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CTA200 Assignment 3

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1. QUESTION 1

For this question, we were tasked with recreating the Mandelbrot set. This set is defined by all $c = x+iy$, $-2 \leq x \leq 2$, $-2 \leq y \leq 2$, such that the sequence in Equation 1 converges absolutely for $z_0 = 0$.

$$z_{i+1} = z_i^2 + c \tag{1}$$

To perform this calculation in python, I first used `numpy.meshgrid` to define (1000×1000) coordinates for c within the desired range.

Next, I created a function (`mandelbrot_iteration`) containing a loop that would repeat the calculation of z_i for a maximum number of iterations `max_iter`. `numpy`'s built-in capabilities allows me to perform the calculation for all c simultaneously. At each iteration, the function checks which $|z|$ have surpassed some `arb_limit`. If $|z|$ has diverged, the function stores the iteration number in a new array (`conv`), mutually indexed to c and z. If the sequence doesn't converge, then `conv` stores `max_iter`.

Once the maximum number of iterations has been reached, my function returns `conv` and the array of z values.

I stored this function in a separate python script `complex_iteration.py`. In my notebook, I called the function for my array of c values, and the parameters `arb_limit = 1e6`, `max_iter = 50`. First I used `plt.contourf` to visualize the number of iterations required for z to converge for each c. This plot is shown in Figure 1.

Next, I used `conv` to display the set of convergent c. Yet again, I obtain the Mandelbrot set, as visualized in Figure 2.

2. QUESTION 2

In this question, we were tasked with recreating the integration of the system described in E. N. Lorenz (1963). The system is given in Equation 2, where σ, r, and b are constants. In my analysis, I integrate this system from $t = 0$ to $t = 60$, with a timestep of $dt = 0.01$.

$$\dot{X} = -\sigma(X - Y) \tag{2}$$
$$\dot{Y} = rX - Y - XZ \tag{3}$$
$$\dot{Z} = -bZ + XY \tag{4}$$
$$\tag{5}$$

First, I create a function in python (`lorenz_system`) that takes in t, X, Y, Z in addition to the parameters σ, r, b. The function returns a vector $\dot{W} = (\dot{X}, \dot{Y}, \dot{Z})$. I set $\sigma = 10.$, $r = 28$, and $b = 8./3.$ as specified in E. N. Lorenz (1963). Next, using the initial guess $W_0 = (0., 1., 0.)$, I used `scipy.optimize.solve_ivp` to develop a solution. In my analysis, I integrate the system from $t = 0$ to $t = 60$, with a timestep of $dt = 0.01$.

Figure 3 contains the solution for Y from $t = 0$ to $t = 30$. This figure is comparable to Lorenz's Fig 1, which displays similar results. My integration, similarly to Lorenz's, displays inconsistent oscillations after $N \approx 1600$. While the shape of these oscillations are the same, the sign differs with Lorenz's results. I attribute these deviations to a difference in integration methods in both analyses.

I can also plot the phase space of my solutions from $t = 14$ to $t = 19$, as shown in Figures 4 and 5. Again, these are visually similar to Lorenz's Figure 2. The deviations in the results are again attributed to difference in integration methods.

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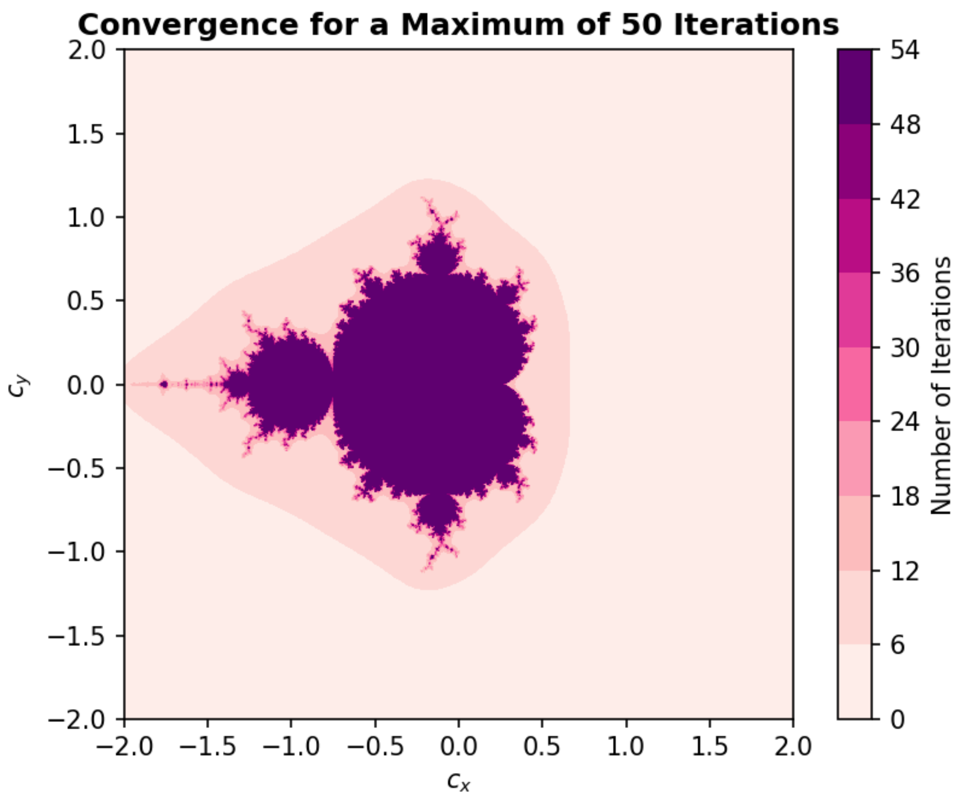

