the integrand, the lower limit of integration, and the upper limit of integration. Notice, that no Δt needs to be specified.

To use quad to estimate E_x, the integrand must first be described.

```
>> x_squared = inline('exp(-2*t).*((t>=0)&(t<1))','t');
```

Estimating E_x immediately follows.

```
>> E_x = quad(x_squared,0,1)
E_x = 0.4323
```

In this case, the relative error is −0.0026%.

The same techniques can be used to estimate the energy of more complex signals. Consider $g(t)$, defined previously. Energy is expressed as $E_g = \int_0^\infty e^{-2t}\cos^2(2\pi t)\,dt$. A closed-form solution exists, but it takes some effort. MATLAB provides an answer more quickly.

```
>> g_squared = inline('exp(-2*t).*(cos(2*pi*t).^2).*(t>=0)','t');
```

Although the upper limit of integration is infinity, the exponentially decaying envelope ensures $g(t)$ is effectively zero well before $t = 100$. Thus, an upper limit of $t = 100$ is used along with $\Delta t = 0.001$.

```
>> t = (0:0.001:100);
>> E_g = sum(g_squared(t)*0.001)
E_g = 0.2567
```

A slightly better approximation is obtained with the quad function.

```
>> E_g = quad(g_squared,0,100)
E_g = 0.2562
```

As an exercise, confirm that the energy of signal $h(t)$, defined previously, is $E_h = 0.3768$.

PROBLEMS

1-1 Find the energies of the signals illustrated in Fig. P1-1. Comment on the effect on energy of sign change, time shifting, or doubling of the signal. What is the effect on the energy if the signal is multiplied by k?

1-2 Repeat Prob. 1-1 for the signals in Fig. P1-2.

1-3 (a) Find the energies of the pair of signals $x(t)$ and $y(t)$ depicted in Fig. P1-3a and P1-3b. Sketch and find the energies of signals $x(t)+y(t)$ and $x(t)-y(t)$. Can you make any observation from these results?

(b) Repeat part (a) for the signal pair illustrated in Fig. P1-3c. Is your observation in part (a) still valid?

1-4 Find the power of the periodic signal $x(t)$ shown in Fig. P1-4. Find also the powers and the rms values of:

(a) $-x(t)$

(b) $2x(t)$

(c) $cx(t)$.

Comment.

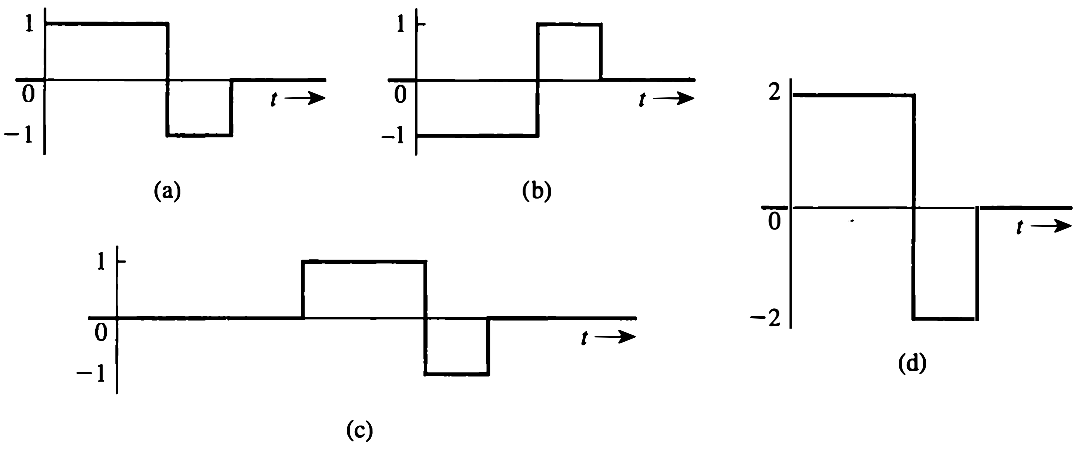

Figure P1-1

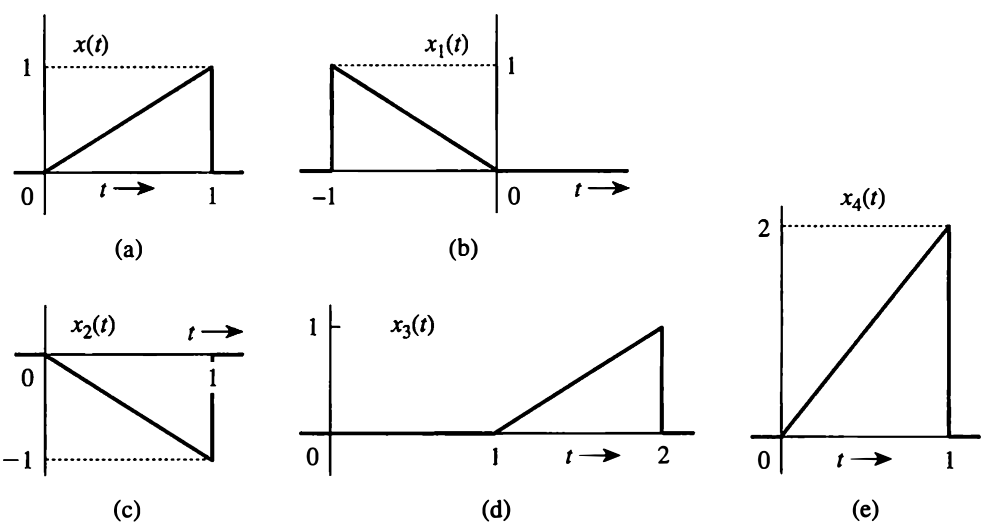

Figure P1-2

1-5 Figure P1-5 shows a periodic 50% duty cycle dc-offset sawtooth wave $x(t)$ with peak amplitude A. Determine the energy and power of $x(t)$.

1-6 (a) There are many useful properties related to signal energy. Prove each of the following statements. In each case, let energy signal $x_1(t)$ have energy $E[x_1(t)]$, let energy signal $x_2(t)$ have energy $E[x_2(t)]$, and let T be a nonzero, finite, real-valued constant.

(i) Prove $E[Tx_1(t)] = T^2 E[x_1(t)]$. That is, amplitude scaling a signal by constant T scales the signal energy by T^2.

