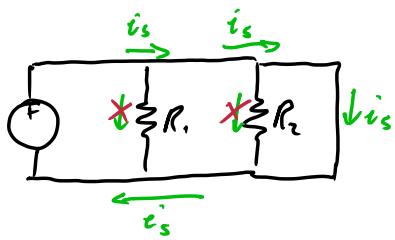
Short Circuit

- where the current ther encounters no (or very low) electrical resistance
 - therefore a short circuit will have too much current flowing through it
- **Remember:** current will always take path of least resistance (like water)

In a series DC Circuit:



In a Parallel DC Circuit:

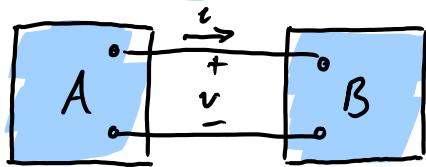


- when a parallel circuit becomes short circuited, there is a sudden + very large increase in circuit current, because the resistance between the power source is basically zero.

Power Transfer

- when elements are connected, power transfers from one element to the other.
- In this case, element A is delivering power, and element B is an extracting power.

Power transferred from A to B.



Example: Power Transfer

a) How much power is being transferred between these elements?

$$P = VI = 200V \times 10A = 2000W = 2kW$$

b) Is the power transfer going from A to B/B to A?

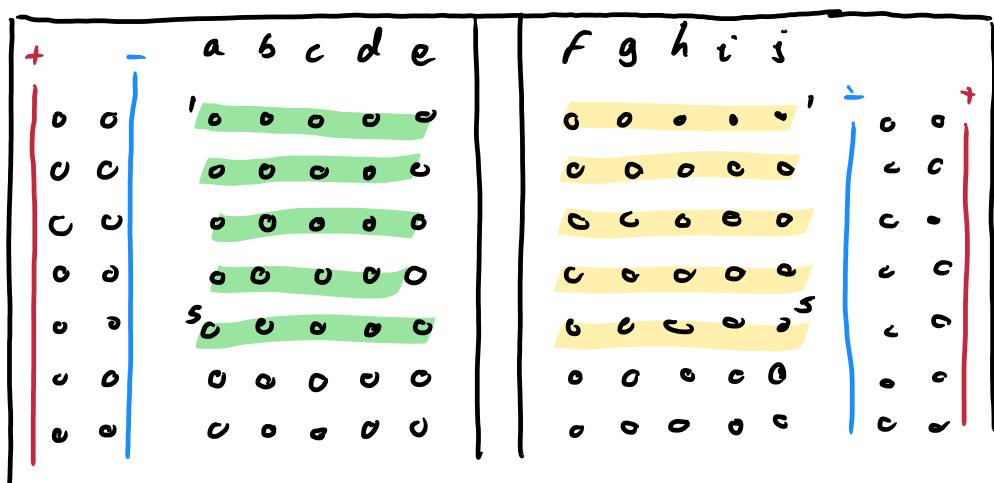
B to A

Power Transfer

- Positive power: power is being dissipated by element
- Negative power: power is being supplied by element
- All power must be equal in a circuit: supplied = dissipated

Project Application

- breadboards will be used in project



- allows you to build circuits without having to solder components together

Project Application

- consider electrical power budget given
- design a system that draws low during the steady-state period
- work backwards to find value of resistors, and ensure you have the right type of resistors
- make sure you understand you need to find a way to distribute voltage and current between motors

Read up on Zener Diodes and Voltage Regulators.

