An analysis of the North Sea International Bottom Trawl Survey Data

Abstract

In this research we present non-parametric estimation procedures for calculating abundance at age indices, and investigate the sensitivity of these estimates with respect to the number of otholits collected at sea. The procedures presented are applied on the North Sea International Bottom Trawls Survey data for cod (*gadus morhua*) and saithe (*pollachius virens*), which is public available on the ICES web page. We demonstrate how much information that would have been historically lost if the survey design was defined such that fewer otholits were collected. The abundance at age indices introduced differ from non-parametric indices provided by IBTS by how the age given length relation (ALK) is included. We use ALK's without the assumption of constant ALK over pre defined areas. All abundance at age indices are presented with variance estimates. At current time, such variance estimates are *only* utilized for assessment of Herring (*Clupea harengus*) in the North Sea, even thou they may include valuable information from the survey.

1 Introduction

There are two separate cost full aspects of the North Sea International Bottom Trawl Survey (IBTS) for generation abundance indices per age. The first consist of calculating indices per *length* class, which are obtained by trawling in a predefined procedure and counting the number of fish caught. Then that knowledge is transform to indices with respect to age. The latter part is achieved with an age-length key (ALK), which is constructed by sampling otholits in a stratified procedure from each haul and/or sub-area. To our best knowledge, there has been no research on how much the uncertainty of the abundance indices is related to these two distinct parts. The main contribution of this article is to share light on how the indices estimates and its associated uncertainty estimates changes if less effort was spent on collection of otholits. We achieve the reduction of otholits by mimicking a defined sampling procedure with less effort. We also focus on the spatial distribution of the ALK, spatial structures in the ALK has previously been investigated in Berg and Kristensen (2012); Hirst et al. (2012).

Fish stock assessments are used by fishery managers for making management decisions regarding catch quotas. The assessments provide fundamental information about the status of the stock, for instance, whether the stock is increasing and support for increased levels of harvest should be given, or whether the stock is decreasing and stricter control on harvest should be implemented. Associated with the parameters used in fish stock assessment is their uncertainty, which should not be ignored when formulating management policies (Walters and Ludwig, 1981; Ludwig and Walters, 1981). This uncertainty can arise from many sources including natural variability, estimation procedures and lack of knowledge regarding the parameter (Ehrhardt and Legault, 1997). The North Sea International Bottom Trawl Survey (IBTS) data, coordinated by the International Council for the Exploration of the Sea (ICES), provides information on seasonal distribution of stocks and estimates of abundance indices and catch in numbers of fish per age-class without an assessment of the accuracy of these estimates. As pointed out by Ludwig and Walters (1981) estimates of parameters relating to stock size are of little value unless they are accompanied by uncertainty estimates.

Indices of abundance at age from IBTS are based on data obtained from a stratified semi-random sampling approach of trawl stations, and it is essential to account for the sampling approach so as to produce reliable variance estimates (Lehtonen and Pahkinen, 2004). If the sampling approach is ignored, the effect on the variance of the parameters could be substantial. In particular, the variance could be greatly inflated due to the clustering effect, which involves intra-cluster correlation of the variables (Aanes and Vølstad, 2015; Lehtonen and Pahkinen, 2004).

Abundance indices from IBTS are reported in DATRAS (ICES, 2018b) using an age-length key (ALK) (Fridriksson, 1934) which is assumed to be constant over relatively large areas. In this paper we propose two ALKs which accounts for spatial variation: i) a nonparametric haul based ALK, and ii) a spatial model-based

ALK. These ALKs are described in section 2, and the results from the model based ALK gives a strong case for assuming variation in the ALK within RFAs. A spatial model based ALK (Berg and Kristensen, 2012) is currently used for assessment in the North Sea with use of IBTS data. The model introduced in Berg and Kristensen (2012) is similar as the model used in this research, the main difference is that we include the spatial structure trough a spatial random field (Lindgren et al., 2011) and not trough two dimensional splines (Wood, 2017).

Section 1.1 gives an overview of the North Sea International Bottom Trawl Surveys. A brief description of the data is given in Section 3. The current estimators for ALK and catch per unit effort (CPUE) used by ICES in their database for trawl surveys (DATRAS) and our proposed ALK estimators are given in Section 2. The results are given in Section 4 and a discussion is given in Section 5.

1.1 Overview of the North Sea International Bottom Trawl Surveys

The North Sea International Bottom Trawl Survey was formed in 1991, which is a combination of the International Young Herring Survey (IYHS) and eight national surveys in the North Sea, Skagerrak and Kattegat areas. These surveys began in the 1960's, and the 1970's and 1980's, respectively. The IYHS was developed with the aim of obtaining annual recruitment indices for the combined North Sea herring *Clupea harengus* stock (ICES, 2012), but yielded valuable information on other fish species such as cod *Gadus morhua* and haddock *Melanogrammus aeglefinus*.

The North Sea IBTS began with quarterly surveys providing information on seasonal distribution of stocks sampled, hydrography and the environment, which allows changes in fish stock to be monitored and abundance of all fish species (Table 1) to be determined. These quarterly surveys, however became difficult to sustain as countries experienced budget cuts making it impossible to maintain high levels of research vessel effort. As such, in 1997 countries carried out a survey only twice a year; a first quarter survey (January-February) and a third quarter survey (August-September). Table 1 gives the common names (scientific names in parentheses) of the target species that are sampled during the quarterly North Sea International Bottom Trawl Surveys. The common names of the species in parentheses will be used in the rest of paper.

Table 1: Species fished in the NS-IBTS from 1991-2017.

Standard Pelagic	Standard Roundfish	By-Catch Gadoid
Herring (Clupea harengus)	Cod (Gadus morhua)	Pollock (Pollachius)
Sprat (Sprattus sprattus)	Haddock (Melanogrammus aeglefinus)	Pouting (Trisopterus luscus)
Mackerel (Scomber scombrus)	Norway Pout (Trisopterus esmarkii)	Trisopterus minutus (Poor Cod)
	Saithe (Pollachius virens)	Blue Whiting (Micromesistius poutassou)
	Whiting (Merlangius merlangus)	Hake (Merluccius merluccius)
		Ling (Molva molva)
		Tusk (Brosme brosme)

Research vessels from seven nations in the first quarter (Q1) and six nations in the third quarter (Q3) are used for conducting surveys on all finfish species in the North Sea during January-February and July-August, respectively, between 1997-2017 (see Table 7 in appendix B). The sampling frame is defined by the ICES index or roundfish areas (RFA) as shown in Figure 1 numbered 1 to 10, and which we refer to as superstrata (Nottestad et al., 2015; Fuller, 2011). These roundfish areas were substratified into small strata defined by non-overlapping statistical rectangles of roughly 30×30 nautical miles (1^o Longitude $\times$ 0.5^o Latitude), and were convenient to use for North Sea IBTS as they were already being used for fisheries management purposes. Most statistical rectangles contain a number of possible tows that are deemed free of obstructions, and vessels are free to choose any position in the rectangles as long as the hauls are separated by at least 10 nautical miles within and between rectangles. However, all countries select tows based on a semi-random approach from datababes of national safe tows or DATRAS or commercial fishing data, except Sweden who uses fixed stations and in some cases depth-stratified semi-random sampling design (ICES, 2018a), and England who also uses fixed stations and only conduct surveys during the third quarter. In some rectangles, sampling may be further stratified due to significant changes in seabed depth which may, in turn, cause variations in the fish population. In particular, the North Sea IBTS herring, saithe and sprat data are weighted by depth strata in the statistical rectangle (see Table 9 in appendix E). It is also a requirement that countries avoid clustering their stations between adjacent rectangles in order to reduce positive serial correlation, and thereby maximize survey precision. The latest major reallocation of rectangles occurred in 1991, but since then the survey has tried to keep at least one vessel in every subarea in which it had fished in the most recent years. Minor reallocation of rectangles between Norway, Scotland and Germany was done in 2013. Each rectangle was typically sampled twice by two different countries before 1997, but after that target coverage of two trawl hauls per rectangle per survey was introduced because of national financial constraints (ICES, 2015). But in some rectangles in the Eastern English Channel, Southern North Sea and Central North Sea intensified sampling is carried out.

