Practice exercise

① Let $a \in \mathbb{Z}_+$. Show that if a is odd then a^2 must be odd.

② Proof using induction that $1+3+5+7+\ldots+2(k+1)$ is k^2

② Inductive Predicate:

$1^2+3+5+7+\ldots+2(i-1) = i^2$

Base case:

$P(1)$, LHS = 1, RHS = 1

Inductive step:

If $1+3+5+7+\ldots+2(i-1) = i^2$

then $\underbrace{1+3+5+7+\ldots}_{i^2}+2(i+1) = (i+1)^2$

$LHS = i^2+2(i+1)$

$= \cancel{i^2+2i+2} = (i+1)^2$

~~RHS = i^2+i+i+~~ = RHS

① If n^2 is even then n is even

n is even so we can write $n=2k$

$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$

~~Which means~~ $2k^2$ is an integer, so we can write $2(2k^2)$ as $2p$.

so, $n^2 = 2p$ ~~which~~

this means n^2 is even.