Tutorial Week 10 - Designing Electrical Systems

Today's Tutorial

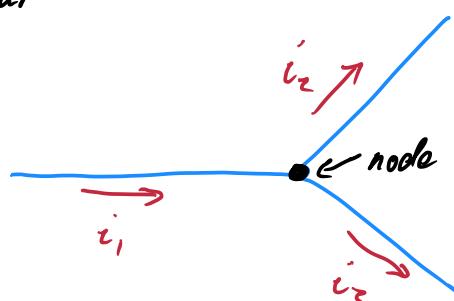
- learn additional circuit analysis techniques, and methods to control behaviour of DC Motors:
 - Kirchoff's Voltage (KUL) + Current Law (KOL)
 - Derived laws
 - Power, Equivalent Resistance and Voltage + current dividers
 - Motor control
 - Voltage Regulators
 - Project Application

Kirchoff's Current law (KOL)

- sum of all current entering a node **must** equal zero

$$\sum \text{currents in} = \sum \text{currents out}$$

$$i_1 = i_2 + i_3$$



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- the voltage law- "sum of voltages around every closed loop in circuit **must** equal zero"

$$\sum \text{Voltage drops} = \sum \text{Voltage rises}$$



Derived Circuit laws

Electrical Power for a Resistor:

$$P = V_i i$$

$$P = i^2 R$$

$$P = \frac{V^2}{R}$$

$$P = V_i$$

$$P = \frac{V}{R}$$

Equivalent Resistance

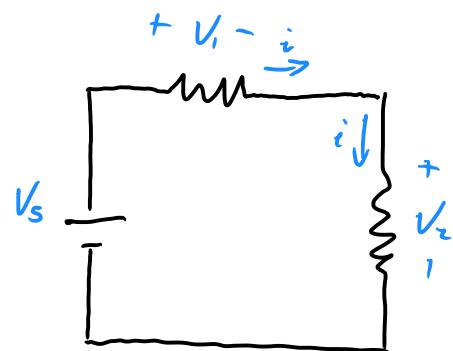
- equivalent resistance: to replace multiple resistors w/ one resistor

Equivalent Series Resistance

$$V_{in} = V_1 + V_2$$

$$V_{in} = iR_1 + iR_2 = i(R_1 + R_2)$$

$$\therefore R_{total} = R_1 + R_2$$



Problem 1

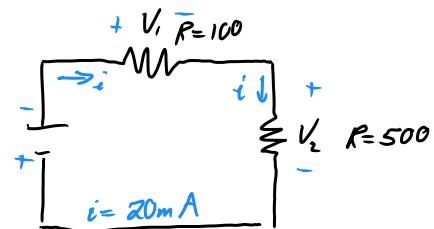
Calculate:

a) The equivalent resistance of this circuit.

$$R_T = R_1 + R_2 = 100 + 500 = 600 \Omega$$

b) If the current is 20mA, what is the input voltage?

$$V = iR = 0.02 \times 600 = 12V$$



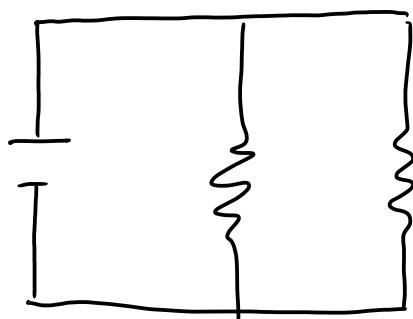
Equivalent Parallel Resistance

$$i_{in} = i_1 + i_2 = \frac{V_s}{R_1} + \frac{V_s}{R_2}$$

$$\therefore i_{in} = V_s \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{R_{\parallel}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

For two parallel resistors:



For two parallel resistors:

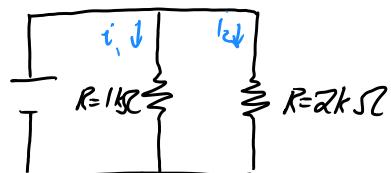
$$R_{\parallel} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Problem 2

a) Find the equivalent resistance of this circuit.

$$R_{\parallel} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{2000000 \Omega}{1000 + 2000} = \frac{2000000}{3000} \cancel{\Omega}$$

$$= 666.67 \Omega$$



b) If the input voltage is 12V, what is the input current i_{in} ?

