Chapter 3: Mixed Slip Flow

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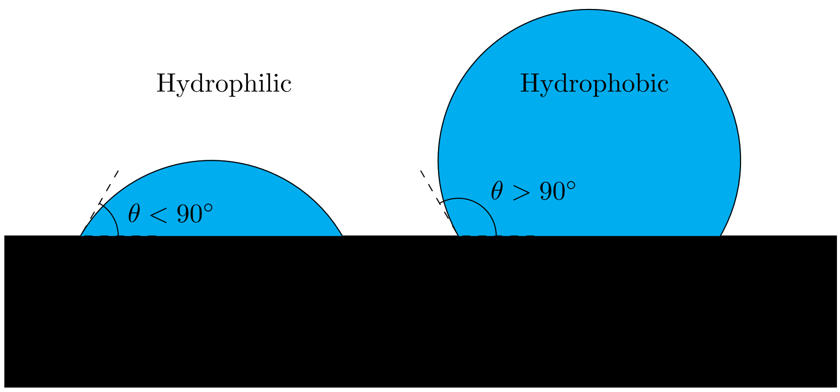
If water flows over a surface covered with air pockets, then the water experiences a boundary that is partly ~~fresh air~~ and should experience reduced drag. That is, the air-water interface should exhibit a large slip length, and such air-pocketed surface should have a larger effective slip length than the unadorned surface. It is tempting to accept this as self-evident, and immediately proceed to finding an effective slip length in terms of the intrinsic slip lengths. However, there exist experimental results in which such a surface has a *reduced* effective slip compared to the pure surface. Furthermore, there are claims that roughness both increases *and* decreases slip. The paradox in these findings is ultimately resolved by taking appropriate care in defining the location of the $z = 0$ plane. Since this a key part of any mathematical model, we shall spend this chapter looking at the nuances of this issue. We trace an approximate historical development of results for flow over heterogeneous surfaces: that is, surfaces that are rough, or mixed-slip surfaces, whose intrinsic slip length varies across the surface.

0.1 Mixed Slip Surfaces

While there could in principle be an infinite variety of mixed-slip surfaces, only two broad classes are important. This is because physical mixed-slip surfaces almost always feature a liquid-*air* interface, along with the the more familiar liquid-solid interface. And there are two canonical configurations: superhydrophobic, and nanobubbles.

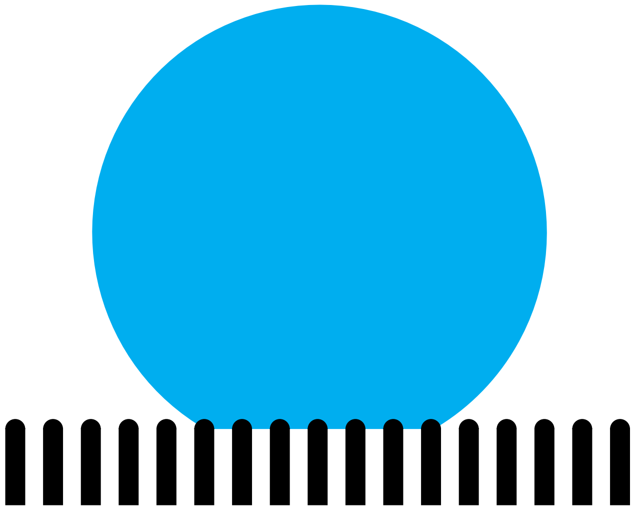
0.1.1 Superhydrophobic Surfaces

Recall that when a droplet of water sits on a surface, a contact angle θ is defined:

