Estimativa do erro para a fórmula de Newton-Cotes aberta com polinômio de grau 2

$$I_e = h\int_{-2}^{2}\left(f(\bar{x}) + f'(\bar{x})(\xi h) + \frac{1}{2!}f''(\bar{x})(\xi h)^2 + \frac{1}{3!}f'''(\bar{x})(\xi h)^3 + \frac{1}{4!}f^{iv}(\bar{x})(\xi h)^4 + \dots\right)d\xi$$

$$I_f = \frac{\Delta x}{3}\cdot\left(2f\left(a+\frac{\Delta x}{4}\right) - f\left(a+\frac{\Delta x\cdot 2}{2}\right) + 2f\left(a+\frac{\Delta x\cdot 3}{2}\right)\right)$$

$$h = \frac{\Delta x}{2}, \quad \bar{x} = \frac{a+b}{2}$$

Usando a série de Taylor nos três pontos de interpolação, temos

$$-f\left(a+\frac{\Delta x}{4}\right) = f\left(\bar{x}+\left(-\frac{h}{2}\right)\right) = f(\bar{x}) - \frac{f'(\bar{x})h}{2} + \frac{f''(\bar{x})h^2}{2!\,4} - \frac{f'''(\bar{x})h^3}{3!\,8} + \frac{f^{iv}(\bar{x})h^4}{4!\,16} + \dots$$

$$-f\left(a+\frac{\Delta x}{2}\right) = f(\bar{x}+0) = f(\bar{x})$$

$$-f\left(a+\frac{3\Delta x}{4}\right) = f\left(\bar{x}+\frac{h}{2}\right) = f(\bar{x}) + \frac{f'(\bar{x})h}{2} + \frac{f''(\bar{x})h^2}{2!\,4} + \frac{f'''(\bar{x})h^3}{3!\,8} + \frac{f^{iv}(\bar{x})h^4}{4!\,16} + \dots$$

Substituindo esses valores em I_f temos

$$I_f = \frac{\Delta x}{3}\Big(2f(\bar{x}) - f(\bar{x}) + 2f(\bar{x}) - 2f'(\bar{x})\frac{h}{2} + 2f'(\bar{x})\frac{h}{2} +$$
$$2f''(\bar{x})\frac{h^2}{2!\,4} + 2f''(\bar{x})\frac{h^2}{2!\,4} - 2f'''(\bar{x})\frac{h^3}{3!\,8} + 2f'''(\bar{x})\frac{h^3}{3!\,8} +$$
$$2f^{iv}(\bar{x})\frac{h^4}{4!\,16} + 2f^{iv}(\bar{x})\frac{h^4}{4!\,16} + \dots\Big) =$$

$$= \frac{\Delta x}{3}\left(3f(\bar{x}) + \frac{f''(\bar{x})h^2}{2} + \frac{f^{iv}(\bar{x})h^4}{96} + \dots\right)$$

Integrando I_e obtemos

$$I_e = h\left(2f(\bar{x}) + \frac{h^2}{2!}f''(\bar{x}) + \frac{h^4}{4!}f^{iv}(\bar{x})\frac{2}{5} + \dots\right)$$

O erro é dado por $E_a = I_e - I_f$. Substituindo os valores obtidos acima temos

$$E_a = 2hf(\bar{x}) + \frac{h^3}{2!}f''(\bar{x})\frac{2}{3} + \frac{h^5}{4!}f^{iv}(\bar{x})\frac{2}{5} + \dots$$
$$- \Delta x f(\bar{x}) - \frac{\Delta x}{3}\cdot\frac{f''(\bar{x})h^2}{2} - \frac{\Delta x}{3}\cdot\frac{f^{iv}(\bar{x})h^4}{96} - \dots$$

$$= \Delta x f(\bar{x}) - \Delta x f(\bar{x}) + \frac{\Delta x^3}{3.8}f''(\bar{x}) - \frac{\Delta x^3}{3.24}f''(\bar{x}) + \frac{h^5}{4!}f(\bar{x})$$
$$- \frac{\Delta x}{3}\cdot\frac{f^{iv}h^4}{96} - \dots = \frac{2\cdot\Delta x^5}{24\cdot 5.32}\cdot f^{iv}(\bar{x}) - \frac{\Delta x^5}{96.3.16}\cdot f^{iv}(\bar{x}) + \dots$$
$$= \frac{\Delta x^5 f^{iv}(\bar{x})}{1920} - \frac{\Delta x^5 f^{iv}(\bar{x})}{4608} = \Delta x^5 f^{iv}(\bar{x})\left(\frac{12-5}{23040}\right) =$$

$$\boxed{\frac{7\cdot\Delta x^5}{23040}\cdot f^{iv}(\bar{x})}$$ Essa é a estimativa do erro.