ICCS310: Assignment 3

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February 2021

1: NFA vs DFA Expressiveness

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(1)
Let construct an NFA, $M = (Q, \Sigma, \delta, q_0, F)$ where $Q = \{0, 1, ..., k\}$. Let $\delta(0, b) = 0, \delta(0, 1) = \{0, a\} = \{0, 1\}$ and $\delta(i-1, a) = i$, for $2 \leq i \leq k$. Then set $q_0 = 0$ and $F = \{k\}$. We know that the machine will start at state 0 (starting state). When the machine locate an a it wil guess that it is a kth character to the right and will move to state 1. When it reaches state k, it will only accept if there are exactly $k-1$ bits following the one that move from b to a.

(2)
Let the input have k character. We know that the characters can be either a or b. Let x and y be a string with k bit such that $x, y \in \Sigma^*$ and that $|x| = |y| = k$. Let i be a position such that $x_i \neq y_i$. Hence either x or y contain an a at the ith position. Let $z = b^{i-1}$, then z distinguish x and y as one of the xz and yz has an a at kth postion from the right. Since there are 2^k string of length k that are all distinguishable from the above prove, a DFA that accept this language need to have 2^k states.

2: Regular or Not

(L_1)
We know that y can be any string in Σ^*. We only need to detect x and x^r.

(L_2)
AFSOC, let assume that L_2 is regular. This mean their is a pumping length $l \geq 1$. Consider string $S = p^l qqp^l$, $S \in L_2, |S| \geq l$. Since $S = ww^R$, where $w = p^l q$ then $S \in L_2$. From this we know that:

1. S can be split into $S = xyz$
2. $|xy| \leq l$
3. $xy^i z \in L_2, i \geq 0$
4. $xy = p^j$, $j \leq l$
5. $y = p^k, k \geq 1$

If we pump y 0 times then the string S will be $S = xz$. $xz = p^{l-k} qqp^l$. We state that $k \geq 1$, this mean $xz \notin L_2$. This contradict, therefore L_2 is not regular.

3: Nonregular

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(1)
AFSOC, let assume that L is regular then their is a pumping length $p \geq 1$. Consider a string $S = 10^{2^p}, S \in L$. From this we know that:

1. S can be split into $S = xyz$, and xy be arbitary element in L.
2. $|xy| \leq p$
3. $xy^iz \in L, i \geq 0$
4. $x = 10^{2^i}$
5. $y = 10^{2^j}, 1 \leq j \leq i$
6. Let $z = 10^{2^i}$

From this we know that $xz = 10^{2^i} \cdot 10^{2^i} = 10^{2^{i+1}}$, $xz \in L$. However $xy = 10^{2^i} \cdot 10^{2^j} = 10^{2^i+2^j}, xy \notin L$ as $i \neq j$. This contradict, therefore L is not regular.

4: HackerRank Challenge

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