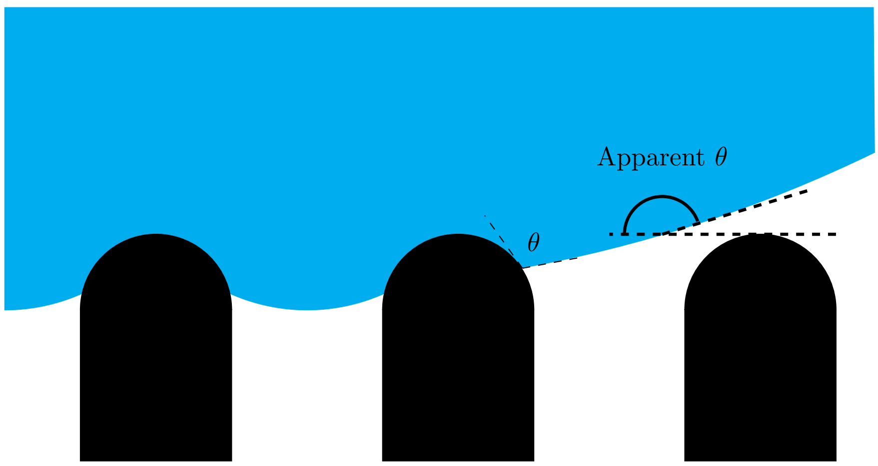


Usually, a surface is defined as *hydrophobic* when the contact angle is more than 90°. ~~This means that the water molecules are more attracted to each other than they are to the surface.~~ (If $\theta < 90°$, the surface is *hydrophilic*. If $\theta \sim 0°$, then *complete wetting* occurs: the water spreads out as far as it can.)

Tiny, micron or nanometer scale pillars can be constructed out of hydrophobic material. A collection of theses hydrophobic nanopillars can be affixed to a suitable substrate, forming a 'nanoforest'. (Or, more practically, a nanoforest can be constructed, then chemically treated to become hydrophobic.) If a water droplet is placed on top of the nanoforest, two curious things happen: First, the droplet sits on the tops of the nanopillars, supported by surface tension.



Second, the apparent contact angle is very large. This second effect is caused by the fact that at the point where the contact line is located, the solid surface is at an angle to the plane of the nanoforest.



Due to the extremely high contact angle, these nano or micro-structured surfaces are known as *superhydrophobic* surfaces.

Such surfaces were constructed as early as 1996. Onda *et al* [14] discussed the theoretical contact angle of such a surface, and demonstrated a "super-water-repellent fractal surface" made of alkylketene dimer, with a remarkable contact angle of 174°.

Enormous contact angles are routinely quoted for static droplets. The contact angle is slightly different if the droplet is advancing (or retreating). This hysteresis was studied, for example, by Kusumaatmaja and Yeomans in 2007 [10].

But perhaps more interestingly, superhydrophobic surfaces were first observed in nature. The sacred lotus is an aquatic plant (not a water lily, but similar) whose water-repellent qualities have been noted since antiquity. A passage in the Baghavad Gita states "One who performs his duty without attachment, ... is unaffected by sinful action, as the lotus is unaffected by water." In 1993, Barthlott and Neinhuis were taking scanning electron micrographs of the leaf surfaces of some 10,000 plant species. They noticed that *flat* surfaces always had to be cleaned before examination, while certain rough waxy surfaces did not. They characterised these self-cleaning surfaces as covered with wax crystalloids "in a regular microrelief of about 1 - 5 μm" – i.e. superhydrophobic. They describe the cleaning mechanism: Water beads into near-spherical droplets, which easily roll off the leaf. Dirt particles tend to be hydrophilic, and only weakly bound to the tops of the roughness. Thus the dirt particles are captured by the water droplets, and move with them off the leaf. More from antiquity: a Confucian scholar wrote "I love the lotus, because while growing in mud, it is unstained." In a pair of papers in 1997 [1, 13], Barthlott and Neinhuis describe their studies of what they dub the 'lotus effect'.

The image of a droplet supported by thin spikes inspires another metaphor: a Fakir (malnourished Yoga practitioner) sitting on a bed of nails. David Quéré's article 'Fakir droplets' gives a very readable summary of the state of affairs in 2002 [15]. The quote of relevance to this thesis is the last few sentences of the article: "On a superhydrophobic solid, however, drops seem to move over a dynamic film of air — which makes the friction comparable to that experienced by a raindrop falling in air. But what happens if these textured solids are fully immersed in a pool of water? Will the water still slide on them? Except for a few controversial studies, this question still remains open, and designers of boats and swimsuits impatiently await an answer."

This question heads a line of research leading to this thesis.