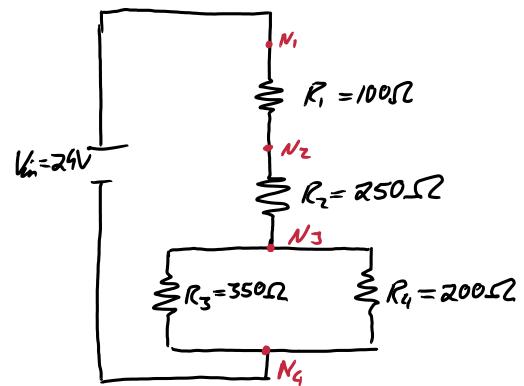
$$V = iR \rightarrow i = \frac{V}{R} = \frac{12}{2000} = \frac{12 \times 3}{2000} = \frac{36}{2000} = \frac{18}{1000} = 0.018 A$$

Problem 3

Find the source current using equivalent resistance by:

a) adding series resistors R_1 and R_2 together

b)



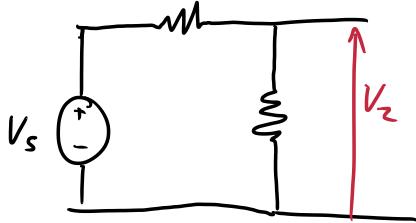
Series Resistors: Voltage Divider

V_s is divided into $V_1:V_2$ in the proportions $R_1:R_2$

Approx. Voltage Divider

If $i_d = 0$, then:

$$V_2 = \frac{R_2}{R_T} V_s$$



If i_d is very small, then:

$$V_2 \approx \frac{R_2}{R_T} V_s$$

- use a voltage divider to find all voltages in the circuit

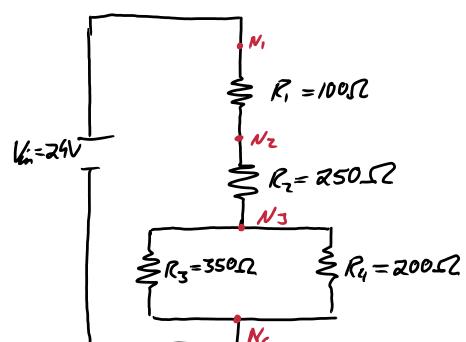
$$R_{Total} = 477.27 \Omega$$

$$V_1 = \frac{R_1}{R_T} \times V_s$$

$$= \frac{100}{477.27} \times 24 = 5.03 \text{ V}$$

$$V_2 = \frac{R_2}{R_T} \times V_s = \frac{250}{477.27} \times 24 = 12.57 \text{ V}$$

$$V_3 = \frac{R_3}{R_T} \times V_s = \frac{350}{477.27} \times 24 = 17.6 \text{ V}$$



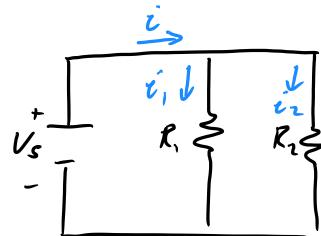
$$V_s = 477.27$$

$$V_A = \frac{300}{477.27} \times 24 = 10.08V$$

Parallel Resistors: Current Divider

i_1 is divided into $i_1 : i_2$ in the ratios $\frac{1}{R_1} : \frac{1}{R_2}$

$$i_2 = i \frac{R_1}{R_1 + R_2}$$



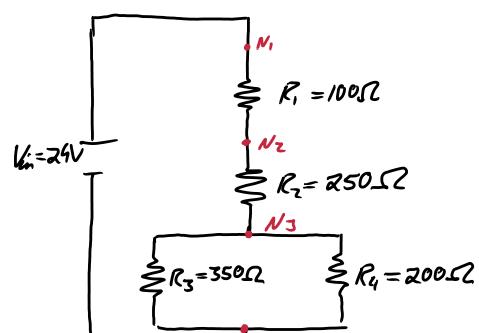
- use a current divider to find the current through R(3) and R(4)

$$i = 0.05A$$

$$i_3 = \frac{200}{350+200} \times 0.05 = 0.018A$$

$$i_4 = \frac{350}{350+200} \times 0.05 = 0.032A$$

$$\text{check: } i = i_3 + i_4 = 0.018 + 0.032 \\ 0.05 = 0.05A \quad \checkmark$$