Figure 1. Visualization of the number of iterations required for z to converge for each c. I obtain the Mandelbrot set as visualized in S. Avalos-Bock (2009).

Lastly, I examine the effects of small perturbations on the solution. I reintegrate the system, this time with initial condition $W'_0 = (0.1. + 1e - 8, 0.)$. I calculate the distance between the corresponding solution W' and the original solution W. A plot of the distance between the two solutions over time is shown in 6. The result is roughly linear growth in the perturbation over time. The growth of the perturbation indicates that the equilibrium state of the solution is unstable.

3. FLOATS

REFERENCES

Avalos-Bock, S. 2009,

Lorenz, E. N. 1963, https://journals.ametsoc.org/view/journals/atsc/20/2/1520-0469_1963_020_0130_dnf_2_0_co_2.xml

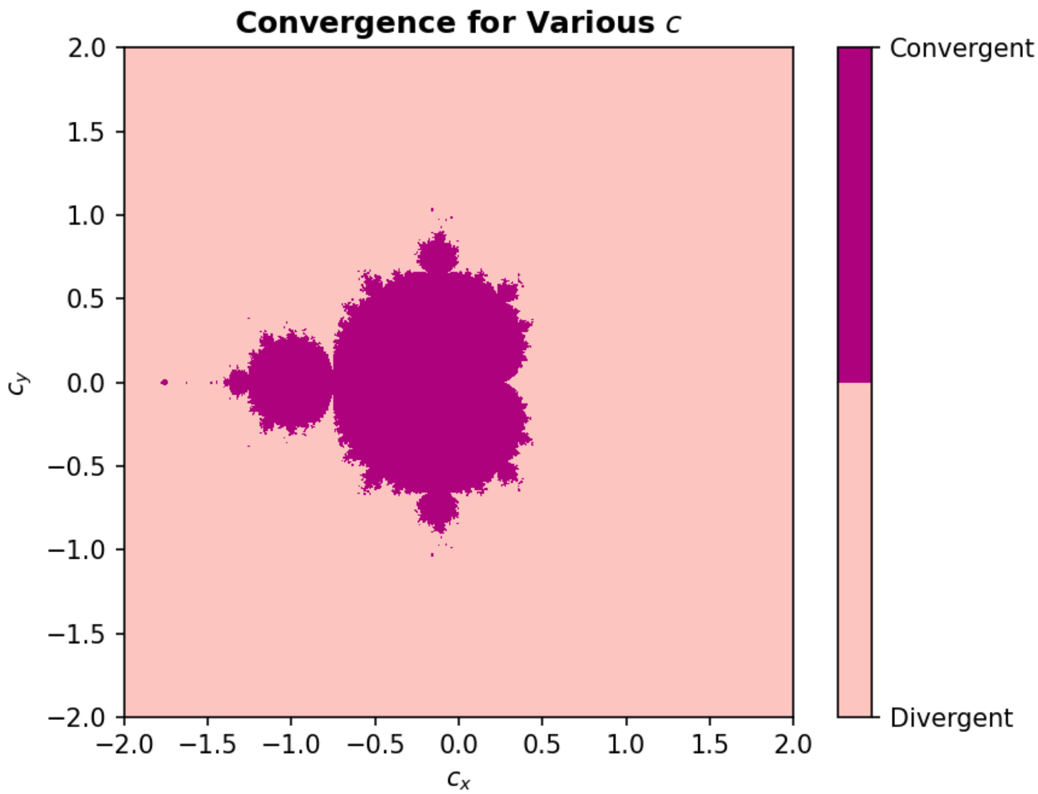

Figure 2. Visualization of the the Mandelbrot set. The darker magenta represents c such that z converges, while the light pink is the divergent region.

