SCPY 394: Advanced Physics Laboratory II
Lab 1: Measuring k_B from resistors

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1 Introduction

In conductors, a random motion from electrons leads to the appearance of noise (called Johnson's noise). In this experiment, we will measure the noise, from the random motion of electrons, of voltage on a resistor. Using different resistors with different resistance, Boltzmann constant k_B can be determined from this experiment.

2 Relevant Theory

Root mean square of noise from the resistor can be calculated by using

$$V_{rms} = \sqrt{4k_B TR\Delta f} \tag{1}$$

where
$k_B = 1.3806 \times 10^{-23}$ $\mathrm{m^2 kg \cdot s^{-2} K^{-1}}$ is Boltzmann constant,
T is a temperature in K,
R is a resistance in Ω,
Δf is a bandwidth, a specific range of used frequency, in Hz.

The experiment is designed to find the Boltzmann constant by measuring noise from resistors with different resistance. Doing the experiment together with using the equation (1) to analyze is applicable.

3 Part 0: Before the Experiment

The effective gain bandwidth f_e is used instead of Δf using the relation

$$f_e = \int_0^\infty |G(f)|^2 \, df \tag{2}$$

where $G(f)$ is a frequency-dependent gain (a ratio between the output voltage and input voltage to determine the amplification).

Hence equation (1) becomes

$$V_{rms} = \sqrt{4k_B TRf_e} \tag{3}$$

where f_e is determined by equation(4)

