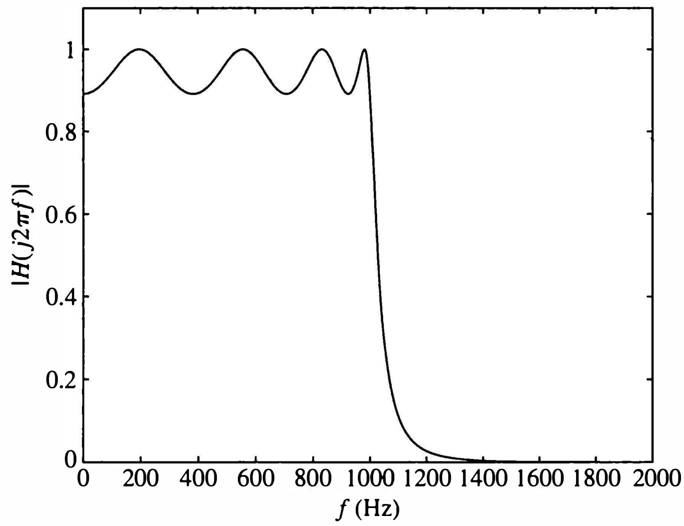


Figure M4.9 Magnitude responses for an order-8 Chebyshev LPF with $f_c = 1$ kHz and $r = 1$ dB.

For higher-order filters, polynomial rooting may not provide reliable results. Fortunately, Chebyshev roots can also be determined analytically. For

$$\phi_k = \frac{2k+1}{2N}\pi \qquad \text{and} \qquad \xi = \frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)$$

the Chebyshev poles are

$$p_k = \omega_c \sinh(\xi)\sin(\phi_k) + j\omega_c \cosh(\xi)\cos(\phi_k)$$

Continuing the same example, the poles are recomputed and again plotted. The result is identical to Fig. M4.8.

```
>> k = [1:N]; xi = 1/N*asinh(1/epsilon); phi = (k*2-1)/(2*N)*pi;
>> C_poles = omega_c*(-sinh(xi)*sin(phi)+j*cosh(xi)*cos(phi));
>> plot(real(C_poles),imag(C_poles),'kx'); axis equal;
>> axis(omega_c*[-1.1 1.1 -1.1 1.1]);
>> xlabel('\sigma'); ylabel('\omega');
```

As in the case of high-order Butterworth filters, a cascade of second-order filter sections facilitates practical implementation of Chebyshev filters. Problems 4.M-3 and 4.M-6 use second-order Sallen–Key circuit stages to investigate such implementations.

PROBLEMS

4-1 By direct integration [Eq. (4.1)] find the Laplace transforms and the region of convergence of the following functions:

(a) $u(t) - u(t-1)$
(b) $te^{-t}u(t)$
(c) $t\cos\omega_0 t\, u(t)$
(d) $(e^{2t} - 2e^{-t})u(t)$
(e) $\cos\omega_1 t \cos\omega_2 t\, u(t)$
(f) $\cosh(at)\, u(t)$
(g) $\sinh(at)\, u(t)$
(h) $e^{-2t}\cos(5t+\theta)\, u(t)$

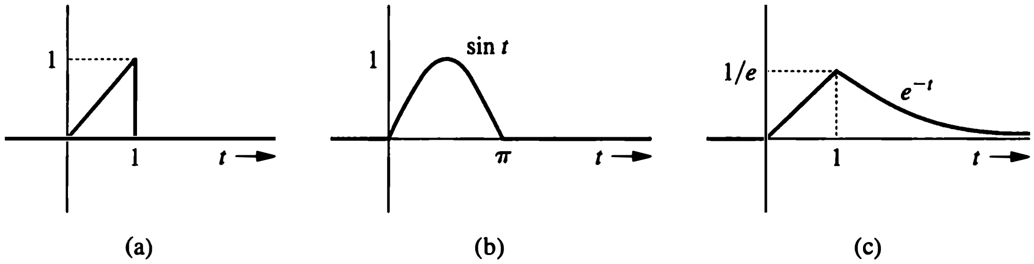


Figure P4-2

4-2 By direct integration find the Laplace transforms of the signals shown in Fig. P4-2.

4-3 Find the inverse (unilateral) Laplace transforms of the following functions:

(a) $\dfrac{2s+5}{s^2+5s+6}$

(b) $\dfrac{3s+5}{s^2+4s+13}$

(c) $\dfrac{(s+1)^2}{s^2-s-6}$

(d) $\dfrac{5}{s^2(s+2)}$

(e) $\dfrac{2s+1}{(s+1)(s^2+2s+2)}$

(f) $\dfrac{s+2}{s(s+1)^2}$

(g) $\dfrac{1}{(s+1)(s+2)^4}$

(h) $\dfrac{s+1}{s(s+2)^2(s^2+4s+5)}$

(i) $\dfrac{s^3}{(s+1)^2(s^2+2s+5)}$

4-4 Find the Laplace transforms of the following functions using only Table 4.1 and the time-shifting property (if needed) of the unilateral Laplace transform:

(a) $u(t) - u(t-1)$

(b) $e^{-(t-\tau)}u(t-\tau)$

(c) $e^{-(t-\tau)}u(t)$

(d) $e^{-t}u(t-\tau)$

(e) $te^{-t}u(t-\tau)$

(f) $\sin[\omega_0(t-\tau)]\, u(t-\tau)$

(g) $\sin[\omega_0(t-\tau)]\, u(t)$

(h) $\sin \omega_0 t\, u(t-\tau)$

4-5 Using only Table 4.1 and the time-shifting property, determine the Laplace transform of the signals in Fig. P4-2. [Hint: See Section 1.4 for discussion of expressing such signals analytically.]

4-6 The Laplace transform of a causal periodic signal can be determined from the knowledge of the Laplace transform of its first cycle (period).

(a) If the Laplace transform of $x(t)$ in Fig. P4-6a is $X(s)$, then show that $G(s)$, the Laplace transform of $g(t)$ (Fig. P4-6b), is

$$G(s) = \frac{X(s)}{1 - e^{-sT_0}} \qquad \operatorname{Re} s > 0$$

(b) Use this result to find the Laplace transform of the signal $p(t)$ illustrated in Fig. P4-6c.

4-7 Starting only with the fact that $\delta(t) \Longleftrightarrow 1$, build pairs 2 through 10b in Table 4.1, using various properties of the Laplace transform.

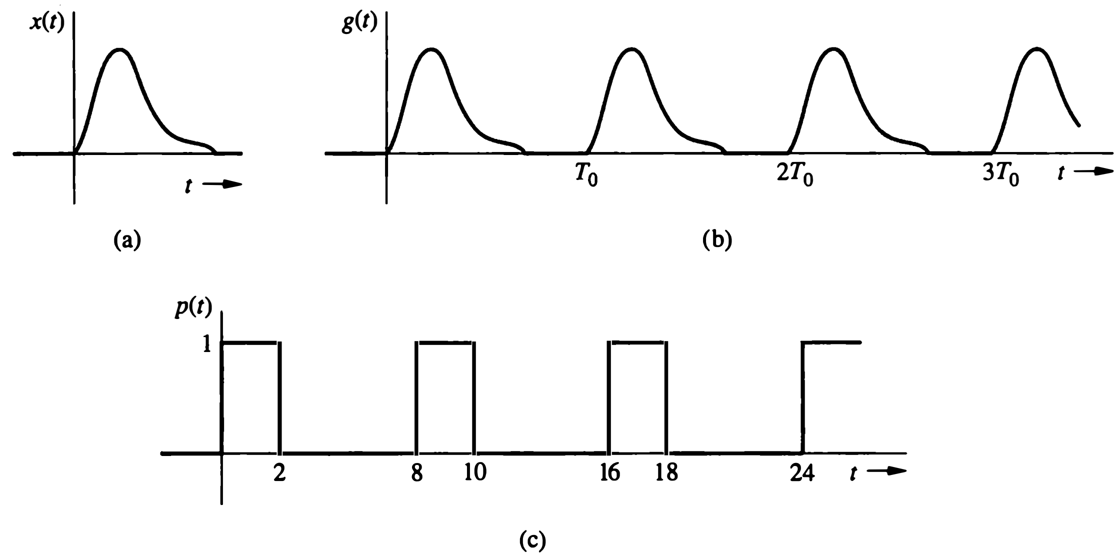


Figure P4-6

4-8 (a) Find the Laplace transform of the pulses in Fig. 4.2 in the text by using only the time-differentiation property, the time-shifting property, and the fact that $\delta(t) \Longleftrightarrow 1$.

