# Outline



- AC Bridges
  - Introduction
  - Measurement of Self Inductance
  - Measurement of Capacitance
  - Measurement of Frequency
  - Measurement of Medium Resistance

# Fundamental

AC bridges are applied to measure inductance, capacitance, storage factor and loss factor of active elements of an ac circuit. This type of bridges are also used for phase shifting, to provide feedback path for oscillators and amplifiers, to filter out unwanted signals.

*Sources:* The source used in ac bridges is power line for measuring at low frequencies in addition to electronic oscillators for measuring at high frequencies.

*Detectors:* Headphones, Vibration Galvanometer and Tunable amplifiers are commonly used as detectors in ac bridges.

# General Equation of AC Bridge

AnACbridge is shown in figure (1). At balance,

$$E_1 = E_2 \Rightarrow I_1 Z_1 = I_2 Z_2 \qquad (1)$$

Similarly, at balance condition the current

$$I_1 = I_3 = \frac{E}{Z_1 + Z_3}, \qquad I_2 = I_4 = \frac{E}{Z_2 + Z_4} \qquad (2)$$

Using equation (1) and (2)

$$\frac{E}{Z_1 + Z_3}.Z_1 = \frac{E}{Z_2 + Z_4}.Z_2 \qquad (3)$$

$$\Rightarrow Z_1 Z_4 = Z_2 Z_3 \qquad (4)$$

$$\Rightarrow Y_1 Y_4 = Y_2 Y_3 \qquad (5)$$

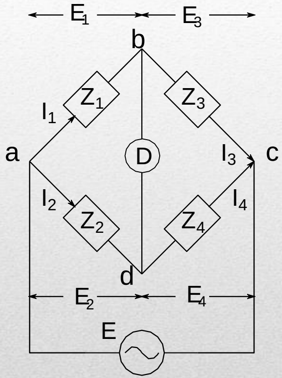


Figure 1:AC bridge

# General Equation of AC Bridge

From equation (4), the polar form of balance equations are

$$Z_1 < \theta_1 . Z_4 < \theta_4 = Z_2 < \theta_2 . Z_3 < \theta_3 \quad (6)$$

$$\Rightarrow Z_1 Z_4 < (\theta_1 + \theta_4) = Z_2 Z_3 < (\theta_2 + \theta_3) \quad (7)$$

If $Z_1 = R_1 + jX_1$, $Z_2 = R_2 + jX_2$, $Z_3 = R_3 + jX_3$ and $Z_4 = R_4 + jX_4$ then, using equation (4)

$$(R_1 + jX_1)(R_4 + jX_4) = (R_2 + jX_2)(R_3 + jX_3) \quad (8)$$

$$R_1R_4 - X_1X_4 + j(R_1X_4 + R_4X_1) = R_2R_3 - X_2X_3 + j(R_2X_3 + R_3X_2) \quad (9)$$

Equating real and imaginary part of equation (9), the balance equations

$$R_1R_4 - X_1X_4 = R_2R_3 - X_2X_3 \quad (10)$$

$$R_1X_4 + R_4X_1 = R_2X_3 + R_3X_2 \quad (11)$$

# General Equation of AC Bridge

As shown in figure (2) and by using balance equation (4)

$$Z_1Z_4 = Z_2Z_3$$

$$(R_1 + j\omega L_1)R_4 = (R_2 + j\omega L_2)R_3$$

$$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega L_2R_3 \quad (12)$$

Equating real and imaginary part of equation (12)

$$R_1 = R_2R_3/R_4 \quad (13)$$

$$L_1 = L_2R_3/R_4 \quad (14)$$

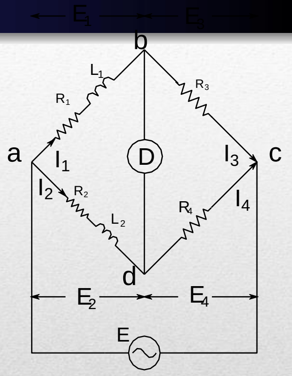


Figure 2:AC bridge

# Maxwell's Inductance-Capacitance Bridge

Here,

$L_1$= unknown inductance, $R_1$= effective resistance of $L_1$, $R_2$, $R_3$, $R_4$= non-inductive resistances, $C_4$= variable standard capacitor.

