PROBLEMS

2-1 An LTIC system is specified by the equation

$$(D^2 + 5D + 6)y(t) = (D + 1)x(t)$$

(a) Find the characteristic polynomial, characteristic equation, characteristic roots, and characteristic modes of this system.

(b) Find $y_0(t)$, the zero-input component of the response $y(t)$ for $t \geq 0$, if the initial conditions are $y_0(0^-) = 2$ and $\dot{y}_0(0^-) = -1$.

2-2 Repeat Prob. 2-1 for

$$(D^2 + 4D + 4)y(t) = Dx(t)$$

and $y_0(0^-) = 3$, $\dot{y}_0(0^-) = -4$.

2-3 Repeat Prob. 2-1 for

$$D(D + 1)y(t) = (D + 2)x(t)$$

and $y_0(0^-) = \dot{y}_0(0^-) = 1$.

2-4 Repeat Prob. 2-1 for

$$(D^2 + 9)y(t) = (3D + 2)x(t)$$

and $y_0(0^-) = 0$, $\dot{y}_0(0^-) = 6$.

2-5 Repeat Prob. 2-1 for

$$(D^2 + 4D + 13)y(t) = 4(D + 2)x(t)$$

with $y_0(0^-) = 5$, $\dot{y}_0(0^-) = 15.98$.

2-6 Repeat Prob. 2-1 for

$$D^2(D + 1)y(t) = (D^2 + 2)x(t)$$

with $y_0(0^-) = 4$, $\dot{y}_0(0^-) = 3$ and $\ddot{y}_0(0^-) = -1$.

2-7 Repeat Prob. 2-1 for

$$(D + 1)(D^2 + 5D + 6)y(t) = Dx(t)$$

with $y_0(0^-) = 2$, $\dot{y}_0(0^-) = -1$ and $\ddot{y}_0(0^-) = 5$.

2-8 A system is described by a constant-coefficient linear differential equation and has zero-input response given by $y_0(t) = 2e^{-t} + 3$.

(a) Is it possible for the system's characteristic equation to be $\lambda + 1 = 0$? Justify your answer.

(b) Is it possible for the system's characteristic equation to be $\sqrt{3}(\lambda^2 + \lambda) = 0$? Justify your answer.

(c) Is it possible for the system's characteristic equation to be $\lambda(\lambda + 1)^2 = 0$? Justify your answer.

2-9 Find the unit impulse response of a system specified by the equation

$$(D^2 + 4D + 3)y(t) = (D + 5)x(t)$$

2-10 Repeat Prob. 2-9 for

$$(D^2 + 5D + 6)y(t) = (D^2 + 7D + 11)x(t)$$

2-11 Repeat Prob. 2-9 for the first-order allpass filter specified by the equation

$$(D + 1)y(t) = -(D - 1)x(t)$$

2-12 Find the unit impulse response of an LTIC system specified by the equation

$$(D^2 + 6D + 9)y(t) = (2D + 9)x(t)$$

2-13 If $c(t) = x(t) * g(t)$, then show that $A_c = A_x A_g$, where A_x, A_g, and A_c are the areas under $x(t)$, $g(t)$, and $c(t)$, respectively. Verify this *area property* of convolution in Examples 2.7 and 2.9.

2-14 If $x(t) * g(t) = c(t)$, then show that $x(at) * g(at) = |1/a|c(at)$. This *time-scaling property* of convolution states that if both $x(t)$ and $g(t)$ are time-scaled by a, their convolution is also time-scaled by a (and multiplied by $|1/a|$).

2-15 Show that the convolution of an odd and an even function is an odd function and the convolution of two odd or two even functions is an even function. [Hint: Use time-scaling property of convolution in Prob. 2-14.]

2-16 Using direct integration, find $e^{-at}u(t) * e^{-bt}u(t)$.

2-17 Using direct integration, find $u(t) * u(t)$, $e^{-at}u(t) * e^{-at}u(t)$, and $tu(t) * u(t)$.

2-18 Using direct integration, find $\sin t\, u(t) * u(t)$ and $\cos t\, u(t) * u(t)$.

2-19 The unit impulse response of an LTIC system is

$$h(t) = e^{-t}u(t)$$

Find this system's (zero-state) response $y(t)$ if the input $x(t)$ is:

(a) $u(t)$
(b) $e^{-t}u(t)$
(c) $e^{-2t}u(t)$
(d) $\sin 3t\, u(t)$

Use the convolution table (Table 2.1) to find your answers.

2-20 Repeat Prob. 2-19 for

$$h(t) = [2e^{-3t} - e^{-2t}]u(t)$$

and if the input $x(t)$ is:

(a) $u(t)$
(b) $e^{-t}u(t)$
(c) $e^{-2t}u(t)$

2-21 Repeat Prob. 2-19 for

$$h(t) = e^{-t}u(t)$$

and each of the following inputs $x(t)$:

(a) $e^{-2t}u(t)$
(b) $e^{-2(t-3)}u(t)$
(c) $e^{-2t}u(t-3)$
(d) The gate pulse depicted in Fig. P2-21—and provide a sketch of $y(t)$.

