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Acknowledgements:

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1. Prove if $y_n \to L$ and $x_n = my_n + b$, then $x_n \to mL + b$.

   Proof.
   Let $y_n \to L$ and $x_n = my_n + b$.
   Then $\exists N \in \mathbb{N} \ni n \geq N \implies |y_n - L| < \frac{\epsilon}{|m|} < \epsilon$
   So $n \geq N \implies |x_n - (mL + b)|$

   $$\begin{aligned}
   &= |(my_n + b) - (mL + b)| \\
   &= |my_n + b - mL - b| \\
   &= |my_n - mL| \\
   &= |m(y_n - L)| \\
   &= |m||y_n - L| \\
   &< |m|\frac{\epsilon}{m} = \epsilon.
   \end{aligned}$$

   $\therefore$ if $y_n \to L$ and $x_n = my_n + b$, then $x_n \to mL + b$ $\square$

2. Prove if $x_n = 1 - 1/n$, then x_n converges to 1 using definitions and the Archimedian Principle.

   Proof.
   Let $\epsilon > 0$. By the AP, select $N \in \mathbb{N} \ni N > \frac{1}{\epsilon}$.
   $n \geq N \implies |x_n - 1|$

   $$\begin{aligned}
   &= |1 - \frac{1}{n} - 1| \\
   &= | - \frac{1}{n}| = |\frac{1}{n}| \\
   &\leq |\frac{1}{N}| \\
   &< \epsilon \qquad \forall n \geq N.
   \end{aligned}$$

   Thus if $x_n = 1 - 1/n$, then x_n converges to 1. $\square$