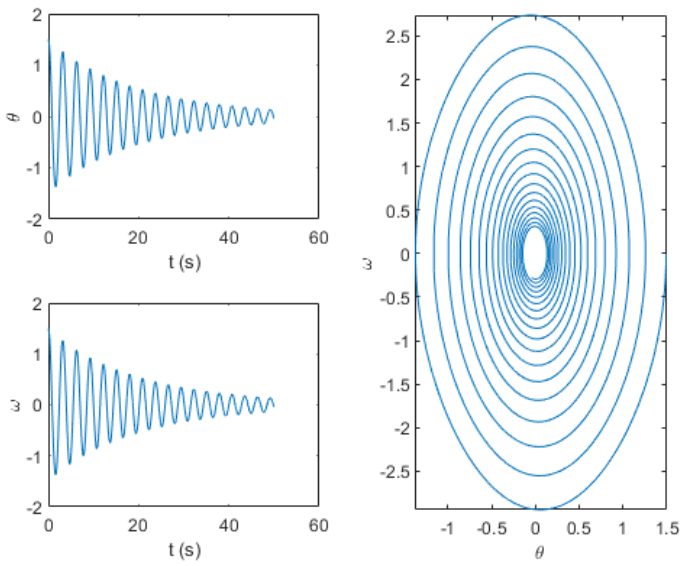
Homework 1

Instruction: Submit your solutions to this assignment as a PDF file via the online submission system in the class website by midnight of September 5, 2022. Show your work in full details.

1. Solve the initial-value problem $\dot{x}(t) = -\ln x,\ \ x(0) = 10$ using the Euler method and the Heun method with h = 1 and 0.1 s in the time interval 0 s - 10 s. Compare the numerical solutions with those computed using the MATLAB function `ode45`. Show the numerical schemes of the Euler and Heun methods for this problem, the graphs of solutions and your source codes. (6 points)
2. Solve the initial-value problem $\ddot{\theta} + 0.1\dot{\theta} + \sin\theta = 0,\ \ \theta(0) = 1.5,\ \ \dot{\theta}(0) = 0.$ using the Euler method and the Heun method with h = 0.1, 0.01, 0.001 s in the time interval 0 s - 10 s. Compare the numerical solutions with those computed using the MATLAB function `ode45`. Show the corresponding system of first-order ODEs, the numerical schemes of the Euler and Heun methods for this problem, the graphs of solutions and your source codes. An example of the solution graphs is shown on the right figure. (6 points)

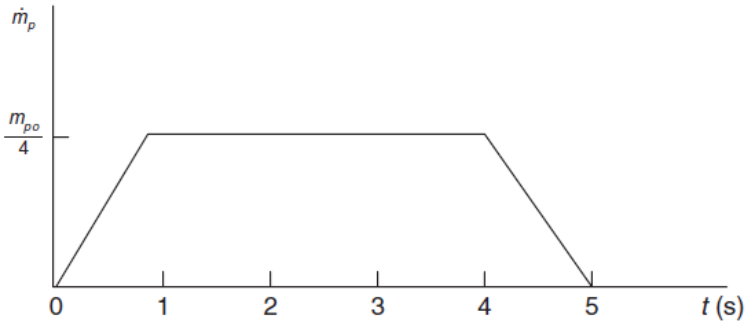


3. The equations of motion for a rocket are

$$\dot{v} = -g + \left(m_p v_e - \tfrac{1}{2} c_d \rho v |v| A\right) \Big/ \left(m_c + m_p\right)$$
$$\dot{z} = v$$

where the gravitational acceleration g = 9.8 m/s^2, the mass of rocket casing m_c = 50 kg, the air density ρ = 1.23 kg/m^3, the maximum cross-sectional area A = 0.1 m^2, the exhaust speed v_e = 360 m/s, the drag coefficient c_d = 0.15. The instantaneous mass of the propellant at time t, $m_p(t)$, is given by

$$m_p(t) = m_{po} - \int_0^t \dot{m}_p dt$$



where the initial weight of the propellant at time t = 0 is m_{po} = 100 kg, and the time-varying burn rate $\dot{m}_p$ is given in the right graph. Solve the equations of motion using the Heun method with h = 0.1 s. Determine the maximum speed of the rocket, the maximum height the rocket can reach, the time and velocity when the rocket hits the ground. Also use the MATLAB function `ode45` to determine these values. (8 points)