Replicating John Philip 1972

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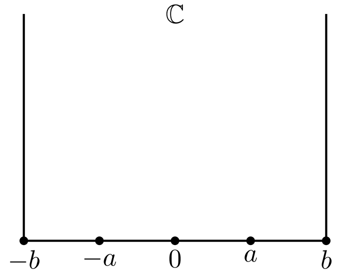
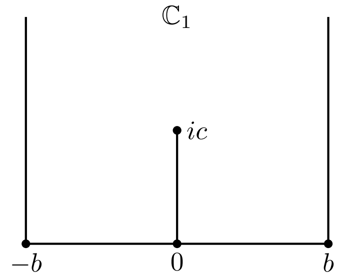
Rough working as we attempt to replicate the double Schwarz-Christoffel transformations in J. R. Philip's 1972 paper in ZAMP.

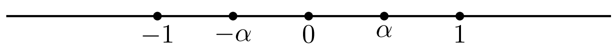
We map:



to



presumably via the upper half plane $\mathbb{H}$:



where $\alpha = \dfrac{a}{b}$

The Schwarz-Christoffel mapping from $\mathbb{H}$ to $\mathbb{C}_1$ should be given by the indefinite integral:

$$z = \int \frac{w}{\sqrt{w^2-1}\sqrt{w^2-\alpha^2}} \tag{1}$$

which according to Wolfram's online Integrator, has solution:

$$z = \ln\left(2\sqrt{w^2-1} + 2\sqrt{w^2-\alpha^2}\right) \tag{2}$$

The factor of 2 can be pulled out into C:

$$z = K\ln\left(\sqrt{w^2-1} + \sqrt{w^2-\alpha^2}\right) + C \tag{3}$$

Let us check:

$$\begin{aligned}
\frac{d}{dw}\ln\left(\sqrt{w^2-1} + \sqrt{w^2-\alpha^2}\right) &= \frac{\frac{d}{dw}\left(\sqrt{w^2-1} + \sqrt{w^2-\alpha^2}\right)}{\sqrt{w^2-1} + \sqrt{w^2-\alpha^2}} \\
&= \frac{\frac{2w}{2\sqrt{w^2-1}} + \frac{2w}{2\sqrt{w^2-\alpha^2}}}{\sqrt{w^2-1} + \sqrt{w^2-\alpha^2}} = \frac{w\left(\frac{1}{\sqrt{w^2-1}} + \frac{1}{\sqrt{w^2-\alpha^2}}\right)}{\sqrt{w^2-1} + \sqrt{w^2-\alpha^2}} \\
&= \frac{w\left(\frac{\sqrt{w^2-1}+\sqrt{w^2-\alpha^2}}{\sqrt{w^2-1}\sqrt{w^2-\alpha^2}}\right)}{\sqrt{w^2-1} + \sqrt{w^2-\alpha^2}} = \frac{w}{\sqrt{w^2-1}\sqrt{w^2-\alpha^2}} \quad \text{tick!}
\end{aligned}$$