Lab Assignment 1

Alex van Rijswijck
EEE4114F
University of Cape Town
VRJALE001

I. Question 1

The following two signals were produced and plotted using MATLAB.

$$x5[n] = 3cos(2\pi/10 \times n)$$

$$x6[n] = 2cos(2\pi/12 \times n + \pi/4)$$

In figure one above the signals X5 and X6 are displayed using the MATLAB stem function. The complete code can be found in the appendix at the end of this document.

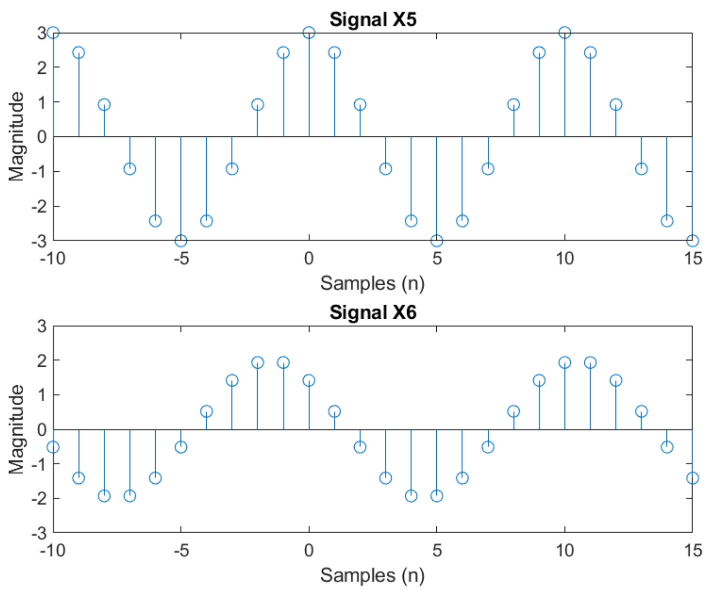


Figure 1. Signals X6 and X5

II. Code

```
%%Sample space:
    step = 1;
    starts = -10;
    ends = 25;
    n = (starts:1:ends)';
 %Signals
    x5 = 3*cos((2*pi)/10*n);
    x6 = 2*cos((2*pi)/12*n+pi/4);
 %%Plot:
    g = figure;
    subplot(2,1,1),stem(n,x5),xlabel('Samples (n)'),ylabel('Magnitude'),title('Signal X5'
    subplot(2,1,2),stem(n,x6),xlabel('Samples (n)'),ylabel('Magnitude'),title('Signal X6'
    print(g, '-dpng', 'Question1.png')
```

III. Question 2

In this question the input and output signals of a moving average low-pass filter system were investigated. The input and output relationship is described in the equation below.

$$y[n] = \sum_{k=0}^{6} h[k]x[n-k]$$

With the impulse response h[n]

$$h[n] = 1/7\Big\{0 <= n <= 6, else = 0\Big\}$$

And the input signal x[n]

$$x[n] = u[n]cos(\omega n)$$

With u[n] being a right sided unit step function and $\omega = \pi/10$

The following plots were produced showing the input and output signals

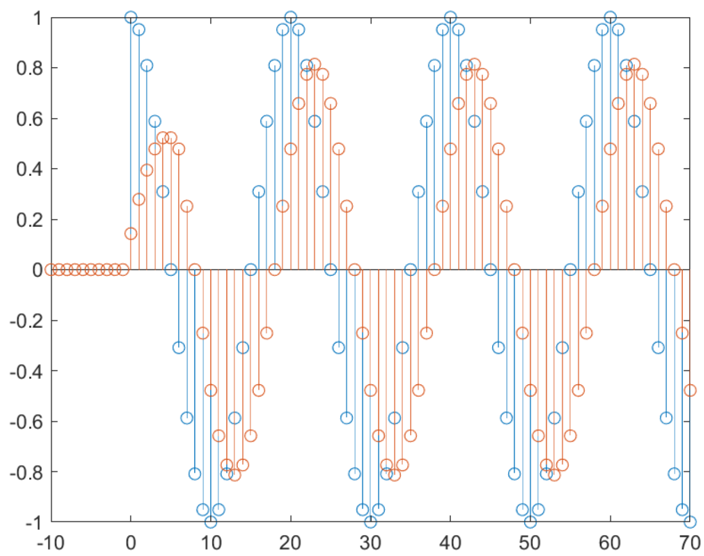


Figure 2. Input (blue) and Output (red) signals of a low pass filter

From the above figure it is clear that the output signal is attenuated and shifted by a few samples. The following results were obtained

