RC lattice analysis

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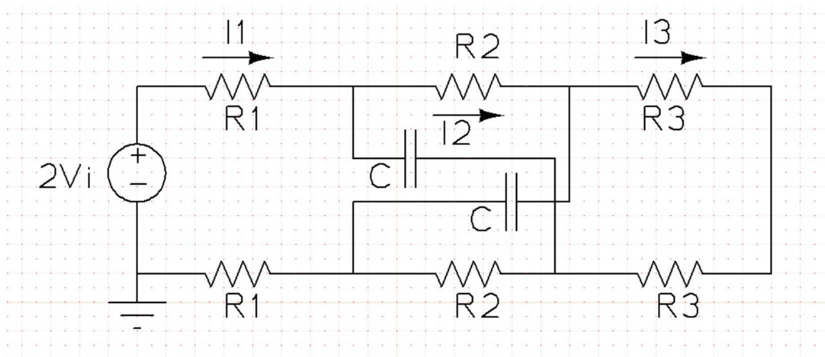
July 5, 2024

Transfer Function Derivation

1x RC lattice



From the above circuit diagram,

$$\frac{I_1 - I_2}{sC} = 2R_3I_3 + R_2I_2 \tag{1}$$

$$\frac{I_2 - I_3}{sC} = 2R_3I_3 + R_2(I_1 - I_2 + I_3) \tag{2}$$

On adding the 2 equations, we get,

$$\frac{I_1 - I_3}{sC} = 4R_3I_3 + R_2(I_1 + I_3) \tag{3}$$

$$I_1(\frac{1}{sC} - R_2) = I_3(4R_3 + \frac{1}{sC} + R_2) \tag{4}$$

Now considering the entire loop,

$$0 = 2V_i - 2R_1I_1 - 2R_3I_3 - R_2(I_1 + I_3) \tag{5}$$

$$2V_i = 2R_1I_1 + 2R_3I_3 + R_2(I_1 + I_3) \tag{6}$$

$$2V_i = I_3(2R_3 + R_2) + I_1(2R_1 + R_2) \tag{7}$$

The output current is,

$$I_{out} = I_3 \tag{8}$$

The transfer function is

$$\begin{aligned}\frac{I_{out}}{V_i} &= \frac{2I_3}{I_3(2R_3+R_2)+I_1(2R_1+R_2)} && (9)\\ &= \frac{2}{(2R_3+R_2)+\frac{(2R_1+R_2)(4R_3+\frac{1}{sC}+R_2)}{(\frac{1}{sC}-R_2)}} && (10)\\ &= \frac{(1-sR_2C)}{(R_1+R_2+R_3+sC(4R_1R_3+R_1R_2+R_2R_3)} && (11)\end{aligned}$$

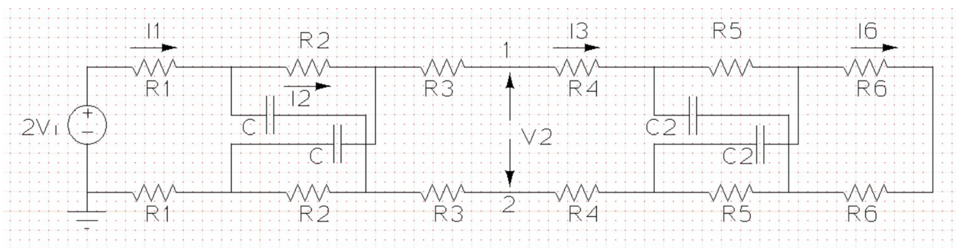
This gives a phase response of,

$$-\tan^{-1}(\omega R_2C)-\tan^{-1}(\omega\frac{(4R_1R_3C+R_1R_2C+R_2R_3C)}{(R_1+R_2+R_3)}) \qquad (12)$$

The time delay is approximately,

$$-\frac{-\omega(R_2C+\frac{(4R_1R_3C+R_1R_2C+R_2R_3C)}{(R_1+R_2+R_3)})}{\omega}=R_2C+\frac{(4R_1R_3C+R_1R_2C+R_2R_3C)}{(R_1+R_2+R_3)} \qquad (13)$$

2x RC lattice



Let the voltage across the points 1 and 2 be V_2.
Now the circuit beyond points 1 and 2 is just a regular 1x RC lattice, from which we get the following relations,

$$\frac{I_6}{V_2} = \frac{2}{(2R_6 + R_5) + \frac{(2R_4+R_5)(4R_6+\frac{1}{sC_2}+R_5)}{(\frac{1}{sC_2}-R_5)}} \tag{14}$$

$$= \frac{(1 - sR_5C_2)}{(R_4 + R_5 + R_6 + sC_2(4R_4R_6 + R_4R_5 + R_5R_6)} \tag{15}$$

$$I_3(\frac{1}{sC_2} - R_5) = I_6(4R_6 + \frac{1}{sC_2} + R_5) \tag{16}$$

Now we find the relation between V_i and I_6, where $I_6 = I_{out}$

$$0 = 2V_i - 2R_1I_1 - 2R_3I_3 - R_2(I_1 + I_3) - V_2 \tag{17}$$

$$2V_i = 2R_1I_1 + 2R_3I_3 + R_2(I_1 + I_3) + V_2 \tag{18}$$

$$2V_i = I_3(2R_3 + R_2) + I_1(2R_1 + R_2) + V_2 \tag{19}$$

$$\frac{I_1 - I_2}{sC} = 2R_3I_3 + R_2I_2 + V_2 \tag{20}$$

$$\frac{I_2 - I_3}{sC} = 2R_3I_3 + R_2(I_1 - I_2 + I_3) + V_2 \tag{21}$$

$$\frac{I_1 - I_3}{sC} = 4R_3I_3 + R_2(I_1 + I_3) + 2V_2 \tag{22}$$

$$I_1(\frac{1}{sC} - R_2) = I_3(4R_3 + \frac{1}{sC} + R_2) + 2V_2 \tag{23}$$

$$\tag{24}$$

The above equations gives us,

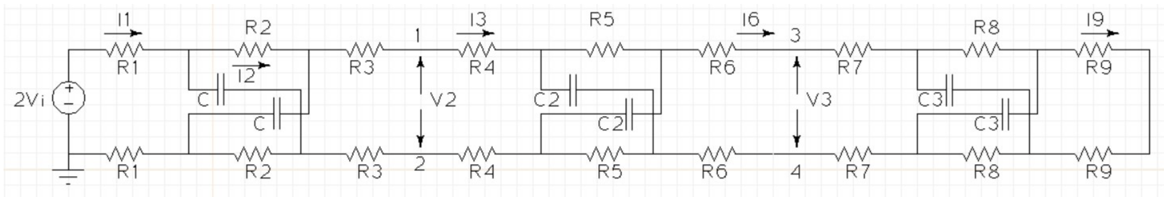
$$2V_i = I_6\frac{(4R_6 + \frac{1}{sC_2} + R_5)(2R_3 + R_2)}{(\frac{1}{sC_2} - R_5)} + (2R_1 + R_2)I_3\frac{(4R_3 + \frac{1}{sC} + R_2) + 2V_2)}{(\frac{1}{sC} - R_2)} + V_2 \tag{25}$$

$$2V_i = I_{out}(\frac{(4R_6 + \frac{1}{sC_2} + R_5)(2R_3 + R_2)}{(\frac{1}{sC_2} - R_5)} + \frac{(2R_1 + R_2)(4R_3 + \frac{1}{sC} + R_2)(4R_6 + \frac{1}{sC_2} + R_5)}{(\frac{1}{sC_2} - R_5)}) \tag{26}$$

$$+ I_{out}(1 + 2\frac{(2R_1 + R_2)}{(\frac{1}{sC} - R_2)})\frac{(R_4 + R_5 + R_6 + sC_2(4R_4R_6 + R_4R_5 + R_5R_6)}{(1 - sR_5C_2)} \tag{27}$$

From the above equation, the transfer function is obtained.

3x RC lattice



We make similar relations by considering the voltage across the points 1 and 2 to be V_2 and across 3 and 4 to be V_3.

$$V_3 = I_9(2R_9 + R_8) + I_6(2R_7 + R_8) \tag{28}$$
$$V_2 = I_6(2R_6 + R_5) + I_3(2R_4 + R_5) + V_3 \tag{29}$$
$$2V_i = I_3(2R_3 + R_2) + I_1(2R_1 + R_2) + V_2 \tag{30}$$

$$I_1(\frac{1}{sC} - R_2) = I_3(4R_3 + \frac{1}{sC} + R_2) + 2V_2 \tag{31}$$
$$I_3(\frac{1}{sC_2} - R_5) = I_6(4R_6 + \frac{1}{sC_2} + R_5) + 2V_3 \tag{32}$$
$$I_6(\frac{1}{sC_3} - R_8) = I_9(4R_9 + \frac{1}{sC_3} + R_8) \tag{33}$$

The output current is,

$$I_{out} = I_9 \tag{34}$$

Using the above relations, the transfer function was defined in MATLAB.

General Procedure

Case - 1 : Resistors are chosen in the ratio 1:2:1 and the capacitance is optimized by minimizing the 2-norm.
Case - 2 : Resistors are constrained to have a sum of 1 and the values of the resistors and capacitors is optimized by minimizing the 2-norm.
Case - 3 : Resistors are constrained to have a sum of 1 and the values of the resistors and capacitors is optimized by minimizing the infinity-norm.

To find the optimal circuit parameters the following objective function is minimized over the bandwidth, for the 2-norm cases,

$$objective = \int_0^{\frac{1.1}{8}} |H(f) - T(f)|^2 df \tag{35}$$

where, H(f) is the transfer function of the circuit and T(f) is the ideal delay line transfer function, given by,

$$T(f) = sinc(f) * \exp(-j2\pi f) \tag{36}$$

The square of the magnitude of the difference is integrated over the bandwidth of $\frac{1.1}{8}$ Hz and the integral is minimized by varying the parameters of the circuit components.

For the infinity norm, we take 1000 points in the range of the bandwidth and minimize the following objective function,

$$objective = max_{\omega\epsilon(0,0.1375]} |H(j\omega) - T(j\omega)| \tag{37}$$

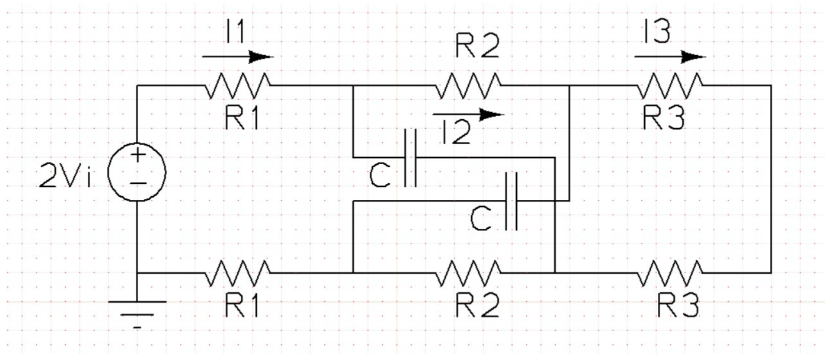
1 1x- RC lattice - 1

1.1 Aim of the experiment

For a general RC lattice, we consider resistors R_1, R_2 and R_3 to be 0.25 Ω, 0.5 Ω and 0.25 Ω and vary the capacitance such that the objective function is minimized.

1.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 = 0.25\Omega \tag{38}$$
$$R_2 = 0.50\Omega \tag{39}$$
$$R_3 = 0.25\Omega \tag{40}$$
$$\tag{41}$$

1.3 Simulation results

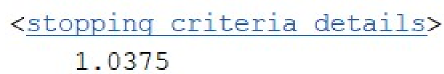
Integral of square of the magnitude of the difference was minimised over the bandwidth.
The transfer function of the circuit is given by,

$$H(s) = \frac{(1 - sR_2C)}{(R_1 + R_2 + R_3 + sC(4R_1R_3 + R_1R_2 + R_2R_3)} \tag{42}$$

