Assignment 4

Information Retrieval

CS 834

Fall 2017

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Contents

Chapter 1

Problem 8.3

1.1 Problem Statement

For one query in the CACM collection (provided at the book website), generate a ranking using Galago, and then calculate average precision, NDCG at 5 and 10, precision at 10, and the reciprocal rank by hand.

1.2 Solution

This problem uses CACM corpus provided in the test collection of the book. The queries ran against the CACM collection have been used from the processed queries section of the CACM test collection.

1.2.1 Query:Interested in articles on robotics motion planning particularly the geometric and combinatorial aspects we are not interested in the dynamics of arm motion

The below XML file shows the query which was used in Galago for batch-search.

```
<parameters>
<query>
  <number>6</number>
  <text>  interested in articles on robotics  motion planning particularly the
    geometric and combinatorial aspects   we are not interested in the
   dynamics of arm motion   </text>
</query>
</parameters>
```

The below commands were run on command prompt for generating the ranked list of query with 10 results.

```
F:\Fall2017\InformationRetreival\Assignment3\Galgo\galagosearch-1.04-bin\galagosearch-1.04\
    bin>galago.bat batch-search --index=F:\Fall2017\InformationRetreival\Assignment4\index.
    idx --count=10 F:\Fall2017\InformationRetreival\Assignment4\onequery.xml > F:\Fall2017\
    InformationRetreival\Assignment4\cacm_onequery.list
```

The below list file was generated by batch search from Galago.

```
6 Q0 CACM-0695 1 -164.66490173 galago
6 Q0 CACM-2826 2 -166.82664490 galago
6 Q0 CACM-2828 3 -167.13014221 galago
```

```
6 Q0 CACM-1664 4 -167.29315186 galago

6 Q0 CACM-1543 5 -167.67100525 galago

6 Q0 CACM-2078 6 -168.12095642 galago

6 Q0 CACM-2176 7 -168.47441101 galago

6 Q0 CACM-1113 8 -169.13491821 galago

6 Q0 CACM-0605 9 -169.25828552 galago

6 Q0 CACM-1517 10 -169.68901062 galago
```

The below commands were run on Galago to evaluate the ranked list for query generated from the previous step.

```
F:\Fall2017\InformationRetreival\Assignment3\Galgo\galagosearch-1.04-bin\galagosearch-1.04\
    bin> galago.bat eval F:\Fall2017\InformationRetreival\Assignment4\cacm_onequery.list  F
    :\Fall2017\InformationRetreival\Assignment4\cacm.rel >F:\Fall2017\InformationRetreiva\
    Assignment4\onequeryoutput.txt
```

The below text file is the result of eval command by using the relevance judgments file provided in cacm test collections.

```
num_ret           6 10

num_rel           6 3

num_rel_ret       6 3

map               6 0.4111

ndcg              6 0.5833

ndcg15            6 0.5833

R-prec            6 0.3333
```

```
bpref             6 0.0000
recip_rank        6 0.3333
P5                6 0.4000
P10               6 0.3000
P15               6 0.2000
P20               6 0.1500
P30               6 0.1000
P100              6 0.0300
P200              6 0.0150
P500              6 0.0060
P1000             6 0.0030
num_ret         all 10
num_rel         all 3
num_rel_ret     all 3
map             all 0.4111
ndcg            all 0.5833
ndcg15          all 0.5833
R-prec          all 0.3333
bpref           all 0.0000
recip_rank      all 0.3333
P5              all 0.4000
```

```
P10             all 0.3000

P15             all 0.2000

P20             all 0.1500

P30             all 0.1000

P100            all 0.0300

P200            all 0.0150

P500            all 0.0060

P1000           all 0.0030
```

The calculation of precision for the ranking was done with the help of relevance judgments provided with the test collections. It contains list of all the documents relevant to a collection. Below is the list of relevant documents for query 6.

```
6 Q0 CACM-1543 1

6 Q0 CACM-2078 1

6 Q0 CACM-2828 1
```

Precision at 10

$Precision(1) = 0/1 = 0$

$Precision(2) = 0/2 = 0$

$Precision(3) = 1/3 = 0.33$

$Precision(4) = 1/4 = 0.25$

$Precision(5) = 2/5 = 0.4$

$Precision(6) = 3/6 = 0.5$

$Precision(7) = 3/7 = 0.429$

$Precision(8) = 3/8 = 0.375$

$Precision(9) = 3/9 = 0.33$

$Precision(10) = 3/10 = 0.3$

where Precision(6) denotes precision at search result 6.

The Precision(10) for this query is 0.3.

Mean Average Precision

$MAP = 1/3(1/3 + 2/5 + 3/6)$

$MAP = 0.41$

Reciprocal Rank

The first relevant document is at position 3.

$RR = 1/3$

$RR = 0.333$

Normalized Discounted Cumulative Gain at 5

Relevance Score: 0, 0, 1, 0, 1

$CG_5 = 0 + 0 + 1 + 0 + 1$

Table 1.1: Discounted Cumulative Gain

i	rel_i	$log_2(i+1)$	$rel_i/log_2(i+1)$
1	0	1	0
2	0	1.585	0
3	1	2	0.5
4	0	2.322	0
5	1	2.585	0.387

$CG_5 = 2$

$$DCG = \sum_{i=1}^{5} rel_i/log_2(i+1)$$

$DCG = 0 + 0 + 0.5 + 0 + 0.387 = 0.887$

Ideal Discounted Cumulative Gain has the document and relevance score as below:

Relevance Score: 1, 1, 0, 0, 0

$CG_5 = 1 + 1 + 0 + 0 + 0$

$CG_5 = 2$

$$IDCG = \sum_{i=1}^{5} rel_i/log_2(i+1)$$

$IDCG = 1 + 0.631 + 0 + 0 + 0 = 1.631$

Normalized Discounted Cumulative Gain (NDCG)= DCG / IDCG

$NDCG = 0.887/1.631 = 0.544$

Table 1.2: Ideal Discounted Cumulative Gain

i	rel_i	$log_2(i+1)$	$rel_i/log_2(i+1)$
1	1	1	1
2	1	1.585	0.631
3	0	2	0
4	0	2.322	0
5	0	2.585	0

Normalized Discounted Cumulative Gain at 10

Relevance Score: 0, 0, 1, 0, 1, 1, 0, 0, 0, 0

$CG_{10} = 0 + 0 + 1 + 0 + 1 + 1 + 0 + 0 + 0 + 0$

$CG_{10} = 3$

$$DCG = \sum_{i=1}^{10} rel_i/log_2(i+1)$$

$DCG = 0 + 0 + 0.5 + 0 + 0.387 + 0.356 + 0 + 0 + 0 + 0 = 1.243$

Ideal Discounted Cumulative Gain has the document and relevance score as below:

Relevance Score: 1, 1, 1, 0, 0, 0, 0, 0 , 0 , 0

$CG_{10} = 1 + 1 + 1 + 0 + 0 + 0 + 0 + 0 + 0 + 0$

$CG_{10} = 3$

$$IDCG = \sum_{i=1}^{10} rel_i/log_2(i+1)$$

$IDCG = 1 + 0.61 + 1 + 0.5 + 0 + 0 + 0 + 0 + 0 + 0 = 2.11$

Normalized Discounted Cumulative Gain (NDCG)= DCG / IDCG

$NDCG = 1.243/2.11 = 0.589$

Table 1.3: Discounted Cumulative Gain

i	rel_i	$log_2(i+1)$	$rel_i/log_2(i+1)$
1	0	1	0
2	0	1.585	0
3	1	2	0.5
4	0	2.322	0
5	1	2.585	0.387
6	1	2.807	0.356
7	0	3	1
8	0	3.17	0
9	0	3.22	0
10	0	3.459	0

Table 1.4: Ideal Discounted Cumulative Gain

i	rel_i	$log_2(i+1)$	$rel_i/log_2(i+1)$
1	1	1	1
2	1	1.585	0.61
3	1	2	0.5
4	0	2.322	0
5	0	2.585	0
6	0	2.807	0
7	0	3	0
8	0	3.17	0
9	0	3.22	0
10	0	3.459	0

Chapter 2

Problem 8.4

2.1 Problem Statement

For two queries in the CACM collection, generate two uninterpolated recall precision graphs, a table of interpolated precision values at standard recall levels, and the average interpolated recall-precision graph.

