

Objectives

- Compare methods of solving ODEs by differential methods and Laplace methods
- Verify solutions of ODEs by solving them with Laplace and inverse Laplace transforms
- Verify 2nd order systems with MATLAB code Chris Lee gave for Day 5 class

Problem 1. In HW 1, problem 1, you found an expression for $v_{out}(t)$ by solving the differential equation that characterized the circuit. Now find $v_{out}(t)$ by applying the Laplace transform to the circuit's differential equation. Compare the two expressions for $v_{out}(t)$. They should be exactly the same!

Problem 2. In HW1, problem 2, you solved a differential equation to find $v_{out}(t)$ for a lead compensator circuit. Solve the differential equation characterizing the circuit for $v_{out}(t)$ by applying the Laplace and inverse Laplace transforms. Again, the two expressions for $v_{out}(t)$ should be the same.

Problem 3. In HW1, problem 3, you found a closed-form analytic solution for $v_c(t)$ for both charging and discharging. Apply Laplace transform methods to the ODE that describes the charging case and solve for $v_{c,charging}(t)$. It should mirror your expression for $v_{c,charging}(t)$ found in HW1, pb3.

Also apply Laplace transform methods to the ODE that describes the discharging case and solve for $v_{c,discharging}(t)$. Check to make sure it agrees with the correct expression for $v_{c,discharging}(t)$ found in HW1, pb3.

(Problem 4 on reverse side ...)

Problem 4. Consider the series RLC circuit, where $R = \frac{7}{12}\Omega$, $L = \frac{1}{12}$, $C = 1F$, and $v_{in}(t) = u(t)$.

The circuit is completely at rest before the input $v_{in}(t) = u(t)$ is applied. Therefore, initial conditions are zero: $v_{out}(0) = 0 = \dot{v}_{out}(0)$.

a. Find an ODE that relates the output voltage $v_{out}(t)$ and input voltage $v_{in}(t)$, i.e., $v_o(t)$ and $v_i(t)$.

b. Apply the Laplace transform to both sides of the ODE found in part (a). Because the Laplace transform is a linear operator, you can apply it to each term in the ODE, i.e., superposition and scaling hold.

c. Find an expression for $V_{out}(s)$.

d. Express $V_{out}(s)$ as a sum of three terms by apply a partial fraction expansion (aka partial fraction decomposition). You can use the method presented in class (also in handouts) to find the values of the constants associated with each of the three terms comprising $V_{out}(s)$.

e. Determine $v_{out}(t)$ by taking the inverse Laplace transform of $V_{out}(s)$.

f. Identify the transient response in $v_{out}(t)$, as well as the steady state response.

g. Find the damping ration ζ .