The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

Table 1: Simulation results

C
1.0375 F



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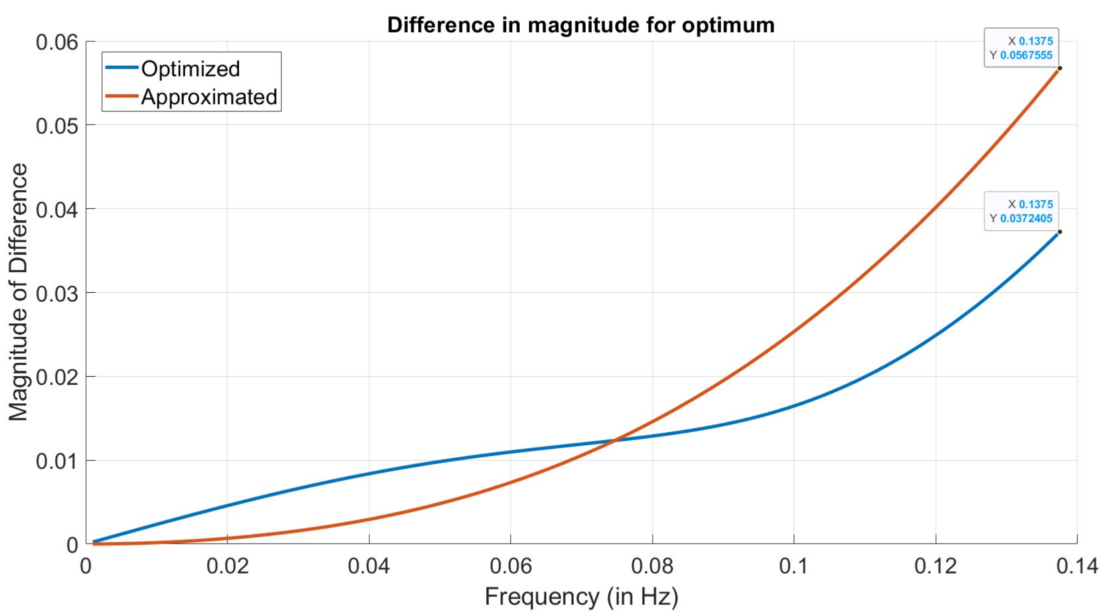
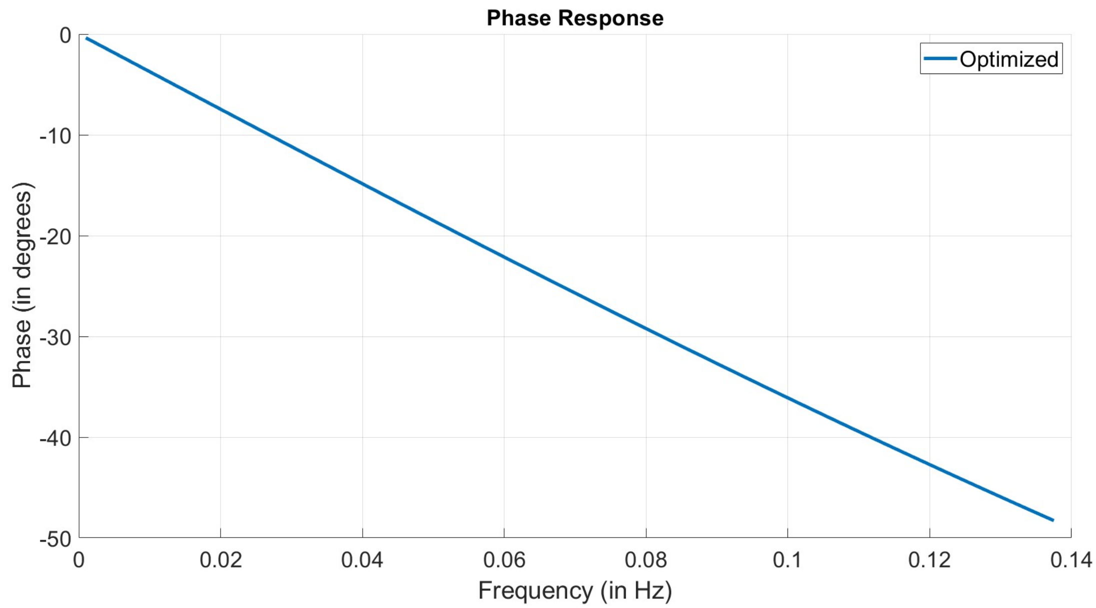
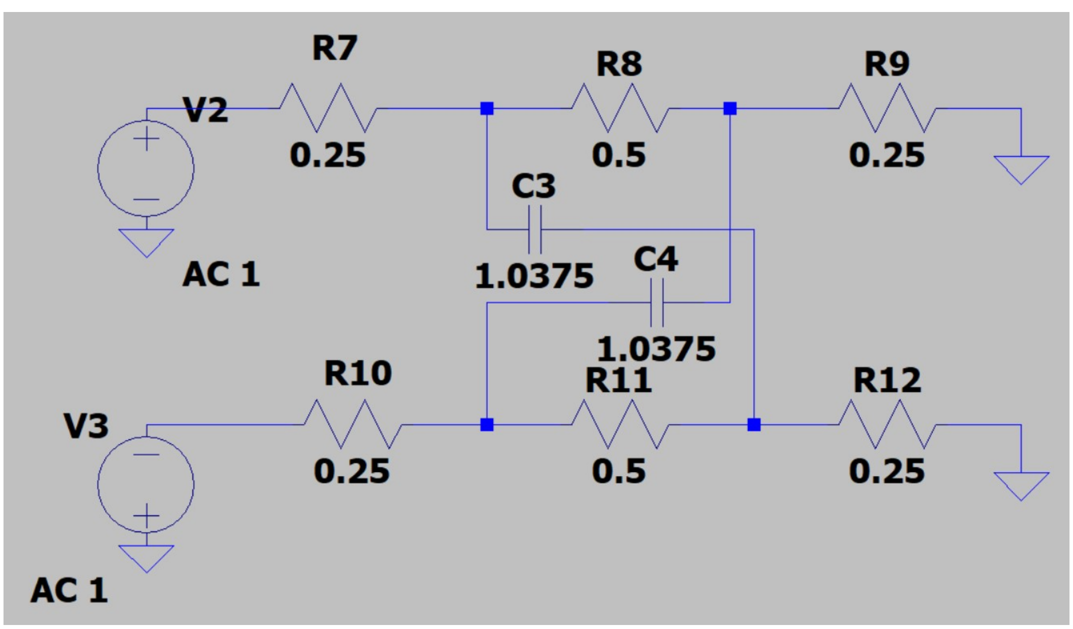
<stopping criteria details>
    1.0375
```



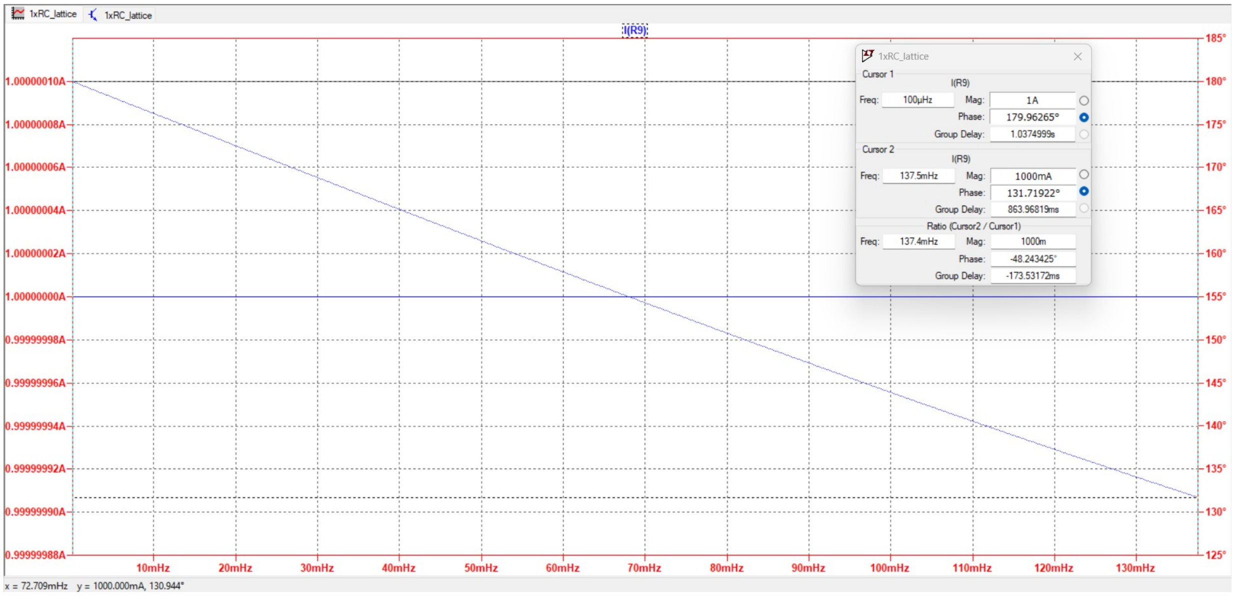
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as “Optimized” and “Approximated” refers to the error obtained when using a capacitor of unit capacitance.



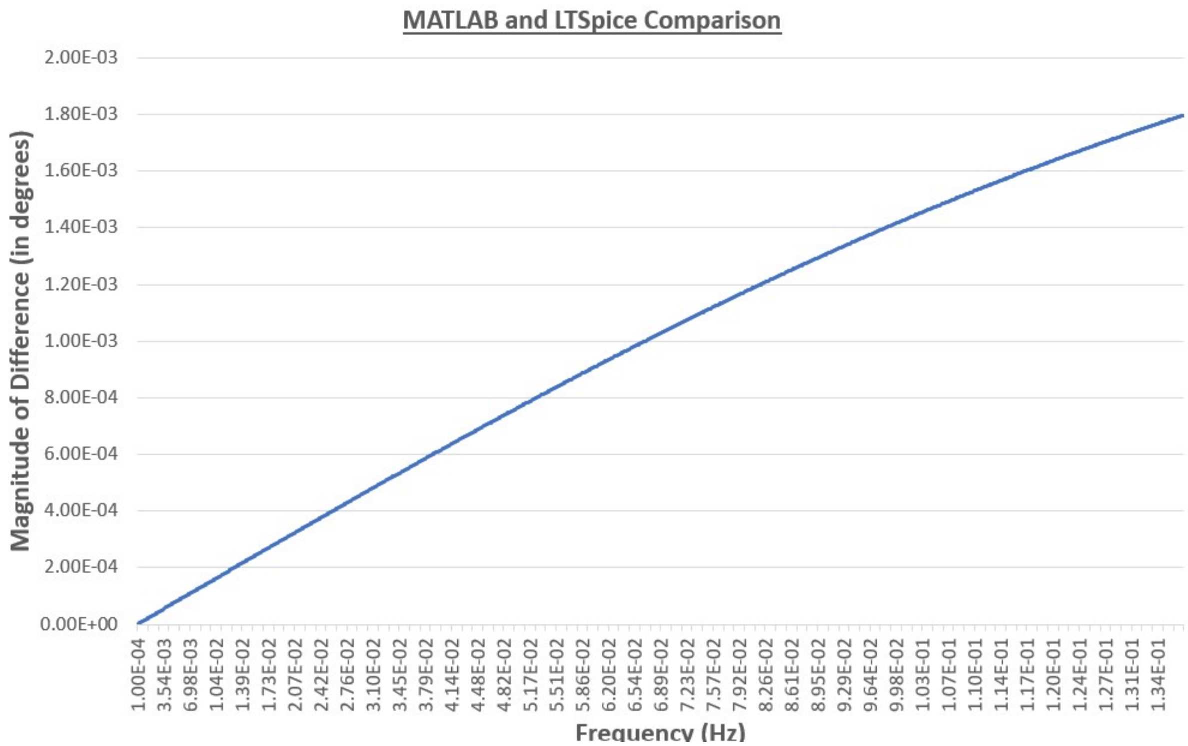
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of 0.0018° over the bandwidth, which gives an error of **0.004**%.

1.4 Experimental results

We observe that the optimized circuit requires the usage of a **1.0375 F** capacitor. The behaviour of the circuit is confirmed by the comparison with the LTSpice simulation.

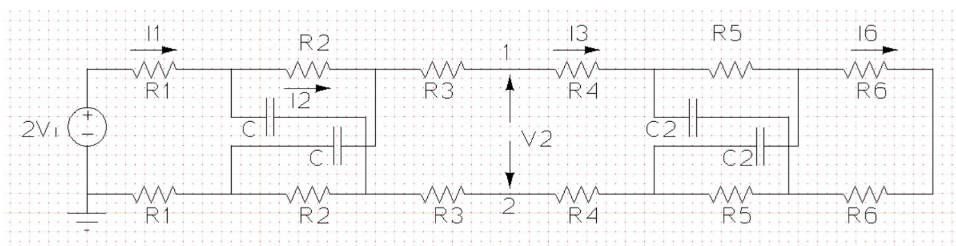
2 2x- cascaded RC lattice - 1

2.1 Aim of the experiment

For a general 2 stage RC lattice, the total resistance is fixed and each segment of the lattice uses resistors in the ratio 1:2:1.

2.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 = R_3 = R_4 = R_6 = 0.125\Omega \tag{43}$$

$$R_2 = R_5 = 0.25\Omega \tag{44}$$

2.3 Simulation results

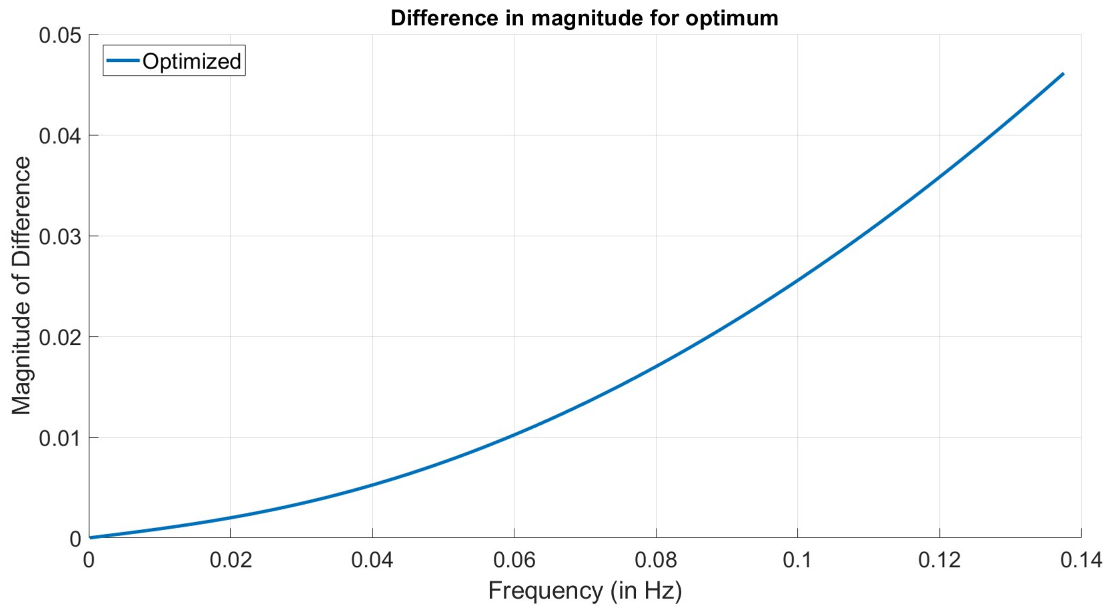
Integral of square of the magnitude of the difference was minimised over the bandwidth. The transfer function obtained from the derivation was used.

The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

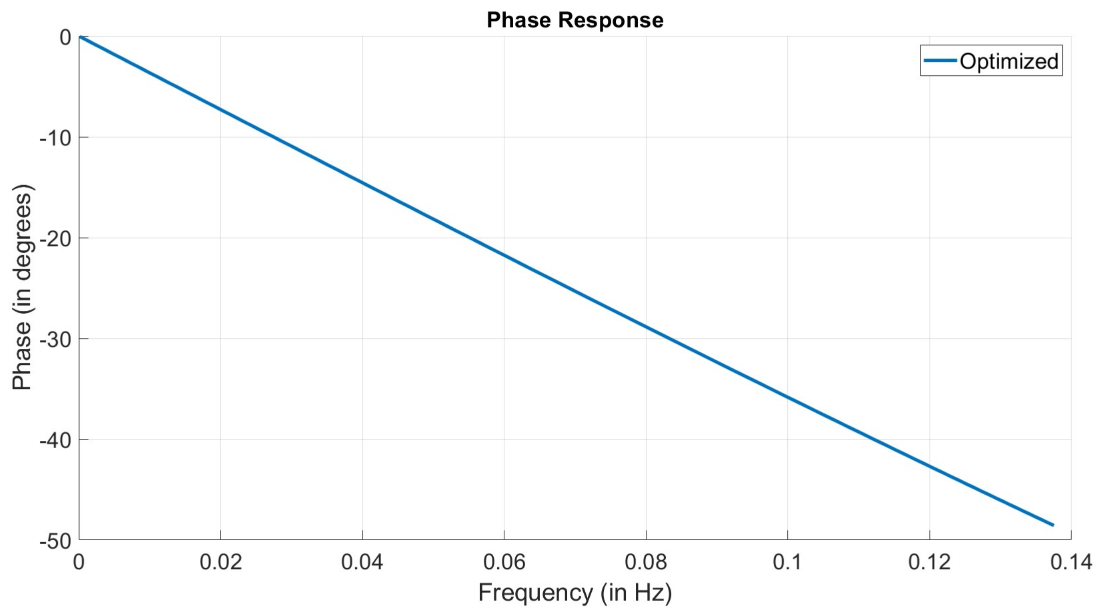
Table 2: Simulation results

C_1	C_2
0.6759 F	0.6759 F

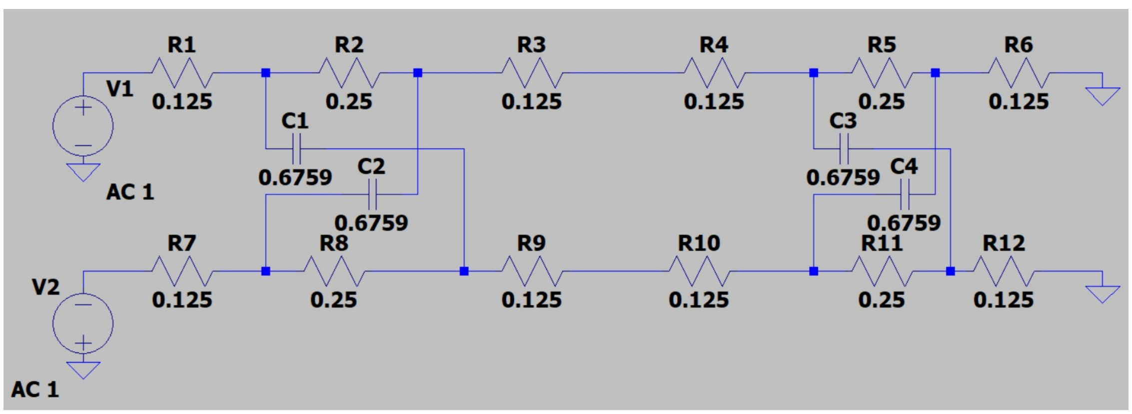
```
<stopping criteria details>
    0.6759    0.6759
```



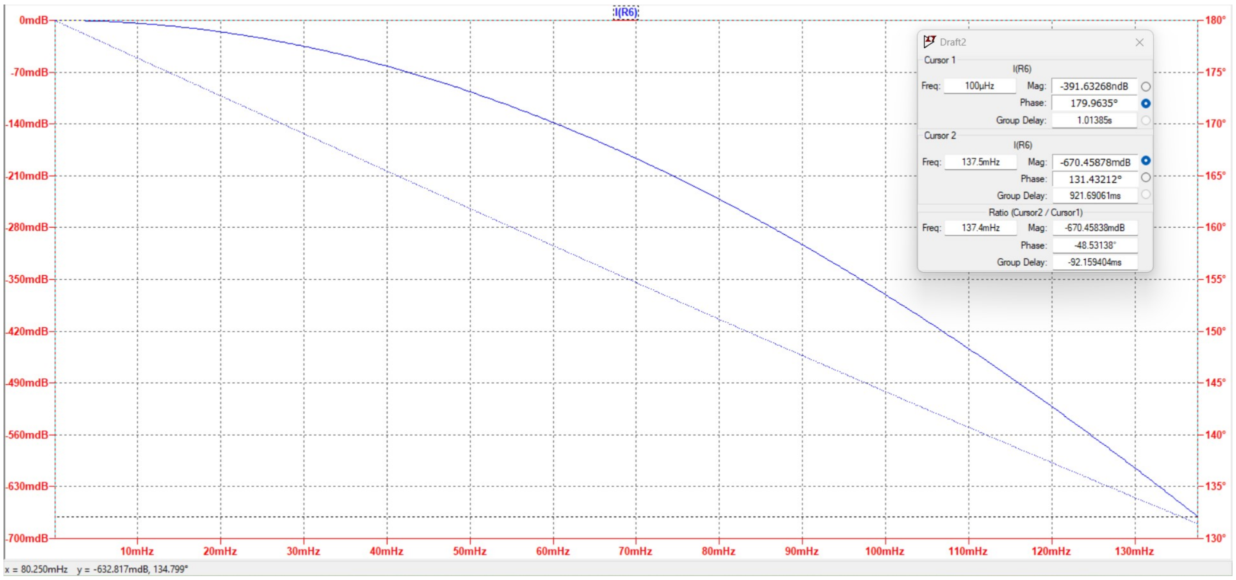
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as “Optimized”.



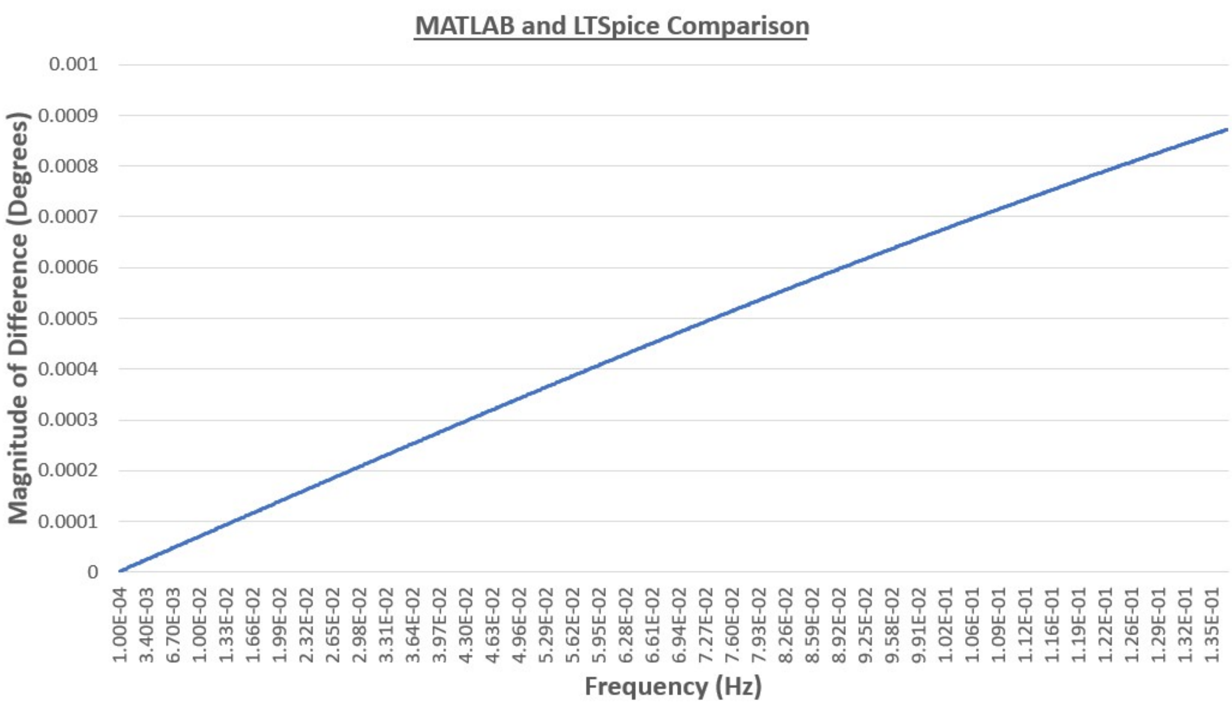
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of $0.0008°$ over the bandwidth, which gives an error of **0.002**%.

2.4 Experimental results

The main observation is that equal capacitors are to be used in order to obtain the optimized circuit. In the optimized circuit, the capacitance used is **0.6759 F**.

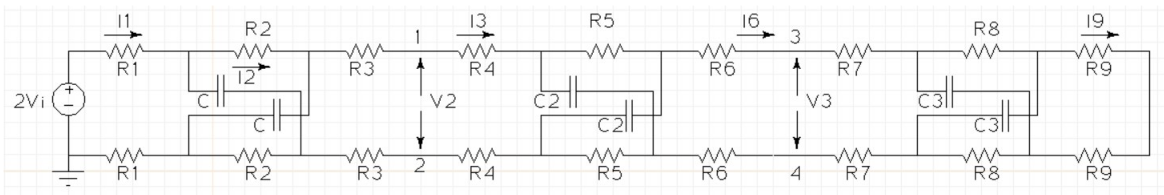
3 3x- cascaded RC lattice - 1

3.1 Aim of the experiment

For a general 3 stage RC lattice, the total resistance is fixed with each section consisting of resistors in the ratio 1:2:1. We find the optimal value of the capacitors for each section of the circuit.

3.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 = R_3 = R_4 = R_6 = R_7 = R_9 = \frac{1}{12}\Omega \tag{45}$$

$$R_2 = R_5 = R_8 = \frac{1}{6}\Omega \tag{46}$$

3.3 Simulation results

Integral of squares of the errors in angles was minimised over the bandwidth.

The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

Table 3: Simulation results

C_1	C_2	C_3
0.8657 F	0.0480 F	0.8657 F

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<stopping criteria details>
    0.8657    0.0480    0.8657
```
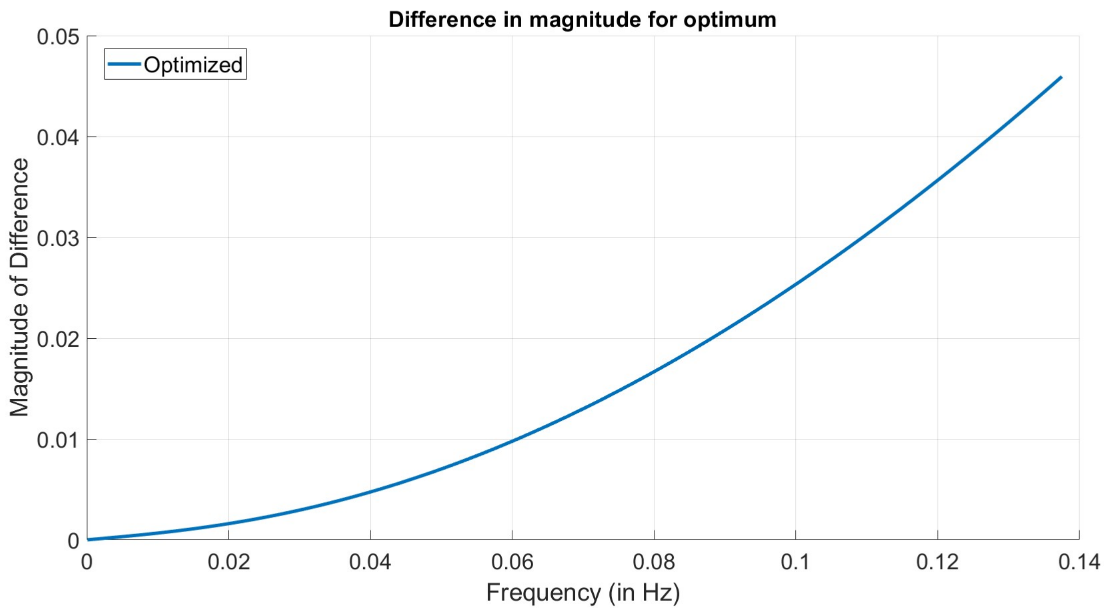
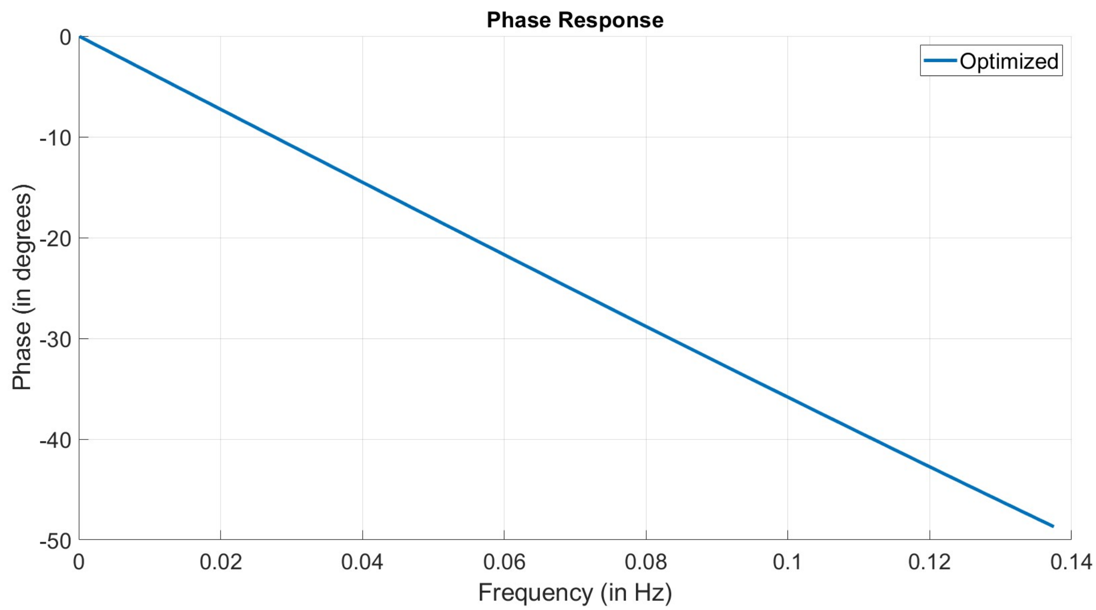
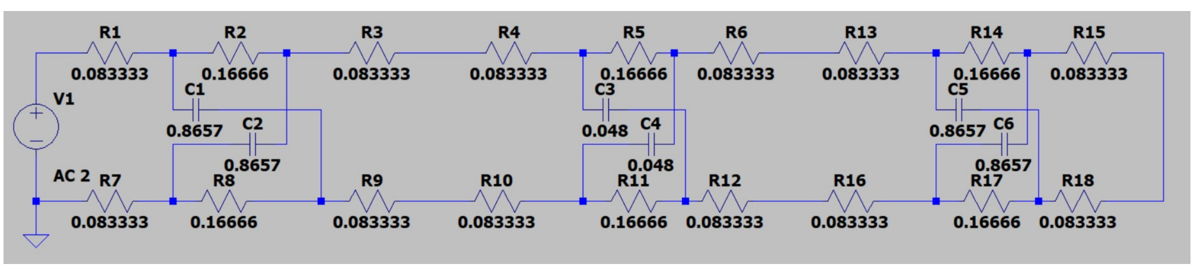