2.2 Solution

This problem uses CACM corpus provided in the test collection of the book. The queries ran against the CACM collection have been used from the processed queries section of the CACM test collection.

Table 2.1: Precision and Recall Table for Query 6

Index	1	2	3	4	5	6	7	8	9	10
Relevant	no	no	yes	no	yes	yes	no	no	no	no
Recall	0	0	0.33	0.33	0.67	1	1	1	1	1
Precision	0	0	0.33	0.25	0.4	0.5	0.429	0.375	0.33	0.3

2.2.1 Query: Interested in articles on robotics motion planning particularly the geometric and combinatorial aspects we are not interested in the dynamics of arm motion

The below list file generated from batch search of Galago has ranked result of 10 for the query.

```
6 Q0 CACM-0695 1 -164.66490173 galago
6 Q0 CACM-2826 2 -166.82664490 galago
6 Q0 CACM-2828 3 -167.13014221 galago
6 Q0 CACM-1664 4 -167.29315186 galago
6 Q0 CACM-1543 5 -167.67100525 galago
6 Q0 CACM-2078 6 -168.12095642 galago
6 Q0 CACM-2176 7 -168.47441101 galago
6 Q0 CACM-1113 8 -169.13491821 galago
6 Q0 CACM-0605 9 -169.25828552 galago
6 Q0 CACM-1517 10 -169.68901062 galago
```

Table 2.2: Precision and Recall Table for Query: Parallel algorithms

Index	1	2	3	4	5	6	7	8	9	10
Relevant	no	no	yes	no	no	yes	no	yes	yes	yes
Recall	0	0	0.2	0.2	0.2	0.4	0.4	0.6	0.8	1
Precision	0	0	0.33	0.25	0.2	0.33	0.286	0.375	0.44	0.5

2.2.2 Query: Parallel algorithms

The below list file generated from batch search of Galago has ranked result of 10 for the query.

```
19 Q0 CACM-2433 1 -10.82248116 galago

19 Q0 CACM-1811 2 -10.83561993 galago

19 Q0 CACM-2973 3 -10.86207390 galago

19 Q0 CACM-2289 4 -10.93358135 galago

19 Q0 CACM-0950 5 -10.96933460 galago

19 Q0 CACM-2266 6 -11.00977135 galago

19 Q0 CACM-2785 7 -11.07456684 galago

19 Q0 CACM-3075 8 -11.07782173 galago

19 Q0 CACM-3156 9 -11.13305855 galago

19 Q0 CACM-2664 10 -11.15161610 galago
```

Both the queries show their highest precision of 0.5 at recall value of 1. This results in a interpolated recall-precision graph to be a straight horizontal line at precision 0.5. Interpolated recall-precision graph is drawn with curves that are descending in manner,

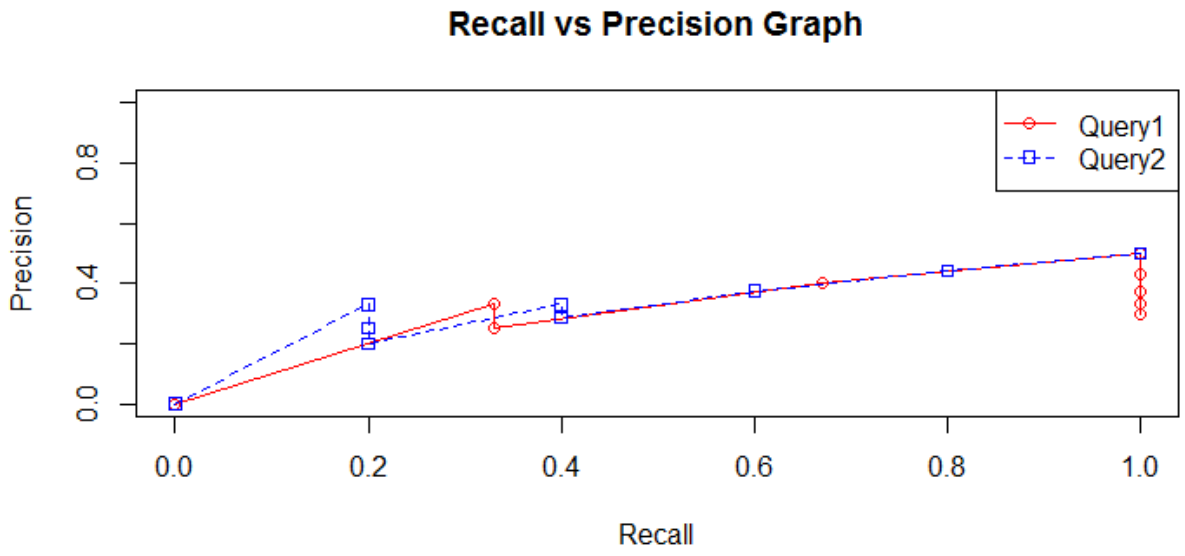

Figure 2.1: Recall vs Precision Graph

starting from highest point towards to lowest. But, for both the queries highest precision was reached at recall 1leading to a straight line.

Table 2.3: Precision values at standard recall levels calculated using interpolation

Recall	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Ranking 1	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Ranking 2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
Average Ranking	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

