Radiation Equilibrium

Disintegration (Decay) Law

$N(t)$: number of atom at time t.

$$\frac{dN}{dt} = -\lambda N$$ (λ : disintegration (decay) constant

$$N(t) = N_0 e^{-\lambda t} = N_0 \left(\frac{1}{2}\right)^{\frac{t}{T}}$$ (T : halflife)

$$\lambda T = \ln 2 = 0.693$$

$A(t)$: radioactivity at time t

$$A(t) = \lambda N = A(0) e^{-\lambda t} = A(0)\left(\frac{1}{2}\right)^{\frac{t}{T}}$$

Disintegration (Decay) Series

$$\text{Nuclide } 1 \xrightarrow{\text{decay}} \text{Nuclide } 2 \xrightarrow{\text{decay}} \text{Nuclide } 3 \cdots$$

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$\therefore N_1(t) = N_{10} e^{-\lambda_1 t}$$

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) + N_{20} e^{-\lambda_2 t}$$

If $N_{20} = 0$ (Nuclide 2 not present at $t=0$),

$$N_2(t) = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

1) If $\lambda_1 < \lambda_2$, after sufficiently long time, $e^{-\lambda_1 t} \gg e^{-\lambda_2 t}$

$$N_2(t) \simeq \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} e^{-\lambda_1 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(t)$$

Nuclide 2 decreases with the half life of Nuclide 1

Transient Equilibrium

Examples:

i) $^{99}_{42}Mo \xrightarrow[66.02h]{\beta^-} {}^{99m}_{43}Tc \xrightarrow[6.02h]{IT} {}^{99}_{43}Tc$

($\lambda_2/\lambda_1 = 11$) $^{99m}_{43}Tc$ used for diagnosis

ii) $^{87}_{39}Y \xrightarrow[80.3hr]{EC} {}^{87m}_{38}Sr \xrightarrow[2.80h]{IT} {}^{87}_{38}Sr$

($\lambda_2/\lambda_1 = 29$)

iii) $^{140}_{56}Ba \xrightarrow[12.79d]{\beta^-} {}^{140}_{57}La \xrightarrow[40.3h]{\beta^-} {}^{140}_{58}Ce$

($\lambda_2/\lambda_1 = 7.6$)

2) If $\lambda_1 << \lambda_2$, after sufficiently long time,

$$N_2(t) \cong \frac{\lambda_1}{\lambda_2 - \lambda_1} N_1(t) \simeq \frac{\lambda_1}{\lambda_2} N_1(t)$$

$$\therefore \lambda_1 N_1(t) = \lambda_2 N_2(t)$$

$$A_1(t) = A_2(t)$$

The radioactivity of parental nuclide and daughter nuclide become equal.

<u>Secure Equilibrium</u>

Examples:

i) $^{90}_{38}Sr \xrightarrow[28.8y]{\beta^-} {}^{90}_{39}Y \xrightarrow[64.1h]{\beta^-} {}^{90}_{40}Zr$

$(\lambda_2 / \lambda_1 = 3.9 \times 10^3)$

ii) $^{137}_{55}Cs \xrightarrow[30.17y]{\beta^-} {}^{137m}_{56}Ba \xrightarrow[2.55min]{IT} {}^{137}_{56}Ba$

$(\lambda_2 / \lambda_1 = 6.2 \times 10^6)$ γ-ray source

iii) $^{226}_{88}Ra \xrightarrow[1600y]{\alpha} {}^{222}_{86}Rn \xrightarrow[3.8235d]{\alpha} {}^{218}_{84}Po$

If the half life of the first nuclide is sufficiently longer than that of others, all the descendant nuclides exist in secure equilibrium with the first.

All nuclides in Thorium series are in secure equilibrium with ^{232}Th and have the same radioactivity with ^{232}Th.

Also applicable to Uranium and Actinium Series.