Let $n \in \mathbb{N}, \mathbf{x} \in \mathbb{N}^n$ and

$$\psi(t) = 2^n \prod_{k=1}^{n} \cos(x_k t) = \sum_{\sigma \in \{-1,1\}^n} \cos t \langle \mathbf{x}, \sigma \rangle \tag{1}$$

where the last equality is easily verified using the sum of angles formula. Put

$$c_k = |\{\sigma \in \{-1,1\}^n \mid \{\langle \mathbf{x}, \sigma \rangle = k\}\}| \tag{2}$$

to be the number of partitions that sum to k, then recalling the cosine being even and that for every partition there exists another partition of the negative sum,

$$\sum_{\sigma \in \{-1,1\}^n} \cos t \langle \mathbf{x}, \sigma \rangle = \sum_{k=-\infty}^{\infty} c_k e^{itk} \tag{3}$$

is simply the Fourier series of ψ. Observe that this sum wrt k is actually finite. Define a sum of N summands as

$$s_N = \frac{1}{N} \sum_{m=1}^{N} \psi\left(\frac{2\pi m}{N}\right) = \frac{1}{N} \sum_{k=-\infty}^{\infty} c_k \sum_{m=1}^{N} e^{ik\frac{2\pi m}{N}} \tag{4}$$

since $\sum_{m=1}^{N} e^{ik\frac{2\pi m}{N}}$ is sum of roots of unity, then it equals zero unless N divides k where on that case it equals N. So we get

$$\frac{1}{N} \sum_{m=1}^{N} \psi\left(\frac{2\pi m}{N}\right) = \sum_{k=-\infty}^{\infty} c_{kN} \tag{5}$$

concluding that s_N is simply the number of partitions that sum to a sum that is divisible by N. Note that a zero sum is divisible by all numbers. So if one knows a number that does not divide any of the result of all possible partitions, then the number of zero partitions can be calculated in N evaluations of ψ at the N'th roots of unity. Also, it should be noted that a zero partition exists if and only if $s_N \neq 0$ for all N. So trying various N would typically hit zero very fast if the set is unpartitioned. Otherwise, we may select the lowest N that we know that doesn't divide any possible partition. Similar development considering the Taylor series of ψ shows that the convergence of s_N towards $c_0 \left(= 2^n \int_0^\pi \prod_{k=1}^n \cos(x_k t)\, dt\right)$ is always exponential in N (TBD).

Now, modify the definition of ψ such that

$$\psi(z) = \prod_{k=1}^{n} \left(z^{x_k} + z^{-x_k}\right) \tag{6}$$

and we're interested at

$$\int_0^\pi \psi\left(e^{it}\right) dt = \int_0^\pi \prod_{k=1}^{n} \left(e^{itx_k} + e^{-itx_k}\right) dt = 2^n \int_0^\pi \prod_{k=1}^{n} \cos(x_k t)\, dt \tag{7}$$

combining (1) and (6) we see that

$$\psi(z) = \sum_{\sigma\in\{-1,1\}^n} \left[z^{\langle \mathbf{x},\sigma\rangle} + z^{-\langle \mathbf{x},\sigma\rangle} \right] \tag{8}$$

differentiating

$$\psi'(z) = \sum_{\sigma\in\{-1,1\}^n} \left[\langle \mathbf{x},\sigma\rangle z^{\langle \mathbf{x},\sigma\rangle - 1} - \langle \mathbf{x},\sigma\rangle z^{-\langle \mathbf{x},\sigma\rangle - 1} \right] \tag{9}$$

$$\implies z\psi'(z) = \sum_{\sigma\in\{-1,1\}^n} \langle \mathbf{x},\sigma\rangle \left[z^{\langle \mathbf{x},\sigma\rangle} - z^{-\langle \mathbf{x},\sigma\rangle} \right] \tag{10}$$

differentiating again

$$\psi'(z) + z\psi''(z) = \sum_{\sigma\in\{-1,1\}^n} \frac{1}{z} \langle \mathbf{x},\sigma\rangle^2 \left[z^{\langle \mathbf{x},\sigma\rangle} + z^{-\langle \mathbf{x},\sigma\rangle} \right] \tag{11}$$

setting $z = 1$

$$\implies \psi'(1) + \psi''(1) = \sum_{\sigma\in\{-1,1\}^n} 2 \langle \mathbf{x},\sigma\rangle^2 \tag{12}$$

therefore, we can pick N as the smallest number that does not divide $\psi'(1) + \psi''(1)$.