The recommended standard trawling gear of the North Sea IBTS is the mulitpurpose chalut à Grande Ouverture Verticale (GOV) trawl (ICES, 2012), which has been used on all participating vessels since 1992, while different pelagic and bottom trawls suitable for fishing finfish species were used before 1992. Standardized trawling protocols were adopted with a towing speed of 4 knots but depending on vessel performance, tide and weather conditions the average towing speed can be at minimum 3.5 and maximum 4.5 knots. From 2000-2018 trawling is done during the daylight hours, which are considered 15 minutes before sunrise to 15 minutes after sunset (ICES, 2012). After each trawl the total catch of the different species is weighed on board and biological parameters such as length for all fish species caught (to 0.1cm below for shellfish, to 0.5cm below for herring and sprat and to 1cm below for all other species) are collected. Where the numbers of individuals are too large for all of them to be measured to obtain the length distribution, a representative subsample of 100 fish is selected. Otoliths are collected on board from a small fraction of all the target species from all round fish areas (RFAs) (Figure 1) to retrieve age reading. Table 8 in appendix **??** gives the minimum sampling levels of otoliths for the target species.

2 METHODS

This section gives the estimators of abundance indices. The estimators are haul time-based and utilizes an ALK approach. We consider the ALK approach used in DATRAS and we propose two ALK estimators. The ALK used in DATRAS for computing abundance indices does not account explicitly for the spatial distribution in the age-length composition, which may be different and would result in a biased ALK. This difference may be caused either by variation in length-at-age distributions or by variations in the relative abundance of age classes, that is age-at-length distributions (Gerritsen et al., 2006). To account for the

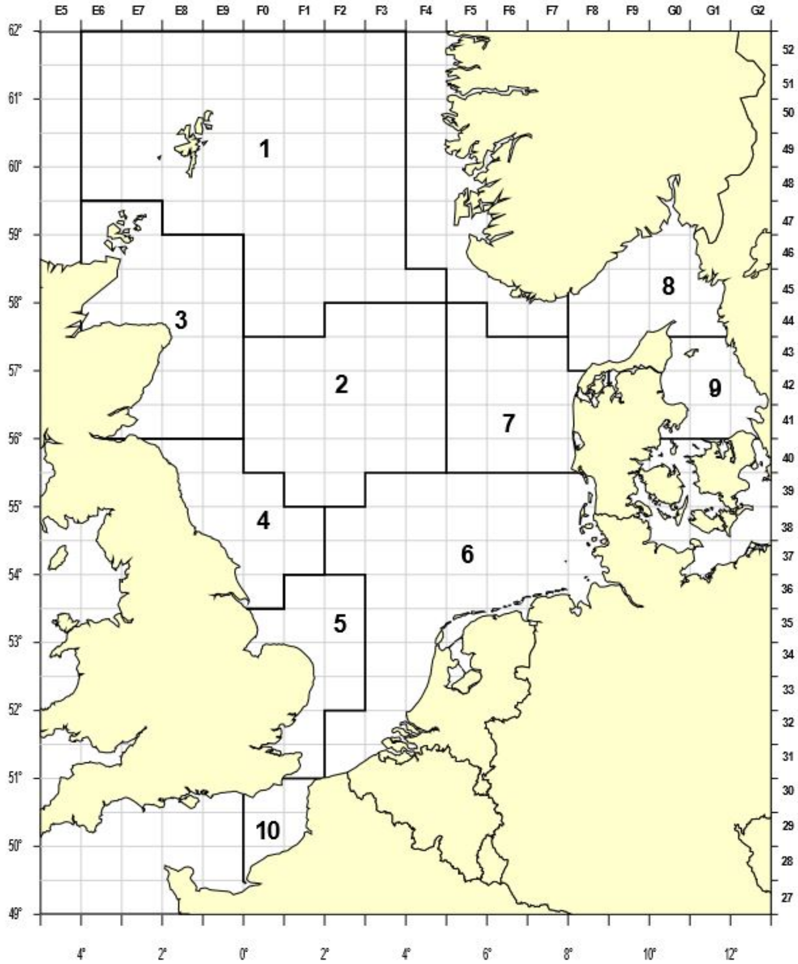

Figure 1: Standard roundfish areas used for roundfish since 1980, for all standard species since 1991. Additional RFA 10 added in 2009. For example, the number 1 indicates ICES Index Area 1, and an ICES Statitical rectangle (ST) in IA 1 is 43F1 (ICES, 2015).

spatial distribution we propose a design-based ALK estimator that is haul dependent (Section 2.2.2) and a model-based ALK estimator (2.2.3).

2.1 Catch per unit effort

In this paper, the catch per unit effort (CPUE) is defined as the number of fish of a certain species and age or length which are caught per hour trawl. In this subsection we define CPUE matematically, which explain how the index is calculated.

For a given species of interest, let $n_{h,l}$ be the number of fish with length l caught by trawl haul h. Define the CPUE for a given length l by trawl haul h as

$$\text{CPUE}_{h,l} = \frac{n_{h,l}}{d_h}, \tag{2.1}$$

were d_h is the duration of the trawl in hours. The CPUE per age class is further defined as

$$\text{CPUE}_{h,a} = \sum_{l \in \mathbf{L}} \text{CPUE}_{h,l} \times ALK_{a,l,h}, \tag{2.2}$$

where $\mathbf{L}$ is the set of all length classes. Further more is $ALK_{a,l,h}$ an age length key which represents the estimated proportion of fish with age a in lth length class in haul h. The mCPUE in a statistical rectangle is further defined as the average of the CPUE for each trawl haul in the rectangle:

$$\text{mCPUE}_{s,a} = \sum_{h \in H_s} \frac{\text{CPUE}_{h,a}}{|H_s|}. \tag{2.3}$$

Here H_s represents the set of trawl hauls taken in statistical rectangle s, and $|H_s|$ is the number of hauls taken in the rectangle. The mCPUE in pth RFA is further defined as

$$\text{mCPUE}_{p,a} = \sum_{s \in S_p} \frac{\text{mCPUE}_{s,a}}{|S_p|} \omega_s, \tag{2.4}$$

where S_p is the set of all statistical rectangles in RFA p, $|S_p|$ is the number of statistical rectangles in RFA p, and ω_s is a weight variable for each statistical rectangle. The weight variable ω_i varies between species. For some species ω equals 1 (e.g. gadus morhua) for all s, and for other species it is the proportion of the statistical rectangle which has depth between 10 to 200 meters (e.g. pollachius virens). The index for abundance at age in the whole study area, $mCPUE_{N,a}$, is further defined by

$$mCPUE_{N,a} = \frac{\sum_{p \in \mathbf{P}} A_p \text{mCPUE}_{p,a}}{A_{\text{total}}}. \tag{2.5}$$

Here $\mathbf{P}$ is the set of RFAs, A_p is the area RFA p, and $A_{\text{total}} = \sum_{p \in \mathbf{P}} A_p$.

