Dimensional Analysis of Perturbation Height in Incompressible Stokes Flow

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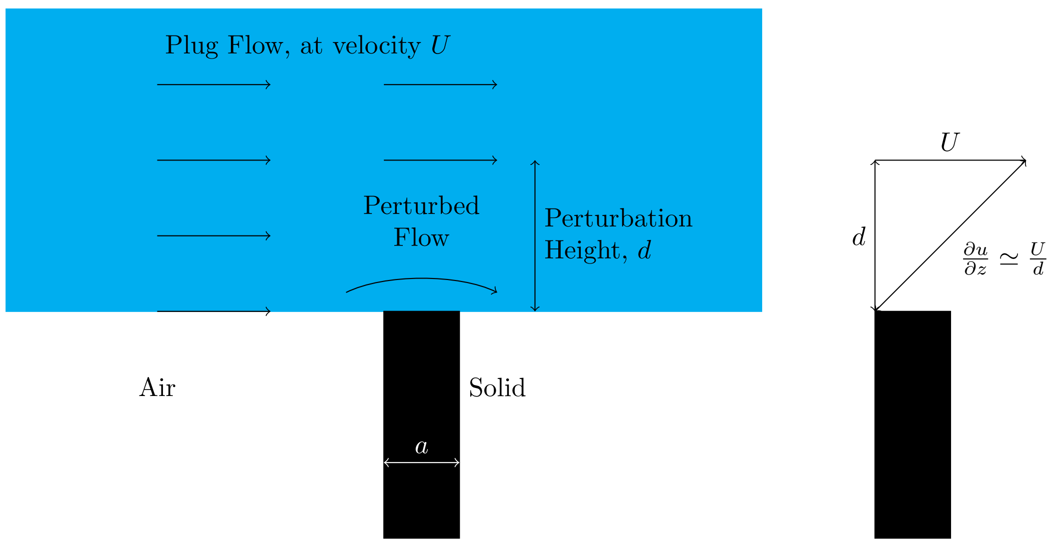
Observe:



What is perturbation height d?
It will be a function of some fundamental physical parameters:

- Ridge width a
- Velocity U
- Viscosity η
- Density ρ
- Pressure p

In other words $d = g(a, U, \eta, \rho, p)$. Or equivalently, $f(d, a, U, \eta, \rho, p) = 0$.

Dimensionally,

$$d = [m], \ \ a = [m], \ \ U = [ms^{-1}], \ \ \eta = [kgs^{-1}m^{-1}], \ \ \rho = [kgm^{-3}], \quad p = [kgm^{-1}s^{-2}]$$

$$d = [m], \quad a = [m], \quad U = \left[\frac{m}{s}\right], \quad \eta = \left[\frac{kg}{ms}\right], \quad \rho = \left[\frac{kg}{m^3}\right], \quad p = \left[\frac{kg}{ms^2}\right]$$

We also know that the fluid is incompressible, creeping flow. As such, it obeys the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0$$

And Stokes equations:

$$\mu \nabla^2 u = \frac{\partial p}{\partial x} \quad \text{and} \quad \mu \nabla^2 v = \frac{\partial p}{\partial z}$$

These equations put a constraint on the relationships between the physical variables...