0.1.2 Nanobubbles

In 2001, Tyrrell and Attard discovered what appeared to be nanobubbles on hydrophobic surfaces [17]. An extract from the abstract of their PRL paper says it all: "Imaging of hydrophobic surfaces in water with tapping mode atomic force microscopy reveals them to be covered with soft domains, apparently nanobubbles, that are close packed and irregular in cross section, have a radius of curvature of the order of 100 nm, and a height above the substrate of 20 – 30 nm."

It had been observed that when two hydrophobic bodies were brought together underwater, at some very close separation, a 'hydrophobic force' suddenly pulled them together. In 2002, Tyrrell and Attard published [18] more AFM images of nanobubbles, and proposed them to be the origin of the 'hydrophobic force'.

The question naturally arises: when are nanobubbles present? Yang and coworkers published in 2007 [20] an exhaustive experimental study on the factors influencing the formation of nanobubbles, such as temperature, dissolved gases etc. It turns out that if a surface had been immersed in ethanol before being immersed in water, then nanobubbles are reliably formed. Using this 'solvent exchange' technique, Yang and coworkers were able to do repeatable studies of nanobubbles. Using infrared spectroscopy, they confirmed the presence of a gas phase 5 – 80 nm thick at the surface — i.e. good evidence that the 'soft domains' really are nanobubbles. The pressure in the bubbles was found to be 1.0 – 1.7 atmospheres, consistent with the Laplace pressure calculated from their radii of curvature. This low pressure allows nanobubbles of air to remain stable for days. They find that nanobubbles form much more easily on rough surfaces, sometimes even without the solvent exchange technique.

Crucially, the solvent exchange technique is also a common cleaning technique. Therefore, nanobubbles may be formed inadvertently in the process of a slip experiment. Given the further fact that rough surfaces may spontaneously form nanobubbles, the question arises: How many ostensibly 'pure' slip experiments are actually experiments on *mixed-slip* surfaces?

A purpose of the research in this thesis is to give some indication of the effect of such nanobubble contamination on a slip experiment.

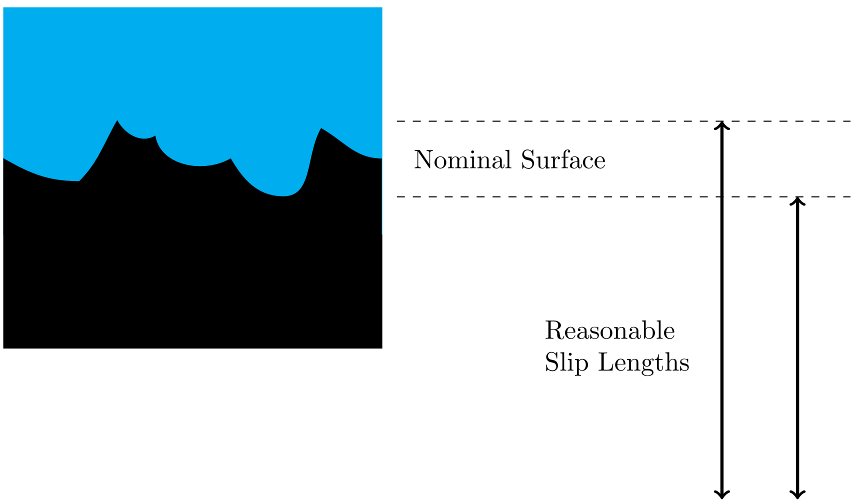
In summary, mixed-slip surfaces tend to fall into the two types described above: either a solid surface interspersed with pockets of air (nanobubble type), or a gas-liquid interface interspersed with islands of solid material (superhydrophobic type). Thus, a superhydrophobic type surface has a contiguous *air-liquid* interface, while a nanobubble type surface has a contiguous *solid* phase.

In the two-dimensional case, the difference disappears. *Neither* the solid-liquid nor air-liquid interfaces are contiguous. Physically, this surface consists of a grating of parallel ridges, with an air gap between the ridges. The liquid sits on the top of the ridges, and the air-liquid interface forms a meniscus between the ridges.

0.2 Rough Surfaces

In the previous section on mixed-slip surfaces, we focused on two archetypes drawn from the real world. In keeping with their real-world nature, they have a hugely important feature: *they are not flat.* In a previous section, the slip length of a mixed-slip surface was defined with respect to a mathematically perfect *flat plane.* The definition of the slip length of a *rough* surface is ambiguous. The question “What is the slip length of this surface?” suddenly acquires a resonance with the old Vaudeville joke “How’s your wife?”; the answer: “Compared to what?”.

