The University of Mindanao

College of Arts and Sciences Education
General Education -Mathematics
2nd Floor, DPT Building, Ma tina Campus, Davao City
Phone No.: (082)300-5456/305-0647 Local 134

DISCRETE MATHEMATICS

Let's check

Raiven Russell B. Suralta **IT2(6605)**

Practice Exercise 2 (Topic: Recurrence Relation)

Solve the following problems related to sequences and recurrence relations.

1.List the first 10 terms of each of these sequences.

c. the sequence whose nth term is 3n − 2n.

n = 1; 3(1) – 2(1) = 1
n = 2; 3(2) – 2(2) = 2
n = 3; 3(3) – 2(3) = 3
n = 4; 3(4) – 2(4) = 4
n = 5; 3(5) – 2(5) = 5
n = 6; 3(6) – 2(6) = 6
n = 7; 3(7) – 2(7) = 7
n = 8; 3(8) – 2(8) = 8
n = 9; 3(9) – 2(9) = 9
n = 10; 3(10) – 2(10) =10 **{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}**

d. the sequence whose nth term is n! − 2n.

n = 1 / (1!) - 2(1) = 1 - 2 = - 1
n = 2 / (2!) - 2(2) = 2 - 4 = - 2
n = 3 / (3!) - 2(3) = 6 - 6 = 0
n = 4 / (4!) - 2(4) = 24 - 8 = 16
n = 5 / (5!) - 2(5) = 120 - 10 = 110
n = 6 / (6!) - 2(6) = 720 - 12 = 708
n = 7 / (7!) - 2(7) = 5040 - 14 = 5026
n = 8 / (8!) - 2(8) = 40320 - 16 = 40304
n = 9 / (9!) - 2(9) = 362880 - 18 = 362862
n = 10 / (10!) - 2(10) = 3628800 - 20 = 3628780

{-1, -2, 0, 16, 110, 708, 5026, 40304, 362862, 3628780}

2. Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

c. $a_n = a_{n-1} + 3a_{n-2}; a_0 = 1 \& a_1 = 2$

Given:

$$a_n = a_{n-1} + 3a_{n-2}$$

$$a_0 = 1, a_1 = 2$$

$$a_2 = a_1 + 3a_0 = 2 + 3(1) = 5$$
$$a_3 = a_2 + 3a_1 = 5 + 3(2) = 11$$
$$a_4 = a_3 + 3a_2 = 11 + 3(5) = 26$$

First 5 terms:
{1, 2, 5, 11, 26}

d. $a_n = na_{n-1} + n^2 a_{n-2}; a_0 = 1 \& a_1 = 1$

Given:

$a_n = na_{n-1} + n^2 a_{n-2}$

$a_0 = 1, a_1 = 1$

$a_2 = 2a_1 + 2^2 a_0 = 2(1) + 4(1) = 6$

$a_3 = 3a_2 + 3^2 a_1 = 3(6) + 9(1) = 18 + 9 = 27$

$a_4 = 4a_3 + 4^2 a_2 = 4(27) + 16(6) = 108 + 96 = 204$

First 5 terms:
{1, 1, 6, 27, 204}

3. What are the values of these sums?

c. $\sum_{j=0}^{4}(-2)^{j}$

$(-2)^0 = 1$

$(-2)^1 = -2$

$(-2)^2 = 4$

$(-2)^3 = -8$

$(-2)^4 = 16$

$1 - 2 + 4 - 8 + 16 = 11$

d. $\sum_{m=0}^{8}[1+(-1)^m]$

$$1+(-1)^0=1+1=2$$

$$1+(-1)^1=1+(-1)=0$$

$$1+(-1)^2=1+1=2$$

$$1+(-1)^3=1+(-1)=0$$

$$1+(-1)^4=1+1=2$$

$$1+(-1)^5=1+(-1)=0$$

$$1+(-1)^6=1+1=2$$

$$1+(-1)^7=1+(-1)=0$$

$$1+(-1)^8=1+1=2$$

$$2+0+2+0+2+0+2+0+2=10$$

4. Compute each of these double sums.

a. $\sum_{i=1}^{2}\sum_{j=1}^{3}(i+j)$

For $i=1$:

- $j=1:1+1=2$
- $j=2:1+2=3$
- $j=3:1+3=4$

Sum: $2+3+4=9$

For $i = 2$:

- $j = 1 : 2 + 1 = 3$
- $j = 2 : 2 + 2 = 4$
- $j = 3 : 2 + 3 = 5$

Sum: $3 + 4 + 5 = 12$

Total sum: $9 + 12 = 21$

b. $\sum_{i=0}^{3}\sum_{j=0}^{3}(2i+3j)$

For $i = 0$:

- $j = 0 : 0 + 0 = 0$
- $j = 1 : 0 + 3 = 3$
- $j = 2 : 0 + 6 = 6$
- $j = 3 : 0 + 9 = 9$ → sum = 18

For $i = 1$ (2i = 2):

- $j = 0 : 2 + 0 = 2$
- $j = 1 : 2 + 3 = 5$
- $j = 2 : 2 + 6 = 8$
- $j = 3 : 2 + 9 = 11$ → sum = 26

For $i = 2$ (2i = 4):

- $j = 0 : 4 + 0 = 4$
- $j = 1 : 4 + 3 = 7$
- $j = 2 : 4 + 6 = 10$
- $j = 3 : 4 + 9 = 13$ → sum = 34

For $i = 3$ (2i = 6):

- $j = 0 : 6 + 0 = 6$
- $j = 1 : 6 + 3 = 9$
- $j = 2 : 6 + 6 = 12$
- $j = 3 : 6 + 9 = 15$ → sum = 42

Now total:

$18 + 26 + 34 + 42 = 120$