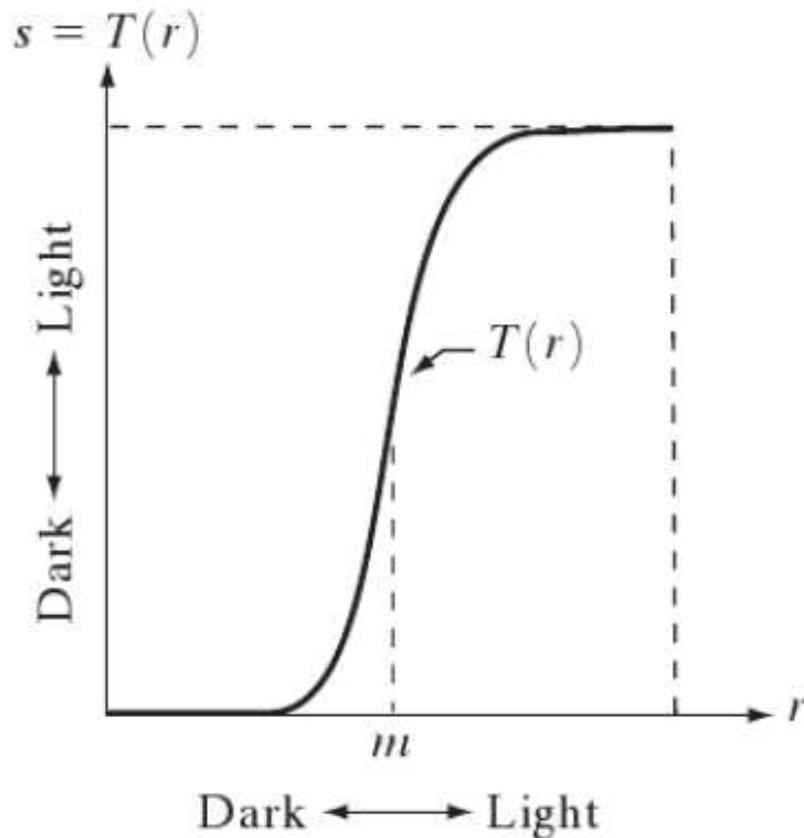


# Ajuste de contraste

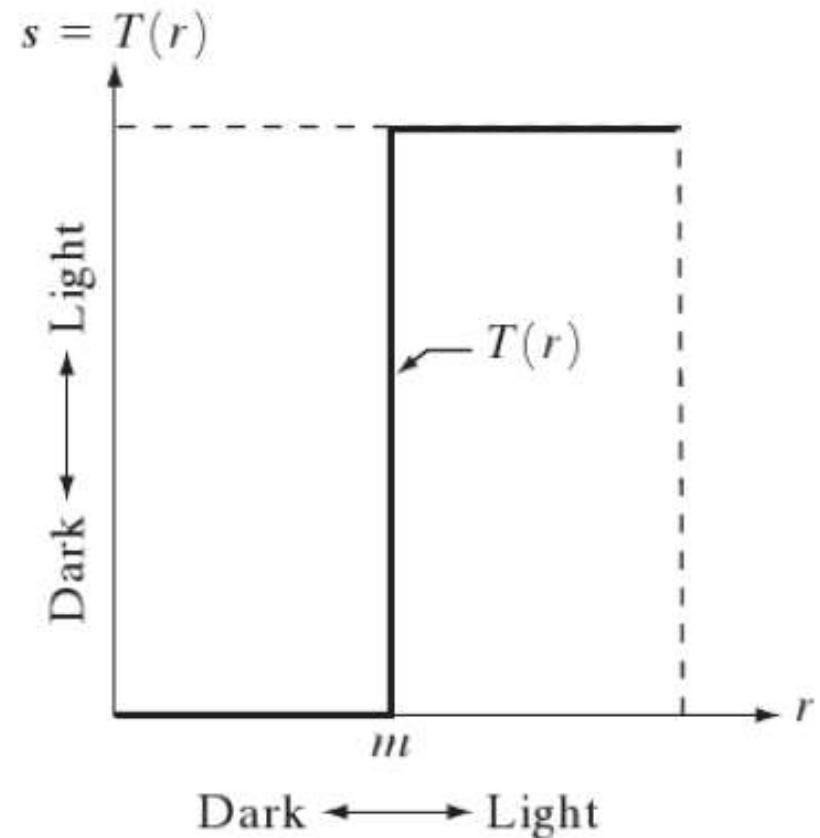
# Ajuste de contraste



Aumento de contraste

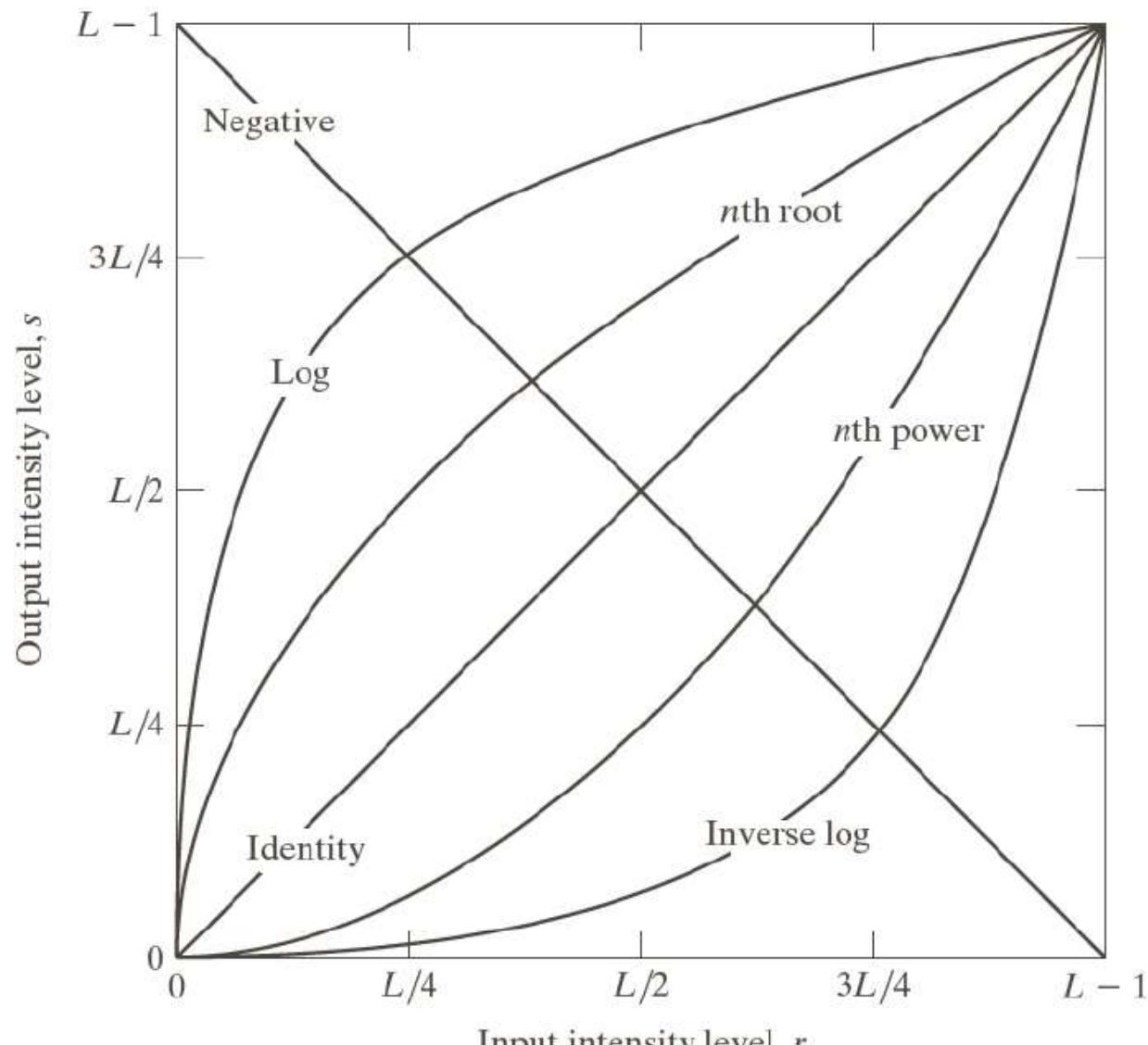
Claro -> mais claro

Escuro -> mais escuro



Caso extremo:  
Binarização da imagem

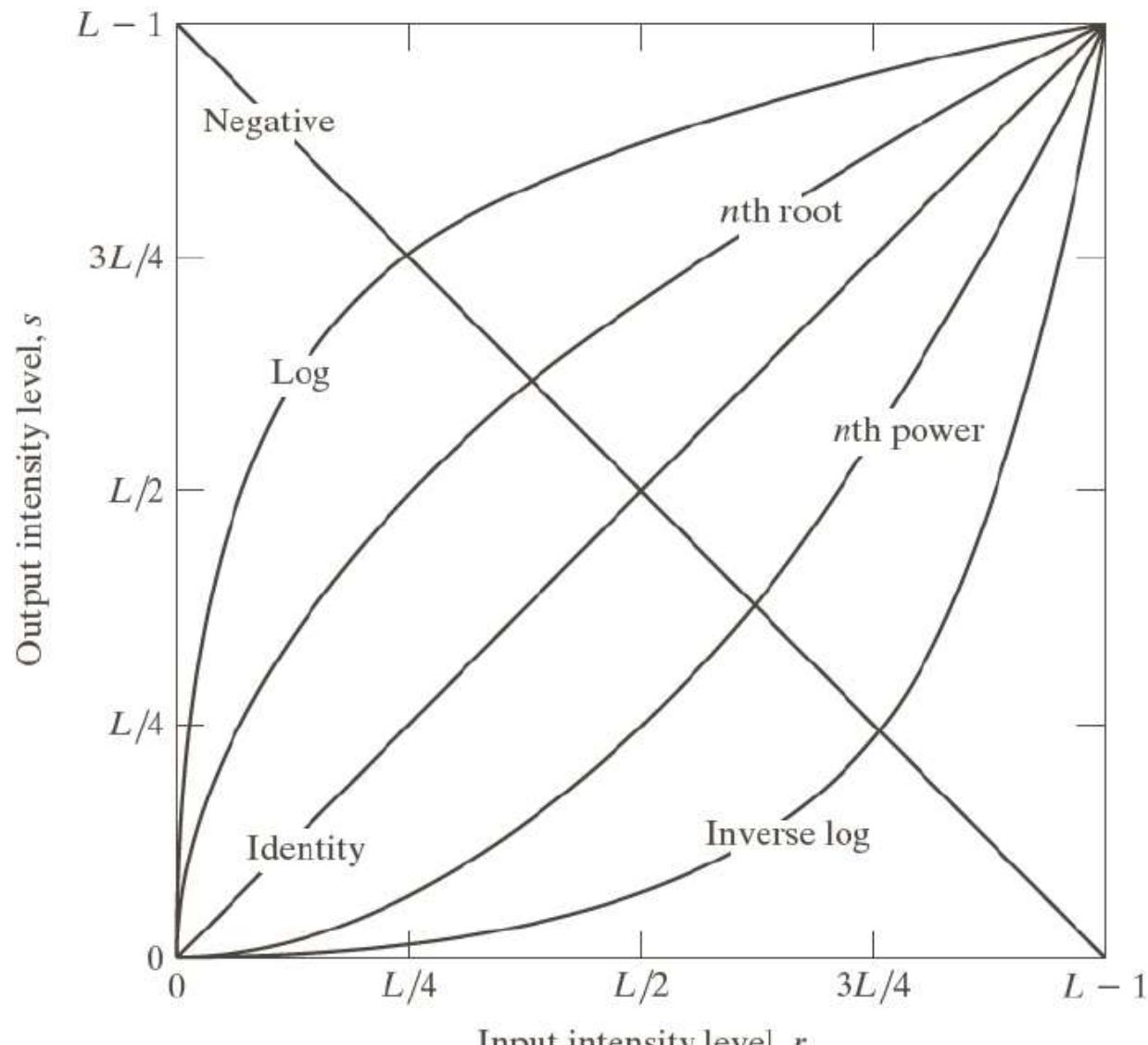
# Transformações



# Transformações

- Logaritmos, potências  $< 1$ 
  - Ajuste de contraste para imagens muito escuras
- Log inverso, potências  $> 1$ 
  - Ajuste de contraste para imagens muito claras

# Transformações



# Ajuste de contraste

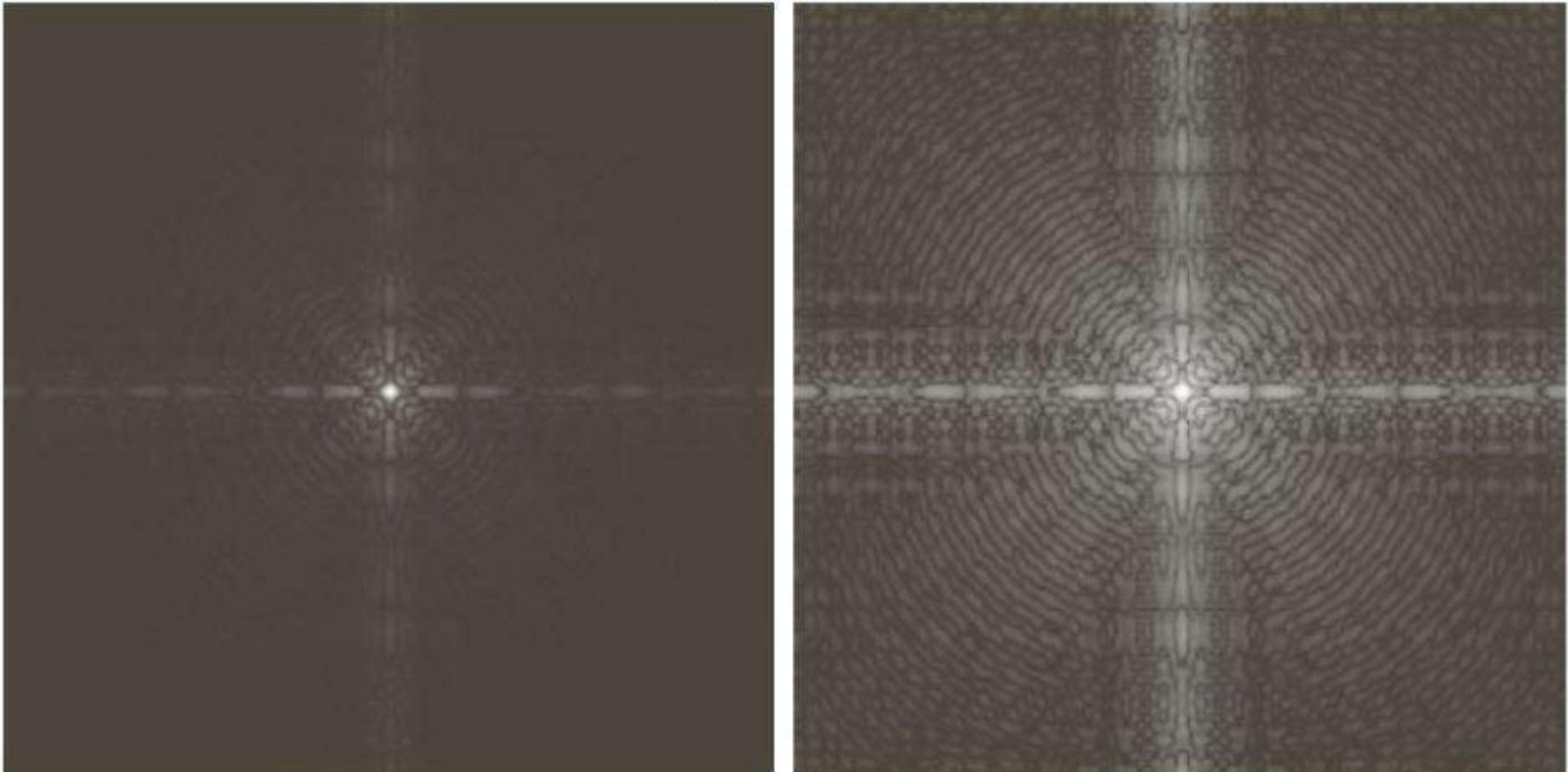
a b

**FIGURE 3.5**

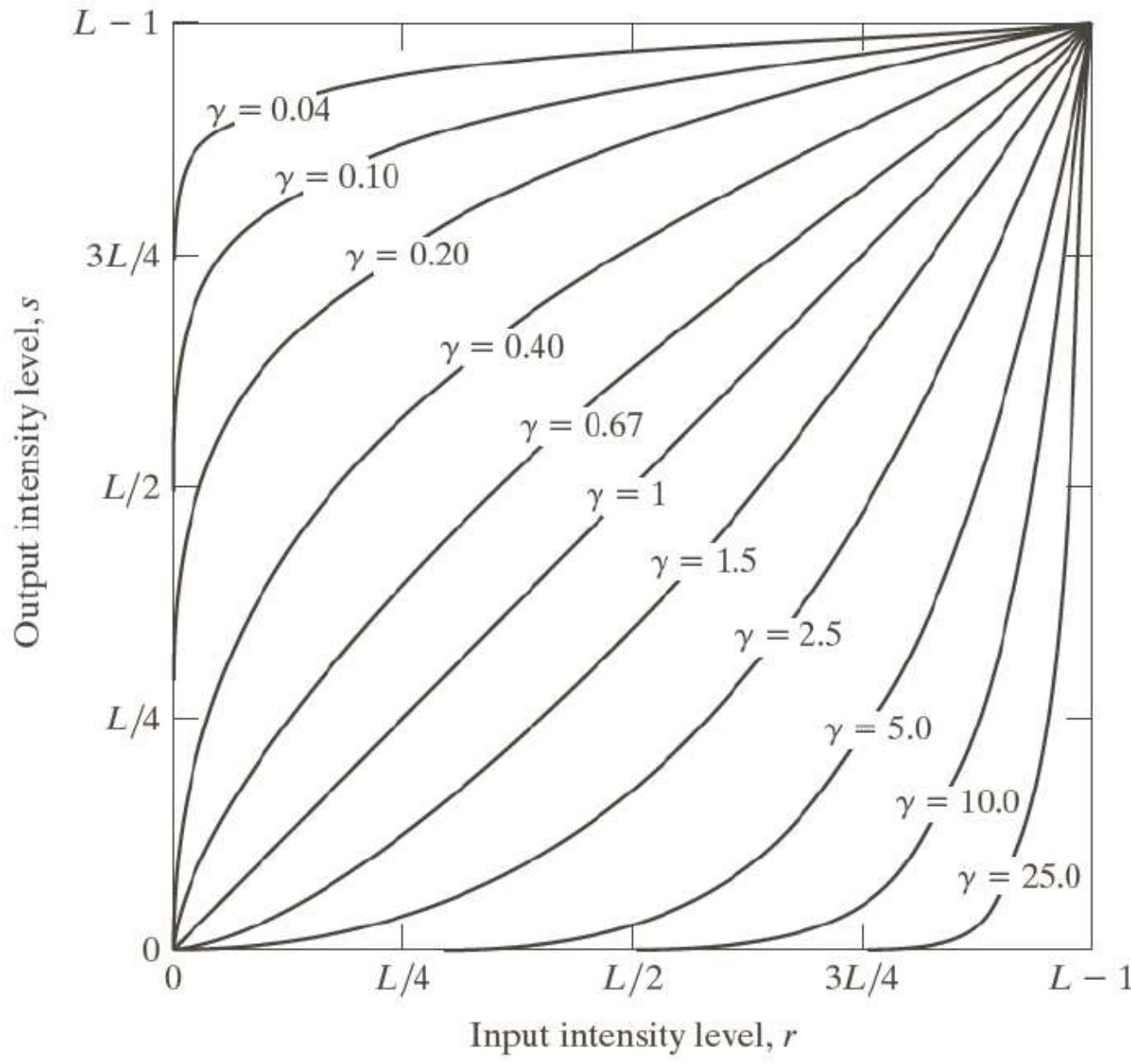
(a) Fourier spectrum.  
(b) Result of applying the log transformation in Eq. (3.2-2) with  $c = 1$ .