(ii) Prove $E[x_1(t)] = E[x_1(t-T)]$. That is, shifting a signal does not affect its energy.

(iii) If $(x_1(t) \neq 0) \Rightarrow (x_2(t) = 0)$ and $(x_2(t) \neq 0) \Rightarrow (x_1(t) = 0)$, then prove $E[x_1(t) + x_2(t)] = E[x_1(t)] + E[x_2(t)]$. That is, the energy of the sum of two nonoverlapping signals is the sum of the two individual energies.

(iv) Prove $E[x_1(Tt)] = (1/|T|)E[x_1(t)]$. That is, time-scaling a signal by T reciprocally scales the signal energy by $1/|T|$.

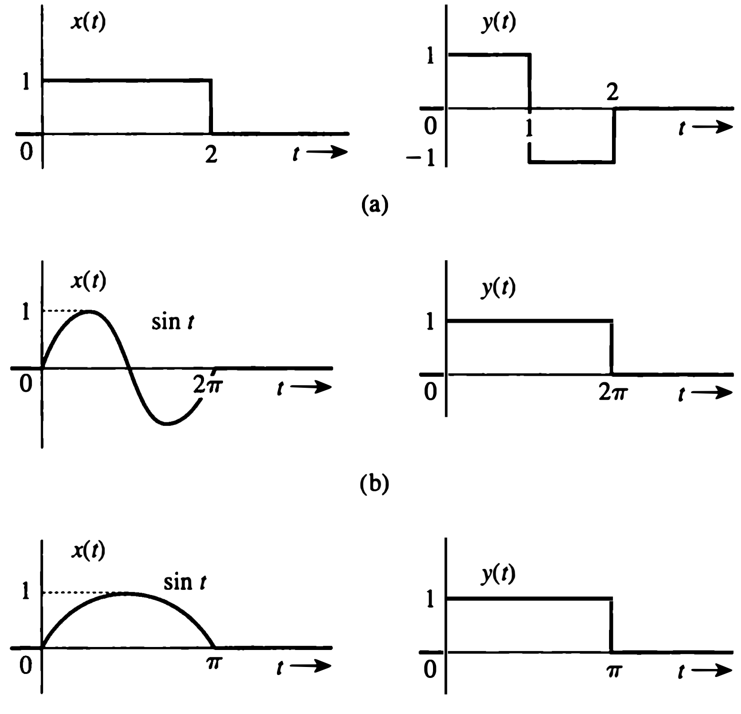

Figure P1-3

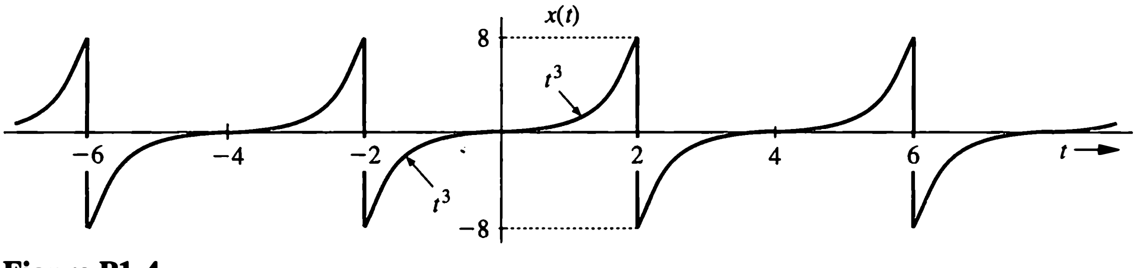

Figure P1-4

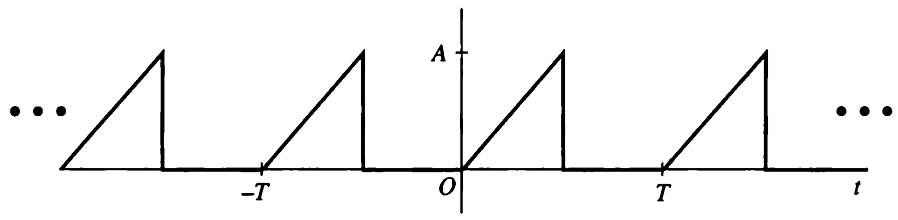

Figure P1-5 50% duty cycle dc-offset sawtooth wave $x(t)$.

(b) Consider the signal $x(t)$ shown in Fig. P1-6. Outside the interval shown, $x(t)$ is zero. Determine the signal energy $E[x(t)]$.

1-7 A binary signal $x(t) = 0$ for $t < 0$. For positive time, $x(t)$ toggles between one and zero as follows: one for 1 second, zero for 1 second, one for 1 second, zero for 2 seconds, one for 1

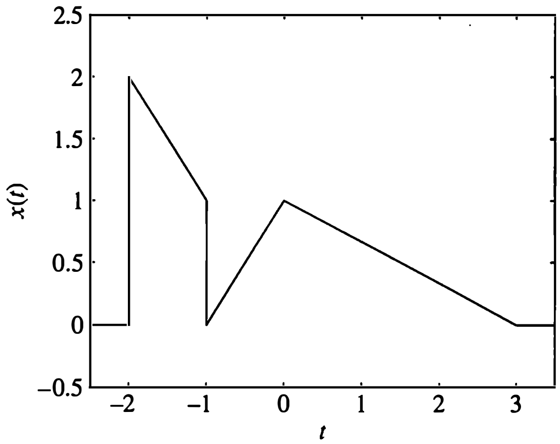

Figure P1-6 Energy signal $x(t)$.

second, zero for 3 seconds, and so forth. That is, the "on" time is always one second but the "off" time successively increases by one second between each toggle. A portion of $x(t)$ is shown in Fig. P1-7. Determine the energy and power of $x(t)$.

1-8 For the signal $x(t)$ depicted in Fig. P1-8, sketch the signals

(a) $x(-t)$

(b) $x(t+6)$

(c) $x(3t)$

(d) $x(t/2)$

1-9 For the signal $x(t)$ illustrated in Fig. P1-9, sketch

(a) $x(t-4)$

(b) $x(t/1.5)$

(c) $x(-t)$

(d) $x(2t-4)$

(e) $x(2-t)$

1-10 In Fig. P1-10, express signals $x_1(t), x_2(t), x_3(t), x_4(t)$, and $x_5(t)$ in terms of signal $x(t)$ and its time-shifted, time-scaled, or time-reversed versions.

