SCPY405: Computational Fluid Dynamics
Homework 2

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1. A MATLAB function for this problem is shown below. When using this function, the flow visualization is displayed using the stream function solved by von Karman method.

```matlab
        % vonkarman.m %
        function [] = vonkarman(U,a,n,zmin,zmax,rmin,rmax,eps)
% von Karman method to simulate a steady-state flow past an ellipsoidal cylinder.
% ----------output----------
% (rp,zp): spatial domain for visualization
% Q: source strength
% (ur,uz): velocity for r-z axes
% ----------input----------
% a: radius
% n: number of segments
% [zmin,zmax]: leftmost,rightmost spatial domain (z-axis)
% [rmin,rmax]: lowermost,uppermost spatial domain (r-axis)
% eps: small positive number
theta=(n*ones(n-1,1)-linspace(1,n-1,n-1)')*pi/n;
r=a*sin(theta);z=cos(theta);

zs=linspace(min(z)+0.1,max(z)-0.1,n+1)'; s=zs(2)-zs(1);

% source strength (Q)
b=[1/2*r.^2; 0];
A=zeros(n);A(n,:)=s;
for i=1:n-1
    for j=1:n
        d=sqrt(r(i)^2+(z(i)-zs(j))^2);
        d_=sqrt(r(i)^2+(z(i)-zs(j+1))^2);
        A(i,j)=d-d_;
    end
end
Q=(A+eps*eye(n))\b;
% spatial domains
hp=(zmax-zmin)/n;
zp=zmin:hp:zmax;rp=rmin:hp:rmax;
nz=length(zp);nr=length(rp);

% flow variable (psi)
psi=zeros(nr,nz);
for i=1:nr
    for j=1:nz
        psik=0;
        for k=1:n
            d_=sqrt(rp(i)^2+(zp(j)-zs(k+1))^2);
            d=sqrt(rp(i)^2+(zp(j)-zs(k))^2);
            psik=psik+Q(k)*U*(d_-d);
        end
        psi(i,j) = 1/2*U*rp(i)^2 + psik;
    end
end

% visualization
figure();
contour(zp,rp,psi,100);colorbar();
xlabel('z');ylabel('r');
title("Flow past ellipsoidal cylinder with a = "+num2str(a)+", n = "+num2str(n)+", ...
    \epsilon = "+num2str(eps));
end
```

For this problem, flow speed $U = 1$ and spatial domain is inside the boundary $-2 < z < 2$ and $0 < r < 2$.

(a) With $a = 0.5$ and $n = 1000$, the result without regularization ($\epsilon = 0$) is shown in below figure.

```
>> vonkarman(1,0.5,1000,-2,2,0,2,0)
```

Figure 1: Using the written function "vonkarman" for $(a, n, \epsilon) = (0.5, 1000, 0)$

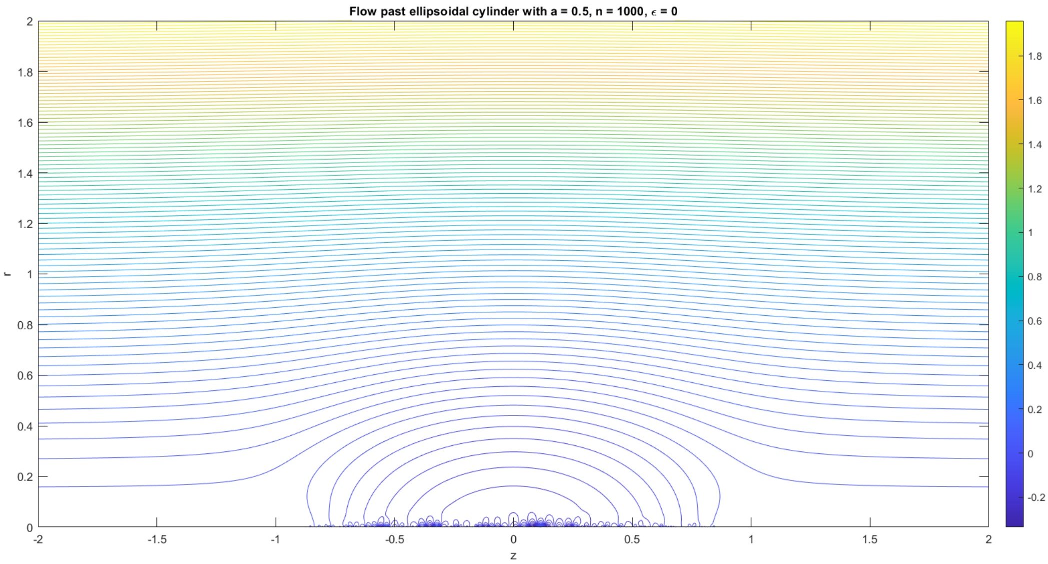

Figure 2: The result of problem 1 using $a = 0.5$, $n = 50$ and $\epsilon = 0$

(b) With $a = 1.5$ and $n = 75$, using $\epsilon = 5 \cdot 10^{-2}$ will obtain the result shown in below figure.

```
>> vonkarman(1,1.5,75,-2,2,0,2,0.05)
```