$$s = c \cdot \log(1+r)$$

Transformação logarítmica ( $c=1$ )



# Correção *gamma*



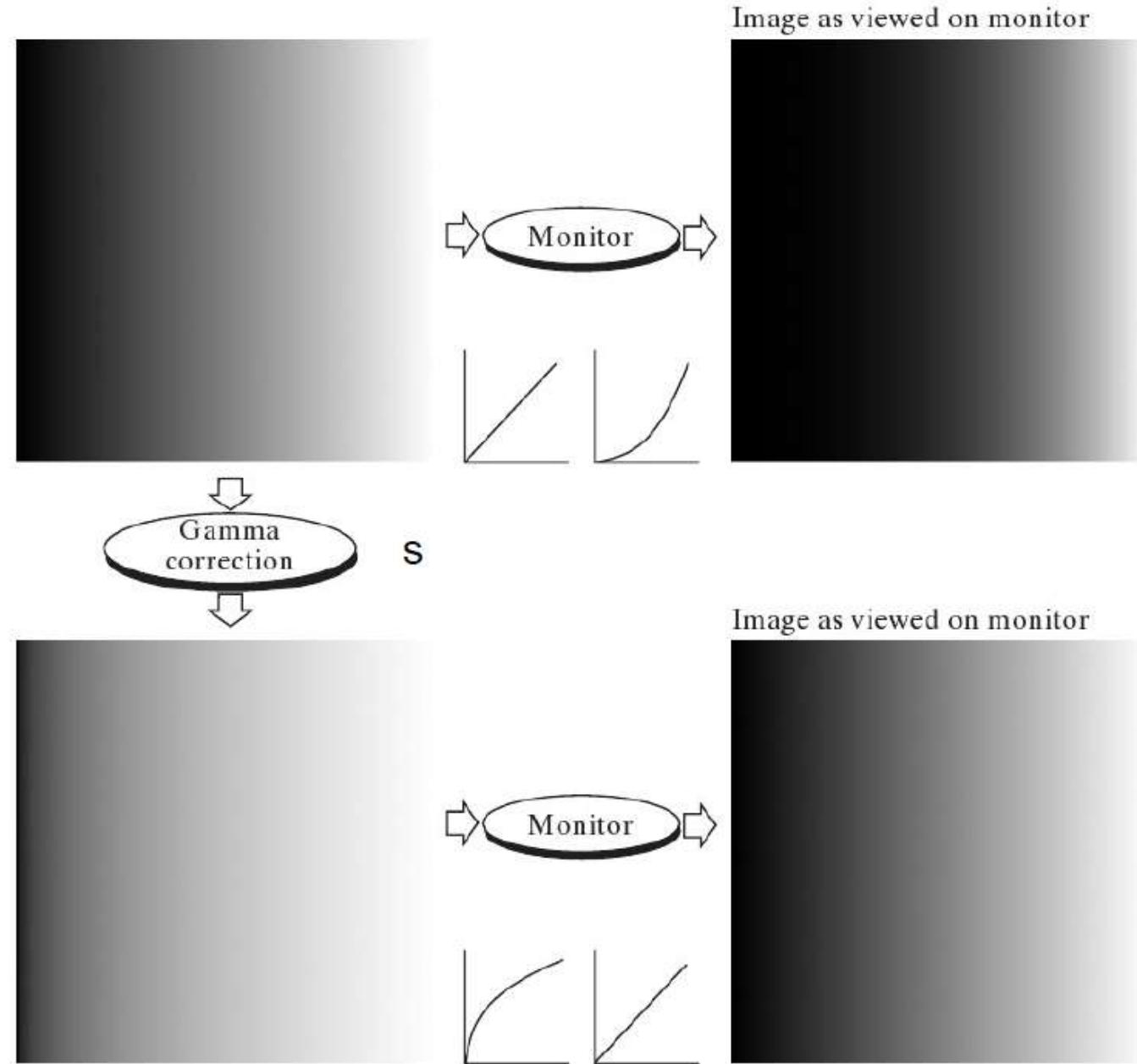
**FIGURE 3.6** Plots of the equation  $s = cr^\gamma$  for various values of  $\gamma$  ( $c = 1$  in all cases). All curves were scaled to fit in the range shown.

# Ex.: correção *gamma* - monitor

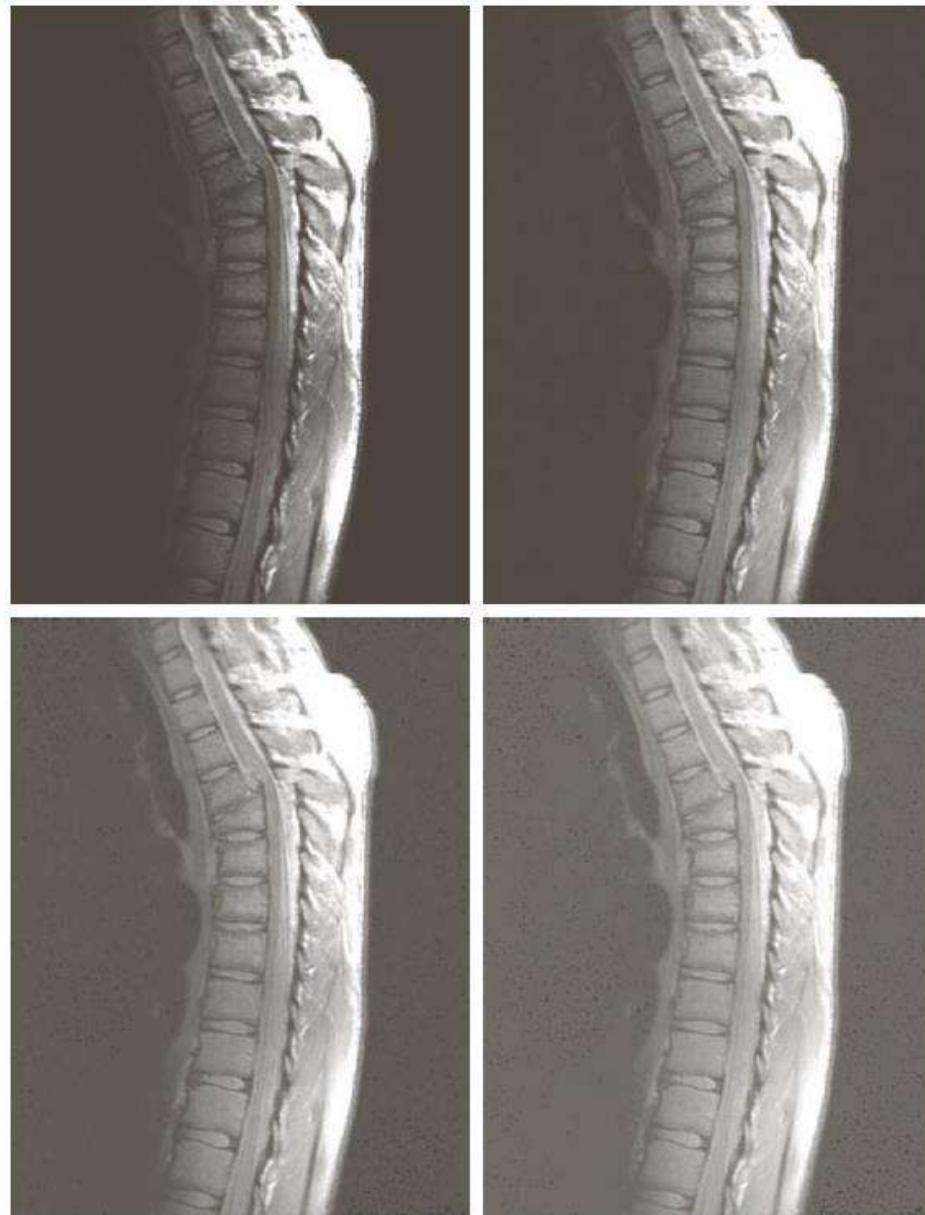
a b  
c d

**FIGURE 3.7**

- (a) Linear-wedge gray-scale image.
- (b) Response of monitor to linear wedge.
- (c) Gamma-corrected wedge.
- (d) Output of monitor.



# Ex.: correção $\gamma$ –MRI



a b  
c d

**FIGURE 3.8**  
(a) Magnetic resonance image (MRI) of a fractured human spine.  
(b)–(d) Results of applying the transformation in Eq. (3.2-3) with  $c = 1$  and  $\gamma = 0.6, 0.4$ , and  $0.3$ , respectively. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

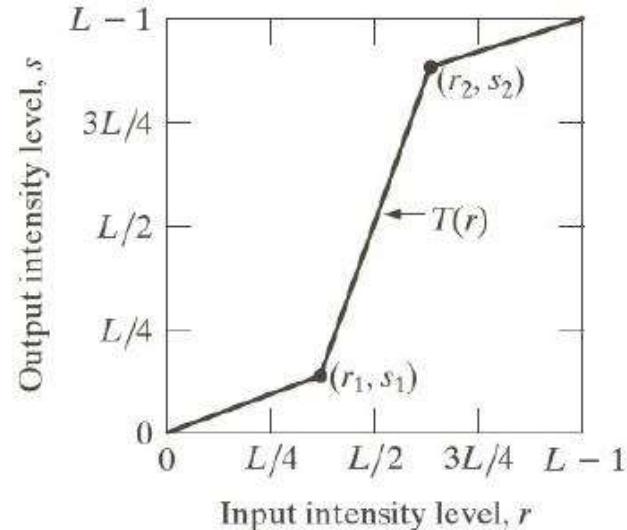
# Ex.: correção *gamma* – imagem aérea



a	b
c	d

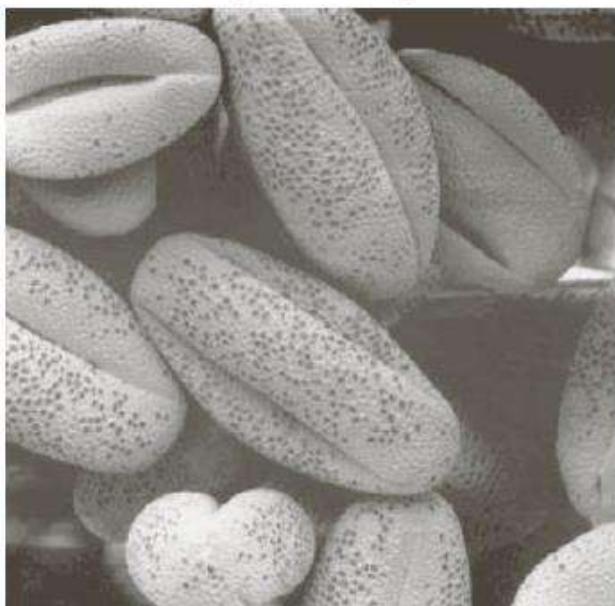
**FIGURE 3.9**  
(a) Aerial image.  
(b)–(d) Results of  
applying the  
transformation in  
Eq. (3.2-3) with  
 $c = 1$  and  
 $\gamma = 3.0, 4.0,$  and  
 $5.0,$  respectively.  
(Original image  
for this example  
courtesy of  
NASA.)

# Transformações lineares por partes



a b  
c d

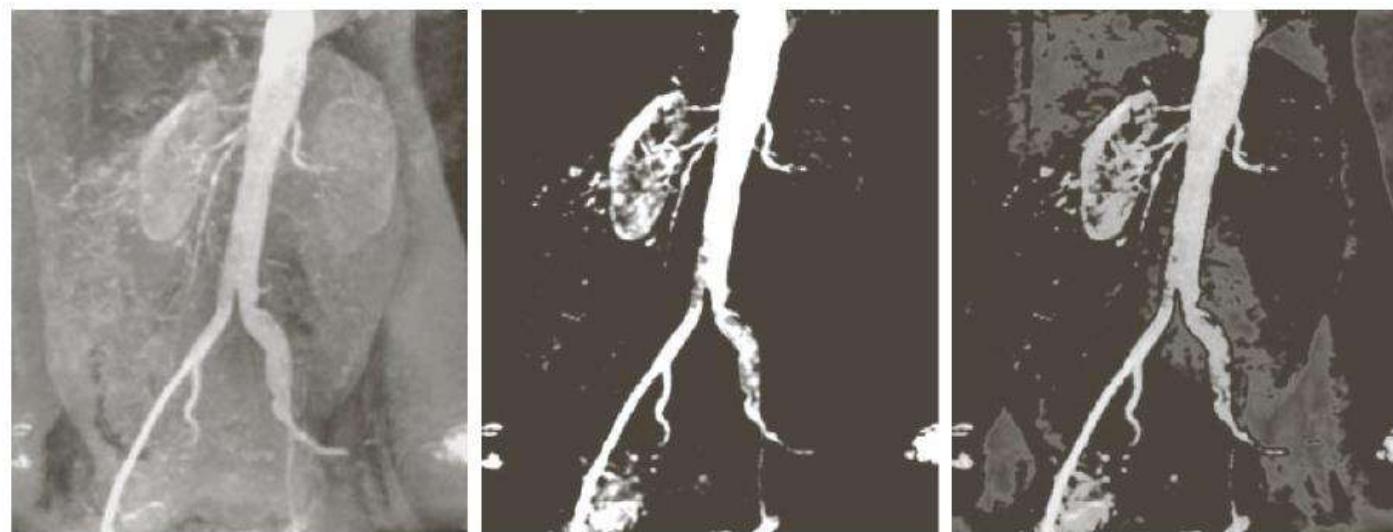
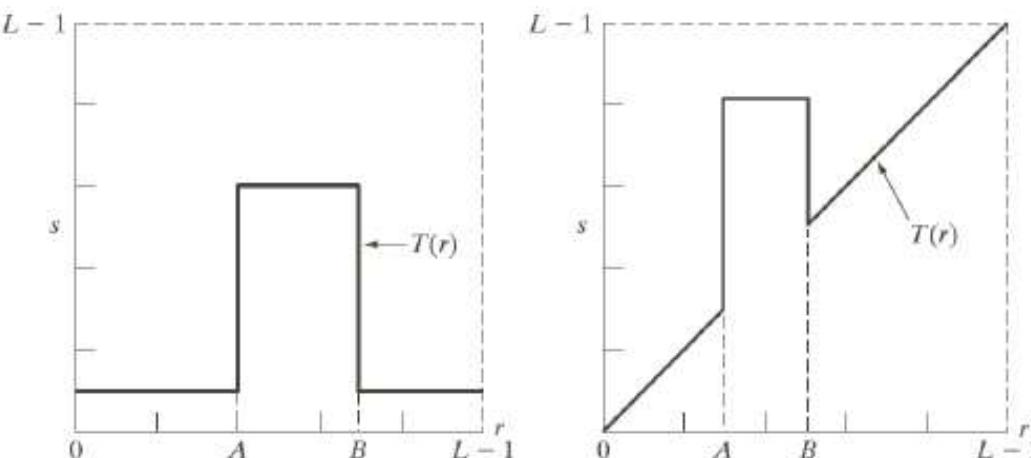
**FIGURE 3.10**  
Contrast stretching.  
(a) Form of transformation function. (b) A low-contrast image.  
(c) Result of contrast stretching.  
(d) Result of thresholding.  
(Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



# Transformações lineares por partes

a b

**FIGURE 3.11** (a) This transformation highlights intensity range  $[A, B]$  and reduces all other intensities to a lower level. (b) This transformation highlights range  $[A, B]$  and preserves all other intensity levels.



a b c

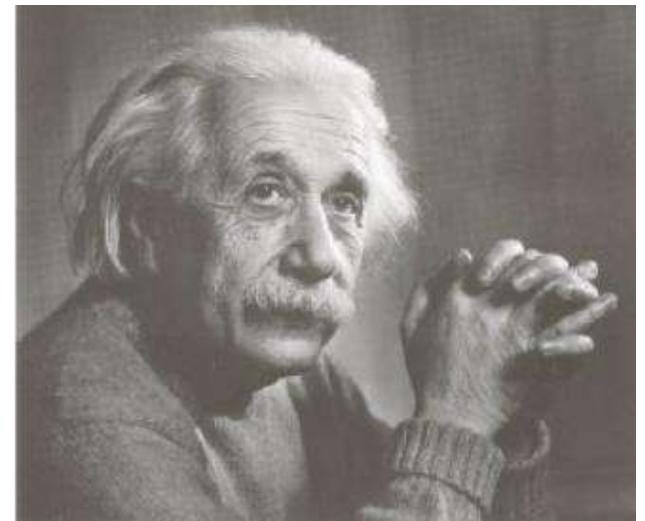
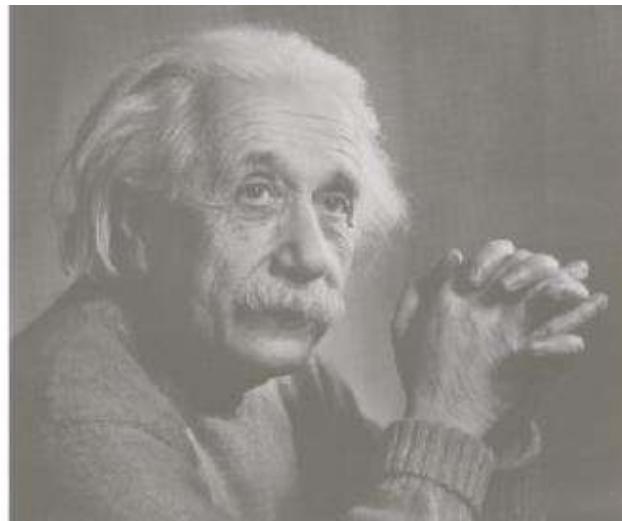
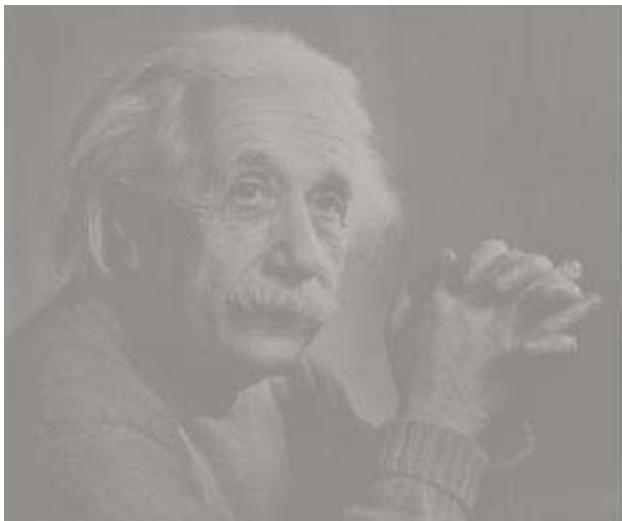
**FIGURE 3.12** (a) Aortic angiogram. (b) Result of using a slicing transformation of the type illustrated in Fig. 3.11(a), with the range of intensities of interest selected in the upper end of the gray scale. (c) Result of using the transformation in Fig. 3.11(b), with the selected area set to black, so that grays in the area of the blood vessels and kidneys were preserved. (Original image courtesy of Dr. Thomas R. Gest, University of Michigan Medical School.)