2-22 A first-order allpass filter impulse response is given by

$$h(t) = -\delta(t) + 2e^{-t}u(t)$$

(a) Find the zero-state response of this filter for the input $e^{t}u(-t)$.

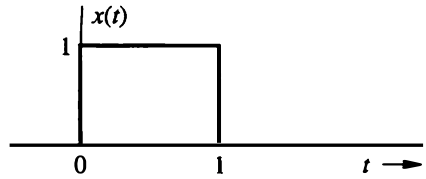


Figure P2-21

(b) Sketch the input and the corresponding zero-state response.

2-23 Figure P2-23 shows the input $x(t)$ and the impulse response $h(t)$ for an LTIC system. Let the output be $y(t)$.

(a) By inspection of $x(t)$ and $h(t)$, find $y(-1)$, $y(0)$, $y(1)$, $y(2)$, $y(3)$, $y(4)$, $y(5)$, and $y(6)$. Thus, by merely examining $x(t)$ and $h(t)$, you are required to see what the result of convolution yields at $t = -1, 0, 1, 2, 3, 4, 5$, and 6.
(b) Find the system response to the input $x(t)$.

2-24 The zero-state response of an LTIC system to an input $x(t) = 2e^{-2t}u(t)$ is $y(t) = [4e^{-2t} + 6e^{-3t}]u(t)$. Find the impulse response of the system. [Hint: We have not yet developed a method of finding $h(t)$ from the knowledge of the input and the corresponding output. Knowing the form of $x(t)$ and $y(t)$, you will have to make the best guess of the general form of $h(t)$.]

2-25 Sketch the functions $x(t) = 1/(t^2 + 1)$ and $u(t)$. Now find $x(t) * u(t)$ and sketch the result.

2-26 Figure P2-26 shows $x(t)$ and $g(t)$. Find and sketch $c(t) = x(t) * g(t)$.

2-27 Find and sketch $c(t) = x(t) * g(t)$ for the functions depicted in Fig. P2-27.

2-28 Find and sketch $c(t) = x_1(t) * x_2(t)$ for the pairs of functions illustrated in Fig. P2-28.