(b) In Example 4.7, the Laplace transform of $x(t)$ is found by finding the Laplace transform of d^2x/dt^2. Find the Laplace transform of $x(t)$ in that example by finding the Laplace transform of dx/dt and using Table 4.1, if necessary.

4-9 Since 13 is such a lucky number, determine the inverse Laplace transform of $X(s) = 1/(s+1)^{13}$ given region of convergence $\sigma > -1$. [Hint: What is the nth derivative of $1/(s+a)$?]

4-10 It is difficult to compute the Laplace transform $X(s)$ of signal

$$x(t) = \frac{1}{t}u(t)$$

by using direct integration. Instead, properties provide a simpler method.

(a) Use Laplace transform properties to express the Laplace transform of $tx(t)$ in terms of the unknown quantity $X(s)$.

(b) Use the definition to determine the Laplace transform of $y(t) = tx(t)$.

(c) Solve for $X(s)$ by using the two pieces from a and b. Simplify your answer.

4-11 Using the Laplace transform, solve the following differential equations:

(a) $(D^2 + 3D + 2)y(t) = Dx(t)$ if $y(0^-) = \dot{y}(0^-) = 0$ and $x(t) = u(t)$

(b) $(D^2 + 4D + 4)y(t) = (D + 1)x(t)$ if $y(0^-) = 2, \dot{y}(0^-) = 1$ and $x(t) = e^{-t}u(t)$

(c) $(D^2 + 6D + 25)y(t) = (D + 2)x(t)$ if $y(0^-) = \dot{y}(0^-) = 1$ and $x(t) = 25u(t)$

4-12 Solve the differential equations in Prob. 4-11 using the Laplace transform. In each case determine the zero-input and zero-state components of the solution.

4-13 Solve the following simultaneous differential equations using the Laplace transform, assuming all initial conditions to be zero and the input $x(t) = u(t)$:

(a) $(D + 3)y_1(t) - 2y_2(t) = x(t)$
$-2y_1(t) + (2D + 4)y_2(t) = 0$

(b) $(D + 2)y_1(t) - (D + 1)y_2(t) = 0$
$-(D + 1)y_1(t) + (2D + 1)y_2(t) = x(t)$

Determine the transfer functions relating outputs $y_1(t)$ and $y_2(t)$ to the input $x(t)$.

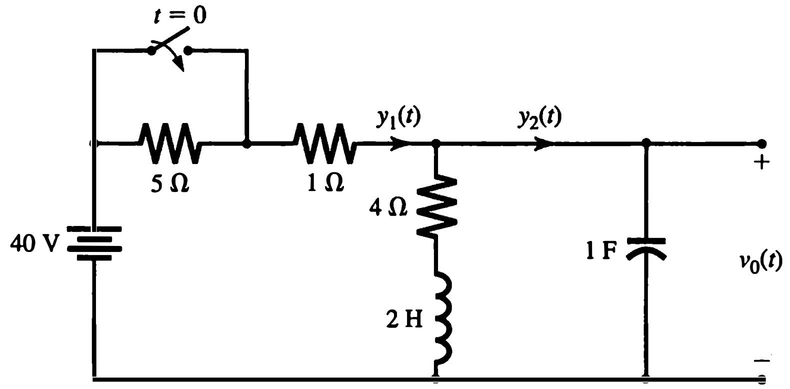


Figure P4-14

4-14 For the circuit in Fig. P4-14, the switch is in open position for a long time before $t = 0$, when it is closed instantaneously.

(a) Write loop equations (in time domain) for $t \geq 0$.

(b) Solve for $y_1(t)$ and $y_2(t)$ by taking the Laplace transform of loop equations found in part (a).

4-15 For each of the systems described by the following differential equations, find the system transfer function:

(a) $\dfrac{d^2y}{dt^2} + 11\dfrac{dy}{dt} + 24y(t) = 5\dfrac{dx}{dt} + 3x(t)$

(b) $\dfrac{d^3y}{dt^3} + 6\dfrac{d^2y}{dt^2} - 11\dfrac{dy}{dt} + 6y(t)$

$= 3\dfrac{d^2x}{dt^2} + 7\dfrac{dx}{dt} + 5x(t)$

(c) $\dfrac{d^4y}{dt^4} + 4\dfrac{dy}{dt} = 3\dfrac{dx}{dt} + 2x(t)$

(d) $\dfrac{d^2y}{dt^2} - y(t) = \dfrac{dx}{dt} - x(t)$

4-16 For each of the systems specified by the following transfer functions, find the differential equation relating the output $y(t)$ to the input $x(t)$ assuming that the systems are controllable and observable:

(a) $H(s) = \dfrac{s+5}{s^2+3s+8}$

(b) $H(s) = \dfrac{s^2+3s+5}{s^3+8s^2+5s+7}$

(c) $H(s) = \dfrac{5s^2+7s+2}{s^2-2s+5}$

4-17 For a system with transfer function

$$H(s) = \frac{2s+3}{s^2+2s+5}$$

(a) Find the (zero-state) response for input $x(t)$ of (i) $10u(t)$ and (ii) $u(t-5)$.

(b) For this system write the differential equation relating the output $y(t)$ to the input $x(t)$ assuming that the systems are controllable and observable.

4-18 For a system with transfer function

$$H(s) = \frac{s}{s^2+9}$$

(a) Find the (zero-state) response if the input $x(t) = (1 - e^{-t})u(t)$

(b) For this system write the differential equation relating the output $y(t)$ to the input $x(t)$ assuming that the systems are controllable and observable.

4-19 For a system with transfer function

$$H(s) = \frac{s+5}{s^2+5s+6}$$

(a) Find the (zero-state) response for the following values of input $x(t)$:

(i) $e^{-3t}u(t)$

(ii) $e^{-4t}u(t)$

(iii) $e^{-4(t-5)}u(t-5)$

(iv) $e^{-4(t-5)}u(t)$

(v) $e^{-4t}u(t-5)$

(b) For this system write the differential equation relating the output $y(t)$ to the input $x(t)$ assuming that the systems are controllable and observable.

4-20 An LTI system has a step response given by $s(t) = e^{-t}u(t) - e^{-2t}u(t)$. Determine the output of this system $y(t)$ given an input $x(t) = \delta(t - \pi) - \cos(\sqrt{3})u(t)$.

4-21 For an LTIC system with zero initial conditions (system initially in zero state), if an input $x(t)$ produces an output $y(t)$, then using the Laplace transform show the following.

(a) The input dx/dt produces an output dy/dt.

(b) The input $\int_0^t x(\tau)\,d\tau$ produces an output $\int_0^t y(\tau)\,d\tau$. Hence, show that the unit step response of a system is an integral of the impulse response; that is, $\int_0^t h(\tau)\,d\tau$.

