RESEARCH PROJECT

on

Wind Turbine Blade Design

Submitted by:

Ms. Nattirat Promwang

DEPARTMENT OF AERONUATICS AND ASTRONUATICS ENGINEERING

國立成功大學航空太空工程學系

NATIONAL CHENG KUNG UNIVERSITY, TAINAN, TAIWAN

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PROJECT ADVISER: Professor Fei-Bin Hsiao

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Nattirat PROMWANG

Abstract

This project is aimed to design a horizontal axis wind turbine blade by applying Blade Element Momentum theory. The blade parameter calculation s is focused on engineering software which is MATLAB and simulated on Solidworks.

I. Introduction

The rapid development of infrastructure and industry in the 21^{st} century results in an increase in pollution and waste. Besides, a high rate of fuel consumption causes a fast rise of toxic gas such as carbon monoxide and dioxide. As a consequence, environment concern has driven mankind to rethink about clean and renewable energy and wind energy can be the right answer to this problem.

Wind power is an unlimited source of energy existing worldwide. In the past, wind energy had been used to generate the energy for water pump and grain milling in rural area (Burton, et al., 2001). Wind has also been used to generate electricity to supply for household in modern society and become an essential part of the source of energy in some countries.

Although the wind flows everywhere, it is important to design the tool which can extract the air to convert the energy efficiently. Wind turbine is a system used to generate the kinetic energy to power and wind turbine blade is one of the three components of windmill that have to be concerned. This project aims to design the blade based on Blade Element Momentum Theory (BEM) to gain an optimum power coefficient.

Power Generation on Wind Turbine

As the air flows through, the yaw motor causes the nacelle of wind turbine to turn which leads wind turbine blade and its rotor to move to the direction of the wind (Coriolis Energy, 2012). The blade is designed as an airfoil shape which can cause the air to move one side faster than the other side. The speed difference can generate the pressure difference between upper and lower surfaces. As a consequence, there is an increase of lift at the lower surface and this, in turn, causes the blade and rotor to rotate. Moreover, the rotational force or torque is generated.

On most wind turbines, the rotor is designed to connect with the gear box and such box is in turn connected to generator in the nacelle (Fig.1) to convert the torque to electric energy. This electricity is fed to the transformer that can increase the amount of electric voltage in order to reduce the loss during the move of electric flow. The electricity is transported by cable wire to a sub electric station.

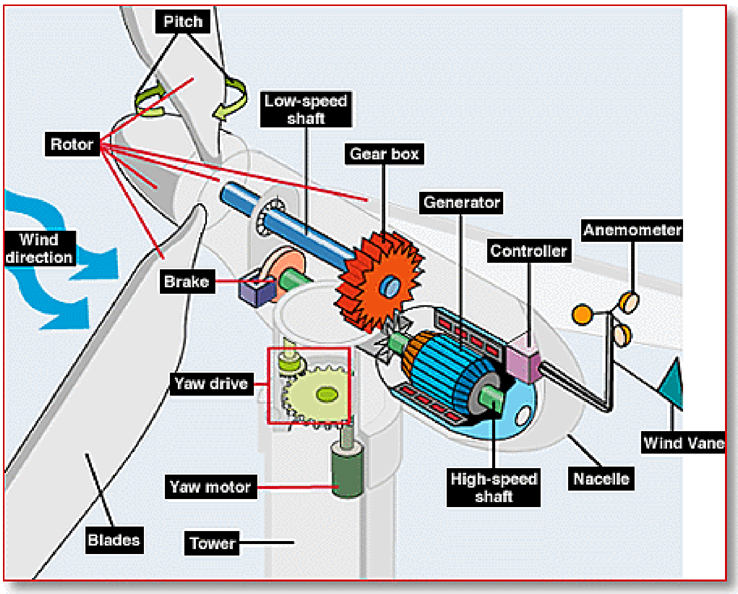

Figure.1Wind turbine inside components

Data source: *How Wind Energy Works* published by Union of Concerned Scientists in 2009

Wind Turbine Classification

Wind turbine is characterized into two main types which is horizontal axis wind turbine (HAWT) and vertical axis wind turbine (VAWT) (TeacherGreek, 2006). Each type of wind turbine has its own characteristics, advantages and disadvantages described as follows

- **Horizontal Axis Wind Turbine (HAWT)**
  The most popular use for wind turbine is horizontal axis type (Hansen, 2008). In this wind turbine, the shaft is placed horizontally to the ground and all of the components such as blades, shaft, and generator are on the top of wind turbine tower. In addition, the wind turbine blades are installed to face the incoming air.

Pros	Cons
• The blades are installed to the side of the center of gravity of turbine (increase stability)	• Cannot extract the low altitude wind
• The blade angle of attack can be changed to reach the optimum value	• Difficult to transport (20% of tool costs)
• Pitch angle can be controlled to decrease a severe damage from an strong wind velocity	• Difficult for the installation (require a large crane and skill workers)
• Wind turbine tower can allow to access stronger wind in sites with wind shear	• Effect radar operation in proximity
• Wind turbine tower can be installed in the fluctuation areas or offshore	• Difficult to repair
• Most horizontal axis wind turbine has self-starting ability	

Table1. The advantages and disadvantages of horizontal axis of wind turbine (HAWT)

- **Vertical Axis Wind Turbine (VAWT)**
  Although the horizontal axis wind turbine is the most use in wind energy application, vertical axis wind turbine is attractive for low cost and simple design application. The rotor of this type of wind turbine is placed perpendicular to free stream direction (Castillo, 2011). Moreover, the parts of wind turbine such as gear box and generator are installed to the ground.

Pros	Cons
• The wind turbine can be maintained easily	• The wind turbine blades constantly spin back to the wind direction which can cause an increase in drag
• The costs for construction and transportation are low	• Low efficiency
• The wind turbine can work effectively at hilltops or high building	• Low starting torque and may require some energy to turn the blades

Table2. The advantages and disadvantages of vertical axis of wind turbine (VAWT)

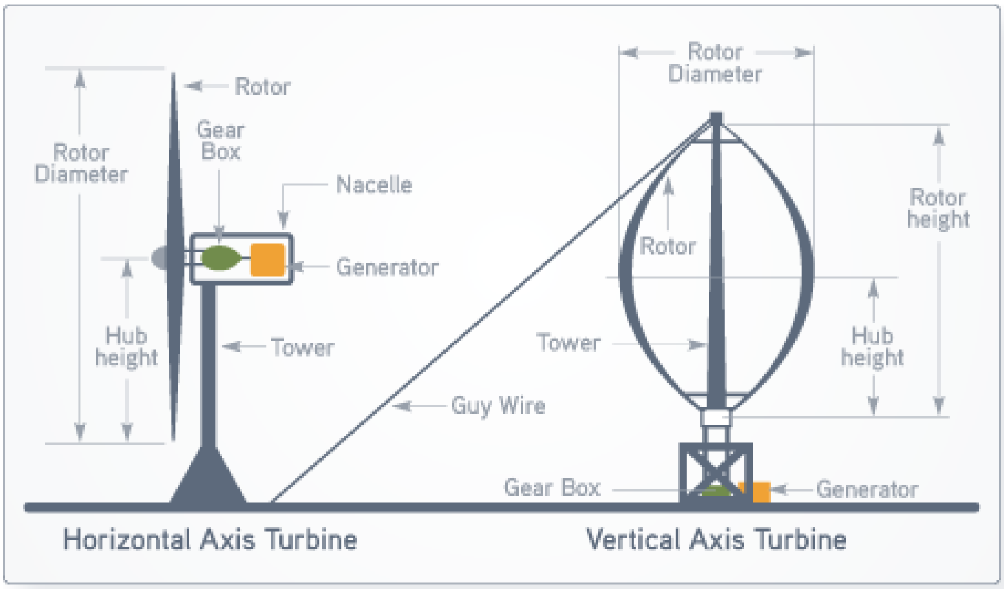

Figure2. Two different type of wind turbine (horizontal axis and vertical axis wind turbines)
Data source: *Wind Basics* published by Hill Country Wind Power (Organization) in 2013

II. Methods for Designing Wind Turbine Blade

In this project, the designed methods are based mainly on Blade Element Momentum Theory (BEM) which is involved with two different approaches to examine the performance of the wind turbine, the momentum balance on a rotating stream tube pass across the wind turbine and the aerodynamics forces created by airfoil at different section along the blade (Ingram, 2011).

