Practice exercise

① Let $a \in \mathbb{Z}_+$. Show that if a is odd then a^2 must be odd.

② Proof using induction that $1+3+5+7+\dots+2(k+1)$ is k^2

② Inductive Predicate:

$1+3+5+7+\dots+2(i-1) = i^2$

Base case:

$P(1)$, LHS $= 1$, RHS $= 1$

Inductive step:

If $1+3+5+7+\dots+2(i-1) = i^2$

then $\underbrace{1+3+5+7+\dots}_{i^2}+2(i+1) = (i+1)^2$

$LHS = i^2 + 2(i+1)$

$\cancel{= i^2 + 2i + 2}$ $= (i+1)^2$

$\cancel{RHS = i^2 + i + i +}$ $= RHS$

1) If n^2 is even then n is even → by contrapositive

n is even so we can write $n = 2k$

$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$

~~Which means~~ $2k^2$ is an integer, so we can write $2(2k^2)$ as $2p$.

So, $n^2 = 2p$ which

this means n^2 is even.