ICCS310: Assignment 6

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March 2021

1: The Meaning of Things

(1) Give a definition of the class NP
Complexity class used to classify problems. It is a set of problem that can be check if true within polynomial time.

(2) Explain how one can prove that a problem belongs to the class NP
Show that the problems have certificate and verifier and that the problem can be check in polynomial time.

(3) What is NP-complete?
The complexity class of decision problems in NP and no other NP problem is harder.

(4) Describe a startegy for showing that a problem is NP-complete
Show that the problem is in NP and that the problem can be reduces to alredy known np-complete problem (NP-hard).

2: Closure of NP

(1)$A \cap B$ must be in NP
Let $A = L_1$ and $B = L_2$. For i = 1,2 let $V_i(x, c)$ be an algorithm such that x is a string, c is a possible certificate and this algorithm will verify whether c is a certificate for $x \in L_i$. If certificates c verifies $x \in L_i$ then $V_i(x, c) = 1$. Else $V_i(x, c) = 0$. Since we know that both L_1 and L_2 are in NP. Then we know that algorithm $V_i(x, c)$ terminates in polynomial time which is $O(|x^d|)$. Where d is a constant. Let construct another verifier called V_3 which verify $L_1 \cap L_2$. Let $L_1 \cap L_2 = L_3$. Then $V_3 = V_1 \cap V_2$. This clearly indicate that $x \in L_3$ if and only if there is a certificate c suchthat $V_3(x, c) = 1$. Then this verifier will run in $O(2(|x|^d))$ which is in polynomial time. Therefore L_3 is also in NP. So, $A \cap B$ is in NP.

(2)$A \cup B$ must be in NP
Let $A = L_1$ and $B = L_2$ For i = 1,2 let $V_i(x, c)$ be an algorithm such that x is a string, c is a possible certificate and this algorithm will verify whether c is a certificate for $x \in L_i$. If certificates c verifies $x \in L_i$ then $V_i(x, c) = 1$. Else $V_i(x, c) = 0$. Since we know that both L_1 and L_2 are in NP. Then we know that algorithm $V_i(x, c)$ terminates in polynomial time which is $O(|x^d|)$. Where d is a constant. Let construct another verifier called V_3 which verify $L_1 \cup L_2$. Let $L_1 \cup L_2 = L_3$. Then $V_3 = V_1 \cup V_2$. This clearly indicate that $x \in L_3$ if and only if there is a certificate c suchthat $V_3(x, c) = 1$. Then this verifier will run in $O(2(|x|^d))$ which is in polynomial time. Therefore L_3 is also in NP. So, $A \cup B$ is in NP.