2.2 ALK Estimators

In the definition of the CPUE of age includes an ALK, see (2.2). Three ALK estimators are included in this research. The three ALKs estimators included are named *i*) DATRAS ALK, *ii*) haul based ALK and *iii*) model based ALK. In this subsection we define these three ALK estimators.

2.2.1 DATRAS ALK

Let ALK^{D} denote the DATRAS ALK. The ALK^{D} is defined as constant within each RFA, and is calculated for each RFA by aggregating the age observation from each RFA. $ALK^{\text{D}}_{a,l,h}$ used in equation (2.2) is defined as the proportion of observed fish with age a in length class l in the RFA h. If there are no observed fish in length class l in the RFA, ages from length classes close to l is used. The details of the procedure for borrowing strength from neighbouring length classes are given in appendix D.1. The underlying assumption of this age length is that age-length compositions are homogeneous within the RFAs. This is a rather strong assumption, and any violation have an unknown impact on the estimates of abundance indices. Aanes and Vølstad (2015) illustrated that violation of the assumption of constant ALK leads to biased estimates of CPUEs.

2.2.2 Haul based ALK

We define a haul dependent ALK by ALK^{H}. The $ALK^{H}_{a,l,h}$ is defined as the average proportion of observed fish with age a in a pooled length class l in haul h. If there are no observed ages of fish in a length class l in the haul, ages from the same length class in the haul close by is used, see appendix D.2 for detail.

2.2.3 Spatial Model-Based ALK Estimator

In this section we introduce a spatial model based ALK. Including such a model enables us to utilize smooth structures in the distribution of age given length. It further enables us to utilize spatial latent effects. Spatial model-based approach of age-lengths are widely used (Berg and Kristensen, 2012; Rindorf and Lewy, 2001; Hirst et al., 2012), and is used for assessment in the North Sea (Berg et al., 2014).

Let the response variable of the age group of a fish be $a = M, ..., A$ where M is the youngest age and A is the oldest age, which is typically defined as a "plus group". Suppose $y(l, \mathbf{s}, h)$ is the age of a fish with length l caught at location $\mathbf{s}$. Just as in Berg and Kristensen (2012) we use a continuous ration model for the spatial age given length model. That is, let

$$\pi_a(y(l, \mathbf{s})) = P(y = a | y \geq a, l, \mathbf{s}) = \frac{p_a(l, \mathbf{s})}{p_a(l, \mathbf{s}) + \cdots p_{A-1}(l, \mathbf{s})} \text{for } a = M, ..., A-1, \tag{2.6}$$

were $p_a(l,s)$ is the probability of a fish with length l at location $\mathbf{s}$ to be of age a. Further is

$$\text{logit}\pi_a(y(l,\mathbf{s},h)) = \beta_a + f_a(l) + \gamma_a(\mathbf{s}). \tag{2.7}$$

Here β_a is an intercept, $f_a^l(l)$ is a continuous function of length and $\boldsymbol{\gamma}$ is a mean zero Gaussian spatial random field with Matérn covariance function. The spatial random field is intended to capture any spatial variation in the ALK.

The continous function $f_a^l(l)$ is modelled with usage of P-splines, (Wood, 2017). This is achieved by including the spline regression coefficients as a Gaussian random effect. The precision matrix for the spline regression coefficients is constructed such that wigglyness in the spline is penalized, see Wood (2017, page 239) for details. The R package mgcv (Wood, 2015) is used for extracting the precision matrix needed for the spline regression coefficients.

We assume that the spatially Gaussian random field in (2.7), $\boldsymbol{\gamma}$, follows a stationary Matérn covariance structure:

$$\text{Cov}(\gamma(\mathbf{s}_1), \gamma(\mathbf{s}_2)) = \frac{\sigma_\gamma^2}{2^{\nu-1}\Gamma(\nu)}(\kappa_\gamma||\mathbf{s}_1 - \mathbf{s}_2||)^\nu K_\nu(\kappa_\gamma||\mathbf{s}_1 - \mathbf{s}_2||), \tag{2.8}$$

where σ_γ^2 is the marginal variance, $||\cdot||$ is the Euclidean distance measure in kilometres, ν is a smoothing parameter, κ_γ is a spatial scale parameter and $K_\nu(\cdot)$ is the modified Bessel function of the second kind with $\nu = 1$. The spatial range parameter and marginal variances in the spatial fields are assumed to be equal across ages. The spatial field is estimated with the stochastic partial differential equation (SPDE) procedure described in Lindgren et al. (2011). The theory behind the SPDE procedure is based on that the precision matrix of a spatial field with Matérn covariance function can be approximated by a sparse matrix on a grid covering the area of interest. Such a grid and sparse precision matrix are constructed with use of the R-INLA package (Rue et al., 2009).

The model based ALK estimate is obtained by maximizing the likelihood. We maximize the likelihood with usage of the R-package TMB (Kristensen et al., 2015) combined with the optimizing function *nlminb* in R. Advantages of using TMB in this application is that it utilizes sparse structures in the latent fields, it Laplace approximate the latent fields, and uses automatic derivation. A laptop with processor intel(R) Core(TM) i5-6300 CPU @ 2,40 GHz, used approximately 2 minutes to estimate the model.

2.3 *Uncertainty estimation*

In this subsection we describe how the unceratinty of the CPUE estimates are calculated. We use nonparametric bootstrapping to quantify the uncertainty of the CPUEs. Nonparametric resampling allows us to estimate the sampling distribution of the CPUE empirically without making assumptions concerning the data. The percentile method is used to estimate 95% confidence intervals of the estimated CPUEs.

In (ICES, 2013) it was suggested a bootstrap procedure for estimating the uncertainty of CPUEs in the North Sea. That procedure is denote it as DATRAS bootstrap in the rest of this paper. The DATRAS procedure is divided into two parts, one part which samples CPUE per length (2.1), and another part which samples the ALK used in (2.2). The DATRAS bootstrap in based on an assumption of homogeneous CPUE within RFAs. That assumption is likely to be wrong, and will typically cause an overestimation of the uncertainty. We have therefore also included a bootstrap procedure which we denote as the stratified bootstrap procedure, which instead assumes constant CPUE within each statistical rectangle.

2.3.1 *DATRAS and stratified bootstrap*

In this subsection we elaborate the bootstrap procedure for catch at length proposed by *DATRAS* (ICES, 2013) and the stratified procedure, and elaborate how the ALK is simulated. Assume there are N_{s} trawl hauls in a statistical rectangle. The DATRAS bootstrap procedure consists of sampling with replacement N_{s} of all trawl hauls in the corresponding RFA, and place them in the statistical rectangle. This procedure is performed independently across all statistical rectangles. Not that this procedure is based on that the CPUE is homogeneous in the whole RFA. The stratified bootstrap procedure is simply to modify the DATRAS bootstrap to sample the N_{s} hauls from the hauls within the same statistical rectangle. If there are only one trawl hauls within an statistical rectangle, we sample either that haul or the closest haul in air distance.

