SCPY 394: Advanced Physics Laboratory II
Lab 2: Four-Point-Probe Conductivity Measurement

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1 Objective

To find the sheet resistivity using **Collinear Four-Point Probe method** and **van der Pauw method**.

2 Sheet Resistance

The resistance R of a conductive sheet with width w, length L and thickness t, and resistivity ρ is

$$R = \frac{\rho L}{tW} \tag{1}$$

Since R is shape-dependent (depend on the dimension of the object), we define

$$R_s = \frac{\rho}{t} \tag{2}$$

as **sheet resistance** which is material-dependent but shape-independent.

3 Collinear Four-Point Probe method

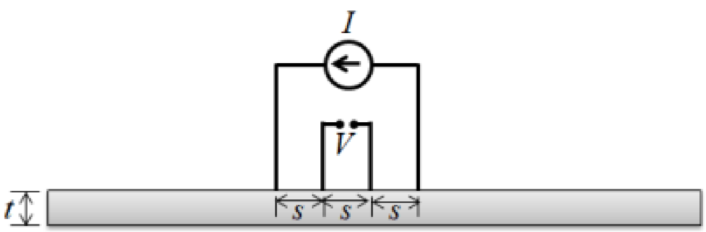

Figure 1: Illustration of implementing collinear four-point probe method (From lab manual)

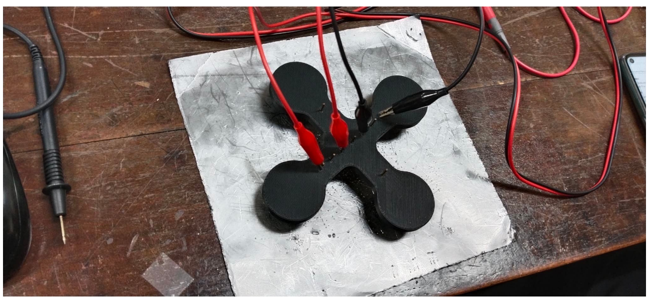

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Figure 2: Experimental setup for collinear four-point probe method

Using two points connecting with a current source and two points connecting a coordinate in the boundary of the object, we can measure the resistance of the object, as shown in figure 2. In this experiment, graphite which the resistivity is $\rho = 1380 \times 10^{-8}\Omega\cdot\text{m} = 1.380 \times 10^{-5}\Omega\cdot\text{m}$[1] is used.

[1] From http://www.troelsgravesen.dk/graphite.htm

With thin film ($t << s$),

$$R_s = \frac{\rho}{t} = \frac{\pi}{\ln(2)}\frac{V}{I} \tag{3}$$

Rearranging equation(3) to obtain

$$V = \frac{\rho \ln(2)}{\pi t} I \tag{4}$$

which we can collect the data of V with some chosen I using the setup shown in figure 2. We find the resistivity of sheets with different shapes: big square sheet, small square sheet, and circular sheet, all with $t = 0.5$ mm. Fitting the linear equation (4), we can obtain the resistivity by considering its slope (that is $\rho = \frac{(\text{SLOPE})\pi t}{\ln(2)}$).

The raw data table is shown below. V_{big} is the voltage measured at the bigger square sheet,V_{square} is the voltage measured at the smaller square sheet, and V_{circle} is the voltage measured at the circular sheet.

I (±0.001 A)	V_{big} (±0.002 mV)	V_{square} (±0.002 mV)	V_{circle} (±0.002 mV)
0.600	2.978	6.367	5.562
0.620	3.086	6.563	5.743
0.640	3.194	6.776	5.926
0.660	3.287	6.988	6.110
0.680	3.392	7.197	6.295
0.700	3.470	7.410	6.475
0.720	3.576	7.615	6.656
0.740	3.733	7.822	6.836
0.760	3.813	8.033	7.017
0.780	3.866	8.236	7.197
0.800	3.943	8.443	7.374
0.820	4.076	8.656	7.533
0.840	4.168	8.852	7.710
0.860	4.253	9.047	7.879
0.880	4.330	9.236	8.043
0.900	4.361	9.427	8.209
0.920	4.495	9.620	8.371
0.940	4.573	9.809	8.540
0.960	4.652	10.006	8.707
0.980	4.741	10.190	8.873
1.000	4.763	10.414	9.047