# Histogramas

## Transformações de Histograma

# Histograma

- Como diferenciar, quantitativamente, estas imagens?

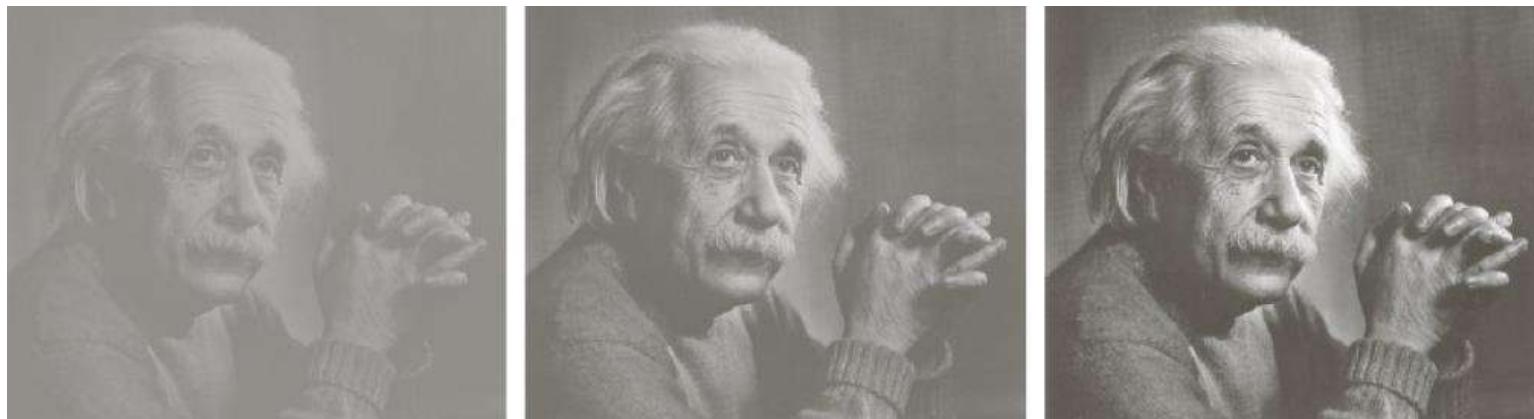


# Histograma

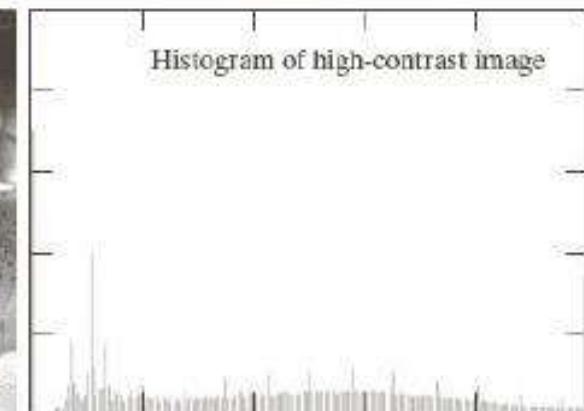
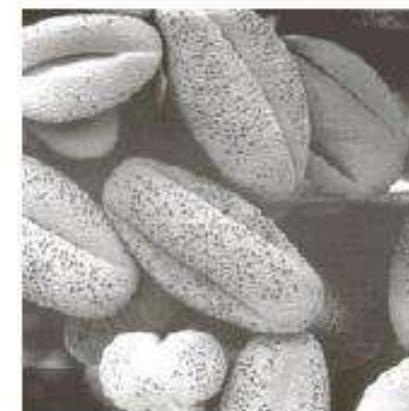
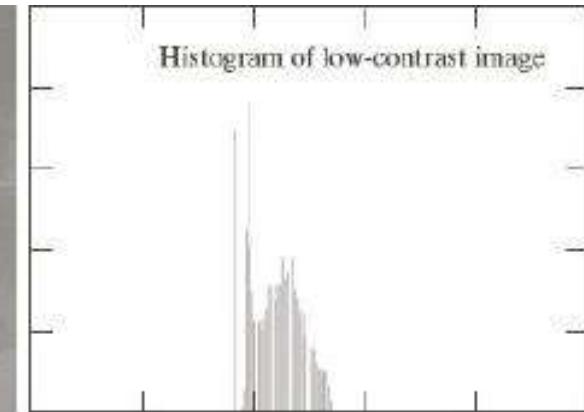
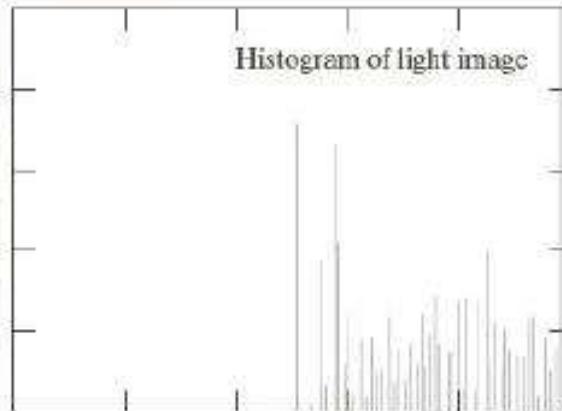
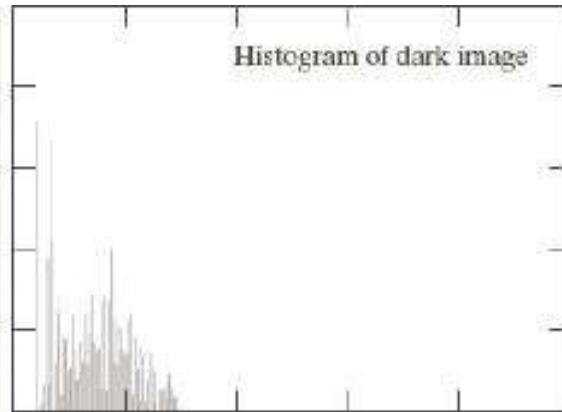
- Dado um nível de amplitude  $r_k$ , podemos contar a quantidade  $n_k$  de amostras (pixels) possuindo este nível
- Isto é:  $n_k = h(r_k)$ 
  - $r_k$ : k-ésima amplitude
  - $n_k$ : qte de pixels com amplitude  $r_k$

# Histograma

- Histograma normalizado
  - Divide-se pela quantidade de pixels  $N_{pixels}$
  - Frequência relativa da amplitude  $r_k$
  - $N_{pixels}$  suficientemente grande:  $h(r_k) = p(r_k)$
  - Soma das frequências relativas = 1



# Histograma



# Histograma

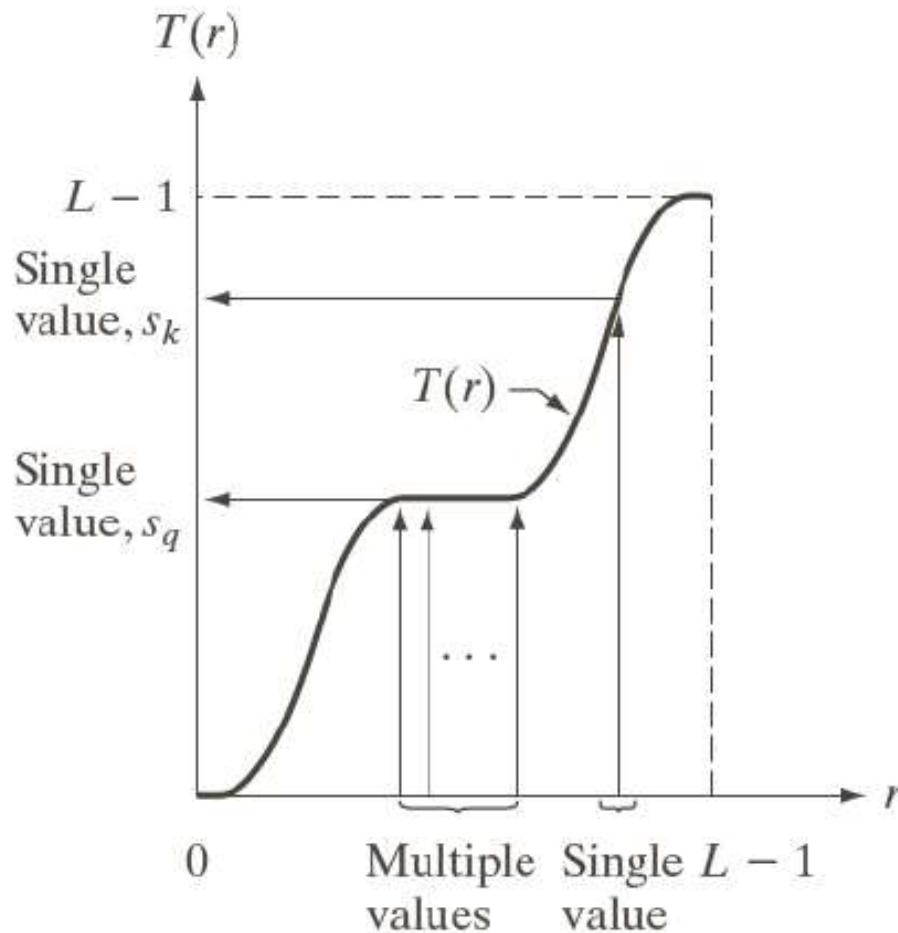
- Nem sempre uma distribuição é adequada ao nosso interesse
- Transformação de Histograma
  - Podemos aplicar transformações de amplitude na imagem de modo a alterar sua distribuição, conseguindo (ou aproximando) um Histograma de interesse

# Histograma

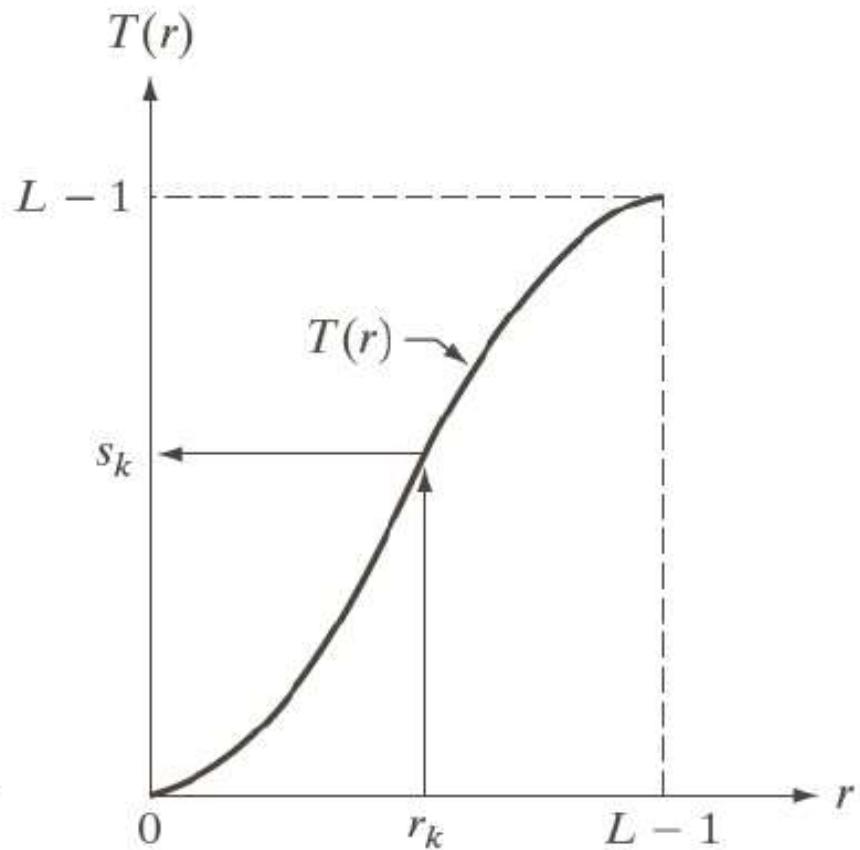
- Transformação de Histograma
  - Função  $s = T(r)$
  - Pixels com amplitude  $r$  passam a ter amplitude  $s$
- Requisitos gerais
  - A transformação  $T()$  deve ser não decrescente no intervalo  $[0,L-1]$  (ou  $[0,1]$ )
    - Se  $r_1 \geq r_2$  então  $T(r_1) \geq T(r_2)$
  - $0 \leq T(r) \leq L-1$  para todo  $r$  (ou  $0 \leq T(r) \leq 1$ )

# Histograma

$T(r)$  não decrescente



$T(r)$  (estritamente) crescente



# Histograma

- Temos  $s = T(r)$  em função de  $r$

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

- Se  $s$  tem distribuição (função densidade de probab.)  $p_s(s)$  e  $r$  tem distribuição  $p_r(r)$ , qual a relação entre  $p_s(s)$  e  $p_r(r)$ ?

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$$

# Histograma

- A amplitude  $s$  obtida será uniformemente distribuída

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L - 1)p_r(r)$$

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = p_r(r) \left| \frac{1}{(L - 1)p_r(r)} \right| = \frac{1}{L - 1}$$

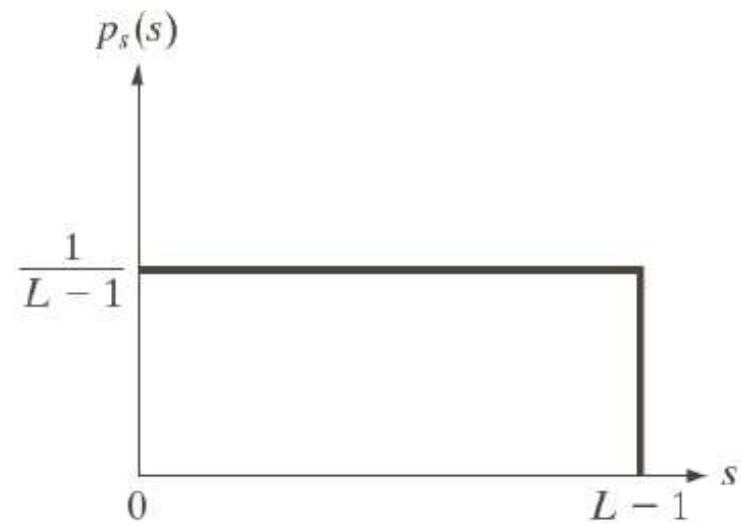
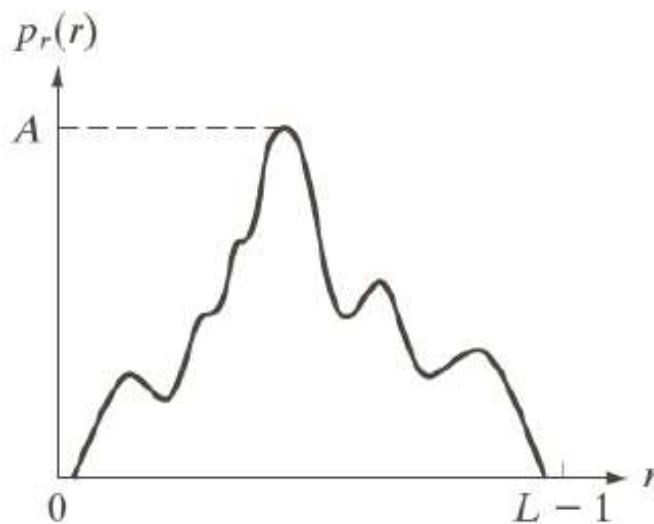
Uniforme!

# Histograma – Equalização

- Exemplo

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

$$p_r(r) \Rightarrow p_s(s)$$



Distribuição resultante de  $s$  é uniforme, independente da distribuição original de  $r$

# Equalização de Histogramas

- Equalização no caso contínuo

$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

- No caso discreto

$$\begin{aligned} s_k &= T(r_k) = (L - 1) \sum_{j=0}^K p_r(r_j) & K &= 0, 1, 2, \dots, N - 1. \\ &= (L - 1) \sum_{j=0}^K \frac{n_j}{MN} = \frac{(L - 1)}{MN} \sum_{j=0}^K n_j \end{aligned}$$

# Equalização de Histogramas

- Ex.: (Equalização discreta)
  - 3 bits  $\rightarrow L = 2^3 = 8$  níveis de cinza
  - $MN = 64 \times 64 = 4096$  pixels