For simulating the DATRAS ALK we sample with replacement age observations within each RFA stratified with respect to length. If there is only one observed age from a given length class, we sample either that age or at random one age of the closest length class with observed ages. For the haul based ALK, we use the observed ages in the sampled hauls when simulating the CPUE per length.

2.4 Reducing sampling effort

The current sampling procedure for NS-IBTS sample one otholit from every observed length group in every trawl. We investigate how the estimated mCPUE change if the sampling procedrue were changed such that fewer otholits were collected. To achieve such a comparison we remove otholits in a stratified procedure, mimicking a sampling procedure with fewer otholits collected. For sampling fewer otholits we define wider length groups and simulate the otholit collection such that only one otholit is collected from every wider length group. Estimated mCPUE's with summary statistics, based on the simulated reduced data sets, are then compared with usage of all data. In principle we are free to define any length class to reduce the number of observed otholits. For simplicity we propose to select two procedures: i) at random sample one otholit from every 2 cm length group, and ii) at random sample one otholit from every 5 cm length group.

3 Case studies

In this section the method will be applied to data from the International Bottom Trawl Survey for the year 2018, which is obtained from the DATRAS database (ICES, 2018b). We choose year 2018 because in that year it was introduced a new sampling procedure of otholits explained in table 8. The species of interest are cod and saithe and the samples are collected in the first and third quarters. All samples are caught using the standard GOV gear described in Section 1.1. Section 3.1 gives a brief description of the data.

3.1 Summary of cod and saithe data

Otoliths are usually collected from a fraction of the fish sampled (see Table 8 in appendix C), but in some cases only a small number of fish are caught so otoliths are taken from all catches. (**include legend on plots and number of trawl hauls with length only and age data**)

Table 2: Summary of North Sea IBTS cod and saithe (in parentheses) data for first quarter in year 2018.

Data	**Description**
Trawl hauls	Total of 357 trawl hauls in Q1 of 2018, 238 (83) with length and 231 (82) with age information
Age	The age of cod varied between 1 to 8 years, while saithe age ranged from 1 to ....
Length	Length information in cm of each cod varied between 8 to 112 cm while saithe varied between ... to ... cm
Date	Date of catch in Q1 varied between 13.01.2015 to 19.02.2015 and in Q3 between 26.07.2015 to 06.09.2015
Statistical rectangle	The stratum in which at least two trawl hauls are made
Coordinates	Geographic coordinates of each trawl haul in a statistical rectangle
Duration of haul	Mean duration is 25.9 minutes, with 15 to 30 minutes as 90% coverage interval.
Total count for all ages	7605 cod in Q1 of 2015 and ....saithe in Q3 of 2015.

4 RESULTS

In this section we show differences between the estimated CPUE with the proposed procedures. We further show how much information which would typically have been lost if the number of otholits investigated were reduced as explained in section 2.4.

Before we show the estimated CPUE we want to highlight some results from the model based ALK.

4.1 Estimates of abundance-at-age and their uncertainty

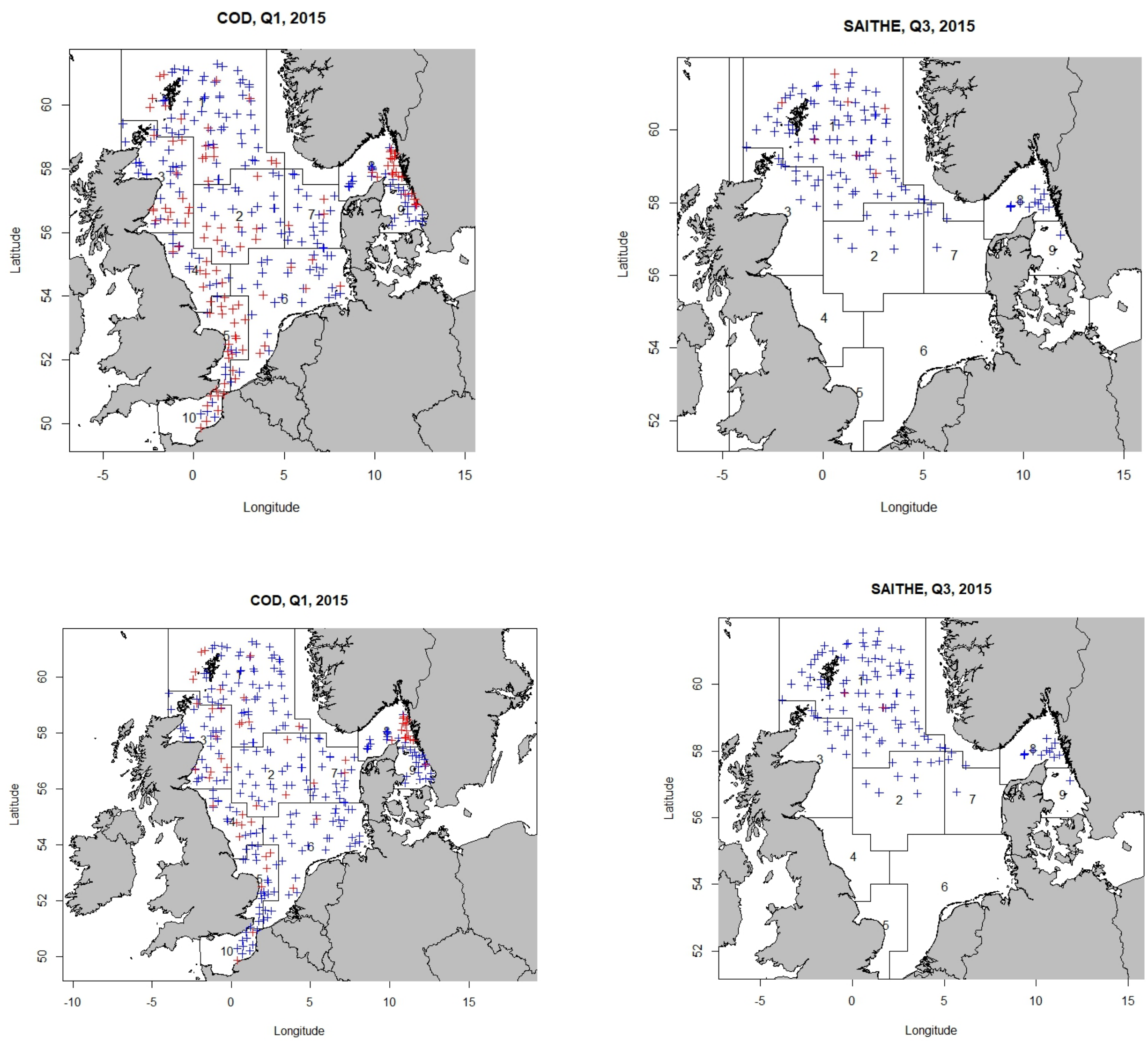

Figure 2: cod with length class 1 cm (upper left) and length class 5 cm (lower left), and saithe with length class 1cm (upper right) and length class 5 cm (lower right)