At balance,

$$(R_1 + j\omega L_1)\,\frac{R_4}{1 + j\omega C_4 R_4} = R_2 R_3 \qquad (15)$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_2 R_3 R_4 C_4 \qquad (16)$$

Equating real and imaginary part of equation (16)

$$R_1 = R_2 R_3 / R_4 \qquad (17)$$

$$L_1 = R_2 R_3 C_4 \qquad (18)$$

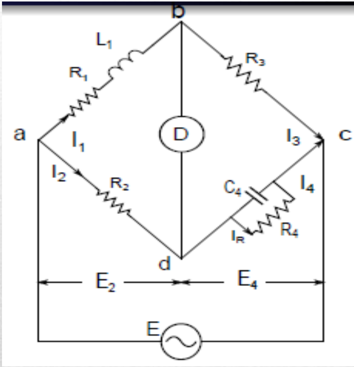


Figure 3:Maxwell's AC Bridge

# Maxwell's Inductance-Capacitance Bridge

Q factor of the coil,

$$Q = \frac{\omega L_1}{R_1} = \omega R_4 C_4 \qquad (19)$$

*Advantages:*

i) Two balance equations are independent if $R_4$ and $C_4$ are chosen as variable

ii) Independent of frequency

iii) Simple expression of $R_1$ and $L_1$

iv) Useful for wide range measurement

*Disadvantages:*

i) Expensive variable capacitor

ii) Low $Q$ factor ( around $1 < Q < 10$)

# Hay's Bridge

$L_1$=Unknown inductance with resistance $R_1$ $R_2$, $R_3$, $R_4$= Known non-inductive resistance
$C_4$= Standard capacitance
At balance condition of the bridge,

$$(R_1 + j\omega L_1)(R_4 - j/\omega C_4) = R_2R_3 \qquad (20)$$

$$R_1R_4 + \frac{L_1}{C_4} + j\omega L_1R_4 - \frac{jR_1}{\omega C_4} = R_2R_3 \qquad (21)$$

Equating real and imaginary part of equation (21)

$$R_1R_4 + L_1/C_4 = R_2R_3 \qquad (22)$$

$$L_1 = \frac{R_1}{\omega^2 R_4 C_4} \qquad (23)$$

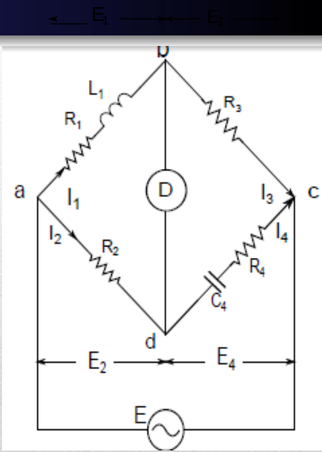


Figure 4:Hay's Bridge

# Hay's Bridge

Solving equation (22) yields

$$L_1 = \frac{R_2R_3C_4}{1+\omega^2C_4^2R_4^2} \qquad R_1 = \frac{\omega^2R_2R_3R_4C_4^2}{1+\omega^2C_4^2R_4^2} \tag{24}$$

Q factor of the coil,

$$Q = \frac{\omega L_1}{R_1} = 1/\omega C_4R_4 \tag{25}$$

The expression of $L_1$ is dependent of frequency. In terms of $Q$, $L_1$ can be re-expressed as

$$L_1 = \frac{R_2R_3C_4}{1+(1/Q)^2} \tag{26}$$

For high value of $Q$, the frequency dependency can be neglected, and hence $L_1 \approx R_2R_3C_4$ which is same as Maxwell's inductance capacitance bridge

