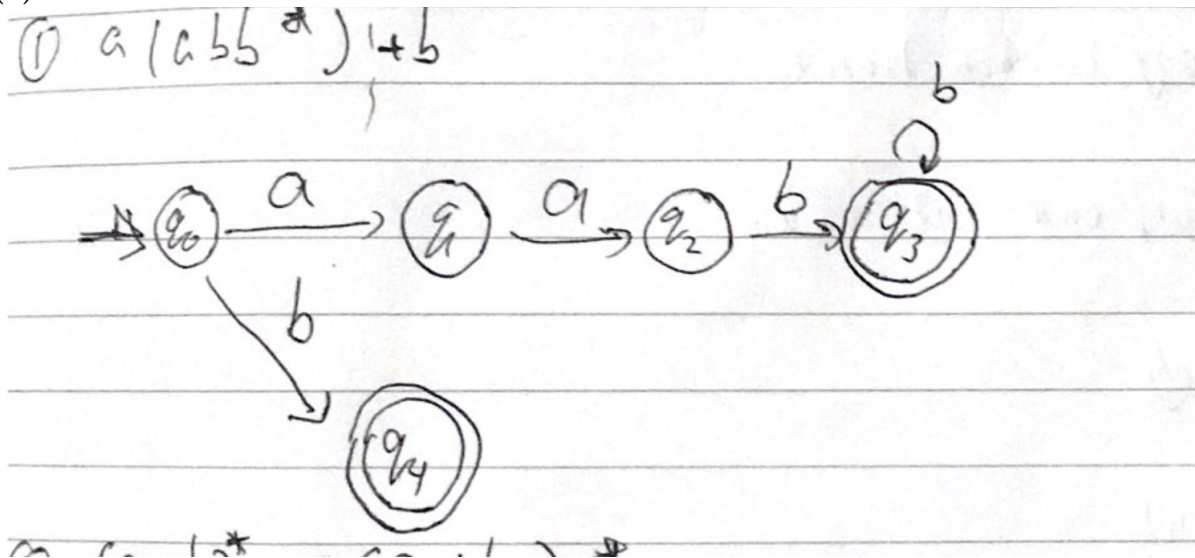
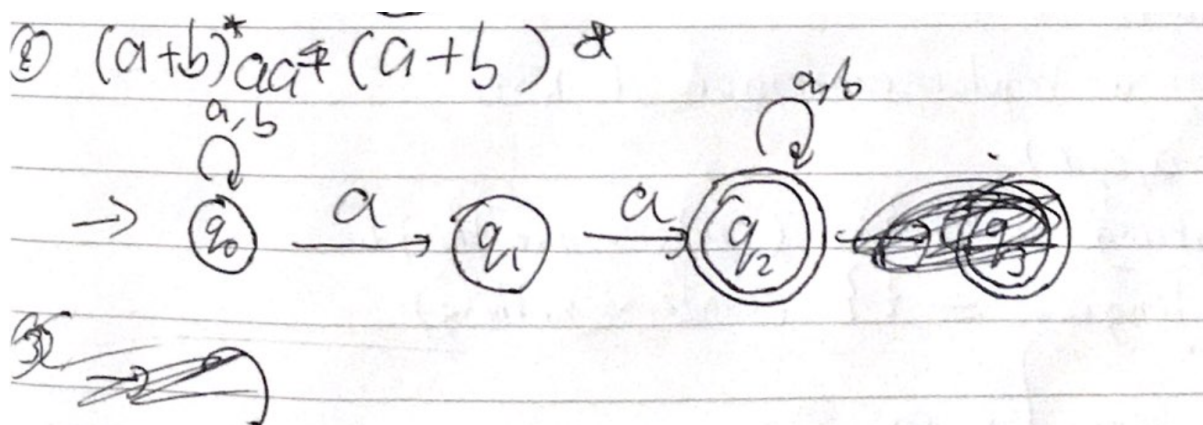