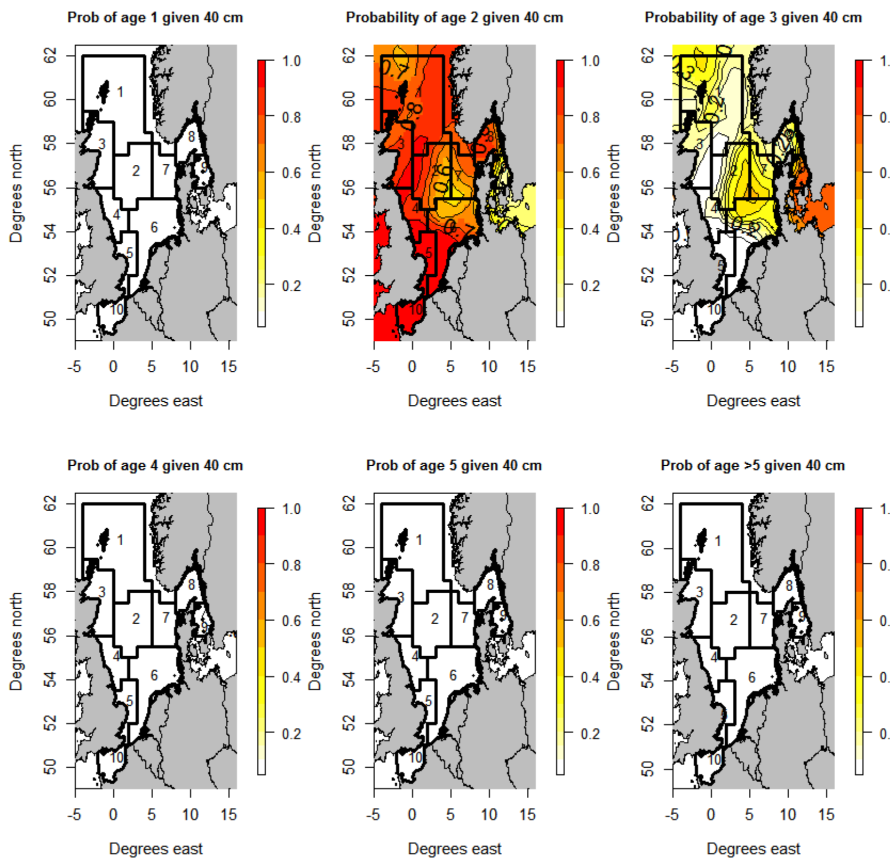

Figure 3: Estimated probability of age of a 40 cm long cod in the first quarter of year 2015. The probability of age three or older is approximately zero. The polygones marked 1 to 10 is the round fish areas (RFAs) where the ALK is assumed constant in the currently used estimators of the official CPUEs.

Table 3: Estimates of abundance indices (Index), and estimated standard errors for 400 bootstrap samples from the following bootstrap procedures: DATRAS, Naive, Stratified, and Hierarchical for cod in RFA 1 in the first quarter of year 2015. Approximate 95% confidence intervals (CI) are also given.

	DATRAS ALK DATRAS Bootstrap			**Haul-based ALK** Stratified Bootstrap			**Model-based ALK** Stratified Bootstrap		
Age (a)	Abundance estimate	Standard error	Relative standard error	Abundance estimate	Standard error	Relative standard error	Abundance Estimate	Standard error	Relative standard error
0	0	0	–	0	0	–	0	0	–
1	0.764	0.26	34%	0.60	0.24	40%	0.70	0.36	51%
2	21.989	6.76	31%	22.21	4.15	19%	22.11	4.28	19%
3	11.285	2.19	19%	10.58	1.20	11%	10.99	1.77	16%
4	3.265	0.71	22%	3.67	1.28	35%	3.50	0.87	25%
5	1.147	0.34	30%	1.27	0.42	33%	1.20	0.48	40%
6+	1.276	0.38	30%	1.40	0.70	50%	1.21	0.42	35%
				Approximate 95% CI from bootstrap procedures					
0	0	(0, 0)		0	(0, 0)		0	(0, 0)	
1	0.764	(0.31, 1.33)		0.60	(0.31, 0.91)		0.70	(0.35, 1.48)	
2	21.898	(12.73, 37.15)		22.21	(15.64, 30.72)		22.11	(14.76, 30.36)	
3	11.285	(6.31, 15.02)		10.58	(8.74, 13.65)		10.99	(8.61, 15.42)	
4	3.265	(1.49, 4.21)		3.67	(2.81, 4.74)		3.50	(1.96, 5.60)	
5	1.147	(0.40, 1.75)		1.27	(0.67, 2.31)		1.20	(0.56, 2.78)	
6+	1.276	(0.44, 1.82)		1.40	(0.78, 2.69)		1.21	(0.70, 2.43)	

Table 4: Estimates of abundance indices (Index), and estimated standard errors for 400 bootstrap samples from the following bootstrap procedures: DATRAS, Naive, Stratified, and Hierarchical for cod in RFA 1 in the first quarter of year 2015. Approximate 95% confidence intervals (CI) are also given.

	DATRAS ALK			**Haul-based ALK**				**Model-based ALK**			
Age (a)	Index	DATRAS	Naive	Index	Naive	Stratified	Hierarchical	Index	Naive	Stratified	Hierarchical
1	0	0	0	0	0	0	0	0	0	0	0
2	0.764	0.26	0.23	0.60	0.24	0.16	0.81	0.70	0.36	0.31	0.78
3	21.989	6.76	4.08	22.21	4.15	4.20	13.23	22.11	4.28	4.26	10.69
4	11.285	2.19	1.27	10.58	1.20	1.28	5.85	10.99	1.77	1.84	4.53
5	3.265	0.71	0.57	3.67	1.28	0.56	3.02	3.50	0.87	0.94	2.46
6	1.147	0.34	0.33	1.27	0.43	0.43	1.59	1.20	0.48	0.62	0.83
7	1.276	0.38	0.39	1.40	0.70	0.53	2.01	1.21	0.42	0.46	0.85
						Approximate 95% CI from bootstrap procedures					
1	0	(0, 0)	(0, 0)	0	(0, 0)	(0, 0)	(0, 0)	0	(0, 0)	(0, 0)	(0, 0)
2	0.764	(0.31, 1.33)	(0.40, 1.22)	0.60	(0.20, 1.18)	(0.31, 0.91)	(0, 3.81)	0.70	(0.37, 1.81)	(0.35, 1.48)	(0.05, 2.05)
3	21.898	(12.73, 37.15)	(14.90, 30.01)	22.21	(15.01, 30.09)	(15.64, 30.72)	(11.34, 61.57)	22.11	(14.56, 30.61)	(14.76, 30.36)	(7.12, 41.41)
4	11.285	(6.31, 15.02)	(9.63, 14.42)	10.58	(8.75, 13.54)	(8.74, 13.65)	(0, 17.73)	10.99	(8.45, 15.43)	(8.61, 15.42)	(3.90, 19.99)
5	3.265	(1.49, 4.21)	(2.45, 4.50)	3.67	(2.42, 7.35)	(2.81, 4.74)	(0, 8.26)	3.50	(2.11, 5.56)	(1.96, 5.60)	(0.87, 8.10)
6	1.147	(0.40, 1.75)	(0.67, 1.95)	1.27	(0.50, 2.14)	(0.67, 2.31)	(0, 4.35)	1.20	(0.58, 2.50)	(0.56, 2.78)	(0.15, 2.98)
7	1.276	(0.44, 1.82)	(0.72, 2.24)	1.40	(0.71, 3.42)	(0.78, 2.69)	(0, 5.15)	1.21	(0.70, 2.27)	(0.70, 2.43)	(0.09, 3.22)

Table 5: Estimates of abundance indices (Index), and estimated standard errors for 200 bootstrap samples from the following bootstrap procedures: DATRAS, Naive, Stratified, and Hierarchical for cod in RFA 1 in the first quarter of year 2015. Approximate 95% confidence intervals (CI) are also given.