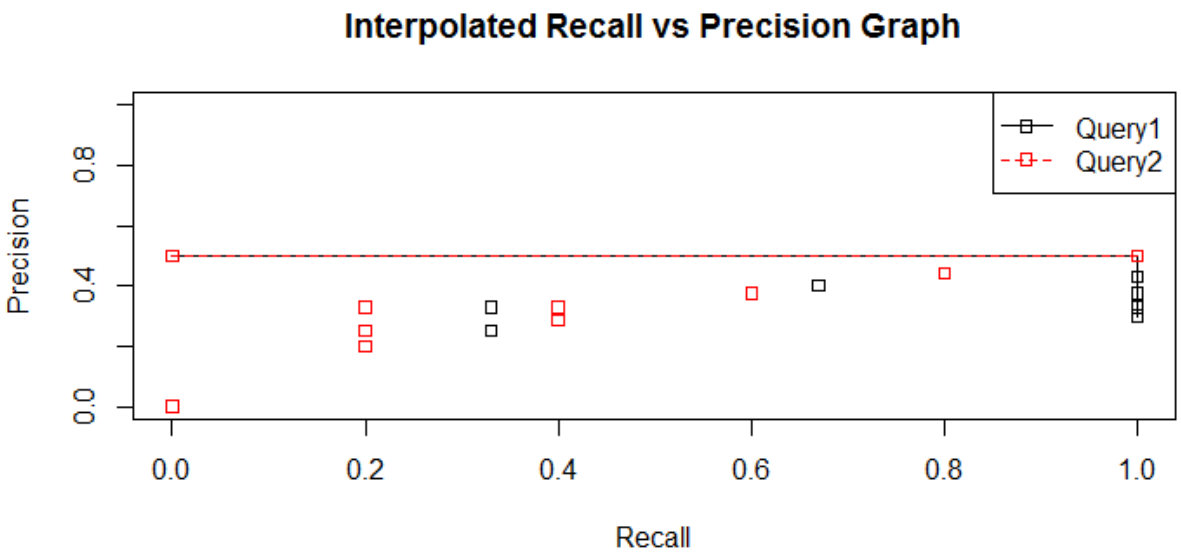

Figure 2.2: Interpolated Recall vs Precision Graph

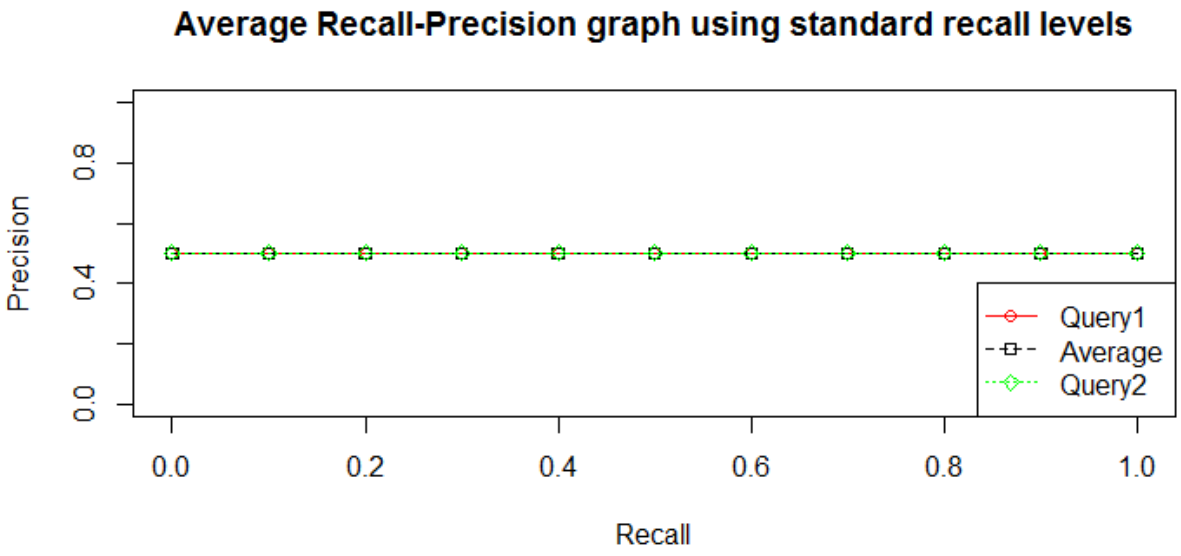

Figure 2.3: Average recall-precision graph using standard recall levels

Chapter 3

Problem 8.5

3.1 Problem

Generate the mean average precision, recall-precision graph, average NDCG at 5 and 10, and precision at 10 for the entire CACM query set.

3.2 Solution

The results have been calculated by Galago batch-search for rank result of 10. Galago provides NDCG 5 and 15 but not NDCG10. So, by making a request for 10 results per query the NDCG15 is same as NDCG10 because all the calculations are based on 10 results for each query.

The values for MAP is lesser than Precision at 10 for the CACM query set. Th query sets having most of their relevant results at top indexes (between 1 and 5) have higher or

equal NCDG5 and NCDG10, but for queries with most of their relevant documents lower in the rank (between 6 and 10) have higher NCDG10 than NCDG5.