1-11 For an energy signal $x(t)$ with energy E_x, show that the energy of any one of the signals $-x(t), x(-t)$, and $x(t-T)$ is E_x. Show also that the energy of $x(at)$ as well as $x(at-b)$ is E_x/a, but the energy of $ax(t)$ is a^2E_x. This shows that time inversion and time shifting do not affect signal energy. On the other hand, time compression of a signal ($a > 1$) reduces the energy, and time expansion of a signal ($a < 1$) increases the energy. What is the

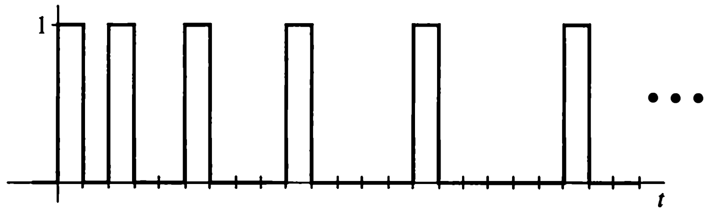

Figure P1-7 Binary signal $x(t)$.

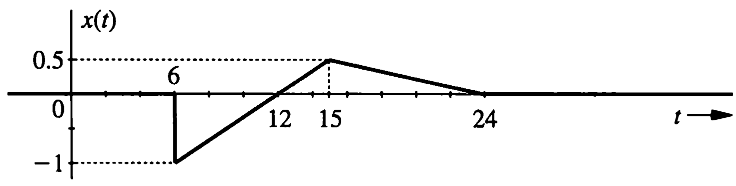

Figure P1-8

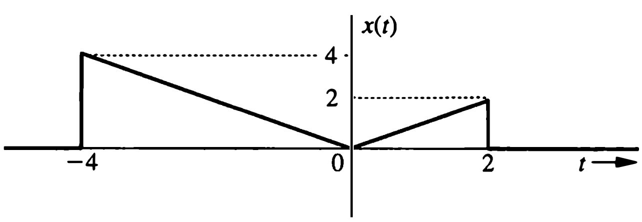

Figure P1-9

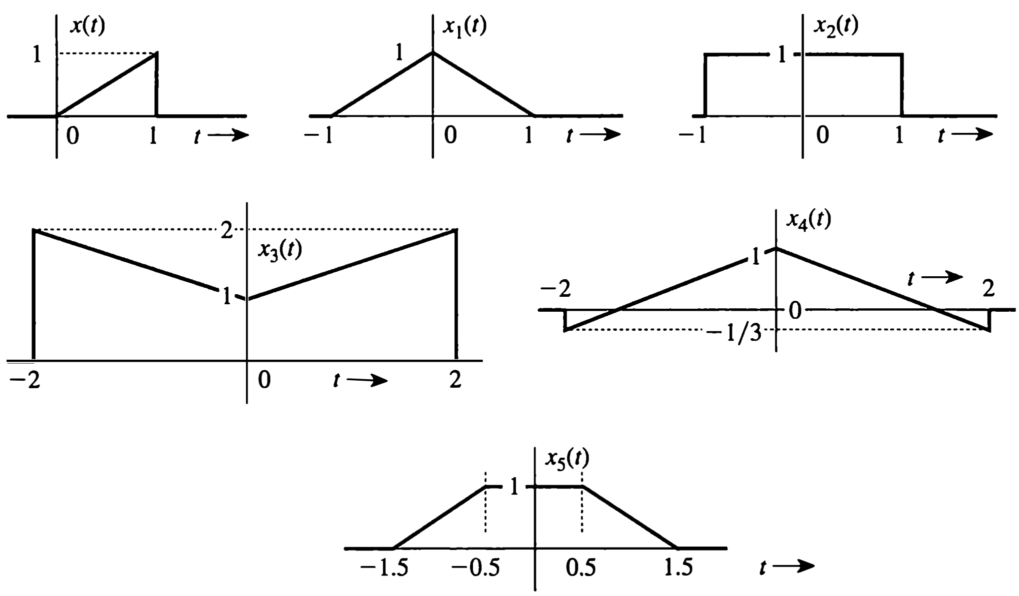

Figure P1-10

effect on signal energy if the signal is multiplied by a constant a?

1-12 Define $2x(-3t+1) = t(u(-t-1)-u(-t+1))$, where $u(t)$ is the unit step function.

(a) Plot $2x(-3t+1)$ over a suitable range of t.
(b) Plot $x(t)$ over a suitable range of t.

1-13 Consider the signal $x(t) = 2^{-tu(t)}$, where $u(t)$ is the unit step function.

(a) Accurately sketch $x(t)$ over $(-1 \leq t \leq 1)$.
(b) Accurately sketch $y(t) = 0.5x(1-2t)$ over $(-1 \leq t \leq 1)$.

1-14 Determine whether each of the following statements is true or false. If the statement is false, demonstrate this by proof or example.

(a) Every continuous-time signal is analog signal.
(b) Every discrete-time signal is digital signal.
(c) If a signal is not an energy signal, then it must be a power signal and vice versa.
(d) An energy signal must be of finite duration.
(e) A power signal cannot be causal.
(f) A periodic signal cannot be anticausal.

1-15 Determine whether each of the following statements is true or false. If the statement is false, demonstrate by proof or example why the statement is false.

(a) Every bounded periodic signal is a power signal.
(b) Every bounded power signal is a periodic signal.
(c) If an energy signal $x(t)$ has energy E, then the energy of $x(at)$ is E/a. Assume a is real and positive.
(d) If a power signal $x(t)$ has power P, then the power of $x(at)$ is P/a. Assume a is real and positive.

1-16 Given $x_1(t) = \cos(t)$, $x_2(t) = \sin(\pi t)$, and $x_3(t) = x_1(t) + x_2(t)$.

(a) Determine the fundamental periods T_1 and T_2 of signals $x_1(t)$ and $x_2(t)$.
(b) Show that $x_3(t)$ is not periodic, which requires $T_3 = k_1 T_1 = k_2 T_2$ for some integers k_1 and k_2.
(c) Determine the powers P_{x_1}, P_{x_2}, and P_{x_3} of signals $x_1(t)$, $x_2(t)$, and $x_3(t)$.