# Hay’s Bridge

*Advantages:*

i) Simple expression of $Q$ and unknown inductance for high $Q > 10$

ii) Requires only low value of $R_4$ for high $Q$

*Disadvantages:*

iii) Expensive variable capacitor

iv) Not suitable for low value of $Q < 10$

# Anderson's Bridge

$L_1$= Self inductance to be measured
$r_1$= Resistance of $L_1$
$R_1$= Resistance connected in series with $L_1$
$r, R_2, R_3, R_4$ = non-inductive resistances
$C$= fixed standard capacitor.
At balance

$$I_1 = I_3 \tag{27}$$

$$I_2 = I_4 + I_c \tag{28}$$

and

$$I_1 R_3 = I_c \frac{1}{j\omega C} \Rightarrow I_c = jI_1 \omega C R_3 \tag{29}$$

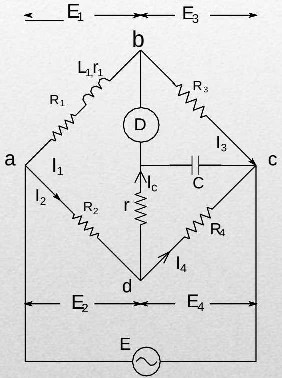


Figure 5:Anderson's ac bridge

# Anderson's Bridge

Another balance equations

$$E_1 = E_2 + I_c r \tag{30}$$

$$I_c(r + \frac{1}{j\omega C}) = (I_2 - I_c)R_4 \tag{31}$$

From equation (30),

$$I_1(R_1 + r_1 + j\omega L_1) = I_2R_2 + I_c r \tag{32}$$

$$\Rightarrow I_1(R_1 + r_1 + j\omega L_1 - j\omega CR_3 r) = I_2R_2 \tag{33}$$

and from equation (31)

$$jI_1\omega CR_3(r + \frac{1}{j\omega C}) = (I_2 - jI_1\omega CR_3)R_4 \tag{34}$$

$$\Rightarrow I_2 = \left[\frac{R_3}{R_4} + \frac{j\omega CR_3 r}{R_4} + j\omega CR_3\right] I_1 \tag{35}$$

# Anderson's Bridge

Using the value of $I_2$ from equation (35) in equation (33) results

$$I_1(R_1 + r_1 + j\omega L_1 - j\omega CR_3 r) = I_1 \left( \frac{R_3}{R_4} + \frac{j\omega CR_3 r}{R_4} + j\omega CR_3 \right) R_2 \quad (36)$$

$$\Rightarrow (r_1 + R_1) + j(\omega L_1 - \omega CR_3 r) = \frac{R_2R_3}{R_4} + j\left( \frac{\omega CR_2R_3 r}{R_4} + \omega CR_2R_3 \right) \quad (37)$$

Equating real part of equation (37)

$$r_1 + R_1 = \frac{R_2R_3}{R_4} \Rightarrow r_1 = \frac{R_2R_3}{R_4} - R_1 \quad (38)$$

similarly, equating imaginary part of equation (37)

$$\omega L_1 - \omega CR_3 r = \frac{\omega CR_2R_3 r}{R_4} + \omega CR_2R_3 \quad (39)$$

$$\Rightarrow L_1 = C\frac{R_3}{R_4}\left[r(R_2 + R_4) + R_2R_4\right] \quad (40)$$

# Anderson's Bridge

*Advantages:*

*i)* $r_1$ and $r$ are independent

ii) Fixed capacitor

iii) May be used for accurate determination of capacitance in terms of inductance

*Disadvantages:*

i) Bridge is complex

ii) Additional junction points increase the difficulty of shielding the bridge

This method is capable of precise measurements of inductances over a wide range of values from a few micro-henrys to several henrys and is one of the commonest and the best bridge methods.