To clarify, a surface with slip behaves as if the no-slip boundary was located some distance — the slip length — below the true surface. For convenience, the plane $z = 0$ in the mathematical model maps to the true surface. But for a rough surface, the physical position of $z = 0$ is a matter of choice. It could be reasonably defined to be at the highest point of the roughness, or the lowest point of the roughness, or at any point in between. Put another way, a rough material has a ‘nominal surface’, which has some non-zero width — essentially the distance between its lowest and highest points. If a rough surface has a slip length that is large compared to this surface width, then all is well, the slip length will be quoted with a small uncertainty equal to this ambiguity. But slip lengths may be of the order of the width of the nominal surface. In this case, the very existence of slip has disappeared into uncertainty about the surface location.



Surprisingly, a failure to consider the precise location of the $z = 0$ point led to considerable confusion and contradiction in the early experimental literature on slip.

Studies were made on the effects of roughness on slip. These involve a surface that is non-flat, but otherwise homogeneous; the material is the same everywhere, so is assumed to have the same intrinsic slip length everywhere.

In 2002, Zhu and Granick published results of drainage force experiments on hydrophobic surfaces of varying roughness [22]. The molecularly smooth surface showed a flow dependent slip length of up to 35 nm, while rougness *suppressed* slip, with a roughness of 6 nm giving no slip at all.

They defined the $z = 0$ level in the surface force appartatus by 'adhesive contact in air'. Therefore, the $z = 0$ level could well be below the tops of the roughness peaks. No effort was made to account for this.

A paper from 2003 by Bonaccurso, Butt and Craig [2] claimed that roughness could *increase* slip, even on a *hydrophilic* (contact angle zero!) surface. They measure drainage forces of a glass sphere approaching a silicon surface roughened up to 12.2 nm rms.

They discuss the importance of defining the zero distance. They end up defining it at the tops of the peaks, as this is the first point of contact. They calculate slip lengths by fitting the data to Vinogradova's model. The best fit is when they fix the slip length on the glass sphere at about 43 nm, and increase the slip length of the substrate as roughness increases. Under 'normal' conditions, they find a slip length of 3.5 nm for maximum roughness of 12.2 nm. But for extremely high approach velocities, for the same roughness they find a slip length of 900 nm!

Vinogradova herself waded into the issue in 2006, with a paper with Yakubov [19]. They used a purpose-built AFM device that tapped a roughened sphere onto a smooth plane. The sphere had an rms roughness of 10 -11 nm, and a maximum peak-to-valley distance of 45 nm. With the surface taken to be at the tops of the peaks, a reduction in drainage force was observed, compared to a smooth sphere of equal diameter. But the reduction was not due to slip: The force was equivalent to that of a smooth sphere whose surface was located at an intermediate position between the peaks and valleys of the roughness.

Thus the issue is resolved: if the boundary is taken to be at the valleys of the roughness, then roughness reduces slip. Conversely, if the boundary is taken to be at the tops of the peaks, then roughness *increases* slip. They note "We believe our paper entirely clarifies the situation with flow past rough surfaces, highlights reasons for existing controversies, and resolves apparent paradoxes."

A couple of numerical studies expand on Vinogradova's clarification.

Kunert and Harting in 2007 [8] formalized the concept with the introduction of an 'effective no slip plane', typically located between the peaks and valleys of the surface roughness. They carried out numerical simulations using lattice Boltzmann methods on different surfaces. Each surface has minimum and maximum heights, h_{min} and h_{max}, and an average height h_{average}. They calculate the position of the effective no-slip plane, h_{eff}. In all cases, h_{eff} was always considerably higher than h_{average}. If the surface has a few very tall but sparsely distributed spikes, then h_{average} can be much smaller than h_{max}, and h_{eff} lies somewhere between them and cannot be well approximated by either.