4 Part 1: Measuring frequency response and gain ratio

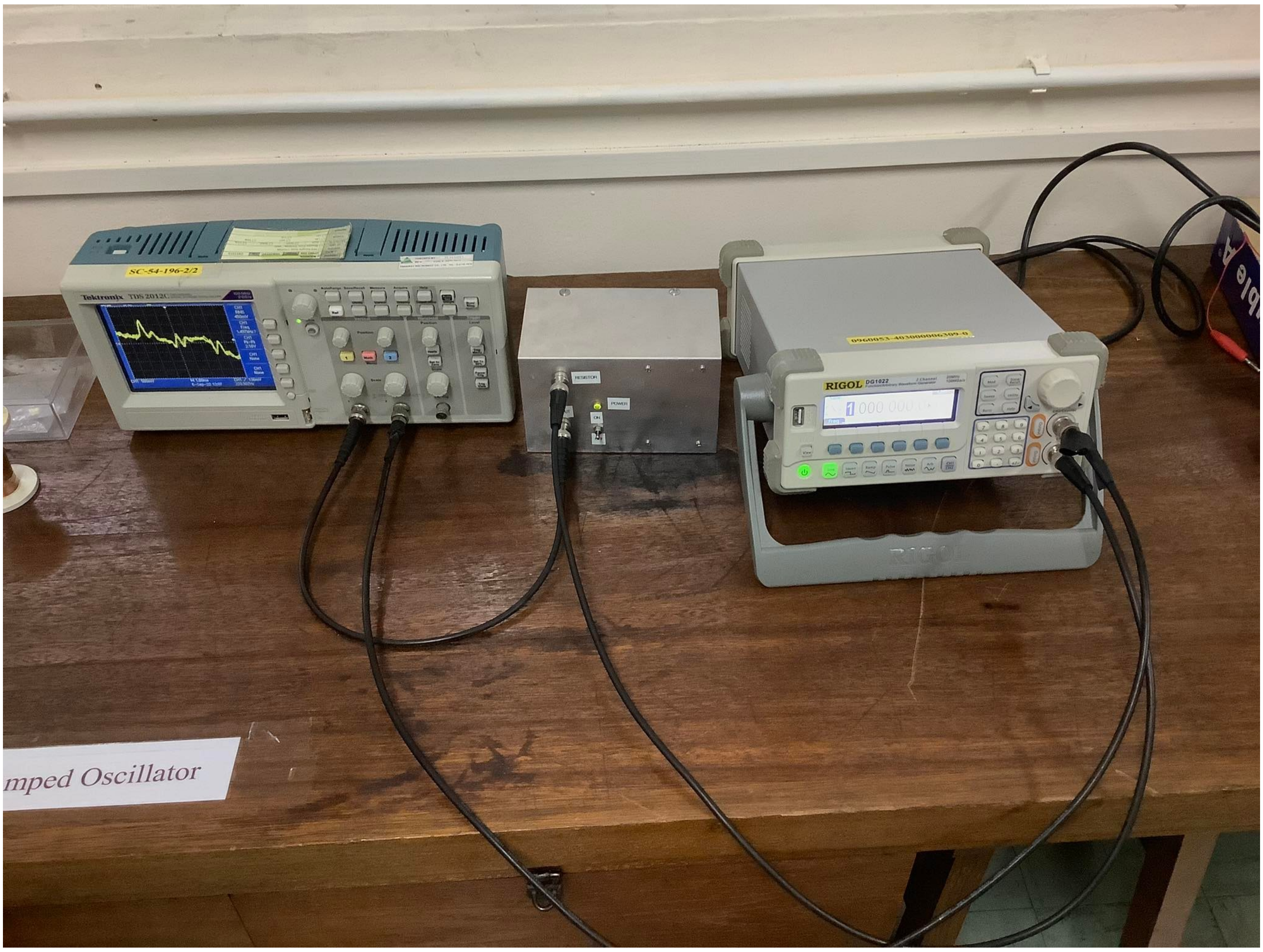

Figure 1: Experimental setup for part 1

The setup for part 1 (as shown in figure 1) consists of a function generator in which the sinusoidal signal is input into the system, an amplifier to amplify the signal (since the noise voltage is significantly small), and an oscilloscope for measuring different values from the amplified signal.

We fix the input amplitude of the wave at $V_{in} = 4.0$ mV. We then measure the output signal V_{out} as a root mean square (RMS) voltage measure by using a "Measure" mode in the oscilloscope.

Different frequencies will show the different V_{out} and also show different gain $G = V_{out}/V_{in}$, as shown in the below table.

f (Hz)	G (± 1)
10	104
20	105
40	132
70	1596
100	2745
200	6063
400	9581
700	11402
1000	12021
2000	12410
4000	12198
7000	11455
10000	10430
20000	7372
40000	3730
70000	1566
100000	804
200000	222

Table 1: Data table for part 1

A graph of frequency and square of gain is shown below.