# De-Sauty Bridge

Consider figure (6)(a) where,

$C_1$= Capacitor whose capacitance to be measured

$C_2$= Standard capacitor

$R_1$= resistance connected in series with $C_1$

$R_2$, $R_4$= non-inductive resistances

At balance

$$Z_1 Z_4 = Z_2 Z_3 \tag{41}$$

$$\Rightarrow \frac{1}{j\omega C_1}.R_4 = \frac{1}{j\omega C_2}.R_3 \tag{42}$$

$$\Rightarrow C_1 = C_2.\frac{R_4}{R_3} \tag{43}$$

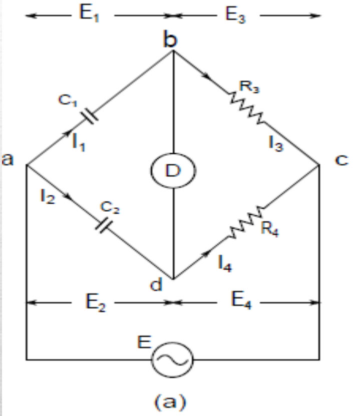


Figure 6:De-Sauty ac bridge

Though the circuit is simple, it is used only for loss-less capacitor.

# Modified De-Sauty Bridge

The bridge is modified then by De-Sauty as shown in figure (7) where, $r_1$ and $r_2$ are the loss component of capacitor $C_1$ and $C_2$ respectively.
Here, at balance condition

$$Z_1 Z_4 = Z_2 Z_3 \quad (44)$$

$$(R_1 + r_1 - j\frac{1}{\omega C_1})R_4 = (R_2 + r_2 - j\frac{1}{\omega C_2})R_3 \quad (45)$$

Equating the imaginary and real part of equation (45) results

$$\frac{R_4}{\omega C_1} = \frac{R_3}{\omega C_2} \Rightarrow \frac{C_1}{C_2} = \frac{R_4}{R_3} \quad (46)$$

$$(R_1 + r_1)R_4 = (R_2 + r_2)R_3 \Rightarrow \frac{R_2 + r_2}{R_1 + r_1} = \frac{R_4}{R_3} \quad (47)$$

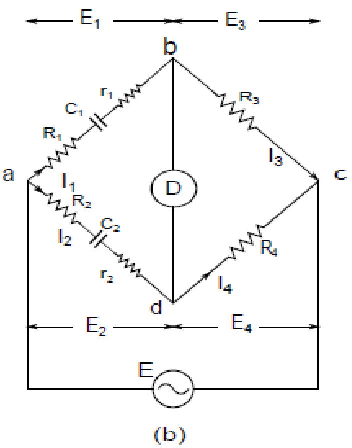


Figure 7:Modified De-Sauty bridge

# Modified De-Sauty Bridge

By using equations (46) and (47)

$$\frac{C_1}{C_2} = \frac{R_2 + r_2}{R_1 + r_1} = \frac{R_4}{R_3} \qquad (48)$$

Dissipation factor of the capacitor

$$C_1 : D_1 = \omega C_1 r_1 \qquad C_2 : D_2 = \omega C_2 r_2$$

From equation (48) it can be written that

$$C_2 r_2 - C_1 r_1 = C_1 R_1 - C_2 R_2 \Rightarrow \omega C_2 r_2 - \omega C_1 r_1 = \omega(C_1 R_1 - C_2 R_2)$$

$$\Rightarrow D_2 - D_1 = \omega(C_1 R_1 - C_2 R_2)$$

$$\Rightarrow D_2 - D_1 = \omega C_2\left(\frac{R_1 R_4}{R_3} - R_2\right) \qquad (49)$$

Therefore, dissipation factor of one capacitor can be obtained if it is known of other. However, due to difference between $\frac{R_1 R_4}{R_3}$ and $R_2$, this method does not give accurate measurement of dissipation factor.

