μSR superconducting gap fitting function

Python code: https://github.com/naufal151/gap-function-python

$$\frac{\lambda_{ac}^{-2}(T)}{\lambda_{ac}^{-2}(0)} = 1 + \frac{1}{\pi}\int_0^{2\pi}\int_{\Delta(\phi,T)}^{\infty}\frac{\delta f}{dE}\frac{EdEd\phi}{\sqrt{E^2 - \Delta_i(\phi,T)^2}}$$
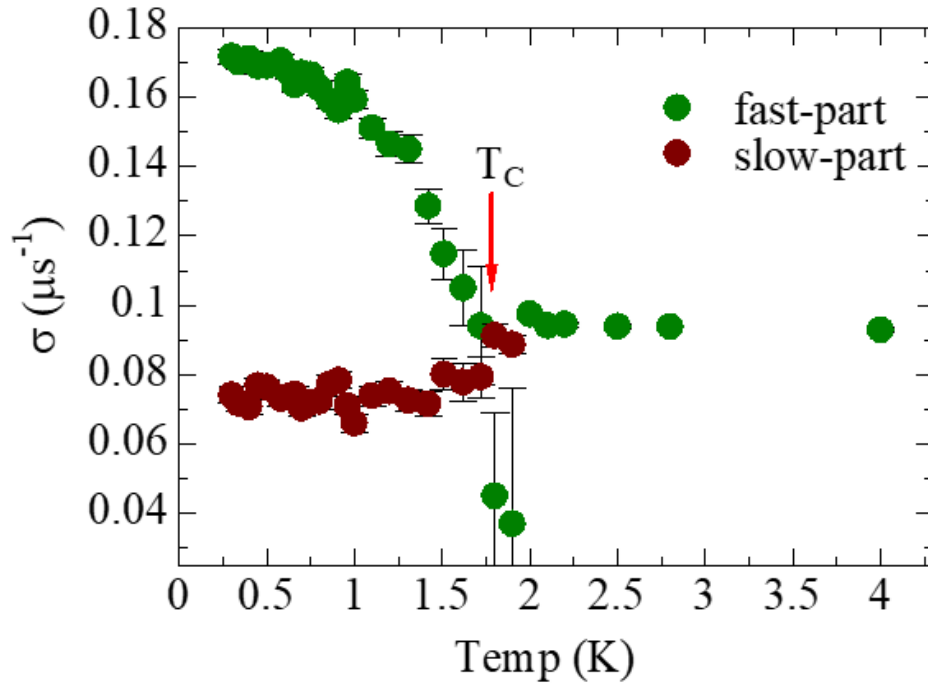
Where

$f = \frac{1}{1+e^{E/k_BT}}$ is the Fermi function

$\Delta_i(\phi,T) = \Delta(T)g(\phi)$ is the gap symmetry, and i indicates each gap symmetry, e.g., s- or d-wave

$\Delta(T) = \Delta(0)\tanh\left[1.82\left(1.018\left(\frac{T_C}{T} - 1\right)\right)^{0.51}\right]$ where $\Delta(0)$ is the fitting parameter

The material's T_C $(PtBi_{1-x}Se_x) = 2.2\ K$ from reference

Whereas from the data $T_C = 1.7\ K$

From the identity $\cosh^2 u - 1 = \sinh^2 u$

$\sqrt{E^2 - \Delta^2} = \sqrt{\Delta^2(\cosh^2 u - 1)} = \Delta\sinh u$

$$\therefore dE = \Delta \sinh u \, du$$

Then

$$\frac{E dE}{\sqrt{E^2 - \Delta^2}} = \frac{\Delta \cosh u \cdot \Delta \sinh u \, du}{\Delta \sinh u} = \Delta \cosh u$$

For the inner integral limits

When $E = \Delta$, then $\cosh u - 1 = 1 \Rightarrow u = 0$

When $E = \infty$, then $\cosh u \to \infty \Rightarrow u \to \infty$

Then the equation became

$$\frac{\lambda_{ac}^{-2}(T)}{\lambda_{ac}^{-2}(0)} = 1 + \frac{1}{\pi} \int_0^{2\pi} \int_0^{\infty} \Delta(\phi, T) \cosh u \, \frac{\delta f}{\delta E} |_{E=\Delta(\phi,T)\cosh u} \, du \, d\phi$$