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

# Equalização de Histogramas

$$s_0 = T(r_0) = 7 \sum_{j=0}^0 p_r(r_j) = 7 \times 0,19 = 1,33 \rightarrow 1$$

$$s_1 = T(r_1) = 7 \sum_{j=0}^1 p_r(r_j) = 7 \times (0,19 + 0,25) = 3,08 \rightarrow 3$$

$$s_2 = T(r_2) = 7 \sum_{j=0}^2 p_r(r_j) = 7 \times (0,19 + 0,25 + 0,21) = 4,55 \rightarrow 5$$

$$s_3 = 5,67 \rightarrow 6$$

$$s_4 = 6,23 \rightarrow 6$$

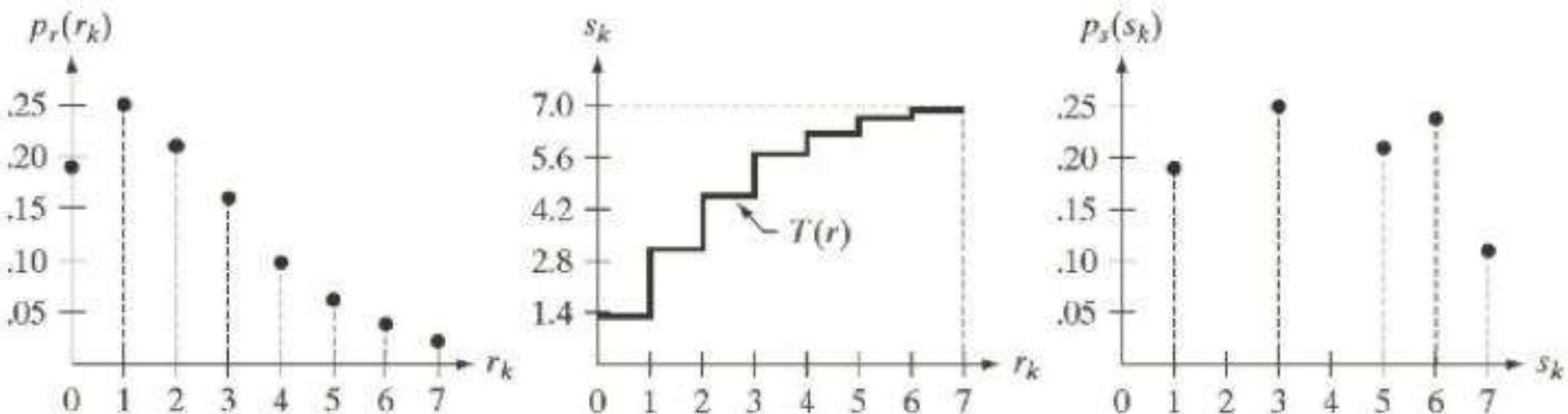
$$s_5 = 6,65 \rightarrow 7$$

$$s_6 = 6,86 \rightarrow 7$$

$$s_7 = 7,00 \rightarrow 7$$

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

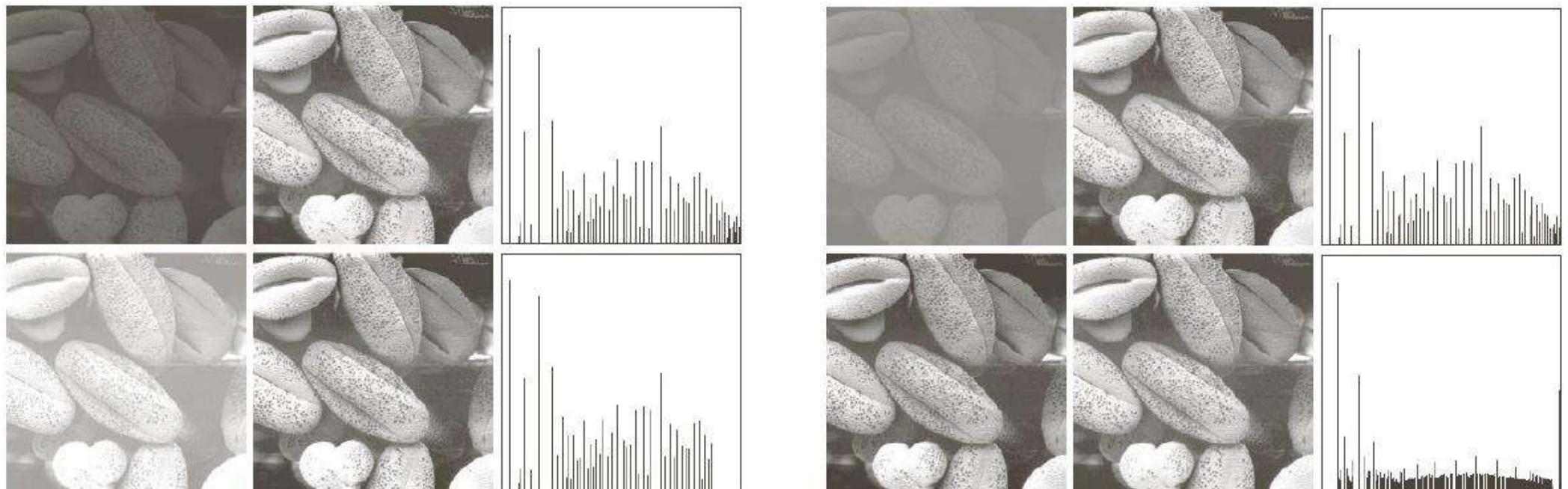
# Equalização de Histogramas



a b c

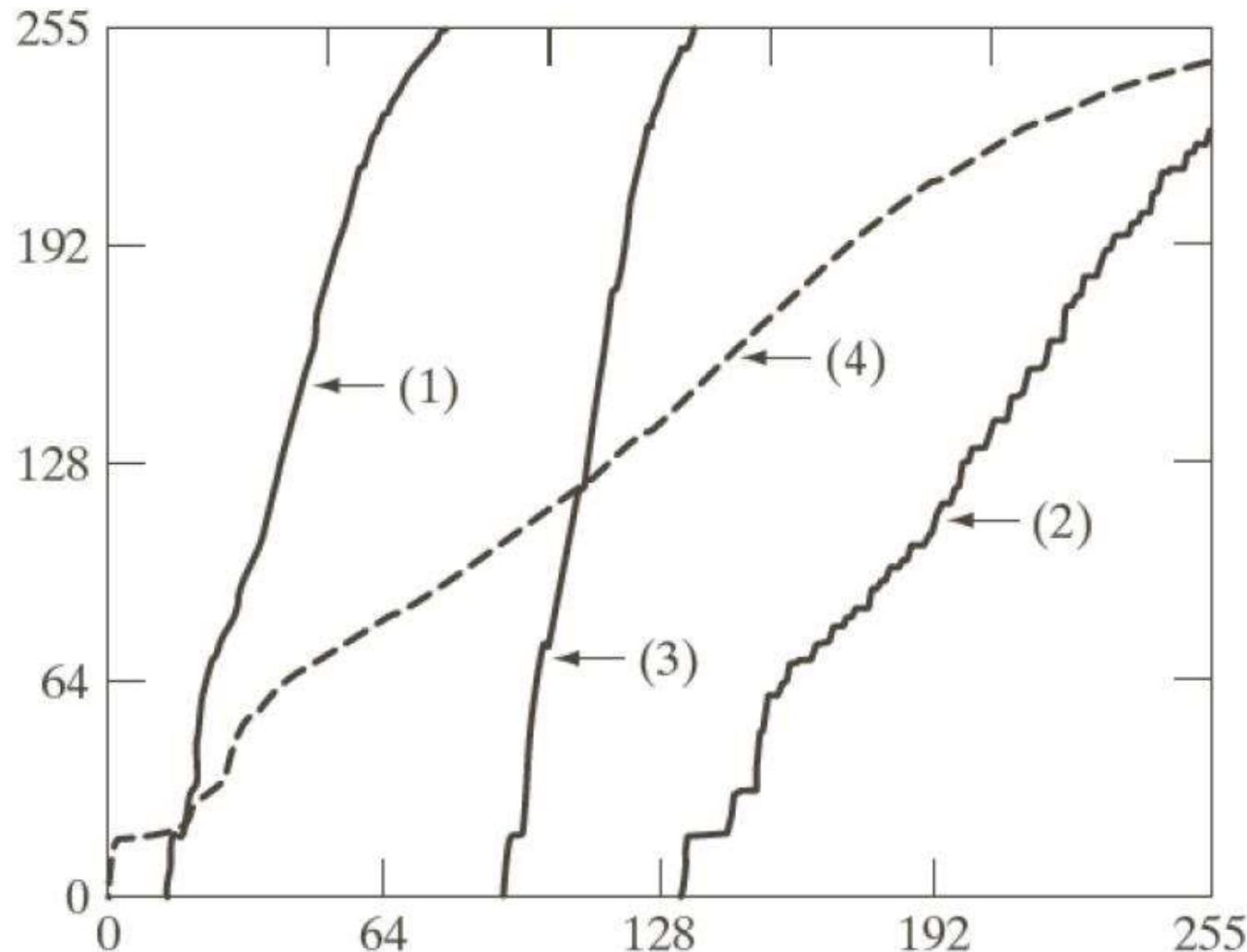
**FIGURE 3.19** Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

# Equalização de Histogramas - exemplos



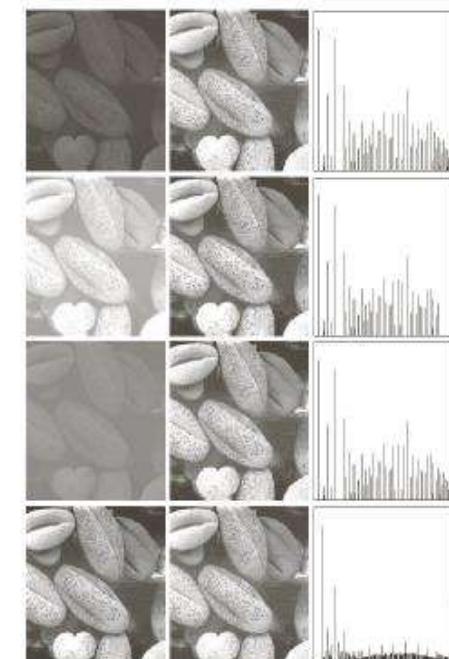
**FIGURE 3.20** Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

# Equalização de Histogramas - exemplos



Transformation functions for histogram equalization. Transformations

(1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using



# Especificação de Histograma

- Nem sempre a equalização de histograma, desta maneira, é desejada
- Podemos especificar histogramas de interesse

# Especificação de Histograma

- Equalização de histograma nem sempre é desejável
- Podemos especificar distribuições de interesse
- PDF da imagem original:  $p_r(r)$
- PDF especificada:  $p_z(z)$

# Especificação de Histograma

- Temos  $s = T(r)$  em função de  $r$

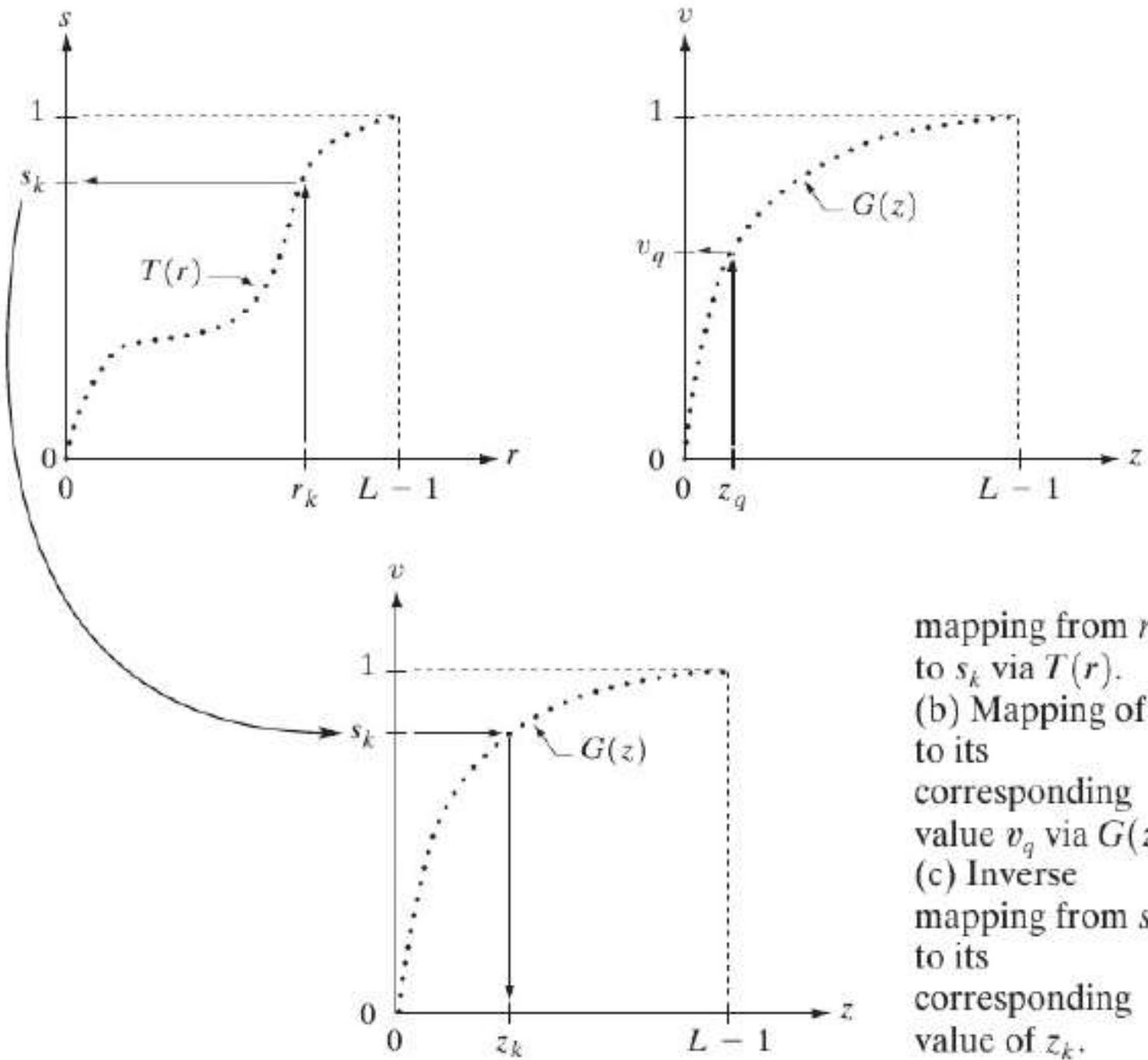
$$s = T(r) = (L - 1) \int_0^r p_r(w) dw$$

- Também podemos obter  $s = G(z)$

$$s = G(z) = (L - 1) \int_0^z p_z(t) dt$$

- Temos então mapeamento de  $r$  para  $z$

$$z = G^{-1}(s) = G^{-1}(T(r))$$



mapping from  $r_k$  to  $s_k$  via  $T(r)$ .  
 (b) Mapping of  $z_q$  to its corresponding value  $v_q$  via  $G(z)$ .  
 (c) Inverse mapping from  $s_k$  to its corresponding value of  $z_k$ .

# Especificação de histogramas - Exemplo

- Suponha que uma imagem possua distribuição dada pela PDF

$$p_r(r) = \begin{cases} \frac{2r}{(L-1)^2}, & \text{for } 0 \leq r \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

- Queremos que a imagem resultante possua distribuição dada pela PDF

$$p_z(z) = \begin{cases} \frac{3z^2}{(L-1)^3}, & \text{for } 0 \leq z \leq (L-1) \\ 0, & \text{otherwise} \end{cases}$$

# Especificação de histogramas - Exemplo

- Temos

$$s = T(r) = (L-1) \int_0^r p_r(w) dw = (L-1) \int_0^r \frac{2w}{(L-1)^2} dw = \frac{r^2}{L-1}$$

- Mas, também

$$s = G(z) = (L-1) \int_0^z p_z(t) dt = (L-1) \int_0^z \frac{3t^2}{(L-1)^3} dt = \frac{z^3}{(L-1)^2}$$

- Então

$$z = G^{-1}(s) = G^{-1}[T(r)] = \left[ (L-1)^2 s \right]^{1/3} = \left[ (L-1)^2 \frac{r^2}{L-1} \right]^{1/3} = \left[ (L-1)r^2 \right]^{1/3}$$

# Especificação de histogramas

- Caso discreto
  - Dada a imagem, obter sua PDF  $p_r(r_j)$
  - Determinar os valores  $s_k$  arredondando para inteiros no intervalo  $[0, L-1]$ 
$$s_k = T(r_k) = (L-1) \sum_{j=0}^k p_r(r_j) = \frac{(L-1)}{MN} \sum_{j=0}^k n_j$$
  - Usando a PDF especificada, obter  $G(z_q)$  arredondando para inteiros no intervalo  $[0, L-1]$ 
$$G(z_q) = (L-1) \sum_{i=0}^q p_z(z_i) = s_k$$
  - Fazer o mapeamento de  $s_k$  para  $z_q$ 
$$z_q = G^{-1}(s_k)$$

# Exemplo

- Imagem com 3 bits ( $L = 8$ ) e  $M \times N = 64 \times 64 = 4096$

$r_k$	$n_k$	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$z_q$	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

# Exemplo

- 1) obter as intensidades no intervalo [0 , L-1]
  - De r para s: procedimento feito anteriormente

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7, \\ s_5 = 7, s_6 = 7, s_7 = 7.$$

- 2) calcular as transformações  $G(z_q)$

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00$$

$$G(z_2) = 0.00$$

$$G(z_3) = 1.05$$

$$G(z_4) = 2.45$$

$$G(z_5) = 4.55$$

$$G(z_6) = 5.95$$

$$G(z_7) = 7.00$$

# Exemplo

- 1) obter as intensidades no intervalo [0 , L-1]
  - De r para s: procedimento feito anteriormente

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

- 2) calcular as transformações  $G(z_q)$

$$G(z_0) = 7 \sum_{j=0}^0 p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00$$

$$G(z_2) = 0.00$$

$$G(z_3) = 1.05 \leftarrow s_0$$

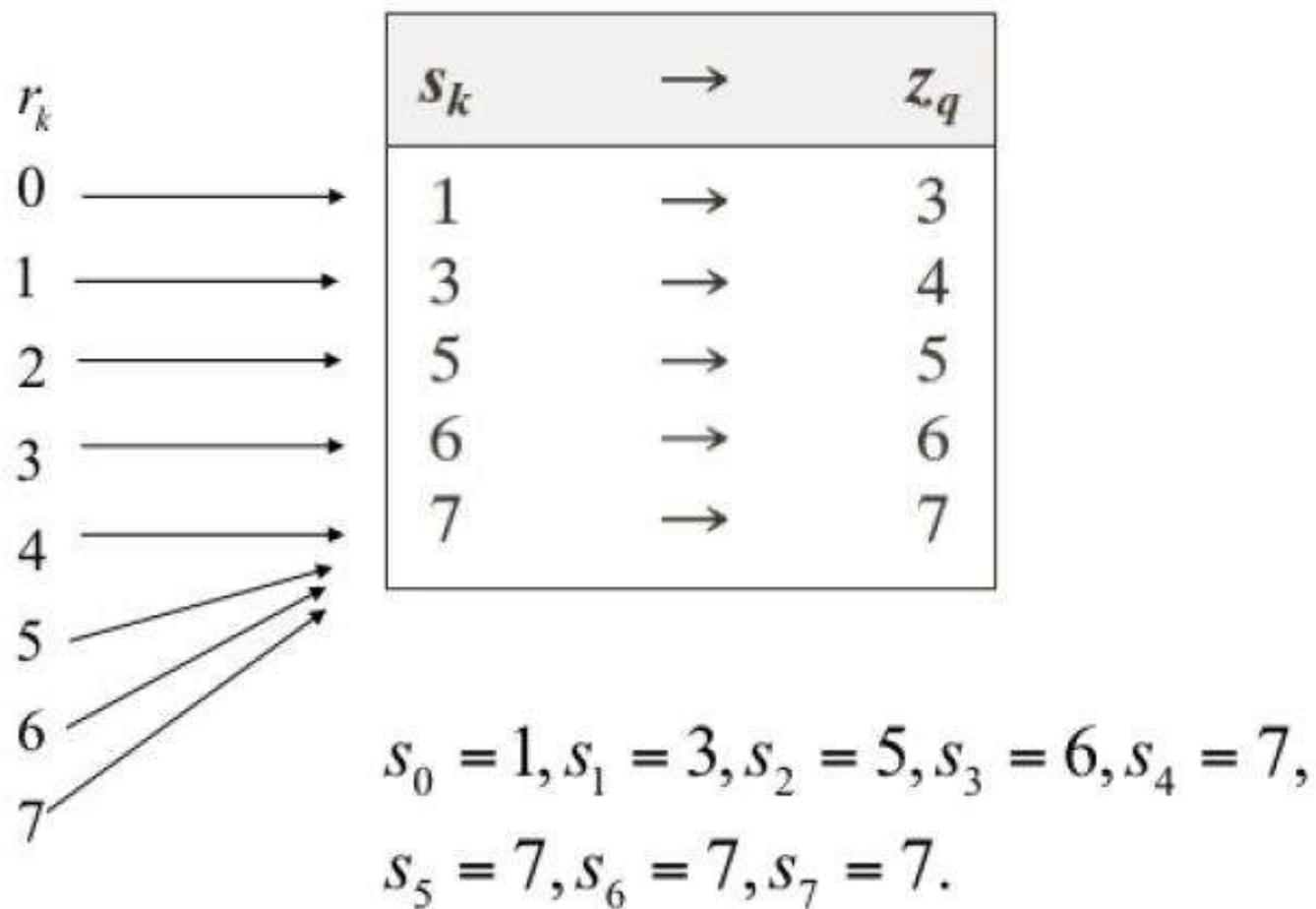
$$G(z_4) = 2.45 \leftarrow s_1$$

$$G(z_5) = 4.55 \leftarrow s_2$$

$$G(z_6) = 5.95 \leftarrow s_3$$

$$G(z_7) = 7.00 \leftarrow s_4, s_5, s_6, s_7$$

# Exemplo



# Exemplo

$$G(z_0) = 0.00$$

$$G(z_1) = 0.00$$

$$G(z_2) = 0.00$$

$$G(z_3) = 1.05 \leftarrow s_0$$

$$G(z_4) = 2.45 \leftarrow s_1$$

$$G(z_5) = 4.55 \leftarrow s_2$$

$$G(z_6) = 5.95 \leftarrow s_3$$

$$G(z_7) = 7.00 \leftarrow s_4, s_5, s_6, s_7$$

$r_k$	$s_k$	$\rightarrow$	$z_q$
0	1	$\rightarrow$	3
1	3	$\rightarrow$	4
2	5	$\rightarrow$	5
3	6	$\rightarrow$	6
4	7	$\rightarrow$	7