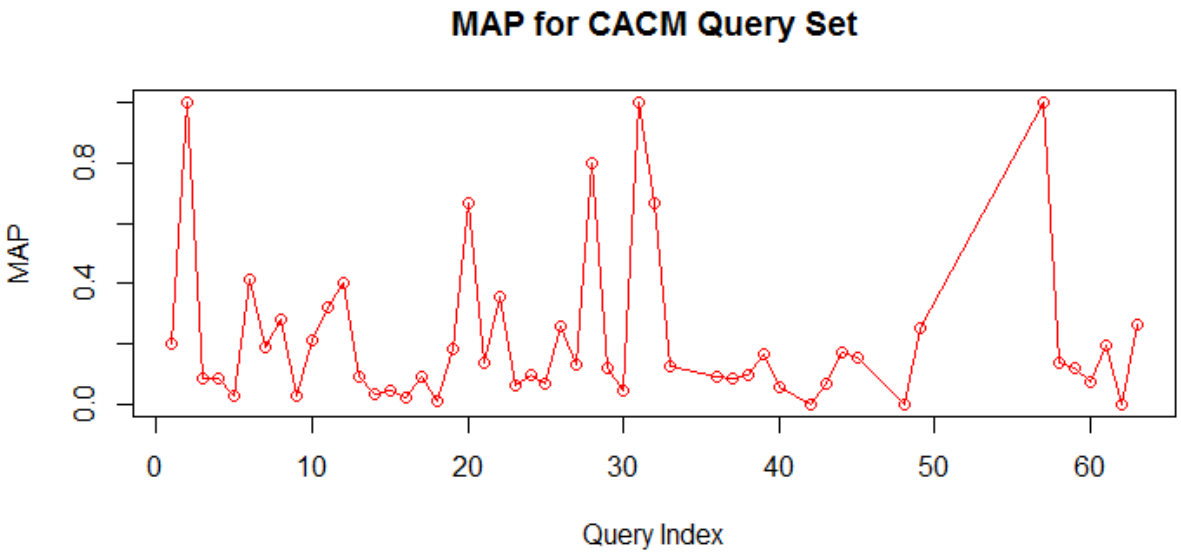

Figure 3.1: MAP values for CACM Query Set

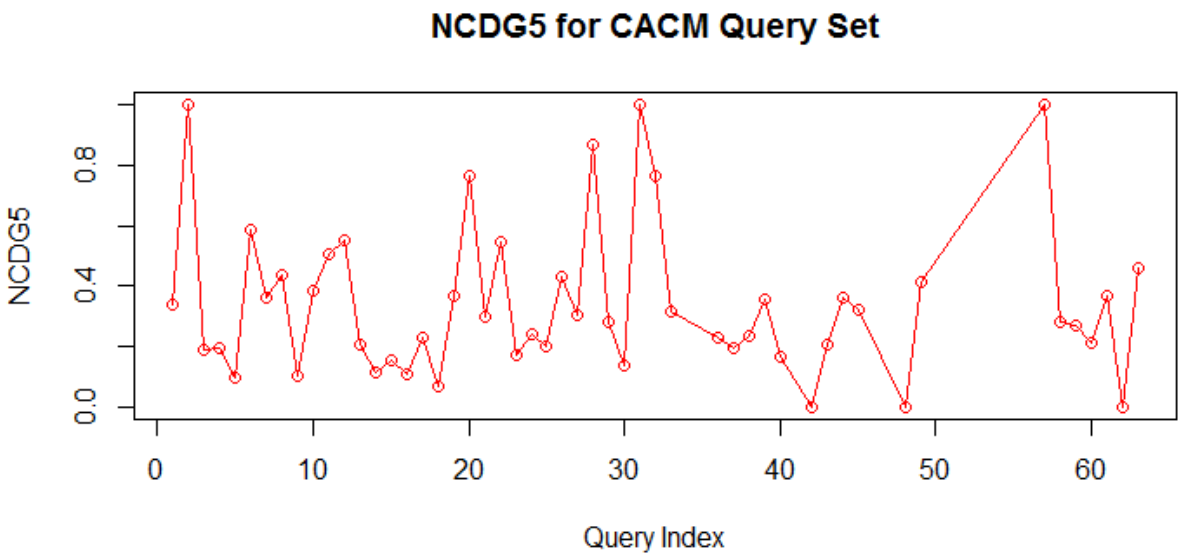

Figure 3.2: NCDG5 values for CACM Query Set

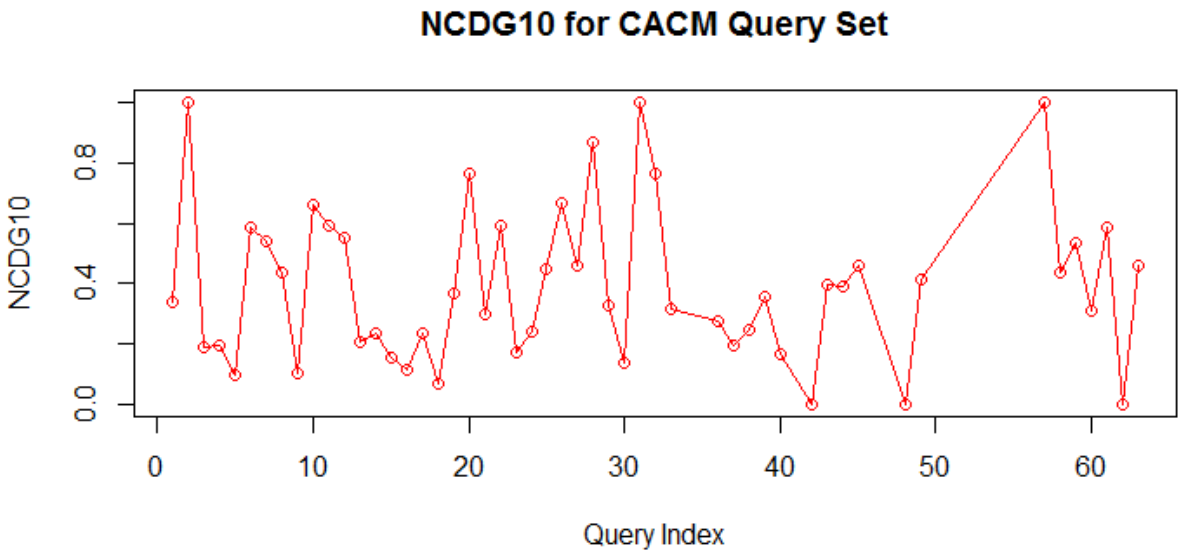

Figure 3.3: NCDG10 values for CACM Query Set

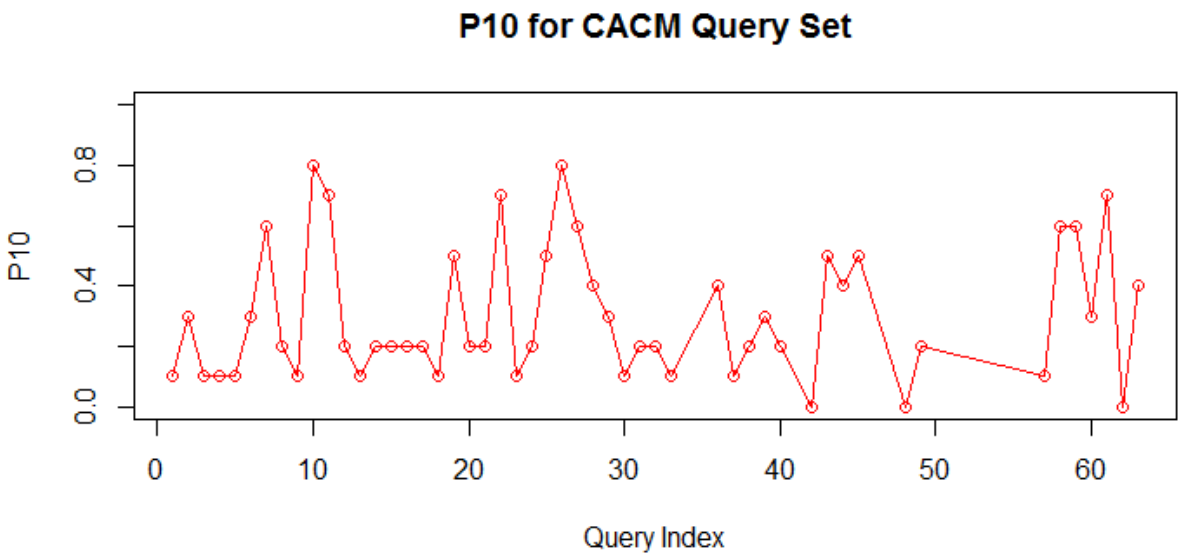

Figure 3.4: Precision at 10 values for CACM Query Set

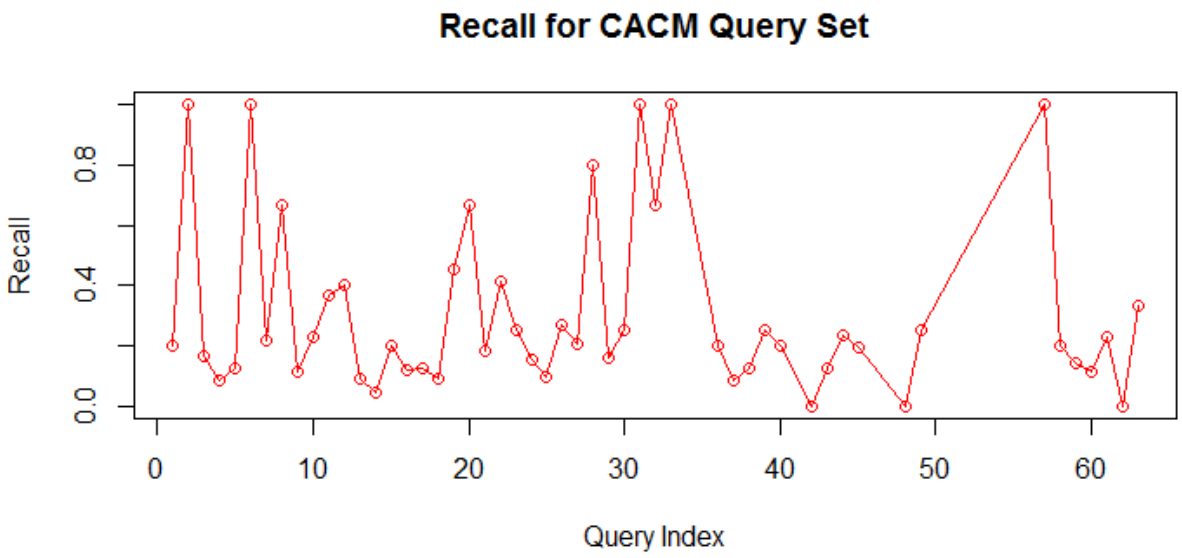

Figure 3.5: Recall values for CACM Query Set

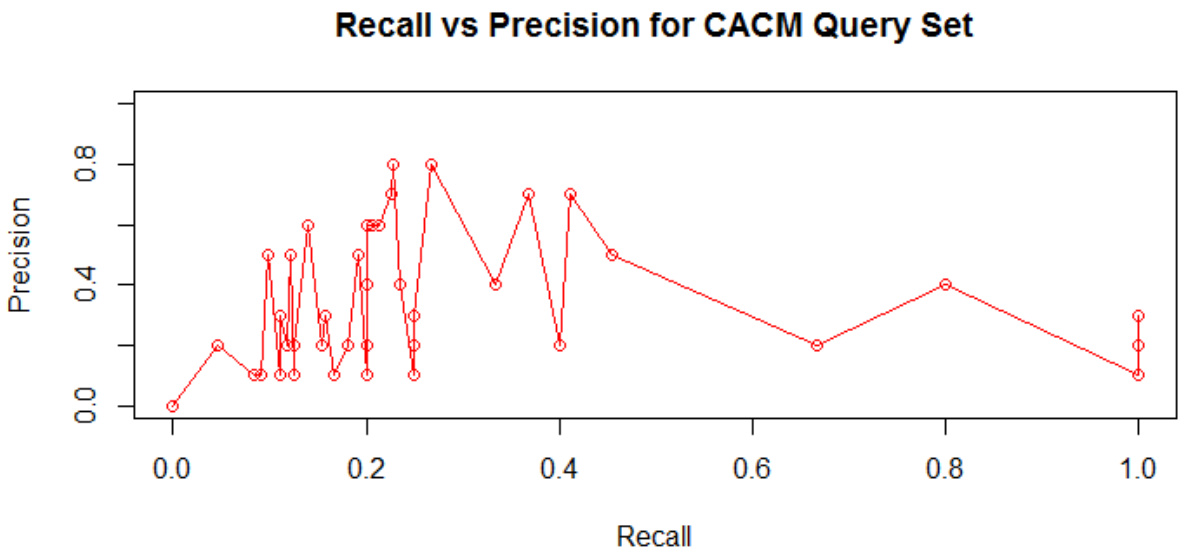

Figure 3.6: Recall vs Precision for CACM Query Set

Chapter 4

Problem 8.7

4.1 Problem

Another measure that has been used in a number of evaluations is R-precision. This is defined as the precision at R documents, where R is the number of relevant documents for a query. It is used in situations where there is a large variation in the number of relevant documents per query. Calculate the average R-precision for the CACM query set and compare it to the other measures.

4.2 Solution

R-precision is defined as precision at R documents, where R is the number of relevant documents for a query. In context to the CACM query set, the relevance judgments file suggests that all the queries have varied number of relevant documents. It ranges from 0 to 51. So, for

Table 4.1: CACM Query Set measured against different measures

map	0.2840
ndcg	0.4419
ndcg15	0.4734
R-prec	0.3192
bpref	0.0000
recip_rank	0.7296
P5	0.3922
P10	0.2980
P15	0.2614
P20	0.2373
P30	0.1843

calculating R-precision number of relevant documents per query is set to 51 and evaluated in the Galago. The results for the different measures inclusive of R-precision at relevant document 51 are shown in the table:

Chapter 5

Problem 8.9

5.1 Problem

For one query in the CACM collection, generate a ranking and calculate BPREF. Show that the two formulations of BPREF give the same value.

5.2 Solution

The solution is based on relevance results for query, interested in articles on robotics motion planning particularly the geometric and combinatorial aspects we are not interested in the dynamics of arm motion. The Table 5.1 shows the relevance results for the query. The relevance results show the search results had 3 relevant documents and 997 non- relevant documents. The ranked list for the query is generated from Galago without providing any explicit value for the nmber of ranked results. The output of ranked list for the query

contains 3 relevant and 997 non-relevant documents.

```
<parameters>
<query>
  <number>6</number>
  <text>  interested in articles on robotics  motion planning particularly the geometric
</query>
</parameters>
```

Below is the list of relevant documents to the query.

```
6 Q0 CACM-1543 1
6 Q0 CACM-2078 1
6 Q0 CACM-2828 1
```

5.2.1 Calculating BPREF

$BPREF = 1/R \sum_{d_r}(1 - (N_{d_r}/min(R, N))$

where R is the number of relevant documents that are considered

N is the number of non-relevant documents that are considered

d_r is the number of relevant documents

For a query result with 3 relevant documents R is 3 implying that first 3 non-relevant documents are considered. The relevance table for the query is manipulated for BPREF as,

Table 5.1: Relevance table showing R relevant and non-relevant documents

Index	Relevance
1	No
2	No
3	Yes
4	No
