# Effective slip lengths for Stokes flow over rough, mixed-slip surfaces

## PhD Defense Presentation

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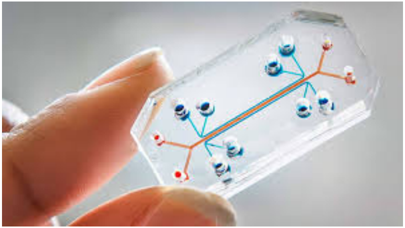
- Elevator speech
- Regimes of Applicability, and What Next?
- Limitations of Homogenization

# Motivation

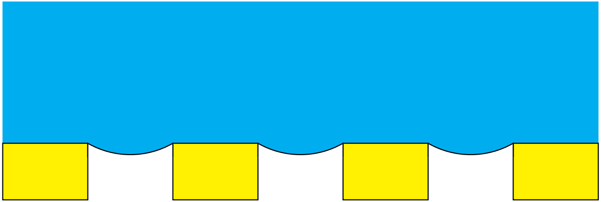
Lab-on-a-chip



Very small pipe: friction dominates

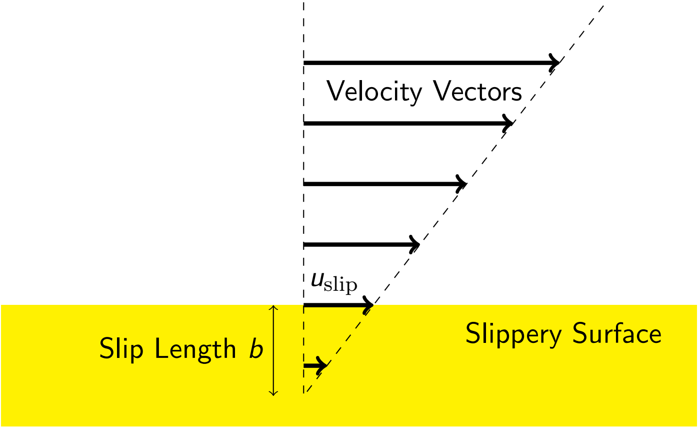
How to reduce friction?

# Slippery Surfaces



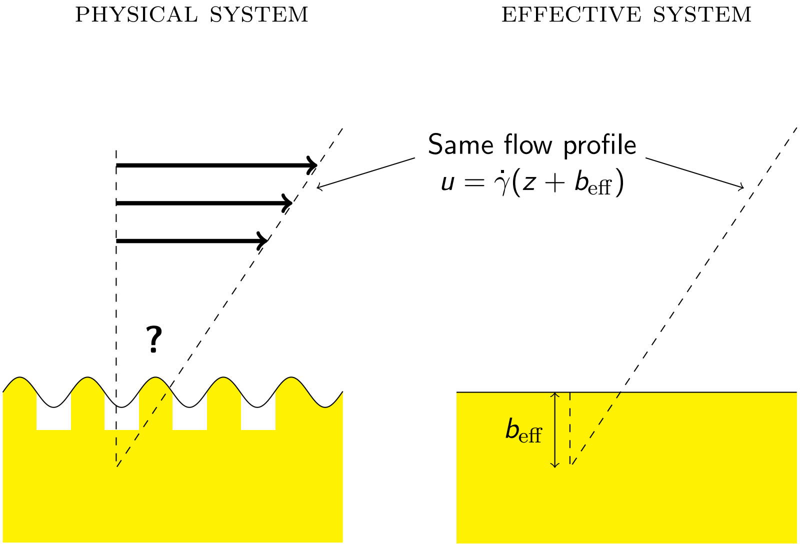
- Holes on the wall of the pipe
- Air bubbles trapped in holes
- Water slips over top of air bubble
- Friction reduced: How much?

# Slip Length

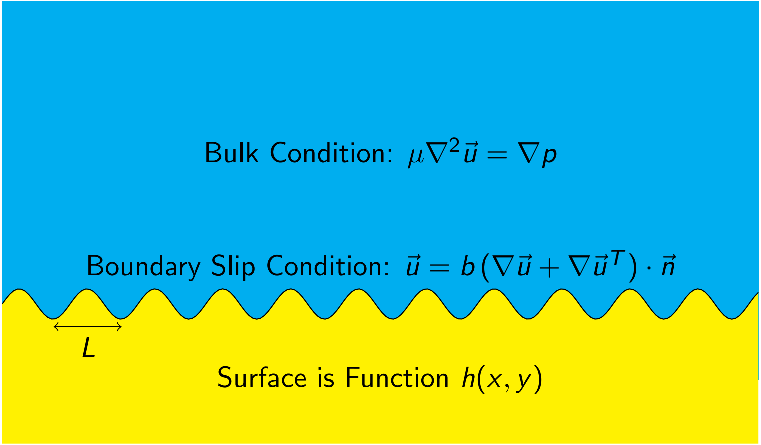



Single parameter to express the friction of a surface.

