ℝose-Hulman Undergraduate Mathematics Journal
VOLUME ∞, ISSUE π, 2112

An Approach to Computing Magic Squares Using High-Performance Computing and Group Theory

By *Nathan H. Keough**

Abstract. This paper introduces a novel algorithm for computing magic squares, exploiting group theory concepts such as permutation representation, group operations, and group actions to encode symmetries. By defining the group operation as composition, the set as a subset of the group of magic squares in a specific order, we may systematically explore the permutations of the group and extrapolate information about the magic squares to generate new magic squares not in the originating set. This vastly reduces computation times for enumerating the solutions to magic squares, while also encoding the symmetries in a manner that is easier to analyze programmatically. Overall, this study reveals the profound connection between magic squares and group theory, offering promising avenues for symmetry-driven algorithms and applications in combinatorial mathematics.

1 Introduction

1.1 Something Subsection

Ideally, the introduction should introduce the background of the problem at hand and motivate why mathematicians (and/or others) are interested in studying it. This should be aimed at the level of undergraduates who are not experts in your area. We'd also like the introduction to contain a road map for the article; i.e., a description of what to expect in each section.

2 The title of the second section

The class file contains definitions for several environments (e.g., `thm`, `corollary`, `lemma`, etc). Examples follow below.

Important definitions should be set in the `defin` environment.

Definition 2.1. Let $p \in \mathbf{N}$ and suppose the following hold:

*Thanks to all who encouraged and supported me.
Mathematics Subject Classification. 68V99
Keywords. magic squares, computational group theory, rust

P1. If $d \in \mathbf{N}$ such that $d \mid p$, then $d = 1$ or $d = p$.

P2. $p \neq 1$.

Then we say that p is a *prime number*.

Use the `label` command for reference to these environments later on in your document. The class file uses the `cleveref` package which determines the type of label being referenced:

	LaTeX code	Result
Using `ref`:	`\textbf{definition \ref{prim-def}}`	**definition 2.1**
Using `cref`:	`\cref{prim-def}`	**definition 2.1**

Lemma 2.2. *If $n \in \mathbf{N}$, then there exists a prime number p such that $p \mid n$.*

Proof. Suppose, for contradiction, that there exists $n \in \mathbb{N}$ such that $n > 1$ and n is not divisible by any prime. Let

$$\mathrm{S} = \{n \in \mathbf{N} : n > 1,\ n \text{ is not divisible by a prime}\}.$$

Then S is a non-empty subset of $\mathbf{N}$. The well-ordering principle[1] fashions a least element of S, say m. Note that m is not prime (otherwise it would be divisible by a prime, namely itself). Since m is not prime, there exists $a \in \mathbf{N}$ such that $a \mid m$ and $a \neq 1$ and $a \neq m$. Let $b \in \mathbf{Z}$ such that $ab = m$. Note that $b \in \mathbf{N}$ since $a, m \in \mathbf{N}$. What's more, $b > 1$ since otherwise $a = m$. So it must be that $1 < a, b < m$. Since divisibility is transitive, it follows that neither a nor b is divisible by a prime. But then $a \in \mathrm{S}$ (and so is b) and $a = m/b < m$ contradicting the minimality of m. Thus it must be that S is empty which proves the lemma. □

Theorem 2.3 (Euclid)**.** *There are infinitely many prime numbers.*

Proof. Let $p_1, p_2, \ldots, p_k$ be primes, and let

$$n = p_1 p_2 \cdots p_k + 1.$$

By **lemma 2.2**, there exists a prime p such that $p \mid n$. Suppose, for contradiction, that $p = p_i$ for some $i = 1, 2, \ldots, k$. It follows that $p \mid n - p_1 p_2 \cdots p_k = 1$, but this is absurd. So it must be that $p \neq p_i$ for any $i = 1, 2, \ldots k$. This shows that a new prime can be constructed from any finite list of primes. Hence the number of primes is not finite. □

Use the `cite` command to cite references. For example, the above proof can be found in Euclid's Elements [1]. See below for how to add citations to the `thebibliography` environment.

[1] An excellent principle.

3 A title for the third section

There is an `example` environment.

Example 3.1. The geometric series

$$\sum_{k=0}^{\infty} \frac{1}{2^k}$$

converges to 2.

When starting a sentence with a reference, use the `Cref` command. **Example 3.1** can be used to prove that 2 is not the only prime.

Theorem 3.2. *The set of primes includes more numbers than just* 2*.*

Proof. If 2 were the only prime number, then the fundamental theorem of arithmetic would give us that

$$\sum_{n=1}^{\infty} \frac{1}{n} = \sum_{k=0}^{\infty} \frac{1}{2^k}.$$

The series on the left diverges whereas the series on the right converges. This is impossible, so there must be more prime numbers beyond the number 2. □

There is a `remark` environment as well.

Remark 3.3. The idea in **theorem 3.2** can be extended to show that the set of primes includes more numbers than just 2 and 3. In fact, Euler went on to prove that if the number of primes were finite, then the harmonic series would converge.

4 A title for the fourth section

There are `prop` and `corollary` environments as well.

Proposition 4.1. *Every natural number can be written uniquely as a product of a square-free number and a square.*

As a corollary to **proposition 4.1** we obtain

Corollary 4.2. *Let* $\pi(n)$ *denote the number of primes less than or equal to* n*. Then*

$$\pi(n) \geq \frac{\log n}{2\log 2}.$$

Proof. There are no more than $2^{\pi(n)}$ square free numbers less than n. Also, there are no more than $\sqrt{n}$ squares less than n. It follows from **proposition 4.1** that

$$n \leq 2^{\pi(n)}\sqrt{n}.$$

The corollary follows by applying log. □

For an unnumbered remark, use the `xrem` environment.

Remark. Note that **corollary 4.2** implies that there are infinitely many primes.

References

[1] Euclid, *Euclid's Elements*, the Thomas L. Heath translation, Green Lion Press, Santa Fe, NM, 2002. MR1932864

[2] Euler, *Foundations of differential calculus*, translated from the Latin by John D. Blanton, Springer-Verlag, New York, 2000. MR1753095

[3] P. G. L. Dirichlet, Gedächtnißrede auf Carl Gustav Jacob Jacobi, in *Nachrufe auf Berliner Mathematiker des 19. Jahrhunderts*, 6–34, Teubner-Arch. Math., 10, Teubner, Leipzig. MR1104895

Nathan H. Keough
Maryville College
nathan.keough@my.maryvillecollege.edu