$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$   
 $s_5 = 7, s_6 = 7, s_7 = 7.$

# Exemplo

$$G(z_0) = 0.00$$

$$G(z_2) = 0.00$$

$$G(z_4) = 2.45 \leftarrow s_1$$

$$G(z_6) = 5.95 \leftarrow s_3$$

$$G(z_1) = 0.00$$

$$G(z_3) = 1.05 \leftarrow s_0$$

$$G(z_5) = 4.55 \leftarrow s_2$$

$$G(z_7) = 7.00 \leftarrow s_4, s_5, s_6, s_7$$

$$r_k \rightarrow z_q$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$

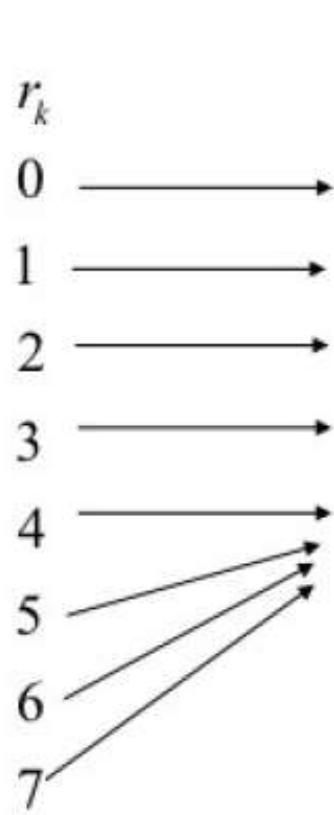
$$3 \rightarrow 6$$

$$4 \rightarrow 7$$

$$5 \rightarrow 7$$

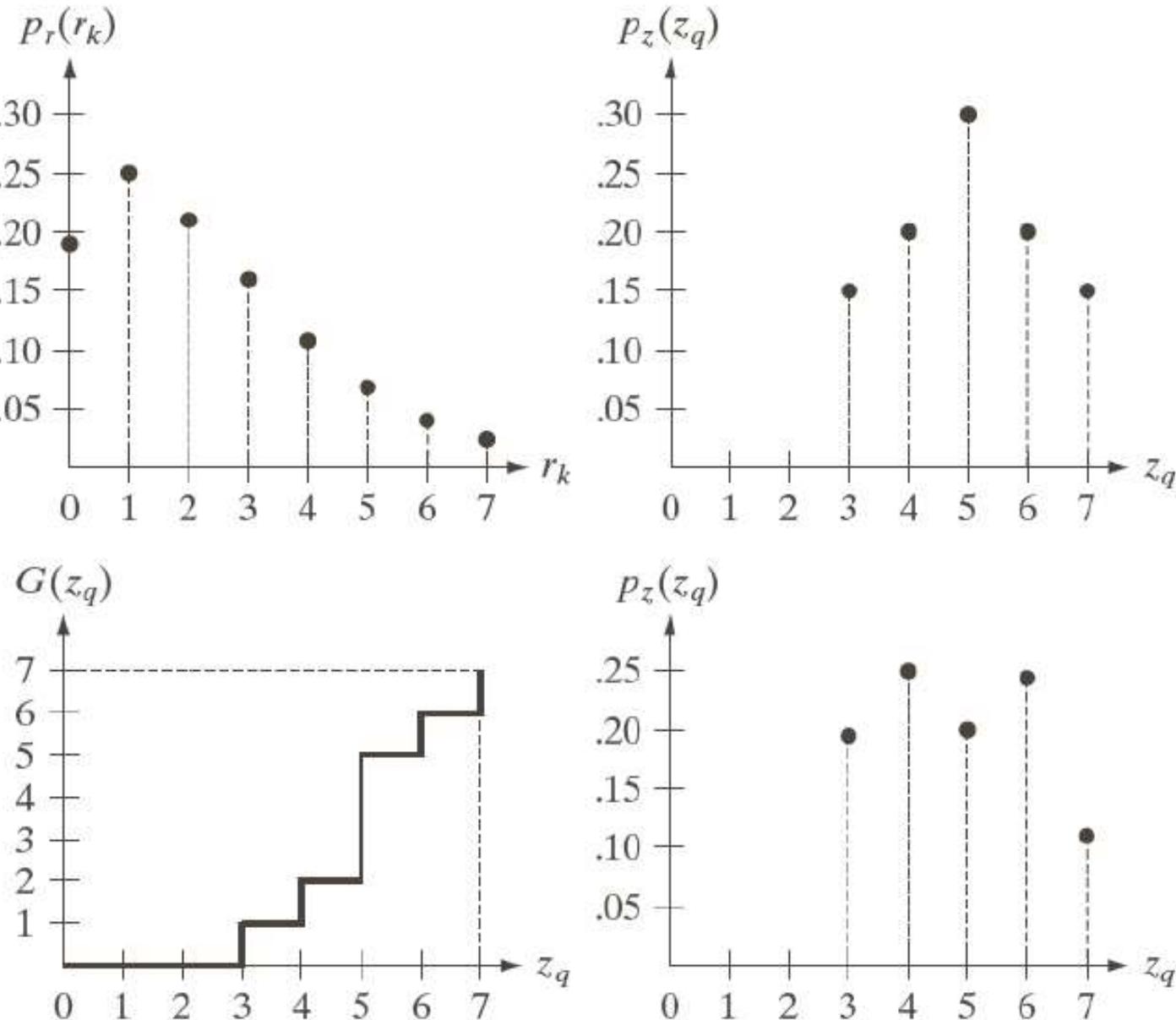
$$6 \rightarrow 7$$

$$7 \rightarrow 7$$



$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$   
 $s_5 = 7, s_6 = 7, s_7 = 7.$

# Exemplo

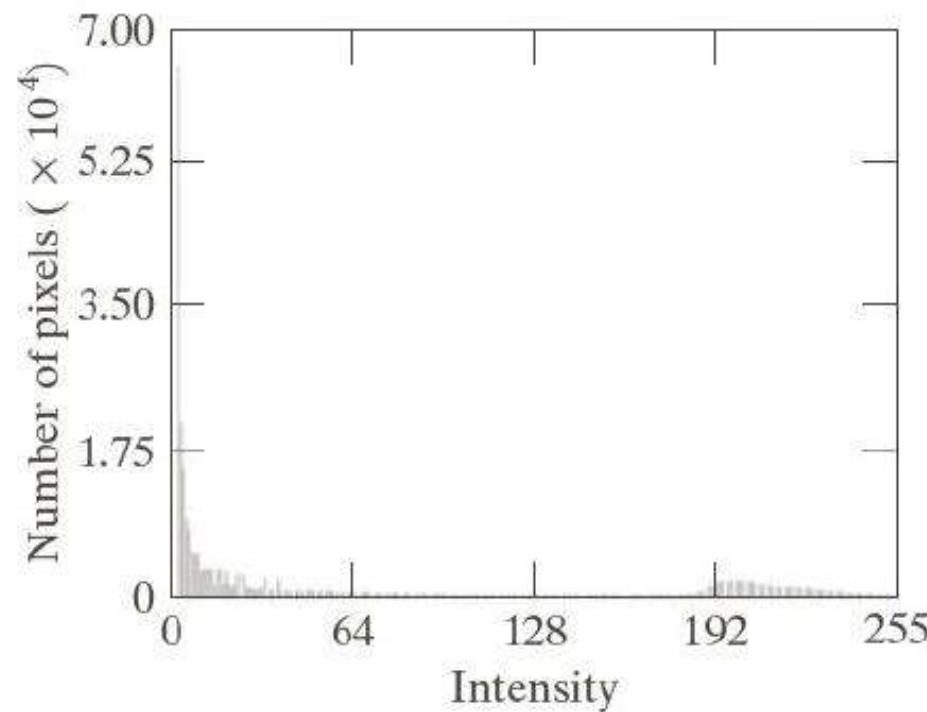


a	b
c	d

**FIGURE 3.22**

- (a) Histogram of a 3-bit image.
- (b) Specified histogram.
- (c) Transformation function obtained from the specified histogram.
- (d) Result of performing histogram specification. Compare (b) and (d).

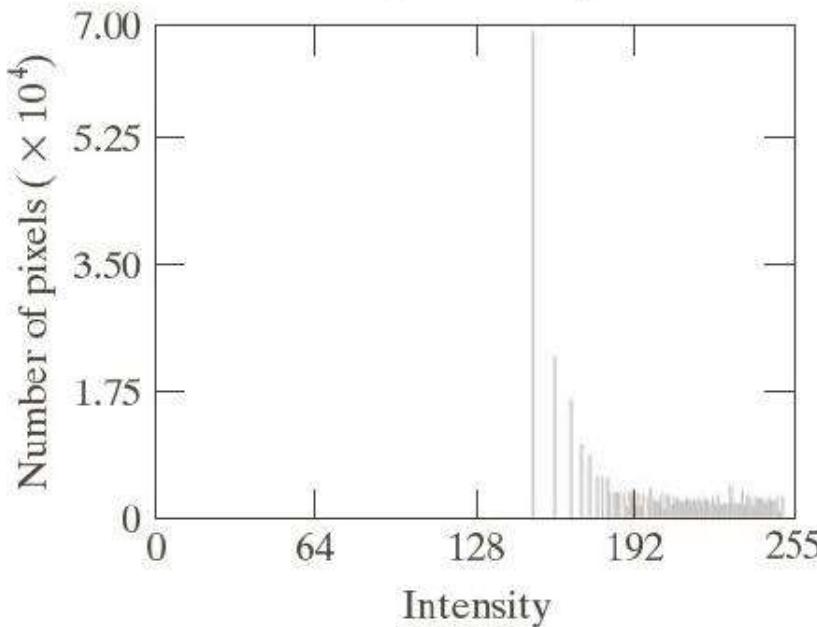
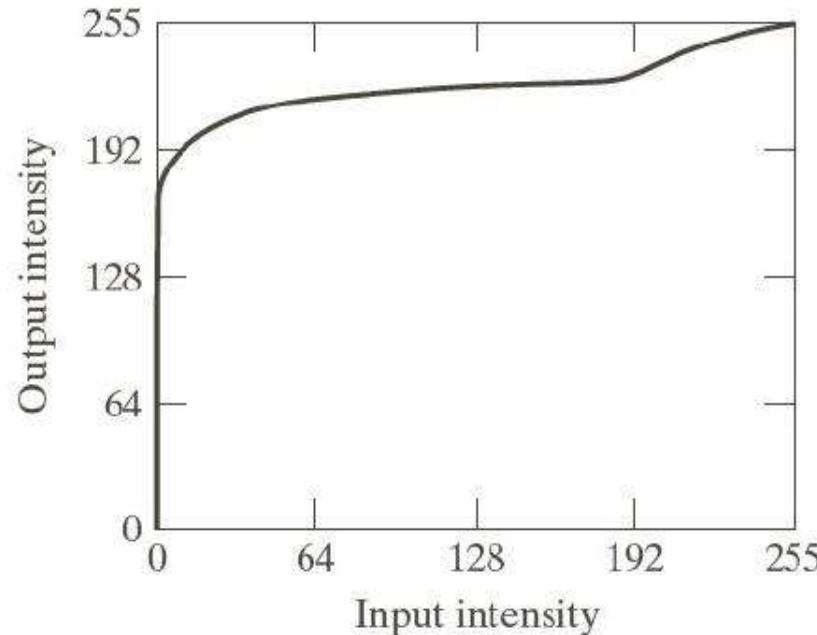
# Outro exemplo



a b

**FIGURE 3.23**  
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*.  
(b) Histogram.  
(Original image courtesy of NASA.)

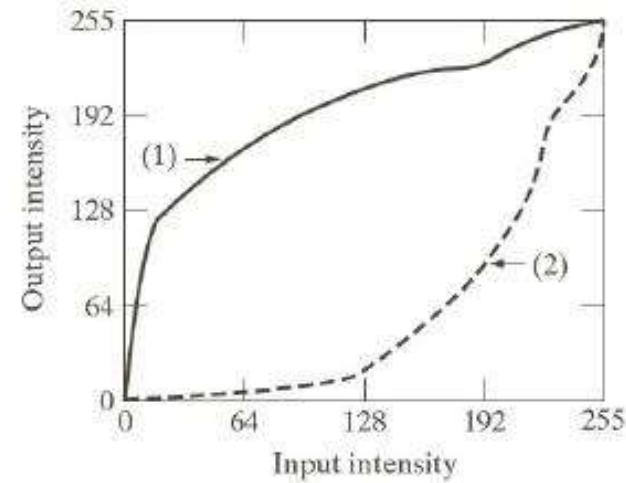
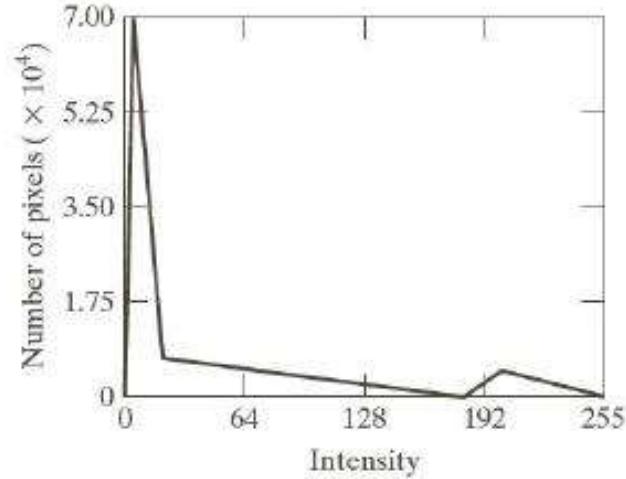
# Outro exemplo



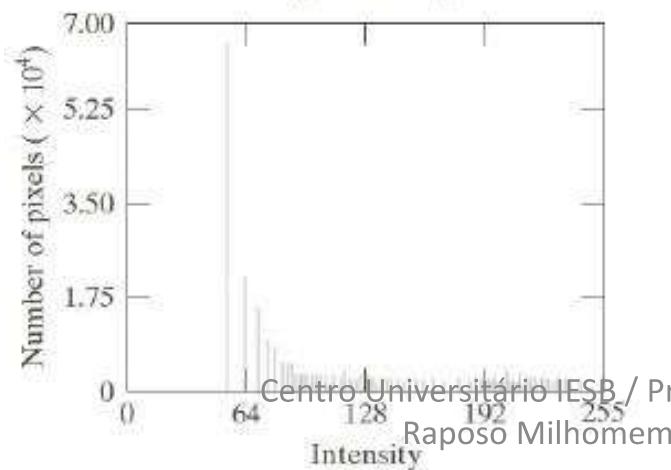
a  
b  
c

**FIGURE 3.24**  
(a) Transformation function for histogram equalization.  
(b) Histogram-equalized image (note the washed-out appearance).  
(c) Histogram of (b).

# Outro exemplo



a  
b  
c  
d

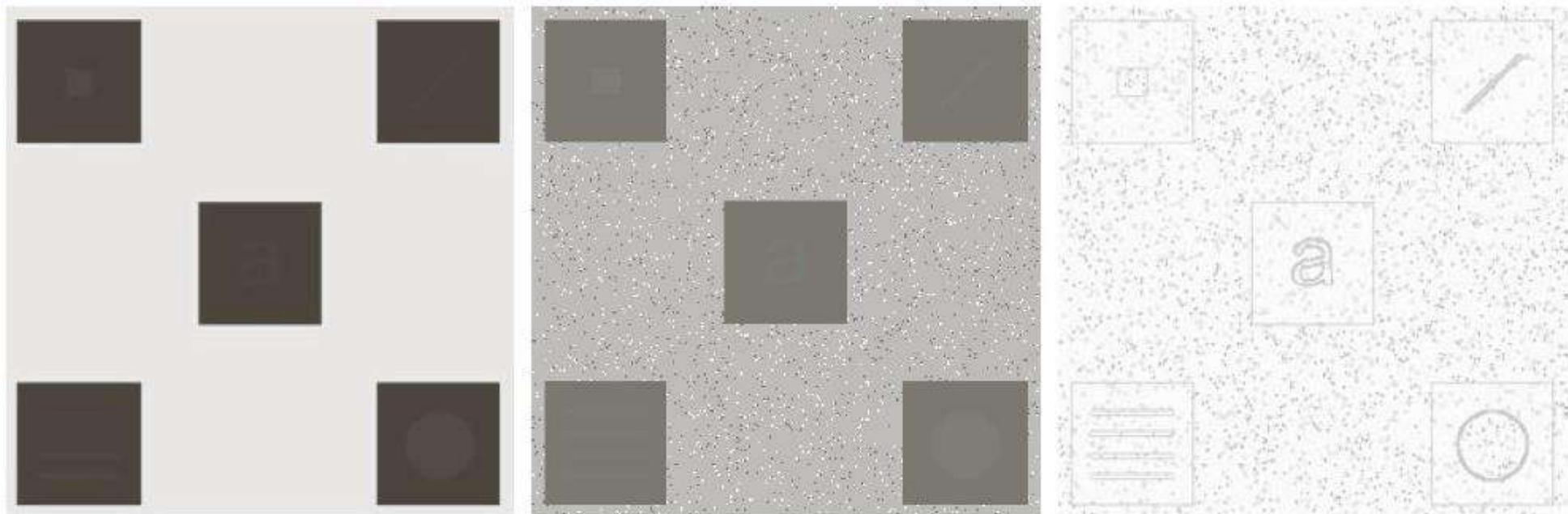


**FIGURE 3.25**  
(a) Specified histogram.  
(b) Transformations.  
(c) Enhanced image using mappings from curve (2).  
(d) Histogram of (c).