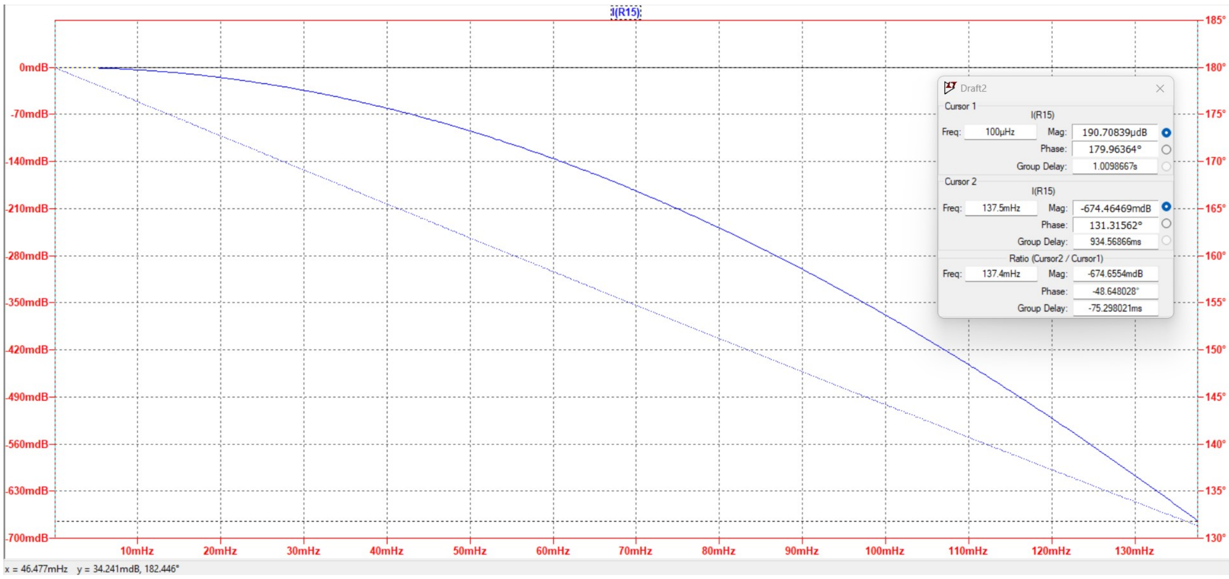
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as “Optimized”.



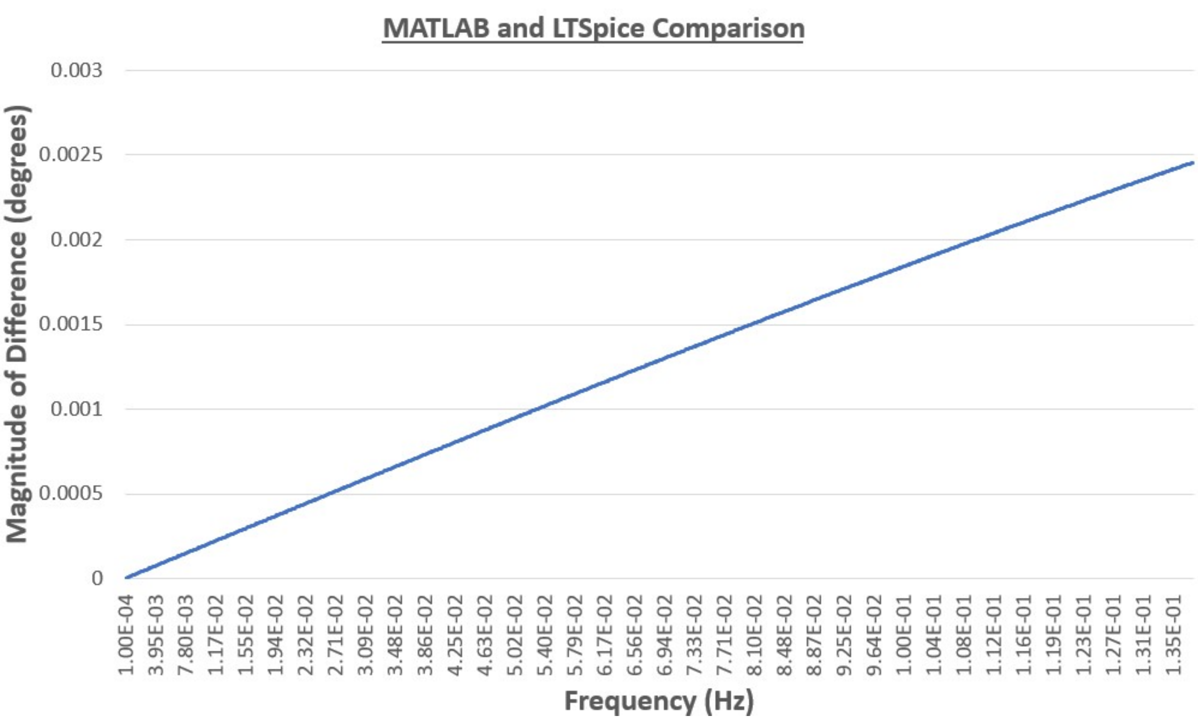
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



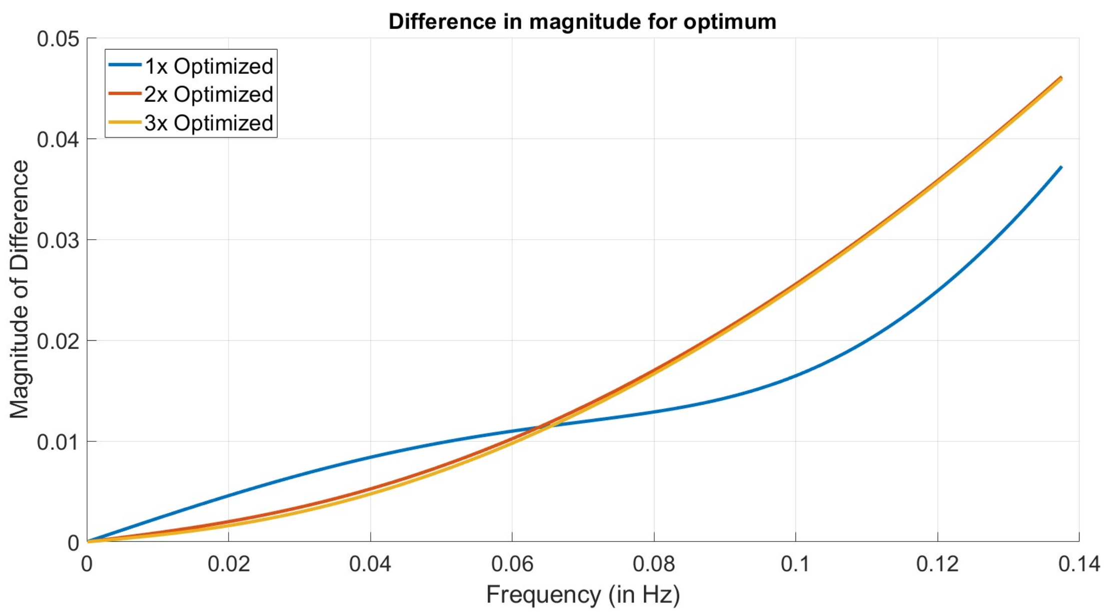
The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of 0.0025° over the bandwidth, which gives an error of **0.005**%.

3.4 Experimental results

Thus we observe that the optimal circuit consists of equal capacitors in the outer sectors and a smaller capacitor in the middle sector. The behaviour of the circuit is verified with the LTSpice simulation.



Above is the comparison for the optimum circuits of the 1x, 2x and 3x RC lattice circuits. For lower frequencies, it is clear that the best circuit is the 3x followed by the 2x and 1x RC lattice circuits.

