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1.2.1 Suppose that $a, b, c \in \mathbb{R}$ and $a \leq b$.

(a) Prove that $a + c \leq b + c$.

Proof. Let $a, b, c \in \mathbb{R}$ and $a \leq b$. By the TP, $a < b$ or $a = b$.
Case 1 [Assume $a < b$]
By the AP, adding c to both sides results in

$$a + c < b + c \tag{1}$$

Case 2 [Assume $a = b$]
By the AP, adding c to both sides results in

$$a + c = b + c \tag{2}$$

In either case, $a + c < b + c$ or $a + c = b + c$
$\therefore a + c \leq b + c$. □

(b) If $c \geq 0$, prove that $a \cdot c \leq b \cdot c$.

Proof. Let $a, b, c \in \mathbb{R}$ and $a \leq b$. By the TP, $a < b$ or $a = b$.
Case 1 [Assume $a < b$]

Now,by TP, $c > 0$ or $c = 0$.

Case i. [Assume $c > 0$]
By MP1, multiplying both sides by $c > 0$ gives us
$a \cdot c < b \cdot c$
Case ii.[Assume $c = 0$]
By MP, multiplying both sides by $c = 0$ gives us
$a \cdot c < b \cdot c$
$a \cdot 0 < b \cdot 0$
$= (0 = 0)$

Case 2 [Assume a = b]

Now,by TP, $c > 0$ or $c = 0$.

Case i. [Assume $c > 0$]
By MP, multiplying both sides by $c > 0$ gives us
$a \cdot c = b \cdot c$
Case ii.[Assume $c = 0$]
By MP, multiplying both sides by $c = 0$ gives us
$a \cdot c = b \cdot c$
$a \cdot 0 = b \cdot 0$
$= (0 = 0)$

In both cases $a < b$ and $a = b$ with $c = 0$, we get $a \cdot c = b \cdot c$.
In both cases $a < b$ and $a = b$ with $c > 0$, we get $a \cdot c < b \cdot c$.
$\therefore a \cdot c \leq b \cdot c$

$\square$