5	Yes
6	Yes

$$BPREF = 1/R \sum_{d_r} (1 - (N_{d_r}/min(R, N))$$

$$BPREF = 1/3[(1 - 2/3) + (1 - 3/3) + (1 - 3/3)]$$

$$BPREF = 1/3[(1/3) + 0 + 0] = 1/9 = 0.11$$

5.2.2 Calculating BPREF based on preference

$$BPREF = P/(P + Q)$$

where P is the number of relevant documents

Q is the number of non-relevant documents

For query: Code optimization for space efficiency

P = 3

Q = 997

$$BPREF = 3/(3 + 997)$$

$$BPREF = 0.003$$

The value of BPREF is 0.11 and 0.003 by computing respectively with relevant documents formula and the clickthrough preference formula. The values computed are not equal because the BPREF for clickthough formula does not take into account the ranking of relevant results and the value for BPREF with relevant documents formula dips due to the fist relevant result showing after two non-relevant result and the rest two relevant result appear after all the non-relevant documents taken in consideration. Thus, the last two relevant documents do not add any value to the BPREF. Using the clickthrough preference approach for calculating BPREF, with 3 relevant and 997 non-relevant documents the value becomes too small.

Chapter 6

Problem 9.8

6.1 Problem

Cluster the following set of two-dimensional instances into three clusters using each of the five agglomerative clustering methods: (4, 2), (3, 2), (2, 2), (1, 2), (1, 1), (1, 1), (2, 3), (3, 2), (3, 4), (4, 3) Discuss the differences in the clusters across methods. Which methods produce the same clusters? How do these clusters compare to how you would manually cluster the points?

6.2 Solution

The code for agglomerate clustering methods generates 3 clusters for the data set.

6.2.1 Discussion of the Methods

Single Linkage: It uses the minimum distance between between elements of two clusters to merge them as one cluster. For two clusters A and B, it finds the minimum euclidean distance between the points of each cluster and compares them against the minimum threshold distance of all the other clusters to merge them in one cluster. The disadvantage of this approach is that it does not consider how far spread each cluster is and focusing only on the minimum distance between the clusters to merge them.

Complete Linkage: It uses the maximum distance between elements of two clusters to merge them as one cluster. For two clusters A and B, it finds the maximum euclidean distance between the points of each cluster and compares them against the minimum distance of all the other clusters to merge them in one cluster. This approach creates a more compact and less spread cluster than single linkage clustering technique.

Average Clustering: It uses average distance of all the elements between two clusters to merge them as one cluster. For two clusters A and B, it finds the average distance by calculating the euclidean distance between all the points in each cluster and dividing them by the number of elements in each cluster. The average distance calculated is compared against average distance of other clusters to merge the minimum average distance clusters in to one cluster. The type of cluster formed by average linkage depend heavily on the structure of clusters, since it is based on the average distance between all the elements in the two clusters.

