Homework 2

Instruction: Submit your solutions to this assignment as a PDF file via the online submission system in the class website by 9:00 AM of October 7, 2022. Show your work in full details.

1. Implement the von Karman method to simulate a steady-state flow past an ellipsoidal cylinder. If the number of line segments is n, the n-1 points on the surface of the cylinder can be computed as follows.

$$\theta_i = (n-i)\pi/n, \qquad i = 1, 2, \ldots, n-1$$
$$r_i = a\sin\theta_i$$
$$z_i = \cos\theta_i$$

The linear system obtained from the von Karman method can be written as **Ax** = **b** where **A** is the coefficient matrix, **x** is the unknown vector, and **b** is the right-hand-side vector. After forming **A** and **b**, you may solve the linear system using the MATLAB command `x = A\b`. If **A** is close to singular, you may use a regularization such that matrix **A** is added by a diagonal matrix ε**I**, that is, **A** + ε**I**, where ε is a small positive constant.

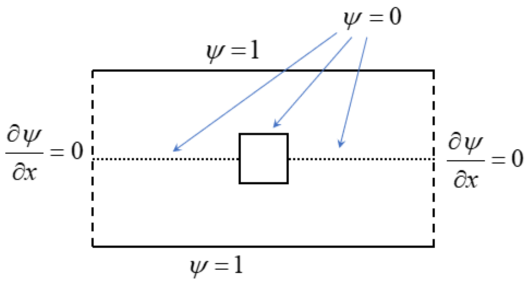
1.1 Write a program to simulate the flow when a = 0.5. Use $n \geq 35$. Show streamlines in the region $-2 \leq z \leq 2$ and $0 \leq r \leq 2$. Also show your code.

1.2 Modify your program to simulate the flow when a = 1.5. Use $n \geq 35$. In this case you may need to use regularization to get a good result. Show streamlines in the same domain as in Problem 1.1. What is the value of ε that you use?

1.3 Modify your program to simulate the flow when a = 0.3. Use $n \geq 90$. In this case you may need to use regularization to get a good result. Show streamlines in the same domain as in Problem 1.1. What is the value of ε that you use?

2. Simulate a steady flow in a channel of height 4 m and length 6 m past a square cylinder of side length 1 m. The cylinder is at the center of the channel as shown in the figure. Solve the Laplace's equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0$$



with the given boundary conditions. Use one of the following methods to solve the resulting linear system: Jacobi method, Gauss-Seidel method, or SOR method. You can choose your own value of the grid spacing h.