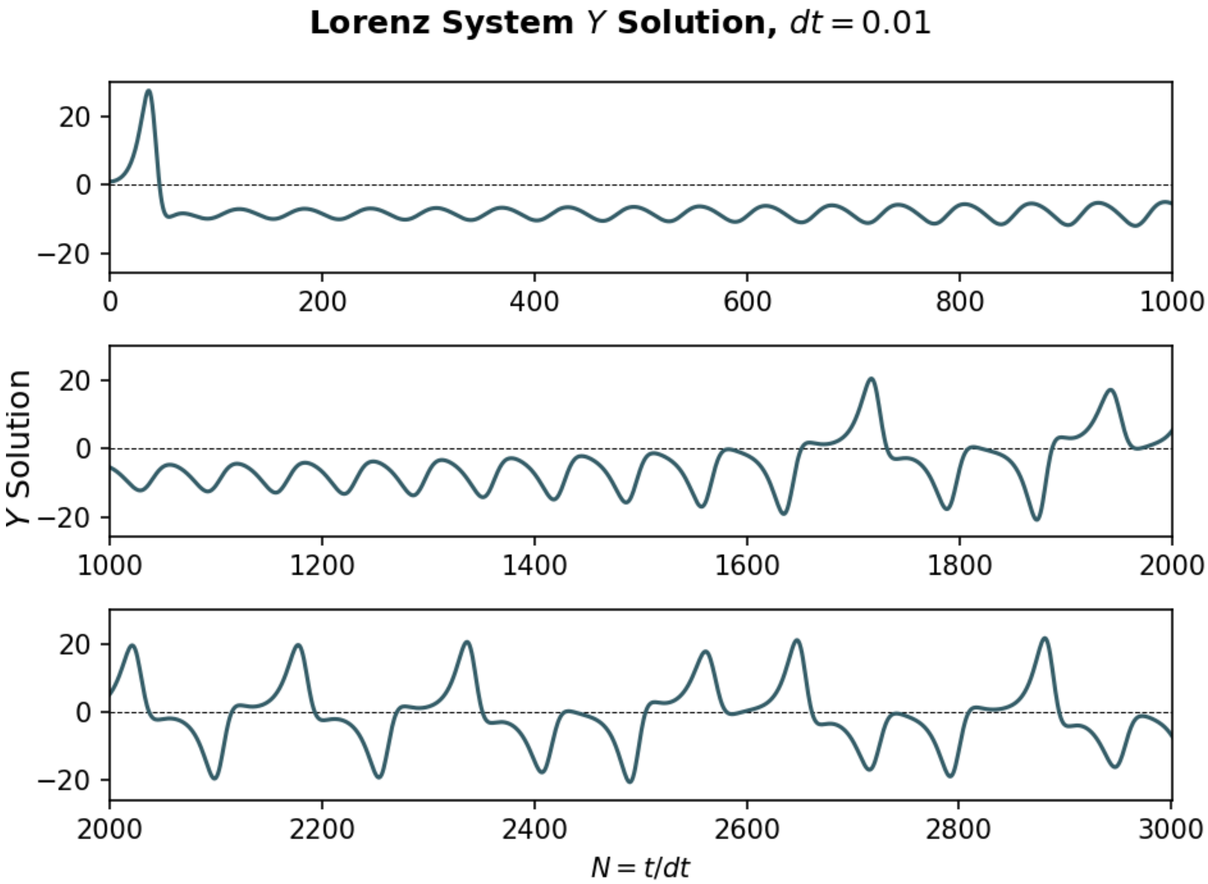

Figure 3. my Recreation of Figure 1 in E. N. Lorenz (1963). We see a slight deviation from the results from the paper after $N \approx 1600$, attributed to the difference in numerical integration techniques.