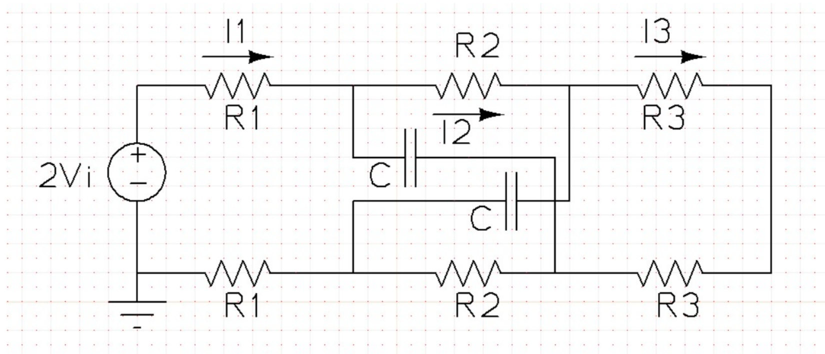
4 1x- RC lattice - 2

4.1 Aim of the experiment

For a general RC lattice, we consider resistors R_1, R_2 and R_3 to have a net resistance of 1 Ω and vary the capacitance such that the objective function is minimized. (2- norm)

4.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 + R_2 + R_3 = 1 \tag{47}$$

4.3 Simulation results

Integral of square of the magnitude of the difference was minimised over the bandwidth.
The objective function of the circuit is given by,

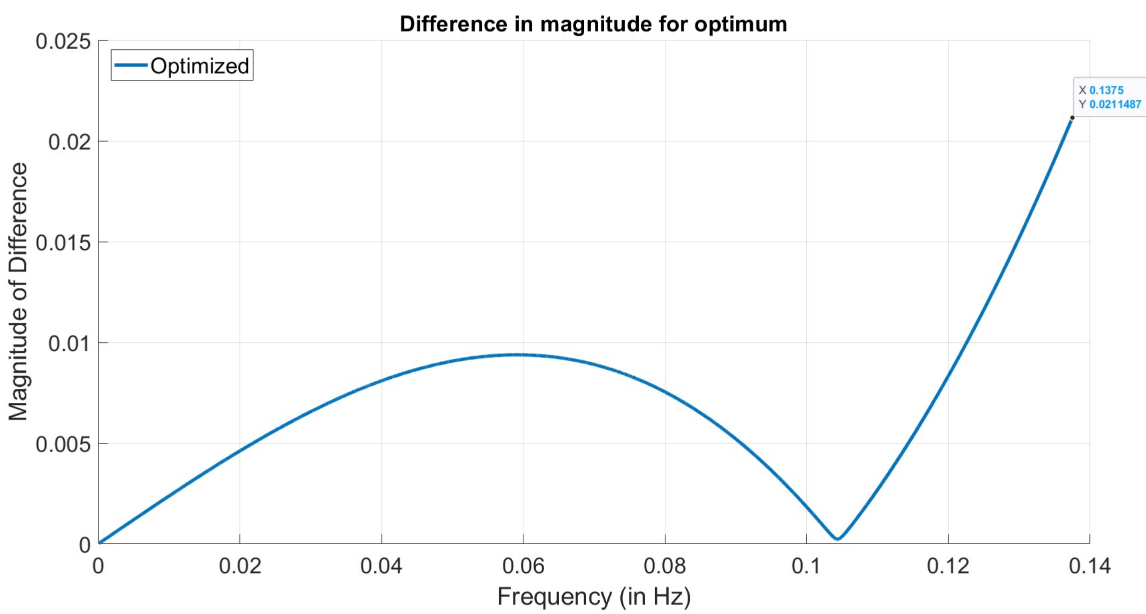
$$objective = \int_0^{\frac{1.1}{8}} |H(f) - T(f)|^2 df \tag{48}$$

The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

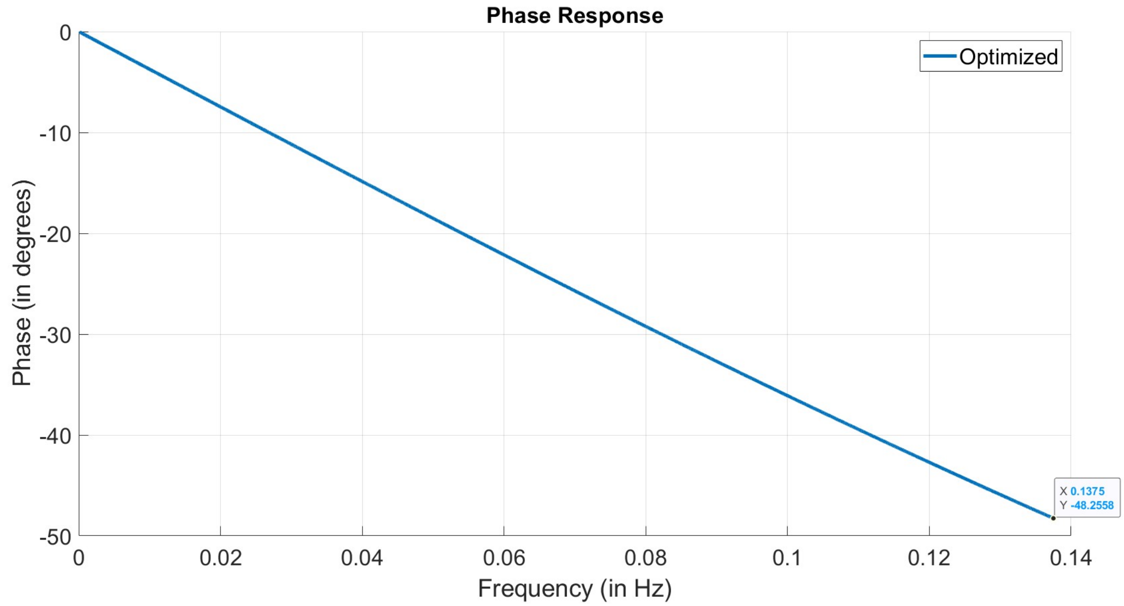
Table 4: Simulation results

R_1	R_2	R_3	C
0.2719 Ω	0.4561 Ω	0.2719 Ω	1.0382 F

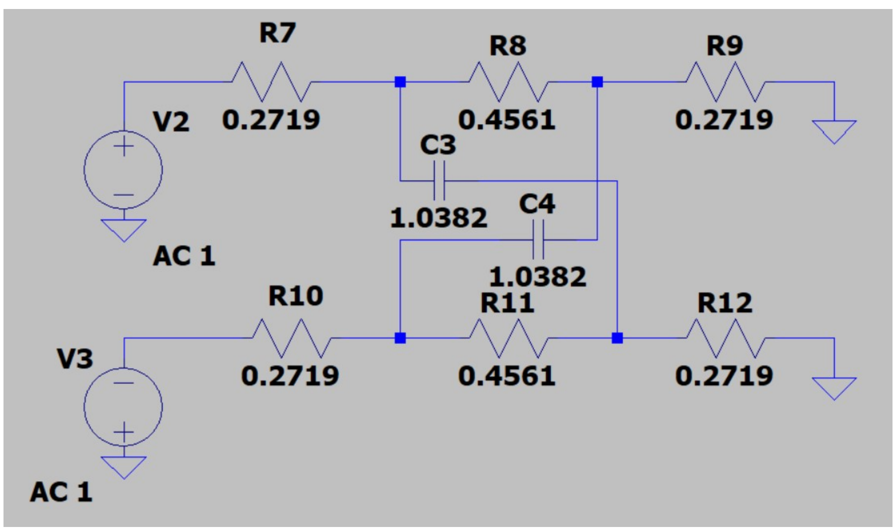
```
<stopping criteria details>
    0.2719    0.4561    0.2719    1.0382
```



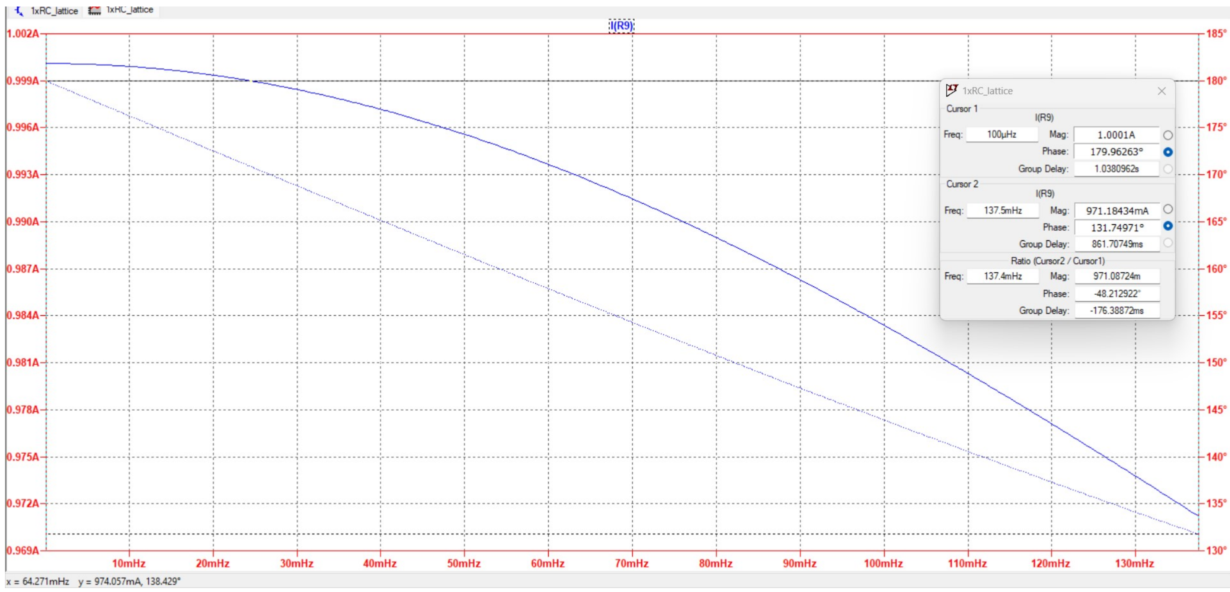
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as “Optimized”.



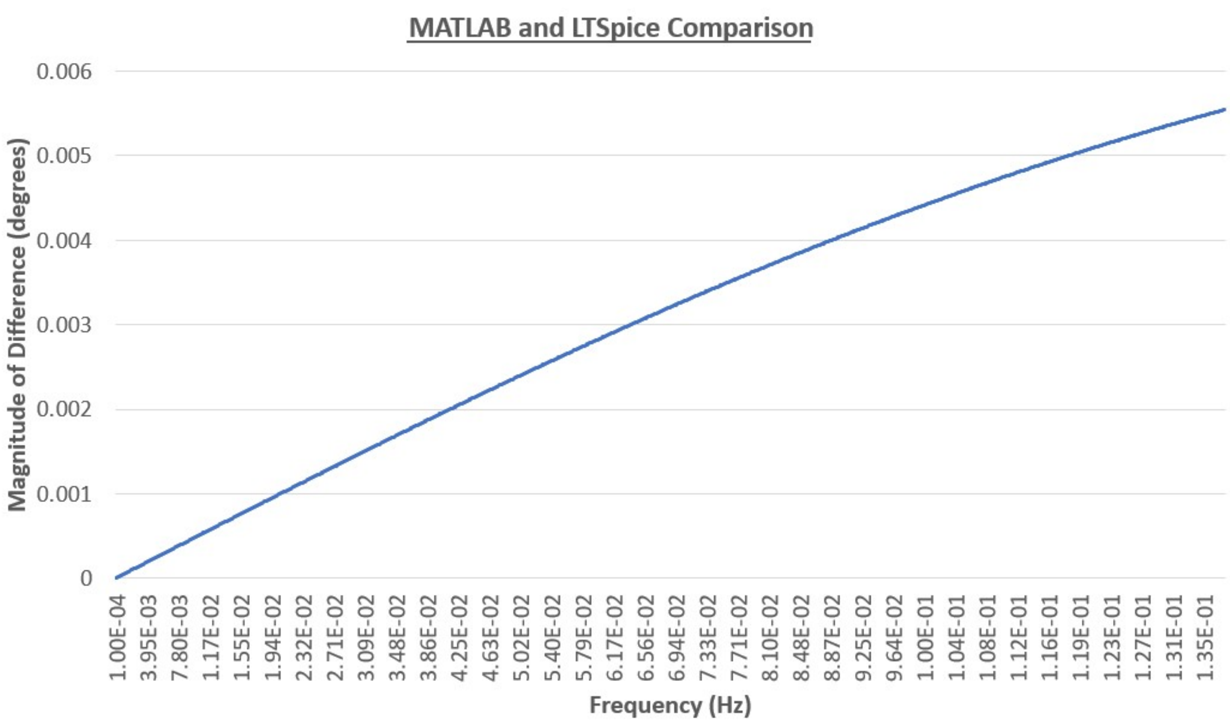
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of $0.0055°$ over the bandwidth, which gives an error of **0.011**%.

4.4 Experimental results

We observe that the optimized circuit requires the usage of equal resistors of resistance **0.2719** Ω for R_1 and R_3 and **0.4561** Ω for R_2. The capacitance is **1.0382** F. The behaviour of the circuit is confirmed by the comparison with the LTSpice simulation.

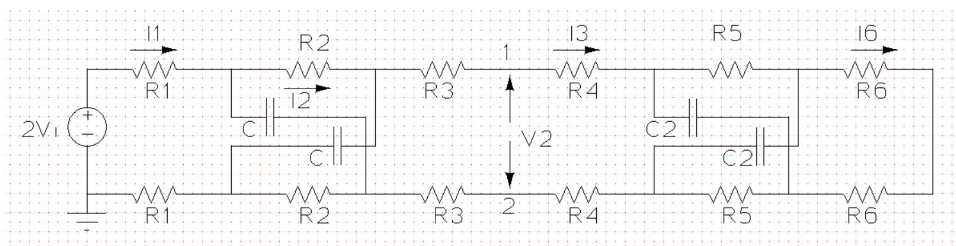
5 2x- cascaded RC lattice - 2

5.1 Aim of the experiment

For a general 2 stage RC lattice, the total resistance is fixed equal to 1 and the component values are varied to optimize the objective function. (2- norm)

5.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 + R_2 + R_3 + R_4 + R_5 + R_6 = 1\Omega \tag{49}$$

5.3 Simulation results

Integral of square of the magnitude of the difference was minimised over the bandwidth. The transfer function obtained from the derivation was used.

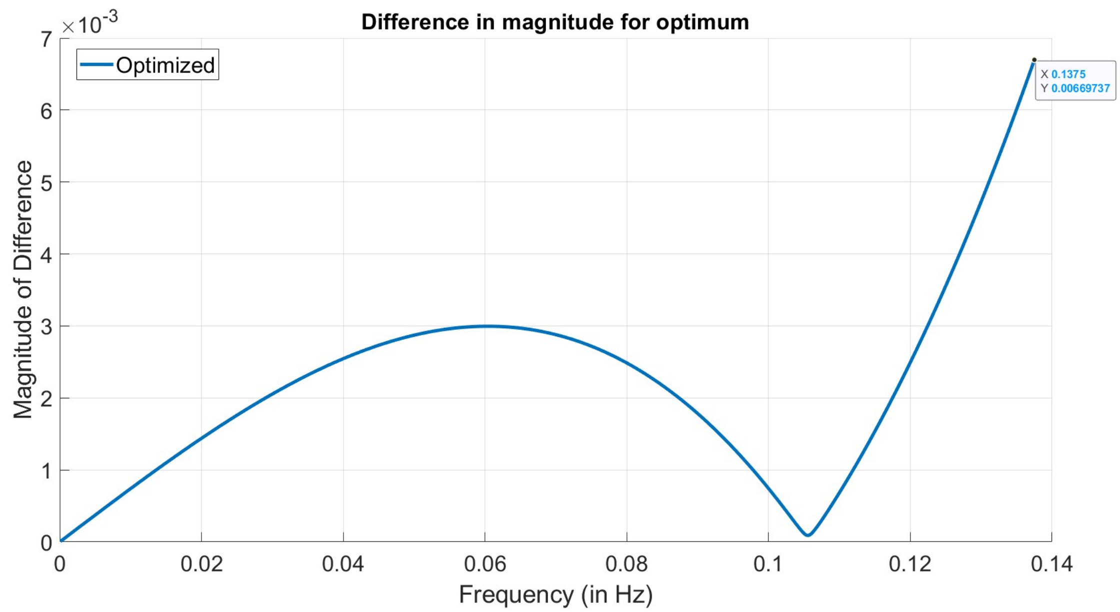
$$objective = \int_0^{\frac{1.1}{8}} |H(f) - T(f)|^2 df \tag{50}$$

