Don't forget to go through the more recurrence relation examples pdf file to see more examples.

Practice with Recurrence Relations

Solve the following recurrence relations using the iteration technique:

1) $T(n) = T(n-1) + 2, \quad T(1) = 1$

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$T(n) = T(n-2) + 2 + 2$

$T(n) = T(n-3) + 2 * 3$

$T(n) = T(n-4) + 2 * 4$

$T(n) = T(n-k) + 2 * k$

$k = n-1$

$T(n) = 2(n-1) + 1 = 2n - 1$

$O(n)$

Substituting Equations

$T(n-1) = T(n-2) + 2$

$T(n-2) = T(n-3) + 2$

$T(n-3) = T(n-4) + 2$

2) $T(n) = 2T(n/2) + n,\quad T(1) = 1$

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$T(n) = 2[2T(n/4) + n/2] + n$

$= 4T(n/4) + 2n$

$T(n) = 4[2T(n/8) + n/4] + 2n$

$= 8T(n/8) + 3n$

$= 16T(n/16) + 4n$

$T(n) = 2^k T(n/2^k) + kn$

$n = 2^k$

$k = \log_2 n$

$T(n) = 2^{\log_2 n} T(1) + (\log_2 n)n$

<u>Substituting Equations</u>

$T(n/2) = 2T(n/4) + n/2$

$T(n/4) = 2T(n/8) + n/4$

$T(n/8) = 2T(n/16) + n/8$

$T(n) = n\,T(1) + n\log n = n + n\log n$

$O(n * \log n)$

3) $T(n) = 2T\left(\frac{n}{2}\right) + 1, T(1) = 1$

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$2[2T(\frac{n}{4}) + 1] + 1$

$4T(\frac{n}{4}) + 2 + 1$

$4[2T(\frac{n}{8}) + 1] + 2 + 1$

$8T\frac{n}{8} + 4 + 2 + 1$

$T(n) = 2^k T(n/2^k) + k$

$k = \log_2 n$

$O(n * \log n)$

<u>Substituting Equations</u>

<u>n → n/2</u>

$T(n/2) = 2T(n/4) + 1$

$T(n/4) = 2T(n/8) + 1$

$T(n/8) = 2T(n/16) + 1$

4) $T(n) = T(n-1) + n, T(1) = 1$

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$Tn = T(n-2) + n-1+n$

$T(n) = T(n-3) + n-2 + n-1 + n$

$T(n) = T(n-4) + n-3 + n-2 + n-1 + n$

$T(n) = T(n-k) + n$ $O(n)$

$k = 2n-1$

<u>Substituting Equations</u>

<u>n $\rightarrow$ n-1</u>

$T(n-1) = T(n-2) + n-1$

$T(n-2) = T(n-3) + n-2$

$T(n-3) = T(n-4) + n-3$

5.

Use the iteration technique to find a Big-Oh bound for the recurrence relation below. Note you may find the following mathematical result helpful: $2^{\log_3 n} = n^{\log_3 2}$, $\sum_{i=0}^{\infty} (2/3)^i = 3$

$T(n) = 2T(n/3) + cn$

$$T(n) = 2\left(2T\left(\frac{n}{9}\right) + c\left(\frac{n}{3}\right)\right) + cn$$

$$4T\left(\frac{n}{9}\right) + c\left(\frac{2n}{3}\right) + n$$

$$4\left(2T\left(\frac{n}{27}\right) + c\left(\frac{n}{9}\right)\right) + c\left(\frac{2n}{3}\right) + n$$

$$8T\left(\frac{n}{27}\right) + c\left(\frac{4n}{9}\right) + \frac{2n}{3} + n$$

$$T(n) = 8T\left(\frac{n}{27}\right) + cn\left(\frac{4}{9}\right) + \frac{2}{3} + 1$$

$$2^{\log_3 n} T(1) + cn$$

$$n^{\log_3 2} + cn = O(n)$$