One Dimensional Momentum Theory Analysis

In this section, the analysis can be used in the case where the rotating rotor creates angular momentum which can be used to determine rotor torque. The concept is associated with a 1D momentum theory and there are some assumptions as follows

- The wind turbine operates within a control volume which its boundaries are the stream tube's surfaces (as shown in Figure 3.)
- The turbine actuator disk is uniform which produces the fluctuation of pressure in the stream tube when the free stream passes across it
- The air is incompressible, homogenous, and in steady state condition
- No air friction involves during the wind turbine operating
- The number of wind turbine blade is infinite
- The thrust generates uniformly from the turbine's rotor area
- The wake is non-rotated
- The static pressure of the upstream and downstream outside the control volume are equal to ambient pressure

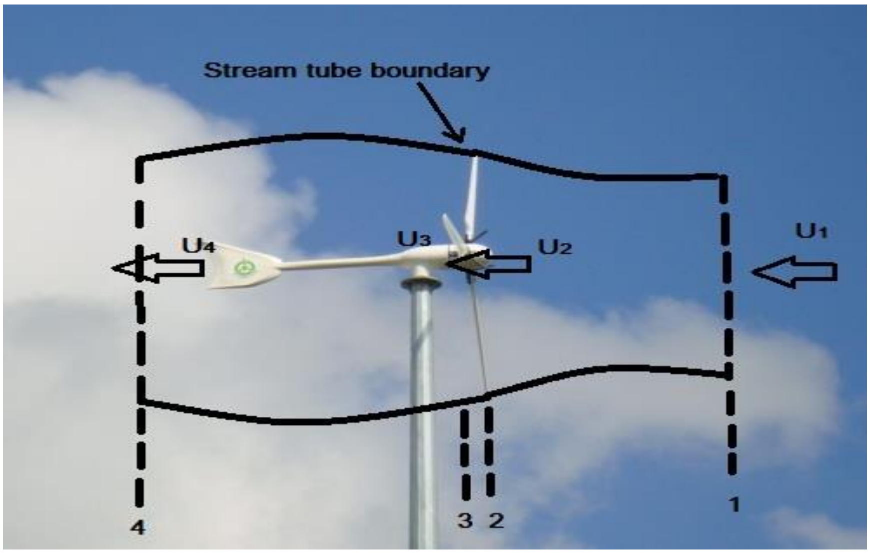

Figure 3.Actuator disk model of a wind turbine (where U = mean velocity; 1, 2, 3, and 4 are locations)

The power (P) calculation for ideal wind turbine is:

$$P = \frac{1}{2}\rho A_2 U_2 (U_1^2 - U_4^2) \quad (1)$$

When the axial induction factor is concerned as the fractional decrease in wind velocity between the free stream and the rotor plane, the equation (1) becomes

$$P = \frac{1}{2}\rho A U^3 4a(1-a)^2 \quad (2)$$

Where ρ is the density of air at sea level (1.225 kg/m^3), A is control volume area at the rotor, U is the air velocity, and a is the axial induction factor.

The power coefficient (C_P) which is used to determine the wind turbine performance is expressed as follows:

$$C_P = \frac{P}{\frac{1}{2}\rho U^3 A} \tag{3}$$

Firstly, the efficient radius (R) is determined by the derivation of power for ideal wind turbine known as Betz limit. According to Betz limit, the maximum power coefficient (C_P) that turbine can generate is 0.593 and, from statistic, the mean wind velocity (U) is 12.5 m/s. The rotor area (A) is πR^2 ; therefore, the efficient radius (R) can be determined from the following equation:

$$R = \sqrt{\frac{P}{\frac{1}{2}\rho U^3 \pi\ Cp}} \tag{4}$$

The equation (4) is then written in the form of MATLAB Codes to find the limitation of the designed blade radius. Secondly, to apply the BEM theory, the blade is needed to be subdivided into a small section (as seen in Figure 4.). In this project, the blade is subdivided into 10 sections and the radius for the first section and the last section are given to be equal to r_{root} and r_{tip} respectively. The sections 2 to 9 are calculated by the following equation;

$$r_i = r_{i-1} + \frac{r_{tip} - r_{root}}{\text{the number of section}} \tag{5}$$

Rotating Wind Turbine Analysis

In the previous section, the wind turbine blade movement is fixed (no rotation) which can relate to rotor torque calculation. However, in this section, the flow behind the rotor moves in the opposite direction to the torque exerted by the flow. The tip speed ratio (λ) is determined which is determined as the ratio between the blade tip velocity and free stream velocity by this analysis. On the design process, the value of tip speed ratio is required for λ= 6

$$\lambda = \frac{\omega R}{U} \tag{6}$$

where ω is the angular velocity of the blade during operating.

However, the tip speed ratio is needed also to calculate for each section of the blade which is called "local tip speed ratio (λ_r)". Thus,

$$\lambda_{r_i} = \frac{\omega r_i}{V_s} \tag{7}$$

Or

$$\lambda_{r_i} = \left(\frac{r_i}{r_{tip}}\right) * \lambda \tag{8}$$

Where i indicates the section on the blade and r_{tip} is the radius of the blade at tip (See Figure4.). For this project, the MATLAB Codes are written to determine the value of local tip speed ratio for each section (λ_{r_i}) with different angular velocities (ω).

Blade Element Momentum Theory (BEM)

BEM refers to the theory which relates the blade shape to the ability of the rotor to generate the power form the wind (wind turbine performance). Moreover, this theory combines the momentum and blade element theory equation together.

Blade Element Theory

Assumptions

- Each blade element has no aerodynamic interaction
- The force on the blades is a function of lift and drag only

The next step is to determine angles which are involved with wind turbine operation. The first is angle of relative wind (φ) and pitch angle (θ_p) which is the angle between chord line and the plane of rotation (see Figure5.) Another is angle of attack ($\propto$) which is the angle between chord line and relative wind velocity (U_{rel}). In this design, the angle of attack ($\propto$) is set at 5°

