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Acknowledgements: *Utilized the study session: Worked independently and checked work cooperatively with other classmates on this practice set.*

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**Wade 2.1.1a**
$2 - \frac{1}{n} \to 2$ *as* $n \to \infty$

*Proof.*
Let $\epsilon > 0$. By the AP, $\exists N \in \mathbb{N} \ni$
$N > \frac{1}{\epsilon}$. Then $n \geq N \implies n > \frac{1}{\epsilon} \implies \frac{1}{n} < \epsilon$.
So $n \geq N \implies |2 - \frac{1}{n} - 2| = \frac{1}{n} < \epsilon$.
$\therefore 2 - \frac{1}{n} \to 2$ *as* $n \to \infty$.

$\square$

**Wade 2.1.1c**
$3\left(1 + \frac{1}{n}\right) \to 3$ *as* $n \to \infty$

*Proof.*
Let $\epsilon > 0$. By the AP, $\exists N \in \mathbb{N} \ni$
$N > \frac{3}{\epsilon}$. Then $n \geq N \implies n \geq \frac{3}{\epsilon} \implies \frac{3}{n} < \epsilon$.
So $n \geq N \implies |3\left(1 + \frac{1}{n}\right) - 3| = \frac{3}{n} < \epsilon$.
$\therefore 3\left(1 + \frac{1}{n}\right) \to 3$.

$\square$

**Wade 2.1.2a**
$1 + 2x_n \to 3$ *as* $n \to \infty$

*Proof.*
Let $\epsilon > 0$. Then since $X_n \to 1$, $\exists N \in \mathbb{N} \ni$
$n \geq N \implies |X_n - 1| < \frac{\epsilon}{2}$
Then $n \geq N \implies$

$$\begin{aligned}
&2|X_n - 1| \\
&= |2||X_n - 1| \\
&= |2(X_n - 1)| \\
&= |2X_n - 2| \\
&= |1 + 2X_n - 3| < \epsilon.
\end{aligned}$$

$\therefore 1 + 2X_n \to 3$

$\square$

**Wade 2.1.2b**
$\frac{\pi x_n - 2}{x_n} \to \pi - 2$ *as* $n \to \infty$

*Proof.*
Let $\epsilon > 0$. Then since $X_n \to 1$, $\exists N_1 \in \mathbb{N} \ni$
$n \geq N_1 \implies |X_n - 1| < \frac{\epsilon}{2X_n}$
Let $\epsilon = 0.5$ so $\exists N_2 \in \mathbb{N} \ni n \geq N_2 \implies |X_n - 1| < 0.5$
So then, $n \geq N_2 \implies X_n > 0.5$.
Now let $N = MAX\{N_1, N_2\}$.
Hence, $n \geq N \implies$

$$\begin{aligned} n &\geq N_1 \\ &\implies |X_n - 1| < \frac{\epsilon}{2X_n} \\ &\implies n \geq N_2 \\ &\implies 0 < \frac{1}{2X_n} < \frac{1}{2} \\ Then \\ &\left| \frac{\pi X_n - 2}{\pi} - \pi - 2 \right| \\ &= \frac{|2X_n - 1|}{X_n} < \epsilon \\ &\implies |X_n - 1| < \frac{\epsilon}{2X_n} \forall n \geq N. \end{aligned}$$

$\therefore \frac{\pi x_n - 2}{x_n} \to \pi - 2$

$\square$