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Wade Exercise 2.3.6
This result is used in Section 6.3 and elsewhere.

a. Suppose that $\{x_n\}$ is a monotone increasing sequence in $\mathbb{R}$ (not necessarily bounded above). Prove that there is an extended real number x such that $x_n \to x$ as $n \to \infty$.

b. State and prove an analogous result for decreasing sequences.

Proof.

a. Let x_n be be a monotone increasing sequence.
If x_n is bounded above, then $\exists x \in \mathbb{R} \ni x_n \to x$ [MCT].
If x_n is not bounded, then given any upper bound $M > 0$, $\exists N \in \mathbb{N} \ni x_N > M$.
So, x_n is increasing and $n \geq N \implies x_n \geq x_N > M$.
Thus if x_n is increasing and not bounded above, then x_n diverges to ∞ as $n \to \infty$.

b. Likewise, $-x_n$ is a decreasing sequence.
If $-x_n$ is bounded below, then $\exists x \in \mathbb{R} \ni -x_n \to -x$ [MCT].
If $-x_n$ is not bounded, then given any lower bound $M < 0$, $\exists N \in \mathbb{N} \ni x_N < M$.
So, $-x_n$ is decreasing and $n \leq N \implies x_n \leq x_N < M$.
Thus if $-x_n$ is decreasing and not bounded below, then $-x_n$ diverges to $-\infty$ as $n \to \infty$.

$\square$