1-17 For any constant ω, is the function $f(t) = \sin(\omega t)$ a periodic function of the independent variable t? Justify your answer.

1-18 The signal shown in Fig. P1-18 is defined as

$$x(t) = \begin{cases} t & 0 \leq t < 1 \\ 0.5 + 0.5\cos(2\pi t) & 1 \leq t < 2 \\ 3 - t & 2 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}$$

The energy of $x(t)$ is $E \approx 1.0417$.

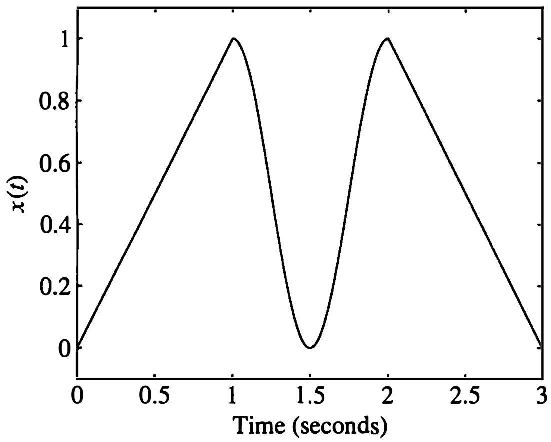

Figure P1-18 Energy signal $x(t)$.

(a) What is the energy of $y_1(t) = (1/3)x(2t)$?
(b) A periodic signal $y_2(t)$ is defined as

$$y_2(t) = \begin{cases} x(t) & 0 \le t < 4 \\ y_2(t+4) & \forall t \end{cases}$$

What is the power of $y_2(t)$?
(c) What is the power of $y_3(t) = (1/3)y_2(2t)$?

1-19 Let $y_1(t) = y_2(t) = t^2$ over $0 \le t \le 1$. Notice, this statement does not require $y_1(t) = y_2(t)$ for all t.

(a) Define $y_1(t)$ as an even, periodic signal with period $T_1 = 2$. Sketch $y_1(t)$ and determine its power.
(b) Design an odd, periodic signal $y_2(t)$ with period $T_2 = 3$ and power equal to unity. Fully describe $y_2(t)$ and sketch the signal over at least one full period. [Hint: There are an infinite number of possible solutions to this problem—you need to find only one of them!]
(c) We can create a complex-valued function $y_3(t) = y_1(t) + jy_2(t)$. Determine whether this signal is periodic. If yes, determine the period T_3. If no, justify why the signal is not periodic.
(d) Determine the power of $y_3(t)$ defined in part (c). The power of a complex-valued function $z(t)$ is

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} z(\tau)z^*(\tau)\,d\tau$$

1-20 Sketch the following signal:

(a) $u(t-5) - u(t-7)$
(b) $u(t-5) + u(t-7)$
(c) $t^2[u(t-1) - u(t-2)]$
(d) $(t-4)[u(t-2) - u(t-4)]$

1-21 Express each of the signals in Fig. P1-21 by a single expression valid for all t.

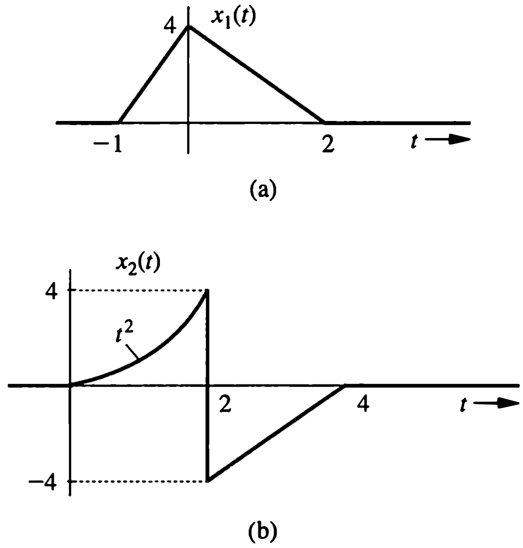

Figure P1-21

1-22 Simplify the following expressions:

(a) $\left(\dfrac{\sin t}{t^2+2}\right)\delta(t)$

(b) $\left(\dfrac{j\omega+2}{\omega^2+9}\right)\delta(\omega)$

(c) $[e^{-t}\cos(3t - 60°)]\delta(t)$

(d) $\left(\dfrac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+4}\right)\delta(1-t)$

(e) $\left(\dfrac{1}{j\omega+2}\right)\delta(\omega+3)$

(f) $\left(\dfrac{\sin k\omega}{\omega}\right)\delta(\omega)$

[Hint: Use Eq. (1.23). For part (f) use L'Hôpital's rule.]

1-23 Evaluate the following integrals:

(a) $\displaystyle\int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)\,d\tau$

(b) $\displaystyle\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$

(c) $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}\, dt$

(d) $\int_{-\infty}^{\infty} \delta(2t-3)\sin \pi t\, dt$

(e) $\int_{-\infty}^{\infty} \delta(t+3)e^{-t}\, dt$

(f) $\int_{-\infty}^{\infty} (t^3+4)\delta(1-t)\, dt$

(g) $\int_{-\infty}^{\infty} x(2-t)\delta(3-t)\, dt$

(h) $\int_{-\infty}^{\infty} e^{(x-1)}\cos\left[\frac{\pi}{2}(x-5)\right]\delta(x-3)\, dx$

1-24 (a) Find and sketch dx/dt for the signal $x(t)$ shown in Fig. P1-9.
(b) Find and sketch d^2x/dt^2 for the signal $x(t)$ depicted in Fig. P1-21a.

1-25 Find and sketch $\int_{-\infty}^{t} x(t)\, dt$ for the signal $x(t)$ illustrated in Fig. P1-25.

