1. Найти предел функции:

а) $\lim\limits_{x\to 6}\dfrac{x^2-36}{x^2-x-30}$:

$$\lim_{x\to 6}\frac{x^2-36}{x^2-x-30}=\left(\frac{0}{0}\right)=\lim_{x\to 6}\frac{(x-6)(x+6)}{(x-6)(x+5)}=\lim_{x\to 6}\frac{x+6}{x+5}=\frac{6+6}{6+5}=\frac{12}{11};$$

б) $\lim\limits_{x\to 7}\dfrac{x^2-49}{x^2-13x+42}$:

$$\lim_{x\to 7}\frac{x^2-49}{x^2-13x+42}=\left(\frac{0}{0}\right)=\lim_{x\to 7}\frac{(x-7)(x+7)}{(x-7)(x-6)}=\lim_{x\to 7}\frac{x+7}{x-6}=\frac{7+7}{7-6}=\frac{14}{1}=14;$$

в*) $\lim\limits_{x\to 7}\dfrac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}$

$$\lim_{x\to 7}\frac{\sqrt{x+2}-\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}=\lim_{x\to 7}\left(\frac{\sqrt{x+2}}{\sqrt[4]{x+9}-2}-\frac{\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}\right)=\lim_{x\to 7}\frac{\sqrt{x+2}}{\sqrt[4]{x+9}-2}-\lim_{x\to 7}\frac{\sqrt[3]{x+20}}{\sqrt[4]{x+9}-2}$$

г) $\lim\limits_{x\to 0}\dfrac{3x\mathrm{tg}4x}{1-\cos 4x}$:

$$\lim_{x\to 0}\frac{3x\mathrm{tg}4x}{1-\cos 4x}=\lim_{x\to 0}\frac{3x\cdot 4x}{2\sin^2\dfrac{4x}{2}}=\lim_{x\to 0}\frac{12x^2}{2\cdot 4x^2}=\frac{12}{8}=\frac{3}{2};$$

$$\lim_{x\to 0}\frac{3x\mathrm{tg}4x}{1-\cos 4x}=\lim_{x\to 0}\frac{12x^2}{\dfrac{16x^2}{2}}=\lim_{x\to 0}\frac{24x^2}{16x^2}=\frac{24}{16}=\frac{3}{2};$$

д**) $\lim\limits_{x\to 0}\dfrac{\sqrt{2}x^2\sin 4x}{(1-\cos 2x)^{\frac{3}{2}}}$

е) $\lim\limits_{x\to\infty}\left(\dfrac{4x}{4x+3}\right)^{\frac{5x^2}{7x-1}}$

$$\lim_{x\to\infty}\left(\frac{4x}{4x+3}\right)^{\frac{5x^2}{7x-1}}=(1)^{+\infty}=\lim_{x\to\infty}\left(\frac{4x+3-3}{4x+3}\right)^{\frac{5x^2}{7x-1}}=\lim_{x\to\infty}\left(\frac{4x}{4x+3}\right)^{\frac{5x^2}{7x-1}}=\lim_{x\to\infty}\left(1+\frac{-3}{4x+3}\right)^{\frac{5x^2}{7x-1}}=$$

$$=\lim_{x\to\infty}\left(1+\frac{-3}{4x+3}\right)^{\frac{4x+3}{-3}\cdot\frac{-3}{4x+3}\cdot\frac{5x^2}{7x-1}}=\lim_{x\to\infty}e^{\frac{-3}{4x+3}\cdot\frac{5x^2}{7x-1}}=\lim_{x\to\infty}e^{\frac{-15x^2}{(4x+3)(7x-1)}}=e^{\frac{-15}{28}};$$

$$\lim_{x\to\infty}\left(\frac{4x}{4x+3}\right)^{\frac{5x^2}{7x-1}}=(1)^{+\infty}=\lim_{x\to\infty}e^{\left(\frac{4x}{4x+3}-1\right)\cdot\frac{5x^2}{7x-1}}=\lim_{x\to\infty}e^{\frac{-3}{4x+3}\cdot\frac{5x^2}{7x-1}}=\lim_{x\to\infty}e^{\frac{-15x^2}{(4x+3)(7x-1)}}=e^{\frac{-15}{28}};$$

ж*) $\lim\limits_{x\to+0}\frac{5^x-1}{x}$

з*) $\lim\limits_{x\to+\infty}\frac{\ln(x^2-x+1)}{\ln(x^{10}+x+1)}$