$$0.05 = 0.05A \checkmark$$

N_1

Problem 4

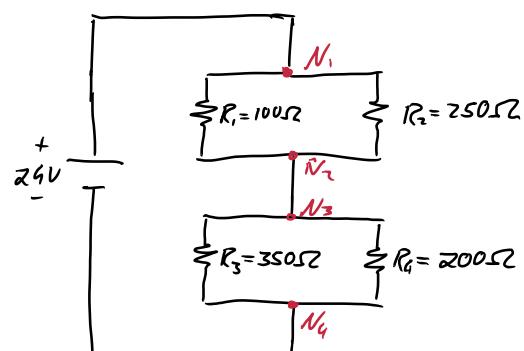
Solve for voltage, current and power of all components in the circuit, and check that power is balanced.

$$R_{1\parallel 2} = \frac{100 \times 250}{100 + 250} = 71.43 \Omega$$

$$R_{3\parallel 4} = \frac{350 \times 200}{350 + 200} = 127.27 \Omega$$

$$\therefore R_T = 71.43 + 127.27 = 198.7 \Omega$$

$$i = \frac{V}{R} = \frac{24}{198.7} = 0.12 A$$



$$V_1 = V_2$$

$$\therefore V_1 = \frac{R_{1\parallel 2}}{R_T} \times V_S = \frac{71.43}{198.7} \times 24 = 8.65 V$$

$$V_2 = 8.65 V$$

$$V_3 = V_4 = \frac{R_{3\parallel 4}}{R_T} \times V_S = \frac{127.27}{198.7} \times 24 = 15.61 V \times 15.37 ?$$

$$i_1 = \frac{V_1}{R_1} = \frac{8.65}{100} = 0.086 A \quad P_1 = i_1 V_1 = 0.086 \times 8.65 = 0.744 W$$

$$i_1 = \frac{V}{R_1} =$$

$$i_2 = \frac{V}{R_2} = \frac{8.65}{250} = 0.035 \text{ A}$$

$$i_3 = \frac{V}{R_3} = \frac{15.37}{350} = 0.044 \text{ A}$$

$$i_4 = \frac{V}{R_4} = \frac{15.37}{200} = 0.077 \text{ A}$$

$$P_2 = i_2 V_2 = 0.035 \times 8.65 = 0.303 \text{ W}$$

$$P_3 = i_3 V_3 = 0.044 \times 15.37 = 0.676 \text{ W}$$

$$P_4 = i_4 V_4 = 0.077 \times 15.37 = 1.183 \text{ W}$$

$$\therefore P_{\text{total}} = P_1 + P_2 + P_3 + P_4 = 0.744 + 0.303 + 0.676 + 1.183 = 2.906 \text{ W}$$

$$P_{\text{in}} = i V = 0.12 \text{ A} \times 24 \text{ V} = 2.88 \text{ W}$$

different possibly due to
heating of wires

Speed Control

speed proportional to voltage

$$V = K_b \omega$$

current is roughly proportional to torque requirement of motor

$$T = K_T i$$

we can use this knowledge to control the speed of the motor.

[more info-voltage divider with Motor]

Problem 5

A gearmotor with a nominal voltage of 12V requires 0.5A of current when it lifts a 1kg mass. If you plan to operate the motor with a 24V battery, what resistor should you connect in series? What is the minimum power rating required for the resistor?

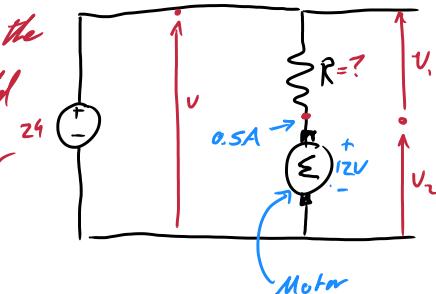
$$24 = 12 + v_i$$

$$\therefore v_i = 12V$$

$$R = \frac{V}{i} = \frac{12}{0.5} = 24\Omega$$

$$P_R = V_i i = 12 \times 0.5 = 6W$$

You go purchase a resistor, but you find that all the resistors are sold out except for three 16Ω, 5W resistors. Will you be able to operate the motor at nominal voltage with



at nominal voltage with your 24V battery?