Average Group Clustering: It uses centroid distance between two clusters to merge them as one cluster. For two clusters A and B, it finds the centroid of the two clusters and merges them together by comparing against the centroid distances of other clusters. It forms similar clusters to the average linkage clusters.

Ward's Method: It uses sum of variance between two clusters to merge them as one cluster.It forms minimum spread clusters around the centroid of the cluster.

```
'''
Created on Nov 22, 2017

@author: nauman
'''
import math

def singleLinkage(clusterPointA, clusterPointB):
    minDistance = math.inf
    for i in range(0, len(clusterPointA)):
        for j in range(0,len(clusterPointB)):
            distance = math.sqrt(math.pow((clusterPointA[i][0]-clusterPointB[j
    ][0]),2) + math.pow((clusterPointA[i][1]-clusterPointB[j][1]),2))
            if minDistance > distance:
                minDistance = distance
```

```
15      return minDistance
16
17  def agglomerativeSingleLinkageCluster(clusterValue, clusterPoints):
18
19      distanceMatrix = []
20      finalClusters = []
21
22      for i in range(0,len(clusterPoints)):
23          finalClusters.append([i])
24
25      for c in range(len(clusterPoints), clusterValue, -1):
26          bestClusterA = []
27          bestClusterB  = []
28          bestCost = math.inf
29          for i in range(0,len(clusterPoints)):
30              temp = []
31              for j in range(0,len(clusterPoints)):
32                  temp.append(0)
33              distanceMatrix.append(temp)
34
35          for i in range(0,len(clusterPoints)):
36              for j in range((i+1),len(clusterPoints)):
37                  distance = singleLinkage(clusterPoints[i], clusterPoints[j])
38                  distanceMatrix[i][j] = (distance)
39                  if bestCost > distance:
40                      bestCost = distance
```

```
41                      if bestClusterA and bestClusterB:
42                          bestClusterA.pop()
43                          bestClusterB.pop()
44                      bestClusterA.append(clusterPoints[i])
45                      bestClusterB.append(clusterPoints[j])
46          clusterPoints.remove(bestClusterB[0])
47          index = clusterPoints.index(bestClusterA[0])
48          for x in range(0,len(bestClusterB[0])):
49              clusterPoints[index].append(bestClusterB[0][x])
50      return (clusterPoints)
51
52  def completeLinkage(clusterPointA, clusterPointB):
53      maxDistance = 0
54      for i in range(0, len(clusterPointA)):
55          for j in range(0,len(clusterPointB)):
56              distance = math.sqrt(math.pow((clusterPointA[i][0]-clusterPointB[j
    ][0]),2) + math.pow((clusterPointA[i][1]-clusterPointB[j][1]),2))
57              if maxDistance < distance:
58                  maxDistance = distance
59      return maxDistance
60
61  def agglomerativeCompleteLinkageCluster(clusterValue, input):
62
63      distanceMatrix = []
64      finalClusters = []
65
```

```
    clusterPoints = input
    for i in range(0,len(clusterPoints)):
        finalClusters.append([i])

    for c in range(len(clusterPoints), clusterValue, -1):
        bestClusterA = []
        bestClusterB  = []
        bestCost = 0
        for i in range(0,len(clusterPoints)):
            temp = []
            for j in range(0,len(clusterPoints)):
                temp.append(0)
            distanceMatrix.append(temp)

        for i in range(0,len(clusterPoints)):
            for j in range((i+1),len(clusterPoints)):
                distance = completeLinkage(clusterPoints[i], clusterPoints[j])
                distanceMatrix[i][j] = (distance)
                if bestCost < distance:
                    bestCost = distance
                    if bestClusterA and bestClusterB:
                        bestClusterA.pop()
                        bestClusterB.pop()
                    bestClusterA.append(clusterPoints[i])
                    bestClusterB.append(clusterPoints[j])
        clusterPoints.remove(bestClusterB[0])
```

```
92          index = clusterPoints.index(bestClusterA[0])
93          for x in range(0,len(bestClusterB[0])):
94              clusterPoints[index].append(bestClusterB[0][x])
95      return (clusterPoints)
96
97  def averageLinkage(clusterPointA, clusterPointB):
98      averageDistance = 0
99      for i in range(0, len(clusterPointA)):
100         for j in range(0,len(clusterPointB)):
101             averageDistance = averageDistance +  math.sqrt(math.pow((
        clusterPointA[i][0]-clusterPointB[j][0]),2) + math.pow((clusterPointA[i
        ][1]-clusterPointB[j][1]),2))
102     averageDistance = averageDistance/(len(clusterPointA) +len(clusterPointB))
103     return averageDistance
104
105
106 def agglomerativeAverageLinkageCluster(clusterValue, clusterPoints):
107
108     distanceMatrix = []
109     finalClusters = []
110
111     for i in range(0,len(clusterPoints)):
112         finalClusters.append([i])
113
114     for c in range(len(clusterPoints), clusterValue, -1):
115         bestClusterA = []
```

```
116         bestClusterB  = []
117         bestCost = math.inf
118         for i in range(0,len(clusterPoints)):
119             temp = []
120             for j in range(0,len(clusterPoints)):
121                 temp.append(0)
122             distanceMatrix.append(temp)
123 
124         for i in range(0,len(clusterPoints)):
125             for j in range((i+1),len(clusterPoints)):
126                 distance = averageLinkage(clusterPoints[i], clusterPoints[j])
127                 distanceMatrix[i][j] = (distance)
128                 if bestCost > distance:
129                     bestCost = distance
130                     if bestClusterA and bestClusterB:
131                         bestClusterA.pop()
132                         bestClusterB.pop()
133                     bestClusterA.append(clusterPoints[i])
134                     bestClusterB.append(clusterPoints[j])
135         clusterPoints.remove(bestClusterB[0])
136         index = clusterPoints.index(bestClusterA[0])
137         for x in range(0,len(bestClusterB[0])):
138             clusterPoints[index].append(bestClusterB[0][x])
139     return (clusterPoints)
140 
141 def averageGroupLinkage(clusterPointA, clusterPointB):
```

```
142     centroidX_A = 0
143     centroidY_A = 0
144     centroidX_B = 0
145     centroidY_B = 0
146     centroidA = []
147     centroidB = []
148     for i in range(0, len(clusterPointA)):
149         centroidX_A = centroidX_A + clusterPointA[i][0]
150         centroidY_A = centroidY_A + clusterPointA[i][1]
151     centroidA.append(centroidX_A/(len(clusterPointA)))
152     centroidA.append(centroidY_A/(len(clusterPointA)))
153     for j in range(0,len(clusterPointB)):
154         centroidX_B = centroidX_B + clusterPointB[j][0]
155         centroidY_B = centroidY_B + clusterPointB[j][1]
156     centroidB.append(centroidX_B/(len(clusterPointB)))
157     centroidB.append(centroidY_B/(len(clusterPointB)))
158     clusterDistance =   math.sqrt(math.pow((centroidA[0]-centroidB[0]),2) +
    math.pow((centroidA[1]-centroidB[1]),2))
159     return clusterDistance
160
161