# Schering Bridge

$C_1$= Capacitor whose capacitance to be measured
$r_1$= loss component of $C_1$
$C_2$= Standard capacitor
$R_3$= Standard non-inductive resistor

$R_4$= non-inductive variable resistor
$C_2$= standard variable capacitor
At balance condition

$$Z_1 Z_4 = Z_2 Z_3 \quad (50)$$

$$\Rightarrow \left(r_1 + \frac{1}{j\omega C_1}\right) \cdot \frac{R_4}{1 + j\omega C_4 R_4} = \frac{1}{j\omega C_2} R_3$$

$$\Rightarrow r_1 R_4 - j\frac{R_4}{\omega C_1} = -j\frac{R_3}{\omega C_2} + \frac{R_3 R_4 C_4}{C_2} \quad (51)$$

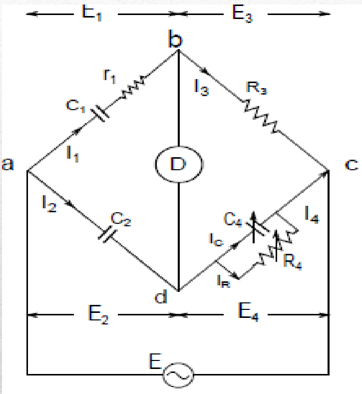


Figure 8:Schering AC bridge

# Schering Bridge

Equating real part of equation (51),

$$r_1R_4 = \frac{R_3R_4C_4}{C_2} \Rightarrow r_1 = \frac{R_3C_4}{C_2} \tag{52}$$

Equating imaginary part of equation (51),

$$\frac{R_4}{\omega C_1} = \frac{R_3}{\omega C_2} \Rightarrow C_1 = C_2\frac{R_4}{R_3} \tag{53}$$

So, the value of $r_1$ and $C_1$ can be obtained independently if $R_4$ and $C_4$ are variable.

Dissipation factor:

$$D = \omega C_1 r_1 = \omega C_2 \frac{R_4}{R_3} \frac{R_3C_4}{C_2} = \omega C_4R_4 \tag{54}$$

# Wien's Bridge

The most important bridge to determine frequency in terms of various bridge elements is Wien bridge as shown in figure (9) that is describe below. This bridge can be used as notch filter as well as audio and RF oscillators.

At balance condition,

$$\left(\frac{R_1}{1+j\omega C_1 R_1}\right) R_4 = \left(R_2 - \frac{j}{\omega C_2}\right) R_3 \qquad (55)$$

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} + j\left(\omega C_1 R_2 - \frac{1}{\omega C_2 R_1}\right) \qquad (56)$$

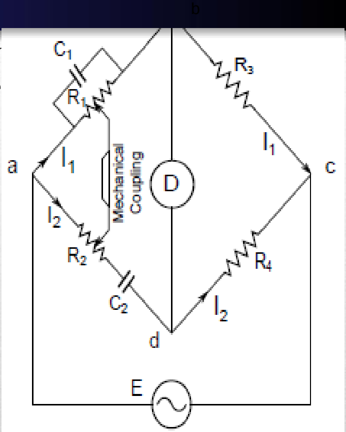


Figure 9: Wien's bridge

Equating real part yields

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} \qquad (57)$$

# Wien's Bridge

Equating imaginary part yields

$$\omega C_1 R_2 - \frac{1}{\omega C_2 R_1} = 0 \Longrightarrow \omega = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{R_1 R_2 C_1 C_2}} \qquad (58)$$

In most Wien's bridge, the components are so chosen that $R_1 = R_2 = R$ and $C_1 = C_2 = C$. Therefore, the frequency becomes,

$$f = \frac{1}{2\pi RC} \qquad (59)$$

Campbell's bridges is self study and Assignment

# Limitations of Wheatstone Bridge

Wheatstone bridge is not suitable for low and high resistance due to the following reasons:

- It can measure from few Ω to several MΩ.
- Upper limit is set by reducing the sensitivity to unbalance caused by resistance values.
- Upper limit can be extended by increasing EMF that causes heat inaccuracy due to leakage out of insulation.
- Contact resistance presents a source of uncertainty that is difficult to overcome.