Amplitude of x: **1**
Amplitude of y: **0.81367**
difference in amplitude: **0.18633**
The output signal was shifted by 3 samples and since there is a frequency of $\pi/10$ radians per sample, this results in a total phase shift of $3\pi/10$ **radians.**

IV. Code

```
%%Sample space:
    step = 1;
    starts = -10;
    ends = 70;
    n = (starts:1:ends)';
%%Signals
    g = figure;
    u = zeros(size(n));
    u(n>=0) = 1;
    x = u.*cos(pi/10*n);
    stem(n,x);
    hold on;
    h = zeros(size(n));
    h(0<=n) = 1/7;
    h(n>6) = 0;
    %z = conv(h,x);
    y = zeros(size(n));
    for i=1:81
        y(i)=0;
        for k=0:6
            if (i-k)>0
                y(i) = y(i)+x(i-k);
```

```
            end

        end
        y(i) = y(i)*1/7;
    end
    stem(n,y);
    hold off;
    disp("Amplitude of x: "+max(x))
    disp("Amplitude of y:"+max(y))
    disp("difference in amplitude: "+(max(x)-max(y)))
    disp("Y is shifted by 3 sample points")
    print(g, '-dpng', 'Question2.png')
```

V. Question 3

In this question we investigate the output signal when a complex exponential is an input to the low pass filter system.

$$x[n] = u[n]e^{j\omega n}$$

With $\omega = \pi/10$ and u[n] a right sided unit step function. While the overall system response is still given by the following relationship

$$y[n] = \sum_{k=0}^{6} h[k]x[n-k]$$

With an impulse response h[n]

$$h[n] = 1/7\Big\{0 <= n <= 6, else = 0\Big\}$$

Below the input and output signals were plotted. It should be noted that since the output is expected to be a complex function, its real and imaginary parts were plotted separately.The same applies for the input signal. From the figure above we can conclude that the output signal is indeed a complex exponential that

