

Effective slip lengths for Stokes flow over rough, mixed-slip surfaces

PhD Defense Presentation

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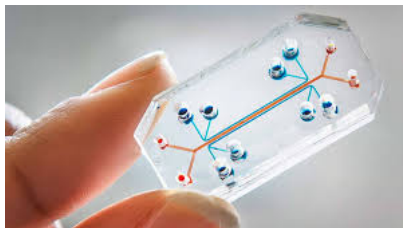
Victoria University of Wellington

Friday 12th September 2014

- ▶ Elevator Speech
- ▶ Regimes of Applicability – can we extend?
- ▶ Limitations of Homogenization
- ▶ What Next?

Motivation

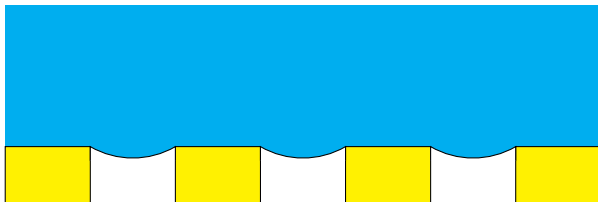
Lab-on-a-chip



Very small pipe: friction dominates

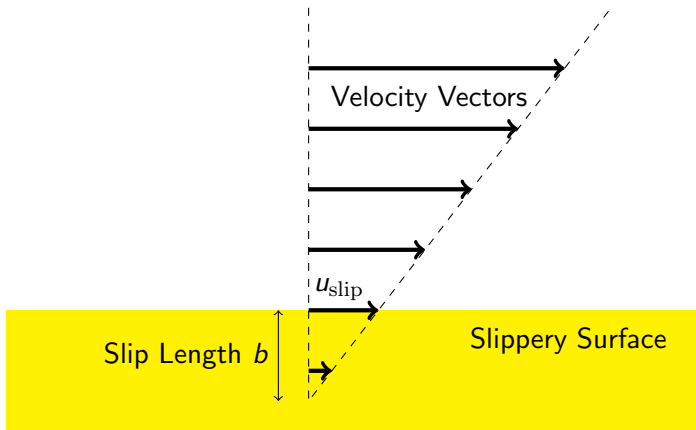
How to reduce friction?

Slippery Surfaces



- ▶ Holes on the wall of the pipe
- ▶ Air bubbles trapped in holes
- ▶ Water slips over top of air bubble
- ▶ Friction reduced: How much?

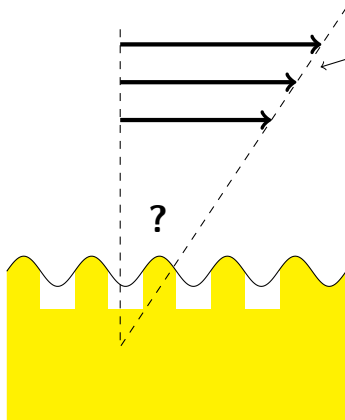
Slip Length



Single parameter to express the friction of a surface.

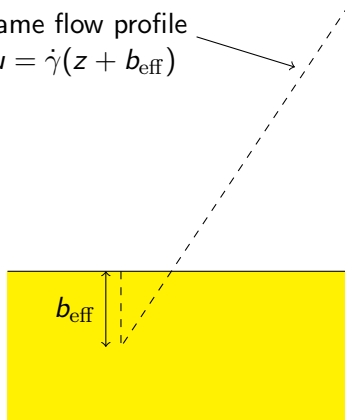
Effective Slip Length

PHYSICAL SYSTEM

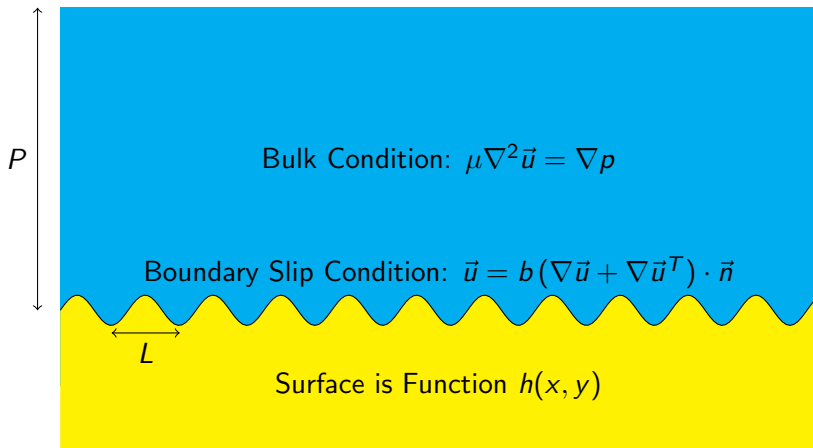


EFFECTIVE SYSTEM

Same flow profile
 $u = \dot{\gamma}(z + b_{\text{eff}})$



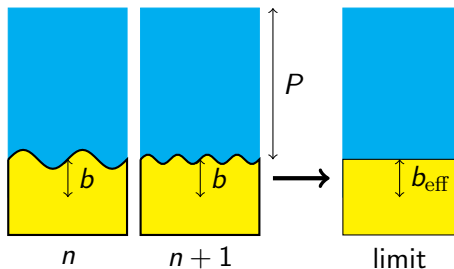
Mathematical Model



Solving

Homogenization:

Thought experiment about what happens when the period L becomes infinitely small.



