11/2

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Acknowledgements: *Utilized the study session: Worked independently and checked work cooperatively with other classmates on this practice set.*

Wade Exercise 3.1.1 Using Definition 3.1, prove that each of the following limits exists.

a. $\lim_{x\to 2} x^2 + 2x - 5 = 3.$

Proof.
Let $f(x) = x^2 + 2x - 5$ and Let $\epsilon > 0$.
Set $\delta = \min\left\{1, \frac{\epsilon}{7}\right\}$.
Assume $0 < |x-2| < \delta$.
Then,

$$\begin{aligned} |x-2| < 1 &\implies -1 < x - 2 < 1 && \text{[FTAV]} \\ &\implies 1 < x < 3 \\ &\implies x \in (1,3) \\ &\text{Also note that } |x-2| < \frac{\epsilon}{7} \end{aligned}$$

Then,

$$\begin{aligned} |f-3| &= |x-2||x+4| && \text{[Algebra/Factoring]} \\ &< |x-2|(|x|+4) \\ &< \frac{\epsilon}{7}((3)+4) \\ &= \frac{\epsilon}{7}7 \\ &= \epsilon && . \end{aligned}$$

$\therefore f \to 3$ as $x \to 2$. $\square$

b. $\lim_{x\to 1} \frac{x^2+x-2}{x-1} = 3$

Proof.
Let $f(x) = \frac{x^2+x-2}{x-1}$ and Let $\epsilon > 0$.
Set $\delta = \epsilon$.
Assume $0 < |x-1| < \delta$.
Notice $|x-1| < \epsilon$.
Then,

$$\begin{aligned} |f-3| &= |x+2-3| && \text{[Substitution/Algebra]} \\ &= |x-1| \\ &< \epsilon. \end{aligned}$$

$\therefore f \to 3$ as $x \to 1$. $\square$