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Completely positive maps for higher-order unraveling of Lindblad master equations

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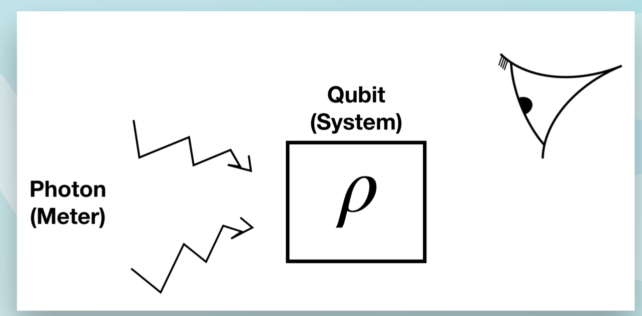
Motivation

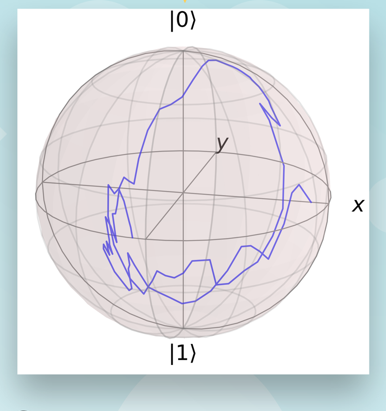
In quantum measurement, the system of interest is measured by interacting with a meter (or a bath). For example, one can measure the evolution of a qubit using photons via homodyne detection.



For continuous measurement, system's dynamics conditioned on a diffusive measurement records can be described by a quantum trajectory. The quantum trajectory can be alternatively simulated by using a map or measurement operator which has been proposed in several approaches [1-3].

Measurement records y_t

$\hat{M}_{y_t}$
Measurement operators



Quantum trajectory

However, the existing measurement operators suffer from finite time step resolution Δt, inducing errors in quantum trajectory simulation. We therefore derive a new approach which yields a measurement operator mitigating the time resolution problem.

Measurement operator properties

The measurement operator should satisfy the following properties:

✅ Agree with Lindblad master equation, i.e.,

$$\int dy \wp_{\rm ost}(y) \hat{M}_y \bullet \hat{M}_y^\dagger = e^{\Delta t \mathcal{L}\bullet} \approx (\bullet + \sum_{k=1}^{n} \frac{\Delta t^k}{k!} \mathcal{L}^k \bullet), \qquad (1)$$

where $\wp_{\rm ost}(y)$ is the "ostensible" probability and $\mathcal{L}$ is Lindblad superoperator. We will use the order of expansion n to measure this property. Noting that, in Eq. (1), one can change $\approx$ to $=$ if $n = \infty$.

✅ Complete positivity (CP): the measurement operator should guarantee the trace preserving, i.e.,

$$\int dy \wp_{\rm ost}(y) \hat{M}_y^\dagger \hat{M}_y = \hat{1} + o(\Delta t^k). \qquad (2)$$

Noting that, for $o(\Delta t^k) = 0$, the map is complete positivity.

✅ Closeness to the "exact" quantum trajectory: the exact individual trajectory ($\rho_{\rm exact}$) can be simulated in literatures for its particular measurement setting. We here calculate the "averaged" trace distance to measure this property, i.e.,

$$\langle D_j \rangle = \tfrac{1}{2} \langle {\rm Tr} |\rho_j - \rho_{\rm exact}| \rangle_{y_{\rm exact}}, \qquad (3)$$

where we average over all the exact records and ρ_j is generated by the maps.

Higher-order completely positive map

Our new measurement operator is derived from a qubit system coupled to a bosonic (harmonic oscillator) bath via a unitary evolution $\hat{U}(t+\Delta t, t)$, projected to a homodyne eigenstate $|{\rm e}_y\rangle$, i.e.,

$$\hat{M}_y = \langle {\rm e}_y | \hat{U}(t+\Delta t, t) | {\rm e}_0 \rangle.$$

To construct $\hat{M}_{\rm W}$, we intuitively select terms which make the map satisfy the first two conditions to $o(\Delta t^2)$.

Comparisons of Maps

Lindblad and CP properties

We make comparisons with 3 different existing maps which has been proposed in literatures in different intuitions, namely, the conventional Ito map ($\hat{M}_{\rm I}$) [1], Rouchon- Ralph map ($\hat{M}_{\rm R}$), having terms quadratic in y [2], and Guevara-Wiseman map ($\hat{M}_{\rm G}$), designed for high-order CP [3], to our new map ($\hat{M}_{\rm W}$). By considering the properties in Eqs. (1)-(2), we have the analytical results of order of satisfaction as the following:

Map	Lindblad $\mathcal{L}^n$: n up to 2	CP: k up to 2
$\hat{M}_{\rm I}$	❌	❌
$\hat{M}_{\rm R}$	❌	❌
$\hat{M}_{\rm G}$	❌	✅
$\hat{M}_{\rm W}$	✅	✅

Quantum trajectory closeness (qubit examples)