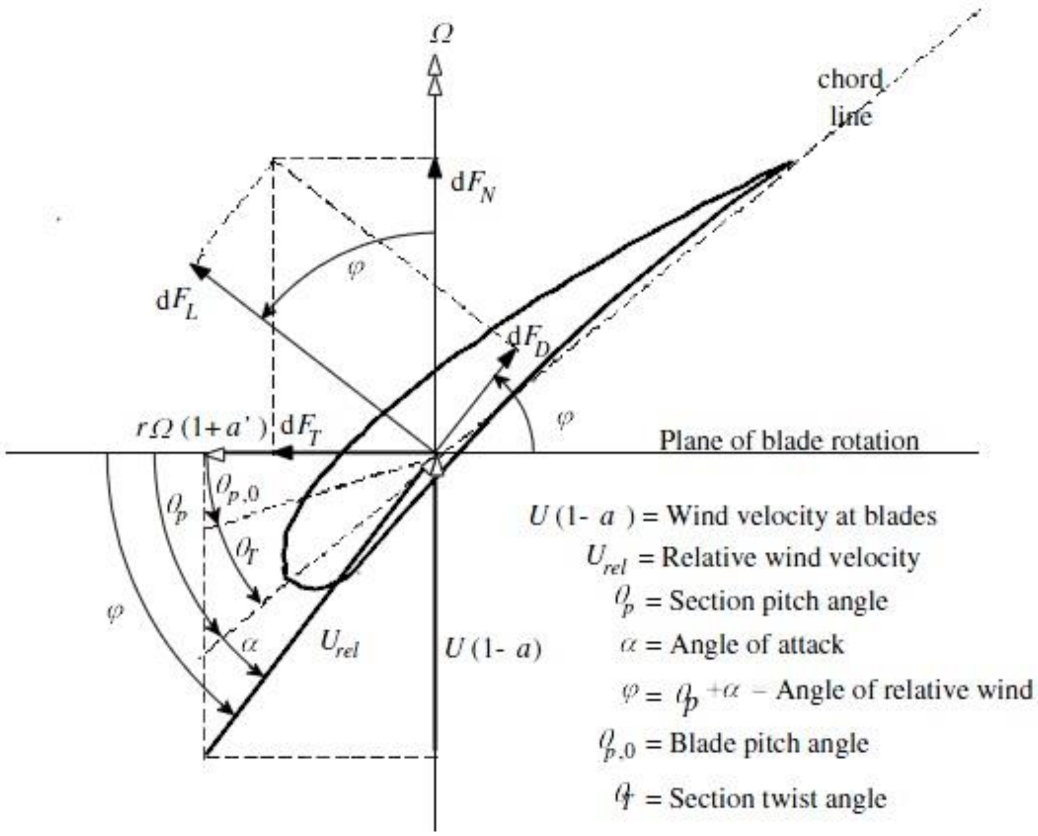

Figure5. Angles description in 2D of wind turbine blade

Data source: *Wind Energy Explained- Theory, Design and Application* by J.F. Manwell et al,

In this design, the angle of attack ($\propto$) is set at 5°. The pitch angles (θ_p) for with and without wake rotations on each section are in Equation (8.)

$$\theta_{p_i} = \varphi_i - \alpha \tag{9}$$

However, the relative wind angle (φ_i) and other parameters are needed to be separately determined for both condition as follows

- **For ideal rotor wind turbine (No wake disturbance)**

Assumptions:

1. The wind turbine produces the maximum possible power coefficient (C_p)
2. The wind turbine is determined at the axial induction factor (a) of 1/3
3. No wake rotation occurs during operation (a' = 0)
4. The air flows ideally (C_d = 0)
5. The losses are not occurs from a finite number of blade

Thus, the relative wind angle (φ) for each section of the blade can be found from Equation (10).

$$\varphi_{\ i} = tan^{-1}(\frac{2}{3\ \lambda_{r_i}}) \tag{10}$$

When the relative wind angle (φ_i) for each blade section is determined, the chord length on each section (c_i) of the Bezt optimum blade can also be calculated as in Equation (11). Suppose that the airfoil has a lift coefficient C_l = 1, the value of C_d/C_l is minimum at a setting angle of attack (α = 5°), and the number of blade (B) of the wind turbine is 3.

$$c_i = \frac{(8\pi sin\varphi_i)}{3BC_l\ \lambda_{r_i}} \tag{11}$$

Solidity (σ) is one of the important parameter in determining the wind turbine performance. According to Manwell et al, (2002), it can be described as “the ratio of the area of the blade to the swept area”. And the local solidity (σ')

$$\sigma_i' = \frac{Bc_i}{2\pi r} \tag{12}$$

Another parameter is relative velocity (U_{rel})

$$U_{rel_i} = \frac{2V_s}{3sin\varphi_i} \tag{13}$$

Where V_s is incoming flow which will be vary from 3 – 12 m/s in this project

In the optimum blade design, the pitch angle for each section (φ_i) is needed to redesign by substituting Equation (7) into (9), we get

$$\varphi_{new_i} = tan^{-1}(\frac{2V_s}{3\ r\omega}) \tag{14}$$

And the new optimum angle of attack for each blade section ($\propto_{new_i}$) which is obtained from Equation (14) and (9) is

$$\propto_{new_i} = \varphi_{new_i} - \theta_p \tag{15}$$

Normally, the wind turbine blade is not on the optimum design. And when an optimum blade operates at a different tip speed ratio, the blade cannot consider to optimum. Therefore, the blade shape has to be designed for the whole range of wind and rotor velocities. The values of axial and angular induced factors must be recalculated as in Equation (16) and (17), respectively.

$$a_i = 1/ [1+sin^2\varphi_i/ (\sigma' C_l cos\varphi_i)] \quad (16)$$

$$a_i' = 1/ [(4\cos\varphi_i/ (\sigma' C_l)) - 1] \quad (17)$$

Moreover, tip speed ratio for each section ($\lambda_{\mathrm{r,new_i}}$) can be recalculate as follows

$$\lambda_{\mathrm{r,new_i}} = (a_i/a_i') \tan\varphi_i \quad (18)$$

From blade element theory, the torque exerted on the wind turbine (see Equation (19) and (20)) is a function of non-linear relation of lifts and drag coefficient (C_l andC_d) and an angle o f attack (α). On the design, the airfoil used for the blade is NACA1014 which the value of lift and drag coefficient at various angle of attack is on Appendix A. The torque is expressed as Q and the differential torque for each section is

$$dQ_i = \mathrm{B}\frac{1}{2}\rho U_{rel_i}^2(C_l \sin\varphi_i\text{-}\ C_d \cos\varphi_i)C_i\ r_i\mathrm{dr} \quad (19)$$

From Momentum Theory, the torque can also be expressed from the annular momentum is

$$dQ_i = 4a_i'\ (1\text{-}\ a_i)\rho\mathrm{U}\pi r_i^3\omega\ \mathrm{dr} \quad (20)$$

When the values of $a_i$are determined, the overall rotor power coefficient (C_p) can be calculated as follows

$$C_p = 8/\ \lambda^2 \int_{\lambda_h}^{\lambda} \lambda_r^3\ a_i'\ (1\text{-}\ a_i)[1\text{-}\ (C_d/\ C_l) \cot\theta_{p_i}]\ \mathrm{d}\lambda_r \quad (21)$$

Tip Loss Effect

The differences of pressure between two sides of blade cause the air to flow around the tip from upper to lower surfaces. This leads to the change of lift coefficient (C_l) and power coefficient (C_p). Hence, the correction factor (F) is concerned in this section

$$\mathrm{F} = (2/\pi)\ cos^{-1}[\exp\ (\text{-}\{\frac{\left(\frac{B}{2}\right)[1-(r_i/\ R)]}{\left(\frac{r_i}{R}\right)\sin\varphi}\})] \quad (22)$$

The values of a_i and a_i' are needed to be recalculate if the tip loss is concerned

$$a_i = 1/ [1+ 4\mathrm{F}sin^2\varphi_i/ (\sigma' C_l cos\varphi_i)] \quad (23)$$

$$a_i' = 1/ [(4\mathrm{F}\cos\varphi_i/ (\sigma' C_l)) - 1] \quad (24)$$

The differential torque for each section becomes (dQ_i)

$$dQ_i = 4\mathrm{F}a_i'\ (1\text{-}\ a_i)\rho\mathrm{U}\pi r_i^3\omega\ \mathrm{dr} \quad (25)$$

The overall power coefficient (C_p) with correction factor (F) is

$$C_p = 8/\ \lambda^2 \int_{\lambda_h}^{\lambda} \mathrm{F}\lambda_r^3\ a_i'\ (1\text{-}\ a_i)[1\text{-}\ (C_d/\ C_l) \cot\theta_{p_i}]\ \mathrm{d}\lambda_r \quad (26)$$

III. Result and Blade Shape Simulation

The MATLAB code is written for the following constrains

- Horizontal Axis Wind Turbine (HAWT)
- Small wind turbine with 3 blades
- 1250 W power input
- 5 degree of initial angle of attack
- Tip speed ratio (λ) of 6
- RPM from 0 to 900
- Wind speed from 3 to 12 m/s

The chord and twist or pitch angle distribution for each section are plotted with respect to blade section radius (r_i) over effective radius (R)