Table 1: Raw data table from collinear four-point probe method

From the data, the V-I graph for different shapes are shown below

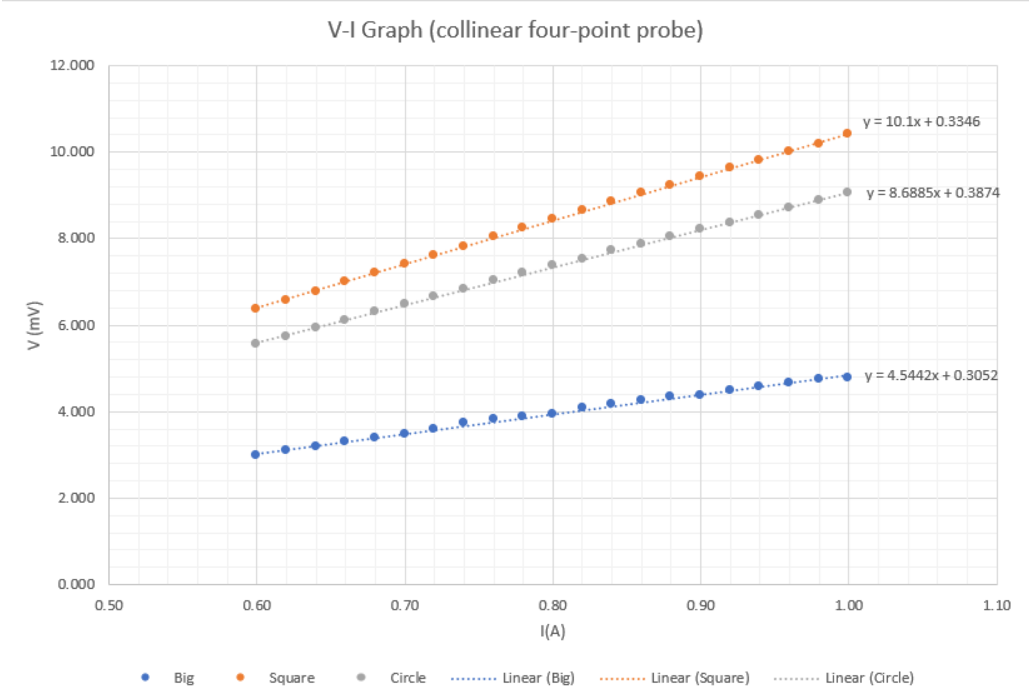

Figure 3: V-I graph (collinear four-point probe)

From the graph, its slope and resistivity is shown below

Object	slope (Ω)	ρ (Ω·m)
Big rectangular sheet	4.54 ± 0.07	(2.29 ± 0.02)·10^{-5}
Square sheet	10.10 ± 0.04	(1.97 ± 0.01)·10^{-5}
Circular sheet	8.69 ± 0.04	(1.029 ± 0.004)·10^{-5}

Table 2: Slope and resistivity

In comparison with the known resistivity of graphite, the smaller sheet results better in measuring resistivity by using collinear four-point probe.

4 Van der Pauw method

Using the setup like one using the collinear four-point probe with different configuration of probe connection as shown in figure 4, we can determine the potential different V_i in i^{th} configuration, called the van der Pauw method. The result is expected to be better than one with the collinear four-point probe method since the van der Pauw method is area-dependent, that area of the object affects the different results of voltage.

The raw data table of V at different configurations for square and circular sheets is shown below.