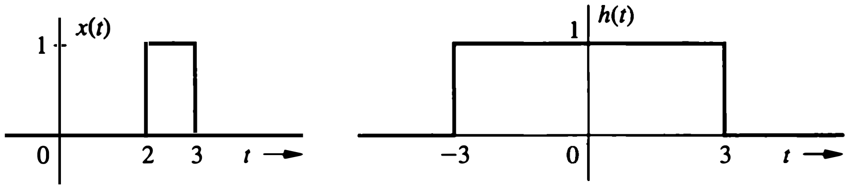


Figure P2-23

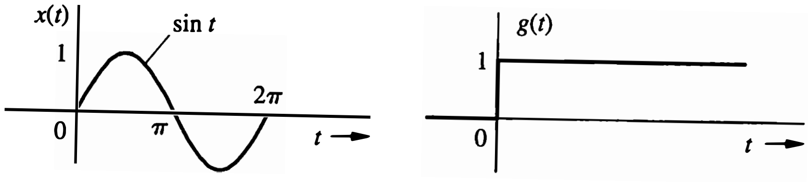


Figure P2-26

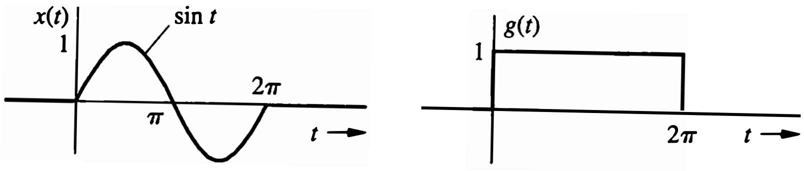


Figure P2-27

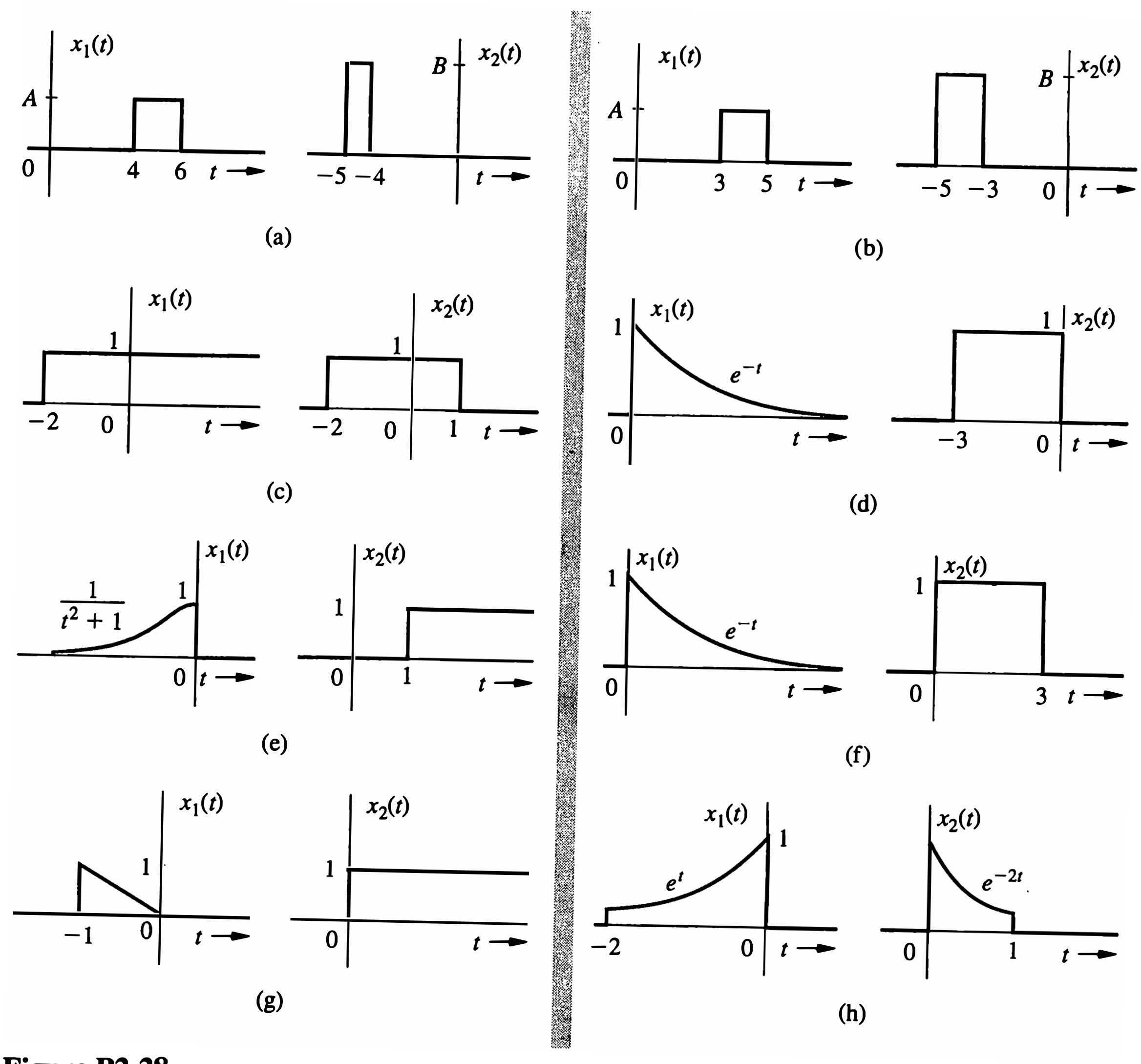
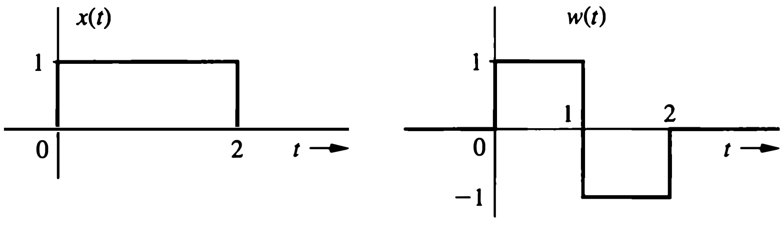


Figure P2-28



(a) (b) **Figure P2-29**

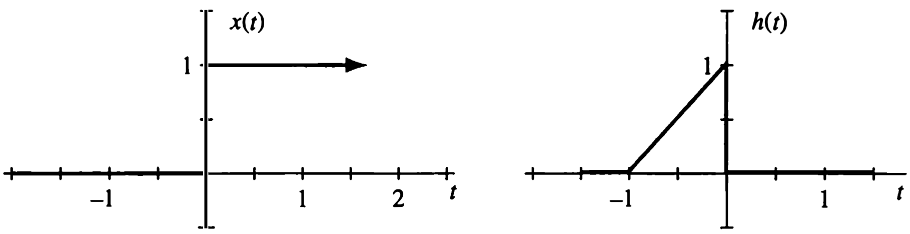


Figure P2-31 Analog signals $x(t)$ and $h(t)$.

