ICCS310: Assignment 4

Natthakan Euaumpon
natthakaneuaumpon@gmail.com
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1: Eh? They Have The Same Cardinality

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(1)
$|[0, \frac{1}{2})| = |[0, 1)|$
Let $X = [0, \frac{1}{2})$ and $Y = [0, 1)$
Claim $|X| = |Y|$
The function is bijective iff it is both injective and surjective.
To prove this let a function f be $f : X \to Y$ where $f(x) = 2x$
First, let prove that the function is injective.
We need to show that $\forall x, y \in X$ if $(f(x) = f(y)) \to (x = y)$.
We can take any arbitary to show that $x, y \in X$. Let choose 1. So $2x = 2y$. Thus $x = y$ and that f is injective.
Second, we need to prove that f is surjective.
We need to show that $\exists x \in X$ and $\forall y \in Y$ such that $f(x) = y$.
WLOG, let assume $y \in Y$. If $x \in X$ then x = $\frac{y}{2}$. This means $f(x) = f(\frac{y}{2}) = 2(\frac{y}{2}) = y$. So, $f(x) = y$ and f is surjective.
We already prove that f is both injective and surjective. Therefore f is bijective.

(2)
$|[0, 1)| = |(-1, 1)|$
Let $X = [0, 1)$ and $Y = (-1, 1)$
To prove this let a function f be $f : X \to Y$ where $f(x) = -x$ and a function g be $g : Y \to X$ where $g(x) = |x|$.
First, let prove that the function f is injective.
We need to show that $\forall x, y \in X$ if $(f(x) = f(y)) \to (x = y)$.
We can take any arbitary to show that $x, y \in X$. Let choose 1. So $-x = -y$. Thus $x = y$ and that f is injective.
So, $|X| \leq |Y|$.
Second, let prove that the function g is injective.
We need to show that $\forall x, y \in Y$ if $(g(x) = g(y)) \to (x = y)$.
We can take any arbitary to show that $x, y \in Y$. Let choose 1. So $|x| = |y|$. Thus $x = y$ and that g is injective.
So, $|X| \geq |Y|$
This means $|X| = |Y|$.

(3)
$|[0, 1)| = |\mathbb{R}|$
From the question above, we know that $[0, 1) = (-1, 1)$.
First, let prove that the function is injective.
Consider a function f, $f : (-1, 1)| \to \mathbb{R}$ where $f(x) = \frac{x}{1-x^2}$
$f'(x) = \frac{1+x^2}{(1-x^2)^2}$
$f'(x) > 0$
This mean a function is injective.
Second, we need to prove that f is surjective.

We need to show that $\exists x \in X$ and $\forall y \in Y$ such that $f(x) = y$.
WLOG, let $y \in \mathbb{R}$ and $y \neq 0$.
$y = \frac{x}{1-x^2}$
$yx^2 + x - y = 0$
x = $\frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot -y}}{2 \cdot y}$
x = $\frac{-1 \pm \sqrt{1+4y}}{2y}$
There are 2 cases:
First case, $y > 0$
x = $\frac{-1-\sqrt{1^2+4y}}{2y} < -\frac{2|y|+1}{2y} < -1$
Second case, $y < 0$
x = $\frac{-1-\sqrt{1^2+4y}}{2y} = -\frac{2|y|+1}{2y} > 1$
This means x is outside of (-1,1).
For $y \in \mathbb{R}$ and $y \neq 0$ there exist a value x = $\frac{-1+\sqrt{1^2+4y}}{2y} \in (-1, 1)$
This means f is surjective.
We already prove that f is both injective and surjective. Therefore f is bijective.

3: Hamming Code

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(1)
$\beta_2 = d_2 \oplus d_3 \oplus d_6 \oplus d_7 \oplus d_10 \oplus d_11$

(2)
data: 01101010, which can be written as $p_1 p_2 d_1 p_4 d_2 d_3 d_4 p_8 d_5 d_6 d_7 d_8$.
$p_1 = 1$
$p_2 = 0$
$p_3 = 0$
$p_4 = 0$
code = 100011001010

(3)(i)
data: 010011111000, which can be written as $p_1 p_2 b_1 p_4 b_2 b_3 b_4 p_8 b_5 b_6 b_7 b_8$
$\beta_1 = p_1 \oplus b_1 \oplus b_2 \oplus b_4 \oplus b_5 \oplus b_7$
$\beta_1 = 0 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 0$
$\beta_1 = 1$
$\beta_2 = p_2 \oplus b_1 \oplus b_3 \oplus b_4 \oplus b_6 \oplus b_7$
$\beta_2 = 1 \oplus 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0$
$\beta_2 = 1$
$\beta_4 = p_4 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_8$
$\beta_4 = 0 \oplus 1 \oplus 1 \oplus 1 \oplus 0$
$\beta_4 = 1$
$\beta_8 = p_8 \oplus b_5 \oplus b_6 \oplus b_7 \oplus b_8$
$\beta_8 = 1 \oplus 1 \oplus 0 \oplus 0 \oplus 0$
$\beta_8 = 0$
0111 = 7
Correct data = 010011011000
data: 011101010010, which can be written as $p_1 p_2 b_1 p_4 b_2 b_3 b_4 p_8 b_5 b_6 b_7 b_8$
$\beta_1 = p_1 \oplus b_1 \oplus b_2 \oplus b_4 \oplus b_5 \oplus b_7$
$\beta_1 = 0 \oplus 1 \oplus 0 \oplus 0 \oplus 0 \oplus 1$

natthakaneuaumpon@gmail.com

$\beta_1 = 0$
$\beta_2 = p_2 \oplus b_1 \oplus b_3 \oplus b_4 \oplus b_6 \oplus b_7$
$\beta_2 = 1 \oplus 1 \oplus 1 \oplus 0 \oplus 0 \oplus 1$
$\beta_2 = 0$
$\beta_4 = p_4 \oplus b_2 \oplus b_3 \oplus b_4 \oplus b_8$
$\beta_4 = 0 = 1 \oplus 0 \oplus 1 \oplus 0 \oplus 0$
$\beta_4 = 0$
$\beta_8 = p_8 \oplus b_5 \oplus b_6 \oplus b_7 \oplus b_8$
$\beta_8 = 1 \oplus 0 \oplus 0 \oplus 1 \oplus 0$
$\beta_8 = 0$
$0000 = 0$
Data is already correct.

4: Same number of 0s and 1s

First we scan the input, starting from the left. For each symbol either 0 or 1 search for the matching symbol. If the start symbol is 0 find 1 and vice versa. Replace the pair with X. If the respective symbol is not found then reject. Else if all the symbol match then accept.

7: β-reduction

(1)
$(\lambda z.z)(\lambda z.zz)(\lambda z.zy)$
$= (\lambda z.zz)(\lambda z.zy)$
$= (\lambda z.zy)(\lambda z.zy)$
$= (\lambda z.zy)y$
$=yy$

(2)
((($\lambda x.\lambda y.(xy)$)(λ y.y))w)
$= (((\lambda x.\lambda y.xy)(\lambda y'.y'))w)$
$= (((\lambda y.(\lambda y'.y')y))w)$
$= (\lambda y'.y')w$
$=w$