An Effective Slip Length for Rough, Mixed-Slip Surfaces

by

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requirements for the degree of
Doctor of Philosophy
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Abstract

An abstract of fewer than 500 words must be included.

Acknowledgments

Any acknowledgments should go in here, between the title page and the table of contents. The acknowledgments do not form a proper chapter, and so don't get a number or appear in the table of contents.

Contents

Chapter 1

Introduction

The vuwthesis style attempts to do 2 things:

1. produce the correct front pages – explained in Chapter 2;
2. meet the library requirement for the deposit of theses – explained in Chapter 3.

We try to set the front pages up after the fashion of the example on p58 of the 2002 PhD handbook. (Available at `http://www2.vuw.ac.nz/home/publications/phd_handbook.pdf`, March 2003).

I assume that you are using report.sty or, preferably, book.sty. The book style has things like `\frontmatter` defined, which make life a little easier.

Although the style alters the margins, and so on, if you'd prefer to do something else then feel free to set them all yourself. In this case you will probably want to look at the layout package.

Chapter 2

The front pages

You can declare:

- an author using `\author{}` – your full name;
- a title using `\title{}`– ALL IN CAPITALS IN THE EXAMPLE, but we use `\Large` and `\bf`, so normal case will do;
- a subject using `\subject{}`– probably Computer Science, but you can pick anything you like;
- an abstract using `\abstract{}`– this should be fewer then 500 words for a PhD. The handbook states "A length of about 300 words is recommended." The `book` style does not have an `abstract` environment defined, so this is a bit of a hack.

You can also specify what degree you are submitting the thesis for by using:

- `\phd`, which produces "... in fulfilment of ... Doctor of Philosophy"
- `\mscthesisonly`, which produces "... in fulfilment of ... Master of Science"
- `\mscwithhonours`, which produces "... in partial fulfilment of ... Master of Science with Honours"

- `\mscbothparts`, which produces "... in partial fulfilment of ... Master of Science"
- `\otherdegree{DEGREE OR DIPLOMA NAME}`, which produces "... in fulfilment of ... DEGREE OR DIPLOMA NAME"

Chapter 3

Library requirements

This Chapter describes some things done to be in accordance with the 'Library Requirements for the Deposit of Theses' booklet, which can be obtained from the VUW library.

3.1 Font size

12 point font is recommended.

Set this yourself with the `12pt` option to the document class.

3.2 Single or double sided

Typing on one side preferred, text on both sides acceptable for longer works.

Use the `oneside` or `twoside` option to the document class. The `book` class default is `twoside`.

3.3 Line spacing

Lines should be double spaced, or at least 1.5 spaces apart.

1.5 spacing for 12 point is set using:

- `\renewcommand{\baselinestretch}{1.24}`

To set anything else use, e.g `\renewcommand{\baselinestretch}{1.0}` in the preamble. See p 53 of [1] for more information.

3.4 Margins

Uniform margins of at least 4cm on binding side, other three sides to have at least 1.5 cm

LaTeX styles often leave a lot of white space, and book is no exception. We only have to worry about the binding side.

For `twoside` we use these settings:

For `oneside`, if we use:

```
\setlength{\textheight}{19.8cm}
\setlength{\oddsidemargin}{1.46cm}
```

Then book gives a margin of 4cm on the binding side, and about 3.2cm elsewhere.

If you want to change these then use the layout package.

3.5 Paper size

High quality A4 paper, all pages of equal size

Set this yourself with the `a4paper` option to the document class.

Chapter 4

Binding

The Library states:

- Thesis must be fully bound and cased in cloth or buckram in optional colour. (I think they mean you can pick the colour!)
- Author's surname and initials and (short) title lettered on spine. (The thesis's title, not the author's, presumably.)
- Author's full name and full title lettered on front cover. (Again, the thesis's title, not the author's, presumably.)
- Two copies of a PhD thesis and one copy of a Master's thesis must be deposited. A signed availability slip must accompany the thesis.
- The principal supervisor is responsible for the deposit.