	DATRAS ALK			**Haul-based ALK**				**Model-based ALK**			
Age (a)	Index	DATRAS	Naive	Index	Naive	Stratified	Hierarchical	Index	Naive	Stratified	Hierarchical
1	0	0	0	0	0	0	0	0	0	0	0
2	0.764	0.26	0.24	0.60	0.23	0.17	2.55	0.70	0.34	0.45	0.48
3	21.989	7.30	3.89	22.21	4.14	4.19	9.61	22.11	4.46	4.10	13.23
4	11.285	2.29	1.30	10.58	1.18	1.29	4.72	10.99	2.37	1.94	5.20
5	3.265	0.74	0.57	3.67	1.18	0.58	2.00	3.50	0.93	0.89	3.28
6	1.147	0.35	0.37	1.27	0.42	0.42	0.91	1.20	0.46	0.56	1.28
7	1.276	0.41	0.40	1.40	0.73	0.52	0.85	1.21	0.42	0.43	2.63
					Approximate 95% **CI from bootstrap procedures**						
1	0	(0, 0)	(0, 0)	0	(0, 0)	(0, 0)	(0, 0)	0	(0, 0)	(0, 0)	(0, 0)
2	0.764	(0.27, 1.29)	(0.40, 1.34)	0.60	(0.31, 1.15)	(0.31, 0.93)	(0, 7.22)	0.70	(0.40, 1.72)	(0.38, 1.89)	(0.02, 1.72)
3	21.898	(12.70, 37.67)	(15.46, 29.63)	22.21	(15.22, 30.25)	(14.65, 30.18)	(3.65, 40.02)	22.11	(14.33, 3097)	(14.71, 29.90)	(3.71, 54.91)
4	11.285	(6.64, 15.33)	(9.47, 14.32)	10.58	(8.94, 13.54)	(9.14, 13.94)	(0, 15.63)	10.99	(8.44, 16.60)	(8.65, 16.07)	(1.84, 21.53)
5	3.265	(1.56, 4.46)	(2.43, 4.39)	3.67	(2.46, 6.59)	(2.75, 4.77)	(0, 7.18)	3.50	(2.18, 5.30)	(1.92, 5.07)	(0.10, 10.07)
6	1.147	(0.39, 1.77)	(0.64, 1.96)	1.27	(0.39, 2.02)	(0.69, 2.27)	(0, 3.83)	1.20	(0.54, 2.39)	(0.60, 2.79)	(0.01, 3.30)
7	1.276	(0.42, 2.13)	(0.75, 2.28)	1.40	(0.62, 3.23)	(0.78, 2.68)	(0, 2.38)	1.21	(0.70, 2.30)	(0.66, 2.26)	(0., 3.09)

Table 6: Estimates of abundance indices (Index), and estimated standard errors from the following bootstrap procedures: DATRAS, Naive, Stratified, and Hierarchical for saithe in RFA 1 in the third quarter of year 2015. Approximate 95% confidence intervals (CI) are also given.

	DATRAS ALK			**Haul-based ALK**				**Model-based ALK**			
Age (a)	Index	DATRAS	Naive	Index	Naive	Stratified	Hierarchical	Index	Naive	Stratified	Hierarchical
1	0	0	0	0	0	0	0	0	0	0	0
2	0.764	0.26	0.24	0.60	0.23	0.17	2.55	0.70	0.34	0.45	0.48
3	21.989	7.30	3.89	22.21	4.14	4.19	9.61	22.11	4.46	4.10	13.23
4	11.285	2.29	1.30	10.58	1.18	1.29	4.72	10.99	2.37	1.94	5.20
5	3.265	0.74	0.57	3.67	1.18	0.58	2.00	3.50	0.93	0.89	3.28
6	1.147	0.35	0.37	1.27	0.42	0.42	0.91	1.20	0.46	0.56	1.28
7	1.276	0.41	0.40	1.40	0.73	0.52	0.85	1.21	0.42	0.43	2.63
					Approximate 95% **CI from bootstrap procedures**						
1	0	(0, 0)	(0, 0)	0	(0, 0)	(0, 0)	(0, 0)	0	(0, 0)	(0, 0)	(0, 0)
2	0.764	(0.27, 1.29)	(0.40, 1.34)	0.60	(0.31, 1.15)	(0.31, 0.93)	(0, 7.22)	0.70	(0.40, 1.72)	(0.38, 1.89)	(0.02, 1.72)
3	21.898	(12.70, 37.67)	(15.46, 29.63)	22.21	(15.22, 30.25)	(14.65, 30.18)	(3.65, 40.02)	22.11	(14.33, 3097)	(14.71, 29.90)	(3.71, 54.91)
4	11.285	(6.64, 15.33)	(9.47, 14.32)	10.58	(8.94, 13.54)	(9.14, 13.94)	(0, 15.63)	10.99	(8.44, 16.60)	(8.65, 16.07)	(1.84, 21.53)
5	3.265	(1.56, 4.46)	(2.43, 4.39)	3.67	(2.46, 6.59)	(2.75, 4.77)	(0, 7.18)	3.50	(2.18, 5.30)	(1.92, 5.07)	(0.10, 10.07)
6	1.147	(0.39, 1.77)	(0.64, 1.96)	1.27	(0.39, 2.02)	(0.69, 2.27)	(0, 3.83)	1.20	(0.54, 2.39)	(0.60, 2.79)	(0.01, 3.30)
7	1.276	(0.42, 2.13)	(0.75, 2.28)	1.40	(0.62, 3.23)	(0.78, 2.68)	(0, 2.38)	1.21	(0.70, 2.30)	(0.66, 2.26)	(0., 3.09)

4.2 Optimum Sampling Effort for North Sea cod and saithe

5 DISCUSSION

- We have investigated three ALK estimators: 1) DATRAS ALK, 2)Haul-based ALK and 3) Model-based ALK
- discuss ALK estimators, which of the three is the most appropriate at this time, discuss model-based ALK and compare with Berg and Kristensen (2012) as they are similar are both used on IBTS data
- How can estimators be improved, also computational time (1000 bootstrapped samples for each of the four estimators took hours (possibly more than ten, needs verification))
- Possibly consider hierarchical bootstrapping as done in Ren et al. (2010) - draft codes are available
- Discuss next steps for example, removal of otoliths or age information and trawl hauls: the effect may be substantial for larger fish (hence older fish) - as shown in table **??** fewer older fish are sampled and many younger ones are sampled so the effect would be marginal for younger fish). Draft codes are available for this. Wieland et al. (2009) found that considerable catches for cod of older ages were made where the IBTS reported low densities or no cod all (*this is based on data from collaborative fishermen-biologists project on cod in the north-eastern central North Sea*). Also smaller sample sizes would also have an effect on estimated bootstrapped confidence intervals. The smaller the original sample the less likely it is to represent the entire population, thus the more difficult it becomes to compute valid confidence intervals. The bootstrap relies heavily on the tails of the estimated sampling distribution when computing confidence intervals, and using small samples may jeopardize the validity of this computation.
- a possible full model-based approach for estimating abundance at age with variance simultaneously?
- *Note to us: include ICES references*

