Final Exam

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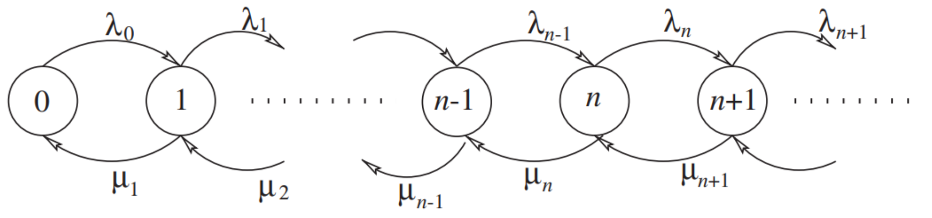

Figure 11.12. State transitions in a birth-death process.

The state transition diagram is shown in Figure 11.12. Notice that we make the assumption that it is impossible to have a death when the population is of size 0 (i.e.,$\mu_0 = 0$) but that one can indeed have a birth when the population is zero (i.e.,$\lambda_0 > 0$). We now need to develop the set of differential-difference equations that define the dynamics of this birth-death process, and to do so, we could proceed as for the M|M|1 queue, by deriving a relationship for $p_n(t + \Delta t)$ and taking the limit of $[p_n(t + \Delta t) - p_n(t)]/\Delta t$ as Δt tends to zero. Instead, we shall use a short cut based on the fact that the rate at which probability "accumulates" in any state n is the difference between the rates at which the system enters and leaves that state. The rate at which probability "flows" into state n at time t is

$$\lambda_{n-1}p_{n-1}(t) + \mu_{n-1}p_{n-1}(t),$$

while the rate at which it "flows" out is

$$(\lambda_n + \mu_n)p_n(t)$$

Subtracting these gives the effective probability flow rate into n, i.e.,

$$\tfrac{dp_n(t)}{dt} = \lambda_{n-1}p_{n-1}(t) + \mu_{n+1}p_{n+1}(t) - (\lambda_n + \mu_n)p_n(t), n \geq 1, \qquad (11.8)$$

and

$$\tfrac{dp_0(t)}{dt} = mu_1 p_1(t) - \lambda_0 p_0(t),$$

which are the forms of the Chapman-Kolmogorov forward equations. This shortcut is illustrated in figure 11.13 where we have separated out state n by surrounding it with dashed line. The net rate of probability flow into n is found by computing the flow across this boundary, using opposing signs for entering and leaving. Notice that if $\lambda_n = \lambda$ and $\mu_n = \mu$ for all n, we get exactly the same equation that we previously derived for the M|M|1 queue.

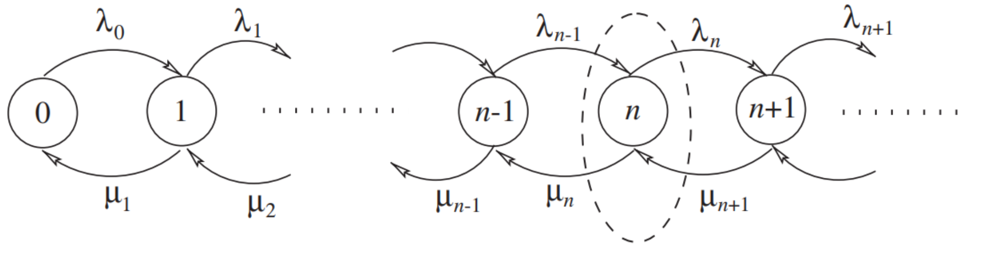

Figure 11.13. Transitions from/to state n.

Example 11.9 A pure birth process.
In pure birth process, there are no deaths, only births. The rates of transition are given by

$$\lambda_n = \lambda \text{ for all n.}$$
$$\mu_n = 0 \text{ for all n}$$

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\renewcommand\makefootrule{\color{}}


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\author{Nauris Silkans}
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\begin{document}

\maketitle
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\includegraphics[width=13cm]{one.PNG}

The state transition diagram is shown in Figure 11.12. Notice that we make the assumption that it
\begin{center}
$\lambda_{n-1}p_{n-1}(t)+\mu_{n-1}p_{n-1}(t)$,\\
\end{center}
\\
while the rate at which it "flows" out is\\
\begin{center}
$(\lambda_n + \mu_n)p_n(t)$\\
\end{center}
\\
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\begin{center}
$\frac{dp_n(t)}{dt}=\lambda_{n-1}p_{n-1}(t)+\mu_{n+1}p_{n+1}(t) - (\lambda_n + \mu_n)p_n(t), n \g
\end{center}
\\
and\\
\begin{center}
$\frac{dp_0(t)}{dt}=mu_{1}p_{1}(t) - \lambda_0p_0(t)$,\\
\end{center}
\\
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\\

\includegraphics[width=13cm]{two.PNG}\\
\textbf{Example 11.9} A pure birth process.\\
In pure birth process, there are no deaths, only births. The rates of transition are given by\\
\begin{center}
$\lambda_n = \lambda$ for all n.\\
 $\mu_n = 0$ for all n
\end{center}
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