Getting to Orbit

1.1 Introduction

Getting a spacecraft to orbit is difficult. Rocket science is one of those things that seems simple at first, and it is. It's not complicated, but it is very difficult. There are several things that have to happen, in a particular order, without failing and without human interaction in order to get a mass to orbit. Making sure that all happens correctly is a matter of good engineering and some trial and error. There is a lot we can learn from physics as to what is involved to get to orbit. This paper explores the physics in a ground up, step-by-step fashion to get to orbit.

1.2 Terms

Lets start be defining terms. There is a glossary at the end, but I prefer to get some of the more confusing and important terms defined first. LEO is low earth orbit. Mass Fraction is a byproduct of the rocket equation. It comes from the fact that the final velocity of a rocket is proportional to the fraction of the vehicle weight that is burned as fuel. I_{sp} is the specific impulse. Another term from the rocket equation that shows up a lot. It's the efficiency of a rocket motor.

2.1 Getting Started

A great place to start is to find an analytical solution for a rocket and solve for the height it will travel. Put everything in the 'up' axis and ignoring a ton of things and making some assumptions will result in an easy place to start. Then we can add in more complicated terms. At some point it will be impossible to make an analytical solution anymore (when you start getting into complex air resistance models) and a numerical approach will be required. At this point the analysis techniques will change a little but a lot will still apply.

2.2 The Simplest Case

The simplest case is a one dimensional ballistic trajectory, all in the z direction with constant gravity and no air resistance. Figure 2.1 shows the forces on the object. Finding an equation of motion is as simple as adding all the forces, which in this case is just gravity.

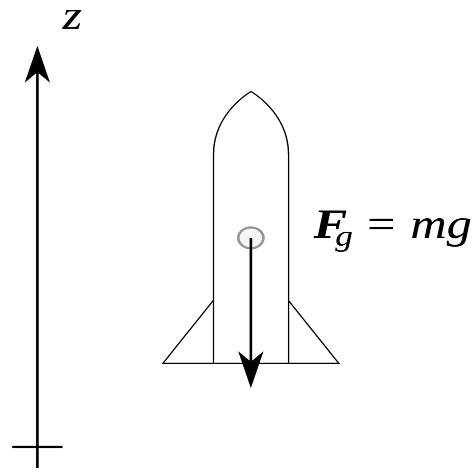

Figure 2.1 Force on a simple rocket

We apply Newtons 2^{nd} law and we get:

$$\boldsymbol{F}=m\boldsymbol{a}=-mg \tag{2.1}$$

$$ma=-mg \tag{2.2}$$

$$a=-g \tag{2.3}$$

Note that we take g to be an acceleration in the negative direction. Integrating once we get a function for the velocity of the projectile.

$$v=-gt+v. \tag{2.4}$$

Integrating twice will get the height.

$$h=-\frac{1}{2}gt^2+v_0t+z_0 \tag{2.5}$$

We can solve for the time it takes because we know it will be at it's apogee when the velocity reaches 0.

$$0=-gt+v_0 \tag{2.6}$$

$$t=\frac{v_0}{g} \tag{2.7}$$

Substituting t into equation 2.5 we get

$$h = -\frac{1}{2} g \left(\frac{v_0}{g} \right)^2 v_0 \left(\frac{v_0}{g} \right) + z_0 \qquad \textbf{(2.8)}$$

$$h = \frac{1}{2} \frac{v_0^2}{g} + z_0 \qquad \textbf{(2.9)}$$

2.3 Simplest Case with Energy

In this case because we are dealing with conservative forces and simple motion so we can work with energy. This should give us the same result as the equation 2.9 so this is a good test for our earlier work.