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1(a) ~~$|(-100,100)| = |(0,50)|$~~

Let ~~$X =$~~

3) $L = \{ca^{3n+1}t^{n+1} \mid n = 0, 1, 2, \dots\}$

AFSOC that L is regular. This means there's a pumping length $l \geq 1$.
Consider a string $ca^{3l+1}t^{l+1}$, $S \in L$. From this we know that

1. S can be split into $S = xyz$
2. $|xy| \leq l$
3. $xy^iz \in L_2$, $i \geq 0$
4. $xy = ca^{3j+1}t^{j+1}$, $j \geq l$
5. $y = ca^{3k+1}t^{k+1}$, $k \geq 1$

If we pump strong 0 time string S will be $S = xz = ca^{l-3k+1}t^{l+1}$,
$xz \notin L$. this contradict. therefore L is not regular.

1(a) $|(-100,100)| = |(0,50)|$

let $X = (-100, 100)$, $Y = (0, 50)$
Let f be $f: X \to Y$ where $f(x) = |x|$

First let prove that a function is injective.
We need to show that $\forall x, y \in X$ if $(f(x) = f(y)) \to (x = y)$
We can take any arbitary to show that $x, y \in X$. Let choose 1, $|x| = |y|$ and that f is injective.
So, $|X| \leq |Y|$.
Second we need to prove that f is surjective.
We need to show that $\exists x \in X$ and $\forall y \in Y$; $f(x) = y$.
let assume $y \in Y$. If $x \in X$ then $x = \frac{y}{2}$. This means $f(x) = f(\frac{y}{2}) = 2(\frac{y}{2}) = y$. So, $f(x) = y$ and f is surjective.
f is both injective and surjective. Therefore f is bijective.

(b) * You can find a bijection between $(0,1)$ and $(1/4, 3/4) \subset [-1,1]$.
Hence $|(0,1)| \leq |([-1,1])|$.
You can also find a bijection between $(1/4, 3/4) \subset (0,1)$ and $[-1,1]$.
Hence $|[-1,1]| \leq [0,1]$
So, $|(0,1)| = |[-1,1]|$.

3) (a) False, Only the language that is Turing recognizable is countable.

(b) False, A "regular" language is Finite. if it is not regular then its not.

(c) ~~True~~ False, it can have more than 1 accepting state.

(d) False, it the same

(e) True, According to the Definition, if the language is regular if there is ~~a DFA~~ if we can construct a DFA. Then we can construct a Turing machine that stimulate that DFA.

(f) True, If it is decidable means a language is regular, So it is ~~conte.~~ countable.

(g) False, it depends on what a machine is program for.

(h) True

(i) False

~~4(b) $(\lambda x. x((\lambda z. zx)(\lambda x. bx)))y$~~

~~$(\lambda x. x(\lambda y. y)x)((\lambda a. a)(\lambda b. b))$~~

4(b) $(\lambda x. x((\lambda z. zx)(\lambda x. bx)))y$

$= \lambda x. xzxy$

$= yzx$

$(\lambda x. x((\lambda y. y)x))((\lambda a. a)(\lambda b. b))$

$= \lambda x. x(\lambda y. yx)(\lambda a. a \lambda b. b)$

$= \lambda a. a \lambda b. b(\lambda y. y(\lambda a. a \lambda b. b))$

$= \lambda b. b(\lambda y. y(\lambda a. a \lambda b. b))$

$= \lambda y. y(\lambda a. a \lambda b. b)$

$= \lambda a. a \lambda b. b$

$= \lambda b. b$

4 (a) $(\lambda x . x\boxed{y})(x \lambda y . y\boxed{x})(\lambda y z : zy)$

$\Box$ = free variable

(b) $(\lambda z . z(\lambda y . y\boxed{zx})\boxed{y})(\lambda x z . (\lambda y \boxed{zx\boxed{y}})\boxed{x})$

$\Box$ = free variable

~~( ) free inside~~ y

free inside ( )