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Appendices

A Probability Plot for saithe in the North Sea in Q1 of 2015

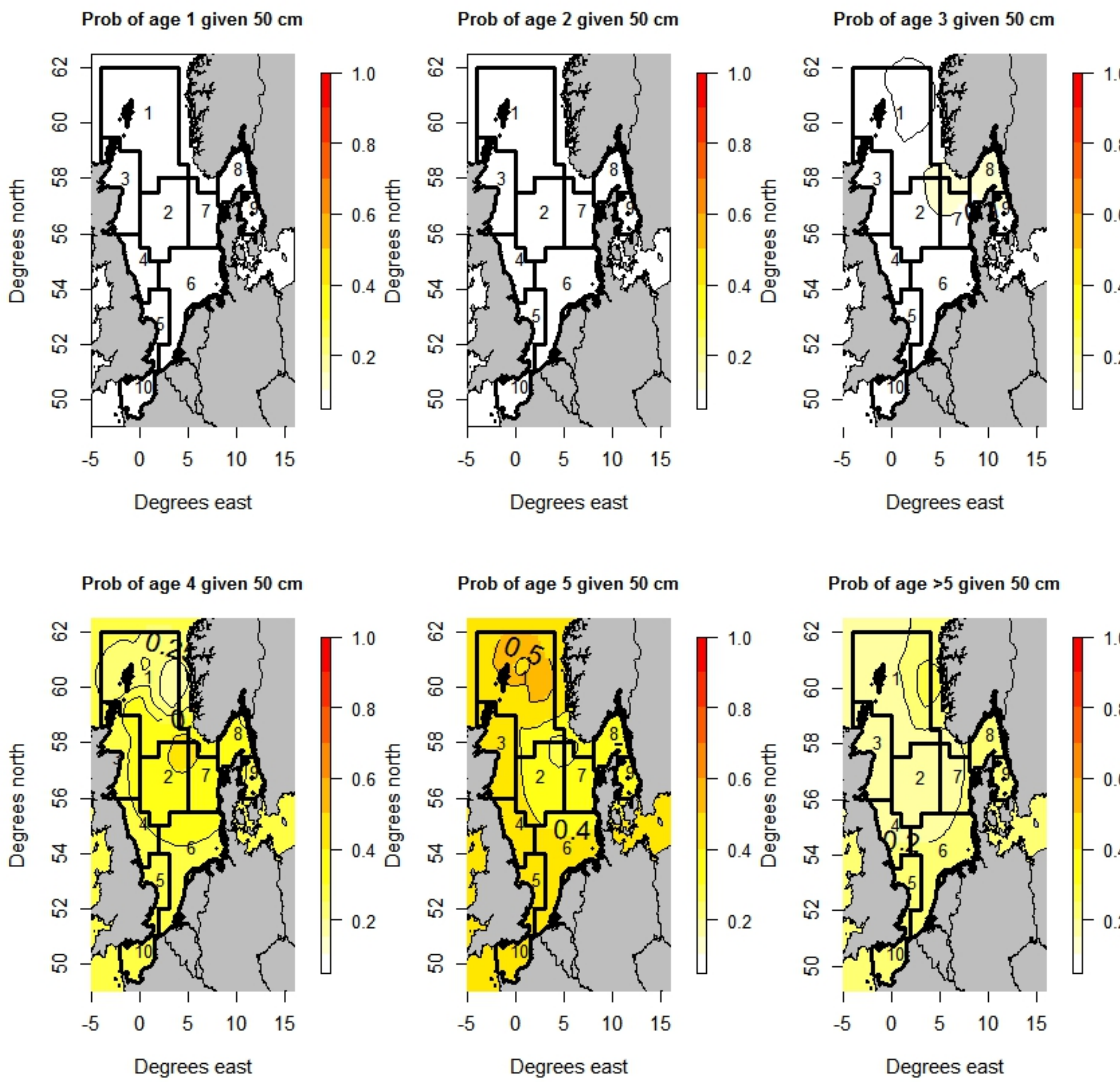

Figure 4: Estimated probability of age of a 50 cm long saithe in the first quarter of year 2015. The probability of age 1 to age 3 is approximately zero. The polygones marked 1 to 10 is the round fish areas (RFAs) where the ALK is assumed constant in the currently used estimators of the official CPUEs. The plots show that the age of saithe is more likely to be 5 given that it is 50 cm, particularly in RFA 1.

B Areas fished by different countries in the North Sea IBTS

Typically, two different countries fish each rectangle so that at least two trawl hauls are made per rectangle. But, intensified sampling is carried out in the following areas: at least 3 hauls per rectangle are taken in statistical rectangles 31F1, 31F2, 32F1, 33F4, 34F2, 34F3, 34F4, 35F3, 35F4; while six or more hauls per rectangle are taken in statistical rectangles 30F1, 32F2, 32F3, 33F2, 33F3 (ICES 1999). The Skagerrak and Kattegat is fished solely by Sweden, who sample more than once in every rectangle while the west of Shetland (in Q1 and Q3) and inshore areas (Q3) is fished solely by Scotland. The edge of the Norwegian Trench is fished solely by Norway, but inshore areas near Denmark is fished by Denmark. The southern North Sea is fished by Denmark, Germany and England. France, typically, is the only country that surveys the western English Channel. Areas are surveyed by a single country because of the large proportion of untrawalable area (and subsequent gear damage issues experienced by other nations) for efficient logistical purposes. Table 7 gives the countries and research vessels participating the North Sea IBTS.

Table 7: Survey countries, vessel name, and period research vessels participating in first quarter (Q1) and third quarter (Q3) during 1997-2017.

	First Quarter (Q1)		**Third Quarter (Q3)**	
Country	Vessel name	Period	Vessel name	Period
Denmark	Dana	January-February	Dana	July-August
France	Thalassa II	January-February	-	-
Germany	Walther Herwig III	January-February	Walther Herwig III	July-August
Netherlands	Tridens 2	January-February	-	-
Norway	G.O. Sars	January-February	Johan Hjort	July
UK England	-	-	Endeavour	August-September
UK Scotland	Scotia III	January-February	Scotia III	July-August
Sweden	Dana	January-February	Dana	August

C Otolith sampling per fish species

From 1991-2017, most countries conducted quota sampling of otoliths per length group in a RFA. But from 2013 Norway has been sampling one otolith per length class from each trawl haul (to 0.1cm below for shellfish, to 0.5cm below for herring and sprat and to 1cm below for all other species). From the first quarter in 2018 all countries are required to sample one otolith per length class per trawl haul. Table 8 gives the minimum sampling levels of otoliths for the target species. However, for the smallest size groups, that presumably contain only one age group, the number of otoliths per length class may be reduced, and more otoliths per length are required for the larger length classes.

Table 8: Minimum sampling levels of otoliths by species for RFA or per trawl haul.