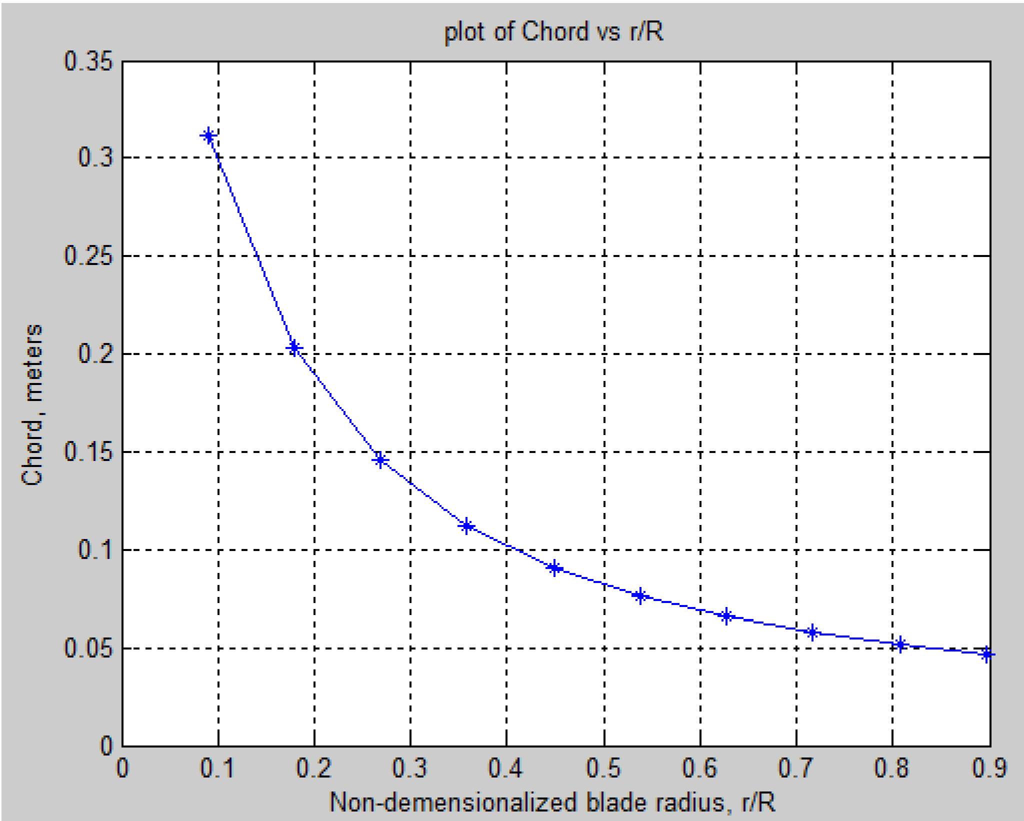

Figure6. Chord distribution

The chord lengths at each section of blade element range from 0.31 m. at 1st section to 0.049 m at section 10.

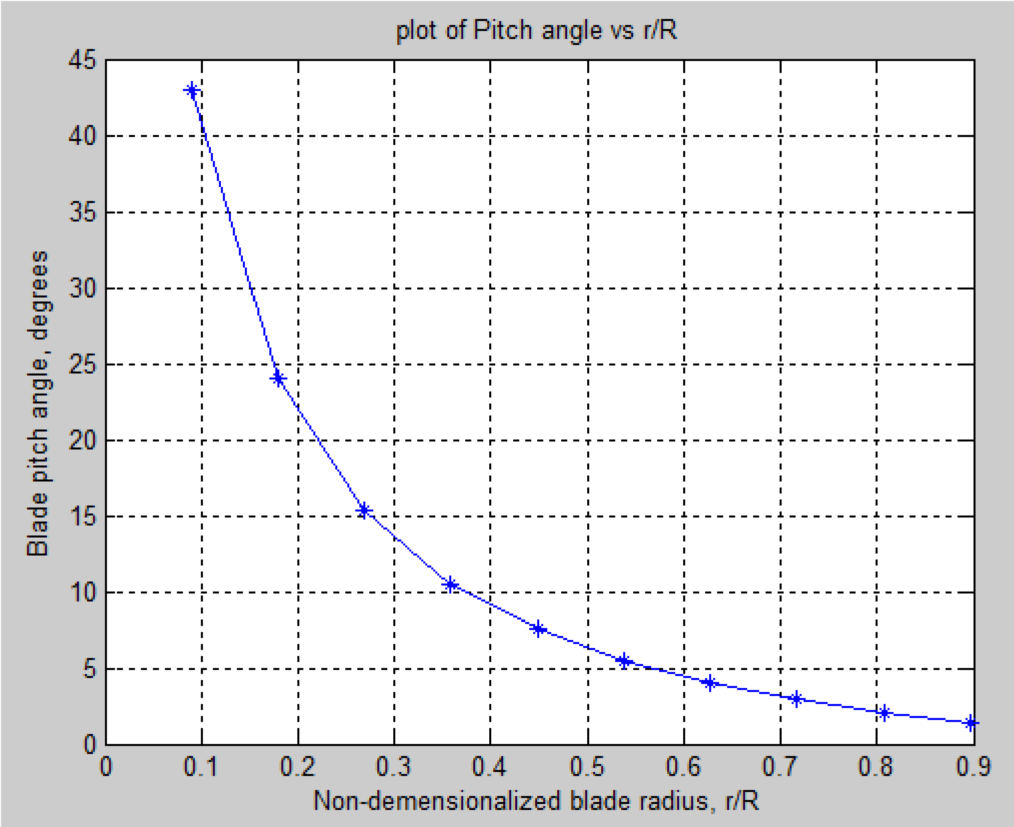

Figure7. Blade pitch angle distribution

The pitch angles along blade element are from 43 degrees at section 1 to 1 degree at section 10.