The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

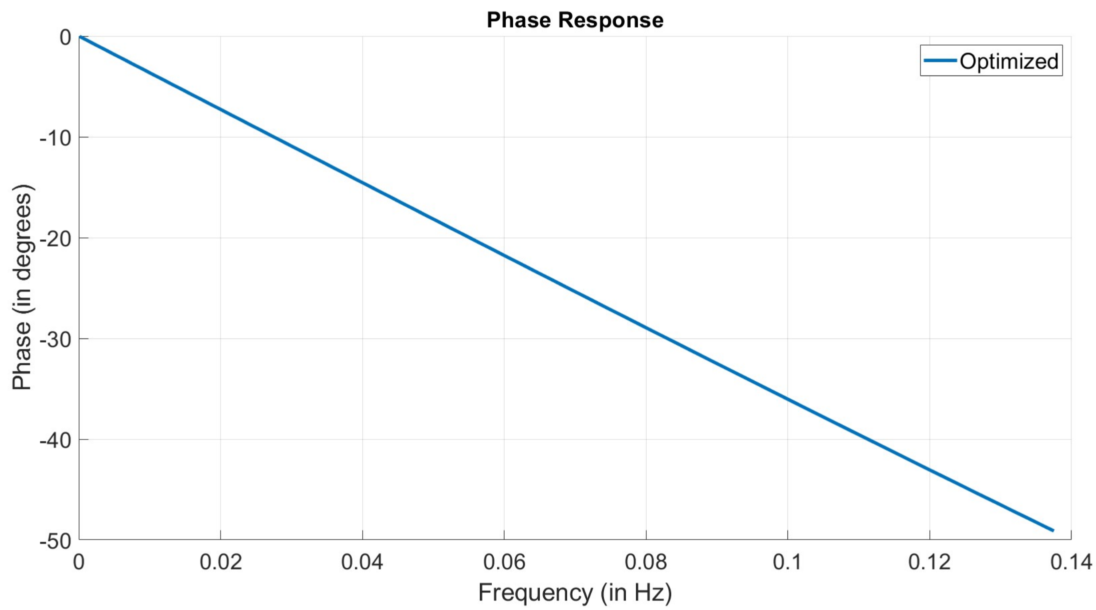
```
<stopping criteria details>
    0.0426    0.2279    0.2295    0.2295    0.2279    0.0426    0.9579    0.9579
```

Table 5: Simulation results

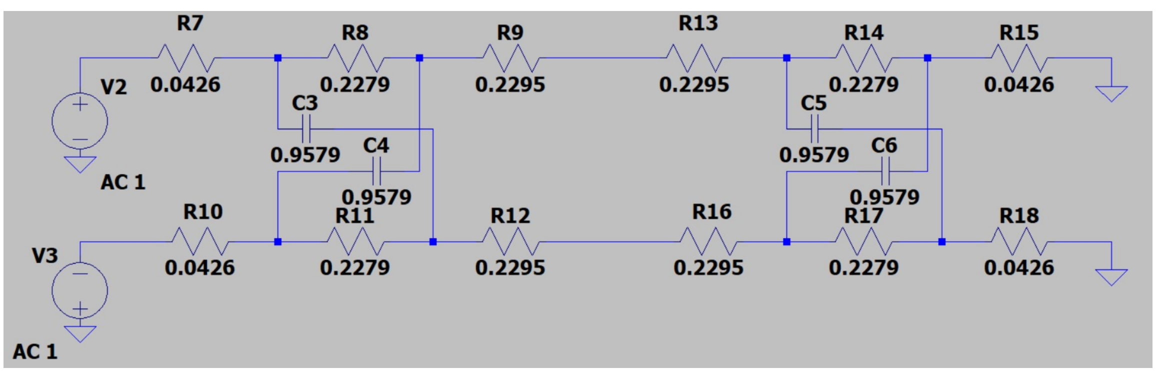
R_1	R_2	R_3	R_4	R_5	R_6	C_1	C_2
0.0426 Ω	0.2279 Ω	0.2295 Ω	0.2295 Ω	0.2279 Ω	0.0426 Ω	0.9579 F	0.9579 F



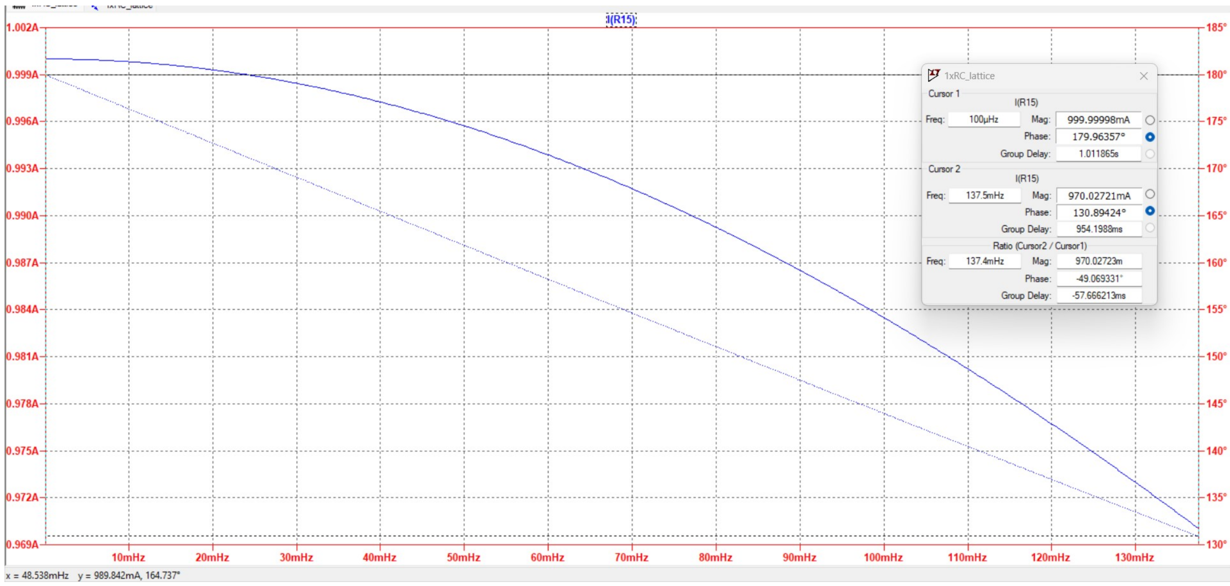
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as "Optimized".



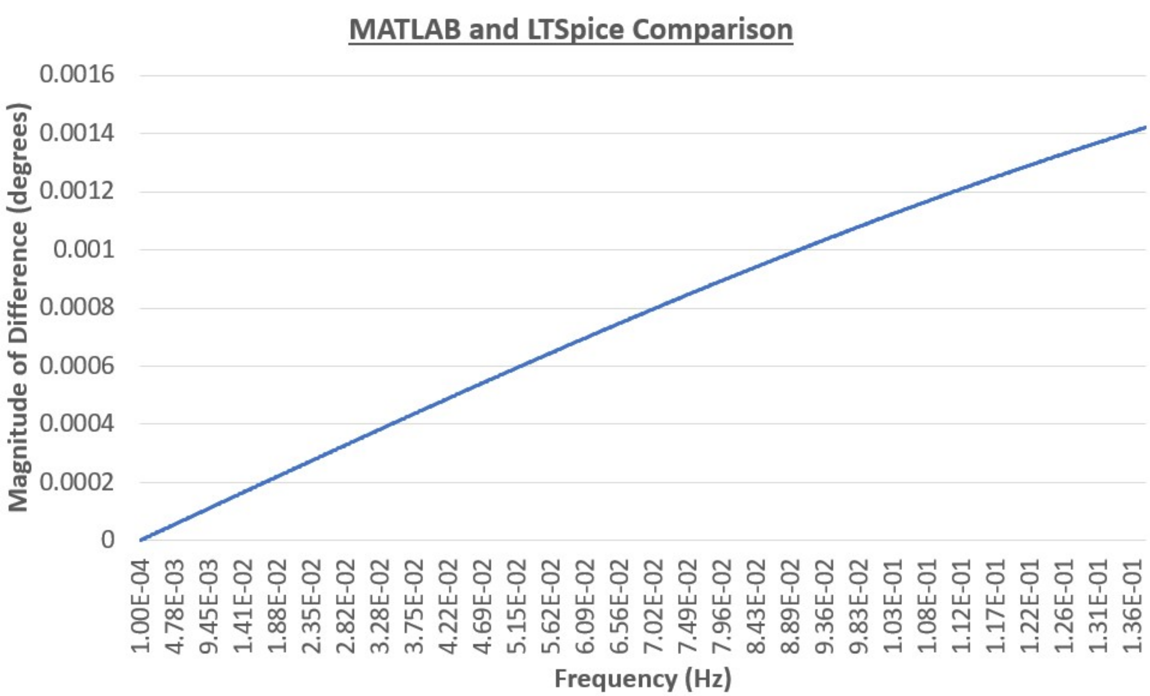
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of $0.0014°$ over the bandwidth, which gives an error of **0.003**%.

5.4 Experimental results

The main observation is the symmetry in the optimized circuit, with equal capacitors in both sections of capacitance **0.9579** F. As well as, $R_1 = R_6 = 0.0426\Omega$, $R_2 = R_5 = 0.2279\Omega$, $R_3 = R_4 = 0.2295\Omega$.

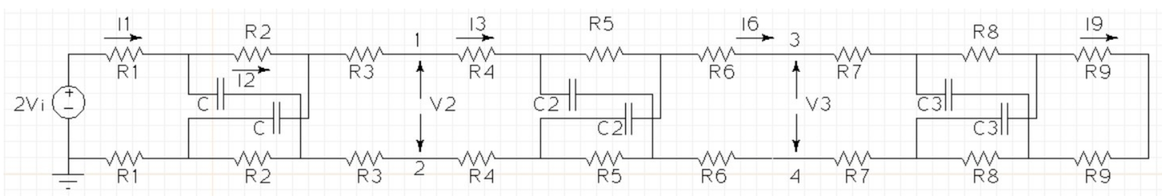
6 3x- cascaded RC lattice - 2

6.1 Aim of the experiment

For a general 3 stage RC lattice, the total resistance is fixed and equal to 1. We find the optimal value of the resistors and capacitors for each section of the circuit. (2- norm)

6.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8 + R_9 = 1\Omega \tag{51}$$

6.3 Simulation results

Integral of squares of the errors in angles was minimised over the bandwidth.

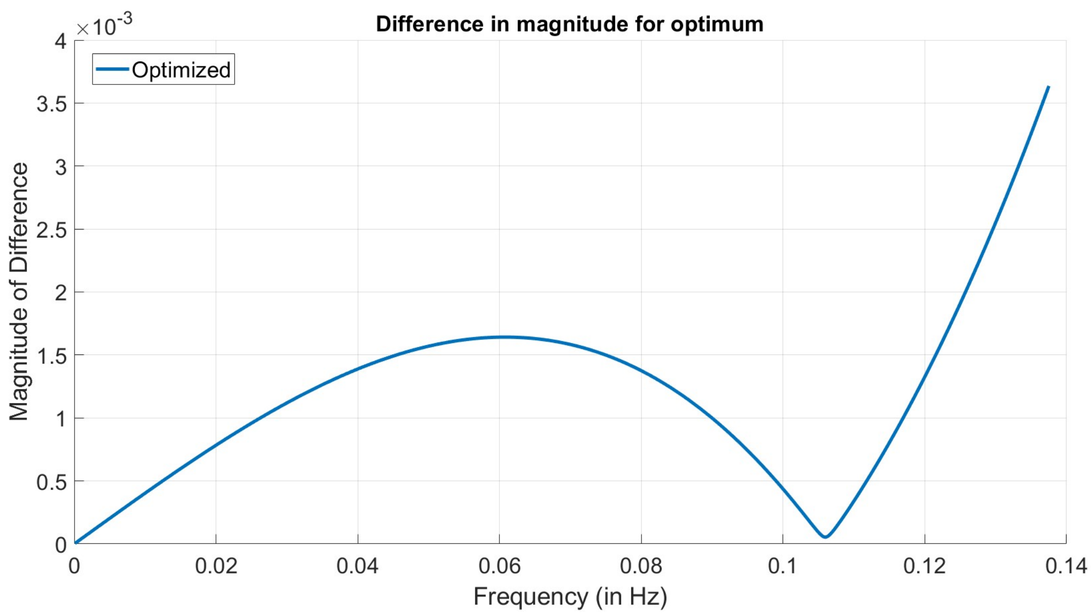
The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

Table 6: Simulation results

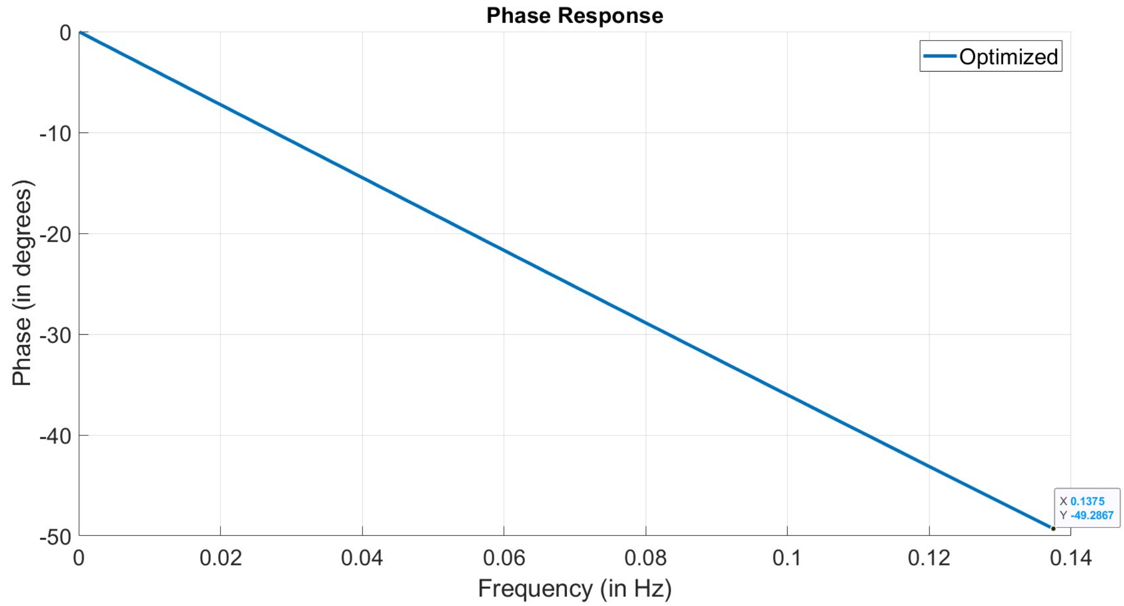
R_1	R_2	R_3	R_4	R_5	R_6
0.0110 Ω	0.0503 Ω	0.1060 Ω	0.1060 Ω	0.4533 Ω	0.1060 Ω

R_7	R_8	R_9	C_1	C_2	C_3
0.1060 Ω	0.0503 Ω	0.0110 Ω	2.2272 F	0.3858 F	2.2272 F

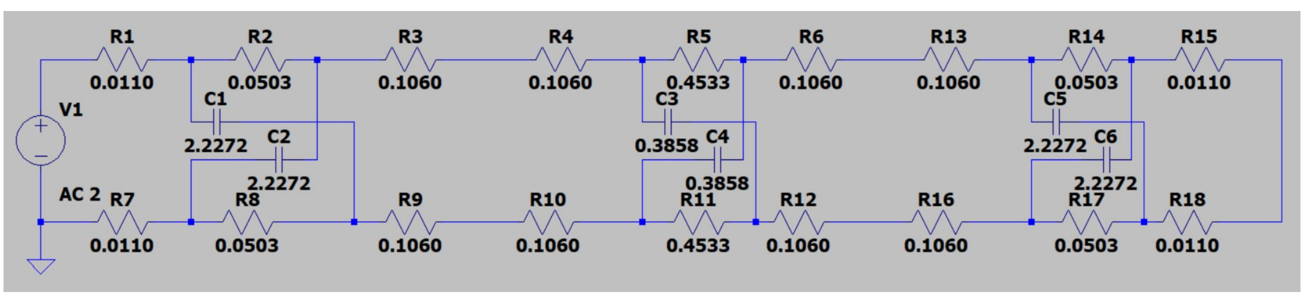
```
<stopping criteria details>
  Columns 1 through 10

    0.0110    0.0503    0.1060    0.1060    0.4533    0.1060    0.1060    0.0503    0.0110    2.2272

  Columns 11 through 12

    0.3858    2.2272
```



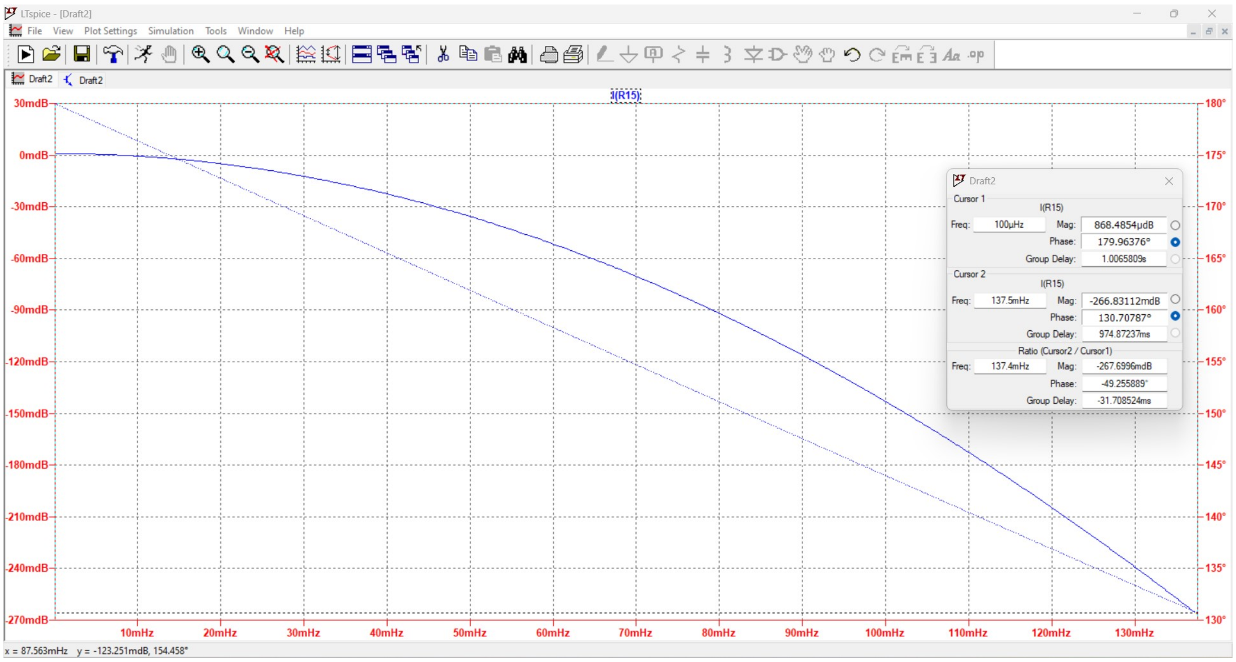
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as “Optimized”.



