$$a^{[n]} = \sigma\left(W^{[n]} a^{[n-1]} + b^{[n]}\right), \quad n = 2, \cdots, L$$

$$\Rightarrow \text{Let } z^{[n]} = W^{[n]} a^{[n-1]} + b^{[n]}, \quad n = 2, \cdots, L$$

$$\delta^{[L]} = \frac{\partial a^{[L]}}{\partial z^{[L]}} = \sigma'\left(z^{[L]}\right)$$

$$\delta^{[L-1]} = \frac{\partial a^{[L]}}{\partial z^{[L-1]}} = \frac{\partial a^{[L]}}{\partial z^{[L]}} \cdot \frac{\partial z^{[L]}}{\partial z^{[L-1]}} = \sigma'\left(z^{[L-1]}\right) \circ \left[\left(W^{[L]}\right)^T \delta^{[L]}\right]$$

$$\vdots$$

$$\delta^{[n]} = \sigma'\left(z^{[n]}\right) \circ \left[\left(W^{[n+1]}\right)^T \delta^{[n+1]}\right], \quad n = 2, \cdots, (L-2), \quad \text{where } \delta^{[n]} = \frac{\partial a^{[L]}}{\partial z^{[n]}}$$

$$\Rightarrow \nabla a^{[L]}(x) = \frac{\partial a^{[L]}}{\partial a^{[1]}} = \frac{\partial a^{[L]}}{\partial z^{[2]}} \cdot \frac{\partial z^{[2]}}{\partial a^{[1]}} = \left(W^{[2]}\right)^T \delta^{[2]}$$