4-22 (a) Discuss asymptotic and BIBO stabilities for the systems described by the following transfer functions assuming that the systems are controllable and observable:

(i) $\dfrac{(s+5)}{s^2+3s+2}$

(ii) $\dfrac{s+5}{s^2(s+2)}$

(iii) $\dfrac{s(s+2)}{s+5}$

(iv) $\dfrac{s+5}{s(s+2)}$

(v) $\dfrac{s+5}{s^2-2s+3}$

(b) Repeat part (a) for systems described by the following differential equations. Systems may be uncontrollable and/or unobservable.

(i) $(D^2+3D+2)\,y(t) = (D+3)\,x(t)$

(ii) $(D^2+3D+2)\,y(t) = (D+1)\,x(t)$

(iii) $(D^2+D-2)\,y(t) = (D-1)\,x(t)$

(iv) $(D^2-3D+2)\,y(t) = (D-1)\,x(t)$

4-23 Find the zero-state response $y(t)$ of the network in Fig. P4-23 if the input voltage $x(t) = te^{-t}u(t)$. Find the transfer function relating the output $y(t)$ to the input $x(t)$. From the transfer function, write the differential equation relating $y(t)$ to $x(t)$.

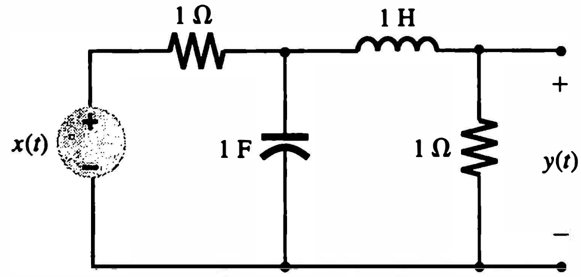


Figure P4-23

4-24 The switch in the circuit of Fig. P4-24 is closed for a long time and then opened instantaneously at $t = 0$. Find and sketch the current $y(t)$.

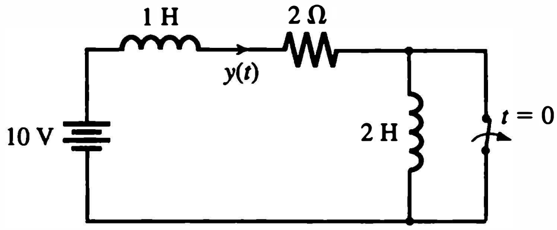


Figure P4-24

4-25 Find the current $y(t)$ for the parallel resonant circuit in Fig. P4-25 if the input is

(a) $x(t) = A\cos\omega_0 t\, u(t)$

(b) $x(t) = A\sin\omega_0 t\, u(t)$

Assume all initial conditions to be zero and, in both cases, $\omega_0^2 = 1/LC$.

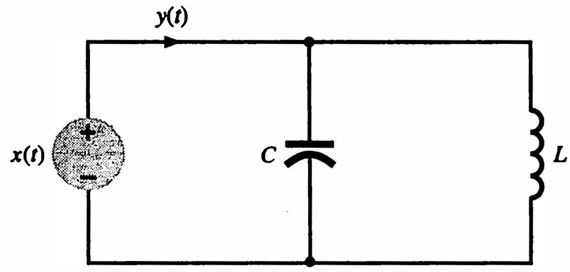


Figure P4-25

4-26 Find the loop currents $y_1(t)$ and $y_2(t)$ for $t \geq 0$ in the circuit of Fig. P4-26a for the input $x(t)$ in Fig. P4-26b.

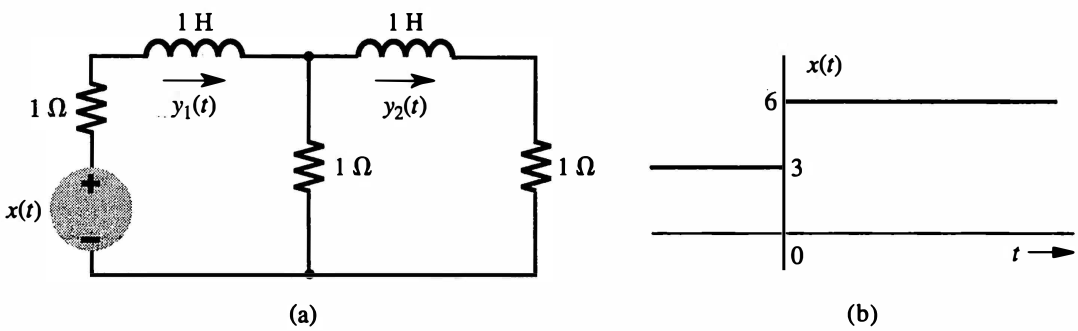


Figure P4-26

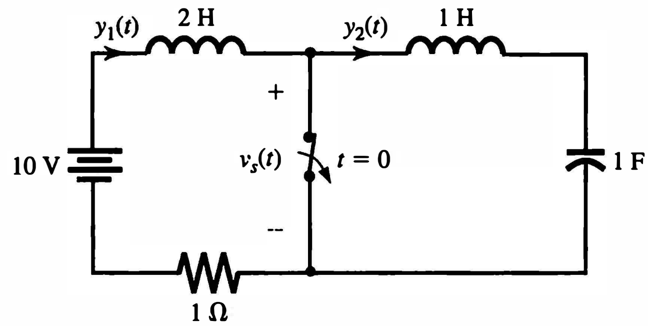


Figure P4-27

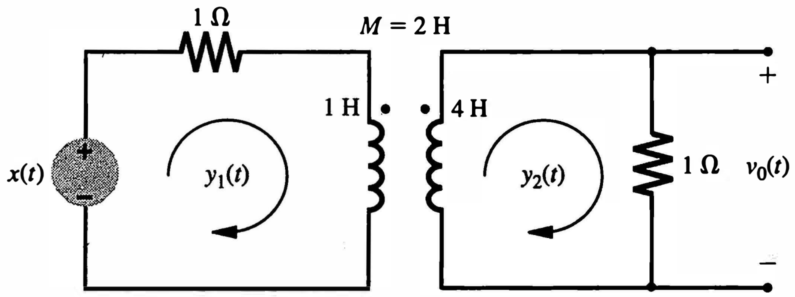


Figure P4-28

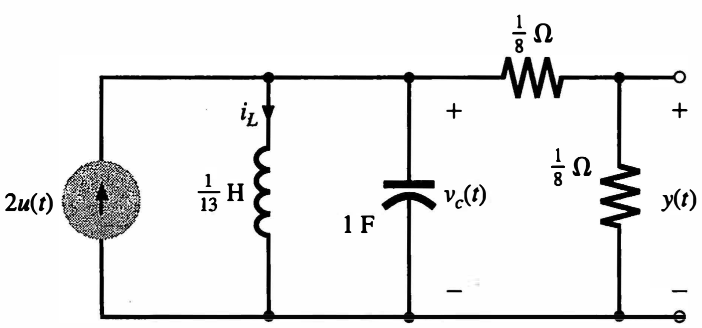


Figure P4-29

4-27 For the network in Fig. P4-27 the switch is in a closed position for a long time before $t = 0$, when it is opened instantaneously. Find $y_1(t)$ and $v_s(t)$ for $t \geq 0$.

4-28 Find the output voltage $v_0(t)$ for $t \geq 0$ for the circuit in Fig. P4-28, if the input $x(t) = 100u(t)$. The system is in the zero state initially.

4-29 Find the output voltage $y(t)$ for the network in Fig. P4-29 for the initial conditions $i_L(0) = 1$ A and $v_C(0) = 3$ V.

4-30 For the network in Fig. P4-30, the switch is in position a for a long time and then is moved to position b instantaneously at $t = 0$. Determine the current $y(t)$ for $t > 0$.