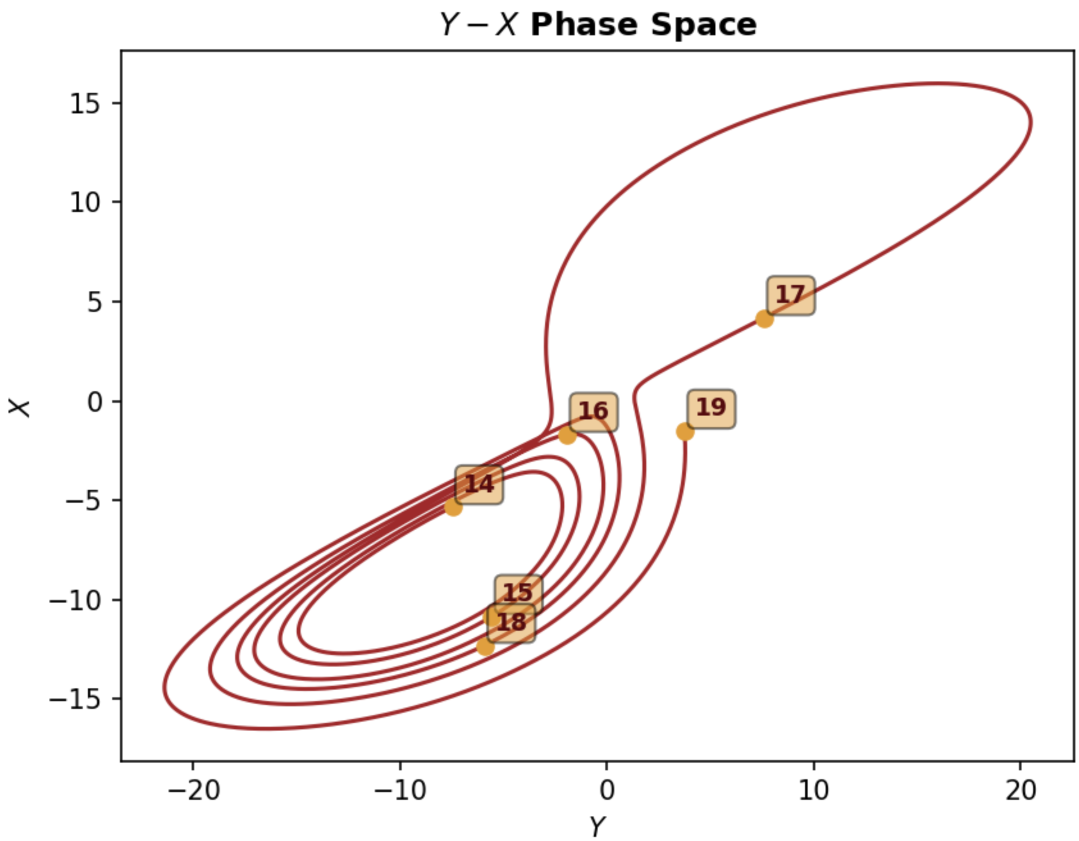

Figure 4. Phase Space of $Y - X$ Solution from $t = 14$ to $tb = 19$.