Perturbation:

Think of system as 'perturbed' slightly away from a well-known system (with known solution).

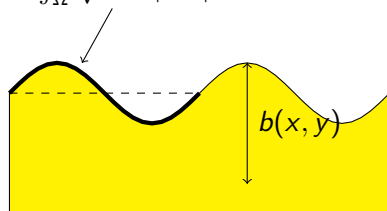
Homogenized Solution

$$b_{\text{eff}} = \left\langle \frac{\sqrt{1 + |\nabla h|^2}}{b} \right\rangle^{-1} \quad (1)$$

Harmonic mean of intrinsic slip lengths, weighted by area of contact:

Fluid-Solid Contact Area

$$\int_{\Omega} \sqrt{1 + |\nabla h|^2}$$



Perturbative Solutions

Replicates homogenized solution for special case of flat surface:

$$b_{\text{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1} \quad (2)$$

Where b is much smaller than other length scales, b_{eff} is simple average:

$$b_{\text{eff}} = \langle b \rangle \quad (3)$$

Regimes of Applicability

Harmonic mean b_{eff} is excellent approximation when period L is much smaller than other lengths, slip lengths b and domain size P .

$$b_{\text{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1} \quad \text{when } L \ll b, P \quad (4)$$

(Still good approximation even if $L \sim b \ll P$.)

Simple mean is good approximation when slip lengths b much smaller than other lengths:

$$b_{\text{eff}} = \langle b \rangle \quad \text{when } b \ll L, P \quad (5)$$

Regimes and Results

DERIVED RESULTS

	2-D Flow (1-D surface pattern)	3-D Flow (2-D surface pattern)	
No-slip/ Perfect-slip Binary Surface	J. R. Philip 1972 Lauga and Stone 2003		FLAT SURFACE
Other Surface	Hendy and Lund 2007	Lund and Hendy 2008	
No-slip/ Perfect-slip Binary Surface	Sbragaglia and Prosperetti 2007 Davis and Lauga 2009a		ROUGH SURFACE
Other Surface	Einzel, Panzer, Liu 1990 Lund <i>et al</i> 2012	This thesis?	


Can we homogenize No-slip/Perfect-slip cases?

No exact results for case of rough surface with $b = 0$ and $b \sim L$.

Can we apply homogenization?

No.

Limitations of Homogenization 1



A diagram showing a rectangular domain Ω . The top boundary is dashed and labeled Γ_0 . The bottom boundary is solid and labeled Γ_b . The domain is labeled Ω in the center.

$$\nabla^2 u = f \quad \text{on } \Omega$$
$$u = b \frac{\partial u}{\partial n} \quad \text{on } \Gamma_b$$

Multiply by test function g and integrate over Ω :

$$\int_{\Omega} g \nabla^2 u = \int_{\Omega} g f \quad (6)$$

Use vector identity and divergence theorem to get:

$$\int_{\Gamma} g \frac{\partial u}{\partial n} - \int_{\Omega} \nabla u \cdot \nabla g = \int_{\Omega} g f \quad (7)$$

Limitations of Homogenization 2

The slip condition on Γ_b implies:

$$\frac{\partial u}{\partial n} = \frac{1}{b}u \quad (8)$$

Substitute this, to get variational form:

$$\int_{\Gamma_b} g \frac{1}{b} u - \int_{\Omega} \nabla u \cdot \nabla g = \int_{\Omega} g f \quad (9)$$

\therefore Require b in form $\frac{1}{b}$.

If $b = 0$ anywhere, $\frac{1}{b}$ is undefined. Cannot homogenize.

Another Subtlety

Mathematically, expect that $b_{\text{eff}} = \langle \frac{1}{b} \rangle^{-1}$ is a good approximation when L smaller than other length scales: $L \ll b, P$.

But it is still a good approximation when $L \sim b \ll P$.

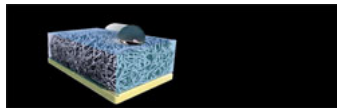
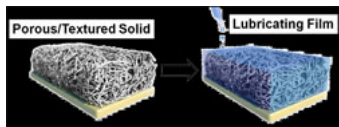
Why?

What Next?

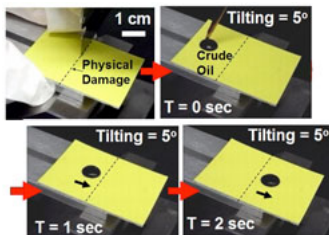
- ▶ Learn FEM. Natural step after having learned Variational Formulation.
- ▶ Apply homogenization to multiscale slip systems.
- ▶ Apply Homogenization to other physical systems.

Superhydrophobic Surfaces Superseded?

“[Harvard’s] SLIPS technology combines a lubricated film on a porous solid...”



“...to create low-cost surfaces that exhibit ultra-liquid repellency, self-healing, optical transparency, pressure stability and self-cleaning.”



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