Q1

$$\frac{dy}{dx} = -2xy^2 \rightarrow \frac{dy}{y^2} = -2x\,dx \rightarrow \int \frac{dy}{y^2} = \int -2x\,dx$$ 5pt 5pt

$$-y^{-1} + C_1 = -2\left(\frac{x^2}{2} + C_2\right) \rightarrow -y^{-1} = -x^2 + \underbrace{2C_2 - C_1}_{C_3} \rightarrow y = \frac{1}{x^2 - C_3}$$ 5pt 5pt 5pt

Q2

$$\frac{dy}{dx} = -6y \rightarrow \frac{dy}{y} = -6\,dx \rightarrow \int \frac{dy}{y} = -6\int dx \rightarrow \ln y + C_1 = -6(x + C_2)$$ 5pt 5pt 5pt

$$\ln y = -6x - \underbrace{6C_2 - C_1}_{C_3} \rightarrow y = e^{-6x}\cdot \underbrace{e^{C_3}}_{C_4} \rightarrow y = C_4 \cdot e^{-6x}$$ 5pt 5pt

Q4

$$\frac{dy}{dx} = \frac{1+y}{1+x} \rightarrow \frac{dy}{1+y} = \frac{dx}{1+x}$$ 5pt

SUBSTITUIÇÃO: $u = 1+y \rightarrow du = dy$; $v = 1+x \rightarrow dv = dx$

$$\int \frac{du}{u} = \int \frac{dv}{v}$$ 5pt

$$\ln u + C_1 = \ln v + C_2 \rightarrow \ln u = \ln v + \underbrace{C_2 - C_1}_{C_3} \rightarrow u = e^{\ln v}\cdot \underbrace{e^{C_3}}_{C_4}$$ 5pt

$$u = v \cdot C_4 \rightarrow y+1 = (1+x)C_4 \rightarrow y = (1+x)C_4 - 1 \rightarrow y = C_4 x + \underbrace{C_4 - 1}_{C_5}$$ 5pt 5pt

Q3

$$\frac{dy}{dx} - 2xy = x \rightarrow \frac{dy}{1+2y} = x\,dx$$ 5pt

SUBSTITUIÇÃO: $u = 1+2y$

$du = 2\,dy \rightarrow dy = \frac{du}{2}$ 5pt

$$\rightarrow \ln u + \underbrace{C_1}_{C_3} = 2\left(\frac{x^2}{2} + \underbrace{C_2}_{C_3}\right)$$

$$\ln u = x^2 + C_3 \rightarrow u = e^{x^2}\cdot \underbrace{e^{C_3}}_{C_4} \rightarrow 2y + 1 = e^{x^2}\cdot C_4 \rightarrow y = \underbrace{2C_4}_{C_5} e^{x^2} - 1/2 \rightarrow y = C_5 e^{x^2} - 1/2$$ 5pt 10pt

Q5

$$(y^2+1)x\,dx + (x+1)y\,dy = 0 \rightarrow \frac{y\,dy}{(y^2+1)} = \frac{-x\,dx}{x-1} \rightarrow \int \frac{y\,dy}{(y^2+1)} = -\int \frac{x\,dx}{x+1}$$ 5pt 5pt

SUBSTITUIÇÃO: $u = y^2+1$; $du = 2y\,dy$; $\frac{du}{2} = y\,dy$ 5pt

$$\int \frac{du/2}{u} = -\int \frac{x\,dx}{x+1} \rightarrow \frac{1}{2}\ln u + C_1 = -1\left(x - \ln(1+x) + C_2\right) \rightarrow \ln u = -2x + 2\ln(1+x) + \underbrace{2C_2 + C_1}_{C_3}$$ 5pt

$$\ln u = -2x + \ln(1+x)^2 + C_3 \rightarrow u = e^{-2x}\cdot e^{\ln(1+x)^2}\cdot \underbrace{e^{C_3}}_{C_4} \rightarrow u = e^{-2x}(1+x)^2 \cdot C_4$$

$$y^2 + 1 = e^{-2x}(1+x)^2 \cdot C_4 \rightarrow y = \pm\sqrt{C_4(1+x)^2 \cdot e^{-2x} - 1}$$ 5pt