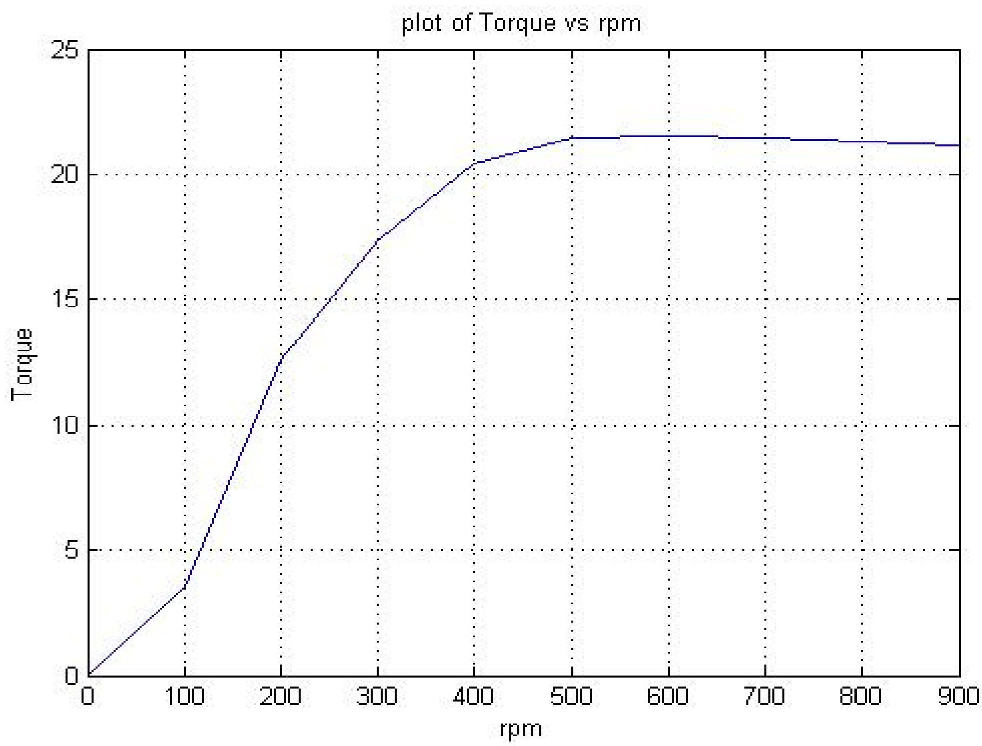

Figure8. Torque distribution at each RPM

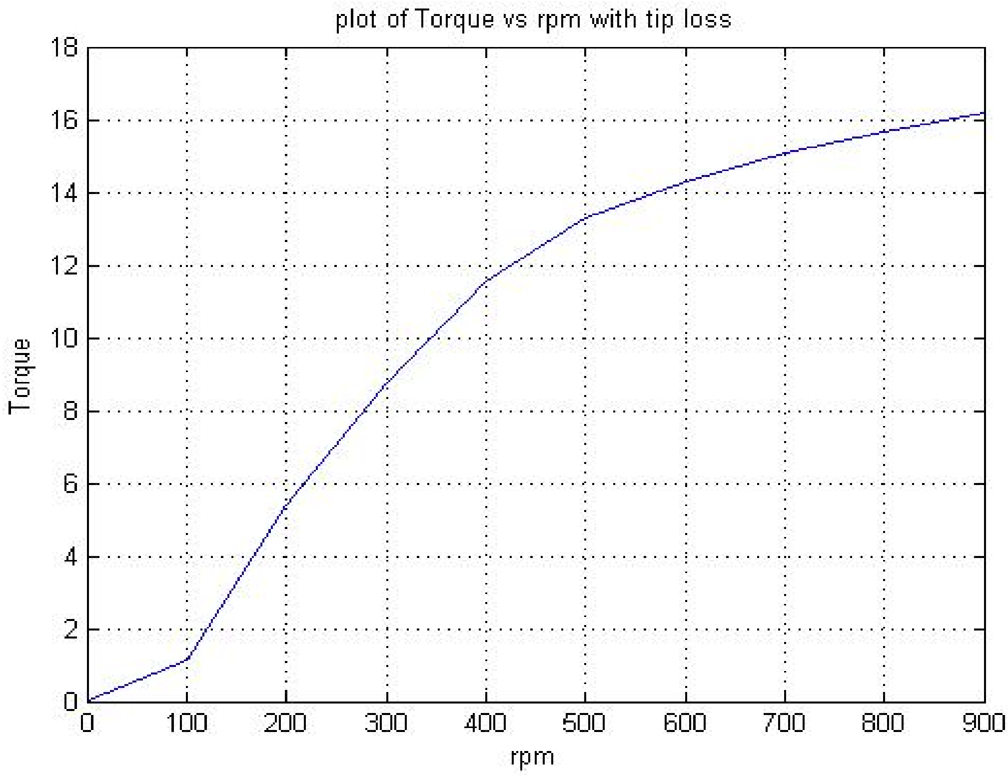

Figure8. Torque with tip loss effect at each RPM

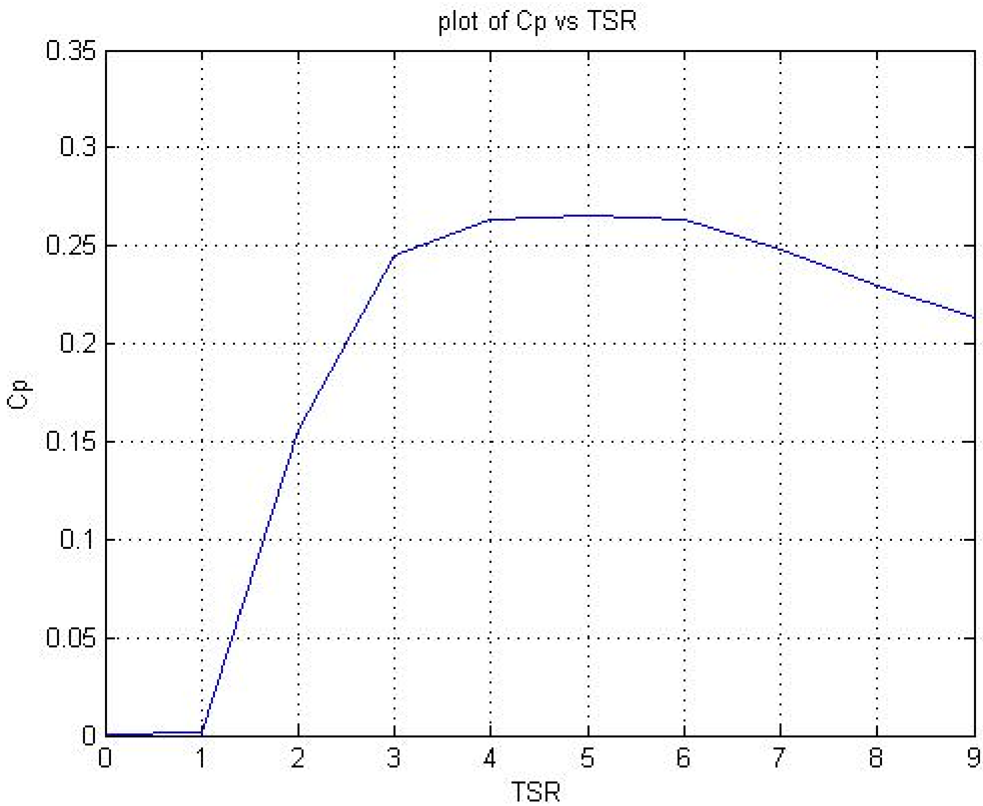

Figure9. Power coefficient (C_p) at different range of tip speed ratios (TSR)

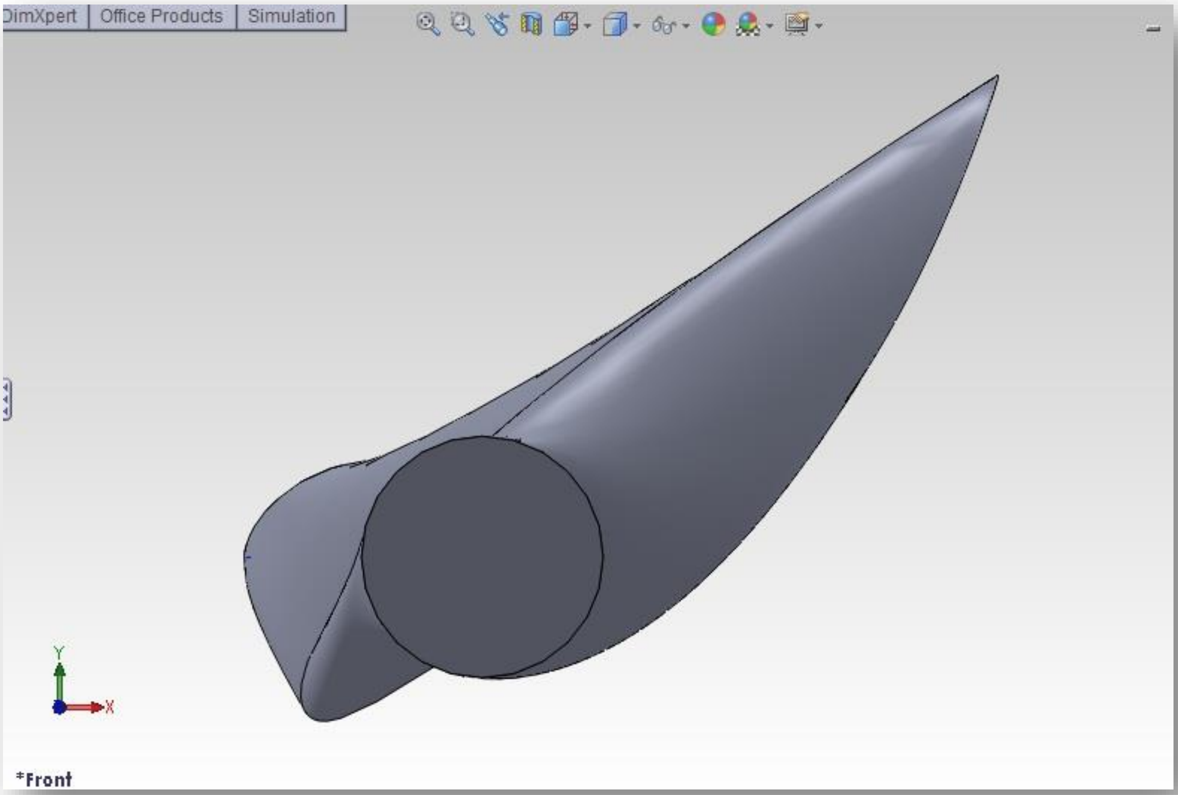

Figure10. Blade shape simulation (Front view)