# Histograma Local

- Histogramas processados globalmente nem sempre são suficientes
- Histograma também pode ser equalizado / especificado localmente
  - Definir uma vizinhança
  - Para cada pixel da imagem, determinar o histograma sobre sua vizinhança e realizar a transformação de intensidade neste pixel (central), mapeando seu novo valor
  - Fazer o procedimento para o próximo pixel

# Histograma Local



a b c

**FIGURE 3.26**(a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size  $3 \times 3$ .

# Estatística de Histograma

- Média e momento de ordem n

$$m = \sum_{i=0}^{L-1} r_i p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

$$u_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

- Variância

$$\sigma^2 = u_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - m]^2$$

# Estatística de Histograma

- Média local

$$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i p_{s_{xy}}(r_i)$$

$s_{xy}$  denotes a neighborhood

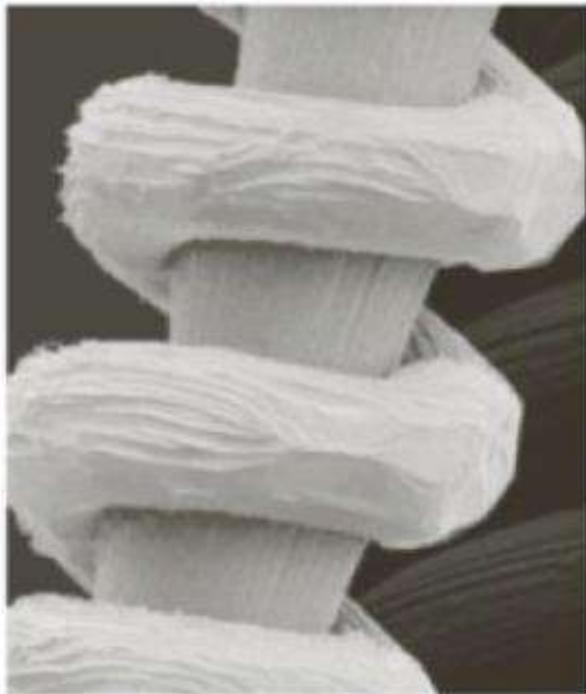
- Variância local

$$\sigma_{s_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{s_{xy}})^2 p_{s_{xy}}(r_i)$$

# Estatística de Histograma

- Exemplo

- Imagem original



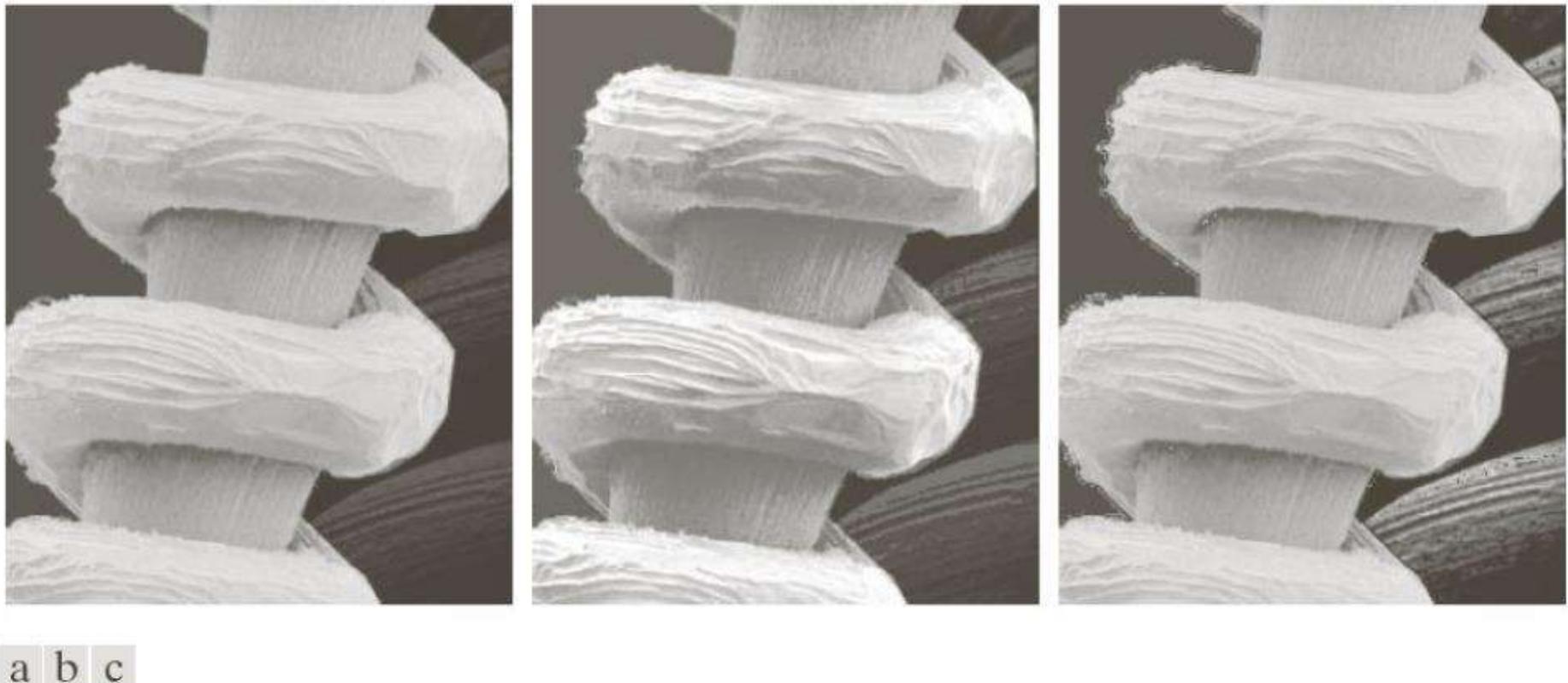
$$g(x, y) = \begin{cases} Egf(x, y), & \text{if } m_{s_{xy}} \leq k_0 m_G \text{ and } k_1 \sigma_G \leq \sigma_{s_{xy}} \leq k_2 \sigma_G \\ f(x, y), & \text{otherwise} \end{cases}$$

$m_G$  : global mean;  $\sigma_G$  : global standard deviation

$k_0 = 0.4$ ;  $k_1 = 0.02$ ;  $k_2 = 0.4$ ;  $E = 4$

# Estatística de Histograma

- Exemplo



a b c

**FIGURE 3.27** (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

# Filtros

# Filtragem Espacial

- Filtragem no domínio espacial
  - Filtragem linear: Correlação ou Convolução 2D
  - Correlação / convolução do filtro com a imagem
  - Filtro
    - *máscara espacial ou resposta ao impulso  $h(x,y)$*
- Filtros passa-baixa
  - Borrar imagens
  - Suavização
  - Redução de ruídos
  - Privilegiam partes da imagem com variações de amplitude suaves
- Filtros passa-alta
  - Detecção de bordas, linhas, pontos isolados etc.
  - Privilegiam partes da imagem com variações abruptas de amplitude

# Filtragem Espacial

- Correlação do filtro com a imagem

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \dots + w_{mn} z_{mn} \\ &= \sum_{i=1}^{mn} w_i z_i \end{aligned}$$

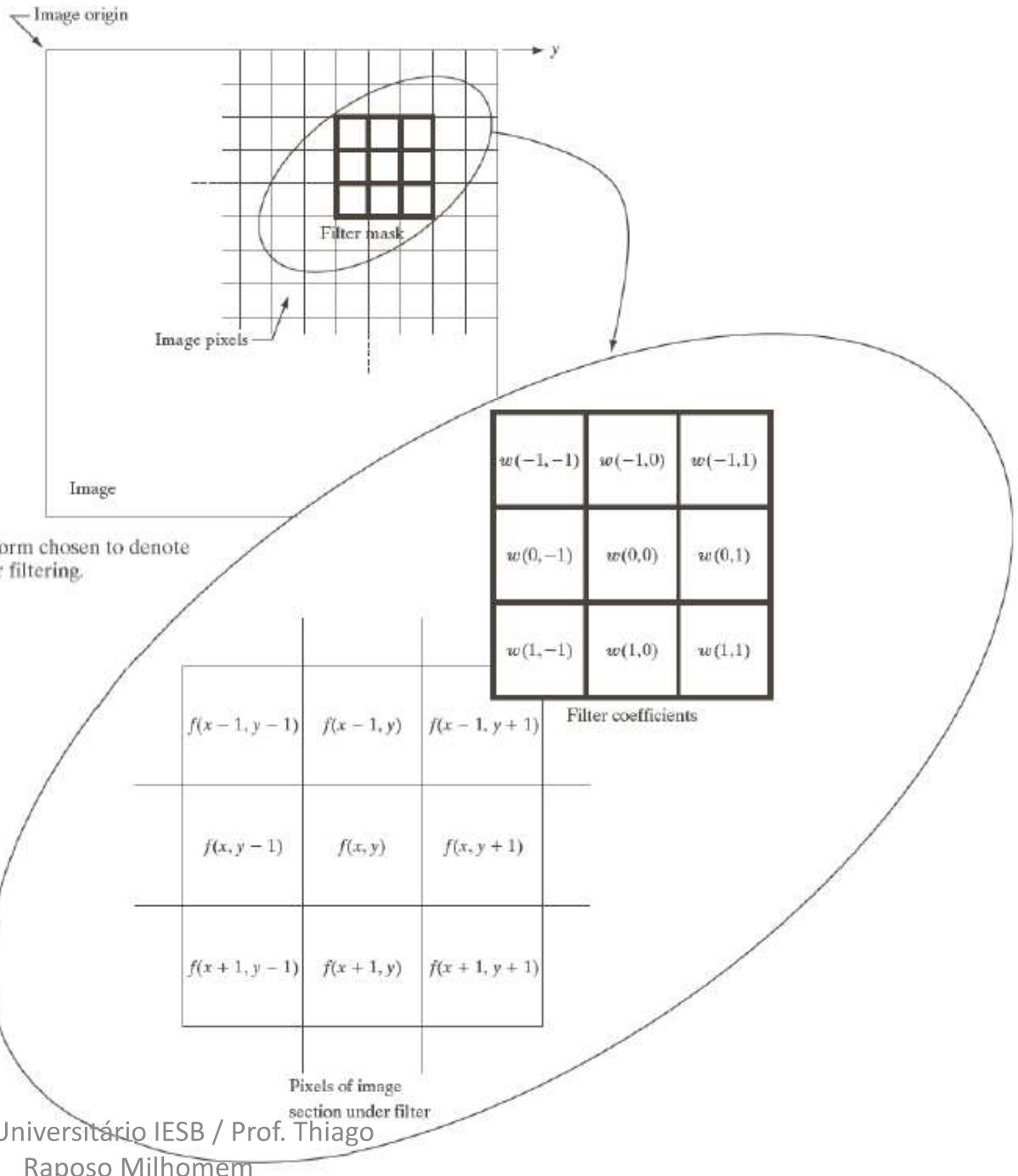
$w_1$	$w_2$	$w_3$
$w_4$	$w_5$	$w_6$
$w_7$	$w_8$	$w_9$

**FIGURE 3.31**  
Another representation of a general  $3 \times 3$  filter mask.

$$\begin{aligned} R &= w(-1, -1)f(x - 1, y - 1) + w(-1, 0)f(x - 1, y) + \dots \\ &\quad + w(0, 0)f(x, y) + \dots + w(1, 0)f(x + 1, y) + w(1, 1)f(x + 1, y + 1) \end{aligned}$$

# Filtragem Espacial

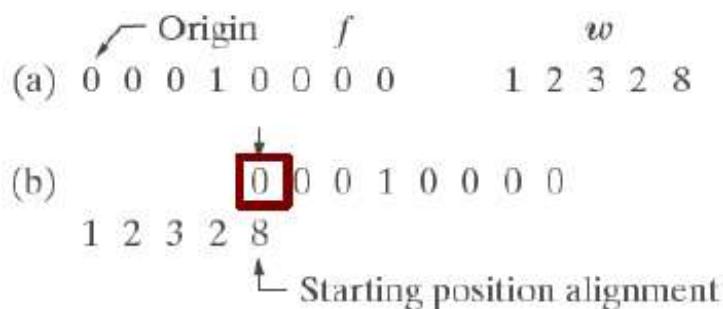
$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$



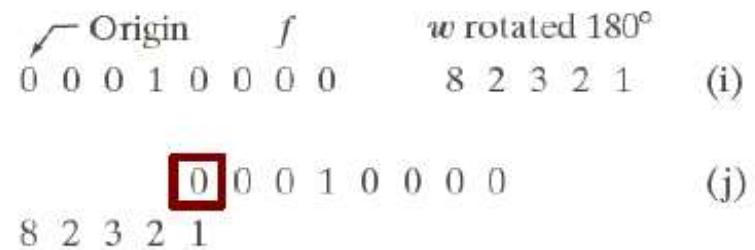
**FIGURE 3.28** The mechanics of linear spatial filtering using a  $3 \times 3$  filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.

## Correlação VS Convolução

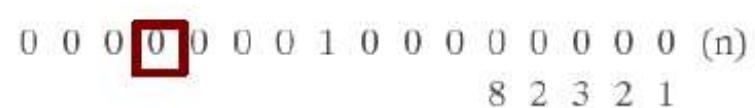
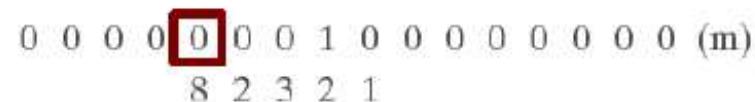
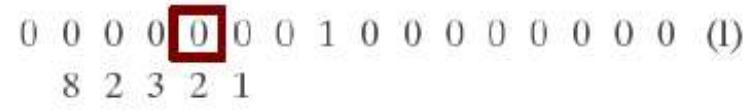
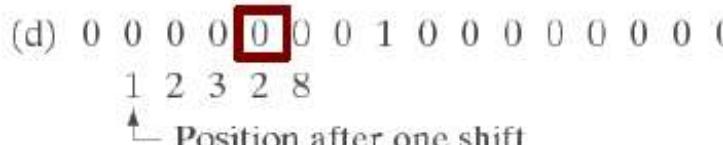
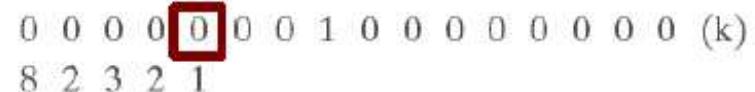
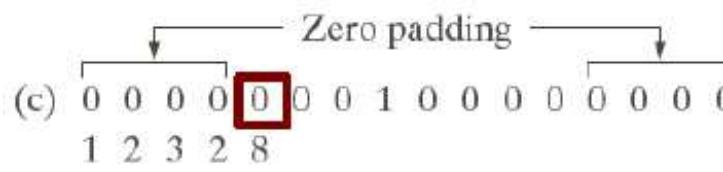
**Correlation**



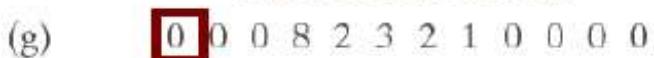
**Convolution**



Zero padding



Full correlation result



Full convolution result



Cropped correlation result



Centro Universitário IESB / Prof. Thiago Raposo Milhomem

Cropped convolution result



# Correlação VS Convolução (2D)

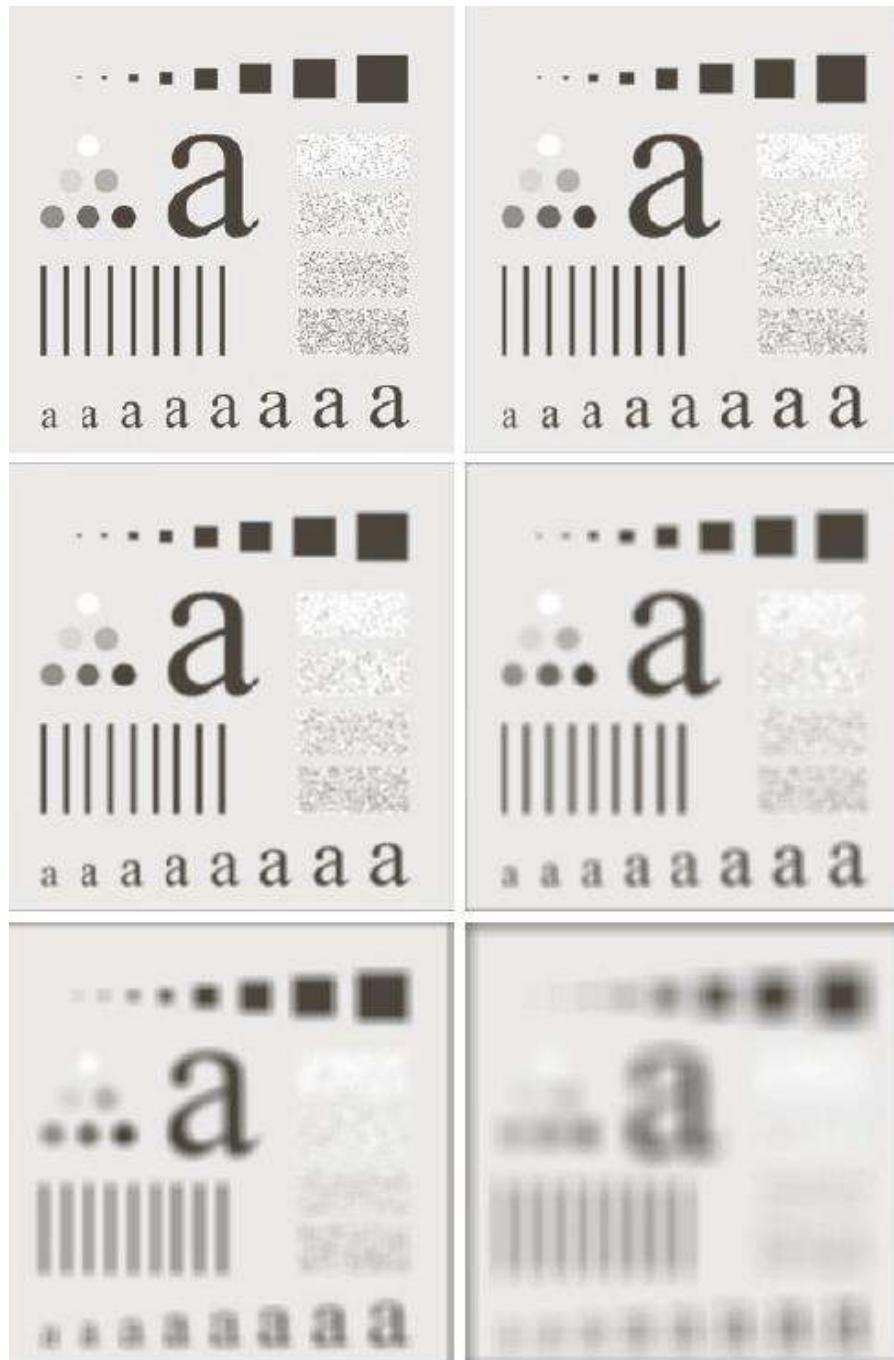
# Filtros passa-baixa

- Suavização, borrado etc.