4-31 Show that the transfer function that relates the output voltage $y(t)$ to the input voltage $x(t)$ for the op-amp circuit in Fig. P4-31a is given by

$$H(s) = \frac{Ka}{s+a} \qquad \text{where}$$

$$K = 1 + \frac{R_b}{R_a} \quad \text{and} \quad a = \frac{1}{RC}$$

and that the transfer function for the circuit in Fig. P4-31b is given by

$$H(s) = \frac{Ks}{s+a}$$

4-32 For the second-order op-amp circuit in Fig. P4-32, show that the transfer function $H(s)$ relating the output voltage $y(t)$ to the input voltage $x(t)$ is given by

$$H(s) = \frac{-s}{s^2 + 8s + 12}$$

4-33 (a) Using the initial and final value theorems, find the initial and final value of the zero-state response of a system with the transfer

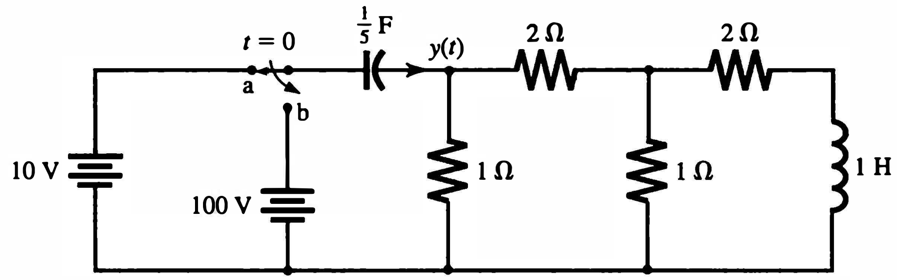


Figure P4-30

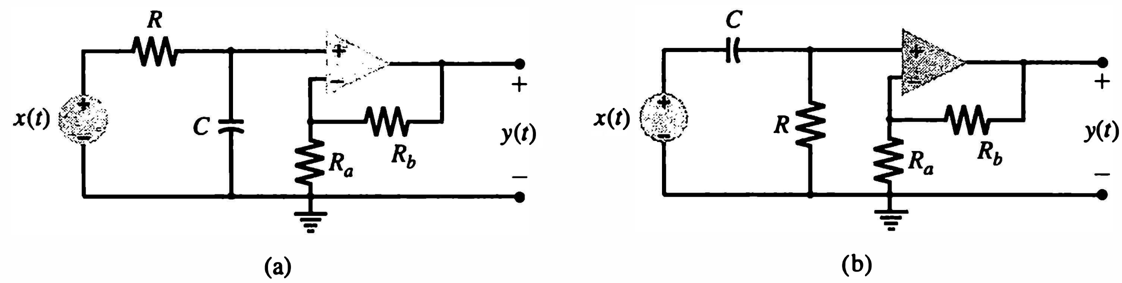


Figure P4-31

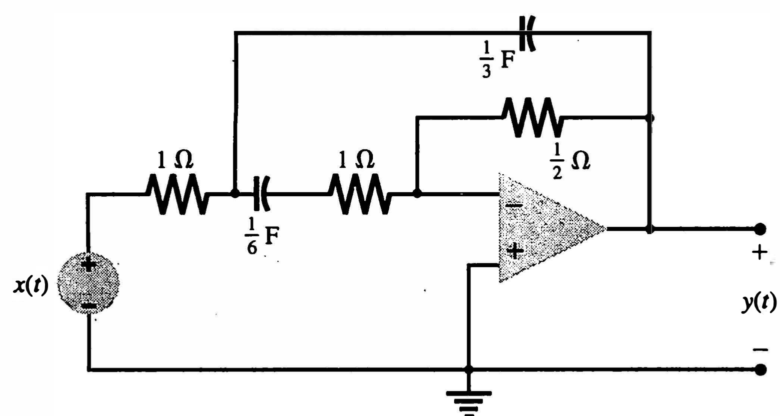


Figure P4-32

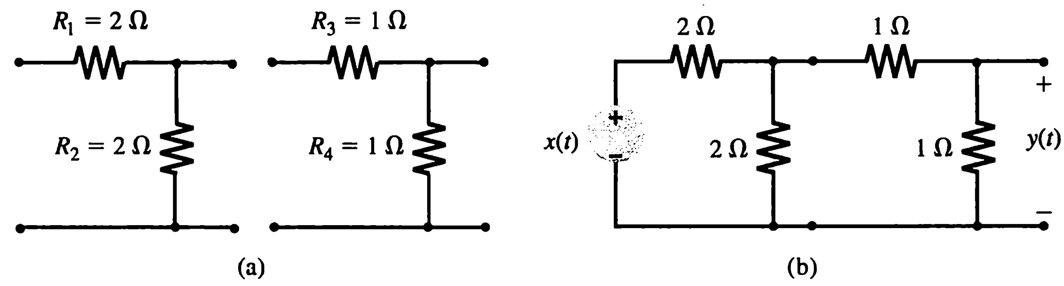


Figure P4-34

function

$$H(s) = \frac{6s^2 + 3s + 10}{2s^2 + 6s + 5}$$

and the input (i) $u(t)$ and (ii) $e^{-t}u(t)$.

(b) Find $y(0^+)$ and $y(\infty)$ if $Y(s)$ is given by

(i) $\dfrac{s^2 + 5s + 6}{s^2 + 3s + 2}$

(ii) $\dfrac{s^3 + 4s^2 + 10s + 7}{s^2 + 2s + 3}$

4-34 Figure P4-34a shows two resistive ladder segments. The transfer function of each segment (ratio of output to input voltage) is 1/2. Figure P4-34b shows these two segments connected in cascade.

(a) Is the transfer function (ratio of output to input voltage) of this cascaded network $(1/2)(1/2) = 1/4$?

(b) If your answer is affirmative, verify the answer by direct computation of the transfer function. Does this computation confirm the earlier value 1/4? If not, why?

(c) Repeat the problem with $R_3 = R_4 = 20\ \text{k}\Omega$. Does this result suggest the answer to the problem in part (b)?

4-35 In communication channels, transmitted signal is propagated simultaneously by several paths of varying lengths. This causes the signal to reach the destination with varying time delays and varying gains. Such a system generally distorts the received signal. For error-free communication, it is necessary to undo this distortion as much as possible by using the system that is inverse of the channel model.

For simplicity, let us assume that a signal is propagated by two paths whose time delays differ by τ seconds. The channel over the intended path has a delay of T seconds and unity gain. The signal over the unintended path has a delay of $T + \tau$ seconds and gain a. Such a channel can be modeled, as shown in Fig. P4-35. Find the inverse system transfer function to correct the delay distortion and show that the inverse system can be realized by a feedback system. The inverse system should be causal to be realizable. [Hint: We want to correct only the distortion caused by the relative delay τ seconds. For distortionless transmission, the signal may be delayed. What is important is to maintain the shape of $x(t)$. Thus a received signal of the form $c\,x(t - T)$ is considered to be distortionless.]

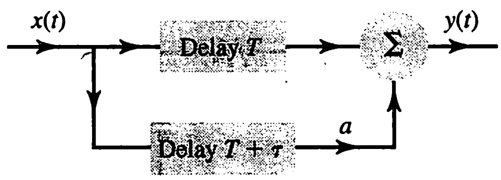


Figure P4-35

4-36 Discuss BIBO stability of the feedback systems depicted in Fig. P4-36. For the case in Fig. P4-36b, consider three cases:

(i) $K = 10$

(ii) $K = 50$

(iii) $K = 48$

4-37 Realize

$$H(s) = \frac{s(s+2)}{(s+1)(s+3)(s+4)}$$

by canonic direct, series, and parallel forms.

4-38 Realize the transfer function in Prob. 4-37 by using the transposed form of the realizations found in Prob. 4-37.