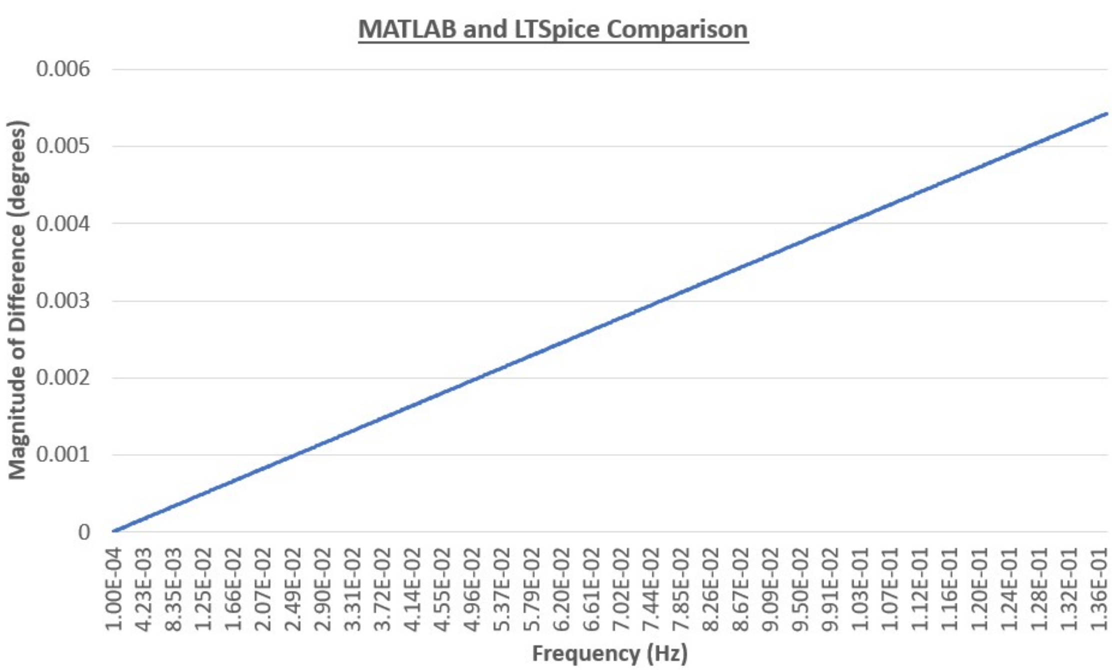
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



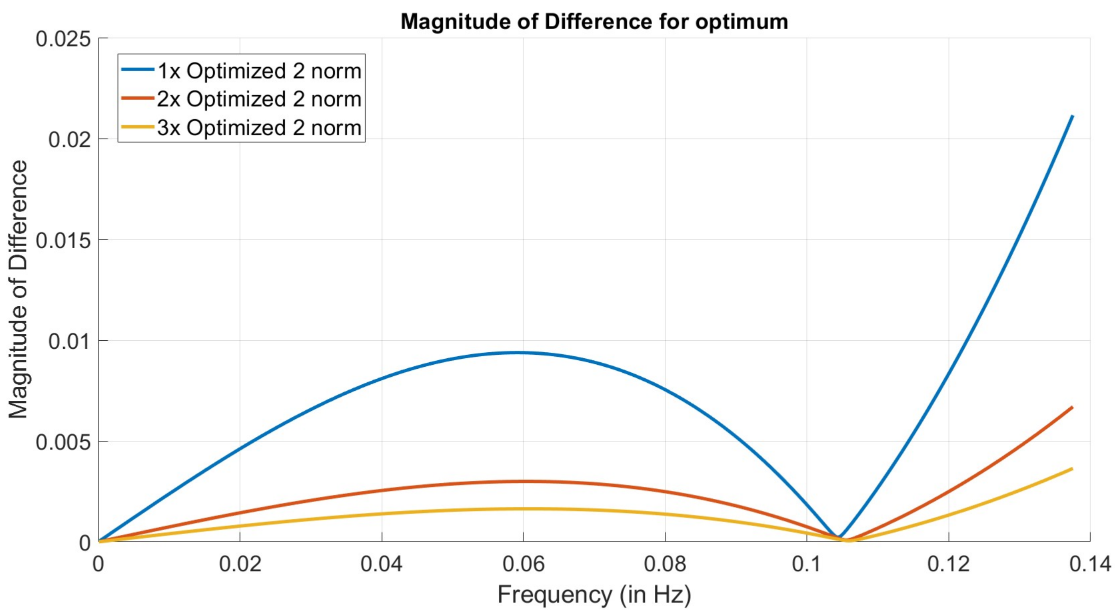
The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of $0.0054°$ over the bandwidth, which gives an error of **0.011**%.

6.4 Experimental results

Thus we observe that the optimal circuit consists of equal capacitors in the outer sectors and a smaller capacitor in the middle sector. It is also symmetric about the middle sector. The behaviour of the circuit is verified with the LTSpice simulation.



Above is the comparison for the optimum circuits of the 1x, 2x and 3x RC lattice circuits. It is clear that the optimized 3x lattice circuit produces the best response, i.e, least error compared to the other optimized circuits.

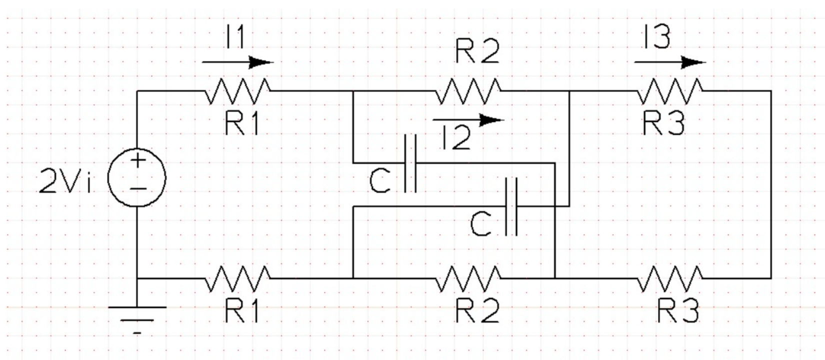
7 1x- RC lattice - 3

7.1 Aim of the experiment

For a general RC lattice, we consider resistors R_1, R_2 and R_3 to have a net resistance of 1 Ω and vary the capacitance such that the objective function is minimized. (Infinity Norm)

7.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 + R_2 + R_3 = 1 \tag{52}$$

7.3 Simulation results

The infinity norm objective function (below) was minimised by varying the resistances and capacitances in the circuit,

$$objective = max_{\omega \epsilon (0,0.1375]} |H(j\omega) - T(j\omega)| \tag{53}$$

The transfer function of the circuit is given by,

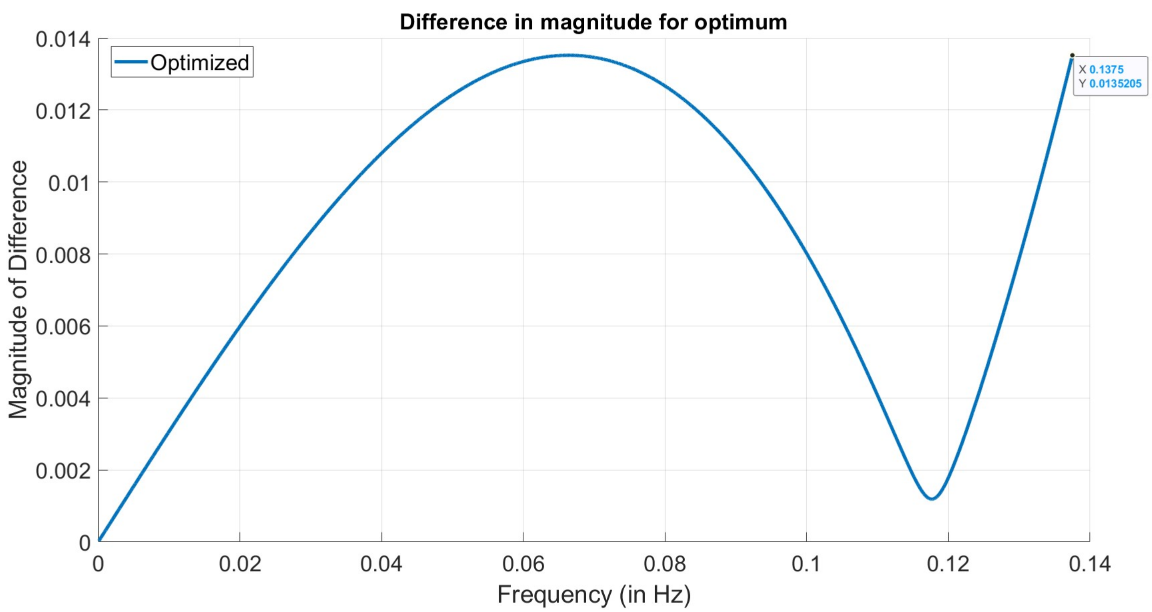
$$H(s) = \frac{(1 - sR_2C)}{(R_1 + R_2 + R_3 + sC(4R_1R_3 + R_1R_2 + R_2R_3)} \tag{54}$$

The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

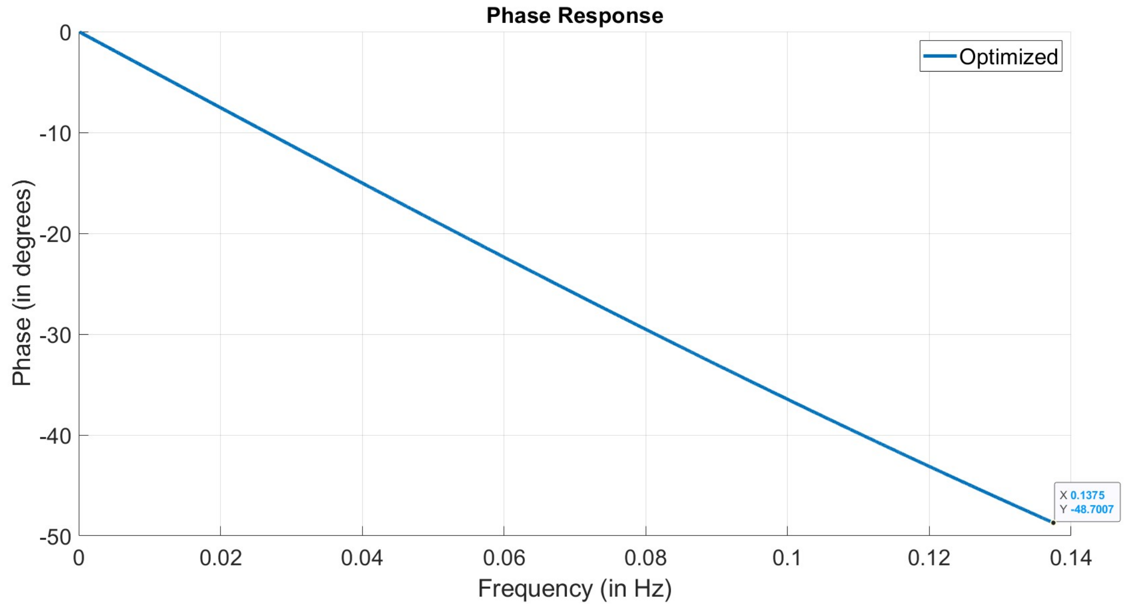
Table 7: Simulation results

R_1	R_2	R_3	C
0.2731 Ω	0.4539 Ω	0.2731 Ω	1.0492 F

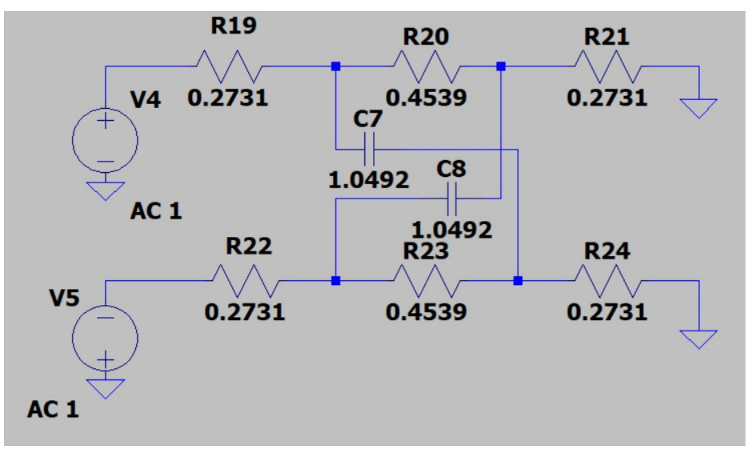
```
<stopping criteria details>
    0.2731    0.4539    0.2731    1.0492
```



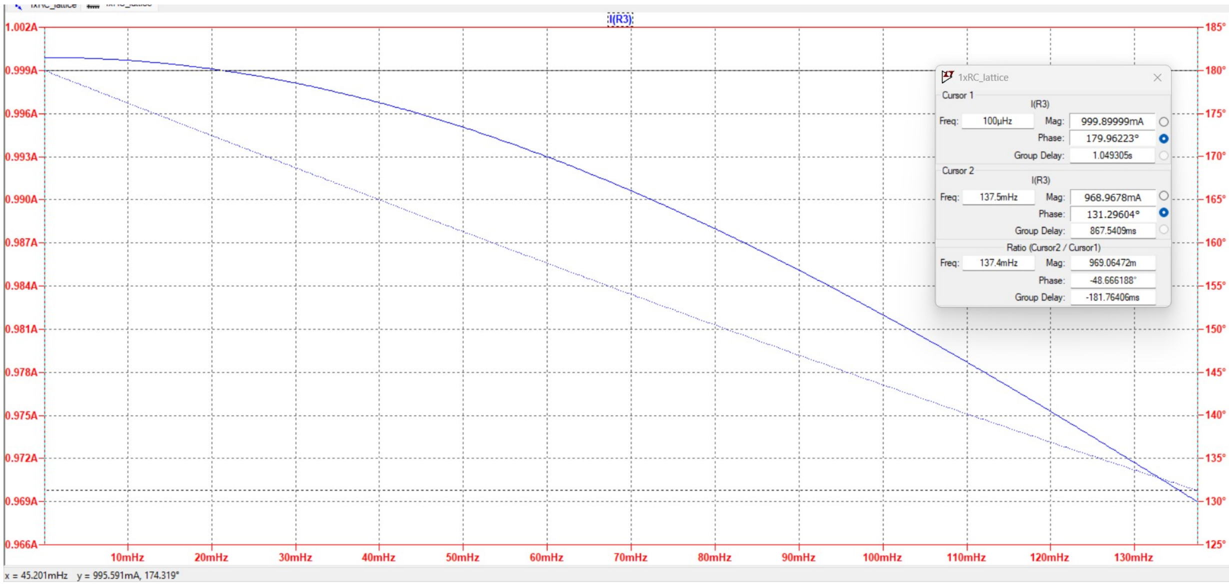
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as “Optimized” and “Approximated” refers to the error obtained when using a capacitor of unit capacitance.



