Monte Carlo Example

Nathan Sanford

November 6, 2019

Consider the problem of computing the one-dimensional integral

$$\mathcal{I} = \int_0^\infty e^{-x} \cos(x) \, dx.$$

This integral can be computed numerically by considering $p(x) = \exp(-x)$ to be a probability distribution (note that it is a valid distribution as it is nonnegative and its total integral on 0 to ∞ is 1), then the above integral is equivalent to the expectation $\mathcal{I} = E[\cos(X)]$ with X drawn from $p(x)$. Therefore, we can compute the value of $\mathcal{I}$ via Monte Carlo integration by repeatedly drawing samples from the exponential distribution, taking the cosine of each sample, and then taking the mean of all the results. After a sufficiently large amount of samples, we obtain an estimate near the true value of $\mathcal{I} = 1/2$.