Assignment 2
CMPUT 474

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1. Are the following languages context-free? Prove your answer in each case.
   (a) $\Sigma = \{0,1\}^*, L = \{xy \,|\, |x| = |y|, x \neq y\}$
   (b) $\Sigma = \{0,1\}^*, L = \{w \,|\, n_0(w) = n_1(w)$ and w includes the string '001'$\}$ where $n_0(w)$ and $n_1(w)$ denote the number of 0s and 1s in w respectively.

2. Let G be a CFG in Chomsky normal form, and $w \in L(G)$. How long is w if there is a derivation of w using p steps? Explain why.

3. Let G be a CFG. Explain an algorithm to determine whether $L(G)$ is finite. (Hint: use the pumping lemma).

4. Convert the CFG

$$\begin{aligned} S &\to E \\ E &\to E + T | T \\ T &\to T \times F | F \\ F &\to (E) | \mathrm{a} \end{aligned}$$

   to an equivalent PDA.

5. Recall that the class of context-free language is closed uner the regular operations, union, concatenation, and star. Prove that every regular language is context free by showing how to convery a regualar expression directly to an equivalent context-free grammar.

6. Consider the CFG G given by

$$S \to \mathsf{a}S\mathsf{b}S \,|\, \mathsf{b}S\mathsf{a}S \,|\, \epsilon$$

   Prove that $L(G)$ is the set of strings with an equal number of as and bs.

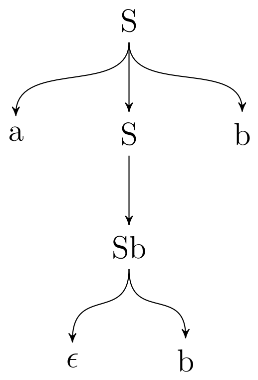
7. Show the following grammar is ambiguous by drawing parse trees. Find an unambiguous grammer for this language.

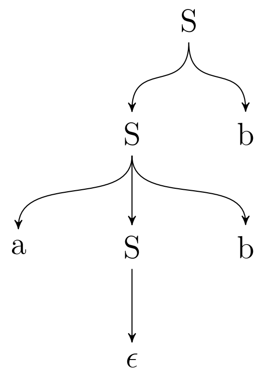
$$S \to \mathsf{a}S\mathsf{b} \,|\, S\mathsf{b} \,|\, \epsilon$$

Solution:

Take this string generated from the grammer and its corresponding parse tree.

$$abb$$





8. Let P be the language of all palindromes over $\{0, 1\}$ containing equal numbers of 0s and 1s. Show that P is not context free.

9. The language $L = \{ww : w \in \{a, b\}^*\}$ is not context free. However, show that the complement language, $\bar{L}$, is context free.

10. Is there a *universal* pusdown automaton? That is, there is a single pushdown automaton, say M such that given a string $s_G w$, where s_g is a string that describes a context free grammar and w is an input string, M accepts $s_G w$ if and only if G generates w? Explain your answer.