

Let $n \in \mathbb{N}$, $\mathbf{x} \in \mathbb{N}^n$ and

$$\psi(t) = 2^n \prod_{k=1}^n \cos(x_k t) = \sum_{\sigma \in \{-1,1\}^n} \cos t \langle \mathbf{x}, \sigma \rangle \quad (1)$$

where the last equality is easily verified using the sum of angles formula. Put

$$c_k = |\{\sigma \in \{-1,1\}^n \mid \langle \mathbf{x}, \sigma \rangle = k\}| \quad (2)$$

to be the number of partitions that sum to k , then recalling the cosine being even and that for every partition there exists another partition of the negative sum,

$$\sum_{\sigma \in \{-1,1\}^n} \cos t \langle \mathbf{x}, \sigma \rangle = \sum_{k=-\infty}^{\infty} c_k e^{itk} \quad (3)$$

is simply the Fourier series of ψ . Observe that this sum wrt k is actually finite. Define a sum of N summands as

$$s_N = \frac{1}{N} \sum_{m=1}^N \psi\left(\frac{2\pi m}{N}\right) = \frac{1}{N} \sum_{k=-\infty}^{\infty} c_k \sum_{m=1}^N e^{ik \frac{2\pi m}{N}} \quad (4)$$

since $\sum_{m=1}^N e^{ik \frac{2\pi m}{N}}$ is sum of roots of unity, then it equals zero unless N divides k where on that case it equals N . So we get

$$\frac{1}{N} \sum_{m=1}^N \psi\left(\frac{2\pi m}{N}\right) = \sum_{k=-\infty}^{\infty} c_{kN} \quad (5)$$

concluding that s_N is simply the number of partitions that sum to a sum that is divisible by N . Note that a zero sum is divisible by all numbers. So if one knows a number that does not divide any of the result of all possible partitions, then the number of zero partitions can be calculated in N evaluations of ψ at the N 'th roots of unity. Also, it should be noted that a zero partition exists if and only if $s_N \neq 0$ for all N . So trying various N would typically hit zero very fast if the set is unpartitioned. Otherwise, we may select the lowest N that we know that doesn't divide any possible partition. Similar development considering the Taylor series of ψ shows that the convergence of s_N towards c_0 ($= 2^n \int_0^\pi \prod_{k=1}^n \cos(x_k t) dt$) is always exponential in N (TBD).