Review of Multiple Comparison Correction methods

- From Bonferroni procedure to Efron's Local FDR -

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1 Introduction

Ever since works on multiple comparison problem (MCP) were started by Tukey (1949) and Scheffé (1953), who have both utilized $t-$statistic for pairwise sample mean comparisons, numerous procedures for controlling family-wise error rate (FWER) has been developed. Achieving higher testing power while controlling FWER for the same level has especially been a major concern in the field of MCP, rulling the field for almost 35 years before Benjamini and Hochberg suggested the concept of false discovery rate (FDR) in 1995.
In this brief report, I would like to review two major concepts in MCP - FWER, FDR - and some of the selected procedures which controls FWER or FDR in certain level. Historical meanings and important proofs were described to better highlight modifications and developments of a statistical concept over time. Simple simulation of multiple testing was also conducted to compare different methods of MCP correction.

1.1 Multiple comparison problem

Multiple comparison problem (MCP), also known as multiple hypothesis tesing problem or multiplicity problem, is a problem which occurs when a family-level hypothesis testing is made based on a set of individual hypotheses. Due to the fact that a family-level type I error rate (family-wise error rate; FWER) is always greater than experiment-level type I error ($\because 1-(1-\alpha)^m > \alpha$, if $m \geq 2$), an appropriate correction for the error control of family-level hypothesis testing is needed.

1.2 Regression dependency of hypotheses

For some family-level error controlling procedure, assumption on dependence structure is needed. Let I_0 be a subset of indices of hypotheses in a set of tests. If there exists no regression dependence between non-rejected test statistics and true-null test statistics, then the hypotheses in the set are said to be independent. If there is a positive regression dependence between non-rejected test statistics and true-null test statistics, then the hypotheses in the set are said to have positive regression dependency on each one from a subset I_0 ($PRDS$ on I_0). Benjamini and Yekutieli (2001) defined $PRDS$ on I_0 as follows: "For any increasing set D, and for each $i \in I_0$, $P(X \in D|X_i = x)$ is non-decreasing in x". More intuitive explanation uses p-values. For each $i \in I_0$, If p-values increases, then probability of corresponding null hypothesis is true does not decreases.

1.3 Notations

	Declared as non-significant	Declared as significant	Total
True H_0	U	V	m_0
False H_0	T	S	$m_1 = m - m_0$
Total	$m - R$	R	m

Table 1: notation of the number of hypotheses in corresponding to each cell

Notations used to define family-level error concepts mathematically are as described on Table 1. This notation follows that of Benjamini and Hochberg (1995). Total m hypotheses in a set is being tested in this situation. $m_0 \leq m$ hypotheses are true null. Note that R, the number of rejected hypotheses, is an observable random variable while U, V, T, S are unobservable random variables.

2 Family-Wise Error Rate (FWER)

Family-wise error rate is

2.1 Bonferroni procedure

2.2 Holm-Bonferroni procedure

2.3 Hochberg procedure

3 False Discovery Rate (FDR)

3.1 Benjamini-Hochberg (B-H) procedure

3.2 Benjamini-Yekutieli (B-Y) procedure

3.3 Brief introduction to local FDR

4 Discussions - Simulation and Comparisons

5 References

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