# Precision Measurement of Medium Resistance

- In case of medium resistance measurements with Wheatstone bridge, the following factors should be taken into consideration:
- Resistance of connecting wire leads to Thermoelectric effects and Temperature effects.
- Contact resistances
- In precision measurements, the accurate comparisons are made on an equal ratio bridge with a fixed standard nominally equal to the resistance under test. <u>The problem is further reduced by determining the exact ratio of *R* to *S* or the difference between them.</u>

# Carey Foster Slide Wire Bridge

Resistance $P$ and $Q$ are adjusted so that the ratio $P/Q$ is approximately equal to the ratio $R/S$. Let, balanced point $d$ is obtained at a distance $l_1$ as shown in

Figure below.

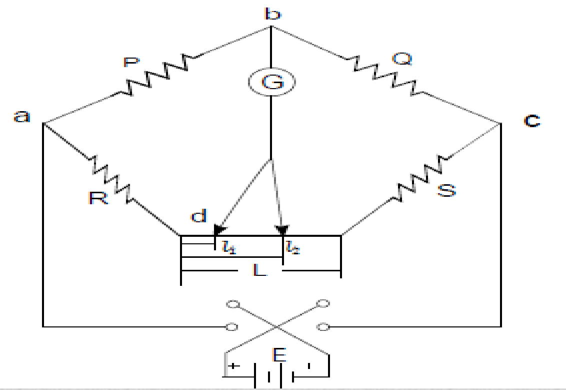


Figure : Carey Foster bridge to measure medium resistance

# Carey Foster Slide Wire Bridge

Therefore at balance condition,

$$\frac{P}{Q} = \frac{R + l_1 r}{S + (L - l_1) r} \tag{34}$$

$$\frac{P}{Q} + 1 = \frac{R + l_1 r + S + (L - l_1) r}{S + (L - l_1) r} = \frac{R + S + Lr}{S + (L - l_1) r} \tag{35}$$

where, $r$ is the resistance per unit length of the slide wire. Then $R$ and $S$ are interchanged and balanced obtained again at a distance $l_2$. Similarly for second balance point,

$$\frac{P}{Q} = \frac{S + l_2 r}{R + (L - l_2) r} \tag{36}$$

$$\frac{P}{Q} + 1 = \frac{S + l_2 r + R + (L - l_2) r}{R + (L - l_2) r} = \frac{R + S + Lr}{R + (L - l_2) r} \tag{37}$$

# Carey Foster Slide Wire Bridge

From equation (35) and (37)

$$S + (L - l_1)r = R + (L - l_2)r \quad (38)$$

$$S - R = (l_1 - l_2)r \quad (39)$$

Thus the difference between $S$ and $R$ is obtained form the resistance per unit length of the slide wire and the difference $(l_1 - l_2)$ between the two slide wire lengths at balance.

The slide wire is calibrated, .i.e. $r$ is obtained by shunting either $S$ or $R$ by a known resistance and again determining the difference in length $(l_1^{\prime} - l_2^{\prime})$.

Suppose, $S$ is known and $S^{\prime}$ is its value when shunted by unknown resistance.

# Carey Foster Slide Wire Bridge

After shunting $S$ equation (39) becomes

$$S' - R = (l'_1 - l'_2)r \tag{40}$$

Therefore, equation (39) and (40) yield

$$\frac{S-R}{l_1-l_2} = \frac{S'-R}{l'_1-l'_2} \tag{41}$$

$$R = \frac{S(l'_1-l'_2)-S'(l_1-l_2)}{l'_1-l'_2-l_1+l_2} \tag{42}$$

The equation (42) shows that this method gives the direct comparison between $R$ and $S$ in terms in terms of length only and the Resistances of $P$ and $Q$ contact, and the Resistances of connecting leads are eliminated.