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$$g(x, y) = \frac{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)}{\sum_{s=-a}^a \sum_{t=-b}^b w(s, t)}$$

# Filtros passa-baixa



**FIGURE 3.33** (a) Original image, of size  $500 \times 500$  pixels (b)–(f) Results of smoothing with square averaging filter masks of sizes  $m = 3, 5, 9, 15, 25, 35, 45$ , and  $55$ , respectively. The black squares at the top are of sizes  $3, 5, 9, 15, 25, 35, 45$ , and  $55$  pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size  $50 \times 120$  pixels.

Efeitos de borda!

# Filtros passa-baixa

Exemplo: Lena



# Filtros passa-baixa

Exemplo: Lena  
Filtro de média



Ordem: 3



Ordem: 9

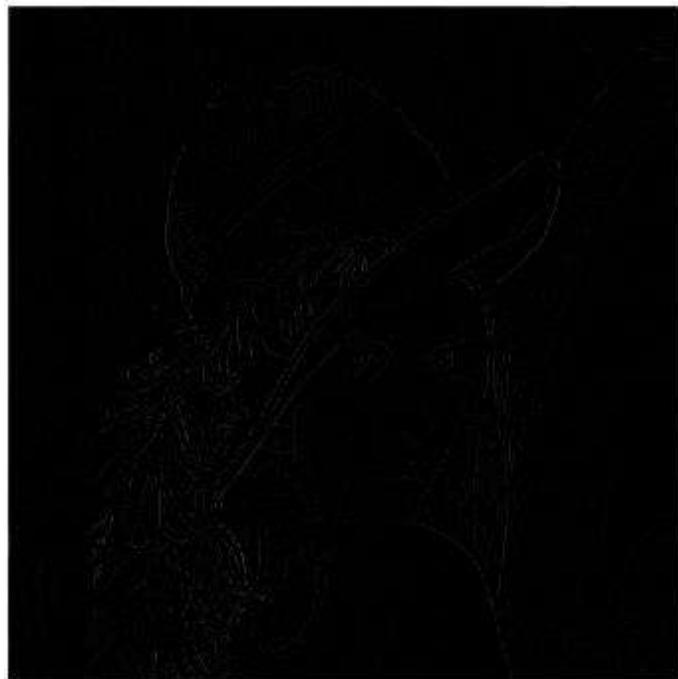


Ordem: 15

# Filtros passa-baixa

Exemplo: Lena

Filtro de média (diferença para a imagem original)



Ordem: 3



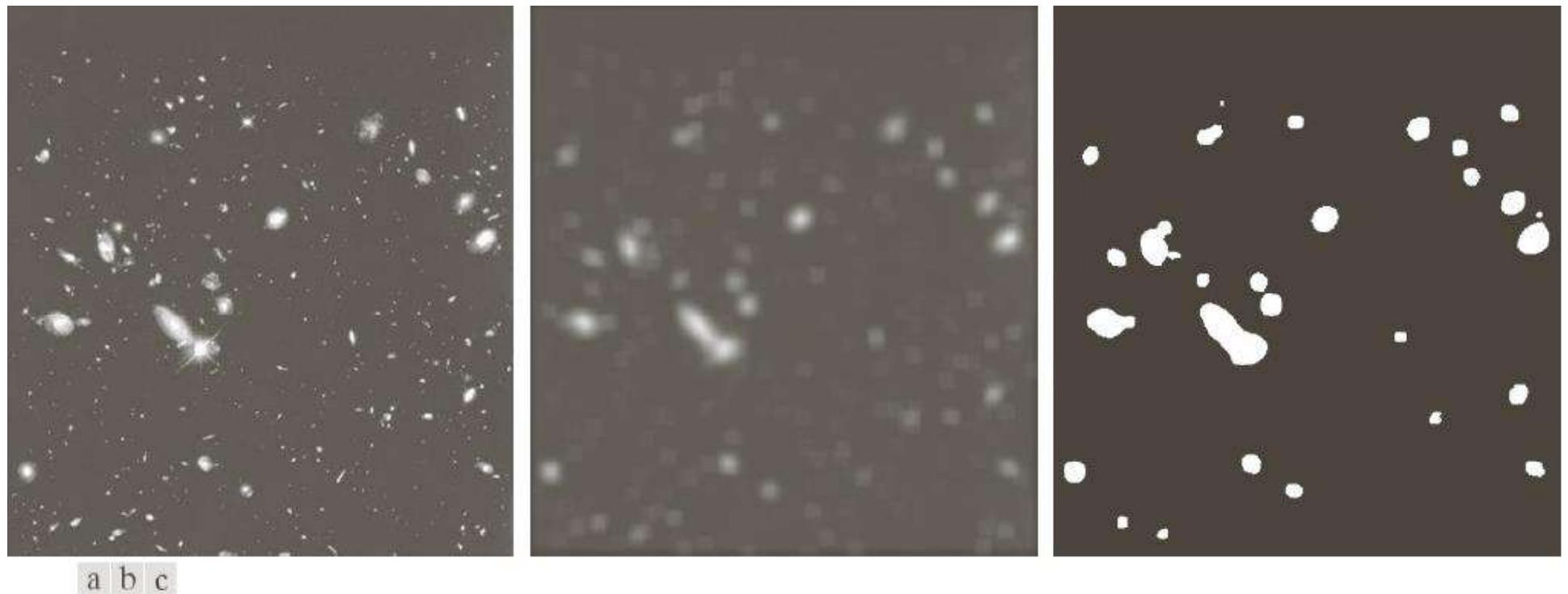
Ordem: 9



Ordem: 15

## Extração de forma / Eliminação de detalhes

Passa-baixa + Thresholding (limiar)



**FIGURE 3.34** (a) Image of size  $528 \times 485$  pixels from the Hubble Space Telescope. (b) Image filtered with a  $15 \times 15$  averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

Extração de formas / Remoção de detalhes  
Filtros passa-baixa + limiarização (thresholding)

Exemplo: Lena

Utilizando filtro de média



Ordem: 3



Ordem: 9



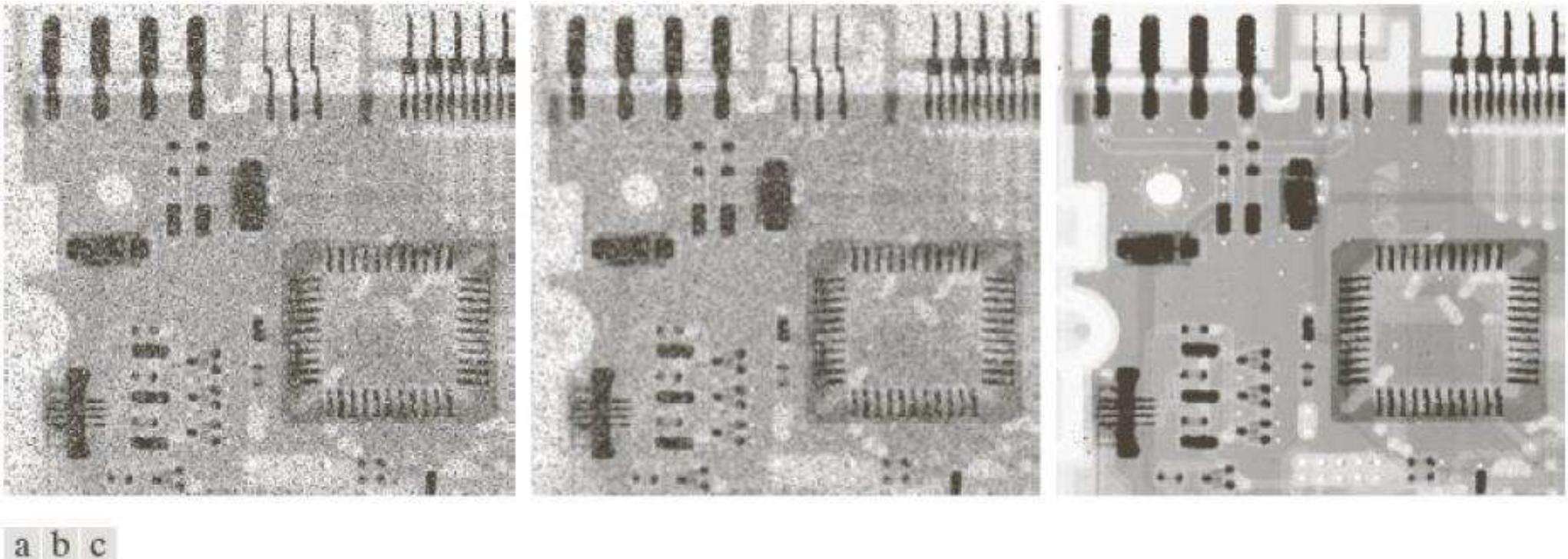
Ordem: 15

# Filtros

- Estatísticos
  - Mediana
    - Altera pixels que diferem muito da mediana da vizinhança
      - Sal e Pimenta
  - Máximo
  - Mínimo
  - Percentual

# Mediana

Ruído sal & pimenta: filtro de média VS filtro de mediana



a b c

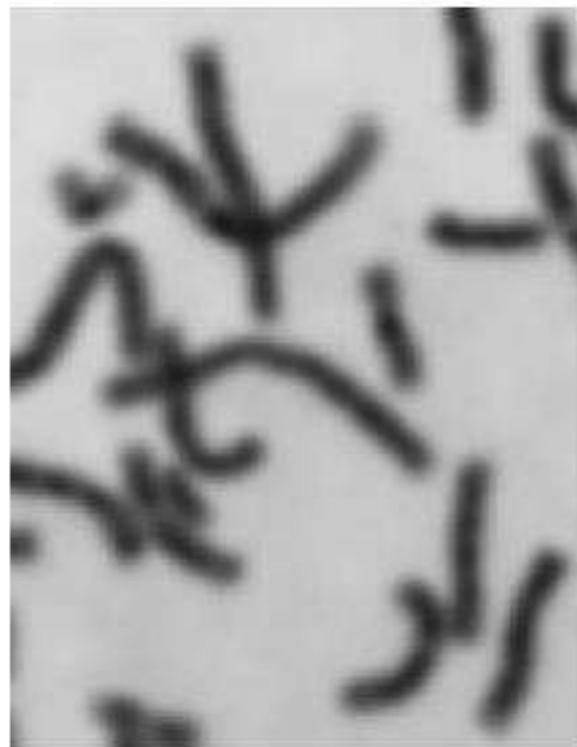
**FIGURE 3.35** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a  $3 \times 3$  averaging mask. (c) Noise reduction with a  $3 \times 3$  median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

# Mediana

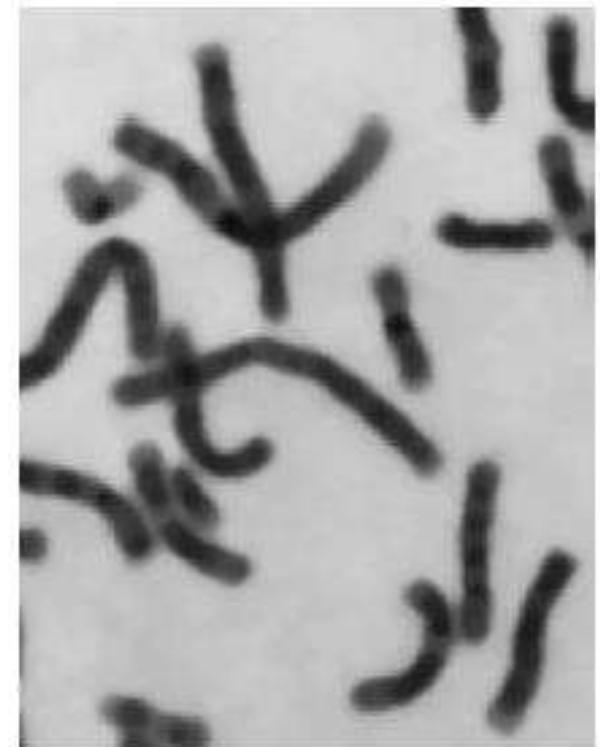
## filtro de média VS filtro de mediana



(a)



(b)



(c)

(a) Original karyotype (chromosome image). (b) Image smoothed with a Gaussian filter. (c) Image filtered with a median filter.

# Filtros passa-alta

- Identificação e aguçamento de picos, bordas etc.
- Derivada discreta (na direção x)

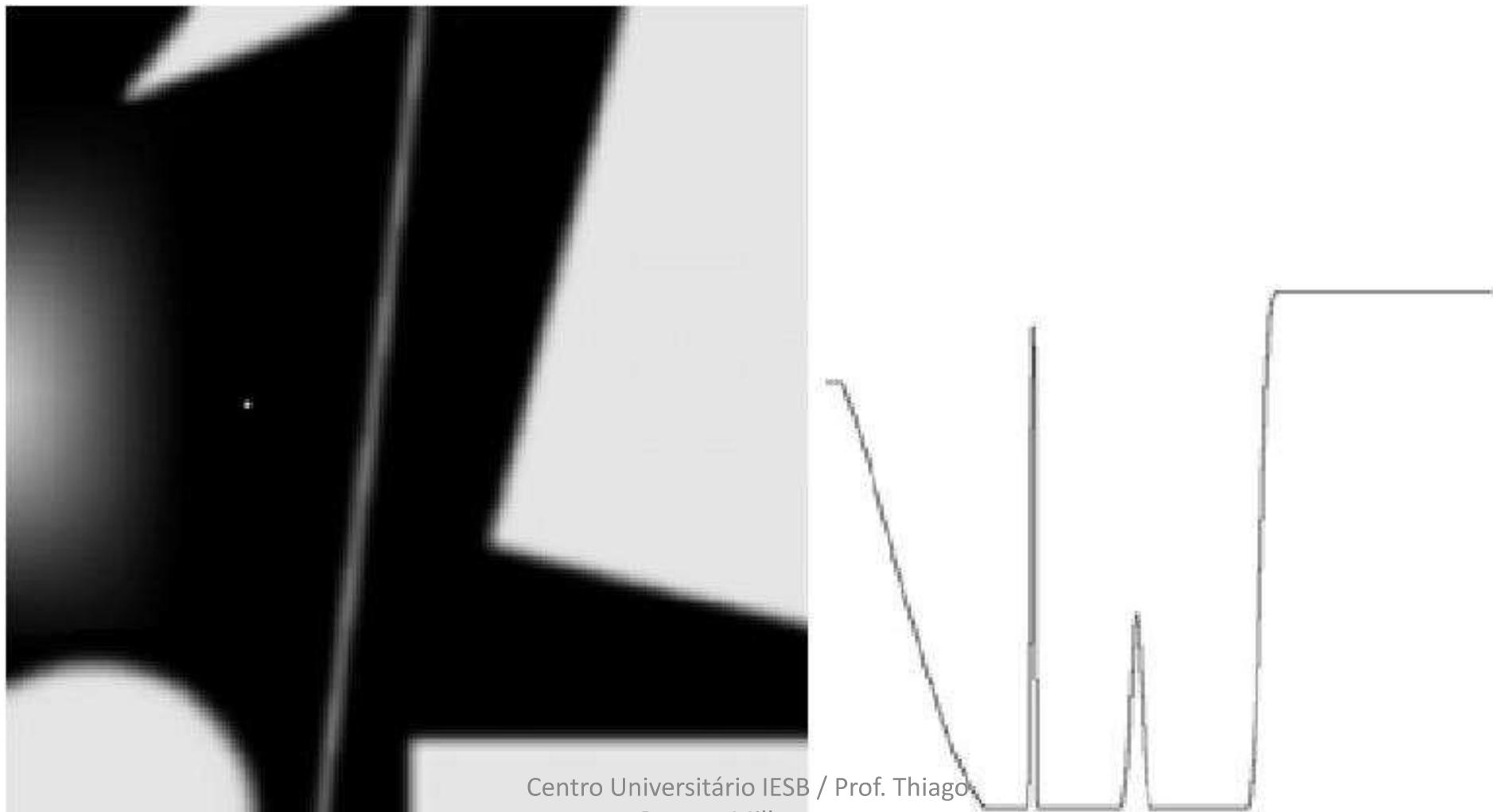
$$\frac{\partial f}{\partial x} = f(x + 1) - f(x)$$

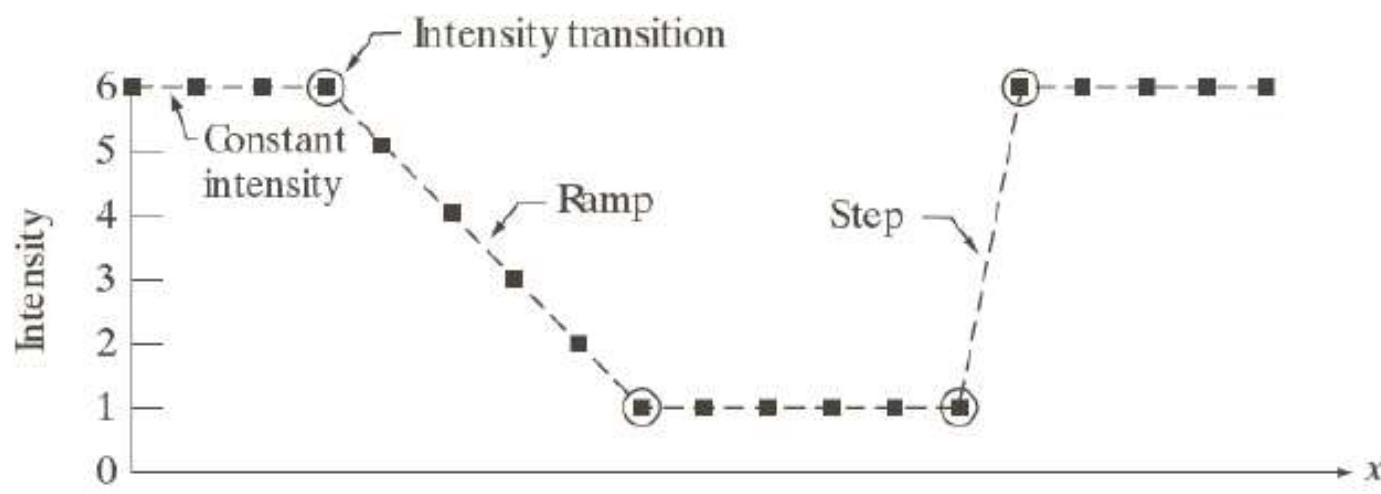
- 2ª derivada discreta (na direção x)

$$\frac{\partial^2 f}{\partial x^2} = f(x + 1) + f(x - 1) - 2f(x)$$

# Filtros passa-alta

Derivada discreta ao longo de uma linha horizontal





a  
b  
c

**FIGURE 3.36**

Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

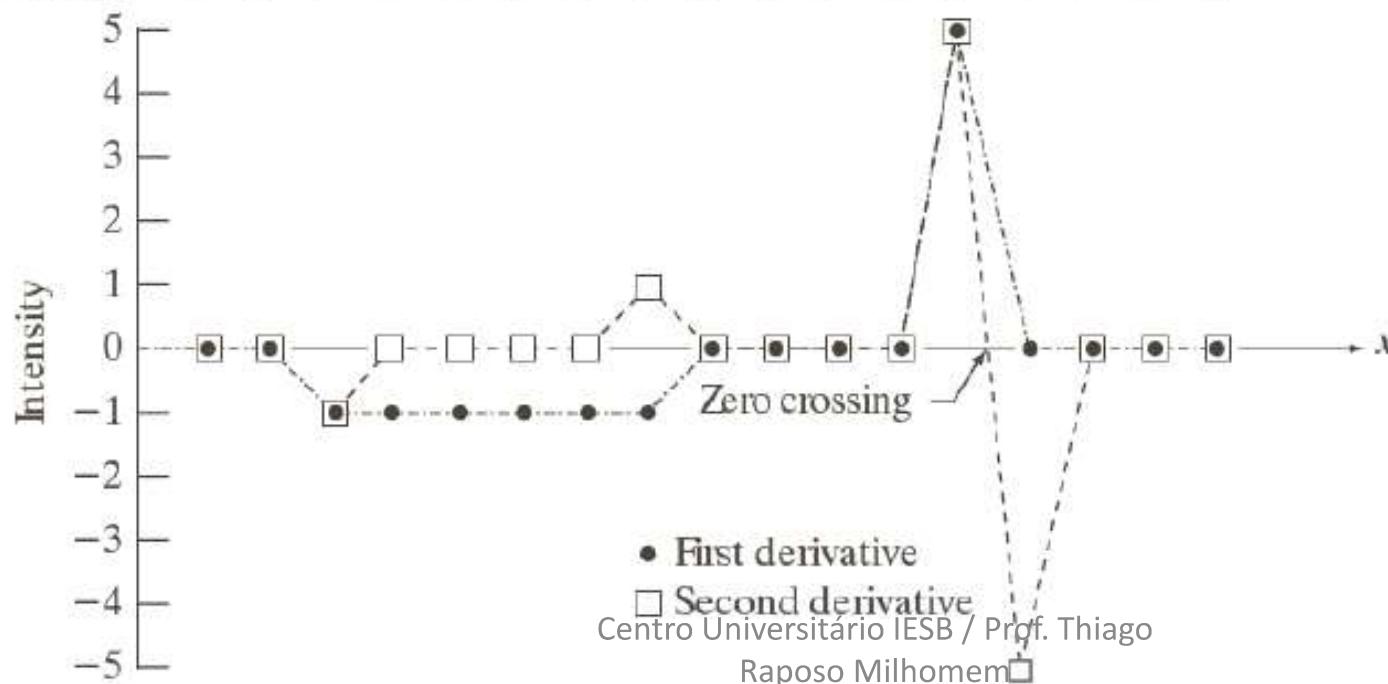
Scan line      

6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

 $\rightarrow x$

1st derivative    0    0   -1   -1   -1   -1   -1   0   0   0   0   0   0   5   0   0   0   0