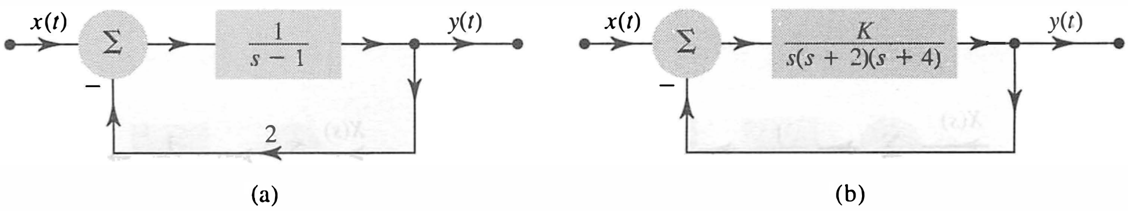


Figure P4-36

4-39 Repeat Prob. 4-37 for

(a) $H(s) = \dfrac{3s(s+2)}{(s+1)(s^2+2s+2)}$

(b) $H(s) = \dfrac{2s-4}{(s+2)(s^2+4)}$

4-40 Realize the transfer functions in Prob. 4-39 by using the transposed form of the realizations found in Prob. 4-39.

4-41 Repeat Prob. 4-37 for

$$H(s) = \frac{2s+3}{5s(s+2)^2(s+3)}$$

4-42 Realize the transfer function in Prob. 4-41 by using the transposed form of the realizations found in Prob. 4-41.

4-43 Repeat Prob. 4-37 for

$$H(s) = \frac{s(s+1)(s+2)}{(s+5)(s+6)(s+8)}$$

4-44 Realize the transfer function in Prob. 4-43 by using the transposed form of the realizations found in Prob. 4-43.

4-45 Repeat Prob. 4-37 for

$$H(s) = \frac{s^3}{(s+1)^2(s+2)(s+3)}$$

4-46 Realize the transfer function in Prob. 4-45 by using the transposed form of the realizations found in Prob. 4-45.

4-47 Repeat Prob. 4-37 for

$$H(s) = \frac{s^3}{(s+1)(s^2+4s+13)}$$

4-48 Realize the transfer function in Prob. 4-47 by using the transposed form of the realizations found in Prob. 4-47.

4-49 In this problem we show how a pair of complex conjugate poles may be realized by using a cascade of two first-order transfer functions and feedback. Show that the transfer functions of the block diagrams in Fig. P4-49a and P4-49b are

(a) $H(s) = \dfrac{1}{(s+a)^2+b^2} = \dfrac{1}{s^2+2as+(a^2+b^2)}$

(b) $H(s) = \dfrac{s+a}{(s+a)^2+b^2} = \dfrac{s+a}{s^2+2as+(a^2+b^2)}$

Hence, show that the transfer function of the block diagram in Fig. P4-49c is

(c) $H(s) = \dfrac{As+B}{(s+a)^2+b^2} = \dfrac{As+B}{s^2+2as+(a^2+b^2)}$

4-50 Show op-amp realizations of the following transfer functions:

(i) $\dfrac{-10}{s+5}$

(ii) $\dfrac{10}{s+5}$

(iii) $\dfrac{s+2}{s+5}$

4-51 Show two different op-amp circuit realizations of the transfer function

$$H(s) = \frac{s+2}{s+5} = 1 - \frac{3}{s+5}$$

4-52 Show an op-amp canonic direct realization of the transfer function

$$H(s) = \frac{3s+7}{s^2+4s+10}$$

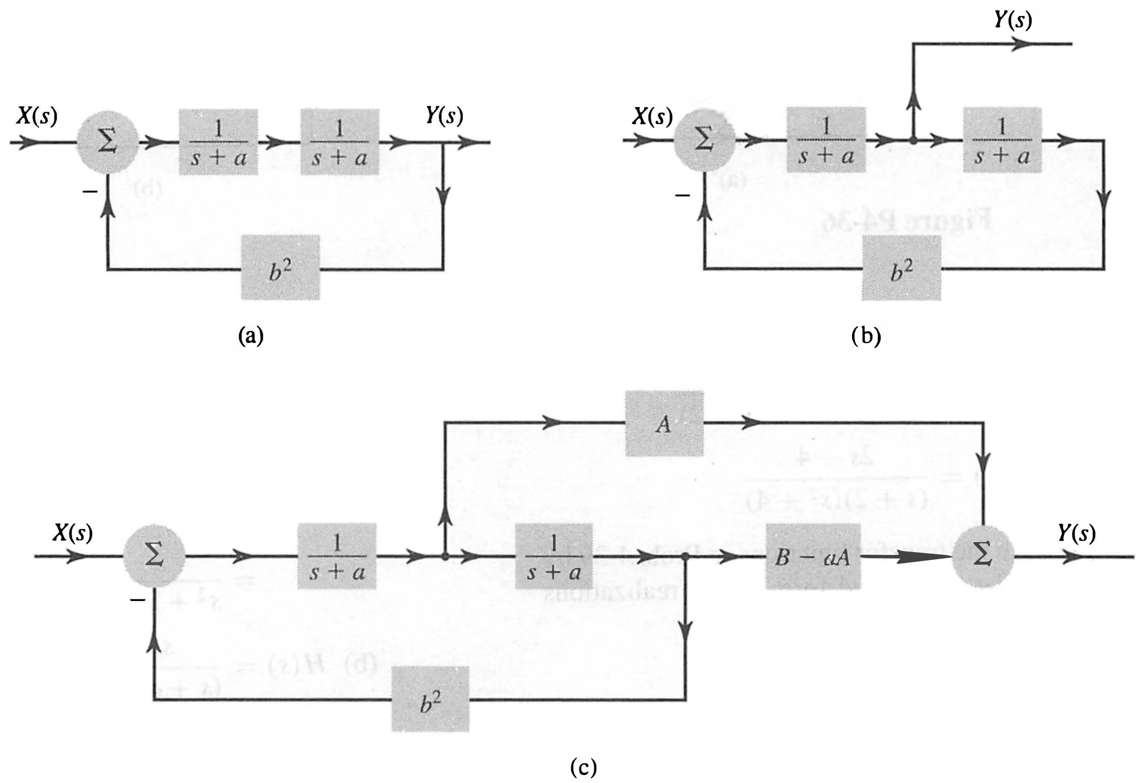


Figure P4-49

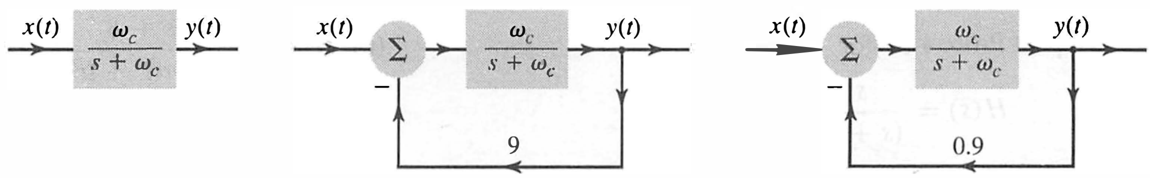


Figure P4-54

4-53 Show an op-amp canonic direct realization of the transfer function

$$H(s) = \frac{s^2 + 5s + 2}{s^2 + 4s + 13}$$

4-54 Feedback can be used to increase (or decrease) the system bandwidth. Consider the system in Fig. P4-54a with transfer function $G(s) = \omega_c/(s + \omega_c)$.

(a) Show that the 3 dB bandwidth of this system is ω_c and the dc gain is unity, that is, $|H(j0)| = 1$.

