Effective slip lengths for Stokes flow over rough, mixed-slip surfaces

PhD Defense Presentation

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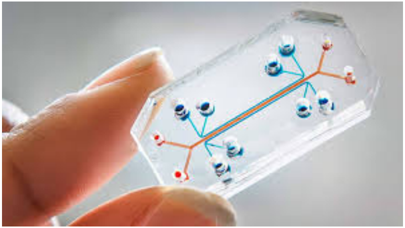
- Elevator Speech
- Regimes of Applicability – can we extend?
- Limitations of Homogenization
- What Next?
- FEM mesh and streamline plots

Motivation

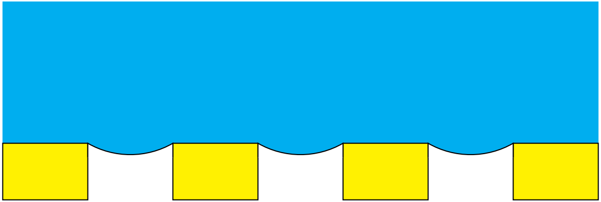
Lab-on-a-chip



Very small pipe: friction dominates

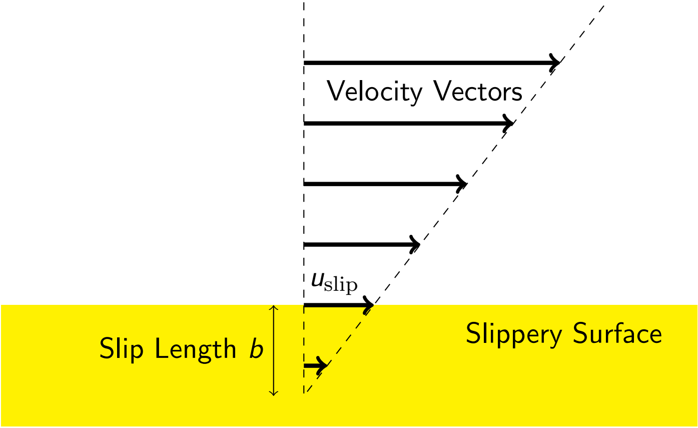
How to reduce friction?

Slippery Surfaces



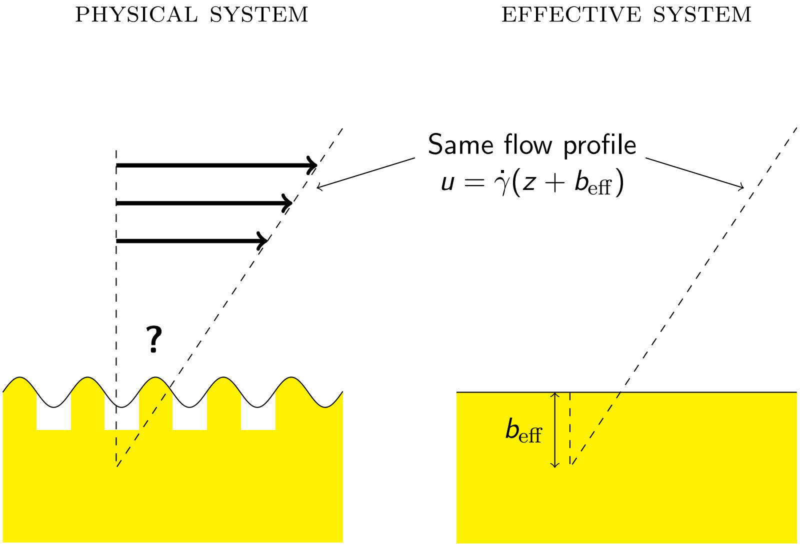
- Holes on the wall of the pipe
- Air bubbles trapped in holes
- Water slips over top of air bubble
- Friction reduced: How much?

Slip Length

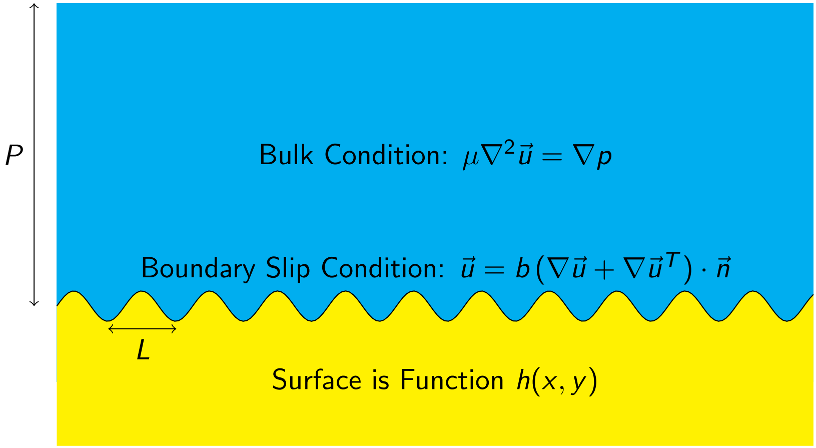


Single parameter to express the friction of a surface.

