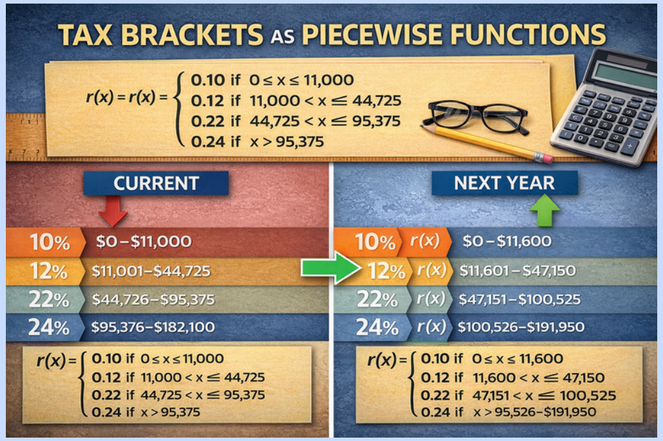
Piecewise Functions in Modelling Pricing

By Natalie Hart-Hawco

Real pricing is often nonlinear.



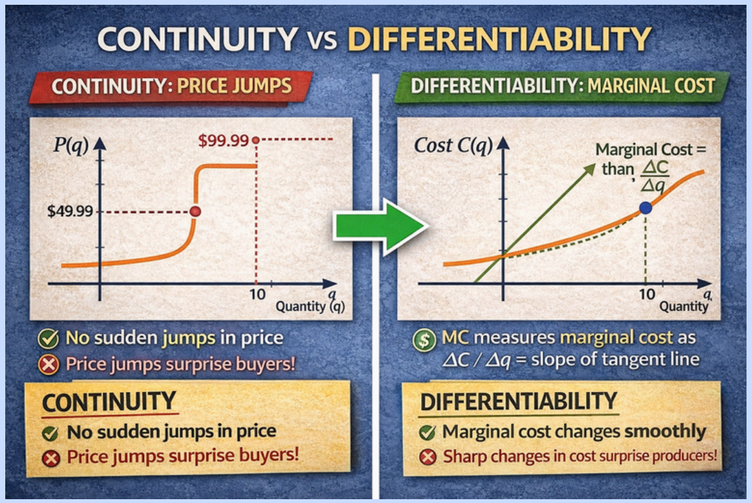
Examples include:

- Subscription tiers.
- Tax brackets.
- Bulk discounts.

How to Model Pricing

Continuity

Showcases sudden price jumps of piecewise functions.



Differentiability

Showcases the marginal change in cost of piecewise functions.

Used to interpret economic relations.

Piecewise Function

$$f(x) = \begin{cases} f_1(x), & x \in D_1, \\ f_2(x), & x \in D_2, \\ \vdots & \vdots \\ f_n(x), & x \in D_n. \end{cases}$$

Single Threshold

$$f(x) = \begin{cases} 10x, & 0 \leq x \leq 10, \\ 8x + 20, & x > 10. \end{cases}$$

Models a bulk discount with one consumption threshold.

