Chapter 1

Conclusion

1.1 Summary

In this PhD thesis we studied the effective slip length of Stokes flow over rough heterogeneous surfaces. In our mathematical model, the rough surface is modelled as a periodic function $h(x, y)$, and the local intrinsic slip length is modelled as a periodic function $b(x, y)$. The period L of both functions is the same. The slip function $b(x, y)$ has a minimum b_{min} and a maximum b_{max}. At some height P above the surface, a fixed velocity or shear rate drives the fluid.

Using the homogenization technique for partial differential equations, we showed that if L is much smaller than other length scales, then the effective slip length is well-approximated by the harmonic mean of intrinsic slip lengths, weighted by area of contact between fluid and surface:

$$b_{\text{eff}} = \left\langle \frac{\sqrt{1 + |\nabla h|^2}}{b(x, y)} \right\rangle^{-1} \tag{1.1}$$

Using a quite different technique, a perturbation method, we replicated this result for the simplified case where the surface is flat, not rough:

$$b_{\text{eff}} = \left\langle \frac{1}{b(x,y)} \right\rangle^{-1} \tag{1.2}$$

The perturbative result reconciles with the homogenized result, since for a flat surface $\sqrt{1+|\nabla h|^2} = 1$. The perturbative result also applies when L is much smaller than other length scales.

Also using the perturbation method, we studied flat surfaces in the limit of vanishing slip length. If $b_{\text{max}} \ll P$, then the slip length is expected to be best approximated by the area-weighted average of intrinsic slip lengths:

$$b_{\text{eff}} = \langle b(x,y) \rangle \tag{1.3}$$

We then tested these effective slip length formulae with numerical simulations using the finite element method. The tests confirmed that the formula are excellent approximations in their respective limits. For example, if L is 5% of P, then $b_{\text{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1}$ is within 1% of the effective slip length calculated from the FEM simulation. The numerics also revealed that the harmonic mean formula is a surprisingly good approximation in the case where L is of the same order as b, and both are smaller than P. Finally, the numerics suggested that the simple mean formula is a good approximation only when b is on the order of two orders of magnitude smaller than L, which itself is much smaller than P.

To summarise:

$$\text{If } L \ll P, b: \qquad b_{\text{eff}} \simeq \left\langle \frac{\sqrt{1+|\nabla h|^2}}{b} \right\rangle^{-1} \tag{1.4}$$

$$\text{If } L \sim b \ll P: \qquad b_{\text{eff}} \approx \left\langle \frac{\sqrt{1+|\nabla h|^2}}{b} \right\rangle^{-1} \tag{1.5}$$

$$\text{If } b \ll L \ll P: \qquad b_{\text{eff}} \simeq \langle b \rangle \tag{1.6}$$

1.2 Consequences

What are the consequences of these formulae for the engineers of, say, nanostructured superhydrophobic surfaces?

Consider a **binary** surface, composed of two different surface types, low-slip regions (eg. Teflon), and high-slip regions (eg. air gaps). Let ϕ be the area fraction of the surface that is occupied the low-slip region. Then given fixed intrinsic slip lengths b_{low} and b_{high} for the two regions, the two effective slip expressions are:

$$b_{\mathrm{eff}} = \left\langle \frac{1}{b} \right\rangle^{-1} = \left[\phi \frac{1}{b_{\mathrm{low}}} + (1-\phi) \frac{1}{b_{\mathrm{high}}} \right]^{-1} \tag{1.7}$$

and

$$b_{\mathrm{eff}} = \langle b \rangle = \phi b_{\mathrm{low}} + (1-\phi) b_{\mathrm{high}} \tag{1.8}$$

We plot the predicted effective slip lengths as a function of ϕ in Figure (1.1).

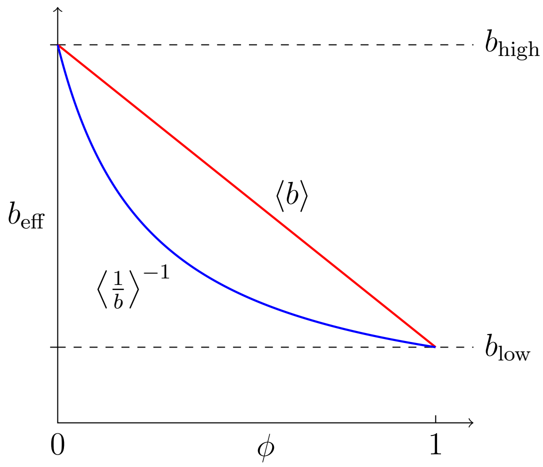

Figure 1.1: The harmonic mean b_{eff} formula (blue), and mean formula (red, straight), as functions of ϕ, for a flat binary surface where ϕ is the area fraction with b_{low}.

The graph of Figure (1.1) is possibly bad news for a nanoengineer aiming to maximise effective slip. A typical nanoengineering effort may involve a nanopatterned surface, say nanogrooves, with a period of tens of nanometers, on the wall of a micron-sized pipe. Air trapped in the nanogrooves creates a liquid-gas interface with a slip length on the order of microns. A credible slip length for the solid surface is perhaps 20 nm (see Chapter 2). Then $L \ll P, b$ or $L \sim b \ll P$, and the harmonic mean formula of Equation (1.7) applies. As Figure (1.1) shows, b_{eff} is **dominated by the lowest slip present,** and a large b_{eff} is achieved only with a very small fraction of low-slip surface.

1.3 Future Work

We have mathematically rigorous results for an approximate b_{eff} that is a good approximation if $L \ll P$, and a progressively better approximation as L/P gets smaller. Assuming $L \ll P$, we have a perturbative approximation that in the limit of b vanishing, b_{eff} is given by the simple average.

We do not have any mathematically rigorous results for regimes where $L \sim P$ or $L > P$. These regimes could apply in some lubrication systems for example. The concept of effective slip length could be different in these situations – it may not arise from the diffusion of momentum, but could be some kind of 'forced' average caused by the constraints of the physical system. Work on these regimes is a possibility for the future.

Finally, the homogenization technique is a very powerful method that can be applied to many problems featuring periodic heterogeneous media. Finding further applications of homogenization is of definite future interest.