162 def agglomerativeAverageGroupLinkageCluster(clusterValue, clusterPoints):
163
164     distanceMatrix = []
165     finalClusters = []
166
```

```
    for i in range(0,len(clusterPoints)):
        finalClusters.append([i])

    for c in range(len(clusterPoints), clusterValue, -1):
        bestClusterA = []
        bestClusterB  = []
        bestCost = math.inf
        for i in range(0,len(clusterPoints)):
            temp = []
            for j in range(0,len(clusterPoints)):
                temp.append(0)
            distanceMatrix.append(temp)

        for i in range(0,len(clusterPoints)):
            for j in range((i+1),len(clusterPoints)):
                distance = averageGroupLinkage(clusterPoints[i], clusterPoints
    [j])
                distanceMatrix[i][j] = (distance)
                if bestCost > distance:
                    bestCost = distance
                    if bestClusterA and bestClusterB:
                        bestClusterA.pop()
                        bestClusterB.pop()
                    bestClusterA.append(clusterPoints[i])
                    bestClusterB.append(clusterPoints[j])
        clusterPoints.remove(bestClusterB[0])
```

```
192         index = clusterPoints.index(bestClusterA[0])
193         for x in range(0,len(bestClusterB[0])):
194             clusterPoints[index].append(bestClusterB[0][x])
195     return (clusterPoints)
196 
197 def wardMethod(clusterPointA, clusterPointB):
198     centroidX_A = 0
199     centroidY_A = 0
200     centroidX_B = 0
201     centroidY_B = 0
202     centroidA = []
203     centroidB = []
204     for i in range(0, len(clusterPointA)):
205         centroidX_A = centroidX_A + clusterPointA[i][0]
206         centroidY_A = centroidY_A + clusterPointA[i][1]
207     centroidA.append(centroidX_A/(len(clusterPointA)))
208     centroidA.append(centroidY_A/(len(clusterPointA)))
209     for j in range(0,len(clusterPointB)):
210         centroidX_B = centroidX_B + clusterPointB[j][0]
211         centroidY_B = centroidY_B + clusterPointB[j][1]
212     centroidB.append(centroidX_B/(len(clusterPointB)))
213     centroidB.append(centroidY_B/(len(clusterPointB)))
214     clusterDistance =   math.sqrt(math.pow((centroidA[0]-centroidB[0]),2) +
    math.pow((centroidA[1]-centroidB[1]),2))
215     variance = len(clusterPointA)* len(clusterPointB)* clusterDistance/ len(
    clusterPointA)+ len(clusterPointB)
```

```
216     return variance
217
218
219 def agglomerativeWardMethod(clusterValue, clusterPoints):
220
221     distanceMatrix = []
222     finalClusters = []
223
224     for i in range(0,len(clusterPoints)):
225         finalClusters.append([i])
226
227     for c in range(len(clusterPoints), clusterValue, -1):
228         bestClusterA = []
229         bestClusterB  = []
230         bestCost = math.inf
231         for i in range(0,len(clusterPoints)):
232             temp = []
233             for j in range(0,len(clusterPoints)):
234                 temp.append(0)
235             distanceMatrix.append(temp)
236
237         for i in range(0,len(clusterPoints)):
238             for j in range((i+1),len(clusterPoints)):
239                 distance = wardMethod(clusterPoints[i], clusterPoints[j])
240                 distanceMatrix[i][j] = (distance)
241                 if bestCost > distance:
```

```
242                     bestCost = distance
243                     if bestClusterA and bestClusterB:
244                         bestClusterA.pop()
245                         bestClusterB.pop()
246                     bestClusterA.append(clusterPoints[i])
247                     bestClusterB.append(clusterPoints[j])
248         clusterPoints.remove(bestClusterB[0])
249         index = clusterPoints.index(bestClusterA[0])
250         for x in range(0,len(bestClusterB[0])):
251             clusterPoints[index].append(bestClusterB[0][x])
252     return (clusterPoints)
253
254 def main():
255     clusters = 3
256     input = [[[-4, -2]], [[-3, -2]], [[-2, -2]], [[-1, -2]], [[1, -1]], [[1,
        1]], [[2, 3]], [[3, 2]], [[3, 4]], [[4,3]]]
257     file = open("ClusteringOutput.txt", "w")
258     clusterPointsSingle = agglomerativeSingleLinkageCluster(clusters,input)
259     file.write("Single Linkage" + "\n")
260     for i in range(0, len(clusterPointsSingle)):
261         file.write("Cluster " + str(i+1) + ": " +str(clusterPointsSingle[i]) +
        "\n")
262     inputComplete =[[[-4, -2]], [[-3, -2]], [[-2, -2]], [[-1, -2]], [[1, -1]],
        [[1, 1]], [[2, 3]], [[3, 2]], [[3, 4]], [[4,3]]]
263     clusterPointsComplete = agglomerativeCompleteLinkageCluster(clusters,
       inputComplete)
```

```
264     file.write("\nComplete Linkage\n")
265     for i in range(0, len(clusterPointsComplete)):
266         file.write("Cluster " + str(i+1) + ": " +str(clusterPointsComplete[i])
    + "\n")
267     inputAverage = [[[-4, -2]], [[-3, -2]], [[-2, -2]], [[-1, -2]], [[1, -1]],
    [[1, 1]], [[2, 3]], [[3, 2]], [[3, 4]], [[4,3]]]
268     clusterPointsAverage = agglomerativeAverageLinkageCluster(clusters,
    inputAverage)
269     file.write("\nAverage Linkage\n")
270     for i in range(0, len(clusterPointsAverage)):
271         file.write("Cluster " + str(i+1) + ": " +str(clusterPointsAverage[i])+
    "\n")
272     inputAverageGroup = [[[-4, -2]], [[-3, -2]], [[-2, -2]], [[-1, -2]], [[1,
    -1]], [[1, 1]], [[2, 3]], [[3, 2]], [[3, 4]], [[4,3]]]
273     clusterPointsAverageGroup = agglomerativeAverageGroupLinkageCluster(
    clusters,inputAverageGroup)
274     file.write("\nAverage Group Linkage\n")
275     for i in range(0, len(clusterPointsAverageGroup)):
276         file.write("Cluster " + str(i+1) + ": " +str(clusterPointsAverageGroup
    [i])+ "\n")
277     inputWard = [[[-4, -2]], [[-3, -2]], [[-2, -2]], [[-1, -2]], [[1, -1]],
    [[1, 1]], [[2, 3]], [[3, 2]], [[3, 4]], [[4,3]]]
278     clusterPointsWard = agglomerativeWardMethod(clusters,inputWard)
279     file.write("\nWard's Algorithm Linkage\n")
280     for i in range(0, len(clusterPointsWard)):
```

```
281         file.write("Cluster " + str(i+1) + ": " +str(clusterPointsWard[i]) + "
    \n")
282     file.close()
283
284 main()
```

```
Single Linkage

Cluster 1: [[-4, -2], [-3, -2], [-2, -2], [-1, -2]]

Cluster 2: [[1, -1], [1, 1]]

Cluster 3: [[2, 3], [3, 2], [3, 4], [4, 3]]


Complete Linkage

Cluster 1: [[-4, -2], [4, 3], [3, 4], [-3, -2], [3, 2], [-2, -2], [2, 3], [-1, -2]]

Cluster 2: [[1, -1]]

Cluster 3: [[1, 1]]


Average Linkage

Cluster 1: [[-4, -2], [-3, -2], [-2, -2], [-1, -2]]

Cluster 2: [[1, -1], [1, 1]]

Cluster 3: [[2, 3], [3, 2], [3, 4], [4, 3]]


Average Group Linkage

Cluster 1: [[-4, -2], [-3, -2], [-2, -2], [-1, -2]]
```

```
Cluster 2: [[1, -1], [1, 1]]

Cluster 3: [[2, 3], [3, 2], [3, 4], [4, 3]]


Ward's Algorithm Linkage

Cluster 1: [[-4, -2], [-3, -2], [-2, -2], [-1, -2]]

Cluster 2: [[1, -1], [1, 1]]

Cluster 3: [[2, 3], [3, 2], [3, 4], [4, 3]]
```

6.2.2 Clustering Results vs Manual Clustering

All the agglomerate clustering techniques except for complete linkage technique produced the same result for the provided data set. The results of the clusters formed by all the clustering methods has been shown above.