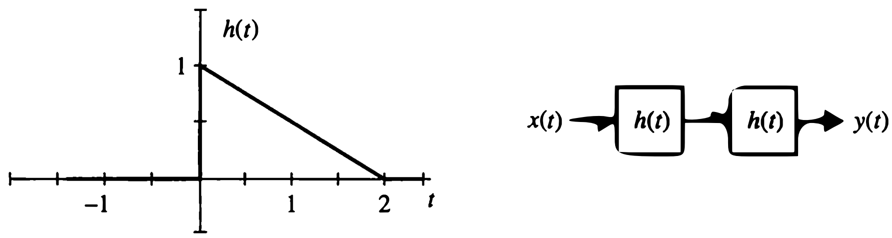


Figure P2-32 Impulse response and cascade system.

2-29 Use Eq. (2.46) to find the convolution of $x(t)$ and $w(t)$, shown in Fig. P2-29.

2-30 Determine $H(s)$, the transfer function of an ideal time delay of T seconds. Find your answer by two methods: using Eq. (2.48) and using Eq. (2.49).

2-31 Determine $y(t) = x(t) * h(t)$ for the signals depicted in Fig. P2-31.

2-32 Two linear time-invariant systems, each with impulse response $h(t)$, are connected in cascade. Refer to Fig. P2-32. Given input $x(t) = u(t)$, determine $y(1)$. That is, determine the step response at time $t = 1$ for the cascaded system shown.

2-33 Consider the electric circuit shown in Fig. P2-33.

(a) Determine the differential equation that relates the input $x(t)$ to output $y(t)$. Recall that

$$i_C(t) = C\frac{dv_C}{dt} \quad \text{and} \quad v_L(t) = L\frac{di_L}{dt}$$

(b) Find the characteristic equation for this circuit, and express the root(s) of the characteristic equation in terms of L and C.

(c) Determine the zero-input response given an initial capacitor voltage of one volt and an initial inductor current of zero amps. That is, find $y_0(t)$ given $v_C(0) = 1$ V and $i_L(0) = 0$ A. [Hint: The coefficient(s) in $y_0(t)$ are independent of L and C.]

(d) Plot $y_0(t)$ for $t \geq 0$. Does the zero-input response, which is caused solely by initial conditions, ever "die" out?

(e) Determine the total response $y(t)$ to the input $x(t) = e^{-t}u(t)$. Assume an initial inductor current of $i_L(0^-) = 0$ A, an

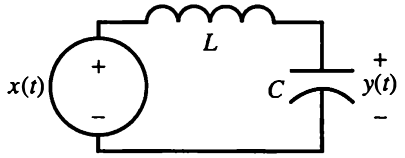


Figure P2-33 *LC* circuit.

initial capacitor voltage of $v_C(0^-) = 1$ V, $L = 1$ H, and $C = 1$ F.

2-34 Two LTIC systems have impulse response functions given by $h_1(t) = (1-t)[u(t) - u(t-1)]$ and $h_2(t) = t[u(t+2) - u(t-2)]$.

(a) Carefully sketch the functions $h_1(t)$ and $h_2(t)$.

(b) Assume that the two systems are connected in parallel as shown in Fig. P2-34a. Carefully plot the equivalent impulse response function, $h_p(t)$.

(c) Assume that the two systems are connected in cascade as shown in Fig. P2-34b. Carefully plot the equivalent impulse response function, $h_s(t)$.

2-35 Consider the circuit shown in Fig. P2-35.

(a) Find the output $y(t)$ given an initial capacitor voltage of $y(0) = 2$ V and an input $x(t) = u(t)$.

(b) Given an input $x(t) = u(t-1)$, determine the initial capacitor voltage $y(0)$ so that the output $y(t)$ is 0.5 volt at $t = 2$ seconds.

2-36 An analog signal is given by $x(t) = t[u(t) - u(t-1)]$, as shown in Fig. P2-36. Determine and plot $y(t) = x(t) * x(2t)$

2-37 Consider the electric circuit shown in Fig. P2-37.

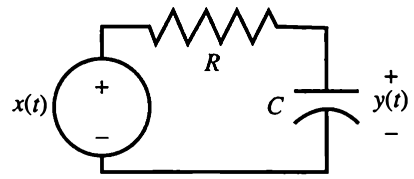


Figure P2-35 *RC* circuit.

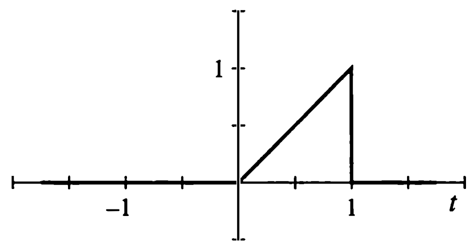


Figure P2-36 Short-duration ramp signal $x(t)$.

(a) Determine the differential equation that relates the input current $x(t)$ to output current $y(t)$. Recall that

$$v_L = L\frac{di_L}{dt}$$

(b) Find the characteristic equation for this circuit, and express the root(s) of the characteristic equation in terms of L_1, L_2, and R.

(c) Determine the zero-input response given initial inductor currents of one ampere each. That is, find $y_0(t)$ given $i_{L_1}(0) = i_{L_2}(0) = 1$ A.