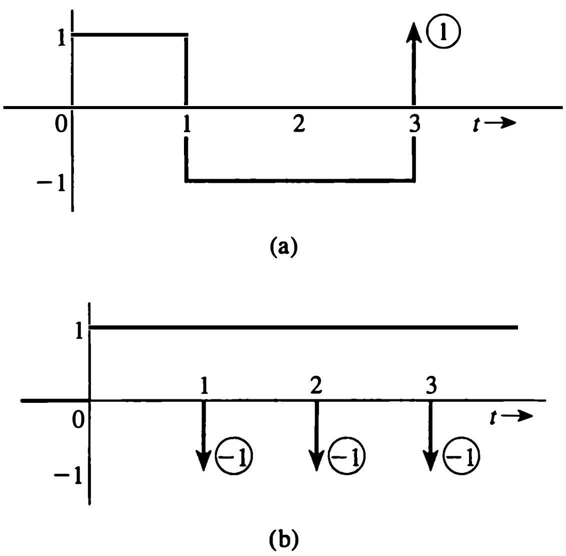

Figure P1-25

1-26 Using the generalized function definition of impulse [Eq. (1.24a)], show that $\delta(t)$ is an even function of t.

1-27 Using the generalized function definition of impulse [Eq. (1.24a)], show that

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

1-28 Show that

$$\int_{-\infty}^{\infty} \dot{\delta}(t)\phi(t)\, dt = -\dot{\phi}(0)$$

where $\phi(t)$ and $\dot{\phi}(t)$ are continuous at $t=0$, and $\phi(t) \to 0$ as $t \to \pm\infty$. This integral defines $\dot{\delta}(t)$ as a generalized function. [Hint: Use integration by parts.]

1-29 A sinusoid $e^{\sigma t}\cos \omega t$ can be expressed as a sum of exponentials e^{st} and e^{-st} [Eq. (1.30c)] with complex frequencies $s = \sigma + j\omega$ and $s = \sigma - j\omega$. Locate in the complex plane the frequencies of the following sinusoids:
(a) $\cos 3t$
(b) $e^{-3t}\cos 3t$
(c) $e^{2t}\cos 3t$
(d) e^{-2t}
(e) e^{2t}
(f) 5

1-30 Find and sketch the odd and the even components of the following:
(a) $u(t)$
(b) $tu(t)$
(c) $\sin \omega_0 t$
(d) $\cos \omega_0 t$
(e) $\cos(\omega_0 t + \theta)$
(f) $\sin \omega_0 t u(t)$
(g) $\cos \omega_0 t u(t)$

1-31 (a) Determine even and odd components of the signal $x(t) = e^{-2t}u(t)$.
(b) Show that the energy of $x(t)$ is the sum of energies of its odd and even components found in part (a).
(c) Generalize the result in part (b) for any finite energy signal.

1-32 (a) If $x_e(t)$ and $x_o(t)$ are even and the odd components of a real signal $x(t)$, then show that

$$\int_{-\infty}^{\infty} x_e(t)x_o(t)\, dt = 0$$

(b) Show that

$$\int_{-\infty}^{\infty} x(t)\, dt = \int_{-\infty}^{\infty} x_e(t)\, dt$$

1-33 An aperiodic signal is defined as $x(t) = \sin(\pi t)u(t)$, where $u(t)$ is the continuous-time step function. Is the odd portion of this signal, $x_o(t)$, periodic? Justify your answer.

1-34 An aperiodic signal is defined as $x(t) = \cos(\pi t)u(t)$, where $u(t)$ is the continuous-time step function. Is the even portion of this signal, $x_e(t)$, periodic? Justify your answer.

1-35 Consider the signal $x(t)$ shown in Fig. P1-35.

(a) Determine and carefully sketch $v(t) = 3x(-(1/2)(t+1))$.
(b) Determine the energy and power of $v(t)$.
(c) Determine and carefully sketch the even portion of $v(t)$, $v_e(t)$.
(d) Let $a = 2$ and $b = 3$, sketch $v(at+b)$, $v(at)+b$, $av(t+b)$, and $av(t)+b$.
(e) Let $a = -3$ and $b = -2$, sketch $v(at+b)$, $v(at)+b$, $av(t+b)$, and $av(t)+b$.

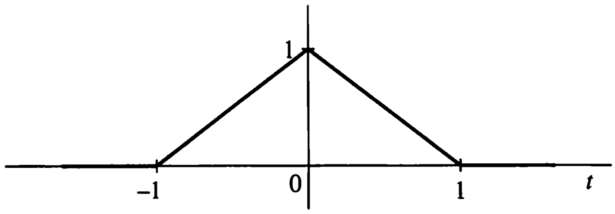

Figure P1-35 Input $x(t)$.

1-36 Consider the signal $y(t) = (1/5)x(-2t-3)$ shown in Figure P1-36.

(a) Does $y(t)$ have an odd portion, $y_o(t)$? If so, determine and carefully sketch $y_o(t)$. Otherwise, explain why no odd portion exists.
(b) Determine and carefully sketch the original signal $x(t)$.

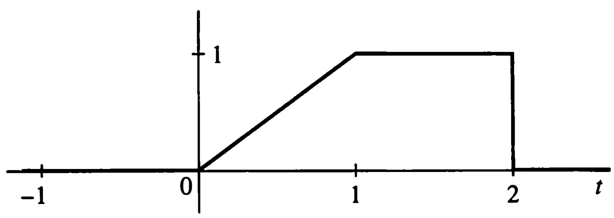

Figure P1-36 $y(t) = \frac{1}{5}x(-2t-3)$.

1-37 Consider the signal $-(1/2)x(-3t+2)$ shown in Fig. P1-37.

(a) Determine and carefully sketch the original signal $x(t)$.
(b) Determine and carefully sketch the even portion of the original signal $x(t)$.
(c) Determine and carefully sketch the odd portion of the original signal $x(t)$.

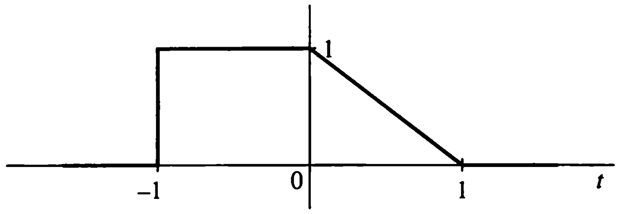

Figure P1-37 $-\frac{1}{2}x(-3t+2)$.

1-38 The conjugate symmetric (or Hermitian) portion of a signal is defined as $w_{cs}(t) = (w(t) + w^*(-t))/2$. Show that the real portion of $w_{cs}(t)$ is even and that the imaginary portion of $w_{cs}(t)$ is odd.