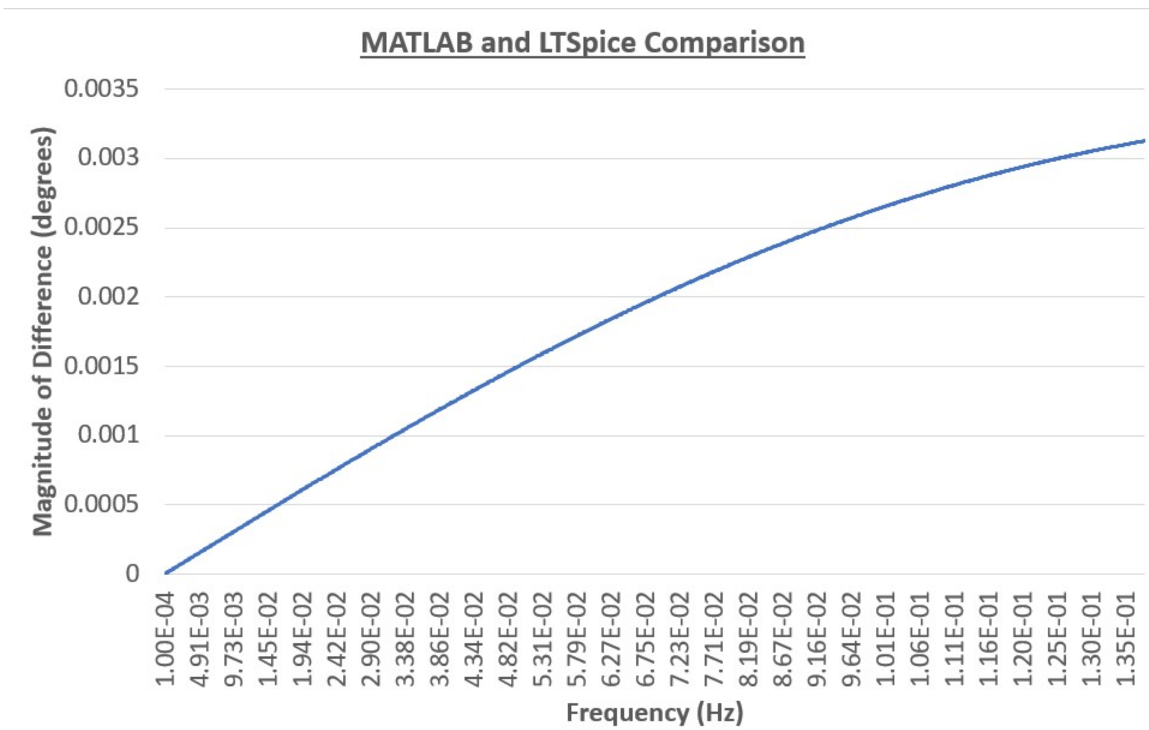
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of $0.0031°$ over the bandwidth, which gives an error of **0.006**%.

7.4 Experimental results

We observe that the optimized circuit requires the usage of a **1.0375 F** capacitor. The behaviour of the circuit is confirmed by the comparison with the LTSpice simulation.

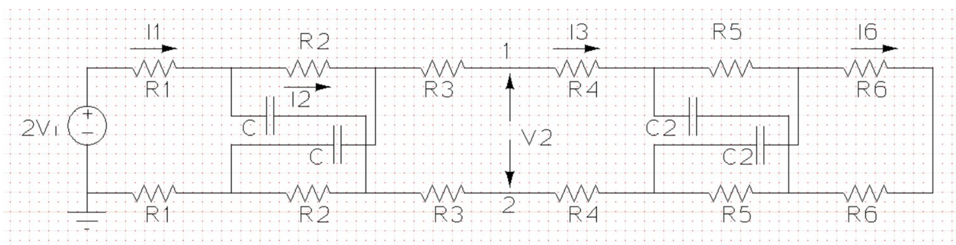
8 2x- cascaded RC lattice - 3

8.1 Aim of the experiment

For a general 2 stage RC lattice, the total resistance is fixed equal to 1 and the component values are varied to optimize the objective function. (Infinity Norm)

8.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 + R_2 + R_3 + R_4 + R_5 + R_6 = 1\Omega \tag{55}$$

8.3 Simulation results

The inifinity norm objective function (below) was minimised by varying the resistances and capacitances in the circuit,

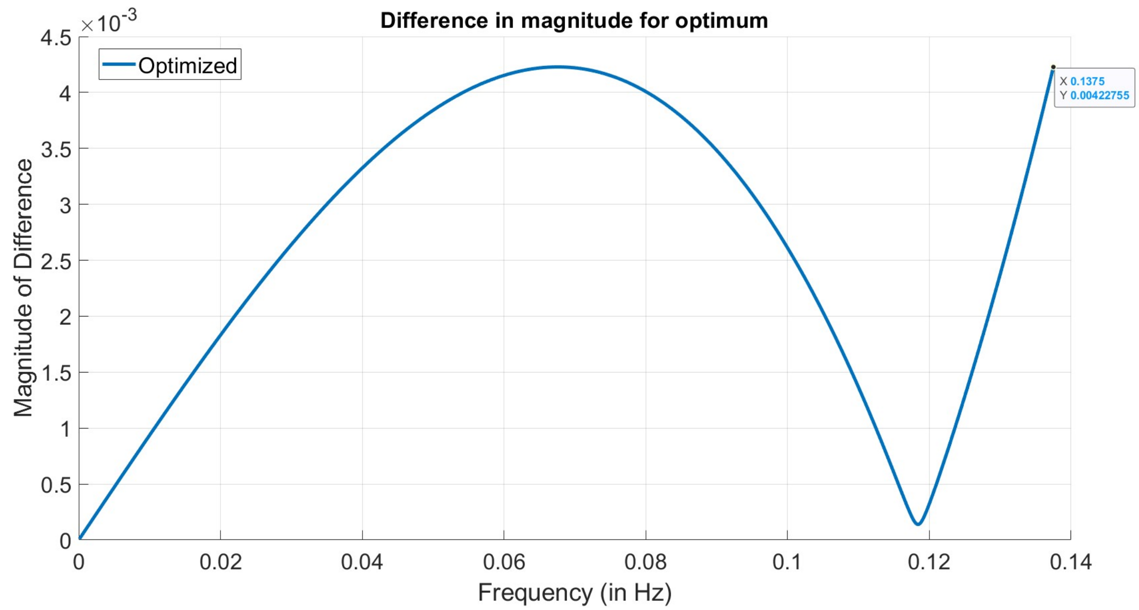
$$objective = max_{\omega\epsilon(0,0.1375]}|H(j\omega) - T(j\omega)| \tag{56}$$

The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

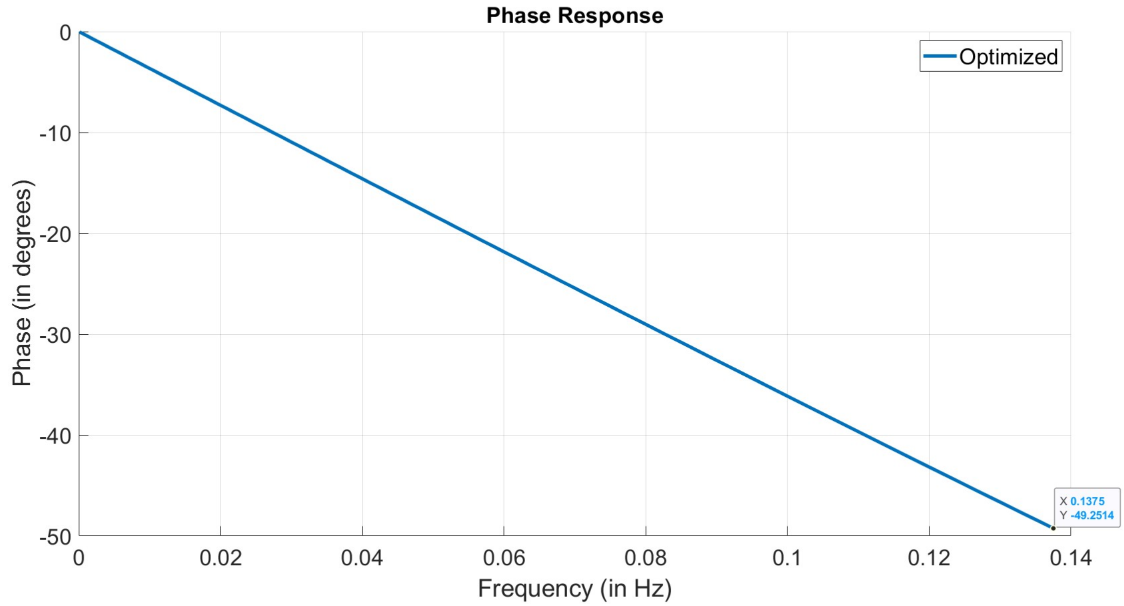
Table 8: Simulation results

R_1	R_2	R_3	R_4	R_5	R_6	C_1	C_2
0.0429 Ω	0.2279 Ω	0.2292 Ω	0.2292 Ω	0.2279 Ω	0.0429 Ω	0.9595 F	0.9595 F

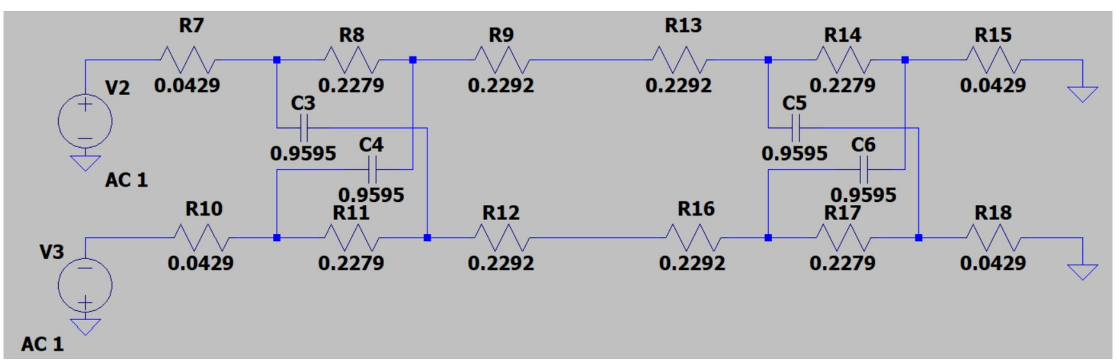
```
<stopping criteria details>
    0.0429    0.2279    0.2292    0.2292    0.2279    0.0429    0.9595    0.9595
```



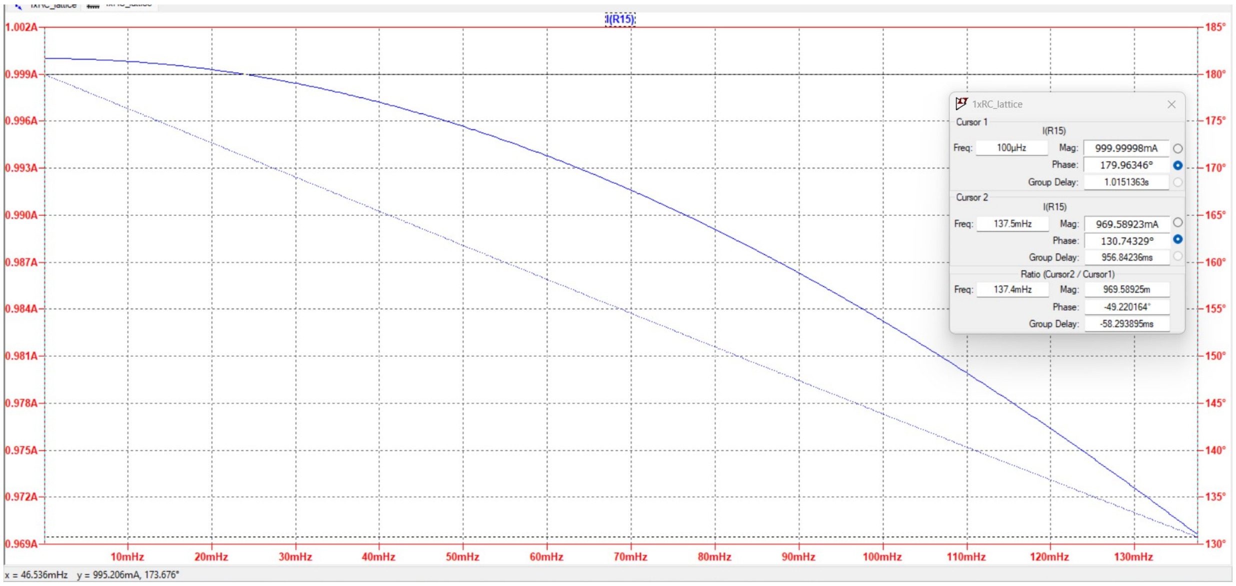
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as “Optimized”.



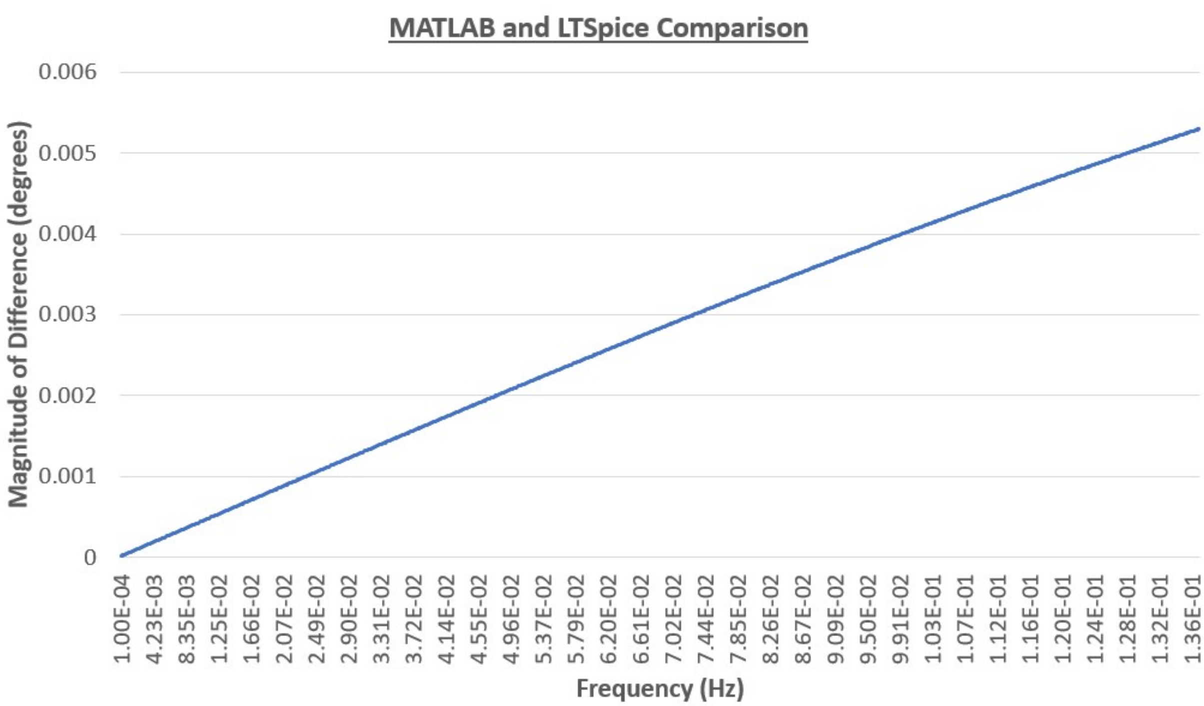
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of $0.0053°$ over the bandwidth, which gives an error of **0.01**%.

8.4 Experimental results

The main observation is the symmetry in the optimized circuit, with equal capacitors in both sections of capacitance 0.9595 F. As well as, $R_1 = R_6 = 0.0429\Omega$, $R_2 = R_5 = 0.2279\Omega$, $R_3 = R_4 = 0.2292\Omega$.
These component values are similar to that obtained by using the 2- norm, differing by less than **0.7 %**.

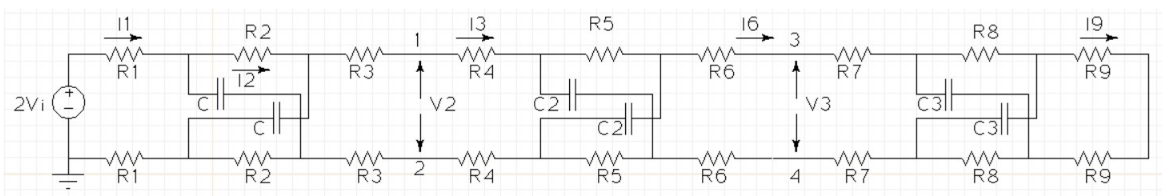
9 3x- cascaded RC lattice - 3

9.1 Aim of the experiment

For a general 3 stage RC lattice, the total resistance is fixed and equal to 1. We find the optimal value of the resistors capacitors for each section of the circuit. (Infinity Norm)

9.2 Design

The circuit diagram is given below



We ensure the following conditions,

$$R_1 + R_2 + R_3 + R_4 + R_5 + R_6 + R_7 + R_8 + R_9 = 1\Omega \tag{57}$$

9.3 Simulation results

The inifinity norm objective function (below) was minimised by varying the resistances and capacitances in the circuit,

$$objective = max_{\omega\epsilon(0,0.1375]}|H(j\omega) - T(j\omega)| \tag{58}$$

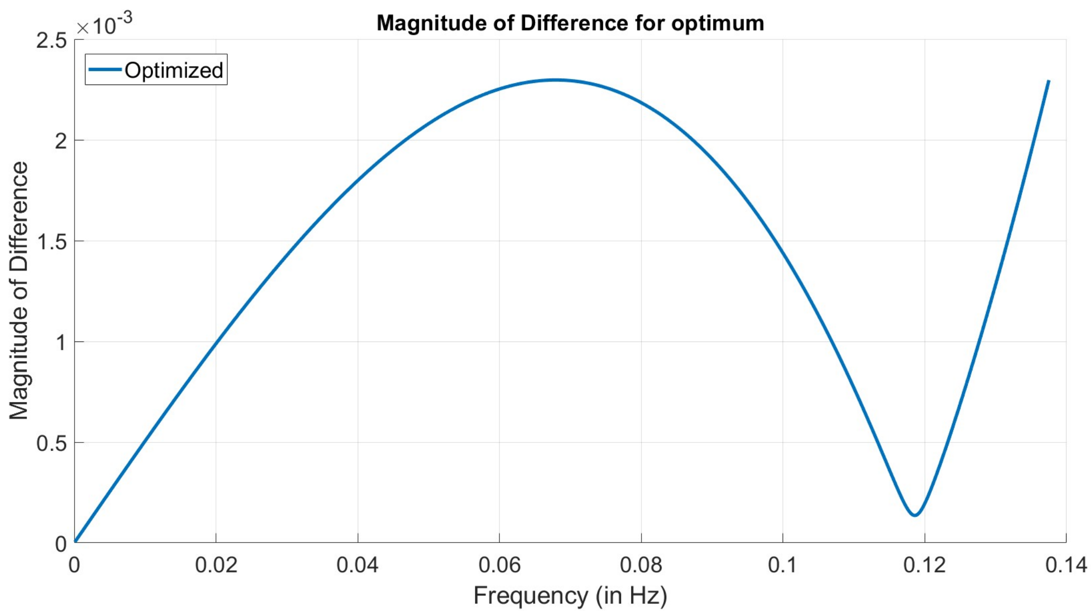
The above conditions were ensured and using the quasi-newton algorithm in the fmincon function of MATLAB the following results were obtained.