Appendix A

Replicating John Philip 1972

The first known expression for an effective slip length appeared in 1972, in a paper in ZAMP by John R. Philip entitled "Flows Satisfying Mixed No-Slip and No-Shear Conditions".

In the paper, John R. Philip says that the limit of

$$W_3 = \Im\left[\alpha^{-1}\cos^{-1}\left\{\frac{\cos(\alpha\Theta)}{\cos\alpha}\right\} - \Theta\right] \tag{A.1}$$

as $y \to \infty$ is

$$W_3 = \alpha^{-1}\ln\sec\alpha \tag{A.2}$$

Let us prove this forthwith.

$\Theta = x + iy$ is a complex number, α is real. Trig identities for *complex* cosine and exponential:

$$\cos z = \frac{e^{iz} + e^{-iz}}{2} \tag{A.3}$$

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{A.4}$$

Expand complex cosine term, dump negligible parts

In Euler's formula $e^{i\theta} = cis(\theta)$, if θ is *real*, then $e^{i\theta}$ traces out the unit circle in $\mathbb{C}$, with θ being the angle.

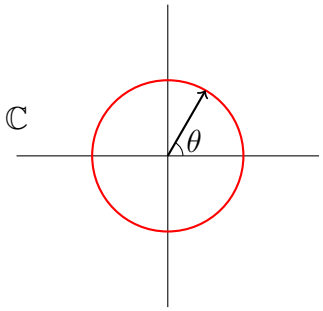

Figure A.1: Euler's formula $e^{i\theta}$ for real θ .

This gives insight into the $\cos z$ function. If z is real, then $\frac{1}{2}e^{iz}$ and $\frac{1}{2}e^{-iz}$ are two vectors of length $\frac{1}{2}$ that cycle in opposite directions, with z being the angle. Then $\cos z$ is the sum of the two vectors, which always ends on the real line between -1 and 1, as shown in Figure (A.2).

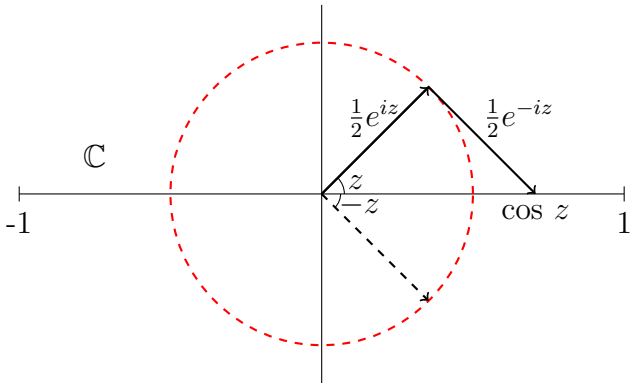

Figure A.2: The complex cosine.

With this insight, it is useful to rewrite $\cos z$ as:

$$\cos(x+iy) = \frac{e^{i(x+iy)} + e^{-i(x+iy)}}{2} = e^{y}\frac{1}{2}e^{-ix} + e^{-y}\frac{1}{2}e^{ix} \tag{A.5}$$

Then it is clear that $\cos(x+iy)$ is the sum of two rotating vectors in $\mathbb{C}$ with amplitudes e^{y} and e^{-y}. A consequence is that for large y, e^{y} is *very*

large, while e^{-y} is negligible, therefore $\cos(x+iy)$ is dominated by the vector $e^y \frac{1}{2} e^{-ix}$. See Figure (A.3).