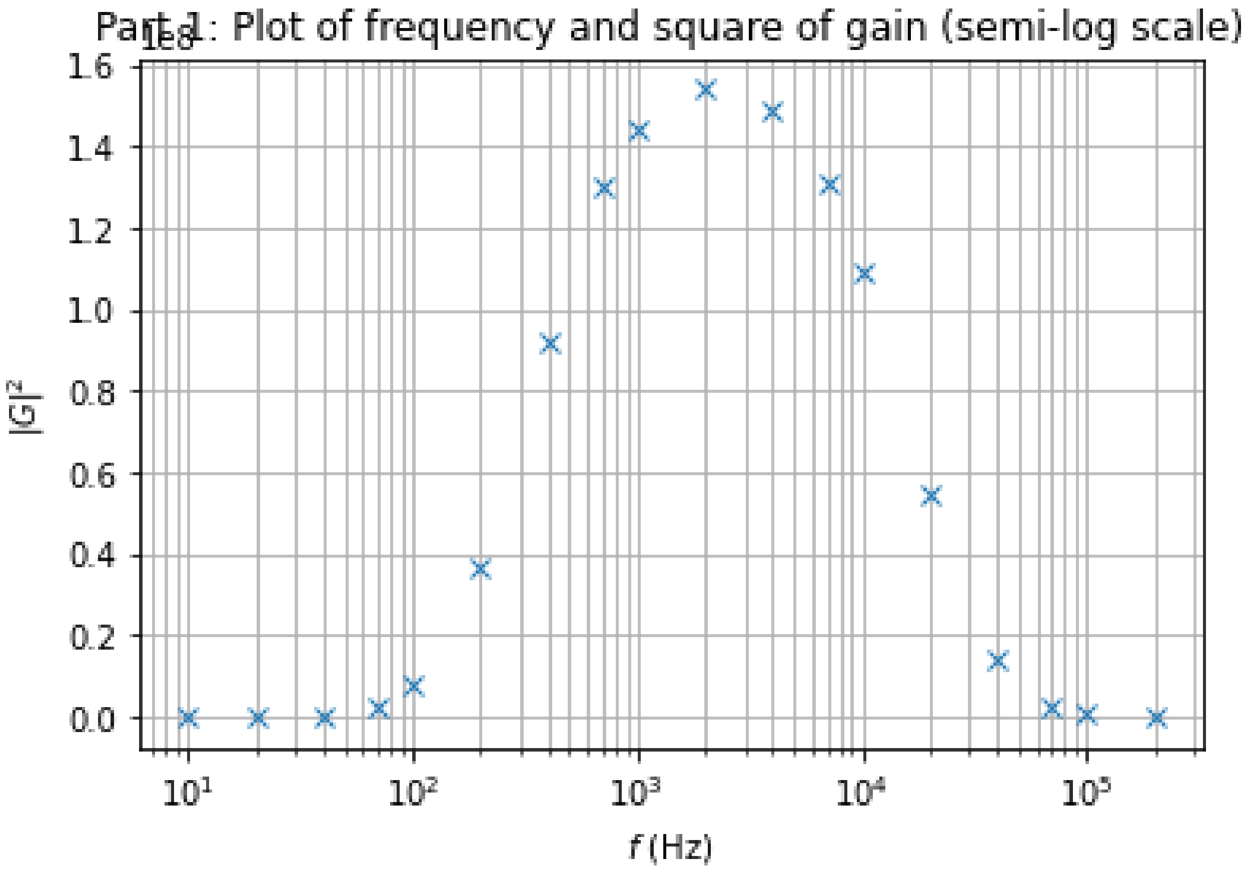

Figure 2: Graph of frequency and gain (Part 1)

From the graph 2, we can calculate the integral. We therefore use Simpson's rule for integration of an array of data evaluated by Python and we obtain

$$f_e = \int_0^\infty |G(f)|^2 \, df = (2.90 \pm 0.01) \times 10^{12} \text{ Hz} \tag{4}$$

5 Part 2: Measuring voltage from the noise of resistors

In part 2, the function generator is excluded. Replacing with a portable circuit with a resistor, it can determine noise voltage V_n. When using $R = 0$ circuit, the noise still appears and discrepancy exists since $V = 0$ when $R = 0$ (from equation (1)).

In this experiment, we obtained $V(R = 0) = 122 \pm 1$ V.

From the experiment, data is shown in the below table

R (Ω)	V_{rms} (± 1 mV)	V_n (± 1 mV)
47	123	1
100	123	1
220	123	1
470	122	1
1000	122	1
2200	124	2
4700	126	4
10000	130	8
22000	138	16
47000	152	30
100000	164	42
220000	200	78
470000	260	138
1000000	320	198

Table 2: Data table for part 2

The graph of resistance and noise voltage is shown below.