In 2010 the same authors teamed up with Vinogradova [9] to carry out lattice Boltzmann simulations of high-speed drainage between a smooth sphere and a randomly rough plane. A sphere of radius R approaches the plane $z = 0$ located at the tops of the roughness. Assuming that the plane $z = 0$ has a single constant slip length, then asymptotically, the drainage force

$$F \to \frac{9R}{32z} \quad \text{as} \quad z \to 0$$

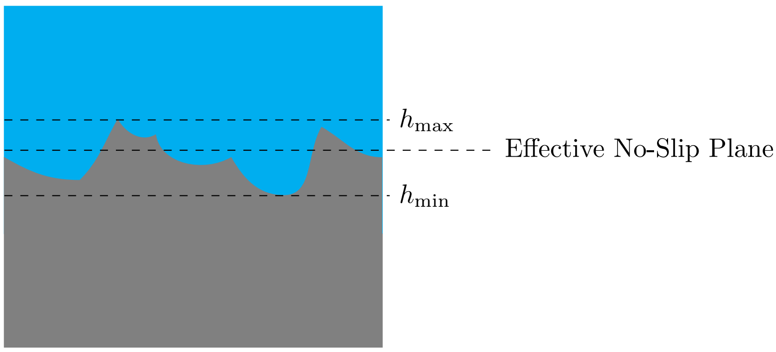
Alternatively, assuming there is a no-slip plane located a distance s below the top of the roughness,

$$F \to \frac{9R}{8s} \quad \text{as} \quad z \to 0$$

At distant separations, the two models agree with each other, but at close separations, the forces predicted by the slip model are higher than observed, while the effective no-slip plane model fits the data very well.

Hence, an effective no-slip plane located between the peaks and valleys of the roughness does a better job at explaining drainage forces than a slip boundary located on top of the roughness. This is not surprising, since the drainage force of a sphere touching a surface (even with slip) will diverge, while the effective no-slip plane model always allows a finite space for fluid to escape.

However, for Couette or Poiseuille type flow, the two models should be equivalent; that is after all, one way of defining the slip length. The important thing is to locate the effective no-slip plane. The distance from there to the surface defined as $z = 0$ determines the slip length. These considerations are obviously still important for any rough surface that also happens to be mixed-slip.



0.3 Mixed Slip Flow

In 2003, Cottin-Bizonne *et al* published a paper [5] in which they stated "Our results show for the first time that, in contrast to common belief, surface friction may be reduced by surface roughness." In fact, what they discovered is that flow over a rough surface may transition into the superhydrophobic state, with the fluid now flowing over vapour pockets. ~~(As previously stated, later that same year, Bonaccurso, Butt and Craig [2] experimentally demonstrated that roughness increases slip. But they did not explicitly consider that their system may have entered the superhydrophobic regime. Had it?)~~

Cottin-Bizonne and coworkers looked at molecular dynamics simulations of a Lennard-Jones fluid flowing over a flat surface decorated with narrow square posts. At sufficiently low pressures, the fluid entered the Cassie state, as a vapour phase spontaneously formed at the surface, leaving the fluid supported on top of the posts. The surface had an intrinsic slip length of 20 - 25 σ (atom diameters). In the Cassie state, slip lengths up to 57 σ appeared. For very narrow posts – 4.9 σ, slip lengths could reach 130 σ. Note that slip lengths were measured from the bottom of the cavity, so the post height — 6 σ — could be added to the slip lengths.

A physical ~~experiment~~ of this superhydrophobic Cassie state flow was presented by Choi and Kim in 2006 [3]. They were probably the first to deliberately engineer a surface for maximum slip: ‘nanoturf’, silicon nanoposts about 1 - 2 μm high, spaced about 0.5 - 1.0 μm apart, rendered hydrophobic by a 10 - 20 nm thick layer of Teflon. They estimated the air fraction of the surface to be 60 %.

A commercial cone-and-plate rheometer was used to measure slip lengths: a collosal 20 μm for water and 50 μ for 30 % glycerine solution. (They expect this, since the viscosity of the glycerine solution is 2.5 times greater than that of water.)

Such ~~suspiciously~~ high slip lengths were not replicated in a more careful study by Joseph *et al* also in 2006 [7]. They did particle image velocimetry on channels coated with carbon nanotubes of diameter 50 - 100 nm, spaced 100 - 250 nm apart. The tops of the nanotubes could be evenly spaced, or clumped together like wet hair, giving inter-clump length scales of 1.7, 3.5 or 6 μm. The derived slip lengths for the three surface morphologies were roughly 0.4, 1.0 and 1.4 μm, respectively.