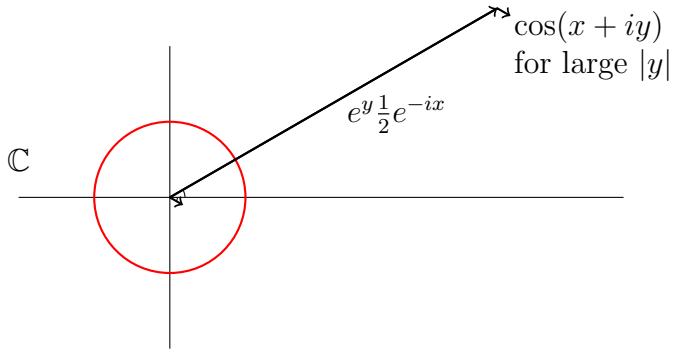

Figure A.3: Complex cosine at large $|y|$.

$$\text{Therefore} \quad \cos(x+iy) \to \frac{e^y e^{-ix}}{2} \quad \text{as} \quad y \to \infty \tag{A.6}$$

$$\cos z \to \frac{1}{2} e^{-iz} \quad \text{as} \quad y \to \infty \tag{A.7}$$

Inverse Cosine at Large y

As $y \to \infty$:

$$w = \cos z \to \frac{1}{2} e^{-iz} \tag{A.8}$$

Solve $w = \cos z$ for z to get:

$$\arccos w = z$$

Likewise solve $w = \frac{1}{2} e^{-iz}$ for z:

$$w = \frac{1}{2} e^{-iz}$$
$$2w = e^{-iz}$$
$$\ln(2w) = -iz$$
$$i \ln(2w) = -i^2 z$$
$$i \ln(2w) = z$$

Equate the two expressions to obtain the inverse cosine in terms of a logarithm:

$$\arccos z = i \ln(2z) \tag{A.9}$$

Put into J. R. Philip's Expression

$$W_3 = \Im \left[\alpha^{-1} \cos^{-1} \left\{ \frac{\cos(\alpha\Theta)}{\cos\alpha} \right\} - \Theta \right] \tag{A.10}$$

As $y \to \infty$, the cosine expression may be substituted:

$$W_3 = \Im \left[\alpha^{-1} \cos^{-1} \left\{ \frac{\frac{1}{2} e^{-i\alpha\Theta}}{\cos\alpha} \right\} - \Theta \right] \tag{A.11}$$

And the inverse cosine expression may also be substituted:

$$W_3 = \Im \left[i\alpha^{-1} \ln \left\{ 2 \frac{\frac{1}{2} e^{-i\alpha\Theta}}{\cos\alpha} \right\} - \Theta \right] \tag{A.12}$$

$$W_3 = \Im \left[i\alpha^{-1} \ln \left\{ e^{-i\alpha\Theta} \frac{1}{\cos\alpha} \right\} - \Theta \right] \tag{A.13}$$

Recall that $\ln ab = \ln a + \ln b$.

$$W_3 = \Im \left[i\alpha^{-1} \ln \left\{ e^{-i\alpha\Theta} \right\} + i\alpha^{-1} \ln \left\{ \frac{1}{\cos\alpha} \right\} - \Theta \right] \tag{A.14}$$

Invoke definition of logarithm: $\ln e^z = z$.

$$W_3 = \Im \left[i\alpha^{-1} \left\{ -i\alpha\Theta \right\} + i\alpha^{-1} \ln \left\{ \frac{1}{\cos\alpha} \right\} - \Theta \right] \tag{A.15}$$

$$W_3 = \Im \left[\Theta + i\alpha^{-1} \ln \left\{ \frac{1}{\cos\alpha} \right\} - \Theta \right] \tag{A.16}$$

$$W_3 = \Im \left[i\alpha^{-1} \ln \left\{ \frac{1}{\cos\alpha} \right\} \right] \tag{A.17}$$

$$W_3 = \alpha^{-1} \ln \left\{ \frac{1}{\cos\alpha} \right\} \tag{A.18}$$

$$W_3 = \alpha^{-1} \ln \sec\alpha \tag{A.19}$$

Appendix B

Conclusions

If all the economists in the world were laid end-to-end they wouldn't reach a conclusion, and neither shall I.

Bibliography

[1] Goossens, M., Mittelbach, F., and Samarin, A. *The LaTeX Companion*. Addison-Wesley, Reading, Massachusetts, USA, 1994.