Yes

What happens if Load changes?

If Load increases:

- current drawn by motor increases
- current through resistor increases
- voltage across resistor increases
- voltage across motor decreases

2 things to check in design

- Motor voltage drop: check motor specs
- Resistor can take current increase: check power rating

Voltage Regulator

- a circuit designed to maintain a voltage across a component (e.g. motor) as current changes

Shunt Regulator

[in Pp]

Problem 6

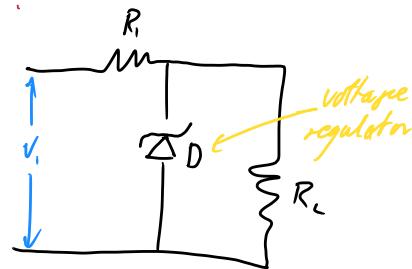
Thanos has a 6V that draws 0.07-0.8A of current, depending on the load. He is trying to get it to run off a 12V battery by incorporating a voltage regulator which requires $\geq 0.01\text{A}$ current to work.

a) What is the desired voltage rating for the regulator?

6V

b) What is the max resistance he can select for R_1 ?

$$R = \frac{V}{i} = \frac{12-6}{0.81} = 7.45\Omega$$



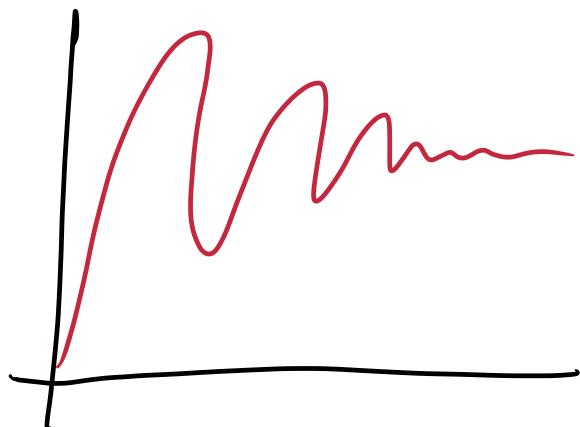
c) What is the minimum power rating required for R_1 and the voltage regulator?

Find max current:

$$\text{Power}_1 = V_i i = (12-6) \times 0.81 = 4.86\text{W}$$

$$\text{Power}_{\text{regulator}} = V I_{\text{max}} = 6 \times (0.81 - 0.07) = 4.64\text{W}$$

- **Transient vs Steady-state**
 - steady-state: where system "settles"
 - Transient: early fluctuations (may contain peaks)



Ohm's Law: $V = I R = \text{current} \times \text{resistance}$

$$v = w \times r$$

Exercise 1

Load Cell Mass = 0.085kg

Mass of Coal = 500g

Total Mass = 385g

Total Force = 3.77N

Middle Pulley radius = 1cm

Shunt Resistor = 1Ω

Exercise 1 Results

Supply Voltage (V)	5	8
Peak Force (N)	4.59	5.36
Steady-state Force (N)	3.84	3.83
Peak Mechanical Power (W)	0.44	0.67
Steady-state Mechanical Power (W)	0.35	0.47
Peak Electrical Power (W)	1.1	1.32
Steady-state Electrical Power (W)	0.98	1.05