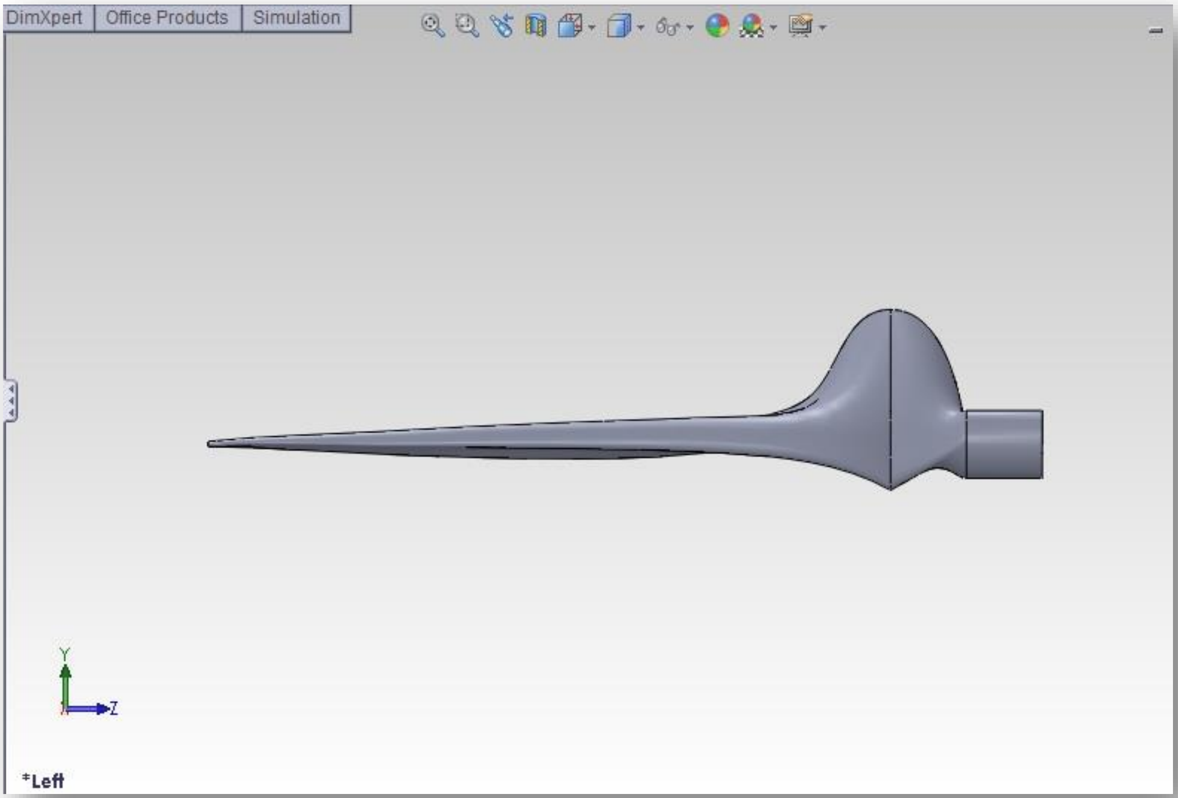

Figure11. Blade shape simulation (Left view)

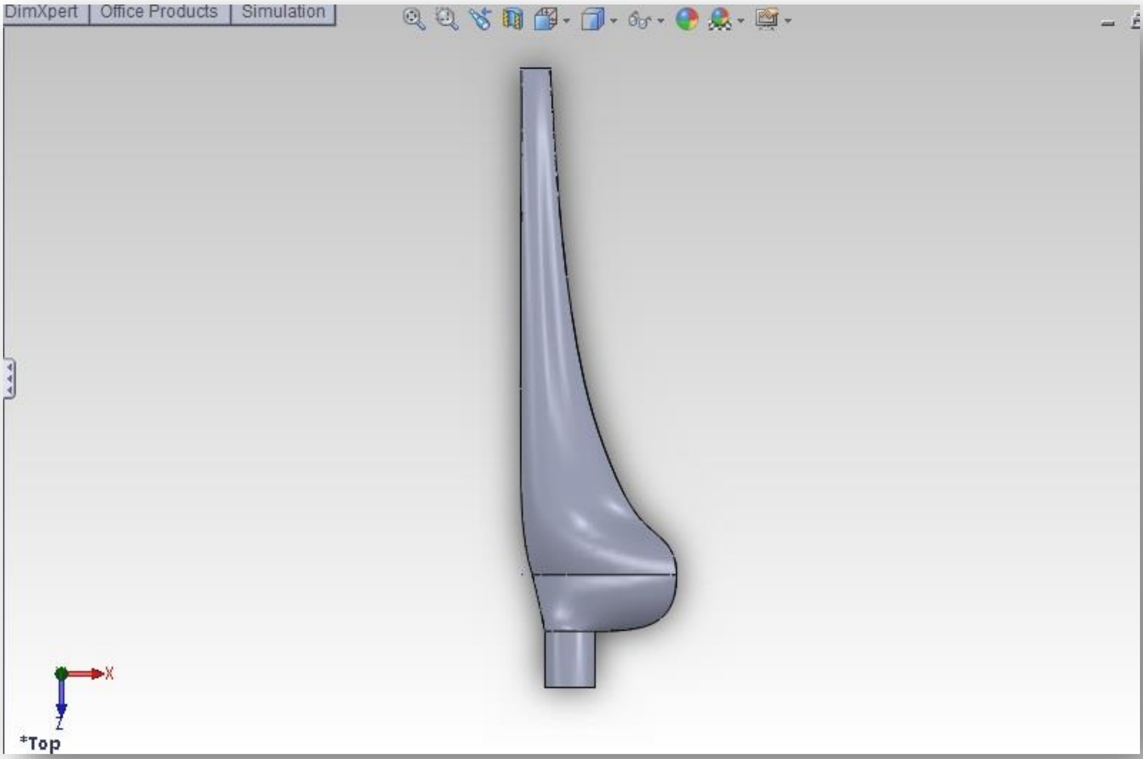

Figure12. Blade shape simulation (Top view)