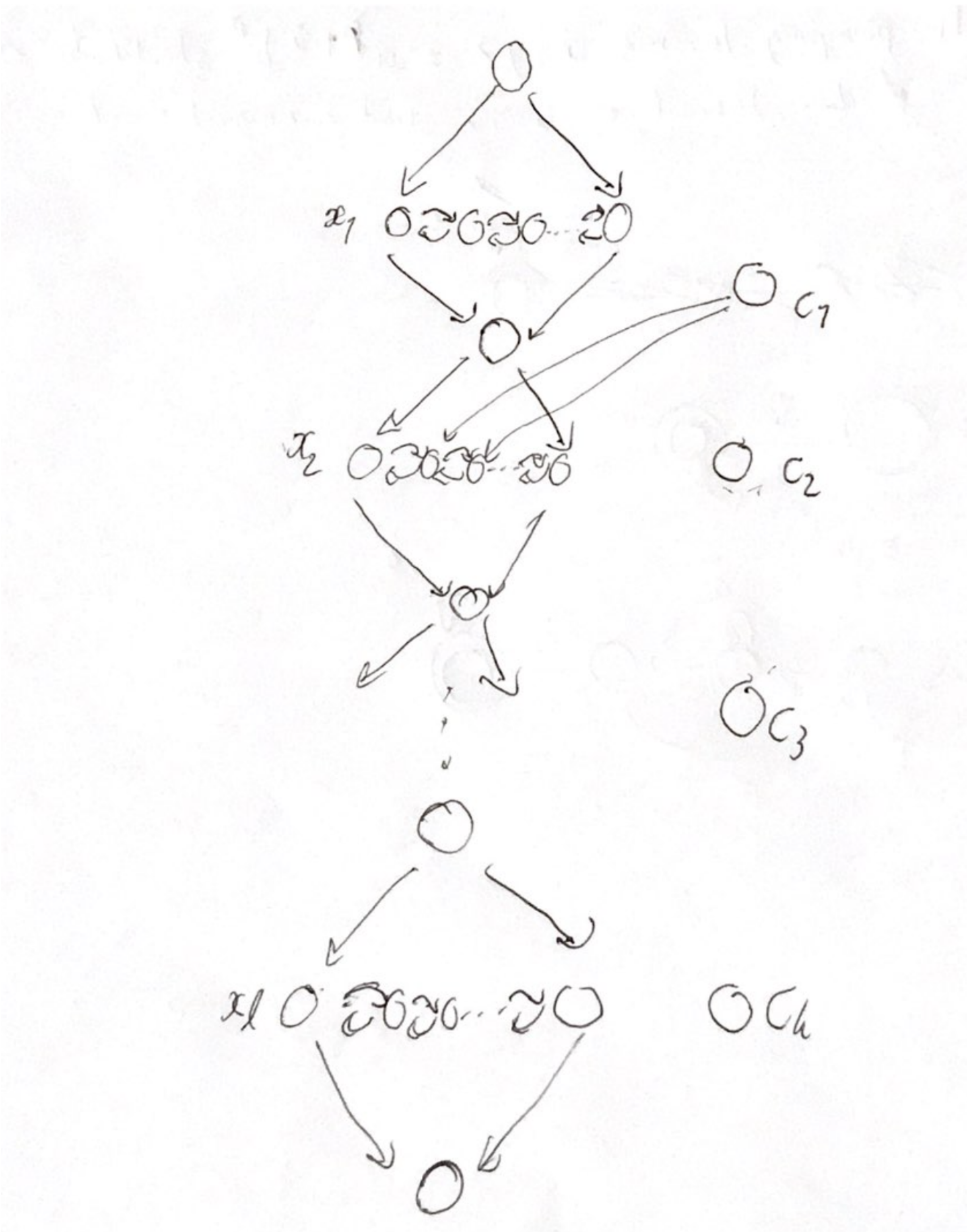
3: This is NP

certificate = colour assignment of each vertex.
verifier = run throgh and check if for each edge (u, v), the colour of u is different from that of

v.

4: NP-Complete

(1) Prove that HAM-PATH is NP-complete From class we know that 3-SAT is in NP-Complete. $HAMPATH = (G, s, t)$ where G is a directed graph with a Hamiltonian path from s to t.
For a given k clauses:
$\phi = (a_1 \vee b_1 \vee c_1) \wedge (a_2 \vee b_2 \vee c_2) \wedge ... \wedge (a_k \vee b_k \vee c_k)$
Let $(a_1 \vee b_1 \vee c_1) = c_1$, $(a_2 \vee b_2 \vee c_2) = c_2$ ... $((a_k \vee b_k \vee c_k)) = c_k$ and a_i, b_i, c_i are literals x or $\overline{x}$, $i = k$. Let $x_1 ... x_l$ be the l variable of ϕ. Let construct graph G where each x_i is represented with a diamond-shaped structure such that each diamond contain a horizontal row of nodes where it is connected by edges running in both direction.The horizontal row contains $2k$ nodes, $k - 1$ extra node in between every 2 node from the clause and 2 nodes on the top and bottom to form a diamond shape. So the total number of node is $2k + (k - 1) + 2 = 3k + 1$ nodes. If x_i appears in the clause then we add two edges from the pair in the ith diamond to the clause node.