(b) To increase the bandwidth of this system, we use negative feedback with $H(s) = 9$, as depicted in Fig. P4-54b. Show that the 3 dB bandwidth of this system is $10\omega_c$. What is the dc gain?

(c) To decrease the bandwidth of this system, we use positive feedback with $H(s) = -0.9$, as illustrated in Fig. P4-54c. Show that the 3 dB bandwidth of this system is $\omega_c/10$. What is the dc gain?

(d) The system gain at dc times its 3 dB bandwidth is the *gain–bandwidth product* of a system. Show that this product is the same for all the three systems in Fig. P4-54. This result shows that if we increase the bandwidth, the gain decreases and vice versa.

4-55 For an LTIC system described by the transfer function

$$H(s) = \frac{s + 2}{s^2 + 5s + 4}$$

find the response to the following everlasting sinusoidal inputs:

(a) $5\cos(2t + 30°)$
(b) $10\sin(2t + 45°)$
(c) $10\cos(3t + 40°)$

Observe that these are everlasting sinusoids.

4-56 For an LTIC system described by the transfer function

$$H(s) = \frac{s+3}{(s+2)^2}$$

find the steady-state system response to the following inputs:

(a) $10u(t)$
(b) $\cos(2t + 60°)u(t)$
(c) $\sin(3t - 45°)u(t)$
(d) $e^{j3t}u(t)$

4-57 For an allpass filter specified by the transfer function

$$H(s) = \frac{-(s-10)}{s+10}$$

find the system response to the following (everlasting) inputs:

(a) $e^{j\omega t}$
(b) $\cos(\omega t + \theta)$
(c) $\cos t$
(d) $\sin 2t$
(e) $\cos 10t$
(f) $\cos 100t$

Comment on the filter response.

4-58 The pole–zero plot of a second-order system $H(s)$ is shown in Fig. P4-58. The dc response of this system is minus one, $H(j0) = -1$.

(a) Letting $H(s) = k(s^2 + b_1 s + b_2)/(s^2 + a_1 s + a_2)$, determine the constants k, b_1, b_2, a_1, and a_2.
(b) What is the output $y(t)$ of this system in response to the input $x(t) = 4 + \cos(t/2 + \pi/3)$?

4-59 Sketch Bode plots for the following transfer functions:

(a) $\dfrac{s(s+100)}{(s+2)(s+20)}$

(b) $\dfrac{(s+10)(s+20)}{s^2(s+100)}$

(c) $\dfrac{(s+10)(s+200)}{(s+20)^2(s+1000)}$

4-60 Repeat Prob. 4-59 for

(a) $\dfrac{s^2}{(s+1)(s^2+4s+16)}$

(b) $\dfrac{s}{(s+1)(s^2+14.14s+100)}$

(c) $\dfrac{(s+10)}{s(s^2+14.14s+100)}$

4-61 Using the lowest order possible, determine a system function $H(s)$ with real-valued roots that matches the frequency response in Fig. P4-61 Verify your answer with MATLAB.

4-62 A graduate student recently implemented an analog phase lock loop (PLL) as part of his thesis. His PLL consists of four basic components: a phase/frequency detector, a charge pump, a loop filter, and a voltage-controlled

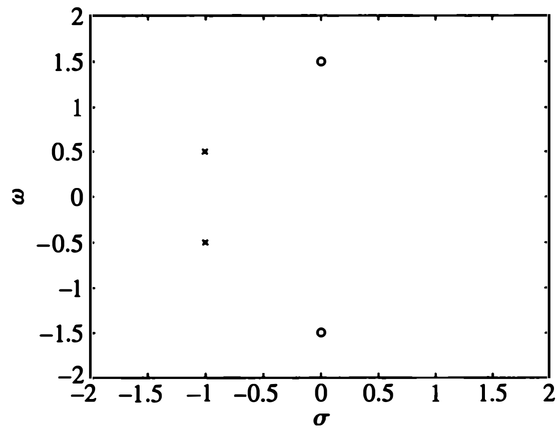


Figure P4-58 System pole–zero plot.

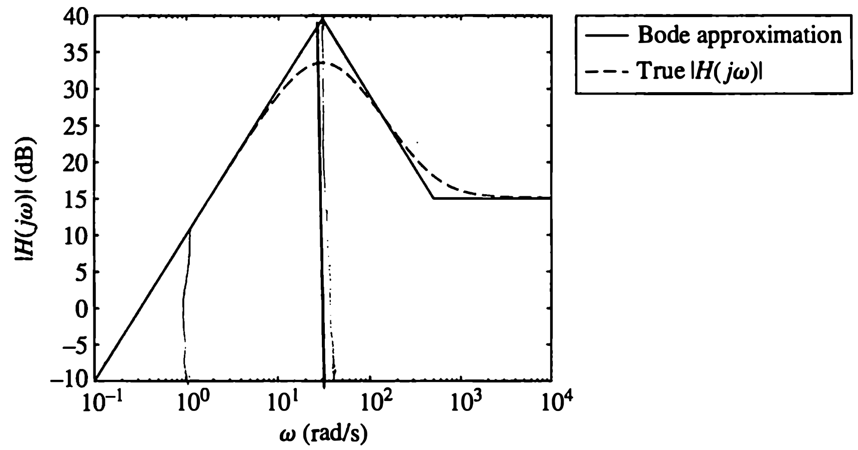
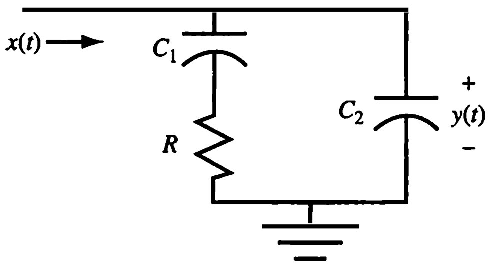


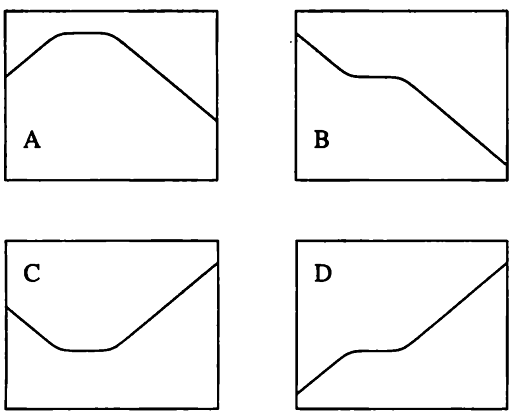
Figure P4-61 Bode plot and frequency response for $H(s)$.

oscillator. This problem considers only the loop filter, which is shown in Fig. P4-62a. The loop filter input is the current $x(t)$, and the output is the voltage $y(t)$.

(a) Derive the loop filter's transfer function $H(s)$. Express $H(s)$ in standard form.



(a)



(b)

Figure P4-62 **(a)** Circuit diagram for PLL loop filter. **(b)** Possible magnitude response plots for PLL loop filter.

(b) Figure P4-62b provides four possible frequency response plots, labeled A through D. Each log–log plot is drawn to the same scale, and line slopes are either 20 dB/decade, 0 dB/decade, or −20 dB/decade. Clearly identify which plot(s), if any, could represent the loop filter.

(c) Holding the other components constant, what is the general effect of increasing the resistance R on the magnitude response for low-frequency inputs?

(d) Holding the other components constant, what is the general effect of increasing the resistance R on the magnitude response for high-frequency inputs?

4-63 Using the graphical method of Section 4.10-1, draw a rough sketch of the amplitude and phase response of an LTIC system described by the transfer function

$$H(s) = \frac{s^2 - 2s + 50}{s^2 + 2s + 50}$$
$$= \frac{(s - 1 - j7)(s - 1 + j7)}{(s + 1 - j7)(s + 1 + j7)}$$

What kind of filter is this?