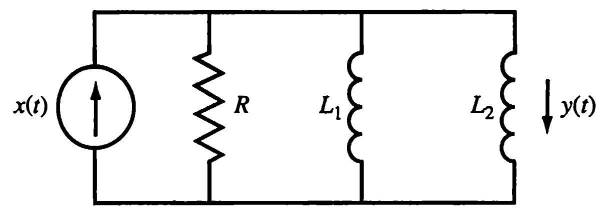


Figure P2-37 *RLL* circuit.

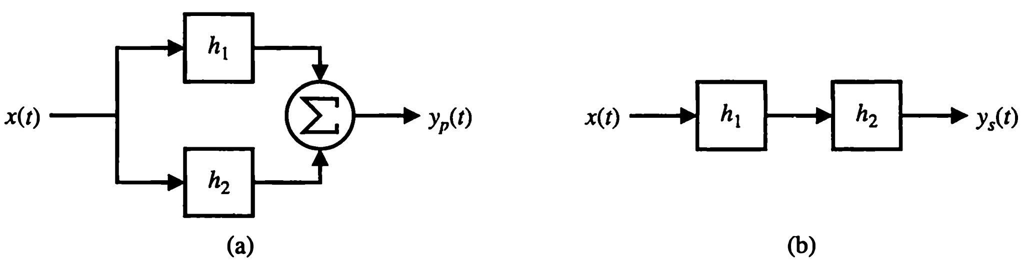


Figure P2-34 **(a)** Parallel and **(b)** series connections.

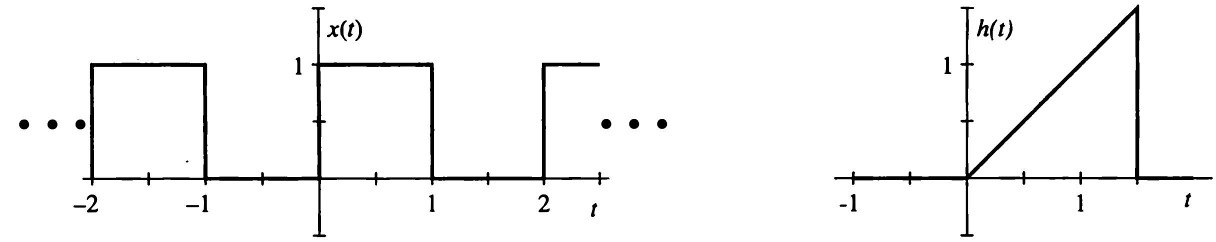


Figure P2-39 Periodic input $x(t)$.

2-38 An LTI system has step response given by $g(t) = e^{-t}u(t) - e^{-2t}u(t)$. Determine the output of this system $y(t)$ given an input $x(t) = \delta(t - \pi) - \cos(\sqrt{3})u(t)$.

2-39 The periodic signal $x(t)$ shown in Fig. P2-39 is input to a system with impulse response function $h(t) = t[u(t) - u(t - 1.5)]$, also shown in Fig. P2-39. Use convolution to determine the output $y(t)$ of this system. Plot $y(t)$ over $(-3 \le t \le 3)$.

2-40 Consider the electric circuit shown in Fig. P2-40.

(a) Determine the differential equation relating input $x(t)$ to output $y(t)$.

(b) Determine the output $y(t)$ in response to the input $x(t) = 4te^{-3t/2}u(t)$. Assume component values of $R = 1\ \Omega$, $C_1 = 1$ F, and $C_2 = 2$ F, and initial capacitor voltages of $V_{C_1} = 2$ V and $V_{C_2} = 1$ V.

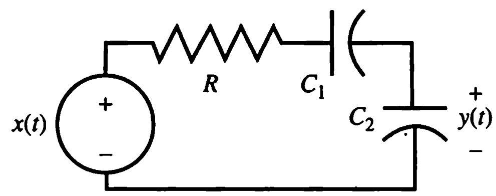


Figure P2-40 *RCC* circuit.

2-41 A cardiovascular researcher is attempting to model the human heart. He has recorded ventricular pressure, which he believes corresponds to the heart's impulse response function $h(t)$, as shown in Fig. P2-41. Comment on the function $h(t)$ shown in Fig. P2-41. Can you establish any system properties, such as causality or stability? Do the data suggest any reason to suspect that the measurement is not a true impulse response?

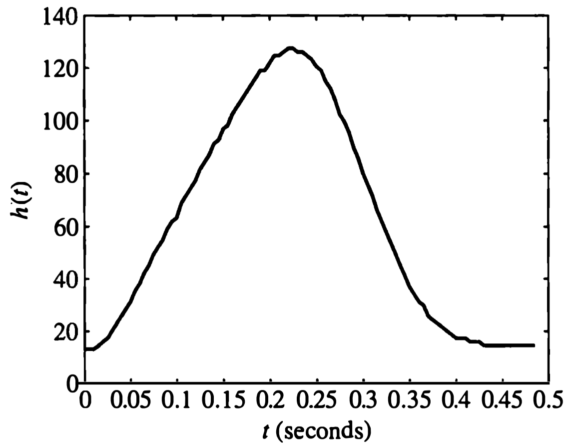


Figure P2-41 Measured impulse response function.