They note that their results are an order of magnitude smaller than the 20 μm slip lengths of Choi and Kim, and point out that rheological methods lack the sensitivity to measure surface effects.

An ~~amusing~~ effort was made by Lee and Kim in 2011 [12] to maximize slip by making a heirarchical structured surface — nanoposts on top of microposts. It worked if area fraction taken up by the microposts was large enough. Below about 10% area fraction — a realistic figure — the advantage began to disappear, and at 4% area fraction, the heirarchical surface gave *lower* slip than conventional unadorned microposts.

A one-dimensional version of the superhydrophobic surface is a nanograting — a surface covered with ridges, with the water supported by surface tension on top of the ridges. In 2006, Choi *et al* [4] presented slip experiments on a “well-defined nanograte”: ridges 500 nm high and 50 nm wide, separated by a gap of 180 nm. Thus the pitch (period) was 230 nm. If the grating was left hydrophilic, they believe that water fully wets the surface, penetrating down into the troughs. After rendering the surface hydrophobic with Teflon, they believe that there is air in the troughs. Experiments were carried out with both states, with fluid flow both parallel to, and transverse to the ridges.

They could measure slip lengths to a resolution of only 30 nm, thus they were unsure if the hydrophobic surface had any intrinsic slip. For flow parallel to the ridges, there was a clear distinction between the slip lengths of hydrophilic and hydrophobic surfaces. 30 ± 15 nm for hydrophilic, and 143 ± 35 nm for hydrophobic. For transverse flow, they found insignificant slip, 0 ± 17 for hydrophilic, and 61 ± 44 for hydrophobic.

One can imagine the difficulties in studying a surface of nanobubbles, given their random, uncontrolled nature. Steinberger *et al* in 2007 [16] addressed the issue by studying flow over a flat surface covered with holes 1.3 μm wide and 3.5 μm deep. Air can be trapped in the holes; they derive slip lengths via drainage force measurements on the resulting microbubble surface.

The plane $z = 0$ is located on the flat surface, at the tops of the holes. In the Wenzel state, with water filling the holes, they measure a slip length of 105 ± 10

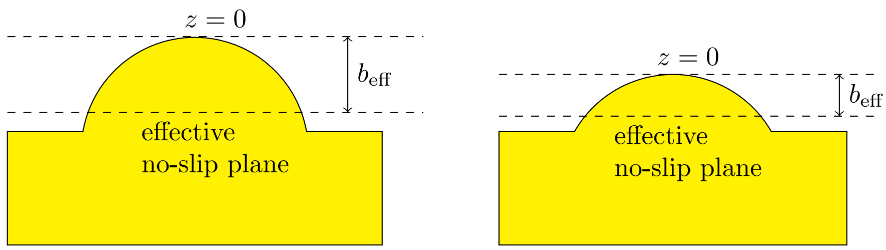
nm. With air trapped in the holes, bizarrely, they find a *lower* slip length: a mere 20 ± 10 nm. ~~Understandably, they are puzzled by this, so they investigate.~~ They discover that the microbubbles protrude an estimated 200 - 400 nm above the flat surface, with the meniscus subtending an angle between 30° and 60° to the flat surface. Is this protrusion into the bulk the cause of low slip lengths?

They test this hypothesis numerically, with a finite element package (Comsol). They find a flat bubble ($\theta = 0°$) gives maximum slip length — about 160 nm. Any increase in θ decreased slip, with $\theta > 45°$ giving a lower slip length than the Wenzel state.

The following year (2008) Hyväluoma and Harting replicate and extend Steinberger's numerics [6]. By using lattice Boltzmann methods, they can model the bubble deforming under stress. They essentially replicate Steinberger: a maximum slip of about 150 nm at zero protrusion angle, plummeting down past zero slip length for a protrusion angle greater than about 70°

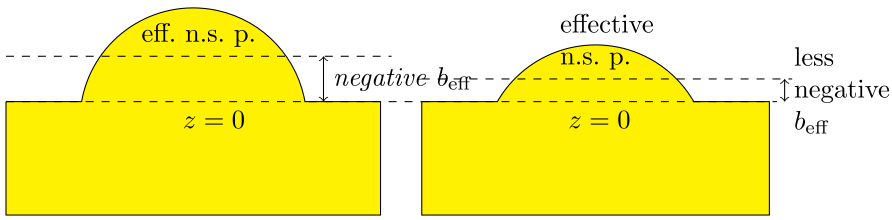
They simulate Couette flow, so are able to investigate shear dependence. In steady state shear-driven flow, they see a *decrease* in slip length with increasing shear. This contradicts some earlier claims. However, higher shear rates deform the microbubbles, reducing the average height of the microbubbles.