Figure 3: Using the written function "vonkarman" for $(a, n, \epsilon) = (1.5, 75, 5 \cdot 10^{-2})$

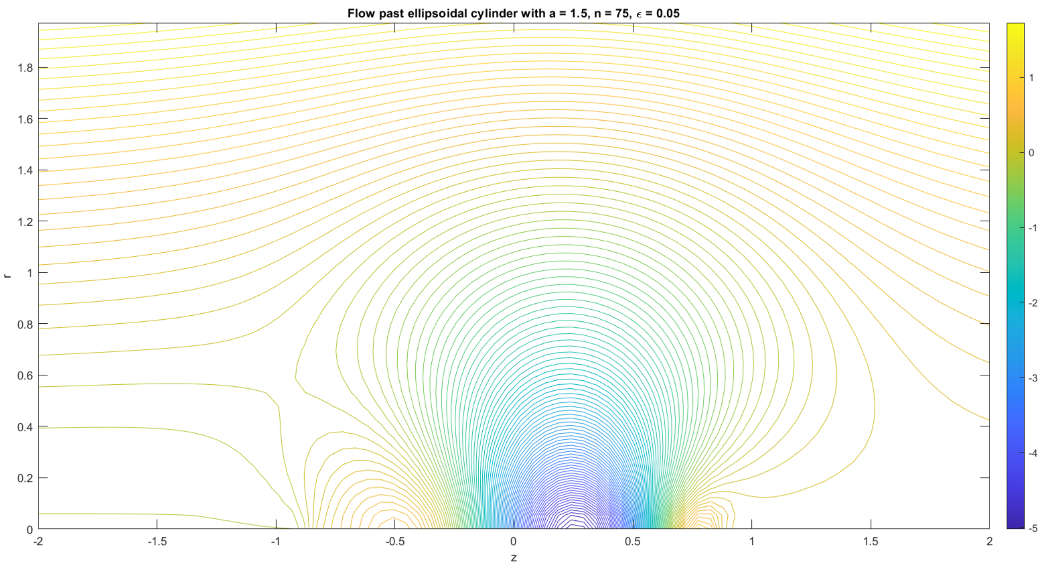

Figure 4: The result of problem 1 using $a = 1.5$, $n = 75$ and $\epsilon = 5 \cdot 10^{-2}$

(c) With $a = 0.3$ and $n = 120$, using $\epsilon = 5 \cdot 10^{-3}$ will obtain the result shown in below figure.

```
>> vonkarman(1,0.3,120,-2,2,0,2,5e-3)
```

Figure 5: Using the written function "vonkarman" for $(a, n, \epsilon) = (0.3, 120, 5 \cdot 10^{-3})$

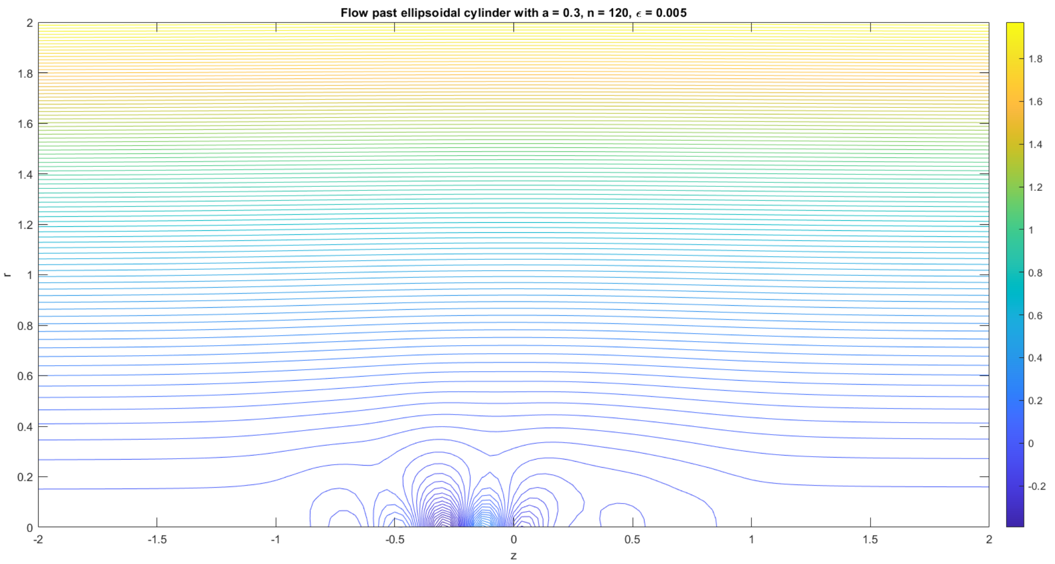

Figure 6: The result of problem 1 using $a = 0.3$, $n = 120$ and $\epsilon = 5 \cdot 10^{-3}$