2-42 The autocorrelation of a function $x(t)$ is given by $r_{xx}(t) = \int_{-\infty}^{\infty} x(\tau)x(\tau - t)\,d\tau$. This equation is computed in a manner nearly identical to convolution.

(a) Show $r_{xx}(t) = x(t) * x(-t)$

(b) Determine and plot $r_{xx}(t)$ for the signal $x(t)$ depicted in Fig. P2-42. [Hint: $r_{xx}(t) = r_{xx}(-t)$.]

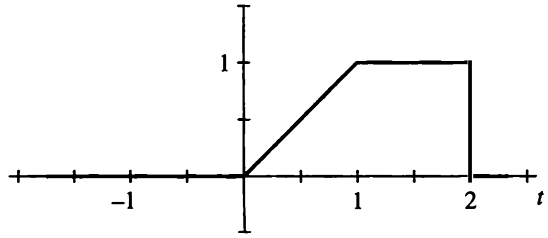


Figure P2-42 Analog signal $x(t)$.

2-43 Consider the circuit shown in Fig. P2-43. This circuit functions as an integrator. Assume ideal op-amp behavior and recall that

$$i_C = C\frac{dV_C}{dt}$$

(a) Determine the differential equation that relates the input $x(t)$ to the output $y(t)$.
(b) This circuit does not behave well at dc. Demonstrate this by computing the zero-state response $y(t)$ for a unit step input $x(t) = u(t)$.

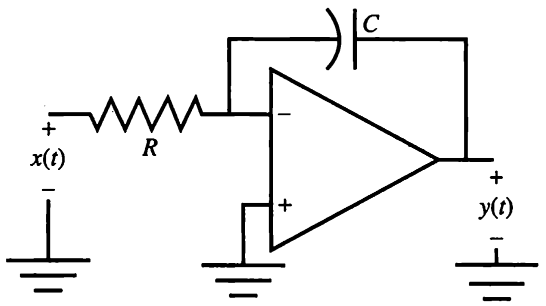


Figure P2-43 Integrator op-amp circuit.

2-44 Derive the result in Eq. (2.46) in another way. As mentioned in Chapter 1 (Fig. 1.27b), it is possible to express an input in terms of its step components, as shown in Fig. P2-44. Find the system response as a sum of the responses to the step components of the input.

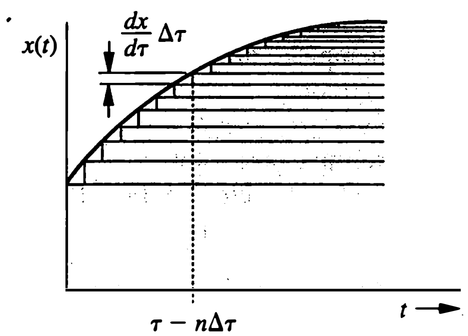


Figure P2-44

2-45 Show that an LTIC system response to an everlasting sinusoid $\cos \omega_0 t$ is given by

$$y(t) = |H(j\omega_0)| \cos [\omega_0 t + \angle H(j\omega_0)]$$

where

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}\, dt$$

assuming the integral on the right-hand side exists.

2-46 A line charge is located along the x axis with a charge density $Q(x)$ coulombs per meter. Show that the electric field $E(x)$ produced by this line charge at a point x is given by

$$E(x) = Q(x) * h(x)$$

where $h(x) = 1/4\pi\epsilon x^2$. [*Hint:* The charge over an interval $\Delta\tau$ located at $\tau = n\Delta\tau$ is $Q(n\Delta\tau)\Delta\tau$. Also by Coulomb's law, the electric field $E(r)$ at a distance r from a charge q coulombs is given by $E(r) = q/4\pi\epsilon r^2$.]

2-47 Consider the circuit shown in Fig. P2-47. Assume ideal op-amp behavior and recall that

$$i_C = C\frac{dV_C}{dt}$$

Without a feedback resistor R_f, the circuit functions as an integrator and is unstable, particularly at dc. A feedback resistor R_f corrects this problem and results in a stable circuit that functions as a "lossy" integrator.
(a) Determine the differential equation that relates the input $x(t)$ to the output $y(t)$. What is the corresponding characteristic equation?
(b) To demonstrate that this "lossy" integrator is well behaved at dc, determine the zero-state response $y(t)$ given a unit step input $x(t) = u(t)$.
(c) Investigate the effect of 10% resistor and 25% capacitor tolerances on the system's characteristic root(s).