They claim that this is consistent with Kunert and Harting 2007 [8], which showed reduced slip from reduced roughness. However a careful reading sheds doubt on this. The upshot of Kunert and Harting 2007 [8] is that if the peaks are lowered, then the position of the effective no-slip plane will be lowered too, but *not as much.* Thus, there is a reduced distance from the effective no-slip plane to the $z = 0$ plane *on the tops of the roughness peaks.* Put another way, the effective no-slip plane has actually *risen* with respect to the $z = 0$ plane. That is to say, the slip length has reduced (caused by reduced roughness).

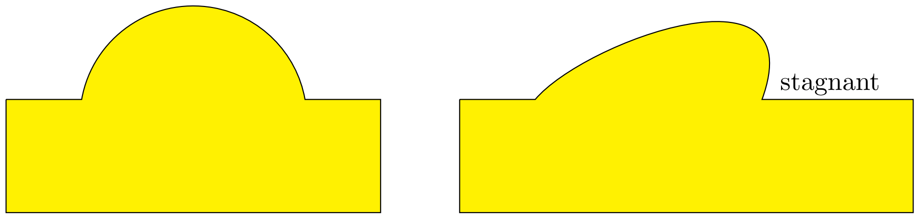


10-1

However, in Hyväluoma and Harting 2008 [6], the $z = 0$ plane is at the 'top of the structured surface' which seems to mean the *bottom* of the roughness. In that case, a lowered effective no-slip plane (for any reason), is always lower with respect to the $z = 0$ plane, which is equivalent to *increased* slip length. (If the effective no-slip plane was above the $z = 0$ plane to start with, then the slip length becomes *less negative.*)



So, the increase in slip from reduced bubble height seems not to be consistent with Harting's earlier work with Kunert. ~~Instead, I hypothesize that a deformed bubble has a steeper wall on the downstream side, causing more stagnancy and turbulence in the lee of the bubble, thus a lower average surface velocity, hence a lower slip length.~~



Incidentally, the idea that slip is reduced by protruding bubbles had been proposed by Lauga and Brenner in 2004 [11]. In a theoretical paper, they present a model to explain the shear-dependent slip found by Zhu and Granick in 2001 [21]. They assume that the surface was (unknown to the experimenters) covered in bubbles. Slip lengths were inferred from the drainage force of a probe slamming into the surface at various speeds. As the probe approach velocity increases, so too does the pressure in front of it. This increased pressure causes the bubbles to shrink, both from compression of the gas and increased diffusion into the liquid. The reduced bubble height widens the channel, making drainage easier, for a given probe-surface distance. Thus, this 'leaking mattress effect' causes a shear-dependent slip effect to appear.

Conclusion

In summary, high slip lengths are possible over mixed-slip surfaces: more than 100 nanometers for nanogratings, and more than 1 micron for nanoforests. However, the effective slip length has a slightly ambiguous definition; the quoted slip length depends on the nominal position of the $z = 0$ plane. Things are clarified by introducing the concept of an effective no-slip plane. This is an objective concept: the surface behaves as if the no-slip plane was located at a given position. Then, the slip length is the distance between $z = 0$ and the effective no-slip plane. Thus, if the no-slip plane becomes lower, then the slip length is increased, and vice versa. A sensible choice for the position of the nominal $z = 0$ plane is the *top of the roughness.* That way, quoted slip lengths will often be positive. Note that if the $z = 0$ plane was chosen to be *below* the no-slip plane to start with, lowering the no-slip plane still *increases* the slip length ~~by making it *less negative.*~~

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2-1

9/11/2013 8:17 am, Shaun Hendy

gas

2-2

9/11/2013 8:17 am, Shaun Hendy

You need to cite these results.

