# PS 9/7

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Acknowledgements: *Utilized the study session: Worked independently and checked work cooperatively with other classmates on this practice set.*

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1. Find all $n \in \mathbb{N}$ so that $\frac{1-n}{1-n^2} < 0.1$.

$$\frac{1-n}{1-n^2} < 0.1$$

$$\begin{aligned}
&= 1-n > \frac{1-n^2}{10} && \text{(MP ii.)}\\
&= 10-10n > 1-n^2 && \text{(MP i.)}\\
&= n^2-10n+9 > 0 && \text{(AP)}\\
&= (n-1)(n-9) > 0 && \text{(Factoring)}\\
&= n > 9, n > 1 && \text{Inequality and}\\
& && \text{Natuarality not satisfied until n} > 9\\
&= \text{So } n > 9 && \{n \in \mathbb{N} | n > 9\}
\end{aligned}$$

2. Prove if $-1 \leq x \leq 2$, then $|x^2+x-2| \leq 4|x-1|$.

*Proof.* Assume $-1 \leq x \leq 2$. Then,

$$\begin{aligned}
&-1 \leq x \leq 2 && (\textit{Given})\\
&\implies -1+2 \leq x+2 \leq 2+2 && \text{(AP)}\\
&= 1 \leq x+2 \leq 4 && \\
&\implies -4 \leq x+2 \leq 4 && \text{(Since -4 < 1)}\\
&\implies |x+2| \leq 4 && \text{(FTAV)}\\
&\implies |x+2||x-1| \leq 4|x-1| && \text{(MP i.)}\\
&= |x^2-x-2| \leq 4|x-1| && \text{(REM 1.5)}
\end{aligned}$$

So $-1 \leq x \leq 2 \implies |x^2+x-2| \leq 4|x-1|$ as desired.

$\therefore$ if $-1 \leq x \leq 2$, then $|x^2+x-2| \leq 4|x-1|$. □

3. Prove $0 \leq a < b$ implies $a^2 < b^2$ and $\sqrt{a} < \sqrt{b}$.

*Proof.* Assume $0 \leq a < b$.
(Showing $0 \leq a < b \implies a^2 < b^2$):
By contradiction: Assume $0 \leq a < b$ and $a^2 \geq b^2$.

$$\begin{aligned}
&\text{Then, } a^2 \geq b^2\\
&\implies \sqrt{a^2} \geq \sqrt{b^2}\\
&= a \geq b
\end{aligned}$$

$a \geq b$ contradicts our assumption.
$\therefore 0 \leq a < b \implies a^2 < b^2$
(Showing $0 \leq a < b \implies \sqrt{a} < \sqrt{b}$):
By contradiction: Assume $0 \leq a < b$ <u>and</u> $\sqrt{a} \geq \sqrt{b}$.

$$\text{Then, } \sqrt{a} \geq \sqrt{b}$$
$$\implies \sqrt{a}^2 \geq \sqrt{b}^2$$
$$= a \geq b$$

$a \geq b$ contradicts our assumption.
$\therefore 0 \leq a < b \implies \sqrt{a} < \sqrt{b}$
Notice, $0 \leq a < b \implies a^2 < b^2$
and $0 \leq a < b \implies \sqrt{a} < \sqrt{b}$.
$\therefore 0 \leq a < b$ implies $a^2 < b^2$ and $\sqrt{a} < \sqrt{b}$ $\square$