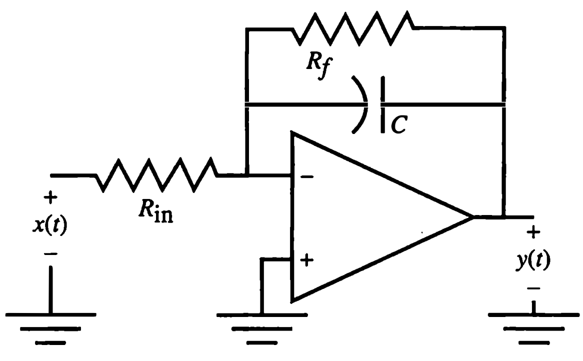


Figure P2-47 Lossy integrator op-amp circuit.

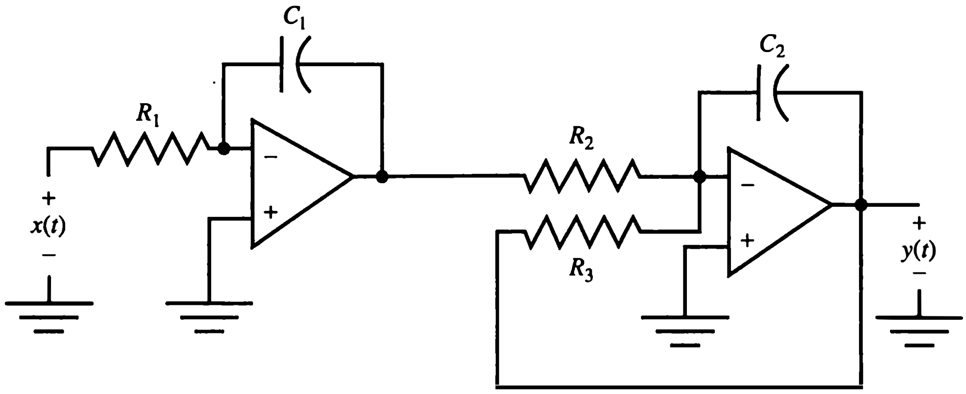


Figure P2-48 Op-amp circuit.

2-48 Consider the electric circuit shown in Fig. P2-48. Let $C_1 = C_2 = 10\ \mu$F, $R_1 = R_2 = 100$ kΩ, and $R_3 = 50$ kΩ.

(a) Determine the corresponding differential equation describing this circuit. Is the circuit BIBO stable?

(b) Determine the zero-input response $y_0(t)$ if the output of each op amp initially reads one volt.

(c) Determine the zero-state response $y(t)$ to a step input $x(t) = u(t)$.

(d) Investigate the effect of 10% resistor and 25% capacitor tolerances on the system's characteristic roots.

2-49 A system is called complex if a real-valued input can produce a complex-valued output. Suppose a linear, time-invariant complex system has impulse response $h(t) = j[u(-t + 2) - u(-t)]$.

(a) Is this system causal? Explain.

(b) Use convolution to determine the zero-state response $y_1(t)$ of this system in response to the unit-duration pulse $x_1(t) = u(t) - u(t - 1)$.

(c) Using the result from part b, determine the zero-state response $y_2(t)$ in response to $x_2(t) = 2u(t-1) - u(t-2) - u(t-3)$.

2-50 Use the classical method to solve

$$(D^2 + 7D + 12)y(t) = (D + 2)x(t)$$

for initial conditions of $y(0^+) = 0, \dot{y}(0^+) = 1$, and input $x(t)$ of

(a) $u(t)$

(b) $e^{-t}u(t)$

(c) $e^{-2t}u(t)$

2-51 Using the classical method, solve

$$(D^2 + 6D + 25)y(t) = (D + 3)x(t)$$

for the initial conditions of $y(0^+) = 0, \dot{y}(0^+) = 2$ and input $x(t) = u(t)$.

2-52 Using the classical method, solve

$$(D^2 + 4D + 4)y(t) = (D + 1)x(t)$$

for initial conditions of $y(0^+) = 9/4, \dot{y}(0^+) = 5$ and input $x(t)$ of

(a) $e^{-3t}u(t)$

(b) $e^{-t}u(t)$

2-53 Using the classical method, solve

$$(D^2 + 2D)y(t) = (D + 1)x(t)$$

for initial conditions of $y(0^+) = 2, \dot{y}(0^+) = 1$ and input of $x(t) = u(t)$.

2-54 Repeat Prob. 2-50 for the input

$$x(t) = e^{-3t}u(t)$$

2-55 Explain, with reasons, whether the LTIC systems described by the following equations are (i) stable or unstable in the BIBO sense;

(ii) asymptotically stable, unstable, or marginally stable. Assume that the systems are controllable and observable.

(a) $(D^2 + 8D + 12)y(t) = (D - 1)x(t)$
(b) $D(D^2 + 3D + 2)y(t) = (D + 5)x(t)$
(c) $D^2(D^2 + 2)y(t) = x(t)$
(d) $(D+1)(D^2-6D+5)y(t) = (3D+1)x(t)$

2-56 Repeat Prob. 2-55 for the following.