2-3

9/11/2013 8:17 am, Shaun Hendy

I don't agree. It all depends on the scale you are interested in. If you are interested in nanoscale flows then variations in implicit slip may be important.

I would say that in general most surfaces have variations in slip length. The general problem is to determine their effective slip length. Then I would say that there are two cases that are most relevant to current experimental work: superhydrophobic surfaces and surfaces with nanobubbles.

2-4

9/11/2013 8:17 am, Shaun Hendy

You need to justify this.

2-5

9/11/2013 8:17 am, Shaun Hendy

It is more complicated than that.

3-1

9/11/2013 8:17 am, Shaun Hendy

This is not very convincing. What happens when the pillars don't have rounded ends? And why don't your other contact lines make an angle of theta to the pillars?

Much easier to use surface energy arguments here, but if you insist on using forces then do it properly.

5-1

9/11/2013 8:42 am, Shaun Hendy

reference

5-2

9/11/2013 8:42 am, Shaun Hendy

How did they measure the pressure?

5-3

9/11/2013 8:42 am, Shaun Hendy

it is not low as such but comparable to atmospheric pressure

6-1

14/12/2013 7:36 am, Shaun Hendy

Figures need a number and a caption

6-2

14/12/2013 7:36 am, Shaun Hendy

Have you established that this is true on a rough surface? I think this is a problematic assumption - one could imagine a situation where a surface of no slip was above a rough surface e.g. If there were eddies etc.

6-3

14/12/2013 7:36 am, Shaun Hendy

I think you are missing the point a bit here - the need to define a z=0 surface comes from experimentalist a trying to interpret their results. They are trying to define an effective z=0 right?

6-4

14/12/2013 7:36 am, Shaun Hendy

I'm not sure this is the right word since z=0 is not a real thing. Rather it depends on the model they are using to fit their force data - is their experimental definition of z=0 the same as that of the model they are using.

7-1

2/12/2013 5:26 pm, Shaun Hendy

This is getting a bit too informal. How do you know that Vinogravoda was the key instigator here and not Yakubov?

7-2 10/12/2013 1:04 pm, Shaun Hendy
Try to write in paragraphs please.

7-3 14/12/2013 7:36 am, Shaun Hendy
This is colourful, but you are making this up. The work might have been done in 2009 for all you know. Stick to being straightforward.

7-4 14/12/2013 7:36 am, Shaun Hendy
Shouldn't this be a ~ rather than an arrow?

7-5 14/12/2013 7:36 am, Shaun Hendy
What is a "no-slip plane"? Need to be more precise.

8-1 14/12/2013 7:36 am, Shaun Hendy
Ok, now you say effective.

8-2 14/12/2013 7:36 am, Shaun Hendy
I found this section confusing. It might be best not to do it as a history. Introduce Vinogradova (2010) *first* and then discuss how earlier work may have had problems.

8-3 14/12/2013 7:36 am, Shaun Hendy
Effective slip now right? It reads as if the intrinsic slip of the surface has changed.

9-1 14/12/2013 7:36 am, Shaun Hendy
Demonstration

9-2 14/12/2013 7:36 am, Shaun Hendy
Compared to what? Surely not colossal compared to slip at a free boundary?

9-3 14/12/2013 7:36 am, Shaun Hendy
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9-4 14/12/2013 7:36 am, Shaun Hendy
You'd better explain what a rheological method is and why it lack sensitivity.

9-5 14/12/2013 7:36 am, Shaun Hendy
Overall I found this section to be confusing. You jump around from experiment to experiment without any systematic progression. It could be help perhaps by some subtitles introducing each set of experiments.

10-1 14/12/2013 7:36 am, Shaun Hendy
Captions and figure labels please!

11-1 14/12/2013 7:36 am, Shaun Hendy
This is a literature review. If you want to say this, you need to say "Later in this thesis, I will show that ..." Otherwise simply note that is possible that bubble deformation plays a role. Also note that it is very unlikely that there is any turbulence in the lee of the bubble! It would have to be a large bubble for that to occur!

11-2 14/12/2013 7:36 am, Shaun Hendy
This chapter is a difficult one for the reader. I wonder if a table of some sort could help pull together all the experiments.