

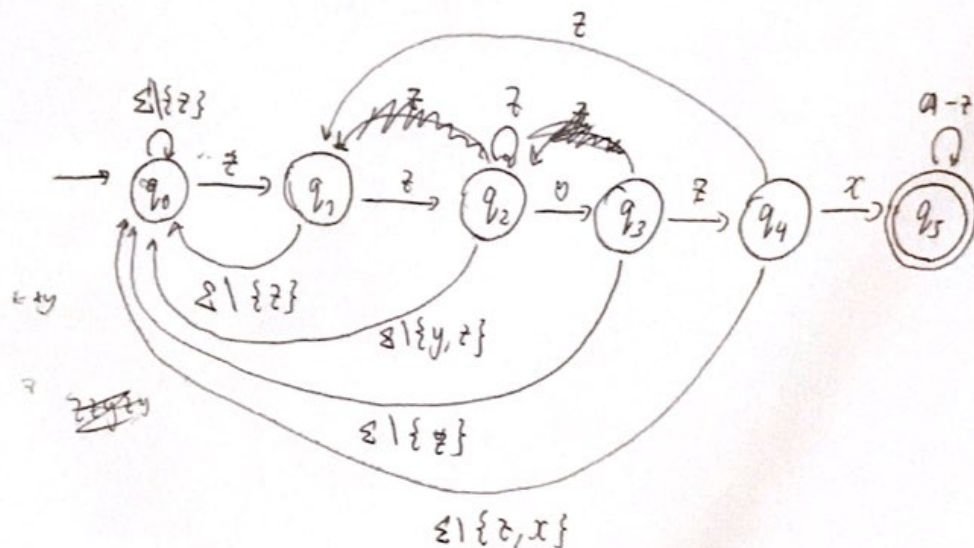
4) (i) A problem is NP-complete if A:

1) $A \in P, \forall A \in NP$ (NP hard)

2) $B \in NP$

(ii) Circuit SAT, 3SAT, 4SAT

1) $zzyzx$



3(ii) AFSOC that $NOYOL_{TM}$ is decidable. This means there is a Turing machine that decides it. Let called machine N.

$N(\langle M, x \rangle)$ where x is in input string (yolo)

$N(\langle M, x \rangle) = \begin{cases} \text{reject if } M \text{ accept string } yolo x \\ \text{accept if } M \text{ rejects string } yolo x \\ \text{reject if } M \text{ loops on } x \end{cases}$

Using N we can create another machine R ~~like~~ $R(\langle M \rangle) = R(\langle M \rangle)$

We then run $N(\langle M, \langle M \rangle \rangle)$. We have:

$R(\langle M \rangle) = \begin{cases} \text{reject if } M \text{ accept } \langle M \rangle \\ \text{accept if } M \text{ reject } \langle M \rangle \\ \text{reject if } M \text{ loop on } \langle M \rangle \end{cases}$

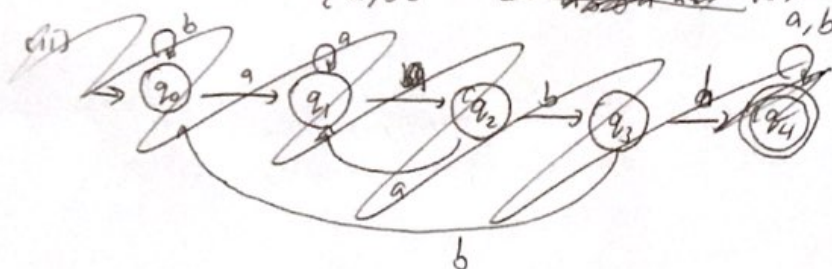
Applying $M=R$ we have

$R(\langle R \rangle) = \begin{cases} \text{reject if } R \text{ accept } \langle R \rangle \\ \text{accept if } R \text{ reject } \langle R \rangle \\ \text{reject if } R \text{ loop on } \langle R \rangle \end{cases}$

This contradict so N and R cannot exist. Hence $NOYOL_{TM}$ is not Turing decidable.

2) (i) 1) $L = \{ w \in \{a,b\}^* : w = a^i b^j \text{ where } i \geq 0 \}$

$L = \{ w \in \{a,b\}^* : w = \text{any string that contains } a^+ b^+ \}$



A language is ~~decidable~~ regular if there's a DFA that recognizes it.

(iii) Assume that B is regular. Then there is a pumping length $P \geq 1$ given by pumping lemma. Consider string $w = a^P b^P$. From this we know that:

1) w can be split into $w = xyz$

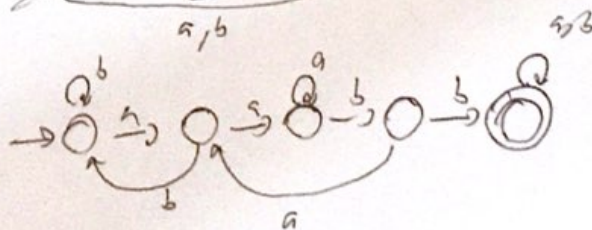
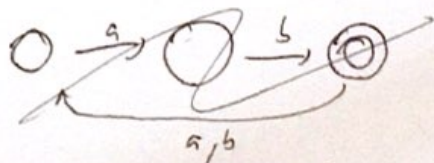
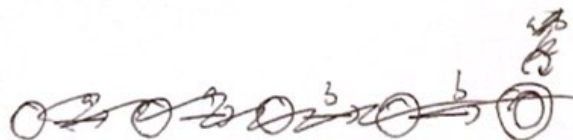
2) $|xy| \leq P, |y| > 0$

3) $y = a^j, 1 \leq j$

According to the pumping lemma, $xyyz = a^{P+j} b^P$ should be in L .

However $xyyz \notin L$. Therefore B is not a regular language.

(ii)



4 ciii) Show that $(k+1)$ coloring is $\in NP$

Certificate Color Assignment o

3(i) A language is Turing Decidable if there is a Turing machine that decides it.
Let construct a Turing machine from the DFA. Let call it M .

$M = \text{on input } x$

check if $x = y_0/0$

if yes, reject

~~if false~~
if true; hcrept

4 (iii) $(k+1)$ coloring where $k \geq 1$ is the same as k coloring where $k \geq 2$.

3 (i) Certificate: a set of $n-1$ edges of G
Verifier: Check if the sum of the weight is equal to k and that the
edges can form a connected graph in $O(n)$ time.
Then check if the sum of the weight is equal to k .

(ii)