2nd derivative    0    0   -1   0   0   0   0   1   0   0   0   0   0   5   -5   0   0   0



# Filtros passa-alta

- Detecção de picos e bordas
  - Laplaciano

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x + 1, y) + f(x - 1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y + 1) + f(x, y - 1) - 2f(x, y)$$

$$\nabla^2 f = [f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1)] - 4f(x, y)$$

# Passa-alta: Laplaciano

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1
0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases}$$

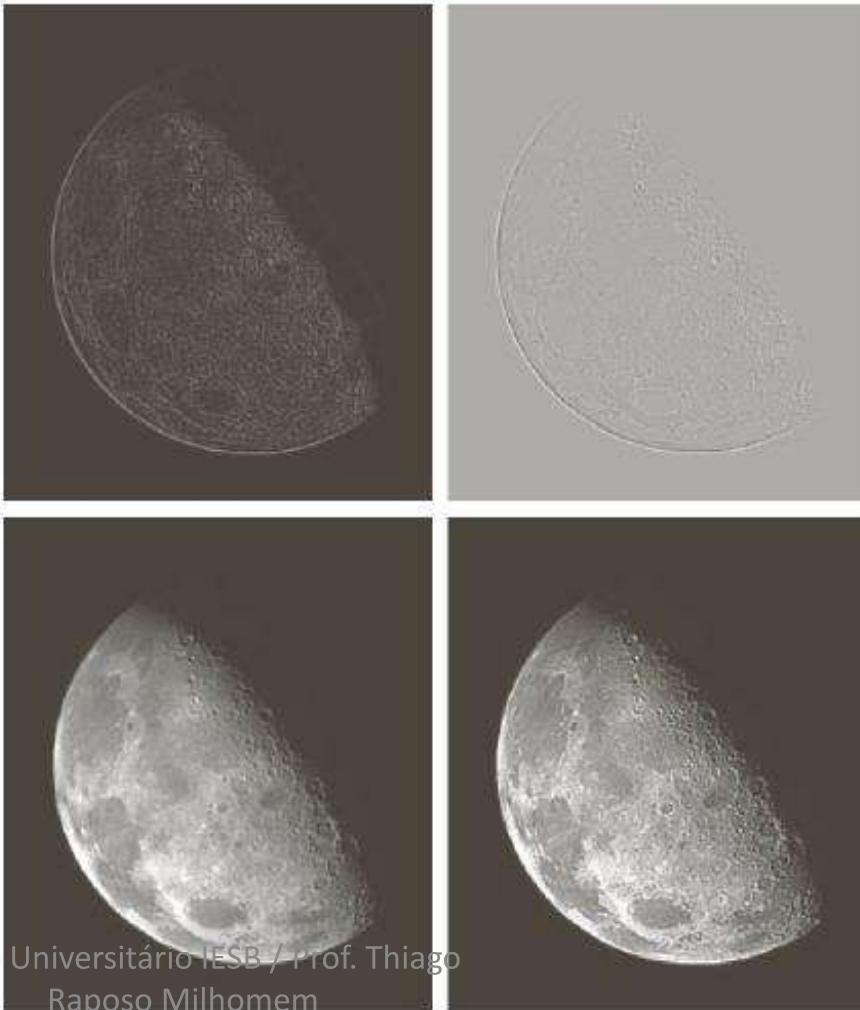
if the center coefficient of the Laplacian mask is negative  
if the center coefficient of the Laplacian mask is positive.

a  
b c  
d e



**FIGURE 3.38**

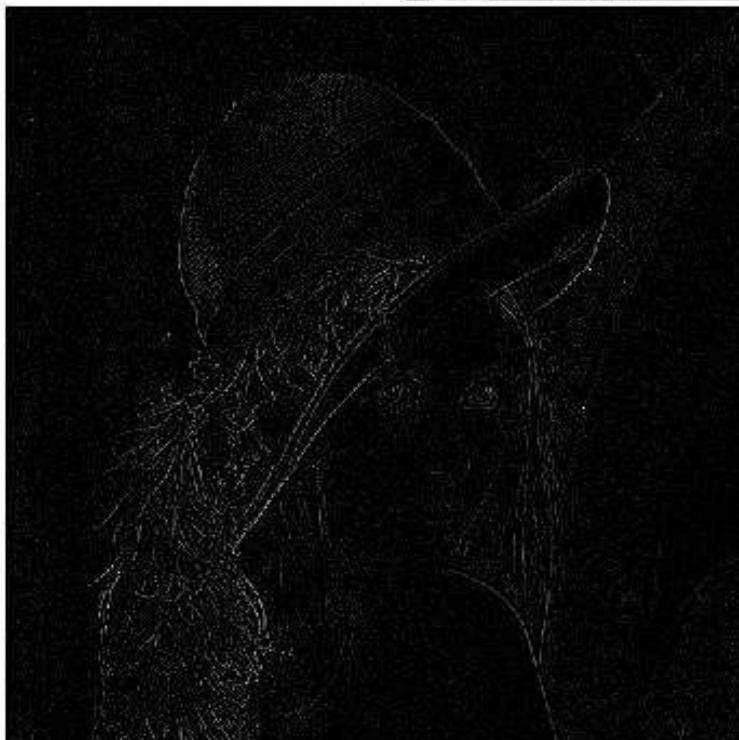
- (a) Blurred image of the North Pole of the moon.  
(b) Laplacian without scaling.  
(c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b).  
(Original image courtesy of NASA.)



Laplaciano



$h(0,0) = 4$



$h(0,0) = 8$



**Lembrando:** Extração de formas / Remoção de detalhes  
Filtros passa-baixa + limiarização (thresholding)

Exemplo: Lena

Utilizando filtro de média



Ordem: 3



Ordem: 9



Ordem: 15

# Obtenção de bordas principais

Filtros passa-baixa + limiarização (thresholding) + passa-alta

Exemplo: Lena

Utilizando filtro de média + Passa-alta Laplaciano (fixo, com  $h(0,0)=8$ )



Ordem: 3



Ordem: 9



Ordem: 15

# Obtenção de bordas principais

Filtros passa-baixa + limiarização (thresholding) + passa-alta

Exemplo: Lena

Utilizando filtro de média + Passa-alta Laplaciano (fixo, com  $h(0,0)=8$ )  
+ valor absoluto (reforço das bordas obtidas com Laplaciano)



Ordem: 3



Ordem: 9



Ordem: 15

# Realce: unsharp masking

$$i) \quad \bar{f}(x, y) = h_{LP}(x, y) * f(x, y)$$

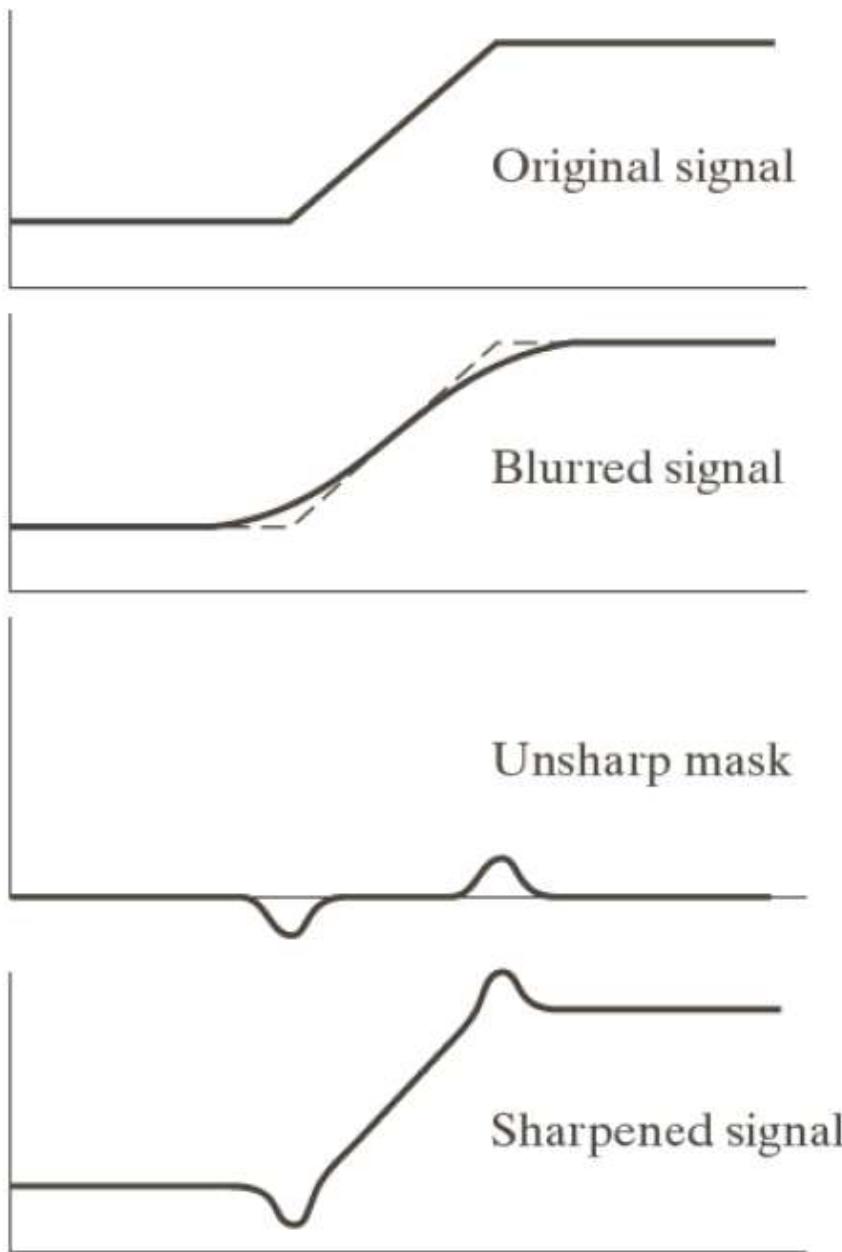
$$ii) \quad m(x, y) = f(x, y) - \bar{f}(x, y)$$

$$iii) \quad g(x, y) = f(x, y) + A \cdot m(x, y)$$

# Realce: unsharp masking

a  
b  
c  
d

**FIGURE 3.39** 1-D illustration of the mechanics of unsharp masking.  
(a) Original signal.  
(b) Blurred signal with original shown dashed for reference.  
(c) Unsharp mask.  
(d) Sharpened signal, obtained by adding (c) to (a).



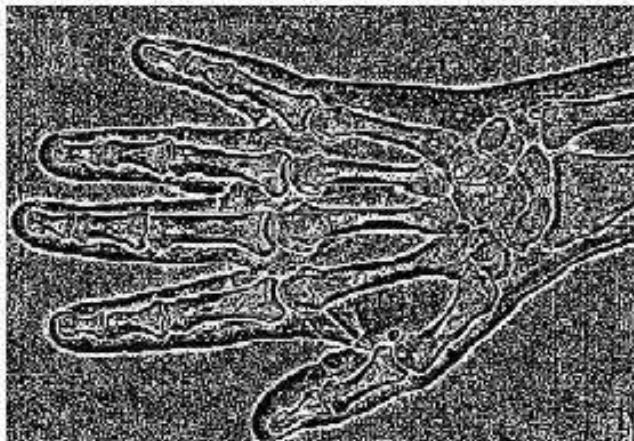
# Realce: unsharp masking



(a)



(b)



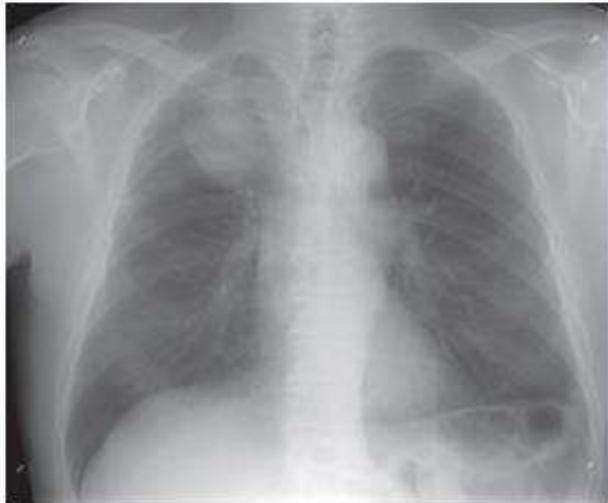
(c)



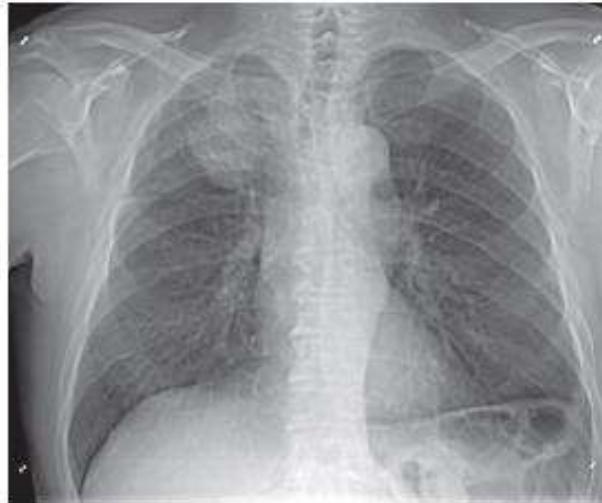
(d)

Radiography of a hand. **(a)** Original image  $I$ . **(b)** Smoothed image  $g * I$  with  $g$  a  $3 \times 3$  averaging filter. **(c)** Edges  $I - g * I$  of the image. **(d)** Unsharp masked image ( $\alpha = 5$ ).

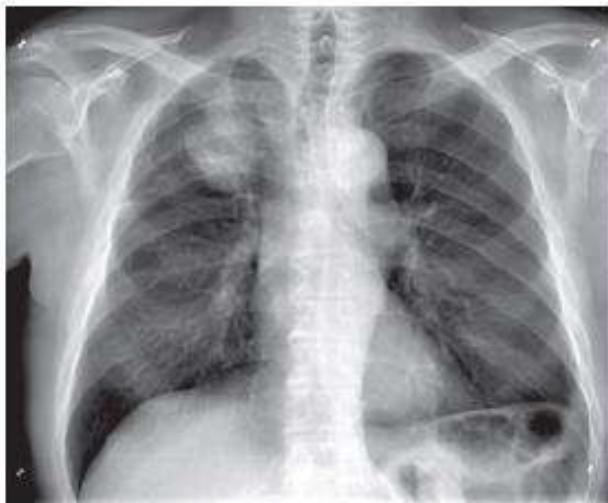
# Realce: unsharp masking



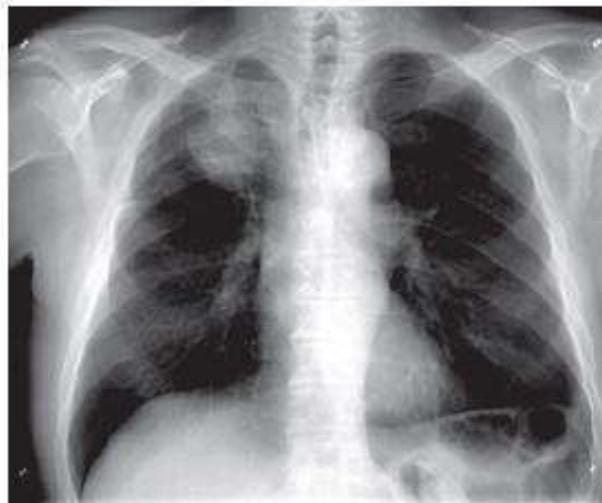
(a)



(b)



(c)



(d)

The effect of the filter size in unsharp masking. (a) Original image ( $1024 \times 1248$  pixels). Unsharp masking with filter size (b) 10, (c) 60, and (d) 125. Image (b) shows enhanced fine details but an overall reduction of the contrast. In image (d), large-scale variations, which correspond to the lungs and the mediastinum, are enhanced, and most of the small details are suppressed. Image (c) shows a case somewhere between (b) and (d).

# Unsharp Masking

Exemplo: Lena

Utilizando filtro de média e  $A=1$



Ordem: 3



Ordem: 9



Ordem: 15

# Unsharp Masking

Exemplo: Lena

Utilizando filtro de média e  $A=3$



Ordem: 3



Ordem: 9



Ordem: 15

# Unsharp Masking

Exemplo: Lena

Utilizando filtro de média e  $A=5$



Ordem: 3



Ordem: 9



Ordem: 15