# ICCS310: Assignment 5

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## 1: Reject TM

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Let AFSOC that REJECT$_{TM}$ is decidable. This mean there is a Turing Machine which decides it. Let the machine $R(< M, x >)$.

$$R(< M, x >) = \begin{cases} \text{reject if M accept x,} \\ \text{accept if M reject x,} \\ \text{reject if M loop on x} \end{cases}$$

Using R we can create another machine D. Where D ¡M¿ where M is a machine. To prove this we run R on $< M, < M >>$. From this we have:

$$D(< M >) = \begin{cases} \text{reject if M accept } < M >, \\ \text{accept if M reject } < M >, \\ \text{reject if M loop on } < M > \end{cases}$$

Applying to $M = D$ we have:

$$D(< D >) = \begin{cases} \text{reject if D accept } < D >, \\ \text{accept if D reject } < D >, \\ \text{reject if D loop on } < D > \end{cases}$$

This contradict, so D and R can't exist. Therefore, REJECT$_{TM}$ is undecidable.

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## 3: Reverse on TM

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AFSOC that T is deciable. Then let $M_T$ be recogniser of T sp that we can construct a recogniser of $A_{TM}$. Let $M' =$ on input $< M, w >$. First, lets construct a TM $M_w =$ on input $x$.

$$M_w = \text{on input } x \begin{cases} \text{reject if } x = rev(w), \\ \text{run M on w and return the result if } x = w, \\ \text{reject if it's not fit on either of the above condition} \end{cases}$$

Then run $M_T$ on $< M_w >$. From this we know that $M^{'}$ accept $< M, w >$ iff $M_T$ accept $< M, w >$ iff M doesn't accept w iff $< M, w > \in \bar{A}_{TM}$. Therefore T is undeciable.

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## 4: Undecidability

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### (1)

Show that TOTAL is undeciable:
AFSOC that TOTAL is decidable. This means there is a Turing Machine T that decides it. Then we can use $T(< M, x >)$ where x is an input String. Let define a new machine N. Where N run on input y. Then we run the machine.

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1. If y is not the same as x hault
2. Feed y to machine M and let M compute x.
3. If M's computation on input x halts and rejects x, loop indefinitely. Else if M's computation on input x halts and accepts x, halt. Else continue looping.

Then feed the string $< N >$ into T. The machine will return output if T accepts $< M, x >$. Which is impossible, so TOTAL is undecidable.

**(2)**
Show that FINITE us undecidable:
AFSOC that FINITE is decideable. Let T be a TM that decide FINITE. Then we can construct another TM called U which will be use to decide $A_{TM}$. Let U = on input $< M, w >$ where w is an input string. Then we construct $< M_w >$ where:

$$M_w = \text{on input } x \begin{cases} \text{reject if M reject w,} \\ \text{accept if M accept w} \end{cases}$$

Run T on $M_w$. If T accept reject and viceversa. If M accept w then it accept all input which means it is indefinite language. Hence T reject M, therefore FINITE is undefineable.

**(3)**
Show that REGULAR is undecidable:
AFSOC that REGULAR is decidable. This mean there is a turing machine T that can decide it. Next, let construct another TM called U which will be use to decide $A_{TM}$. Let U = on input $< M, w >$ where w is an input string. Then we construct $< M_w >$ where:

$$M_w = \text{on input } x \begin{cases} \text{accept if w if in the form of } 0^1 n^1, \\ \text{reject otherwise} \end{cases}$$