Tutorial-4

$T(n) = 3T(n/2) + n^2$

$\Rightarrow c = \cancel{\log_2 a}\ \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

$\Rightarrow n^2 > n^{1.58}$

$\Rightarrow f(n) > n^c$

$\Rightarrow T(n) = \Theta(n^2).$

$T(n) = 4T(n/2) + n^2$

$\Rightarrow c = \log_2 4 = 2$

$\Rightarrow n^c = n^2$

$\Rightarrow n^c = f(n)$

$\Rightarrow T(n) = \Theta(n^2 \log n).$

$T(n) = T(n/2) + 2^n$

$\Rightarrow c = \log_2 1 = 0$

$\Rightarrow n^c = n^0 = 1.$

$f(n) = 2^n$

$\Rightarrow f(n) > n^c$

$\Rightarrow T(n) = \Theta(2^n).$

$T(n) = 2^n T(n/2) + n^n$

$\Rightarrow c = \log_2 2^n$

$\Rightarrow n^c = n^{2^n}$

$\because f(n) = n^n$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \Theta(n^{2^n}).$

$$T(n) = 16\,T(n/4) + n$$

$$\Rightarrow \quad c = \log_4 16 = 2$$

$$n^c = n^2$$

$$f(n) = n$$

$$\Rightarrow \because n^2 > n$$

$$\Rightarrow n^c > f(n)$$

$$T(n) = \Theta(n^2).$$

$$T(n) = 2T(n/2) + n\log n$$

$$\Rightarrow \quad c = \log_2 2 = 1$$

$$n^c = n$$

$$f(n) = n\log n$$

$$\because n\log n > n$$

$$\Rightarrow f(n) > n^c$$

$$\Rightarrow T(n) = \Theta(n\log n)$$

$$T(n) = 2T(n/2) + \frac{n}{\log n}$$

$$c = \log_2 2 = 1$$

$$\Rightarrow n^c = n$$

$$f(n) = \frac{n}{\log n}$$

$$\because \frac{n}{\log n} < n$$

$$\Rightarrow f(n) < n^c$$

$$\Rightarrow T(n) = \Theta(n).$$

$T(n) = 2T(n/4) + n^{0.51}$

$c = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$\because n^{0.5} < n^{0.51}$

$\Rightarrow n^c < f(n)$

$\Rightarrow T(n) = \theta(n^{0.51})$.

$T(n) = 0.5\,T(n/2) + 1/n$

$\Rightarrow$ ~~c = log~~

$\because a = 0.5$

$\Rightarrow a < 1$, for applying master's theorem, condition should be $a \geq 1$.

$\Rightarrow$ Here, Master theorem does not apply.

$T(n) = 16\,T(n/4) + n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$\because n^2 < n!$

$\Rightarrow n^c < f(n)$

$\Rightarrow T(n) = \theta(n!)$

$T(n) = 4T(n/2) + \log n$

$\Rightarrow c = \log_2 4 = 2$

$n^c = n^2$

$\Rightarrow n^2 > \log n$

$n^c > f(n)$

$\Rightarrow T(n) = \theta(n^2)$.

$T(n) = \sqrt{n}\ T(n/2) + \log n$

- $c = \log_2 \sqrt{n}$

$\Rightarrow c = \log_2 n^{1/2}$

$C = \frac{1}{2} \log_2 n$

$\Rightarrow n^c = n^{(\log_2 n)/2}$

$\because n^{(\log_2 n/2)} > \log n$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \theta\left(n^{(\log_2 n/2)}\right)$

$T(n) = 3T(n/2) + n$

$\Rightarrow c = \log_2 3 = 1.584.$

$n^{1.584} > n$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \theta(n^{1.584})$

$T(n) = 3T(n/3) + \text{sqrt}(n).$

$\Rightarrow C = \log_3 3 = 1$

$n^c = n.$, $f(n) = \sqrt{n}$

$\Rightarrow \because n > \sqrt{n}$

$\Rightarrow n^c > f(n)$

$\Rightarrow T(n) = \theta(n).$

$T(n) = 4T(n/2) + cn$

$\Rightarrow K = \log_2 4 = 2$

$\Rightarrow n^k = n^2$

$\because n^2 > cn$ [$\because$ for any constant c]

$\Rightarrow T(n) = \theta(n^2).$

$T(n) = 3T(n/4) + n\log n$

$\Rightarrow a = 3, \quad b = 4$

$c = \log_4 3 = 0.792$

$\Rightarrow n^c = n^{0.792}$

$\because n^{0.792} < n\log n$

$\Rightarrow T(n) = \Theta(n\log n)$

$T(n) = 3T(n/3) + n/2$

$\Rightarrow a = 3, \quad b = 3$

$c = \log_3 3 = 1$

$n^c = n$

$\because n > \frac{n}{2}$

$\Rightarrow$

$n^c > f(n)$

$\Rightarrow T(n) = \Theta(n).$

$T(n) = 6T(n/3) + n^2\log n$

$a = 6, \quad b = 3$

$\Rightarrow c = \log_3 6 = 1.6309$

$\Rightarrow n^c = n^{1.6309}.$

$\because n^{1.6309} < n^2\log n$

$\Rightarrow T(n) = \Theta(n^2\log n).$

$T(n) = 4T(n/2) + n/\log n$

$a = 4, \quad b = 2 \Rightarrow c = \log_2 4 = 2$

$\Rightarrow n^c = n^2.$

$\because n^2 > n/\log n$

$\Rightarrow T(n) = \Theta(n^2).$

$T(n) = 64\,T(n/8) - n^2 \log n$

$a = 64, \quad b = 8$

$c = \log_8 64 = 2$

$\Rightarrow n^c = n^2$, but

$\because \quad f(n) = -n^2 \log n$

$\therefore \quad f(n) < 0 \Rightarrow$ master's theorem can't be applied.

$T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7, \quad b = 3$

$\Rightarrow c = \log_3 7 = 1.771$

$\Rightarrow n^c = n^{1.771}$

$\because \quad n^2 > n^{1.771}$

$\Rightarrow f(n) > n^c$

$\Rightarrow \quad T(n) = \theta(f(n))$

$= \theta(n^2).$

$T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, \quad b = 2$

$c = \log_2 1 = 0$

$\Rightarrow n^c = n^0 = 1.$

now, $f(n) = n(2 - \cos n)$

$\because \quad (2 - \cos n) \in [-1, 1]$

$\Rightarrow$ highest value of $n(2 - \cos n)$ can be,

$n(2 - (-1)) = 3n.$

$\Rightarrow \quad 3n > \cancel{n}\,1$

$\Rightarrow \quad f(n) > 1$

$\Rightarrow \quad T(n) = \theta(3n) = \theta(n).$