Perturbation Height

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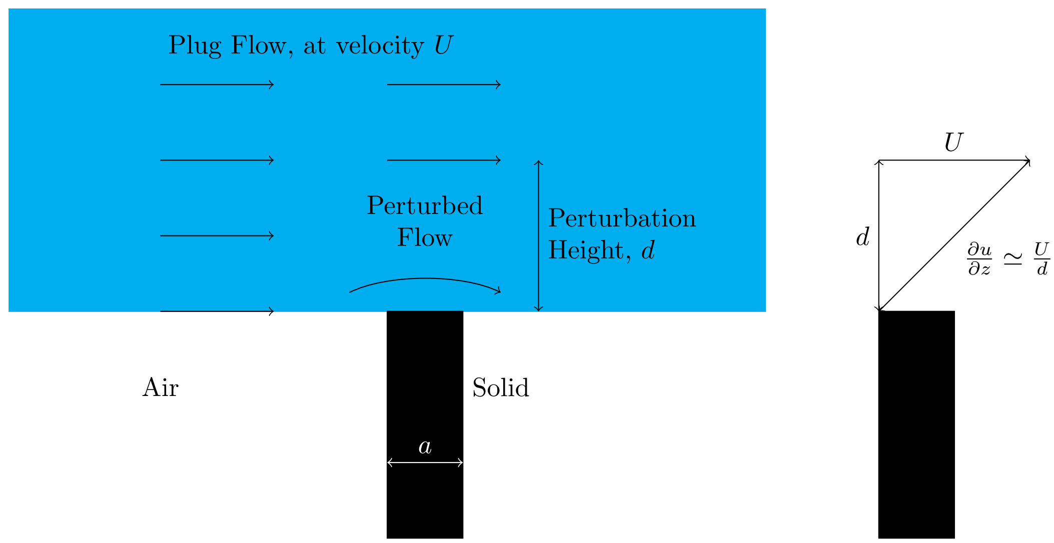
Observe:



What is perturbation height d?
It will be a function of some fundamental physical parameters:

- Ridge width a
- Velocity U
- Viscosity η
- Density ρ
- Pressure p

In other words $d = g(a, U, \eta, \rho, p)$. Or equivalently, $f(d, a, U, \eta, \rho, p) = 0$.
Dimensionally,

$$d = [m], \quad a = [m], \quad U = [ms^{-1}], \quad \eta = [kgs^{-1}m^{-1}], \quad \rho = [kgm^{-3}], \quad p = [kgm^{-1}s^{-2}]$$

$$d = [m], \quad a = [m], \quad U = \left[\frac{m}{s}\right], \quad \eta = \left[\frac{kg}{ms}\right], \quad \rho = \left[\frac{kg}{m^3}\right], \quad p = \left[\frac{kg}{ms^2}\right]$$

Suggestion of Graeme Weir

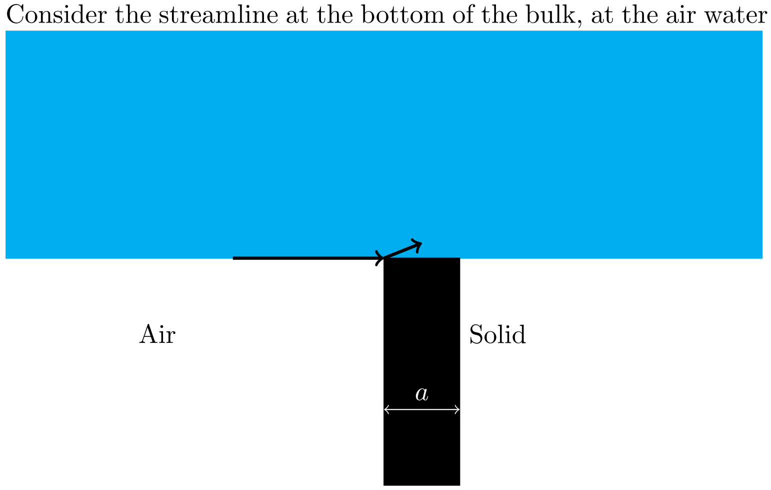
Graeme bypasses the use high-level dimensional analysis, and suggests to solve the problem directly, like so:

Recall Bernoulli's theorem: For a streamline, the sum of static pressure and dynamic pressure remain constant.

$$p + \frac{1}{2}\rho v^2 = P_0$$

Therefore, a change in pressure in the streamline can be *at most* $\Delta p = \frac{1}{2}\rho v^2$

Consider the streamline at the bottom of the bulk, at the air water interface:



Where the streamline encounters the solid post, the velocity will abruptly drop, and there will be a corresponding change in pressure.

The pressure change will be *of the order* $\Delta p \sim \rho v^2$

The fluid is in a state of 'creeping flow', obeying the Stokes equations: for each element of fluid, the pressure gradient balances viscous drag:

$$\mu \nabla^2 \vec{u} = \nabla p$$

For plug flow, only x velocity is significant and interesting. $\vec{u} \simeq (u, 0, 0)$ so that only one equation matters:

$$\mu \nabla^2 u = \nabla p$$

Take divergence of both sides:

$$\nabla \cdot \mu \nabla^2 \vec{u} = \nabla \cdot \nabla p$$