1: Regex to NFA/DFA

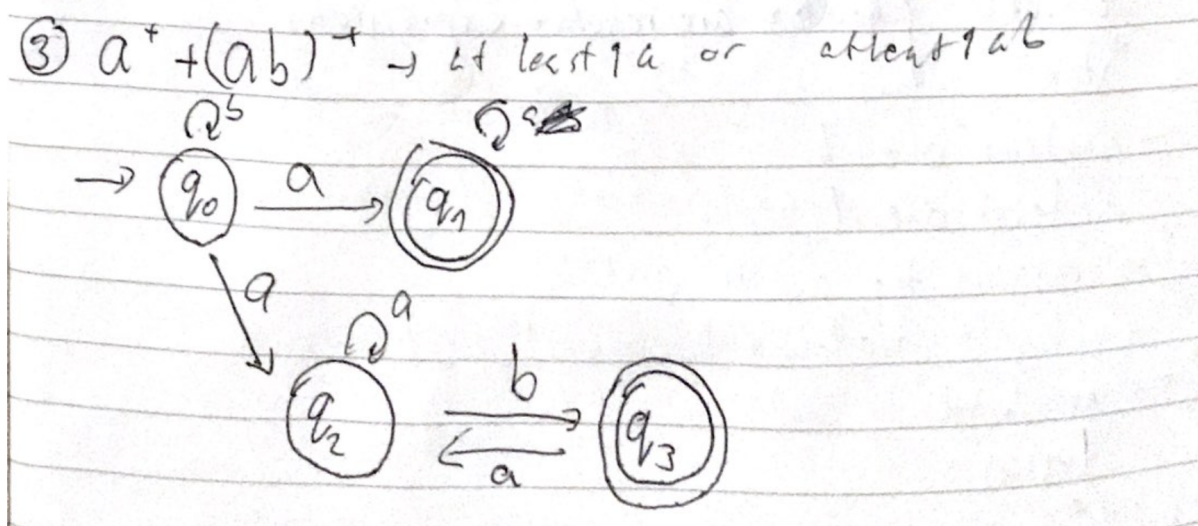
(1)



(2)



(3)



2: Finite-State Machines to Regex

(1)

\emptyset^*

(2)

$a^* + a^*b^+a^+b$

3: Binary Addition

We already proof in class that if L is regular language than L^R is also a regular language. When we add a binary number we start by adding the rightmost bit and in every step of binary addition there is a carry bit which will go to the next step. This carry bit can be either 0 or 1. We will construct a machine that will recognise A^R . This machines will perform the addition of the first two row. In each step the machine will check if the result is equal to the third row or not. We use the states of the machine to remember the carry bit. We can define a function as:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 0$$

We will also have a function that tell us what is the carry bit when we sum the 3 bits together. The machine will have 2 states q_0 and q_1 which will represent the carry bit (q_0 for 0 and q_1 for 1). Where q_0 is both starting state and accepting state because the carry digit need to be zero when the addition is complete. This mean the machine will accept only when the sum of the first 2 row and the carry bit is equal to the third row and that the carry bit is zero.

4: Division Operation?

If L_1 and L_2 are regular then their should be a DFA for both L_1 and L_2 . Then we can construct a DFA $L_1 = L(M)$. Let the DFA be $M = (Q, \Sigma, \sigma, q_0, F)$. We will construct another DFA that

will run with the first DFA. Let it be $\hat{M} = (Q, \Sigma, \sigma, q_0, \hat{F})$. For each node, if there is a walk from that node to the final node using a string in L_2 then put that node in \hat{F} . After looking through every q_i we have constructed \hat{M} . Now we have 2 DFA. Let k be an element in L_1/L_2 . Then there should be $k \in L_2$ such that $a \cdot k \in L_1$. This indicate that $\sigma^*(q_0, a \cdot k) \in F$. So there must be an element $b \in Q$ such that $\sigma^*(q_0, k) = b$ and $\sigma^*(b, a) \in F$. We already show that \hat{M} accept k . This implies that $a \in L_2$. So $a \cdot k \in L_1$ and that L_1/L_2 is regular.

5: Does It Accept Everything?

If we minimize a DFA using regular expression and the it's left with only accepting state then the DFA accept everything. This is because