4-64 Using the graphical method of Section 4.10-1, draw a rough sketch of the amplitude and phase response of LTIC systems whose pole–zero plots are shown in Fig. P4-64.

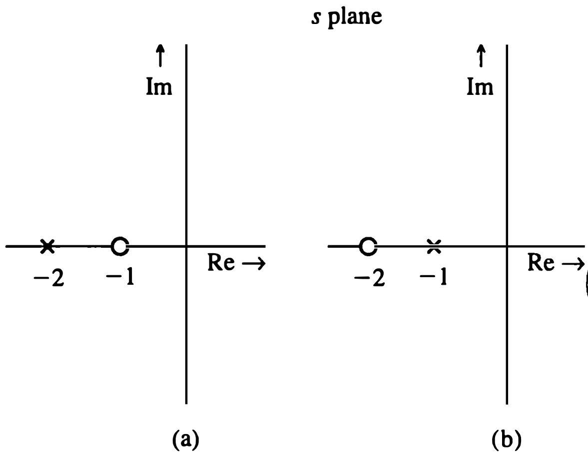


Figure P4-64

4-65 Design a second-order bandpass filter with center frequency $\omega = 10$. The gain should be zero at $\omega = 0$ and at $\omega = \infty$. Select poles at $-a \pm j10$. Leave your answer in terms of a. Explain the influence of a on the frequency response.

4-66 The LTIC system described by $H(s) = (s-1)/(s+1)$ has unity magnitude response $|H(j\omega)| = 1$. Positive Pat claims that the output $y(t)$ of this system is equal the input $x(t)$ since the system is allpass. Cynical Cynthia doesn't think so. "This is *signals and systems* class," she complains. "It *has* to be more complicated!" Who is correct, Pat or Cynthia? Justify your answer.

4-67 Two students, Amy and Jeff, disagree about an analog system function given by $H_1(s) = s$. Sensible Jeff claims the system has a zero at $s = 0$. Rebellious Amy, however, notes that the system function can be rewritten as $H_1(s) = 1/s^{-1}$ and claims that this implies a system pole at $s = \infty$. Who is correct? Why? What are the poles and zeros of the system $H_2(s) = 1/s$?

4-68 A rational transfer function $H(s)$ is often used to represent an analog filter. Why must $H(s)$ be strictly proper for lowpass and bandpass filters? Why must $H(s)$ be proper for highpass and bandstop filters?

4-69 For a given filter order N, why is the stopband attenuation rate of an all-pole lowpass filter better than filters with finite zeros?

4-70 Is it possible, with real coefficients ($[k, b_1, b_2, a_1, a_2] \in \mathcal{R}$), for a system

$$H(s) = k\frac{s^2 + b_1 s + b_2}{s^2 + a_1 s + a_2}$$

to function as a lowpass filter? Explain your answer.

4-71 Nick recently built a simple second-order Butterworth lowpass filter for his home stereo. Although the system performs pretty well, Nick is an overachiever and hopes to improve the system performance. Unfortunately, Nick is pretty lazy and doesn't want to design another filter. Thinking "twice the filtering gives twice the performance," he suggests filtering the audio signal not once but twice with a cascade of two identical filters. His overworked, underpaid signals professor is skeptical and states, "If you are using *identical* filters, it makes no difference whether you filter once or twice!" Who is correct? Why?

4-72 An LTIC system impulse response is given by $h(t) = u(t) - u(t-1)$.

(a) Determine the transfer function $H(s)$. Using $H(s)$, determine and plot the magnitude response $|H(j\omega)|$. Which type of filter most accurately describes the behavior of this system: lowpass, highpass, bandpass, or bandstop?

(b) What are the poles and zeros of $H(s)$? Explain your answer.

(c) Can you determine the impulse response of the inverse system? If so, provide it. If not, suggest a method that could be used to approximate the impulse response of the inverse system.

4-73 An ideal lowpass filter $H_{LP}(s)$ has magnitude response that is unity for low frequencies and zero for high frequencies. An ideal highpass filter $H_{HP}(s)$ has an opposite magnitude response: zero for low frequencies and unity for high frequencies. A student suggests a possible lowpass-to-highpass filter transformation: $H_{HP}(s) = 1 - H_{LP}(s)$. In general, will this transformation work? Explain your answer.

4-74 An LTIC system has a rational transfer function $H(s)$. When appropriate, assume that all initial conditions are zero.

(a) Is is possible for this system to output $y(t) = \sin(100\pi t)u(t)$ in response to an input $x(t) = \cos(100\pi t)u(t)$? Explain.
(b) Is is possible for this system to output $y(t) = \sin(100\pi t)u(t)$ in response to an input $x(t) = \sin(50\pi t)u(t)$? Explain.
(c) Is is possible for this system to output $y(t) = \sin(100\pi t)$ in response to an input $x(t) = \cos(100\pi t)$? Explain.
(d) Is is possible for this system to output $y(t) = \sin(100\pi t)$ in response to an input $x(t) = \sin(50\pi t)$? Explain.

4-75 Find the ROC, if it exists, of the (bilateral) Laplace transform of the following signals:
(a) $e^{tu(t)}$
(b) $e^{-tu(t)}$
(c) $\dfrac{1}{1+t^2}$
(d) $\dfrac{1}{1+e^t}$
(e) e^{-kt^2}

4-76 Find the (bilateral) Laplace transform and the corresponding region of convergence for the following signals:
(a) $e^{-|t|}$
(b) $e^{-|t|}\cos t$
(c) $e^t u(t) + e^{2t}u(-t)$
(d) $e^{-tu(t)}$
(e) $e^{tu(-t)}$
(f) $\cos\omega_0 t\, u(t) + e^t\, u(-t)$

4-77 Find the inverse (bilateral) Laplace transforms of the following functions:
(a) $\dfrac{2s+5}{(s+2)(s+3)}$ $\quad -3 < \sigma < -2$
(b) $\dfrac{2s-5}{(s-2)(s-3)}$ $\quad 2 < \sigma < 3$
(c) $\dfrac{2s+3}{(s+1)(s+2)}$ $\quad \sigma > -1$
(d) $\dfrac{2s+3}{(s+1)(s+2)}$ $\quad \sigma < -2$
(e) $\dfrac{3s^2-2s-17}{(s+1)(s+3)(s-5)}$
$-1 < \sigma < 5$

4-78 Find
$$\mathcal{L}^{-1}\left[\frac{2s^2-2s-6}{(s+1)(s-1)(s+2)}\right]$$
if the ROC is
(a) $\mathrm{Re}\, s > 1$
(b) $\mathrm{Re}\, s < -2$
(c) $-1 < \mathrm{Re}\, s < 1$
(d) $-2 < \mathrm{Re}\, s < -1$

4-79 For a causal LTIC system having a transfer function $H(s) = 1/(s+1)$, find the output $y(t)$ if the input $x(t)$ is given by
(a) $e^{-|t|/2}$
(b) $e^t u(t) + e^{2t}u(-t)$
(c) $e^{-t/2}u(t) + e^{-t/4}u(-t)$
(d) $e^{2t}u(t) + e^t u(-t)$
(e) $e^{-t/4}u(t) + e^{-t/2}u(-t)$
(f) $e^{-3t}u(t) + e^{-2t}u(-t)$

4-80 The autocorrelation function $r_{xx}(t)$ of a signal $x(t)$ is given by
$$r_{xx}(t) = \int_{-\infty}^{\infty} x(\tau)x(\tau+t)\,d\tau$$
Derive an expression for $R_{xx}(s) = \mathcal{L}(r_{xx}(t))$ in terms of $X(s)$, where $X(s) = \mathcal{L}(x(t))$.