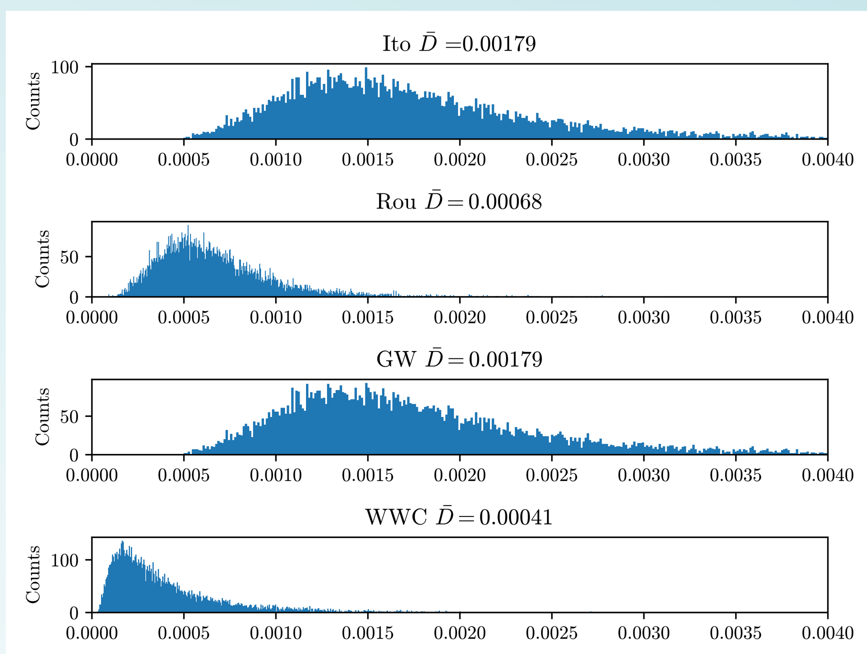
We calculate the averaged trace distance in Eq. (3) of individual trajectories generated by each map (ρ_j) compared to the trajectories generated by the exact map for two particular examples, namely qubit z-measurement with the Lindblad operator $\hat{c} \propto \hat{\sigma}_z$ and the measurement of qubit's fluorescence ($\hat{c} \propto \hat{\sigma}_-$).

Map	Qubit z-measurement	Qubit's fluorescence
$\langle D_{\rm I} \rangle$	0.15α	0.50β
$\langle D_{\rm R} \rangle$	0.09α	0.50β
$\langle D_{\rm G} \rangle$	0.15α	0.50β
$\langle D_{\rm W} \rangle$	0.04α	0.25β

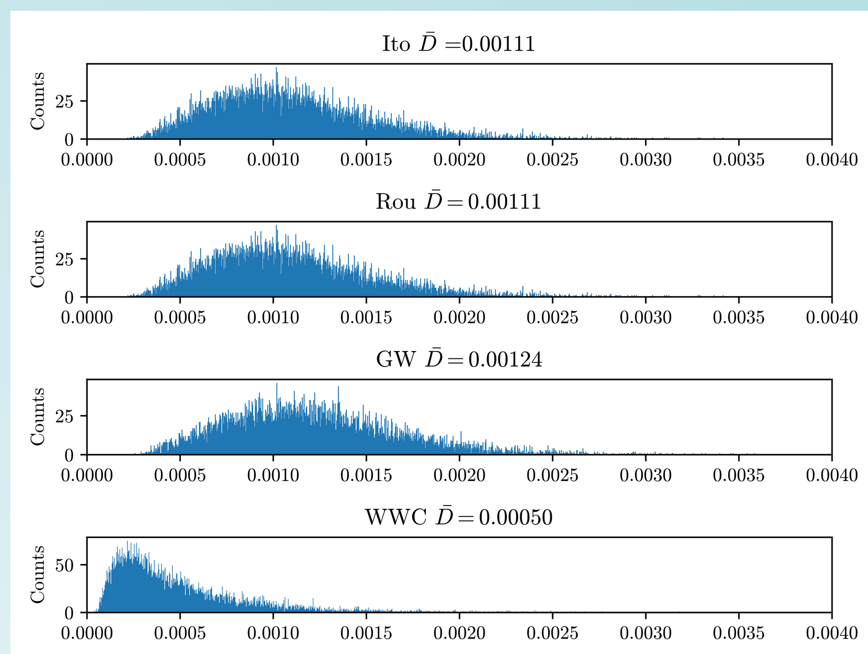
Here α, β are the first order constant with $\alpha, \beta \sim \mathcal{O}(\Delta t^{3/2})$.

Simulation results

The histograms show the numerical calculations of the trace distance of the trajectories generated by each map compared to the exact trajectories for two qubit experiments.



Qubit z-measurement



Measurement of qubit's fluorescence

Discussion

We construct the new map which satisfies analytically both the Lindblad master equation and CP to Δt^2. Interestingly, although the new map derived to satisfy the first two conditions, it gives the closest individual trajectory to the exact one both analytically and numerically despite not high order in Δt.

References

[1] H. M. Wiseman and G. J. Milburn, Quantum measurement and control (Cambridge University Press UK, 2010).
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[3] I. Guevara and H. M. Wiseman, Phys. Rev. A 102, 052217 (2020).

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