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1. Prove if $a < x < b$, then there exists M so that $|x| < M$.

   Proof.
   Let $a, x, b \in \mathbb{R}$.
   $a < x < b \implies x \in (a, b)$,
   So, set $M = \max\{|a|, |b|\} \in \mathbb{R}$.
   $M = \max\{|a|, |b|\} \implies -M < x < M = |x| < M$.
   $\therefore$ if $a < x < b$, then there exists M so that $|x| < M$. $\square$

2. Prove $f(x) = |x|$ is continuous on $\mathbb{R}$.

   Proof.
   Let $f(x) = |x|$ and $a \in \mathbb{R}$. Let $\epsilon > 0$.
   Set $\delta = \epsilon$.
   Assume $|x - a| < \delta$. Notice $|x - a| < \epsilon$.

$$\begin{aligned} \text{Then, } |f(x) - f(a)| &< ||x| - |a|| \\ &\leq |x - a| \\ &< \epsilon \end{aligned}$$

   Thus, $|x - a| < \delta \implies |f(x) - f(a)| < \epsilon$.
   $a \in \mathbb{R}$ is arbitrary, $f(x)$ is continuous at $x = a$,
   $\therefore f(x) = |x|$ is continuous on $\mathbb{R}$. $\square$