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Table Height = 950mm

Requirements

Electrical Sub-System:

$$V_{max} = 24 V$$

$$I_{max} = 3 A$$

$$P_{peak} \leq 10 W$$

FOS between 1 and 2

$$F = g \cdot N = 500 g$$

$$Z = 20 \text{ g cm}$$

$$\gamma = F \times r$$

$$r = \frac{Z}{F} = \frac{20}{500} = \frac{4}{10} = 0.4 \text{ cm}$$

2. Mechanical System Components

Power Supply

$$I_{max} = 3 A$$

DC Motor attached gear box

$$V_{max} = 6.5 V$$

$$V_{max} = 24 V$$

$$N_{no-load} = 12500 \text{ rpm}$$

$$I_{no-load} = 0.25 A$$

$$Z_{stall} = 100 \text{ g cm}$$

$$I_{stall} = 3.337 A$$

Drum

$$r = 1 \text{ cm}$$

Mass

$$g \quad z \\ (-75, 917)$$

Height :

$$\tan \theta = \frac{x}{350}$$

$$350 \tan 50^\circ = x$$

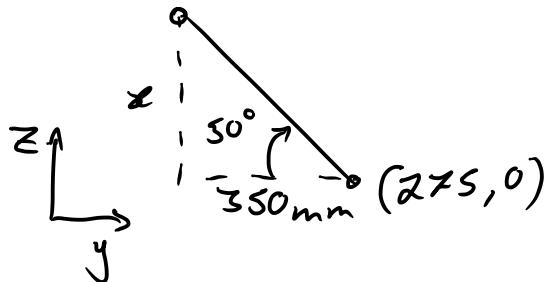
$$x = 917.11376$$

Line 1

$$z = my + c$$

$$0 = \left(-\frac{917}{350}\right) \times 275 + c$$

$$c = 327.843$$



$$z = \left(-\frac{917}{350}\right)y + c$$

$$c = 506.3571$$

Structural Analysis

Free-Body Diagram: shows a single body/subsystem of bodies isolated from its surroundings showing all forces acting on it.



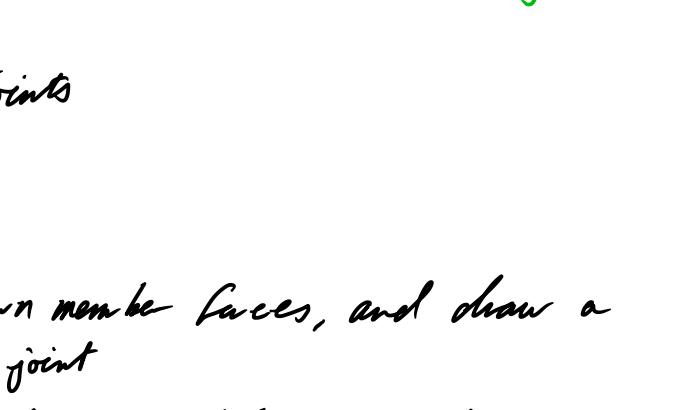
IMPORTANT: an FBD of one side of the truss takes HALF THE LOAD!!!

$$\therefore 8kN = 4kN$$

- Support Types:

