ICCS310: Assignment 19

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1: Prove that $k - Coloring \leq_p (k+1) - Coloring$

We know that k-Coloring is NP-Complete
Let the input to $(k+1)$Coloring be graph G. So now G is $(k+1)$ colored we want to reduce G to $k - Coloring$ by construct a new graph called $G^{'}$. We can done this by removing a node and every edges connected to that node. Hence $(k+1)$Coloring is NP-Complete.

2: Prove that $5 - Coloring \leq_p 4 - Coloring$

We know that $4 - Coloring$ is NP-Complete.
We can reduce 5-Coloring to 4-Coloring by mapping graph G into a new graph $G^{'}$. Where $G \in 4 - Coloring$ iff $G^{'} \in 5 - Coloring$. This can be done by adding a new node y and connect them to each node in $G^{'}$. If G is 4 colorable then $G^{'}$ can be 5 colored exactly as G with a node y being the node that color with an additional color. Thus it is 4-Coloring and 4-Coloring is NP-Complete.

3: Argue that $P \subseteq coNP$, and hence $P \subseteq NP \cap coNP$

WLOG let $x \in P$. We claim that $x \subseteq NP, \forall x \in P$. Then x is use as a verifier in NP. We also claim that $x \subseteq coNP, \forall x \in P$. Then x is use as a verifier in coNP as well. Therfore $P \subseteq coNP$, and hence $P \subseteq NP \cap coNP$.