1-39 The conjugate antisymmetric (or skew-Hermitian) portion of a signal is defined as $w_{ca}(t) = (w(t) - w^*(-t))/2$. Show that the real portion of $w_{ca}(t)$ is odd and that the imaginary portion of $w_{ca}(t)$ is even.

1-40 Figure P1-40 plots a complex signal $w(t)$ in the complex plane over the time range $(0 \le t \le 1)$. The time $t = 0$ corresponds with the origin, while the time $t = 1$ corresponds with the point (2,1).

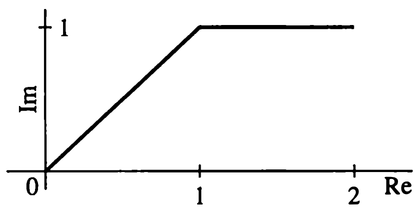

Figure P1-40 $w(t)$ for $(0 \le t \le 1)$.

(a) In the complex plane, plot $w(t)$ over $(-1 \le t \le 1)$ if:
   (i) $w(t)$ is an even signal.
   (ii) $w(t)$ is an odd signal.
   (iii) $w(t)$ is a conjugate symmetric signal. [Hint: See Prob. 1-38.]
   (iv) $w(t)$ is a conjugate antisymmetric signal. [Hint: See Prob. 1-39.]
(b) In the complex plane, plot as much of $w(3t)$ as possible.

1-41 Define complex signal $x(t) = t^2(1+j)$ over interval $(1 \le t \le 2)$. The remaining portion is defined such that $x(t)$ is a minimum-energy, skew-Hermitian signal.

(a) Fully describe $x(t)$ for all t.
(b) Sketch $y(t) = \text{Re}\{x(t)\}$ versus the independent variable t.

(c) Sketch $z(t) = \text{Re}\{jx(-2t+1)\}$ versus the independent variable t.

(d) Determine the energy and power of $x(t)$.

1-42 Write the input–output relationship for an ideal integrator. Determine the zero-input and zero-state components of the response.

1-43 A force $x(t)$ acts on a ball of mass M (Fig. P1-43). Show that the velocity $v(t)$ of the ball at any instant $t > 0$ can be determined if we know the force $x(t)$ over the interval from 0 to t and the ball's initial velocity $v(0)$.

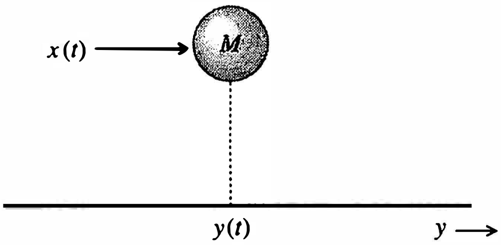

Figure P1-43

1-44 For the systems described by the following equations, with the input $x(t)$ and output $y(t)$, determine which of the systems are linear and which are nonlinear.

(a) $\dfrac{dy}{dt} + 2y(t) = x^2(t)$

(b) $\dfrac{dy}{dt} + 3ty(t) = t^2x(t)$

(c) $3y(t) + 2 = x(t)$

(d) $\dfrac{dy}{dt} + y^2(t) = x(t)$

(e) $\left(\dfrac{dy}{dt}\right)^2 + 2y(t) = x(t)$

(f) $\dfrac{dy}{dt} + (\sin t)y(t) = \dfrac{dx}{dt} + 2x(t)$

(g) $\dfrac{dy}{dt} + 2y(t) = x(t)\dfrac{dx}{dt}$

(h) $y(t) = \displaystyle\int_{-\infty}^{t} x(\tau)\,d\tau$

1-45 For the systems described by the following equations, with the input $x(t)$ and output $y(t)$, explain with reasons which of the systems are time-invariant parameter systems and which are time-varying-parameter systems.

(a) $y(t) = x(t-2)$

(b) $y(t) = x(-t)$

(c) $y(t) = x(at)$

(d) $y(t) = t\,x(t-2)$

(e) $y(t) = \displaystyle\int_{-5}^{5} x(\tau)\,d\tau$

(f) $y(t) = \left(\dfrac{dx}{dt}\right)^2$

1-46 For a certain LTI system with the input $x(t)$, the output $y(t)$ and the two initial conditions $q_1(0)$ and $q_2(0)$, following observations were made:

$x(t)$	$q_1(0)$	$q_2(0)$	$y(t)$
0	1	-1	$e^{-t}u(t)$
0	2	1	$e^{-t}(3t+2)u(t)$
$u(t)$	-1	-1	$2u(t)$

Determine $y(t)$ when both the initial conditions are zero and the input $x(t)$ is as shown in Fig. P1-46. [Hint: There are three causes: the input and each of the two initial conditions. Because of the linearity property, if a cause is increased by a factor k, the response to that cause also increases by the same factor k. Moreover, if causes are added, the corresponding responses add.]

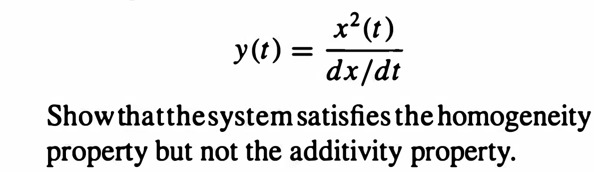

Figure P1-46

1-47 A system is specified by its input–output relationship as

$$y(t) = \frac{x^2(t)}{dx/dt}$$

Show that the system satisfies the homogeneity property but not the additivity property.

1-48 Show that the circuit in Fig. P1-48 is zero-state linear but not zero-input linear. Assume all diodes to have identical (matched) characteristics. The output is the current $y(t)$.

1-49 The inductor L and the capacitor C in Fig. P1-49 are nonlinear, which makes the circuit nonlinear. The remaining three elements are linear. Show that the output $y(t)$ of this nonlinear circuit satisfies the linearity

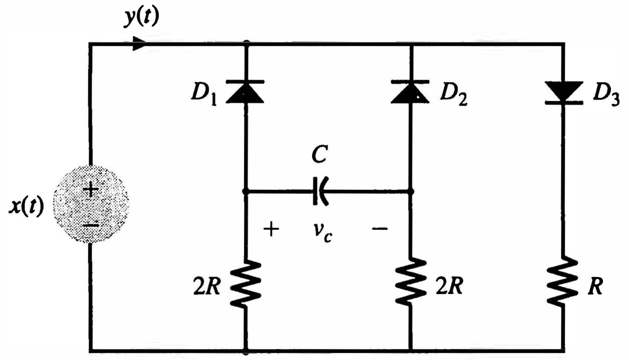

Figure P1-48

conditions with respect to the input $x(t)$ and the initial conditions (all the initial inductor currents and capacitor voltages).