Period	Species	Minimum sampling levels of otoliths per length class
1991-2017		**Number of otoliths per length class in a RFA**
	herring	8 otoliths per $\frac{1}{2}$ cm group
	sprat	16 otoliths per $\frac{1}{2}$ cm length class $8.0 - 11.0$ cm
		12 otoliths per $\frac{1}{2}$ cm length class ≥ 11.0 cm
	mackerel	8 otoliths per $\frac{1}{2}$ cm length class
	cod	8 otoliths per 1 cm length class
	haddock	8 otoliths per 1 cm length class
	whiting	8 otoliths per 1 cm length class
	Norway pout	8 otoliths per 1 cm length class
	saithe	8 otoliths per 1 cm length class
	All target species	From 2013 Norway and Scotland, and Netherlands from 2016 have been sampling 1 otoliths per length class from each trawl haul (to 0.1cm below for shellfish, to 0.5cm below for herring and sprat, and to 1cm below for all other species).
2018		**Number of otoliths per length class per trawl haul**
	herring	1 otoliths per $\frac{1}{2}$ cm group
	sprat	1 otoliths per $\frac{1}{2}$ cm length class $8.0 - 11.0$ cm
		1 otoliths per $\frac{1}{2}$ cm length class ≥ 11.0 cm
	mackerel	1 otoliths per 1 cm length class
	cod	1 otoliths per 1 cm length class
	haddock	2 otoliths per 5 cm length class $11 - 15,\ 16 - 20,\ 21 - 25,\ 26 - 30$ cm
	Norway pout	2 otoliths per 5 cm length class $5 - 10,\ 11 - 15$ cm
		2 otoliths per 1 cm length class > 15 cm
	saithe	1 otoliths per 1 cm length class
	plaice	1 otoliths per 1 cm length class

D Imputation for missing age samples

Catches of the target species are sampled (or subsampled with a size of 100 if the catches are too large) for length, and otoliths are typically collected from a subsample of the individuals sampled for length in the RFA, or per trawl haul as in the case of Norway for determining age of the fish (see Table **??**). In the case of Norway where all trawl hauls are sampled for otoliths, missing age samples would still occur for the following two reasons: 1) the fish is below minimum length for otolith sampling (unreadable otoliths) or 2) otoliths are misplaced. Abundance indices by age group are estimated based on three age-length-keys (ALK): 1) DATRAS ALK estimator, 2) Haul dependent ALK estimator, and 3) Spatial model-based ALK estimator.

D.1 DATRAS ALK Borrowing Approach

The ALK proposed in DATRAS (ICES 2013), which is an aggregation of individual samples from a haul combined over a round fish area (RFA), and missing age samples are imputed as follows:

1. If there is no ALK for a length in the CPUE dataframe, age information is obtained accordingly
    - If length class (CPUE) $<$ minimum length class (ALK), then age=1 for the first quarter and age=0 for all other quarters
    - If minimum length class (ALK) $<$ length class (CPUE) $<$ maximum length (ALK) then age is set to the nearest ALK. If the ALK file contains values at equal distance, a mean is taken from both values.
2. If length class (CPUE) $>$ maximum length (ALK) age is set to the plus group.

The underlying assumption of this ALK approach is that age-length compositions are homogeneous within the superstrata.

D.2 Haul-based ALK Borrowing Approach

The second is an a haul dependent ALK estimator which we propose, and is denoted by ALK^H. Since the age-length composition of fish may be space-variant, that is, there may be variation in age-length compositions between trawl stations within a superstrata, the spatial dependence of the age-length composition must be accounted for to produce reliable estimates of the CPUE per age estimates. If this spatial dependence is ignored not only will estimates of abundance be biased but the impact on the variance may be substantial. So for each trawl haul an ALK^H is produced. Since there are few or none observations of ages for each length class in a trawl haul, length classes are therefore pooled in increasing order such that there are five length classes in each pooled length group. To replace missing values for the age distribution in the pooled length groups the method of "borrowing" ages from length groups in trawl hauls closest in air distance within the RFA is used. If there are no observed ages in the pooled length group in the RFA, missing values for the age distribution are replaced following the procedure outlined in the DATRAS ALK procedure (D.1) in step 1.

E Weightings of Statistical Rectangles

StatRec	Weight	StatRec	Weight	StatRec	Weight	StatRec	Weight	StatRec	Weight
31F1	0.6	38F0	1	41F7	1	44F3	1	48E7	1
31F2	0.8	38F1	1	41F8	0.1	44F4	1	48E8	0.9
31F3	0.05	38F2	1	41G0	0.2	44F5	0.9	48E9	1
32F1	0.8	38F3	1	41G1	0.97	44F8	0.25	48F0	1
32F2	1	38F4	1	41G2	0.53	44F9	0.8	48F1	1
32F3	0.8	38F5	1	42E7	0.4	44G0	0.94	48F2	1
32F4	0.01	38F6	1	42E8	1	44G1	0.6	48F3	0.5
33F1	0.3	38F7	1	42E9	1	45E6	0.4	48G0	0.02
33F2	1	38F8	0.3	42F0	1	45E7	1	49E6	0.8
33F3	1	39E8	0.5	42F1	1	45E8	1	49E7	1
33F4	0.4	39E9	1	42F2	1	45E9	1	49E8	0.4
34F1	0.4	39F0	1	42F3	1	45F0	1	49E9	1
34F2	1	39F1	1	42F4	1	45F1	1	49F0	1
34F3	1	39F2	1	42F5	1	45F2	1	49F1	1
34F4	0.6	39F3	1	42F6	1	45F3	1	49F2	1
35F0	0.8	39F4	1	42F7	1	45F4	0.6	49F3	0.5
35F1	1	39F5	1	42F8	0.2	45F8	0.3	50E6	0.1
35F2	1	39F6	1	42G0	0.32	45F9	0.02	50E7	0.6
35F3	1	39F7	1	42G1	0.89	45G0	0.24	50E8	0.7
35F4	0.9	39F8	0.4	42G2	0.64	45G1	0.55	50E9	0.9
35F5	0.1	40E7	0.04	43E7	0.03	46E6	0.4	50F0	1
36F0	0.9	40E8	0.8	43E8	0.9	46E7	0.9	50F1	1
36F1	1	40E9	1	43E9	1	46E8	1	50F2	1
36F2	1	40F0	1	43F0	1	46E9	1	50F3	0.2
36F3	1	40F1	1	43F1	1	46F0	1	51E6	0
36F4	1	40F2	1	43F2	1	46F1	1	51E7	0
36F5	1	40F3	1	43F3	1	46F2	1	51E8	0.5
36F6	0.9	40F4	1	43F4	1	46F3	0.8	51E9	1
36F7	0.4	40F5	1	43F5	1	46F9	0.3	51F0	1
36F8	0.5	40F6	1	43F6	1	46G0	0.52	51F1	1
37E9	0.2	40F7	1	43F7	1	46G1	0.2	51F2	0.5
37F0	1	40F8	0.1	43F8	0.94	47E6	0.8	51F3	0
37F1	1	41E6	0.03	43F9	0.41	47E7	0.6	52E6	0
37F2	1	41E7	0.8	43G0	0.21	47E8	1	52E7	0
37F3	1	41E8	1	43G1	0.7	47E9	1	52E8	0
37F4	1	41E9	1	43G2	0.3	47F0	1	52E9	0.1
37F5	1	41F0	1	44E6	0.5	47F1	1	52F0	0.2
37F6	1	41F1	1	44E7	0.5	47F2	1	52F1	0.5
37F7	1	41F2	1	44E8	0.9	47F3	0.6	52F2	0.1
37F8	0.8	41F3	1	44E9	1	47F9	0.01		
38E8	0.2	41F4	1	44F0	1	47G0	0.3		
38E9	0.9	41F5	1	44F1	1	47G1	0.02		
52F3	0	41F6	1	44F2	1	48E6	1		

Table 9: *Weights used for pollachius virens in equation (2.3).*