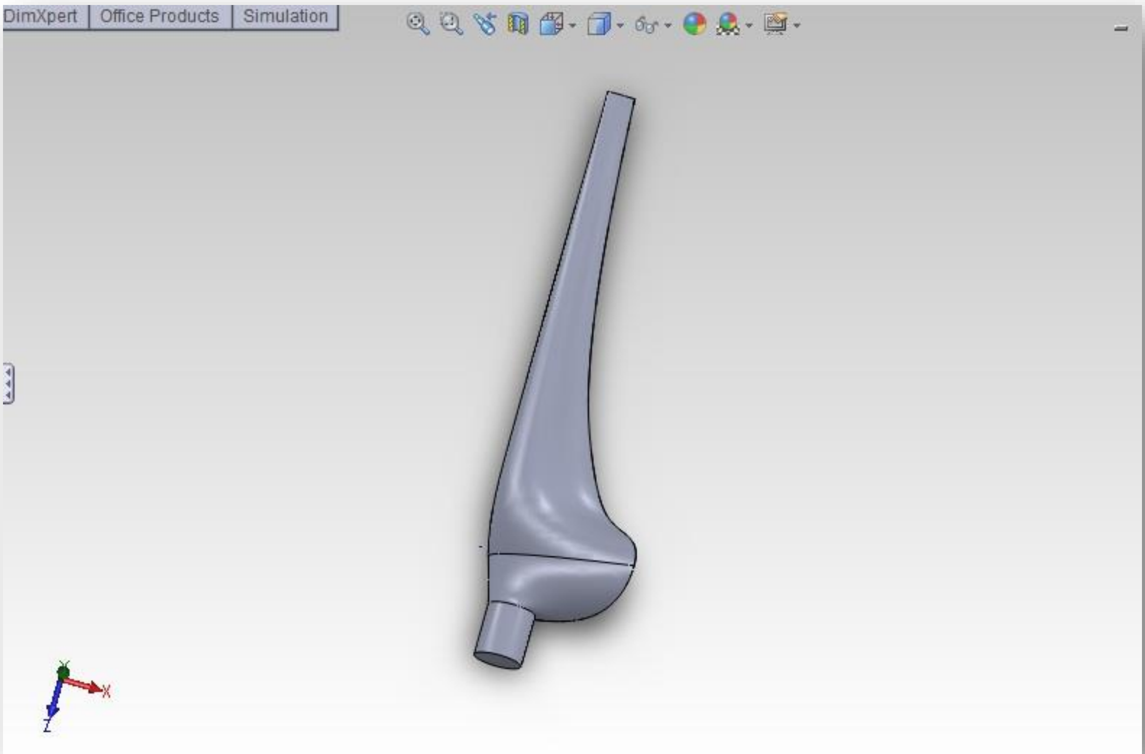

Figure13. Blade shape simulation (Isometric view)

IV. Discussion

In this internship project, the small wind turbine (1250 Watts) blades are designed for a variety range of wind speed; from 3-12 m/s with considering the value of power coefficient (C_p) and torque as objective functions. This section describes some causes of calculation errors in blade design.

The design is based on blade element momentum theory which gives a simplest blade shape. Since this theory covers both blade element and momentum theory, those two theories ignore some important parameters in determining blade characteristics.

Another important parameter that affects the blade design calculation is airfoil lift and drag coefficients. This two coefficients result in axial and angular induction factors (a_i and a_i') and power coefficient (C_p) calculations.

Recommendations

In the process of designing the blade, the optimization for blade parameters should be involved in the design in order to get a best character of blade. The blade designed from this MATLAB code has high different chord length and twist angle between 1st and 2nd blade section; therefore, this should be optimized for more logical value in the real application.

Moreover, the airfoil used in this wind turbine blade is uniform along the element. In fact, each blade element consists of different chord length; as a consequence, there are different values of Reynolds numbers. Each airfoil can operate at its optimum performance in different range of Reynolds number; therefore, the use of different airfoil sections can generate different value of lift to drag ratio and this can increase the value of power coefficient for the whole blade.

V. Conclusion

The code from MATLAB in this project can be use solely for simplest optimal design because, in practical, there are some factors that are needed to concern such as different flow regime over the blade. Furthermore, for an optimal design blade, the wind tunnel test needs to be involved.

