9/14

Nathan Keough

Acknowledgements: *Utilized the study session: Worked independently and checked work cooperatively with other classmates on this practice set.*

---

1. Wade Exercise 1.3.0 Part a This is false:

   Proof. Proof by counterexample:
   If $A \cap B = \varnothing \implies \sup(A \cap B) = DNE$.

   $\square$

2. Wade Exercise 1.3.0 Part b

   Proof. Because $A \neq \varnothing$, and bounded above, $\exists \sup(A)$ by the Completeness Axiom.
   We must show:
   a. $\forall y \in B, y = \varepsilon \sup(A)$.
   b. $forall$ upper bounds N of B, $\varepsilon \sup(A) \leq N$.

   (a) Let $y \in B$. Then $y = \varepsilon a$ for some $a \in A$.
   We know $a \leq \sup(A)$.
   so by MP i, $\varepsilon a \leq \varepsilon \sup(A)$
   Thus $y \leq \varepsilon \sup(A) \forall y \in B$,
   $\therefore \varepsilon \sup(A)$ is an upper bound of B.

   (b) Recall $\forall$ upper bound M of A, $\sup A \leq M$.
   Also notice that by definition of supremum, $a \leq \sup A \leq M$.
   $a \leq \sup A \leq M \implies \varepsilon a \leq \varepsilon \sup A \leq \varepsilon M \implies \varepsilon a \leq \varepsilon M$
   $\implies y \leq \varepsilon M$.
   So, εM is an upper bound of B, and $\varepsilon M \subseteq N$.
   $\therefore \sup A \leq M$, thus $\varepsilon \sup A \leq N$.

   Since both a and b were shown, If $A \neq \varnothing$ and is a bounded subset of $\mathbb{R}$ and $B = \{\varepsilon x : x \in A\}$, then $\sup B = \varepsilon \sup A$

   $\square$

3. Wade Exercise 1.3.1 (no supporting work required)

   (a) $E = \{x \in \mathbb{R} : x^2 + 2x = 3\}$
   $\sup E = 1$
   $\inf E = -3$

   (b) $E = \{x \in \mathbb{R} : x^2 - 2x + 3 > x^2 \text{ and } x > 0\}$
   $\sup E = 3/2$
   $\inf E = 0$

(c) $E = \{p/q \in \mathbb{Q} : p^2 < 5q^2 \text{ and } p, q > 0\}$
$\sup E = \sqrt{5}$
$\inf E = 0$

(d) $E = \{x \in \mathbb{R} : x = 1 + -1^n/n \text{ for } n \in \mathbb{N}\}$
$\sup E = 3/2$
$\inf E = 0$

(e) $E = \{x \in \mathbb{R} : x = 1/n + -1^n \text{ for } n \in \mathbb{N}\}$
$\sup E = 3/2$
$\inf E = 0$

(f) $E = \{2 - (-1)^n/n^2 : n \in \mathbb{N}\}$
$\sup E = 3$
$\inf E = 7/4$