Table 9: Simulation results

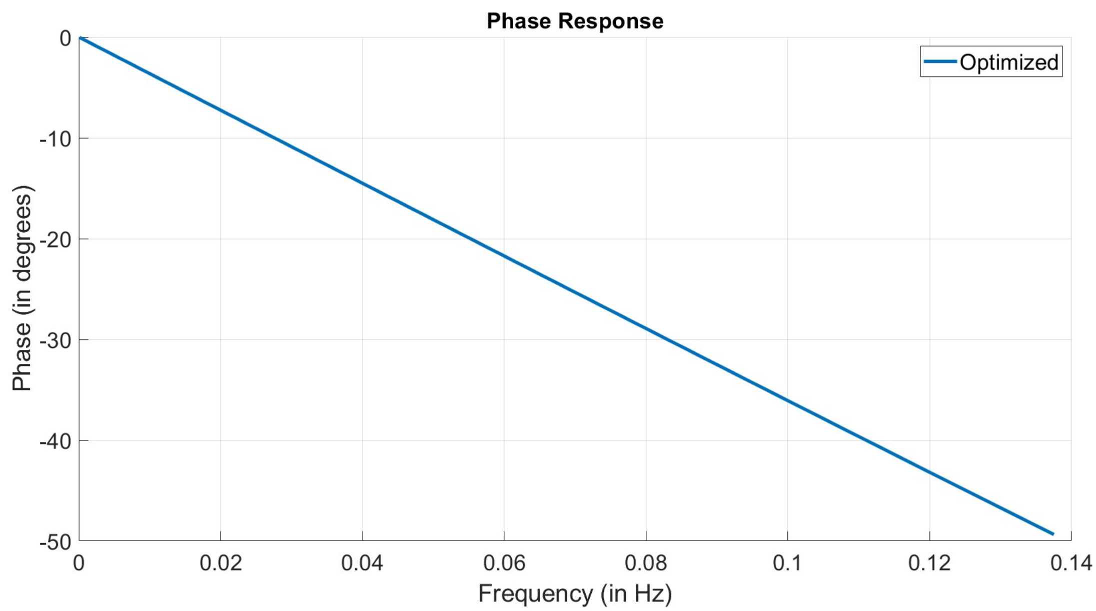
R_1	R_2	R_3	R_4	R_5	R_6
0.0110 Ω	0.0504 Ω	0.1064 Ω	0.1064 Ω	0.4517 Ω	0.1064 Ω

R_7	R_8	R_9	C_1	C_2	C_3
0.1064 Ω	0.0504 Ω	0.0110 Ω	2.2277 F	0.3860 F	2.2277 F

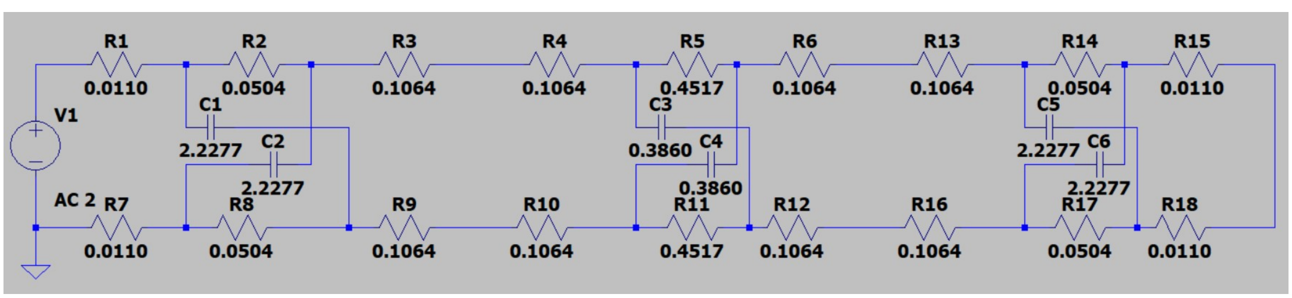
```
<stopping criteria details>
  Columns 1 through 10

    0.0110    0.0504    0.1064    0.1064    0.4517    0.1064    0.1064    0.0504    0.0110    2.2277

  Columns 11 through 12

    0.3860    2.2277
```



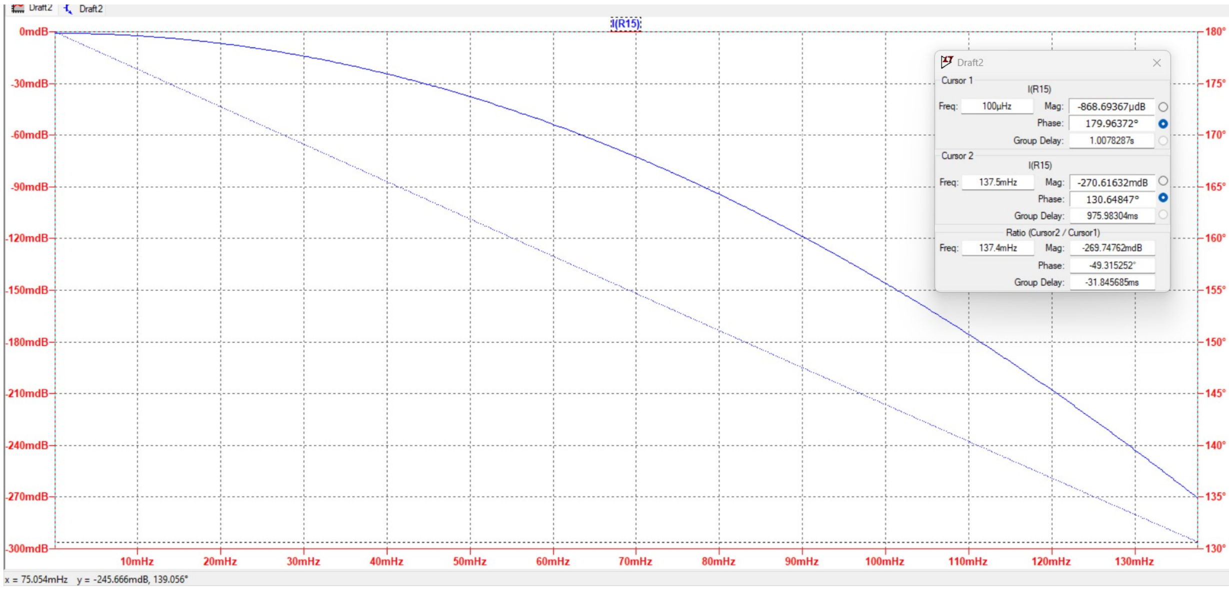
In the above figure, the magnitude of the difference between the optimized circuit and the ideal transfer function is given as “Optimized”.



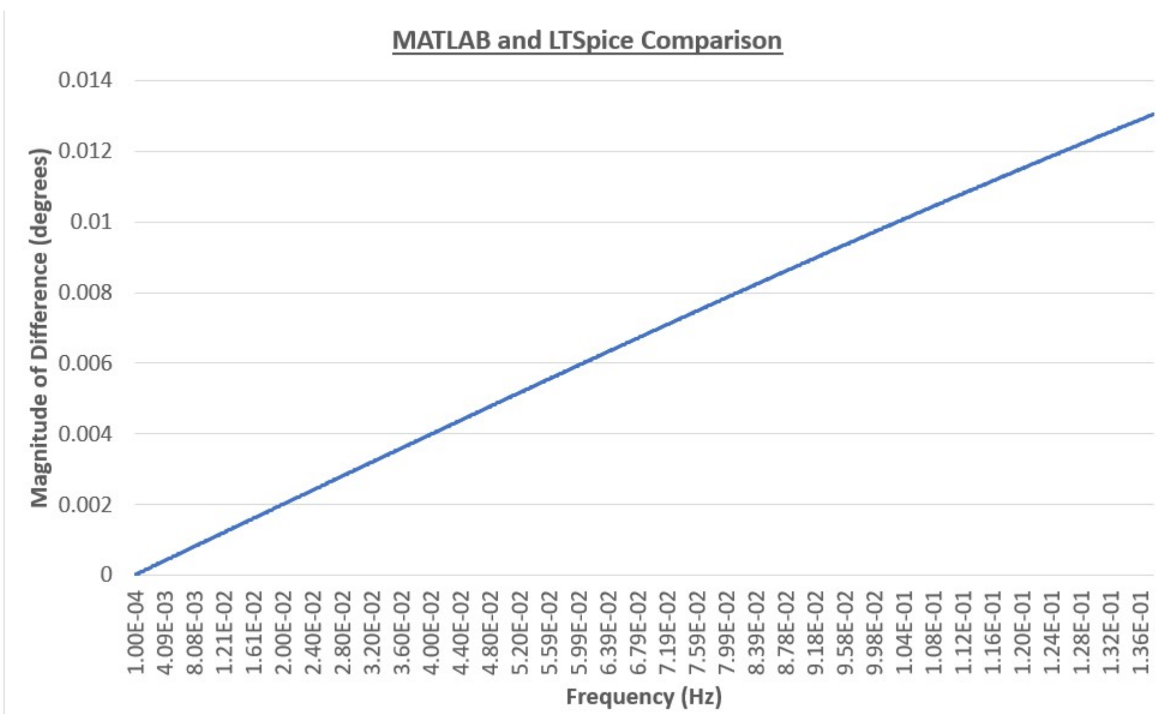
The above plot is the phase response of the optimized circuit obtained from the MATLAB simulation.



The above circuit is constructed in LTSpice and it's behaviour is observed.



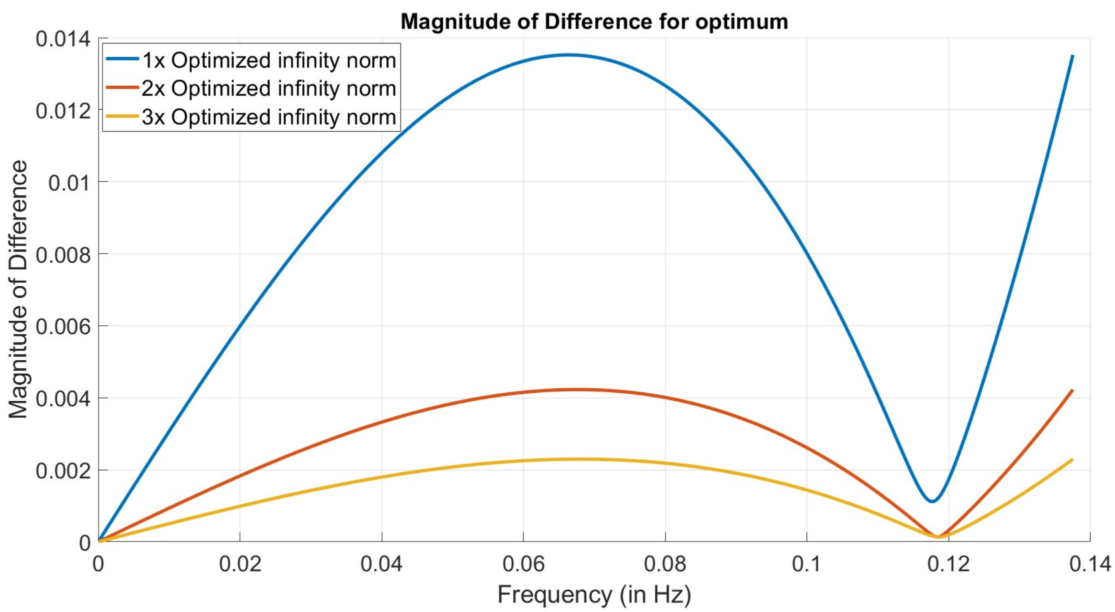
The above plot represents the behaviour of the circuit made in LTSpice.



Above is the plot of the difference in phase response of the MATLAB and LTSpice simulations, there is a maximum error of 0.013° over the bandwidth, which gives an error of **0.026**%.

9.4 Experimental results

Thus we observe that the optimal circuit consists of equal capacitors in the outer sectors and a smaller capacitor in the middle sector. It is also symmetric about the middle sector. The behaviour of the circuit is verified with the LTSpice simulation.



Above is the comparison for the optimum circuits of the 1x, 2x and 3x RC lattice circuits. Again it is clear that the 3x optimized circuit produces the best response.

The optimal circuits obtained from the 2 norm and infinity norm methods are very similar, their components are summarised below,

Table 10: 1x lattice 2 norm and infinity norm optimal circuit comparison

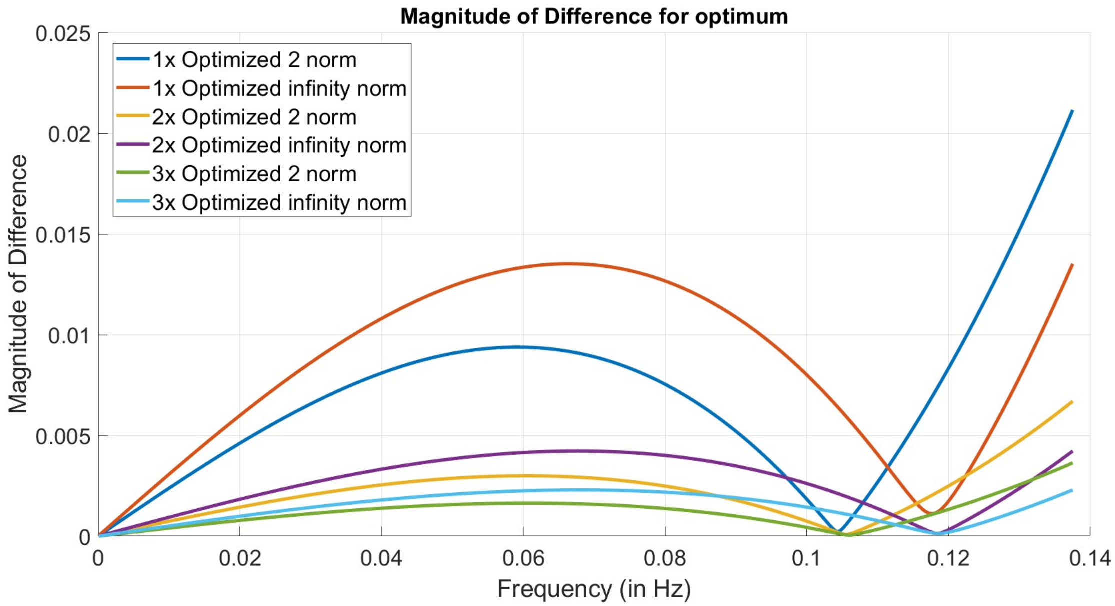
Component	2 norm	Infinity norm	Error (%)
R_1	0.2719	0.2731	0.44
R_2	0.4561	0.4539	0.48
R_3	0.2719	0.2731	0.44
C_1	1.0382	1.0492	1.05

Table 11: 2x lattice 2 norm and infinity norm optimal circuit comparison

Component	2 norm	Infinity norm	Error (%)
R_1	0.0426	0.0429	0.70
R_2	0.2279	0.2279	0
R_3	0.2295	0.2292	0.13
R_4	0.2295	0.2292	0.13
R_5	0.2279	0.2279	0
R_6	0.0426	0.0429	0.70
C_1	0.9579	0.9595	0.17
C_2	0.9579	0.9595	0.17

Table 12: 3x lattice 2 norm and infinity norm optimal circuit comparison

Component	2 norm	Infinity norm	Error (%)
R_1	0.0110	0.0110	0
R_2	0.0503	0.0504	0.20
R_3	0.1060	0.1064	0.38
R_4	0.1060	0.1064	0.38
R_5	0.4533	0.4517	0.35
R_6	0.1060	0.1064	0.38
R_4	0.1060	0.1064	0.38
R_5	0.0503	0.0504	0.20
R_6	0.0110	0.0110	0
C_1	2.2272	2.2277	0.22
C_2	0.3858	0.3860	0.52
C_3	2.2272	2.2277	0.22



Above is the comparison for the optimum circuits of the 1x, 2x and 3x RC lattice circuits got using the 2 norm as well as the infinity norm.
In general, the 3x circuit gives a better response than the 2x circuit followed by the 1x RC lattice circuit.