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Calculando o polinômio de interpolação de grau 4

$$g(s) = \sum_{\substack{k=0 \\ n+4}} \binom{s}{k} \Delta^k x_0 = \sum_{\substack{k=0 \\ n=4}} \frac{s!}{k!(s-k)!} \Delta^k x_0$$

$$= \frac{s!}{0!(s-0)!}\Delta^0 x_0 + \frac{s!}{1!(s-1)!}\Delta^1 x_0 + \frac{s!}{2!(s-2)!}\Delta^2 x_0 + \frac{s!}{3!(s-3)!}\Delta^3 x_0 + \frac{s!}{4!(s-4)!}\Delta^4 x_0$$

$$= \frac{s!}{0!s!}x(0) + \frac{s(s-1)!}{1!(s-1)!}(x(1)-x(0)) + \frac{s(s-1)(s-2)!}{2!(s-2)!}(x(2)-2x(1)+x(0)) +$$

$$+\frac{s(s-1)(s-2)(s-3)!}{3!(s-3)!}(x(3)-3x(2)+3x(1)+x(0)) + \frac{s(s-1)(s-2)(s-3)(s-4)!}{4!(s-4)!}(x(4)-4x(3)+6x(2)-4x(1)+x(0))$$

$$= x(0) + s(x(1)-x(0)) + \frac{1}{2}s(s-1)(x(2)-2x(1)+x(0)) + \frac{1}{6}s(s-1)(s-2)(x(3)-3x(2)+3x(1)-x(0)) + \frac{1}{24}s(s-1)(s-2)(s-3)(x(4)-4x(3)+6x(2)-4x(1)+x(0))$$

$$= x(0)\left(1 - s + \frac{1}{2}s(s-1) - \frac{1}{6}s(s-1)(s-2) + \frac{1}{24}s(s-1)(s-2)(s-3)\right)$$
$$+ x(1)\left(s - 2\left(\frac{1}{2}s(s-1)\right) + 3\left(\frac{1}{6}s(s-1)(s-2)\right) - 4\left(\frac{1}{24}s(s-1)(s-2)(s-3)\right)\right)$$
$$+ x(2)\left(\frac{1}{2}s(s-1) - 3\left(\frac{1}{6}s(s-1)(s-2)\right) + 4\left(\frac{1}{24}s(s-1)(s-2)(s-3)\right)\right)$$
$$+ x(3)\left(\frac{1}{6}s(s-1)(s-2) - 4\left(\frac{1}{24}s(s-1)(s-2)(s-3)\right)\right)$$
$$+ x(4)\frac{1}{24}s(s-1)(s-2)(s-3) = \therefore$$

$$= x(0)\left(1 - \frac{25}{12}s + \frac{35}{24}s^2 - \frac{5}{12}s^3 + \frac{1}{24}s^4\right)$$
$$+ x(1)\frac{1}{6}\left(24s - 26s^2 + 9s^3 - s^4\right)$$
$$+ x(2)\frac{1}{4}\left(-12s + 19s^2 - 8s^3 + s^4\right)$$
$$+ x(3)\frac{1}{6}\left(-8s - 24s^2 + 7s^3 - s^4\right)$$
$$+ x(4)\frac{1}{24}\left(s^4 - 6s^3 + 11s^2 - 6s\right)$$

abordagem fechada:

polinomio deve passar por $x_i$ e $x_f$ e mais tres pontos, todos equidistantes. distancia entre os pontos sera dada por $h = \frac{\Delta x}{4}$.

Assim:

$f(x_i) = f(x(s=0)) = g(0)$, $f(x_i+h) = f(x(s=1)) = g(1)$

$f(x_i+2h) = f(x(s=2)) = g(2)$, $f(x_i+3h) = f(x(s=3)) = g(3)$ e

$f(x_i+4h) = f(x(s=4)) = g(4)$

e $x(s) = x_i + sh$ satisfaz essas relações pois

$x(0) = x_i + 0h = x_i$, $x(1) = x_i + 1h$, $x(2) = x_i + 2h$, $x(3) = x_i + 3h$, $x(4) = x_i + 4h = x_f$

Aplicando a mudança de variável temos:

$$\int_{x_i}^{x_f} f(x)\,dx \approx \int_{x_i}^{x_f} P(x)\,dx = \int_{s_i}^{s_f} P(x(s))\frac{dx(s)}{ds}\,ds = h\int_0^4 P(x(s))\,ds = h\int_0^4 g(s)\,ds$$