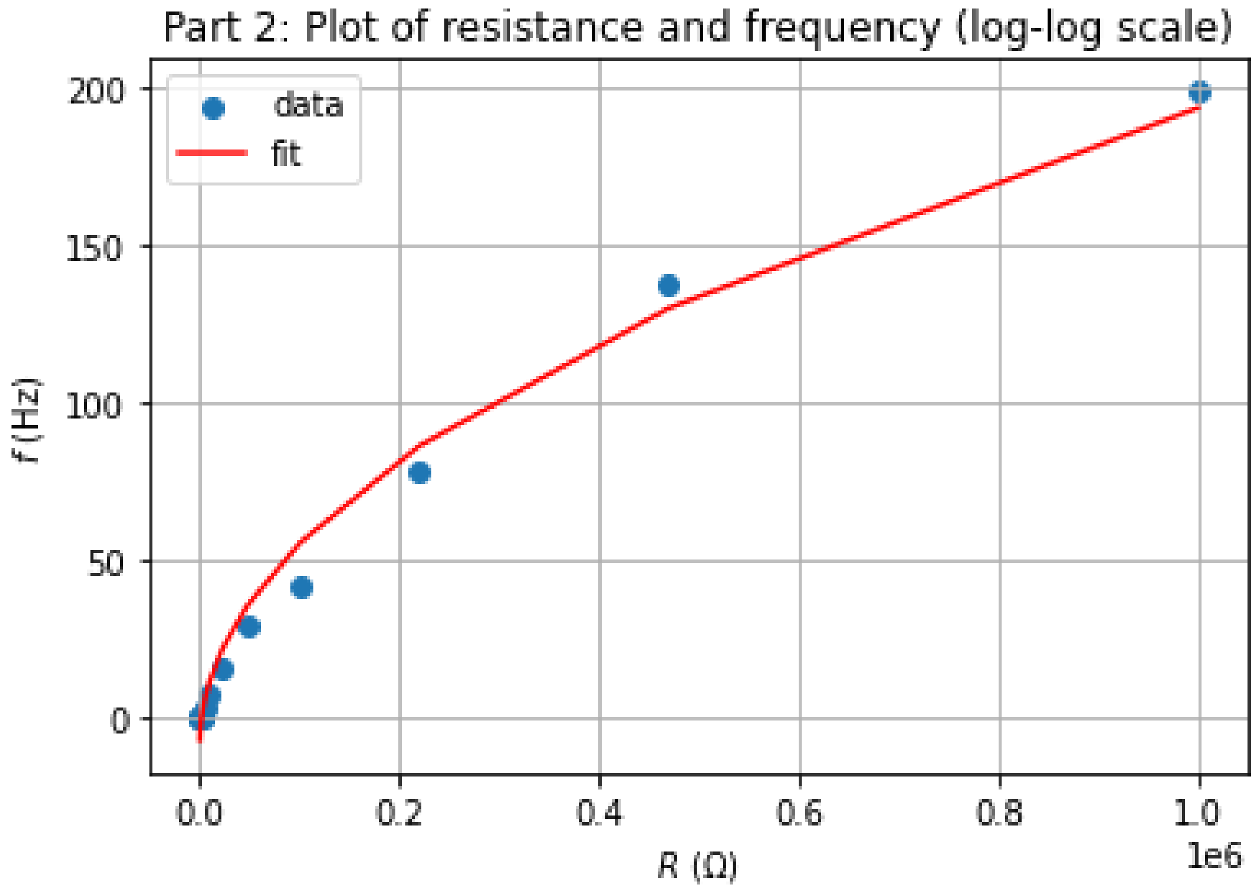

Figure 3: Graph of resistance and noise voltage (Part 2)

6 Part 3: Data Analysis

The data from the table 2 can be used for finding the value of the Boltzmann constant. Using the objective equation $V_{rms} = \sqrt{\alpha R}$, we obtain

$$\alpha = (4052 \pm 6) \times 10^{-6} \text{ mV}^2/\Omega \tag{5}$$

From equation (1), $\alpha = 4k_B T f_e$, we therefore obtain $k_B = \alpha/(4T f_e)$
Hence the Boltzmann constant is

$$k_B = (1.2 \pm 0.2) \times 10^{-23} \text{ V}^2/(\Omega \cdot \text{ K} \cdot \text{Hz}) \tag{6}$$

The real value of k_B is

$$k_B \approx 1.3801 \times 10^{-23} \text{ m}^2\text{kg} \cdot \text{ s}^{-2}\text{K}^{-1} \tag{7}$$

The exact result covers inside the boundary of the obtained value from the experiment.