

ICCS310: Assignment 3  
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**1: NFA vs DFA Expressiveness**

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(1)

Let construct an NFA,  $M = (Q, \Sigma, \delta, q_0, F)$  where  $Q = \{0, 1, \dots, k\}$ . Let  $\delta(0, b) = 0, \delta(0, 1) = \{0, a\} = \{0, 1\}$  and  $\delta(i-1, a) = i$ , for  $2 \leq i \leq k$ . Then set  $q_0 = 0$  and  $F = \{k\}$ . We know that the machine will start at state 0 (starting state). When the machine locate an  $a$  it will guess that it is a  $k$ th character to the right and will move to state 1. When it reaches state  $k$ , it will only accept if there are exactly  $k-1$  bits following the one that move from  $b$  to  $a$ .

(2)

Let the input have  $k$  character. We know that the characters can be either  $a$  or  $b$ . Let  $x$  and  $y$  be a string with  $k$  bit such that  $x, y \in \Sigma^*$  and that  $|x| = |y| = k$ . Let  $i$  be a position such that  $x_i \neq y_i$ . Hence either  $x$  or  $y$  contain an  $a$  at the  $i$ th position. Let  $z = b^{i-1}$ , then  $z$  distinguish  $x$  and  $y$  as one of the  $xz$  and  $yz$  has an  $a$  at  $k$ th position from the right. Since there are  $2^k$  string of length  $k$  that are all distinguishable from the above prove, a DFA that accept this language need to have  $2^k$  states.

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**2: Regular or Not**

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( $L_1$ )

We know that  $y$  can be any string in  $\Sigma^*$ . We only need to detect  $x$  and  $x^r$ .

( $L_2$ )

AFSOC, let assume that  $L_2$  is regular. This mean their is a pumping length  $l \geq 1$ . Consider string  $S = p^l q p^l$ ,  $S \in L_2, |S| \geq l$ . Since  $S = ww^R$ , where  $w = p^l q$  then  $S \in L_2$ . From this we know that:

1.  $S$  can be split into  $S = xyz$
2.  $|xy| \leq l$
3.  $xy^i z \in L_2, i \geq 0$
4.  $xy = p^j, j \leq l$
5.  $y = p^k, k \geq 1$

If we pump  $y$  0 times then the string  $S$  will be  $S = xz$ .  $xz = p^{l-k} q p^l$ . We state that  $k \geq 1$ , this mean  $xz \notin L_2$ . This contradict, therefore  $L_2$  is not regular.

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**3: Nonregular**

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**4: HackerRank Challenge**

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