(a) $(D + 1)(D^2 + 2D + 5)^2y(t) = x(t)$
(b) $(D + 1)(D^2 + 9)y(t) = (2D + 9)x(t)$
(c) $(D + 1)(D^2 + 9)^2y(t) = (2D + 9)x(t)$
(d) $(D^2 + 1)(D^2 + 4)(D^2 + 9)y(t) = 3Dx(t)$

2-57 For a certain LTIC system, the impulse response $h(t) = u(t)$.

(a) Determine the characteristic root(s) of this system.
(b) Is this system asymptotically or marginally stable, or is it unstable?
(c) Is this system BIBO stable?
(d) What can this system be used for?

2-58 In Section 2.6 we demonstrated that for an LTIC system, condition (2.64) is sufficient for BIBO stability. Show that this is also a necessary condition for BIBO stability in such systems. In other words, show that if condition (2.64) is not satisfied, then there exists a bounded input that produces an unbounded output. [Hint: Assume that a system exists for which $h(t)$ violates condition (2.64) and yet produces an output that is bounded for every bounded input. Establish the contradiction in this statement by considering an input $x(t)$ defined by $x(t_1 - \tau) = 1$ when $h(\tau) \geq 0$ and $x(t_1 - \tau) = -1$ when $h(\tau) < 0$, where t_1 is some fixed instant.]

2-59 An analog LTIC system with impulse response function $h(t) = u(t+2) - u(t-2)$ is presented with an input $x(t) = t(u(t) - u(t-2))$.

(a) Determine and plot the system output $y(t) = x(t) * h(t)$.
(b) Is this system stable? Is this system causal? Justify your answers.

2-60 A system has an impulse response function shaped like a rectangular pulse, $h(t) = u(t) - u(t-1)$. Is the system stable? Is the system causal?

2-61 A continuous-time LTI system has impulse response function $h(t) = \sum_{i=0}^{\infty}(0.5)^i\delta(t - i)$.

(a) Is the system causal? Prove your answer.
(b) Is the system stable? Prove your answer.

2-62 Data at a rate of 1 million pulses per second are to be transmitted over a certain communications channel. The unit step response $g(t)$ for this channel is shown in Fig. P2-62.

(a) Can this channel transmit data at the required rate? Explain your answer.
(b) Can an audio signal consisting of components with frequencies up to 15 kHz be transmitted over this channel with reasonable fidelity?

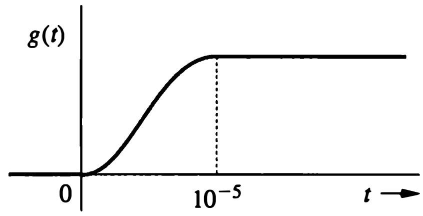


Figure P2-62

2-63 A certain communication channel has a bandwidth of 10 kHz. A pulse of 0.5 ms duration is transmitted over this channel.

(a) Determine the width (duration) of the received pulse.
(b) Find the maximum rate at which these pulses can be transmitted over this channel without interference between the successive pulses.

2-64 A first-order LTIC system has a characteristic root $\lambda = -10^4$.

(a) Determine T_r, the rise time of its unit step input response.
(b) Determine the bandwidth of this system.
(c) Determine the rate at which the information pulses can be transmitted through this system.

2-65 Consider a linear, time-invariant system with impulse response $h(t)$ shown in Fig. P2-65. Outside the interval shown, $h(t) = 0$.

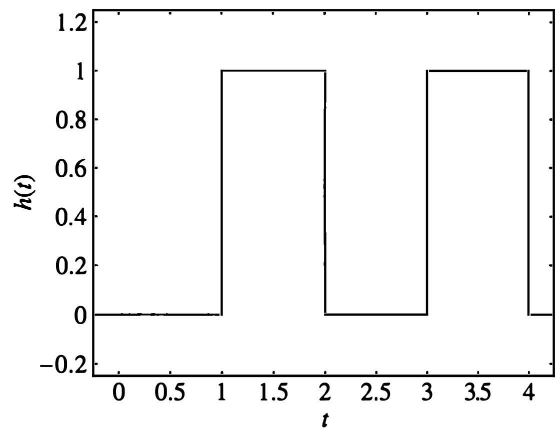


Figure P2-65 Impulse response $h(t)$.

(a) What is the rise time T_r of this system? Remember, rise time is the time between the application of a unit step and the moment at which the system has "fully" responded.

(b) Suppose $h(t)$ represents the response of a communication channel. What conditions might cause the channel to have such an impulse response? What is the maximum average number of pulses per unit time that can be transmitted without causing interference? Justify your answer.

(c) Determine the system output $y(t) = x(t) * h(t)$ for $x(t) = [u(t-2) - u(t)]$. Accurately sketch $y(t)$ over $(0 \le t \le 10)$.