Manual Clustering of the data points on euclidean distance results in the same clusters generated by agglomerative clustering.

Cluster 1: (-4,-2), (-3,-2), (–2,-2), (-1,-2)

Cluster 2: (1,-1), (1,1)

Cluster 3: (2,3), (3,2), (3,4), (4,3)

Cluster 1 is easy to create due to it being far away from other data points. Cluster 2 and 3 have a close margin where the distance between point (1,1) and (-1,1) is equal to 2 and the distance between point (1,1) and (2,3) is $\sqrt{5}$. Therefore the points (1,1) and (-1,1) have been clustered together as Cluster 2.

If the data points are clustered on the basis of the quadrants they lie in:

Cluster 1: 1st Quadrant$->$ (-4,-2), (-3,-2), (–2,-2), (-1,-2)

Cluster 2: 3rd Quadrant$->$ (1,-1)

Cluster 3: 4th Quadrant$->$ (1,1), (2,3), (3,2), (3,4), (4,3)

Chapter 7

Problem 9.9

7.1 Problem

Use K-means and spherical K-means to cluster the data points in Exercise 9.8. How do the clustering differ?

7.2 Solution

```
'''
Created on Nov 24, 2017

@author: nauman
'''
import numpy as np
from sklearn.cluster import KMeans
```

```
8  from spherecluster import SphericalKMeans
9
10 def kMeans(num):
11
12     file = open("Kmeans.txt", "a+")
13     input = np.array ([[-4, -2], [-3, -2], [-2, -2], [-1, -2], [1, -1], [1,
       1], [2, 3], [3, 2], [3, 4], [4,3]])
14     kmeans = KMeans(n_clusters=num, random_state=0).fit(input)
15     file.write("K means output for cluster size : "+ str(num) + "\n")
16     file.write("Clusters index of points" + "\n")
17     file.write(str(kmeans.labels_) + "\n")
18     file.write("Center of Clusters\n")
19     file.write(str(kmeans.cluster_centers_) + "\n")
20     file.close()
21
22 def sphericalKMeans(num):
23     num = 4
24     file = open("Kmeans.txt", "a+")
25     input = np.array ([[-4, -2], [-3, -2], [-2, -2], [-1, -2], [1, -1], [1,
       1], [2, 3], [3, 2], [3, 4], [4,3]])
26     kmeans = SphericalKMeans(n_clusters=num).fit(input)
27     file.write("Spherical K means output for cluster size : "+ str(num) + "\n"
       )
28     file.write("Clusters index of points" + "\n")
29     file.write(str(kmeans.labels_) + "\n")
30     file.write("Center of Clusters\n")
```

```
31      file.write(str(kmeans.cluster_centers_) + "\n")
32      file.close()
33
34
35  kMeans(4)
36  sphericalKMeans(4)
```

```
K means output for cluster size : 3
Clusters index of points
[2 2 2 2 0 0 1 1 1 1]
Center of Clusters
[[  1.00000000e+00   5.55111512e-17]
 [  3.00000000e+00   3.00000000e+00]
 [ -2.50000000e+00  -2.00000000e+00]]
Spherical K means output for cluster size : 4
Clusters index of points
[3 3 3 1 2 0 0 0 0 0]
Center of Clusters
[[ 0.70710678  0.70710678]
 [-0.4472136  -0.89442719]
 [ 0.70710678 -0.70710678]
 [-0.81836024 -0.57470559]]
K means output for cluster size : 2
```

```
Clusters index of points

[0 0 0 0 0 1 1 1 1 1]

Center of Clusters

[[-1.8 -1.8]

 [ 2.6  2.6]]

Spherical K means output for cluster size : 4

Clusters index of points

[3 3 3 0 2 1 1 1 1 1]

Center of Clusters

[[-0.4472136  -0.89442719]

 [ 0.70710678  0.70710678]

 [ 0.70710678 -0.70710678]

 [-0.81836024 -0.57470559]]

K means output for cluster size : 4

Clusters index of points

[2 2 0 0 3 3 1 1 1 1]

Center of Clusters

[[ -1.50000000e+00  -2.00000000e+00]

 [  3.00000000e+00   3.00000000e+00]

 [ -3.50000000e+00  -2.00000000e+00]

 [  1.00000000e+00   5.55111512e-17]]
```

Table 7.1: Index to Data Point relation

Index	0	1	2	3	4	5	6	7	8	9
Data Point	(-4, -2)	(-3,-2)	(-2,-2)	(-1,-2)	(1,-1)	(1,1)	(2,3)	(3,2)	(3,4)	(4,3)

```
Spherical K means output for cluster size : 4

Clusters index of points

[3 3 3 1 2 0 0 0 0 0]

Center of Clusters

[[ 0.70710678  0.70710678]

 [-0.4472136  -0.89442719]

 [ 0.70710678 -0.70710678]

 [-0.81836024 -0.57470559]]
```

The output of the code shows clusters on the basis of the index of data points and center of each cluster.Table 5.1 shows the index to data point relationship.The results for cluster size 2,3 and 4 are show for both the clustering techniques.

The clusters formed by K-means for cluster size 3 are same as the clusters formed by Single Linkage, Average Linkage and Average Group Linkage from the problem 9.8. The clusters formed by spherical K-means for cluster 3 is different from K-means due to the difference between finding similarity between points for clustering. K-means uses euclidean distance to find similarity while spherical K-mean uses cosine similarity to cluster items together. Spherical K-means clusters items on which fall in the same quadrant approach discussed in the previous problem.

Chapter 8

Problem 9.11

8.1 Problem

The K nearest neighbors of a document could be represented by links to those documents. Describe two ways this representation could be used in a search application.

8.2 Solution

The primary advantage of K nearest neighbor compared to K means and agglomerative clustering techniques is that a document can be present in multiple clusters unlike the K means and agglomerative clustering technique where each documents is in a single cluster.

The K nearest neighbors of a documents can be used for tell-text search in search application. All the documents in the corpus can be clustered by K nearest neighbor. The

document can be in multiple clusters based on the features extracted from the cluster. For a corpus of documents containing sports news from USA, it can be classified into multiple clusters as football, baseball, basketball, athletics, Olympics, CONCAF Cup etc. A document that is in cluster soccer can simultaneously be clustered with Olympics and CONCAF Cup. Similarly the queries to the documents can also be clustered on the basis of its features. It presents all the documents that are in the cluster soccer for a query of feature soccer. The same set of documents can also be recalled if the query is about CONCAF Cup. It allows for relevant search results.

The K nearest neighbors can also be used for showing clustered search results. It can be helped to refine a very generic query to a more precise query to return relevant result. For example, a system that employs K nearest neighbor clustering on its queries. For a user input query apple, it will show all the possible clusters where the query term apple appears. For apple, it can be a fruit, it can be the company Apple, it can be products of Apple Company etc. A user on the basis of suggestions can select the particular suggestion to find search results that fall under the specified feature cluster.