C1-W6-FirstExam-Final

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Evaluating the first and second derivatives of V_{n+1} , we obtain
$V'_{n+1} = (1-x^2)^{1/2}U'_n - x(x-x^2)^{-1/2}U_n$
$V''_{n+1} = (1-x^2)^{1/2}U"_n - 2x(1-x^2)^{-1/2}U'_n - (1-x^2)^{-1/2}U_n - x^2(1-x^2)^{-3/2}U_n.$
Substituting these expressions into (18.60) and dividing through by $(1-x^2)^{1/2}$, we find

$$(1-x^2)U"_n - 3xU'_n - U_n + (n+1)^2U_n = 0,$$

which immediately simplifies to give the required result (18.59).

18.4.1 Properties of Chebyshev polynomials

The Chebyshev polynomials $T_n(x)$ and $U_n(x)$ have their principal applications in numerical analysis. Their use in representing other functions over the range $|x| < 1$ plays an important role in numerical integration; Gauss-Chebyshev integration is of particular value for the accurate evaluation of integrals whose integrands contain factors $(1-x^2)^{\pm 1/2}$. It is therefore worthwhile outlining some of the main proporties.

Rodrigues' formula

The Chebyshev polynomials $T_n(x)$ and $U_n(x)$ may be expressed in terms of a Rodrigues' formula, in similar way to that used for the Legendre polynomials discussed in section 18.1.2. For the Chebyshev polynomials, we have

$$T_n(x) = \frac{(-1)^n\sqrt{\pi}(1-x^2)^{1/2}}{2^n(n-\frac{1}{2})!}\frac{d^n}{dx^n}(1-x^2)^{n-\frac{1}{2}}$$
$$U_n(x) = \frac{(-1)^n\sqrt{\pi}(n+1)}{2^{n+1}(n+\frac{1}{2})!(1-x^2)^{1/2}}\frac{d^n}{dx^n}(1-x^2)^{n+\frac{1}{2}}$$

These Rodrigues' formula may be proved in an analogous manner to that used in section 18.1.2 when establishing the corresponding expression for the Legendre polynomials.

Mutual orthogonality

In section 17.4, we noted that Chebyshev's equation could be put into Strum- Liouville form with $p = (1-x^2)^1/2$,$q = 0$, $\lambda = n^2$ and $p = (1-x^2)^-1/2$, and its natural interval is thus [=1,1]. Since the Chebyshev polynomials of the first kind, $T_n(x)$, are solutins of the Chebyshev equation and are regular at the end-points $x = \pm 1$, they must be mutually orthogonal over this interval with respect to the weight function $p = (1-x^2)^-1/2$, i.e.

$$\int_{-1}^{1} T_n(x)T_m(x)(1-x^2)^{-1/2}dx = 0 \; if n \neq m \; (18.62)$$

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\documentclass[4pt]{extreport}
\usepackage[utf8]{inputenc}
\usepackage[paperheight=22cm,paperwidth=18cm,rmargin=2cm,lmargin=2cm, tmargi
\usepackage{xcolor}
\usepackage{titleps}
\usepackage{flexisym}
\usepackage{listings}
\newpagestyle{ruled}
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\title{ C1 W6 FirstExam Final }
\author{nauris.silkans }
\date{March 2019}

\begin{document}

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