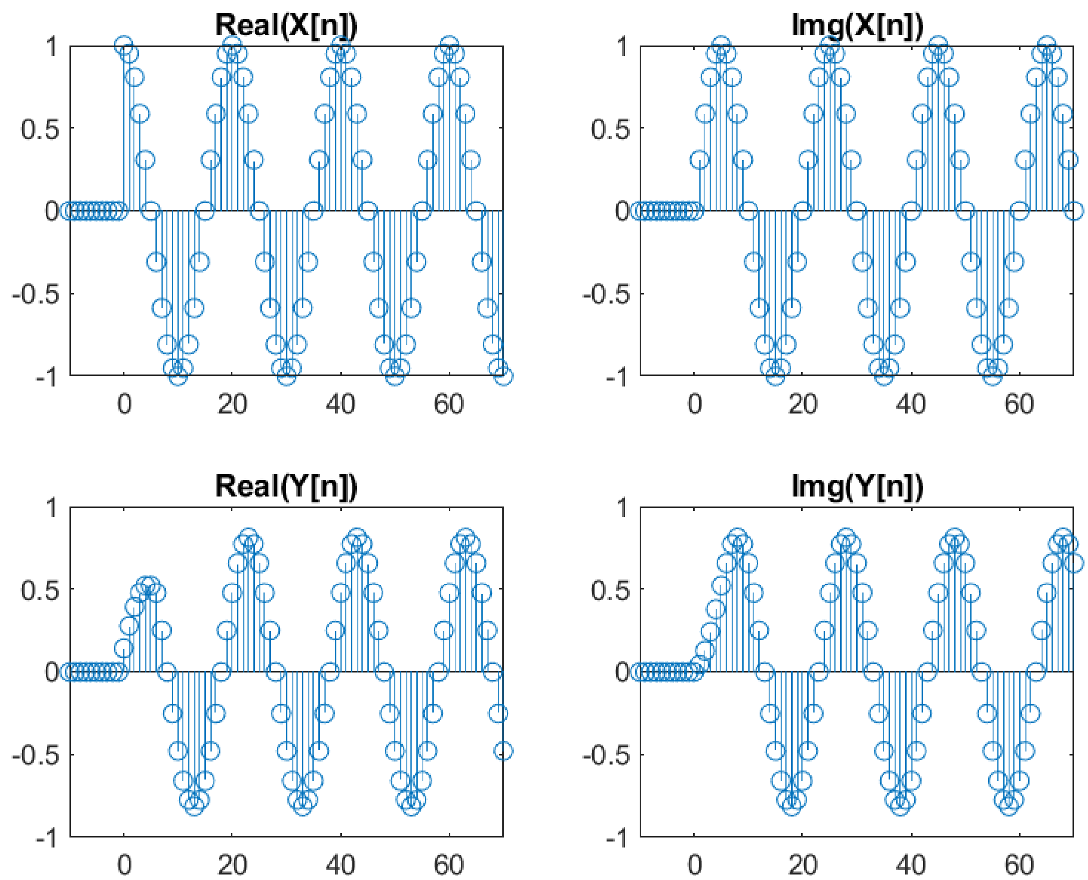


Figure 3. Input (blue) and Output (red) signals of a low pass filter

has been scaled and shifted relative to the input signal. The following relationship describes the result.

$$y[n] = Ae^{j\phi}e^{j\omega n}$$

The values for A and ϕ can then be determined experimentally, where A can be determined by taking the Absolute value of y[n]. It is important to note that this returns an array of values and taking a large value of this array (to avoid transients) results in an estimated value of A. ϕ can be determined by taking the phase of the division of Y[n] by X[n], since the $e^{j\omega n}$ component cancels leaving only $e^{j\phi}$. In both the calculation of phi and A the built in MATLAB functions were used.

A **= 0.81367**
ϕ **= -0.94248**

VI. Code

```
%%Sample space:
    step = 1;
    starts = -10;
    ends = 70;
    n = (starts:1:ends)';
%%Signals
```

```
g = figure;
u = zeros(size(n));
u(n>=0) = 1;
x = u.*exp(1j*pi/10*n);
subplot(2,2,1),stem(n,real(x)),title('Real(X[n])');
subplot(2,2,2),stem(n,imag(x)),title('Img(X[n])');
y = zeros(size(n));
for i=1:81
    y(i)=0;
    for k=0:6
        if (i-k)>0
           y(i) = y(i)+x(i-k);
        end

    end
    y(i) = y(i)*1/7;
end
subplot(2,2,3),stem(n,real(y)),title('Real(Y[n])');
subplot(2,2,4),stem(n,imag(y)),title('Img(Y[n])');
print(g, '-dpng', 'Question3.png');
%%Calculating A and phi
A = abs(y); %since original magnitude is 1, tested by abs(y./x) == abs(y)
phi = angle(y./x);
%% choosing large value of array to avoid transients
A_num = A(50);
phi_num = phi(50);
disp("Magnitude of frequency response: "+A_num);
disp("Phase of frequency response: "+phi_num);
```

VII. Question 4

In this question we investigate the formal definition of the frequency response when the input signal is a complex exponential $x[n] = u[n]e^{j\omega n}$. The response is given by

$$y[n] = \left(\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}\right)e^{j\omega n}$$

$$y[n] = H(e^{j\omega})e^{j\omega n}$$

From the above definition a the formula for the frequency response of the system is given by

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

Using MATLAB the frequency response was calculated for the moving average low-pass filter system at $\omega = \pi/10$, $\omega = 2\pi/5$, $\omega = 4\pi/5$

