STRAIGHT LINES Exercise 7.1

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1 Problem

Q3. AD and BC are equal perpendiculars to a line segment. Show that CD bisects AB.

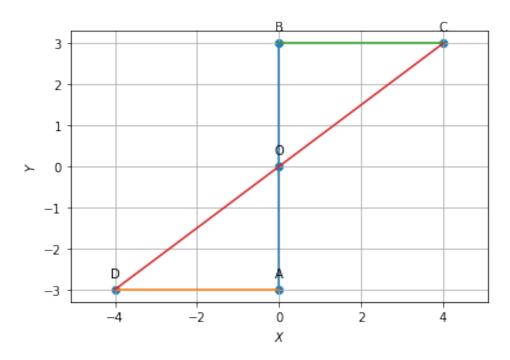


Figure 1:

2 Construction

The input parameters for this construction are

Symbol	Values	Description
A	$\begin{pmatrix} 0 \\ -3 \end{pmatrix}$	point A
В	$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	point B
C	$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$	point C
D	$\begin{pmatrix} -4 \\ -3 \end{pmatrix}$	point D
О	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	point O

Table 2:

Solution 3

Given:

$$AD = BC (1)$$

$$\angle CBO = \angle DAO$$
 (2)

To prove:

$$\angle ODA = \angle OCB$$
 (3)

Proof

The directional vectors are:

$$m_1 = O - D \tag{4}$$

$$\mathbf{m_2} = \mathbf{D} - \mathbf{A} \tag{5}$$

The Normal vectors are:

$$\mathbf{n_1} = \mathbf{O} - \mathbf{C} \tag{6}$$

$$\mathbf{n_2} = \mathbf{C} - \mathbf{B} \tag{7}$$

(8)

$$\theta_1 = \cos^{-1} \left(\frac{m_1^T m_2}{\|m_1\| \|m_2\|} \right) \tag{9}$$

$$\theta_{1} = \cos^{-1} \left(\frac{m_{1}^{T} m_{2}}{\|m_{1}\| \|m_{2}\|} \right)$$

$$\theta_{2} = \cos^{-1} \left(\frac{n_{1}^{T} n_{2}}{\|n_{1}\| \|n_{2}\|} \right)$$
(9)
$$(10)$$

If $\theta_1 = \theta_2$

$$\triangle OBC \cong \triangle OAD \tag{11}$$

$$OA = OB (12)$$