# Effective Slip Length

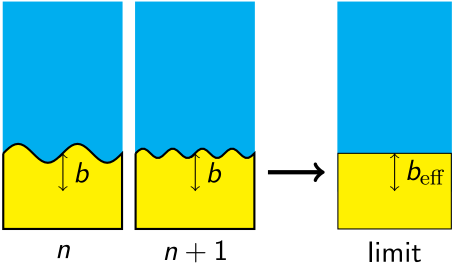


# Mathematical Model



# Solving

**Homogenization:**
Thought experiment about what happens when the period $L$ becomes infinitely small.

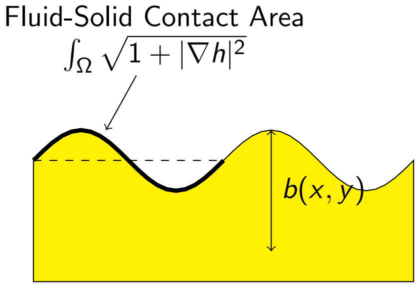



**Perturbation:**
Think of system as ‘perturbed’ slightly away from a well-known system (with known solution).

# Homogenized Solution

$$b_{\text{eff}} = \left\langle \frac{\sqrt{1 + |\nabla h|^2}}{b} \right\rangle^{-1} \tag{1}$$

Harmonic mean of intrinsic slip lengths, weighted by area of contact:



# Perturbative Solutions

Replicates homogenized solution for special case of flat surface:

$$b_{\mathrm{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1} \tag{2}$$

Where $b$ is much smaller than other length scales, $b_{\mathrm{eff}}$ is simple average:

$$b_{\mathrm{eff}} = \langle b \rangle \tag{3}$$

# Regimes of Applicability

Harmonic mean $b_{\text{eff}}$ is excellent approximation when period $L$ is much smaller than other lengths, slip lengths $b$ and domain size $P$.

$$b_{\text{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1} \quad \text{when} \quad L \ll b, P \tag{4}$$

(Still good approximation even if $L \sim b \ll P$.)

Simple mean good approximation when slip lengths $b$ much smaller than other lengths:

$$b_{\text{eff}} = \langle b \rangle \quad \text{when} \quad b \ll L, P \tag{5}$$

# Regimes and Results

DERIVED RESULTS

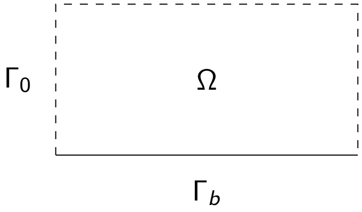
| | **2-D Flow**<br>(1-D surface pattern) | **3-D Flow**<br>(2-D surface pattern) | |
|---|---|---|---|
| **No-slip/ Perfect-slip Binary Surface** | J. R. Philip 1972<br>Lauga and Stone 2003 | | FLAT SURFACE |
| **Other Surface** | Hendy and Lund 2007 | Lund and Hendy 2008 | |
| **No-slip/ Perfect-slip Binary Surface** | Sbragaglia and Prosperetti 2007<br>Davis and Lauga 2009a | | ROUGH SURFACE |
| **Other Surface** | Einzel, Panzer, Liu 1990<br>Lund *et al* 2012 | This thesis? | |

# What next?

No exact results for case of rough surface with $b = 0$ and $b \sim L$.

Can we apply homogenization?
No.

# Limitations of Homogenization 1




$$\nabla^2 u = f \quad \text{on } \Omega$$

$$u = b\frac{\partial u}{\partial n} \quad \text{on } \Gamma_b$$

Multiply by test function $g$ and integrate over $\Omega$:

$$\int_\Omega g \nabla^2 u = \int_\Omega gf \tag{6}$$

Use vector identity and divergence theorem to get:

$$\int_\Gamma g \frac{\partial u}{\partial n} - \int_\Omega \nabla u \cdot \nabla g = \int_\Omega gf \tag{7}$$

# Limitations of Homogenization 2

The slip condition on $\Gamma_b$ implies:

$$\frac{\partial u}{\partial n} = \frac{1}{b} u \tag{8}$$

Substitute this, to get variational form:

$$\int_{\Gamma_b} g \frac{1}{b} u - \int_{\Omega} \nabla u \cdot \nabla g = \int_{\Omega} gf \tag{9}$$

$\therefore$ Require $b$ in form $\dfrac{1}{b}$.

If $b = 0$ anywhere, $\frac{1}{b}$ is undefined.