1-50 For the systems described by the following equations, with the input $x(t)$ and output $y(t)$, determine which are causal and which are noncausal.

(a) $y(t) = x(t-2)$
(b) $y(t) = x(-t)$
(c) $y(t) = x(at) \quad a > 1$
(d) $y(t) = x(at) \quad a < 1$

1-51 For the systems described by the following equations, with the input $x(t)$ and output $y(t)$, determine which are invertible and which are noninvertible. For the invertible systems, find the input–output relationship of the inverse system.

(a) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$
(b) $y(t) = x^n(t) \quad x(t)$ real and n integer
(c) $y(t) = \dfrac{dx(t)}{dt}$
(d) $y(t) = x(3t-6)$
(e) $y(t) = \cos[x(t)]$
(f) $y(t) = e^{x(t)} \quad x(t)$ real

1-52 Consider a system that multiplies a given input by a ramp function, $r(t) = tu(t)$. That is, $y(t) = x(t)r(t)$.

(a) Is the system linear? Justify your answer.
(b) Is the system memoryless? Justify your answer.
(c) Is the system causal? Justify your answer.
(d) Is the system time invariant? Justify your answer.

1-53 A continuous-time system is given by

$$y(t) = 0.5\int_{-\infty}^{\infty} x(\tau)[\delta(t-\tau) - \delta(t+\tau)]\,d\tau$$

Recall that $\delta(t)$ designates the Dirac delta function.

(a) Explain what this system does.
(b) Is the system BIBO stable? Justify your answer.
(c) Is the system linear? Justify your answer.
(d) Is the system memoryless? Justify your answer.
(e) Is the system causal? Justify your answer.
(f) Is the system time invariant? Justify your answer.

1-54 A system is given by

$$y(t) = \frac{d}{dt}x(t-1)$$

(a) Is the system BIBO stable? [Hint: Let system input $x(t)$ be a square wave.]
(b) Is the system linear? Justify your answer.
(c) Is the system memoryless? Justify your answer.
(d) Is the system causal? Justify your answer.
(e) Is the system time invariant? Justify your answer.

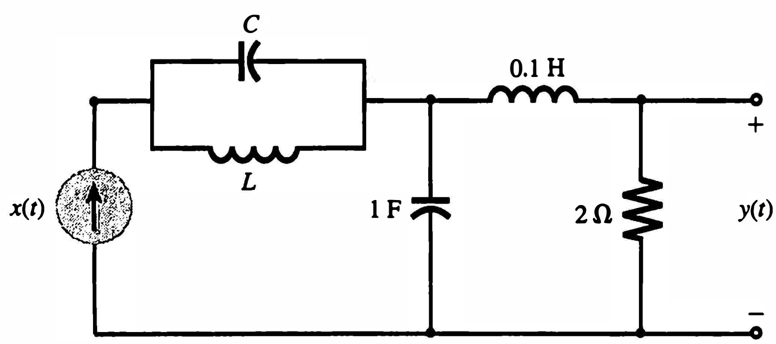

Figure P1-49

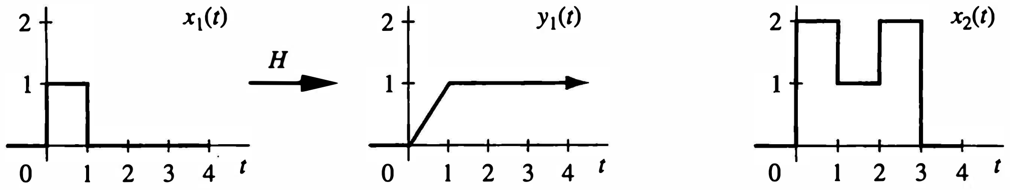

Figure P1-56 $x_1(t) \xrightarrow{H} y_1(t)$ and $x_2(t)$.

1-55 A system is given by

$$y(t) = \begin{cases} x(t) & \text{if } x(t) > 0 \\ 0 & \text{if } x(t) \leq 0 \end{cases}$$

(a) Is the system BIBO stable? Justify your answer.
(b) Is the system linear? Justify your answer.
(c) Is the system memoryless? Justify your answer.
(d) Is the system causal? Justify your answer.
(e) Is the system time invariant? Justify your answer.

1-56 Figure P1-56 displays an input $x_1(t)$ to a linear time-invariant (LTI) system H, the corresponding output $y_1(t)$, and a second input $x_2(t)$.
(a) Bill suggests that $x_2(t) = 2x_1(3t) - x_1(t-1)$. Is Bill correct? If yes, prove it. If not, correct his error.
(b) Hoping to impress Bill, Sue wants to know the output $y_2(t)$ in response to the input $x_2(t)$. Provide her with an expression for $y_2(t)$ in terms of $y_1(t)$. Use MATLAB to plot $y_2(t)$.

1-57 For the circuit depicted in Fig. P1-57, find the differential equations relating outputs $y_1(t)$ and $y_2(t)$ to the input $x(t)$.

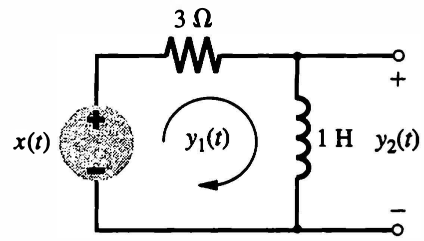

Figure P1-57

1-58 Repeat Prob. 1-57 for the circuit in Fig. P1-58.

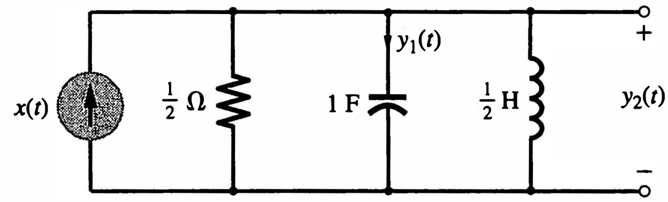

Figure P1-58

1-59 A simplified (one-dimensional) model of an automobile suspension system is shown in Fig. P1-59. In this case, the input is not a force but a displacement $x(t)$ (the road contour). Find the differential equation relating the output $y(t)$ (auto body displacement) to the input $x(t)$ (the road contour).