V_i	V_{square} (±0.002 mV)	V_{circle} (±0.002 mV)
V_1	4.323	4.201
V_2	-4.652	-4.523
V_3	4.258	4.923
V_4	-4.583	-5.263
V_5	4.285	4.195
V_6	-4.645	-4.532
V_7	4.230	4.915
V_8	-4.595	-5.261

Table 3: Raw data table from van der Pauw method

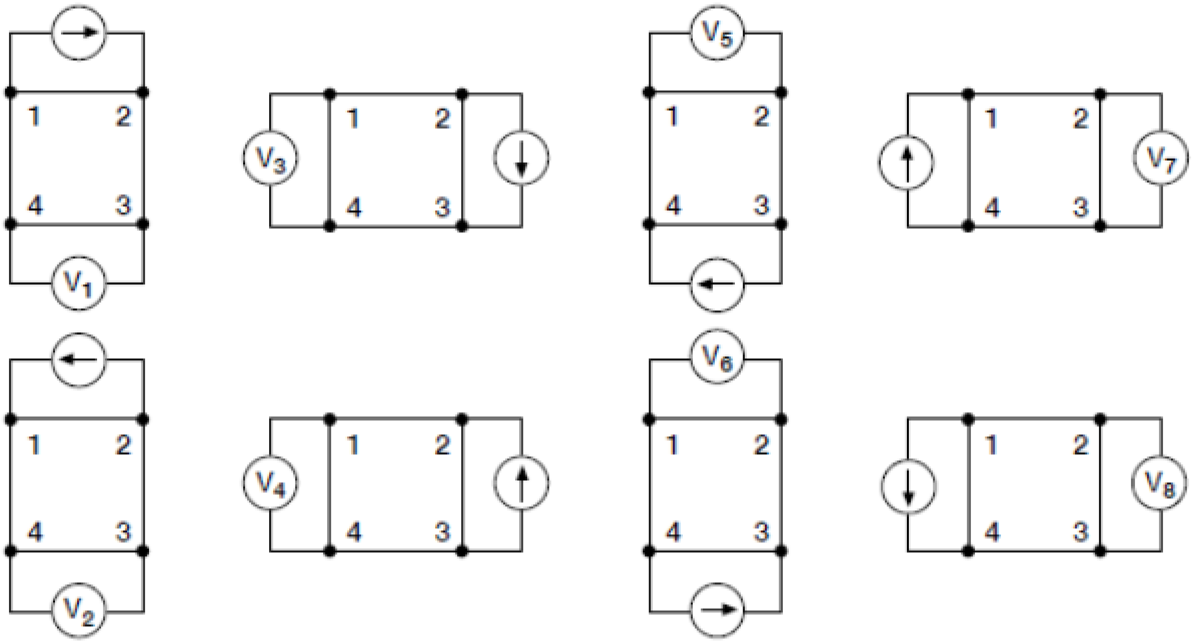

Figure 4: Configurations of probe connection (From: lab manual)

From the van der Pauw method, the resistivity ρ is calculated using the following equations.

$$\begin{cases} \rho_a = \dfrac{\pi}{\ln(2)} f_a t \dfrac{V_1 - V_2 + V_3 - V_4}{4I} \\ \rho_b = \dfrac{\pi}{\ln(2)} f_b t \dfrac{V_5 - V_6 + V_7 - V_8}{4I} \\ \rho = \dfrac{\rho_a + \rho_b}{2} \end{cases} \tag{5}$$

where f_i $(i = A,B)$ is the solution of

$$\frac{Q_i - 1}{Q_i + 1} = \frac{f_i}{0.693} \cosh^{-1}\left(\frac{e^{0.693/f_i}}{2}\right) \tag{6}$$

where

$$Q_i = \begin{cases} \frac{V_1 - V_2}{V_3 - V_4} & \text{if } i = \text{A} \\ \frac{V_5 - V_6}{V_7 - V_8} & \text{if } i = \text{B} \end{cases} \tag{7}$$

Using the van der Pauw method, resistivity ρ is

Object	ρ ($\Omega\cdot$m)
Square sheet	$(1.007 \pm 0.002)\cdot 10^{-5}$
Circular sheet	$(1.07 \pm 0.01)\cdot 10^{-5}$

Table 4: Table of calculated values including the resulting resistivity

From the values in the aforementioned table, the van der Pauw method shows better results than the collinear four-point probe since the reported value is nearly to the known resistivity of graphite.