Substituindo o polinomio de substituição de grau 4 nessa integral temos

$$\int_{x_i}^{x_f} f(x)\,dx \approx h\int_0^4 \Big( g(0)\frac{1}{24}(24 - 50s + 35s^2 - 10s^3 + s^4) +$$

$$+ g(1)\frac{2}{6}(24s - 26s^2 + 9s^3 - s^4) +$$

$$+ g(2)\frac{2}{4}(-12s + 19s^2 - 8s^3 + s^4) +$$

$$+ g(3)\frac{2}{6}(8s - 14s^2 + 7s^3 - s^4) +$$

$$+ g(4)\frac{1}{24}(-6s + 11s^2 - 6s^3 + s^4)\Big)\,ds$$

$$=$$

$$= h\left(g(0)\frac{4}{24}\int_0^4 (24 - 50x + 35x^2 - 10x^3 + x^4)\,dx + \right.$$

$$+ g(1)\frac{4}{6}\int_0^4 (24x - 26x^2 + 9x^3 - x^4)\,dx +$$

$$+ g(2)\frac{4}{4}\int_0^4 (-12x + 19x^2 - 8x^3 + x^4)\,dx +$$

$$+ g(3)\frac{4}{6}\int_0^4 (8x - 14x^2 + 7x^3 - x^4)\,dx +$$

$$\left. + g(4)\frac{4}{24}\int_0^4 (-6x + 21x^2 - 6x^3 + x^4)\,dx \right) =:$$

$$= h\left(\frac{24}{45}g_0 + \frac{64}{45}g(1) + \frac{8}{15}g(2) + \frac{64}{45}g(3) + \frac{24}{45}g(4)\right) =$$

$$\therefore = \frac{2}{45}h\left(7g(0) + 32g(1) + 12g(2) + 32g(3) + 7g(4)\right)$$

Por fim, a fórmula que gera a integral do nosso problema usando a abordagem fechada:

$$\int_{x_i}^{x_{i+4}} f(x)\,dx \approx \frac{2}{45}h\left(7f(x_i) + 32f(x_i + h) + 12f(x_i + 2h) + 32f(x_i + 3h) + 7f(x_i + 4h)\right).$$

Abordagem Aberta

O polinômio de interpolação para por 5 pontos, e $x_i$ e $x_f$ então proibidos nessa abordagem. $x_i, x_0, x_1, x_2, x_3, x_4$ e $x_f$ devem ser equidistantes, portanto a distância entre esses pontos será dada por $h = \frac{\Delta x}{5}$

Assim
$f(x_0) = f(x_i + h) = f(x(s=0)) = g(0)$, $f(x_i + 2h) = f(x(s=1)) = g(1)$,
$f(x_i + 3h) = f(x(s=2)) = g(2)$, $f(x_i + 4h) = f(x(s=3)) = g(3)$ e
$f(x_i + 5h) = f(x(s=4)) = g(4)$.

$x(s) = x_i + h + sh$ satisfaz essas relações pois

$x(0) = x_i + h + 0h = x_0$, $x(1) = x_i + h + 1h = x_1$, $x(2) = x_i + h + 2h = x_2$,
$x(3) = x_i + h + 3h = x_3$ e $x(4) = x_i + h + 4h = x_4$.

Aplicando a mudança de variável temos

$$\int_{x_i}^{x_f} f(x)\,dx \simeq \int_{x_i}^{x_f} p(x)\,dx = \int_{s_i}^{s_f} p(x(s)) \frac{dx(s)}{ds} = h \int_{-1}^{5} p(x(s))\,ds = h \int_{-1}^{5} g(s)\,ds$$

Substituindo o polinômio de substituição na integral temos

$$\int_{x_i}^{x_f} f(x)\,dx = h \int_{-1}^{5} \Big[ g(0) \frac{1}{24}(24 - 50s + 35s^2 - 10s^3 + s^4) +$$
$$+ g(1) \frac{1}{6}(24s - 26s^2 + 9s^3 - s^4) +$$
$$+ g(2) \frac{1}{4}(-12s + 19s^2 - 8s^3 + s^4) +$$
$$+ g(3) \frac{1}{6}(8s - 14s^2 + 7s^3 - s^4) +$$
$$+ g(4) \frac{1}{24}(-6s + 11s^2 - 6s^3 + s^4) \Big] ds =$$

$$= h\left(g(0)\frac{1}{24}\int_{-1}^{5} 24 - 50s + 35s^2 - 10s^3 + s^4\,ds + \right.$$

$$+ g(1)\frac{1}{6}\int_{-1}^{5} 24s - 26s^2 + 9s^3 - s^4\,ds +$$

$$+ g(2)\frac{1}{4}\int_{-1}^{5} -12s + 19s^2 - 8s^3 + s^4\,ds +$$

$$+ g(3)\frac{1}{6}\int_{-1}^{5} 8s - 14s^2 + 7s^3 - s^4\,ds +$$

$$\left. + g(4)\frac{1}{24}\int_{-1}^{5} -6s + 11s^2 - 6s^3 + s^4\,ds\right) =$$

$$\therefore = h\left(\frac{297}{90}g(0) - \frac{289}{45}g(1) + \frac{1117}{25}g(2) - \frac{289}{45}g(3) + \frac{297}{90}g(4)\right)$$

$$\therefore = \frac{6}{20}h\left(11g(0) - 24g(1) + 26g(2) - 24g(3) + 11g(4)\right)$$

Por fim, obtivemos a fórmula que estima a integral do nosso problema usando a abordagem abaixo.

$$\int_{x_i}^{x_{i+4}} f(x)\,dx \approx \frac{6}{20}h\left(11f(x_i + h) - 24f(x_i + 2h) + 26f(x_i + 3h) - 24f(x_i + 4h) + 11f(x_i + 5h)\right)$$