Practice:

~~① $L = \{ww \mid w \in \Sigma^*\}$~~

~~Assu~~me

② Let $A = \{0^n 1^n 2^n \mid n \geq 0\}$

AFSOC that A is regular, there is a pumping length $p \geq 1$ (given by pumping lemma).

Consider $s = 0^p 1^p 2^p$.

$\rightarrow s$ can be split into $s = xyz$ & $|xy| \leq p$

$\rightarrow xy = 0^i$, $i \leq p$

$\rightarrow y = 0^j$, $1 \leq j \leq i$ $(\leq p)$

$\rightarrow z = 0^k 1^p 2^p$

The number of 0's 1's, 2's in s given by $i + j + k = p$.

Let $a = 0$ such that $s' = xy^a z = xz$.

Number of 1's in s' is p, number of 0's is $i+k$.

$s' \in A$,

number of 0's in s' must equal to ~~the~~ number of 1's in s', $i+k=p$.

$i + k = i + j + k$.

$|y| > 0$ and $|y| = j$, $j > 0$

Thus $s' \notin A$; contradiction, A = non regular

③ $B = \{0^n 1^m 0^n \mid n \geq 0, m \geq 0\}$

Assume B is regular, there is a pumping ~~lemma~~ length $p \geq 1$ (given by pumping lemma)

Consider $S = 0^p 1 0^p$. $|xy| \leq p$, ~~xy~~

$\rightarrow xy = 0^i$, $i \leq p$

$\rightarrow y = 0^j \; , \; 1 \leq j \leq i (\leq p)$

~~Let i~~ $= 0 \quad xy^0 z \in B$

$xy^0 z = xz = 0^{(p-j)} 1 0^p \quad \notin B$

Contradiction, B = nonregular.

4) $L_3 = \{ ww \mid w \in \Sigma^* \}$

AFSOC that L_3 is regular. This mean their exist a DFA $M = (Q, \Sigma, \delta, q_0, F)$ deciding L_3 using $k \geq 1$ state. Let $r_i = \delta^*(q_0, 0^i)$ for $i = 0, 1, \ldots, k$

By pigeon hole principle, repeat among r_i's : $r_s = r_t$ for some $0 \leq s \neq t \leq k$

Let $x = 0^s 1 0^s 1 \in L_3$

$q_s = \delta^*(q_0, 0^s 1 0^s 1) = \delta^*(\delta^*(q_0, 0^s 1), 0^s 1) = \delta^*(r_s, 0^s 1)$,

$q_s \in F$

$y = 0^t 1 0^s 1$

$q_t = \delta^*(q_0, 0^t 1 0^s 1) = \delta^*(\delta^*(q_0, 0^t 1), 0^s 1) = \delta^*(r_s, 1^t)$

$q_t \notin F$

Contradict, hence L_3 is non-regular.