TRIANGLES
Exercise 7.1

Contents

1 Problem

Q3. AD and BC are equal perpendiculars to a line segment. Show that CD bisects AB.

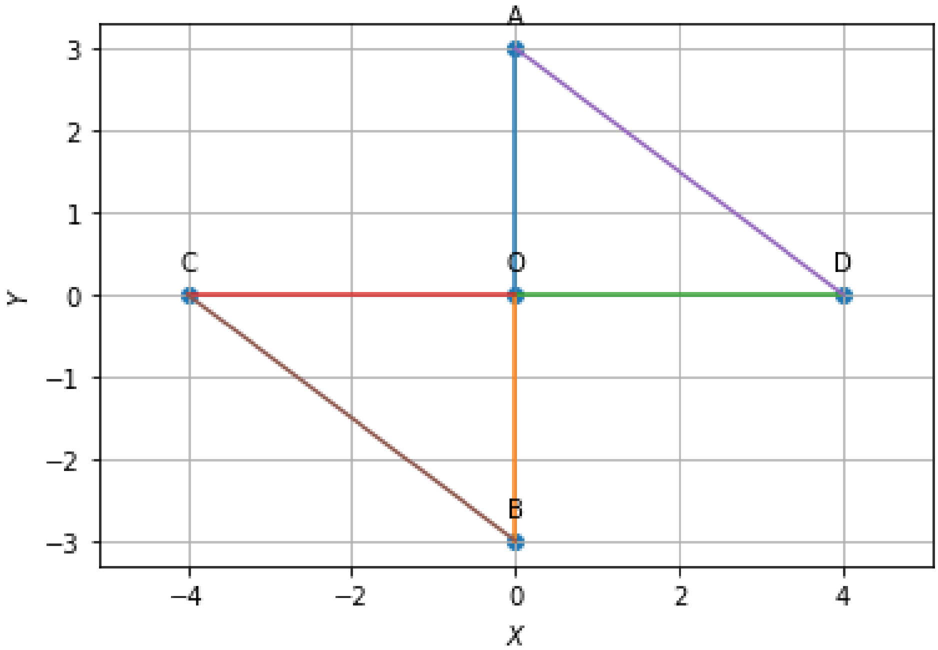

Figure 1:

2 Construction

The input parameters are the lengths a and c.

Symbol	Value	Description
a	4	AD=BC
c	3	OA=OB
θ	arctan$\left(\frac{c}{a}\right)$	$\angle D = \angle C$

3 Solution

Given:

$$AD = BC \tag{1}$$
$$\angle CBO = \angle DAO \tag{2}$$

To prove :

$$\angle ODA = \angle OCB \tag{3}$$

Proof
From the given information Vectors are
$\mathbf{A} = \begin{pmatrix} 0 \\ c \end{pmatrix}, \mathbf{D} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ -c \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} -a \\ 0 \end{pmatrix}$
The directional vectors are:

$$\mathbf{m_1} = \mathbf{O} - \mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{4}$$
$$\mathbf{m_2} = \mathbf{D} - \mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \tag{5}$$

The Normal vectors are:

$$\mathbf{n_1} = \mathbf{O} - \mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{6}$$
$$\mathbf{n_2} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ -3 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{7}$$

$$\theta_1 = \cos^{-1}\left(\frac{\mathbf{m_1}^T\mathbf{m_2}}{\|\mathbf{m_1}\|\|\mathbf{m_2}\|}\right) \tag{8}$$
$$\theta_2 = \cos^{-1}\left(\frac{\mathbf{n_1}^T\mathbf{n_2}}{\|\mathbf{n_1}\|\|\mathbf{n_2}\|}\right) \tag{9}$$

Substitue (4) and (5) in (8)

$$\theta_1 = \cos^{-1}\left(\frac{\begin{pmatrix} -4 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix}}{(\sqrt{16})(\sqrt{25})}\right) = \cos^{-1}\left(\frac{-16}{20}\right) \tag{10}$$

Substitue (6) and (7) in (9)

$$\theta_2 = \cos^{-1}\left(\frac{\begin{pmatrix} 4 & 0 \end{pmatrix}\begin{pmatrix} -4 \\ 3 \end{pmatrix}}{(\sqrt{16})(\sqrt{25})}\right) = \cos^{-1}\left(\frac{-16}{20}\right) \tag{11}$$

From the (10) and (11) $\theta_1 = \theta_2$, so

$$\triangle OBC \cong \triangle OAD \tag{12}$$
$$OA = OB \tag{13}$$