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Appendix A

MATLAB Code

```
clc
clear all
close all

                %% Defined parameter

        P = 1250; % Power (W) :Defined
        Density = 1.225; % kg/m^3
        U = 12.5; % Wind velocity (m/s)
        Cl_i = 1; % Ideal lift coefficient
        Cd_i = 0.03; % Ideal drag coefficient
        TSR = 6; % Tip speed ratio of the entire blade
        Vs = input('Wind speed (m/s) : '); % Freestream velocity



                %% Initial setting parameters
        format long
        AOA = 5; % Angle of attack (deg)
        Cp = 0.33; % Power coefficient for blade :Assumed
        B = 3; % Number of blade
        R = sqrt(P/(0.5*Density*(U^3)*pi*Cp)); % Efficient radius (m)
        R_tip = 0.9; % (m)
        R_root = 0; % (m)
        R_hub = 0.09; %(m)

        % Naca 1014
        %cla = [-20 -18 -16 -14 -12 -10 -8 -6 -4 -2 0 2 4 6 8 10 12 14 16
18 20 30 40 50 60 70 80 90]; % attack angle
        %clf = [-1.524 -1.298 -1.07 -0.839 -0.606 -0.371 -0.134 0.103 0.341
0.571 0.699 0.797 0.868 0.912 0.928 0.918 0.879 0.81 0.708 0.694 0.678
0.678 0.678 0.678 0.678 0.678 0.678 0.678]; % cl coefficient
        %cdf = [0.052 0.0415 0.0294 0.0239 0.0187 0.0145 0.0119 0.0106
0.0103 0.0108 0.0106 0.0109 0.014 0.017 0.0208 0.0256 0.0329 0.0411 0.0511
0.0629 0.0528 0.0528 0.0528 0.0528 0.0528 0.0528 0.0528 0.0528];

        cla = [-20 -19 -18  -17 -16 -15 -14 -13 -12 -11 -10 -9  -8  -7  -6
-5 -4 -3    -2  -1  0   1   2   3   4   5   6   7   8   9   10  11  12  13
14  15  16  17  18  19  20  21  22  23  24  25  26  27  28  29  30  31  32
33  34  35  36  37  38  39  40  41  42  43  44  45  46  47  48  49  50  51
52  53  54  55  56  57  58  59  60  61  62  63  64  65  66  67  68  69  70
71  72  73  74  75  76  77  78  79  80  81  82  83  84  85  86  87  88  89
90];
        clf = [-0.995   -1.026  -1.05   -1.058  -1.052  -1.037  -1.012  -
0.977  -0.933  -0.878  -0.815  -0.743  -0.665  -0.582  -0.604  -0.491  -
0.373  -0.254  -0.132  -0.01   0.112   0.234   0.356   0.476   0.595
0.712   0.825   0.932   1.023   1.085   1.124   1.156   1.2 1.229   1.256
1.276   1.282   1.276   1.263   1.24    1.209   1.17    1.129   1.083
1.035   0.986   0.937   0.888   0.841   0.796   0.752   0.711   0.672
0.636   0.601   0.569   0.539   0.511   0.485   0.461   0.439   0.418
0.396   0.374   0.355   0.337   0.32    0.305   0.29    0.277   0.265
0.253   0.243   0.233   0.224   0.215   0.207   0.2 0.193   0.187   0.181
0.175   0.17    0.165   0.16    0.156   0.152   0.148   0.145   0.142
0.139   0.136   0.133   0.131   0.128   0.126   0.124   0.123   0.121
0.119   0.118   0.117   0.115   0.114   0.113   0.113   0.112   0.111
0.111   0.11    0.11];
        cdf = [0.24309  0.21756 0.19407 0.17521 0.15493 0.13748 0.12394
0.10941 0.09609 0.08503 0.07473 0.06619 0.05876 0.05184 0.01996 0.01886
```

```
0.01519 0.01711 0.01662 0.01632 0.01642 0.01672 0.01734 0.01568 0.02015
0.02117 0.02247 0.02414 0.02677 0.03315 0.04601 0.06407 0.07928 0.09658
0.11226 0.12863 0.14693 0.16516 0.18456 0.20509 0.22897 0.25159 0.28195
0.31023 0.33588 0.37672 0.40662 0.44247 0.47834 0.52363 0.56434 0.61964
0.65847 0.71659 0.75716 0.8168  0.90276 0.93777 0.99492 1.09367 1.10333
1.17569 1.2897  1.39055 1.45956 1.52692 1.58156 1.60377 1.80767 1.87087
1.94008 2.01519 2.09758 2.19022 2.30146 2.42801 2.52088 2.62019 2.72678
2.84609 2.90476 3.04221 3.19932 3.10502 3.31501 3.27036 3.43354 3.61613
3.60457 3.82219 3.73971 4.02847 4.03665 3.96525 4.22588 4.20179 4.17597
4.54428 3.59633 3.49519 4.40395 3.44383 3.70535 3.75529 3.63308 3.55113
3.57938 3.84484 3.7541  3.6465  3.64974];


      %% Blade Shape for ideal rotor without wake
      % Assumption
        % There is no wake rotation, a' = 0
        % There is no drag, Cd = 0
        % There is no losses from blades
        % For the Betz optimum rotor, a = 1/3
i = 0; k =0;
for W = 0:100:900 % an angular velocity (rpm)
    w = W*(pi/30);
    k = k+1;
% The length of blade is subdivided into 10 sections
  for r = R_hub:(R_tip-R_root)/10:R_tip
     i = i+1;
    TSRr(i) = (r/R_tip)*TSR;
    Phi_rad(i) = atan(2/(3*TSRr(i))); % Phi is an angle of relaive wind
    Phi_deg(i) = Phi_rad(i)*(180/pi);
    Pitch_angle(i) = Phi_deg(i)-AOA;
    c(i) = (8*pi*r*sin(Phi_rad(i)))/(3*B*Cl_i*TSRr(i));
    %calU(i) = sqrt((r*w)^2*(1+a_dash)^2+(2/3*Vs)^2);


       %% Performance prediction with wake rotation
           % The analysis includes wake rotation, drag, losses
    % find solidity for each section
    Sol(i) = (B*c(i))/(2*pi*r);
    % find velocity of relative wind
    calU(i) = sqrt((r*w)^2*+(2/3*Vs)^2);

    % find induced factor for each section
    %TSRr_new(i) = r*w/calU(i);
    Phi_new_rad(i) = atan(2*Vs/(3*r*w));
    Phi_new_deg(i) = Phi_new_rad(i)*(180/pi);
    AOA_new(i) = Phi_new_deg(i)- Pitch_angle(i);

    % find Cl and Cd from airfoil data using curve fitting
    Cl(i) = spline(cla,clf,AOA_new(i));
    Cd(i) = spline(cla,cdf,AOA_new(i));

    % the valve of a and a' in an induced factor
    a_up(i) = 1+4*(sin(Phi_new_rad(i)))^2;
    a_down(i) = Sol(i)*Cl(i)*cos(Phi_new_rad(i));
    a(i) = 1/(a_up(i)/a_down(i));
    a_dash_in(i) = (4*cos(Phi_new_rad(i)))/(Sol(i)*Cl(i));
    a_dash_det(i) =  a_dash_in(i)-1;
    a_dash(i) = 1/a_dash_det(i);
    TSRr_new(i) = (a(i)/a_dash(i))*tan(Phi_new_rad(i));
```

```
      %% Momentum Theory and Blade Element Theory
    T_b(k,i) = B*0.5*Density*calU(i)^2*c(i)*(R-
R_hub/10)*r*(Cl(i)*sin(Phi_new_rad(i))-Cd(i)*cos(Phi_new_rad(i)));
    %T(k,i) = Sol(i)*pi*Density*(Vs^2*(1-
a(i))^2/(sin(Phi_new_rad(i)))^2)*((Cl(i)*sin(Phi_new_rad(i)))-
(Cd(i)*cos(Phi_new_rad(i))))*r^2*(R-R_hub/10);
    T_m(k,i) = 4*a_dash(i)*(1-a(i))*Density*(2/3*Vs)*pi*r^3*w*(R-R_hub/10);
    Torque_b(k) = T_b(k,i);
    Torque_m(k) = T_m(k,i);

      %% Tip loss
% find mean radius of each section
    r_c(i) = r;
    if i == 1
        r_m(i) = r_c(i)/2;
    else
        r_m(i) = (r_c(i)+r_c(i-1))/2;
    end
    % find the correction factor
    factorf_1(i) = 1-r_m(i)/R_tip;
    factorf_2(i) = factorf_1(i)*(B/2);
    factorf_3(i) = (r_m(i)/R_tip)*sin(Phi_new_rad(i));
    factorf_4(i) = factorf_2(i)/factorf_3(i);
    factorf_5(i) = exp(-factorf_4(i));
    Fa(i) = (2/pi)*acos(factorf_5(i));

    % find Cl and Cd with correction factor from airfoil data using curve
fitting
    Cl_c(i) = Fa(i)*Cl(i);
    Cd_c(i) = Fa(i)*Cd(i);

   % the valve of a and a' with correction factor
    a_up_c(i) = 1+4*Fa(i)*(sin(Phi_new_rad(i)))^2;
    a_down_c(i) = Sol(i)*Cl_c(i)*cos(Phi_new_rad(i));
    a_c(i) = 1/(a_up_c(i)/a_down_c(i));
    a_dash_in_c(i) = (4*Fa(i)*cos(Phi_new_rad(i)))/(Sol(i)*Cl_c(i));
    a_dash_det_c(i) =  a_dash_in_c(i)-1;
    a_dash_c(i) = 1/a_dash_det_c(i);

      %% Torque with tip loss
    T_c(k,i) = Fa(i)*4*a_dash_c(i)*(1-a_c(i))*Density*(2/3*Vs)*pi*r^3*w*(R-
R_hub/10);
    Torque_c(k) = T_c(k,i);
    SumTorque_c = sum(Torque_c);

      %% Power coefficient
    if i > 1 && k > 1
    cp(k-1,i-1) = (8/TSR^2)*a_dash(i)*(1-a(i))*(1-
(Cd(i)/Cl(i))*cot(Pitch_angle(i)))*((TSRr(i)^4-TSRr(1)^4)/4);
    %cp(k,i-1) = (8/TSR*10)*(sin(Phi_new_rad(i)))^2*(cos(Phi_new_rad(i))-
TSRr(i)*sin(Phi_new_rad(i)))*(sin(Phi_new_rad(i))+TSRr(i)*cos(Phi_new_rad(i
)))*(1-(Cd(i)/Cl(i))*cot(Phi_new_rad(i)))*TSRr(i)^2;
    %x(i) = linspace(TSRr_new(1),TSRr_new(i),100);
    %y = a_dash(i)*(1-a(i))*(1-(Cd(i)/Cl(i))*cot(Pitch_angle(i)))*x(i)^3;
    %z(i) = trapz(x,y);
    %cp(k,i) = z(i)*(8/TSR^2);
    %cp(k,i) = 4*a(i)*(1-a(i))^2;
    calCp(k) = cp(k-1,i-1);
    % Cp for each section wih correction factor
```

```
    cp_c(k-1,i-1) = Fa(i)*(8/TSR^2)*a_dash_c(i)*(1-a_c(i))*(1-
(Cd_c(i)/Cl_c(i))*cot(Pitch_angle(i)))*((TSRr(i)^4-TSRr(1)^4)/4);
    %cp_c(k,i) = (8/(TSRr_new(i)^2))*F(i)*Factor_a_dash_c(i)*(1-
Factor_a_c(i))*(1-Cd(i)/Cl(i)*cot(Pitch_angle(i)))*((TSRr_new(i)^4-
TSRr_new(i-1)^4)/4);
    calCp_c(k) = cp_c(k-1,i-1);
    %else
    end
  end
  %z(k) = trapz(x,y);
  %cp(k) = z(k)*(8/TSR^2);
end
  rpm = 0:100:900;
  plot(rpm,Torque_m);
  grid on;
  title('plot of Torque vs rpm');
  xlabel('rpm');
  ylabel('Torque');
  figure;
  plot(rpm,Torque_c);
  grid on;
  title('plot of Torque vs rpm with tip loss');
  xlabel('rpm');
  ylabel('Torque');
  figure;
  tsr = 0:1:9;
  plot(tsr,calCp_c);
  grid on;
  title('plot of Cp vs TSR');
  xlabel('TSR');
  ylabel('Cp');
```