Suppose that ϕ is satisfiable, then a Hamiltonian path exists from s to t. To show this (Follow the blue arrow):

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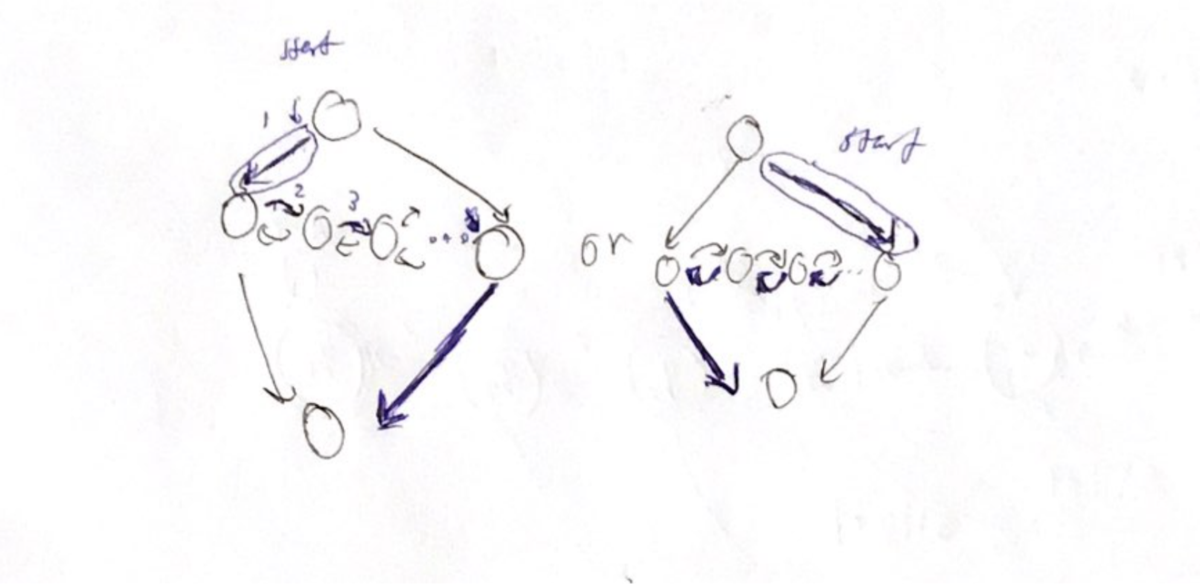
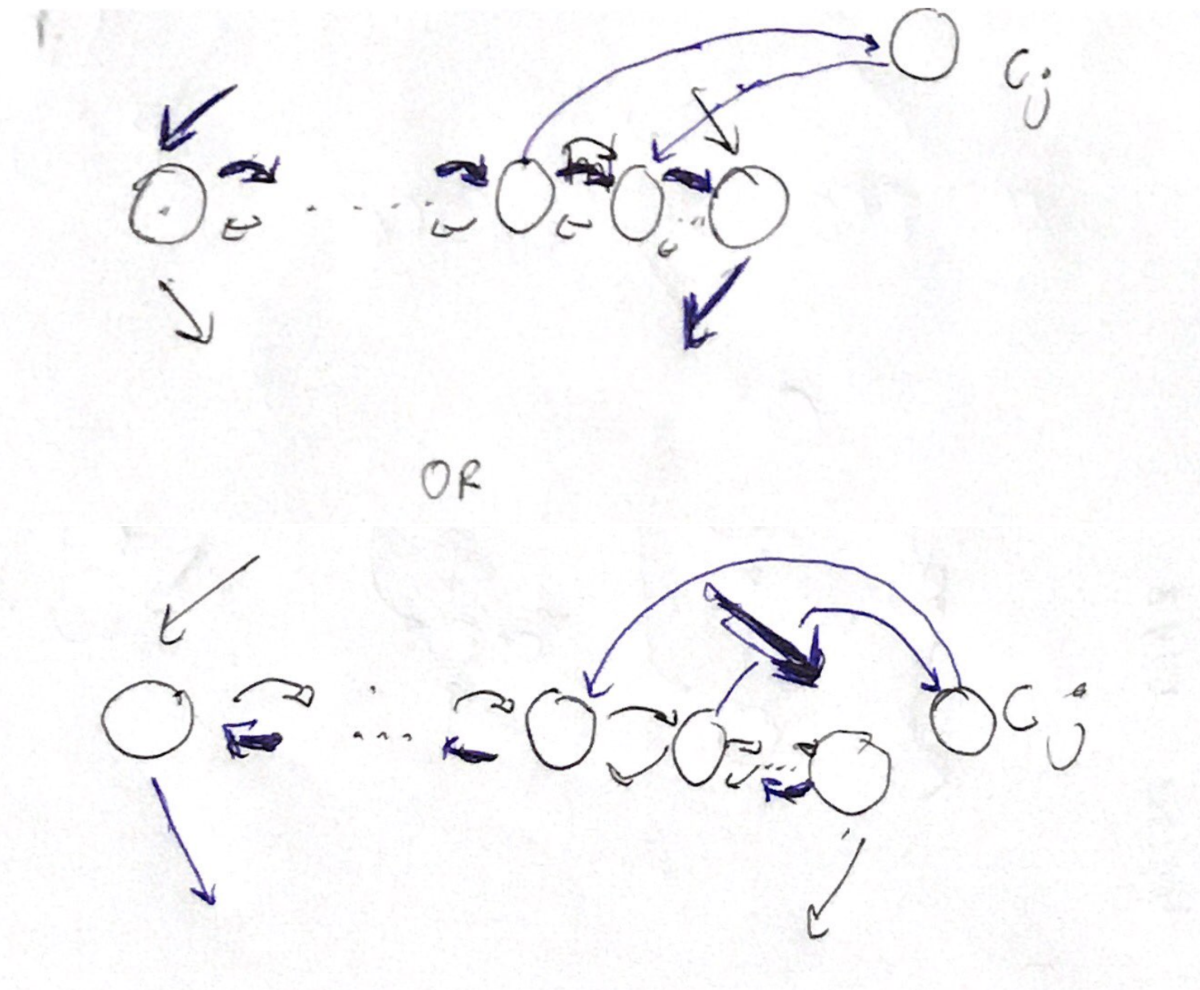
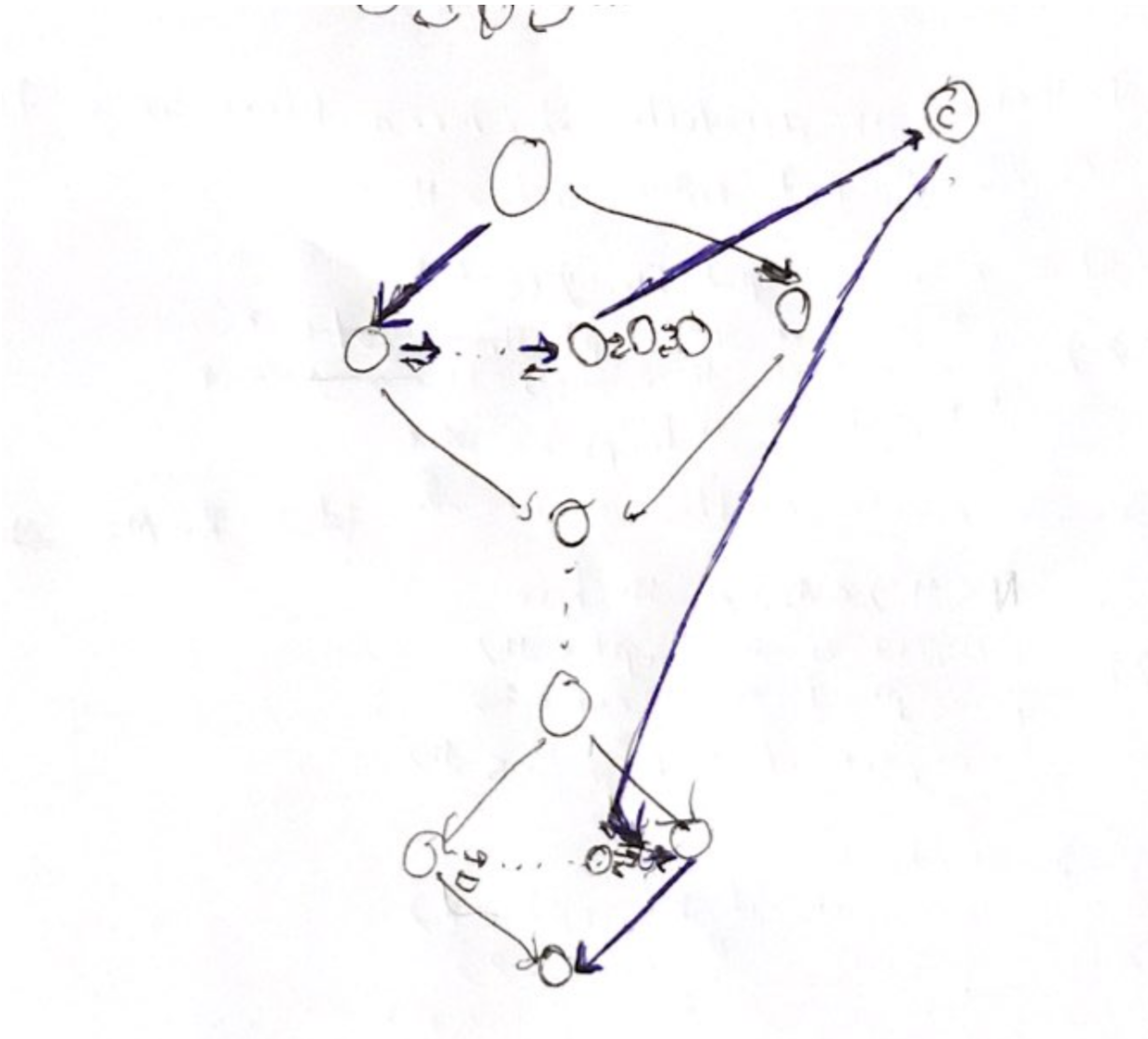