Now solve for $\frac{\delta f}{\delta E}$

$$\frac{\delta f}{dE} = -\frac{e^{E/k_B T}}{(1 + e^{E/k_B T})^2} \frac{1}{k_B T} = -\frac{1}{4k_B T} \text{sech}^2 \left(\frac{E}{2k_B T} \right)$$

Now substitute the solved $\frac{\delta f}{\delta E}$ and $\Delta_i(\phi, T) = \Delta(T)g(\phi)$ to the equation

The equation explicitly became

$$\frac{\lambda_{ac}^{-2}(T)}{\lambda_{ac}^{-2}(0)} = 1 - \frac{1}{4\pi k_B T} \int_0^{2\pi} \int_0^{\infty} \left[\Delta(0)g(\phi) \tanh \left(1.82 \left(1.018 \left(\frac{T_c}{T} - 1 \right) \right)^{0.51} \right) \right]$$

$$\times \cosh u \, \text{sech}^2 \frac{\Delta(0)g(\phi) \tanh \left(1.82 \left(1.018 \left(\frac{T_c}{T} - 1 \right) \right)^{0.51} \right) \cosh u}{2k_B T} \, du d\phi$$
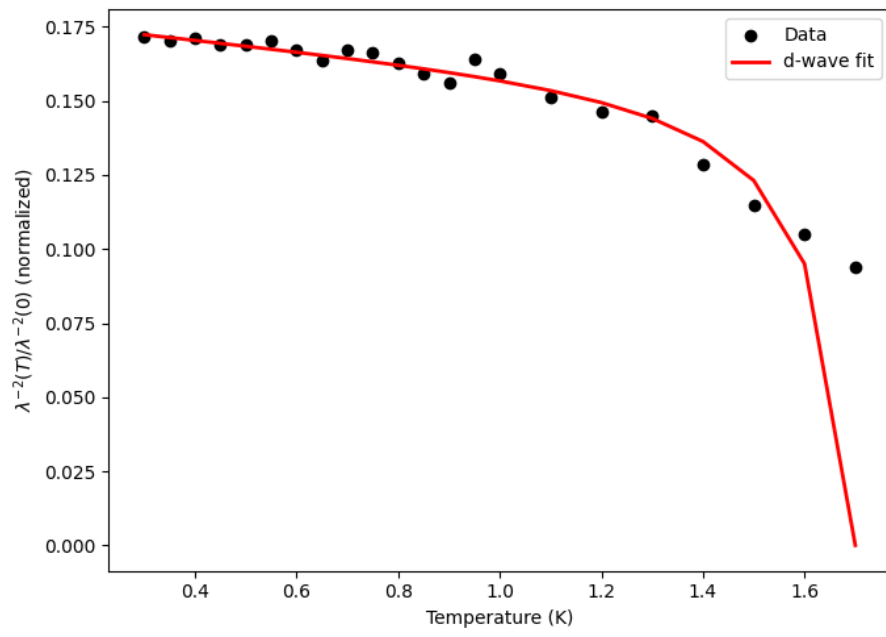
Define $\tanh \left(1.82 \left(1.018 \left(\frac{T_c}{T} - 1 \right) \right)^{0.51} \right) = t(T)$

The equation became

$$\frac{\lambda_{ac}^{-2}(T)}{\lambda_{ac}^{-2}(0)}$$

$$= 1 - \frac{1}{4\pi k_B T} \int_0^{2\pi} \int_0^{\infty} \Delta(0)g(\phi)t(T) \cosh u \, \text{sech}^2 \left(\frac{\Delta(0)g(\phi)t(T) \cosh u}{2k_B T} \right) du d\phi$$

Symmetry	$g(\phi)$
d-wave	$|\cos 2\phi|$
s-wave	1
s+d-wave	$\omega_s + (1 - \omega_s)|\cos 2\phi|$

The data was fitted using d-wave symmetry with Python code

And the gap obtained from fitting was $\Delta_d = 1.106\ meV$