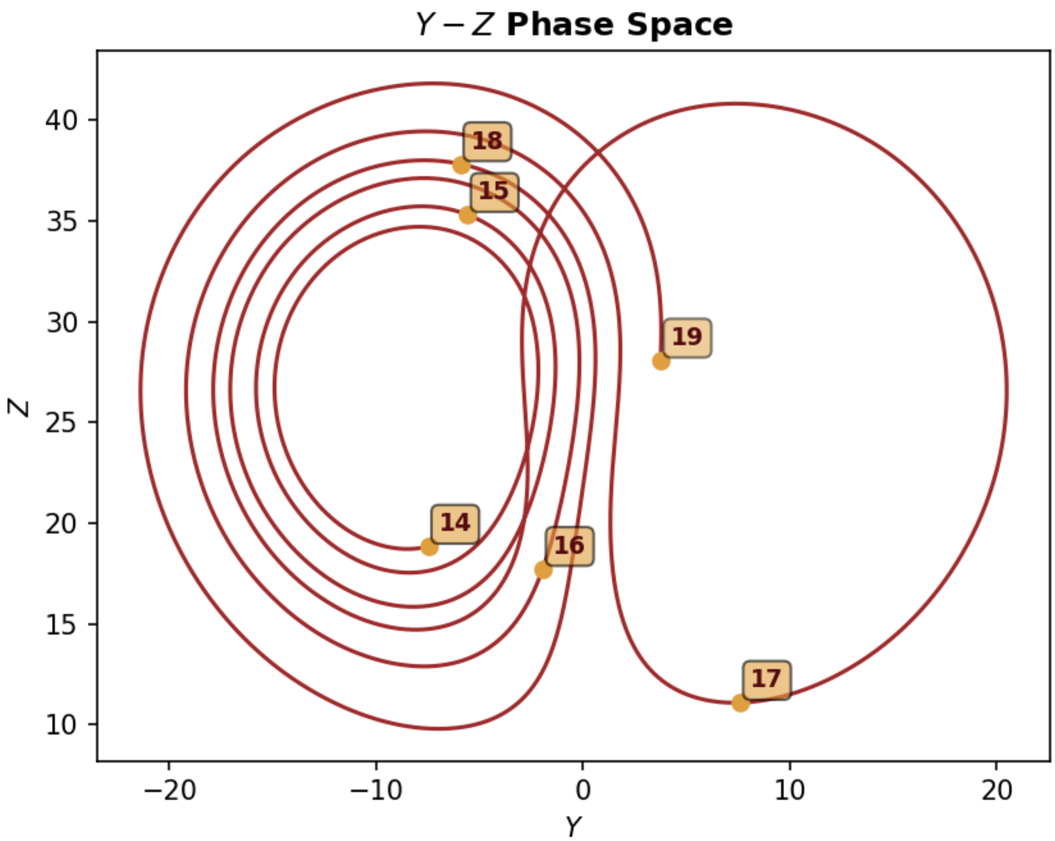

Figure 5. Phase Space of $Y - Y$ Solution from $t = 14$ to $tb = 19$.

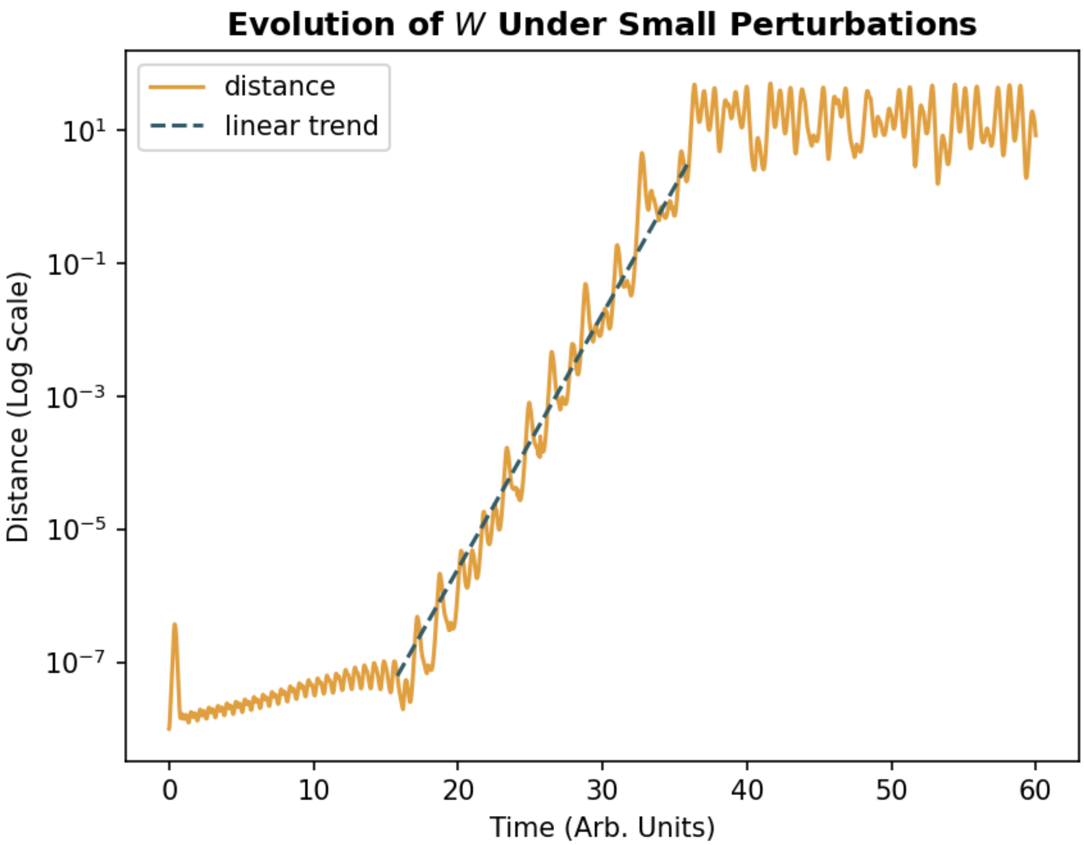

Figure 6. Distance between solutions of the system under small perturbations. The roughly linear behaviour indicates that the equilibrium of the system is unstable.