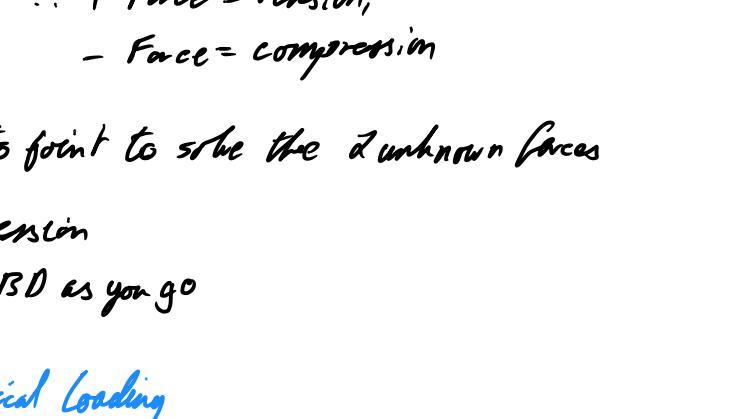
1. Fixed Support

- 2 reaction forces
- 1 reaction moment



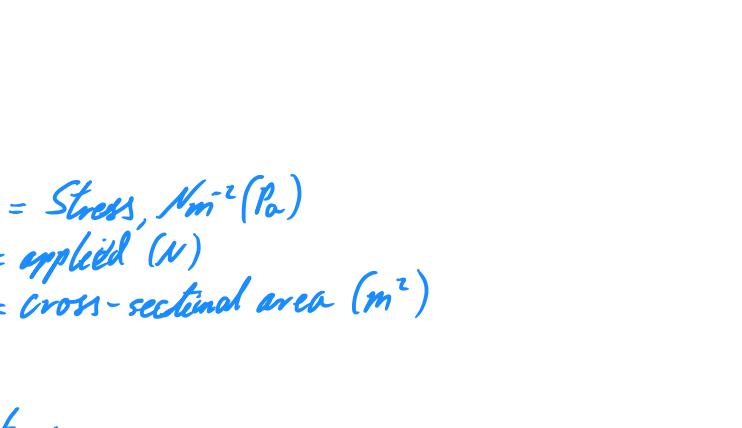
2. Pinned Support

- 2 reaction forces



3. Roller Support

- 1 reaction force



- Truss Analysis: Method of Joints

1. $\sum M = 0, \sum F_x = 0, \sum F_y = 0$

to find Reaction Forces

2. Find a joint w/ ≤ 2 unknown member forces, and draw a Free Body Diagram of the joint

→ always assume forces act in tension so all forces are pointing away from the joint.

3. Apply $\sum F_x = 0$ and $\sum F_y = 0$ to joint to solve the 2 unknown forces

→ T = tension, C = compression

→ update forces onto FBD as you go

- Torsion: caused by asymmetrical loading



- Structural Materials

→ (Axial) Stress

$$\sigma = \frac{F}{A}$$

σ = Stress, Nm^{-2} (Pa)

F = applied (N)

A = cross-sectional area (m^2)

→ (Axial) Strain

$$\epsilon = \frac{\Delta L}{L_0}$$

ϵ = strain

ΔL = change in length (m)

L_0 = initial length (m)

→ Young's Modulus

$$E = \frac{\sigma}{\epsilon}$$

also known as modulus of elasticity

→ measures ability of a material to withstand changes in length under longitudinal tension/compression

- Buckling: when a straight column undergoes bending

→ Critical Buckling Load:

$$F_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

E = Young's Modulus
 I = moment of inertia about axis of buckling
 $I = \frac{\pi r^4}{4}$
 K = effective length factor based on end boundary conditions
 Both ends pinned: $K=1$, fixed: $K=0.5$
 L = length of column

- Brittle: stress = fracture stress σ_f

- Ductile: ultimate tensile stress (UTS) > yield stress

- Mechanical Properties

→ Yield Stress (σ_y): the stress level at which deformation begins
 o stress < yield stress \Rightarrow plastic

o stress > yield stress \Rightarrow elastic

→ Ultimate Tensile Strength (σ_{uts}): stress that causes material to fail

- Types of Failure Modes:

→ Tension (exceeding UTS)

→ Compression (Buckling)

- Failure of Axially Loaded Materials

→ Tension, $\sigma_{y,T} < \sigma < \sigma_{uts,T}$: material plastically deforms

→ $\sigma > \sigma_{uts,T}$: material will break

Mechanical Analysis

- Rotational velocity: $\omega = \frac{v}{r}$ (rad/s)

- Angular acceleration (rad/s²):

$$\alpha = \frac{dv}{dt}$$

- A structure in equilibrium means it's accelerating

- Power: rate of work done (Watts)

$$P = Fv = V_i \quad \text{for mechanical motor}$$

- Electrical Power + Efficiency

$$\eta = \frac{P_{out}}{P_{in}}$$

↳ %

- What is Torque?

→ the moment of a force; the tendency of a force to rotate the body to which it is applied; the engine's rotational force

- Power Train Diagram

- Gear Analysis

$$\frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = -\frac{r_2}{r_1} = -\frac{N_2}{N_1}$$

$$P = iV = i^2 R = \frac{V^2}{R}$$