§3.5. Categories with Finite Products

Det.

C: category.

E has finite products

Let Y { Ci} C & : finite family of objs. E &,

> TT C; € C

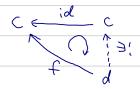
Ren.

@ OTE o object o product &id. terminal object o ix.

 $/\!/$

± €-----C

1 (15) o bject of product it. the object itself.



Prop. 3.5.1

Cicat. [] t e C; terminal

Va, b e C, Jaxbe E 233.

⇒ E has finite products.

と5に、

i) exe e (# bifurctor.

ii)] Xa,b,c: (a x b) xc \(a x (b x c) , natural in a, b, c \(e \).

iii)] \a: a \text{ txa , } } \begin{array}{c} & \text{ a \text{ \text{ a \text{ \text{ txa}}}} & \text{ natural in a \text{ \text{ \text{ b}}}}

し、 上の Rem. より、とかの、1、2 (国 nobject(s) aproduct を持つことははかる トラ3 (= 対しては、

 $(\cdots((a_1 \times a_2) \times a_3) \times \cdots) \times a_n$

to ar, ..., an or product letas. The induction on h zint.

axan 5" an,..., an o product 2" \$3 2 TE = 2 H" & 11 (18:8n-1) Vc+e, fi:c→ a: (1818n), @ a a universality sis, F! F; c→a s.t. f; = Pio F (∀i≤n-1). @ axan on universality bis. $\exists ', \Upsilon : C \longrightarrow A \times An \quad s.\tau.$ $f_n = p_n \circ \widetilde{f}$ $f' = p \circ \widetilde{f}$ F = Pof ti), F = (piop) of (4 : 5 m-1). to?, axan is a product of agrican. .". & has finite products $\exists ! \{xg: axb \longrightarrow axb' \text{ s.t. } \{po(\{xg\}) = \{ep\}\}$ $a \stackrel{P}{\longleftrightarrow} a \times b \stackrel{q}{\longleftrightarrow} b$ $q' \circ (F \times g) = g \circ q$ $f = \begin{cases} 1 & \text{if } \\ 1 & \text{if } \\ 2 & \text{if } \\ 3 & \text{if } \\ 4 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \\ 3 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \\ 2 & \text{if } \end{cases}$ $f' = \begin{cases} 1 & \text{if } \end{cases}$ も明らか. よっ、と×と→とはbifunctionを定める。 ii) te同様のギロンドハ (axb)×cシa×(b×c) (シaxb×c)は分かる. into natural zitazie, i.e., $(a \times b) \times c \xrightarrow{\alpha_{a,b,c}} a \times (b \times c)$ $\forall (f, g, h): (a, b, c)$ $(+x^2) \times h \downarrow \qquad \qquad \downarrow +x(^2 \times h)$ \rightarrow $\langle a', b', c' \rangle$ (a'x b') x c' ~ a'x (bx c') (t) & map or uniqueness から徒う. iii) λ (こついて示す、(Paも同様.)

a := (... (a1 x a z) x...) x an-1 5" an-1 o product 2 \$ 3 2 t.

- ⊕ txa o universality F"

 ,
 - $\exists \lambda_a : a \longrightarrow t \times a \quad s.t.$

$$\int P \circ \lambda a = i da$$

$$\int q \circ \lambda a = F$$

on titerminal obj. In 9 = fop.

idexa = \a . P.

$$452$$
, $po\lambda a = ida$, $\lambda a \circ p = idexa = 41)$, $a \cong + xa$.

In naturality (t. & map or uniqueness siste).

Coproducts に関する Jual prop. も同様に成り立り、

E has both finite products & finite coproducts led 8.

a, ..., an e & (= 7+LZ,

F: □a; → c (t)

Fj:=focj:aj->c (15j5n) (=foc unique に定まる.

特に、

tic, Fzee: null rate.

$$g: \bigcup_{\alpha_i} \alpha_i \longrightarrow \bigcap_{i=1}^{\infty} \alpha_i \quad \xi,$$

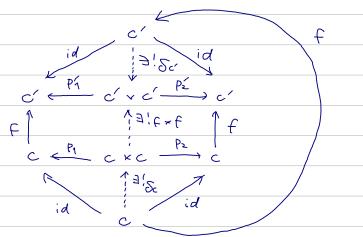
$$g: = \begin{cases} id \quad \text{if } j=k \\ 0 \quad \text{if } j\neq k \end{cases} \quad \text{e.c.} \quad \text{for } \alpha_i = k$$

$$(0: \alpha_j \longrightarrow 2 \longrightarrow \alpha_k)$$

Exs.



○ c + c とすると、



から行う。

(5) \mathcal{B} has finite products \Longrightarrow 50 door \mathcal{B}^{e} .

Prof. 3.5.1 & AU A d &.

@ Be has a terminal obj.

B has finite products \$1), \$\frac{1}{2} t \in \mathbb{B}; \terminal.

T: \mathbb{B} \in \mathref{F}; \text{C} \in \mathref{B}; \text{function, } \frac{1}{2} t \in \mathref{E}; \text{C} \in \text{T} t \in \mathref{E} \in \text{T} t \in \text{T} t \in \text{T} t \in \text{T} \text{T} \in \text{T} \text{T} \in \text{T} \text{T} \text{T} \in \text{T} \text

@ YF, G: E→B: functors, FF×Ge&E

(F×G)
$$c := Fc \times Gc$$
,
 $(F\times G) f := Ff \times GF$ (F:c→c) $z \Rightarrow z \Rightarrow z$,
 $F\times G : \mathcal{E} \to \mathcal{B} : Fanctor$.
 $tt : F\times G \to F$, $P: F\times G \to G$ \mathcal{E} ,
 $tt : F\times G \to F$, $f: F\times G \to G$ (projections)
 $f: F \to G \to Fc$ (projections)

部以 At: c→c, FC = TC × GC PC GC

FF GF

FC × GC PC GC

FC × GC PC

FC × GC PC

FC × GC PC

FC × GC PC

FC × GC PC $\forall \sigma: H \xrightarrow{\cdot} F, \tau: H \xrightarrow{\cdot} G,$ こhit natural transformation である。 実際、 ∀f: c→ C, HF Jπc10 (Uc10 HF) = σ0,0 HF = Ff 0 60, F1), Per 0 (U20Hf) = Toro Hf = GFO Tor (Ff×Gf) = ve' + Hf. LX E & Prop. 3.5.7 2%. BE has finite products.