1-60 A field-controlled dc motor is shown in Fig. P1-60. Its armature current i_a is maintained constant. The torque generated by this motor is proportional to the field current i_f (torque $= K_f i_f$). Find the differential equation relating the output position θ to the input voltage $x(t)$. The motor and load together have a moment of inertia J.

1-61 Water flows into a tank at a rate of q_i units/s and flows out through the outflow valve at a rate of q_0 units/s (Fig. P1-61). Determine the equation relating the outflow q_0 to the input q_i. The outflow rate is proportional to the head h. Thus $q_0 = Rh$, where R is the valve resistance. Determine also the differential equation relating the head h to the input q_i. [Hint: The net inflow of water in time Δt is $(q_i - q_0)\Delta t$. This inflow is also $A\Delta h$ where A is the cross section of the tank.]

1-62 Consider the circuit shown in Fig. P1-62, with input voltage $x(t)$ and output currents $y_1(t)$, $y_2(t)$, and $y_3(t)$.
(a) What is the order of this system? Explain your answer.
(b) Determine the matrix representation for this system.

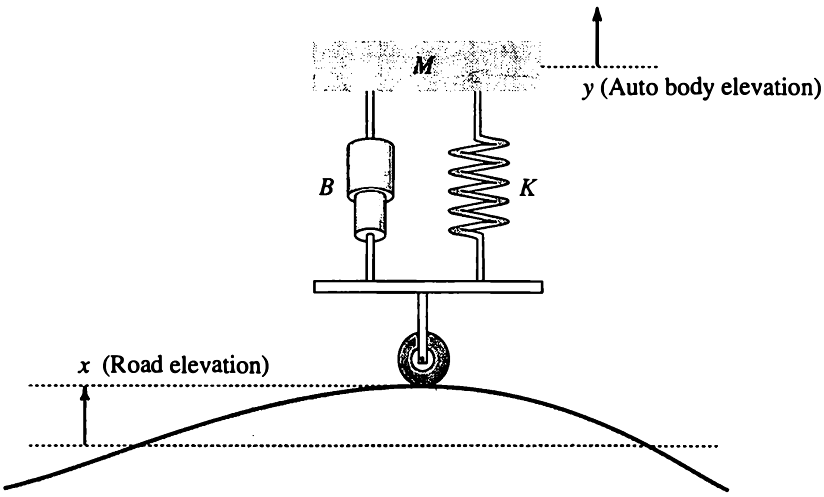

Figure P1-59

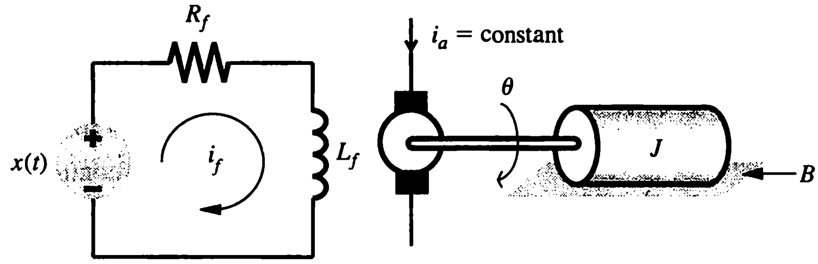

Figure P1-60

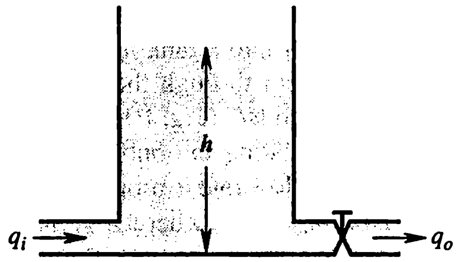

Figure P1-61

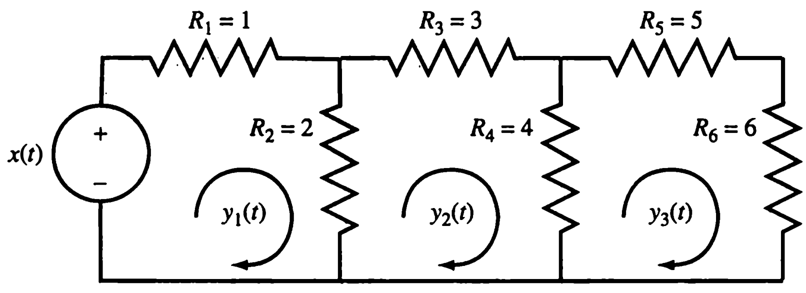

Figure P1-62 Resistor circuit.

(c) Use Cramer's rule to determine the output current $y_3(t)$ for the input voltage $x(t) = (2 - |\cos(t)|)u(t-1)$.

1-63 Write state equations for the parallel RLC circuit in Fig. P1-58. Use the capacitor voltage q_1 and the inductor current q_2 as your state variables. Show that every possible current or voltage in the circuit can be expressed in terms of q_1, q_2 and the input $x(t)$.

1-64 Write state equations for the third-order circuit shown in Fig. P1-64, using the inductor currents q_1, q_2 and the capacitor voltage q_3 as state variables. Show that every possible voltage or current in this circuit can be expressed as a linear combination of q_1, q_2, q_3, and the input $x(t)$. Also, at some instant t it was found that $q_1 = 5$, $q_2 = 1$, $q_3 = 2$, and $x = 10$. Determine the voltage across and the current through every element in this circuit.

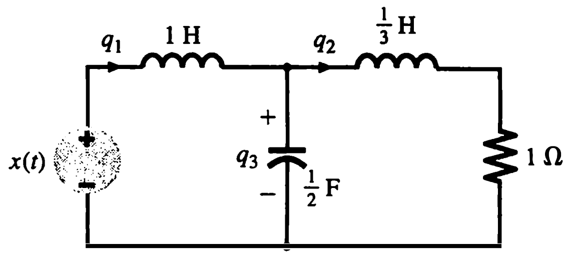

Figure P1-64