4-81 Determine the inverse Laplace transform of
$$X(s) = \frac{2}{s} + \frac{s}{2}$$
given that the region of convergence is $\sigma < 0$.

4-82 An absolutely integrable signal $x(t)$ has a pole at $s = \pi$. It is possible that other poles may be present. Recall that an absolutely integrable signal satisfies
$$\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$$
(a) Can $x(t)$ be left sided? Explain.
(b) Can $x(t)$ be right sided? Explain.
(c) Can $x(t)$ be two sided? Explain.
(d) Can $x(t)$ be of finite duration? Explain.

4-83 Using the definition, compute the bilateral Laplace transform, including the region of convergence (ROC), of the following complex-valued functions:

(a) $x_1(t) = (j + e^{jt})u(t)$

(b) $x_2(t) = j\cosh(t)u(-t)$

(c) $x_3(t) = e^{j(\frac{\pi}{4})}u(-t+1) + j\delta(t-5)$

(d) $x_4(t) = j^t u(-t) + \delta(t-\pi)$

4-84 A bounded-amplitude signal $x(t)$ has bilateral Laplace transform $X(s)$ given by

$$X(s) = \frac{s2^s}{(s-1)(s+1)}$$

(a) Determine the corresponding region of convergence.

(b) Determine the time-domain signal $x(t)$.

4.M-1 Express the polynomial $C_{20}(x)$ in standard form. That is, determine the coefficients a_k of $C_{20}(x) = \sum_{k=0}^{20} a_k x^{20-k}$.

4.M-2 Design an order-12 Butterworth lowpass filter with a cutoff frequency of $\omega_c = 2\pi 5000$ by completing the following.

(a) Locate and plot the filter's poles and zeros in the complex plane. Plot the corresponding magnitude response $|H_{\text{LP}}(j\omega)|$ to verify proper design.

(b) Setting all resistor values to 100,000, determine the capacitor values to implement the filter using a cascade of six second-order Sallen–Key circuit sections. The form of a Sallen–Key stage is shown in Fig. P4.M-2. On a single plot, plot the magnitude response of each section as well as the overall magnitude response. Identify the poles that correspond to each section's magnitude response curve. Are the capacitor values realistic?

4.M-3 Rather than a Butterworth filter, repeat Prob. P4.M-2 for a Chebyshev LPF with $R = 3$ dB of passband ripple. Since each Sallen–Key stage is constrained to have unity gain at dc, an overall gain error of $1/\sqrt{1+\epsilon^2}$ is acceptable.

4.M-4 An analog lowpass filter with cutoff frequency ω_c can be transformed into a highpass filter with cutoff frequency ω_c by using an RC–CR transformation rule: each resistor R_i is replaced by a capacitor $C_i' = 1/R_i\omega_c$ and each capacitor C_i is replaced by a resistor $R_i' = 1/C_i\omega_c$.

Use this rule to design an order-8 Butterworth highpass filter with $\omega_c = 2\pi 4000$ by completing the following.

(a) Design an order-8 Butterworth lowpass filter with $\omega_c = 2\pi 4000$ by using four second-order Sallen–Key circuit stages, the form of which is shown in Fig. P4.M-2. Give resistor and capacitor values for each stage. Choose the resistors so that the RC–CR transformation will result in 1 nF capacitors. At this point, are the component values realistic?

(b) Draw an RC–CR transformed Sallen–Key circuit stage. Determine the transfer function $H(s)$ of the transformed stage in terms of the variables R_1', R_2', C_1', and C_2'.

(c) Transform the LPF designed in part a by using an RC–CR transformation. Give the resistor and capacitor values for each stage. Are the component values realistic?

Using $H(s)$ derived in part b, plot the magnitude response of each section as well as the overall magnitude response. Does the overall response look like a highpass Butterworth filter?

Plot the HPF system poles and zeros in the complex s plane. How do these

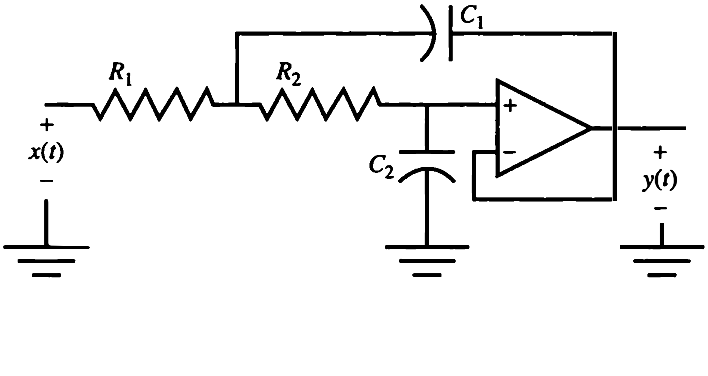


Figure P4.M-2 Sallen–Key filter stage.

locations compare with those of the Butterworth LPF?

4.M-5 Repeat Prob. P4.M-4, using $\omega_c = 2\pi 1500$ and an order-16 filter. That is, eight second-order stages need to be designed.

4.M-6 Rather than a Butterworth filter, repeat Prob. P4.M-4 for a Chebyshev LPF with $R = 3\,\text{dB}$ of passband ripple. Since each transformed Sallen–Key stage is constrained to have unity gain at $\omega = \infty$, an overall gain error of $1/\sqrt{1+\epsilon^2}$ is acceptable.

4.M-7 The MATLAB signal processing toolbox function `butter` helps design analog Butterworth filters. Use MATLAB help to learn how `butter` works. For each of the following cases, design the filter, plot the filter's poles and zeros in the complex s plane, and plot the decibel magnitude response $20\log_{10}|H(j\omega)|$.

(a) Design a sixth-order analog lowpass filter with $\omega_c = 2\pi 3500$.

(b) Design a sixth-order analog highpass filter with $\omega_c = 2\pi 3500$.

(c) Design a sixth-order analog bandpass filter with a passband between 2 and 4 kHz.

(d) Design a sixth-order analog bandstop filter with a stopband between 2 and 4 kHz.

4.M-8 The MATLAB signal processing toolbox function `cheby1` helps design analog Chebyshev type I filters. A Chebyshev type I filter has a passband ripple and a smooth stopband. Setting the passband ripple to $R_p = 3\,\text{dB}$, repeat Prob. P4.M-7 using the `cheby1` command. With all other parameters held constant, what is the general effect of reducing R_p, the allowable passband ripple?

4.M-9 The MATLAB signal processing toolbox function `cheby2` helps design analog Chebyshev type II filters. A Chebyshev type II filter has a smooth passband and ripple in the stopband. Setting the stopband ripple $R_s = 20\,\text{dB}$ down, repeat Prob. P4.M-7 using the `cheby2` command. With all other parameters held constant, what is the general effect of increasing R_s, the minimum stopband attenuation?

4.M-10 The MATLAB signal processing toolbox function `ellip` helps design analog elliptic filters. An elliptic filter has ripple in both the passband and the stopband. Setting the passband ripple to $R_p = 3\,\text{dB}$ and the stopband ripple $R_s = 20\,\text{dB}$ down, repeat Prob. P4.M-7 using the `ellip` command.

4.M-11 Using the definition $C_N(x) = \cosh(N \cosh^{-1}(x))$, prove the recursive relation $C_N(x) = 2xC_{N-1}(x) - C_{N-2}(x)$.

4.M-12 Prove that the poles of a Chebyshev filter, which are located at $p_k = \omega_c \sinh(\xi)\sin(\phi_k) + j\omega_c \cosh(\xi)\cos(\phi_k)$, lie on an ellipse. [Hint: The equation of an ellipse in the x–y plane is $(x/a)^2 + (y/b)^2 = 1$, where constants a and b define the major and minor axes of the ellipse.]