Continuity at Threshold

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x),$$

Preventing Threshold Avoidance

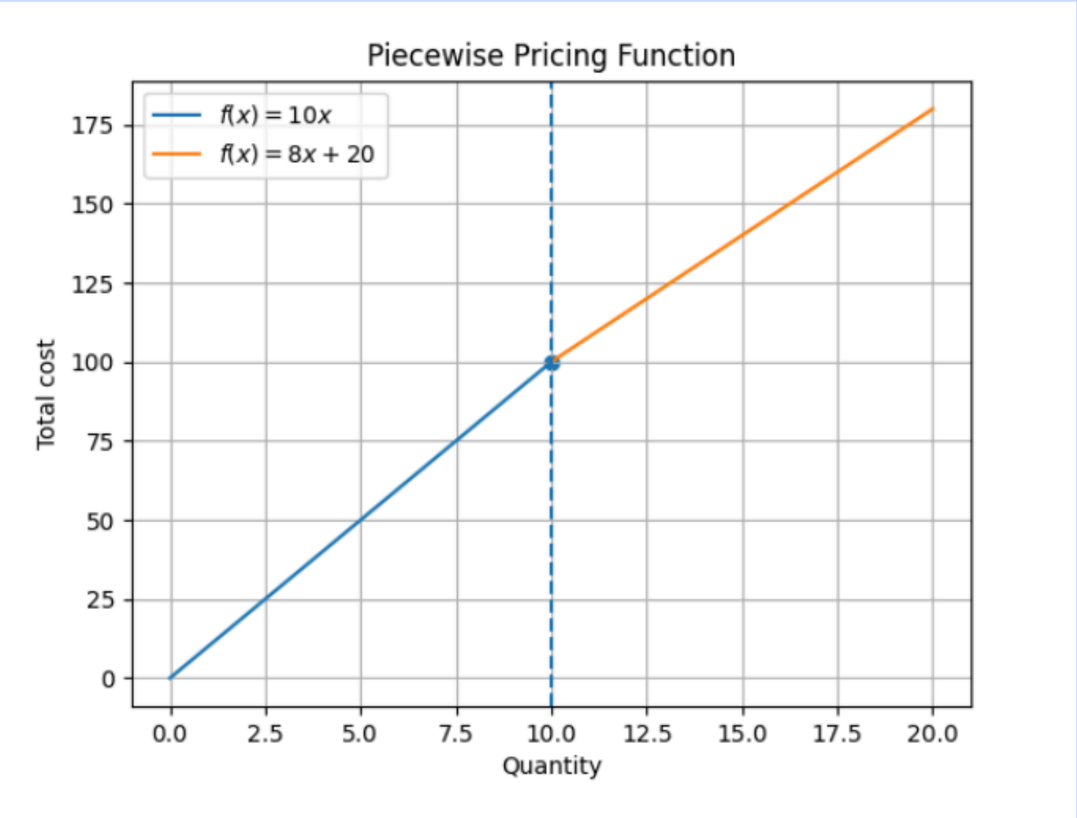
Marginal Cost

$$\lim_{x \to c^-} f'(x) \neq \lim_{x \to c^+} f'(x)$$

Creates a kink that reflects the marginal price change.

Graph

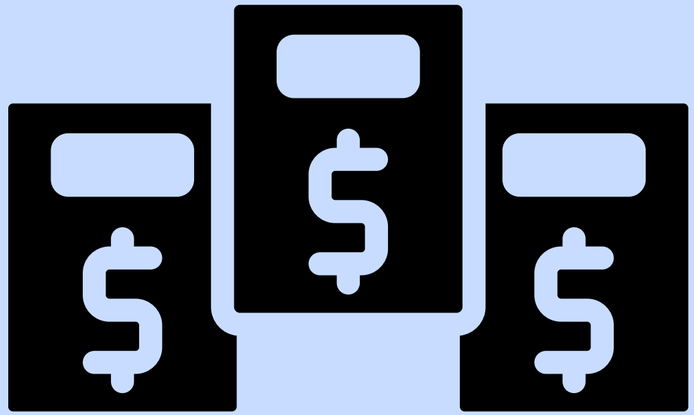
$$f(x) = \begin{cases} 10x, & 0 \leq x \leq 10, \\ 8x + 20, & x > 10. \end{cases}$$



Multiple Thresholds

$$f(x) = \begin{cases} a_1 x, & 0 \le x \le c_1, \\ a_2 x + d_1, & c_1 < x \le c_2, \\ a_3 x + d_2, & x > c_2, \end{cases} \qquad a_i,\ c_i,\ d_i > 0.$$

Tiered pricing system with two consumption thresholds.



Comparison

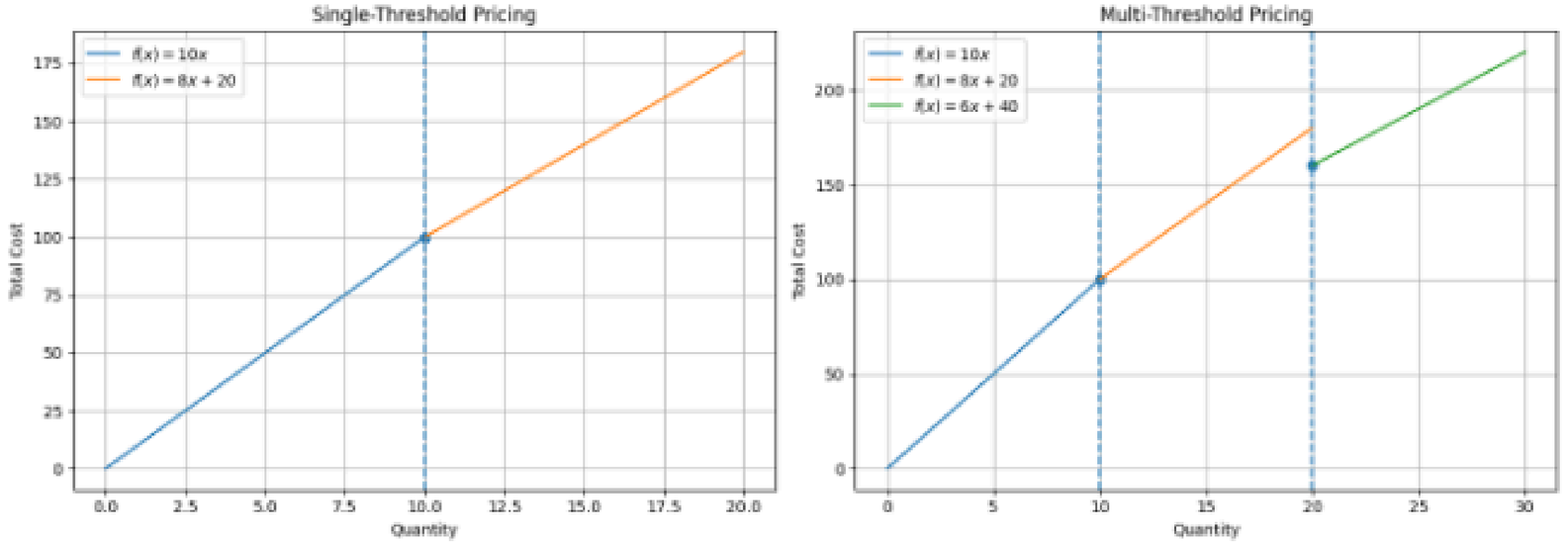


Figure 2: Single-Threshold Vs. Multiple-Thresholds

Conclusion

Piecewise functions can serve as a powerful economic tool to study the effects of tiered pricing on consumer behaviour!

Generative AI was used to create some pictures in this PowerPoint