$$H(e^{j\pi/10}) = 0.81367e^{-j0.94248}$$

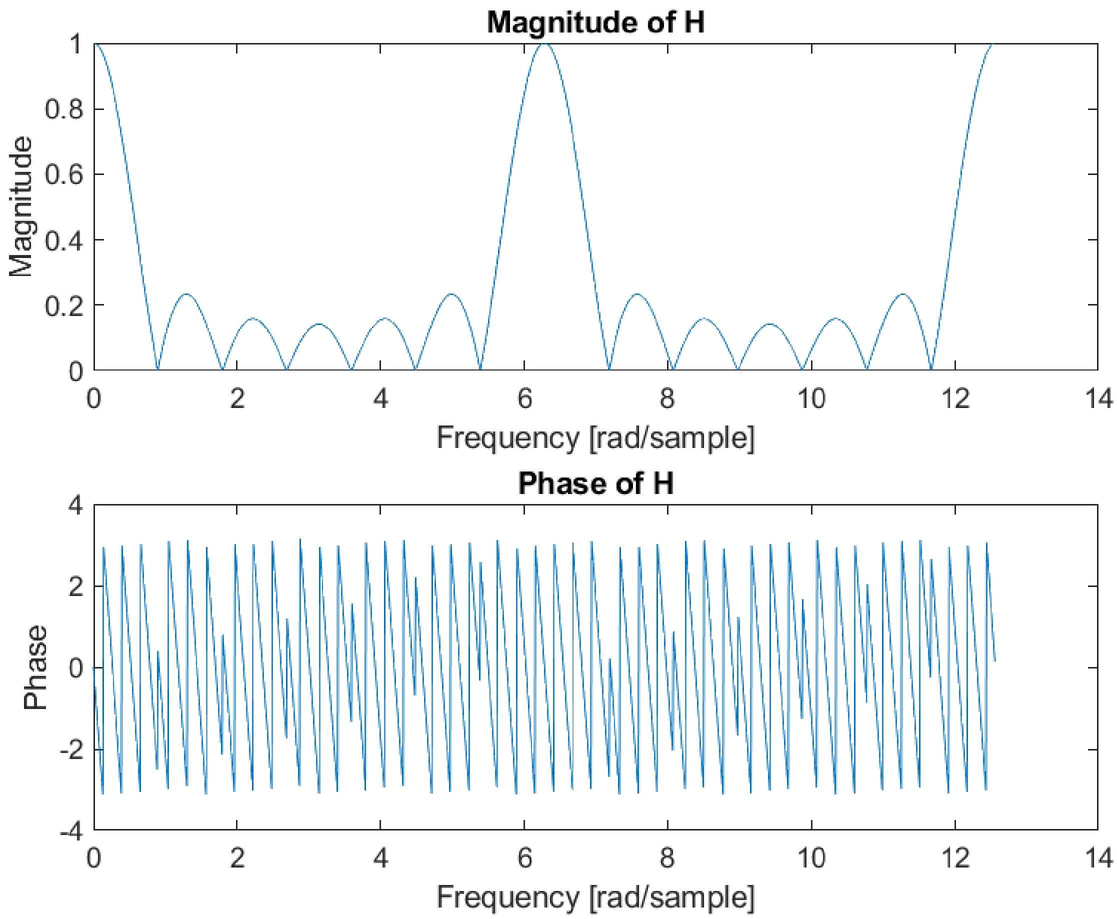
$$H(e^{j2\pi/5}) = 0.23115e^{-j0.62832}$$

$$H(e^{j4\pi/5}) = 0.088291^{-j1.2566}$$

Clearly we can see that as the frequency increases the value of A decreases, thus confirming that the system is acting as a low pass filter.

A function was created in order to plot the frequency response that takes in a impulse response h[n] and a frequency ω, and returns the value of $H(e^{j\omega})$. The impulse response supplied to the function was that of the moving average filter.

The magnitude and phase of the frequency response is shown below



VIII. Code

```
%%frequency response function
g = figure;
starts = -10;
ends = 70;
n = (starts:1:ends)';
h = zeros(size(n));
h(0<=n) = 1/7;
h(n>6) = 0;
omega = (0:0.01:4*pi)';
H = freqresp(h,omega,starts);
subplot(2,1,1),plot(omega,abs(H)),xlabel('Frequency [rad/sample]'),ylabel('Magnitude'),t
subplot(2,1,2),plot(omega,angle(H)),xlabel('Frequency [rad/sample]'),ylabel('Phase'),tit
print(g, '-dpng', 'Question4.png');
function H = freqresp(h, omega, startindex)
    H = 0;
    for k=1:size(h)
        H = H + h(k)*exp(-1j*omega*(k-startindex));
    end
end
```