2. A MATLAB source code for this problem is shown below.

```
    % sq_cylinder.m %
clc;clear

% solve psi_xx + psi_yy =0
% with
% psi_x(x=0)=0
% psi_x(x=6)=1
% psi(y=0)=0=psi(y=4)
% psi(x,y)=0 if (2.5,1.5)<(x,y)<(3.5,2.5) (past cylinder)

H=4;W=6; % width,length
s=1;xl=2.5;yl=1.5; % side_of_square_cylinder,bottom_left_corner_of_cylinder (xl,yl)
h=0.1;
x=0:h:W;nx=length(x);
y=0:h:H;ny=length(y);

nt=500;
psi=zeros(ny,nx);
for it=1:nt
    psi(1,:)=1;psi(ny,:)=1;
    psi(y==H/2,:)=0;
    for ix=2:nx-1
        for iy=2:ny-1
            if (x(ix)≥xl && x(ix)≤xl+s) && (y(iy)≥yl && y(iy)≤yl+s)
                psi(iy,ix)=0;
            else
                if y(iy)≠H/2
                    a=cos(pi/nx)+cos(pi/ny);
                    beta = (8 - 4*sqrt(4-a^2))/a^2;
                    psi(iy,ix)=psi(iy,ix)+beta*(psi(iy,ix+1)+ ...
                        psi(iy,ix-1)+psi(iy+1,ix)+psi(iy-1,ix)-4*psi(iy,ix))/4;
                end
            end
        end
    end
    psi(:,1)=psi(:,2);psi(:,nx)=psi(:,nx-1);
```

```
end
u=diff(psi,1,1)/h;u=[u;u(end,:)];
v=-diff(psi,1,2)/h;v=[v v(:,end)];

figure(1);
contour(x,y,psi,50);colorbar();hold on;quiver(x,y,u,v);
xlabel('x');ylabel('y');title('Flow past square cylinder');hold off;
```

When executing the program, the flow visualization is shown.

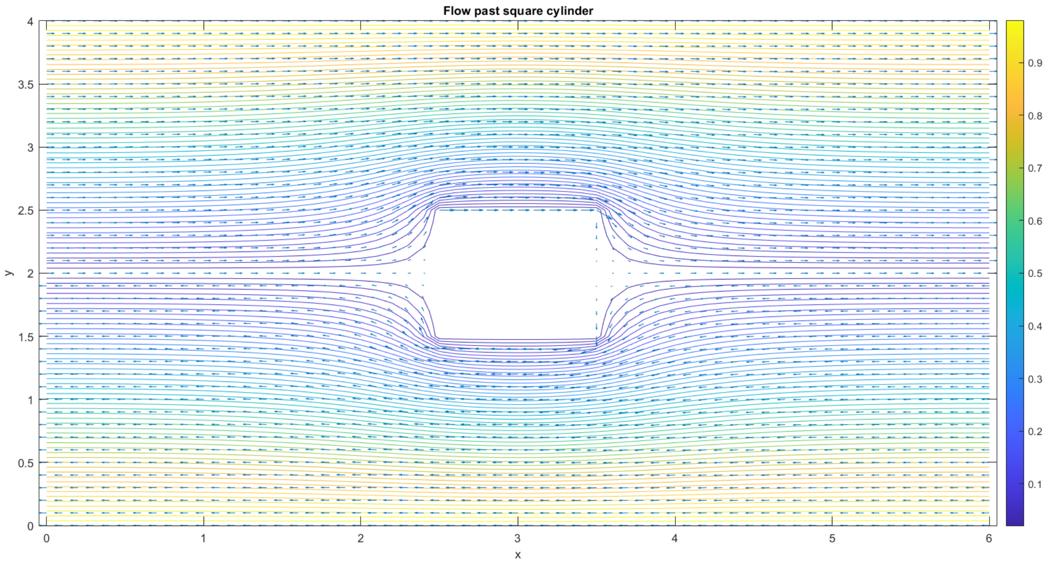

Figure 7: The result of problem 2