Let $n \in \mathbb{N}, \mathbf{x} \in \mathbb{N}^n$ and consider the formula $2\cos a\cos b = \cos(a+b) + \cos(a-b)$ and the cosine being even function to see that:

$$\psi(t) = 2^n \prod_{k=1}^n \cos(x_k t) = \sum_{\sigma \in \{-1,1\}^n} \cos t \langle \mathbf{x}, \sigma \rangle = 2 \sum_{\sigma \in \{-1,1\}^n} e^{it \langle \mathbf{x}, \sigma \rangle}$$
(1)

where $\langle \mathbf{x}, \sigma \rangle = \sum_{k=1}^{n} \sigma_k x_k$ and counting the number of $\sigma \in \{-1, 1\}^n$ satisfying $\langle \mathbf{x}, \sigma \rangle = 0$ is a #P problem. We write down the following sum just for fun and substitute (1) in it:

$$S = \frac{1}{2n+2} \sum_{m=1}^{n+1} \psi \left(2\pi \frac{m}{n+1} + i \ln 2 \right) = \sum_{\sigma \in \{-1,1\}^n} 2^{-\langle \mathbf{x}, \sigma \rangle} \sum_{m=1}^{n+1} \frac{1}{n+1} e^{2\pi i \frac{m}{n+1} \langle \mathbf{x}, \sigma \rangle}$$
(2)

the summation of roots of unity equals zero iff n+1 does not divide $\langle \mathbf{x}, \sigma \rangle$, and if it does divide then it sums to n+1. Using this fact and denoting the number of partitions that sum to k by $c_k = |\{\sigma \in \{-1,1\}^n \mid \langle \mathbf{x}, \sigma \rangle = k\}|$, we get

$$S = \sum_{k=-\infty}^{\infty} c_{k(n+1)} 2^{-(n+1)k}$$
(3)

recalling that $\sum_{k=-\infty}^{\infty} c_k = 2^n$, we have proved that the number of zero partitions in **x**

$$\left| \frac{2^{n-1}}{n+1} \sum_{m=1}^{n+1} \prod_{k=1}^{n} \cos \left(2\pi \frac{m}{n+1} + i \ln 2 \right) \right| \tag{4}$$