Effective Slip Length

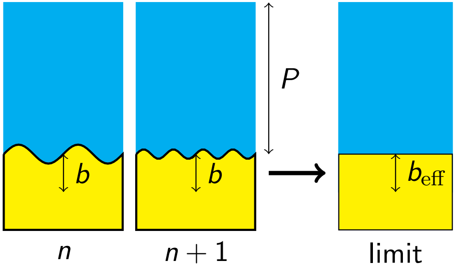


Mathematical Model



Solving

Homogenization:
Thought experiment about what happens when the period L becomes infinitely small.

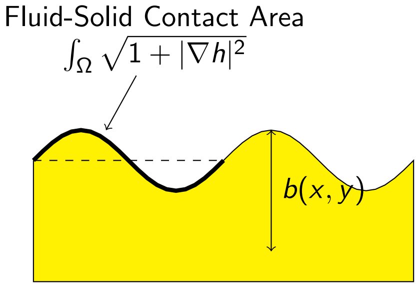


Perturbation:
Think of system as 'perturbed' slightly away from a well-known system (with known solution).

Homogenized Solution

$$b_{\mathrm{eff}} = \left\langle \frac{\sqrt{1 + |\nabla h|^2}}{b} \right\rangle^{-1} \tag{1}$$

Harmonic mean of intrinsic slip lengths, weighted by area of contact:



Perturbative Solutions

Replicates homogenized solution for special case of flat surface:

$$b_{\mathrm{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1} \tag{2}$$

Where b is much smaller than other length scales, b_{eff} is simple average:

$$b_{\mathrm{eff}} = \langle b \rangle \tag{3}$$

Regimes of Applicability

Harmonic mean b_{eff} is excellent approximation when period L is much smaller than other lengths, slip lengths b and domain size P.

$$b_{\text{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1} \quad \text{when} \quad L \ll b, P \tag{4}$$

(Still good approximation even if $L \sim b \ll P$.)

Simple mean is good approximation when slip lengths b much smaller than other lengths:

$$b_{\text{eff}} = \langle b \rangle \quad \text{when} \quad b \ll L, P \tag{5}$$

Regimes and Results

DERIVED RESULTS

	2-D Flow (1-D surface pattern)	3-D Flow (2-D surface pattern)	
No-slip/ Perfect-slip Binary Surface	J. R. Philip 1972 Lauga and Stone 2003		FLAT SURFACE
Other Surface	Hendy and Lund 2007	Lund and Hendy 2008	FLAT SURFACE
No-slip/ Perfect-slip Binary Surface	Sbragaglia and Prosperetti 2007 Davis and Lauga 2009a		ROUGH SURFACE
Other Surface	Einzel, Panzer, Liu 1990 Lund *et al* 2012	This thesis?	ROUGH SURFACE

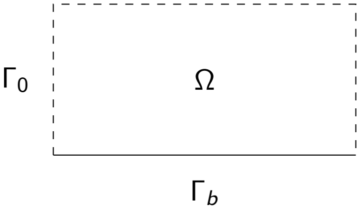
Can we homogenize No-slip/Perfect-slip cases?

No exact results for case of rough surface with $b = 0$ and $b \sim L$.

Can we apply homogenization?

No.

Limitations of Homogenization 1



$$\nabla^2 u = f \quad \text{on } \Omega$$

$$u = b\frac{\partial u}{\partial n} \quad \text{on } \Gamma_b$$

Multiply by test function g and integrate over Ω:

$$\int_\Omega g\nabla^2 u = \int_\Omega gf \tag{6}$$

Use vector identity and divergence theorem to get:

$$\int_\Gamma g\frac{\partial u}{\partial n} - \int_\Omega \nabla u \cdot \nabla g = \int_\Omega gf \tag{7}$$

Limitations of Homogenization 2

The slip condition on Γ_b implies:

$$\frac{\partial u}{\partial n} = \frac{1}{b} u \tag{8}$$

Substitute this, to get variational form:

$$\int_{\Gamma_b} g \frac{1}{b} u - \int_{\Omega} \nabla u \cdot \nabla g = \int_{\Omega} g f \tag{9}$$

$\therefore$ Require b in form $\dfrac{1}{b}$.