To cover the clause nodes c_j then we can make a detour as follow:



This case cannot happend:



This prove that this reduction works. Hence $3-SAT \leq_m HAM-PATH$.

(2) Prove that UNDIRECTED-HAM-PATH is NP-complete
We just prove that HAM-PATH is NP-complete.
$UNDIRECTEDHAMPATH = (G, s, t)$ where G is a directed graph with a Hamiltonian path from s to t.
Let s in G map to s^{out} in G' and t in G map to t^{in} in G'. Other node u_i in G become edges incident on $u_i^{in}, u_i^{middle}, u_i^{out}$ in G'. Any HAMPATH between s^{out} and t^{in} must go through the triple nodes excepth for the start and end nodes. From this we reduce UNDIRECTED-HAM-PATH to HAM-PATH. Therefore, $HAM-PATH \leq_m UNDIRECTED-HAM-PATH$, UNDIRECTED-HAM-PATH is in NP-Complete.

5: Silver Lining If P = NP

If $P = NP$ then $coNP = NP$. Then we can show that $\overline{SPC} = coNP$ where $\overline{SPC}$ check if the logic is not the smallest possible circuit.
Certificate: A logic circuit
Verifier: Check by reducing logic circuit
From this we know that $\overline{SPC} = coNP$, then $SPC = NP$. From the question we know that $NP = P$ therefore $SPC = P$.