

Radiation Equilibrium

Disintegration (Decay) Law

$N(t)$: number of atom at time t .

$$\frac{dN}{dt} = -\lambda N \quad (\lambda : \text{disintegration (decay) constant})$$

$$N(t) = N_0 e^{-\lambda t} = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T}} \quad (T : \text{half life})$$

$$\lambda T = \ln 2 = 0.693$$

$A(t)$: radioactivity at time t

$$A(t) = \lambda N = A(0) e^{-\lambda t} = A(0) \left(\frac{1}{2}\right)^{\frac{t}{T}}$$

Disintegration (Decay) Series

Nuclide 1 $\xrightarrow{\text{decay}}$ Nuclide 2 $\xrightarrow{\text{decay}}$ Nuclide 3 \dots

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\therefore N_1(t) = N_{10} e^{-\lambda_1 t}$$

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) + N_{20} e^{-\lambda_2 t}$$

If $N_{20} = 0$ (Nuclide 2 not present at $t=0$),

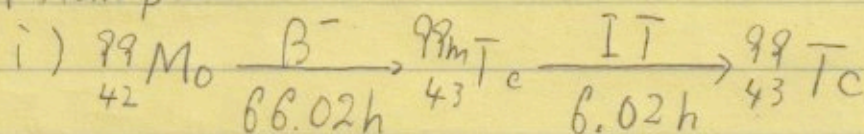
$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

1) If $\lambda_1 < \lambda_2$, after sufficiently long time, $e^{-\lambda_1 t} \gg e^{-\lambda_2 t}$

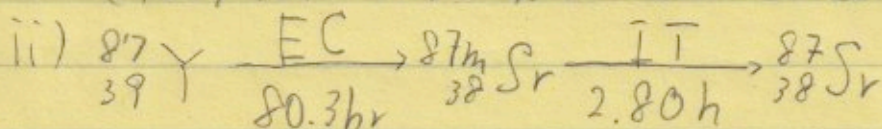
$$N_2(t) \approx \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} e^{-\lambda_1 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(t)$$

Nuclide 2 decreases with the half life of Nuclide 1
Transient Equilibrium

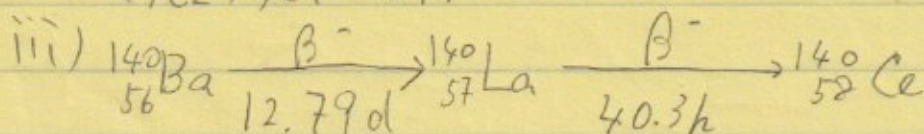
Examples



($\lambda_2 / \lambda_1 = 11$) ${}^{99m}_{43}\text{Tc}$ used for diagnosis



($\lambda_2 / \lambda_1 = 29$)



($\lambda_2 / \lambda_1 = 7.6$)

2) If $\lambda_1 < \lambda_2$, after sufficiently long time,

$$N_2(t) \approx \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(t) \approx \frac{\lambda_1}{\lambda_2} N_1(t)$$

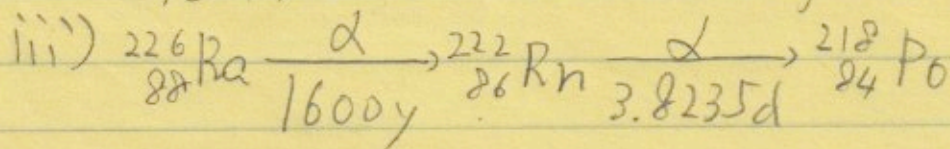
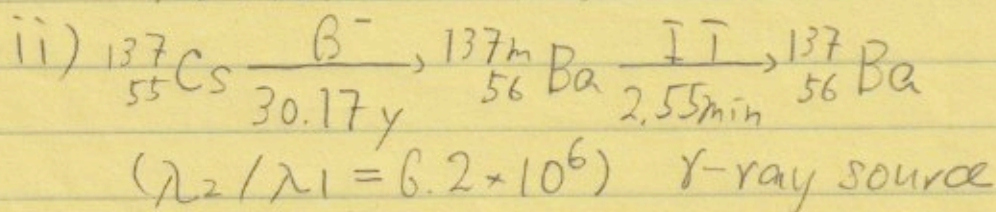
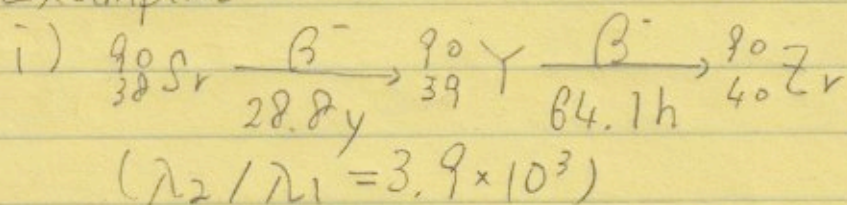
$$\therefore \lambda_1 N_1(t) = \lambda_2 N_2(t)$$

$$A_1(t) = A_2(t)$$

The radioactivity of parental nuclide and daughter nuclide become equal.

Secure Equilibrium

Examples:



If the half life of the first nuclide is sufficiently longer than that of others, all the descendant nuclides exist in secure equilibrium with the first.

All nuclides in Thorium series are in secure equilibrium with ${}^{232}\text{Th}$ and have the same radioactivity with ${}^{232}\text{Th}$.

Also applicable to Uranium and Actinium Series.