If $b = 0$ anywhere, $\frac{1}{b}$ is undefined. Cannot homogenize.

Another Subtlety

Mathematically, expect that $b_{\mathrm{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1}$ is a good approximation when L smaller than other length scales: $L \ll b, P$.

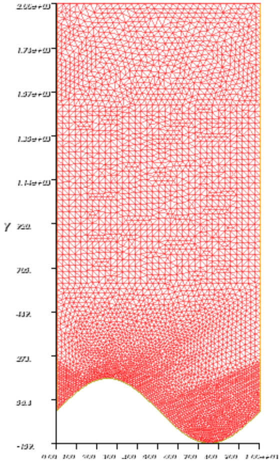
But it is still a good approximation when $L \sim b \ll P$.

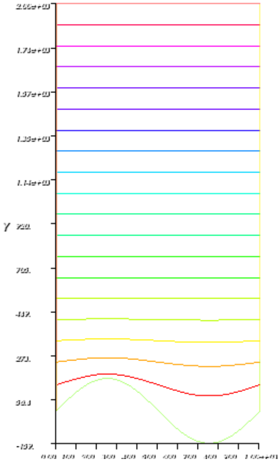
Why?

What Next?

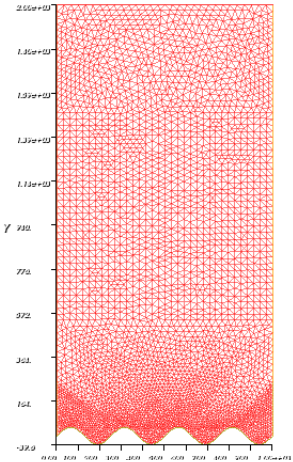
- Learn FEM. Natural step after having learned Variational Formulation.
- Apply homogenization to multiscale slip systems.
- Apply homogenization to other physical systems.

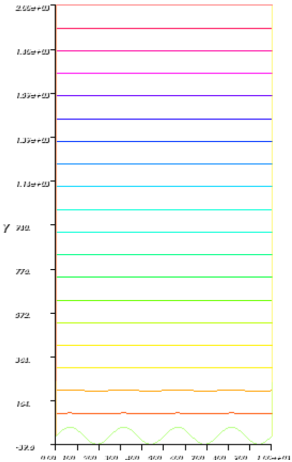
FEM mesh and streamlines: 1 period



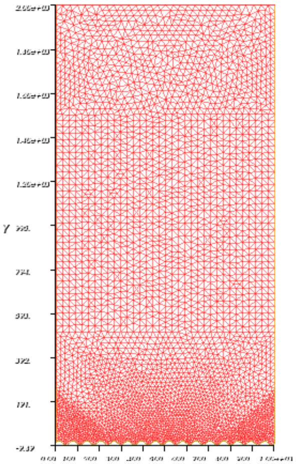


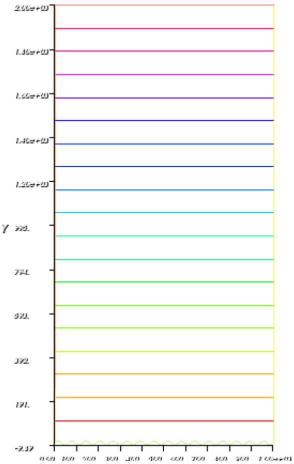
FEM mesh and streamlines: 4 periods





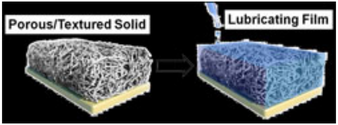
FEM mesh and streamlines: 16 periods

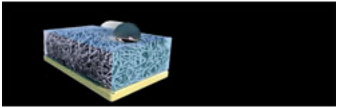




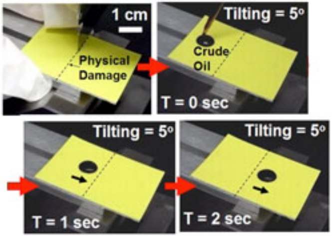
Superhydrophobic Surfaces Superseded?

"[Harvard's] SLIPS technology combines a lubricated film on a porous solid..."





"...to create low-cost surfaces that exhibit ultra-liquid repellency, self-healing, optical transparency, pressure stability and self-cleaning."



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