

STRAIGHT LINES
Exercise 7.1

Contents

1 Problem	1
2 Construction	2
3 Solution	2

1 Problem

Q3. AD and BC are equal perpendiculars to a line segment. Show that CD bisects AB.

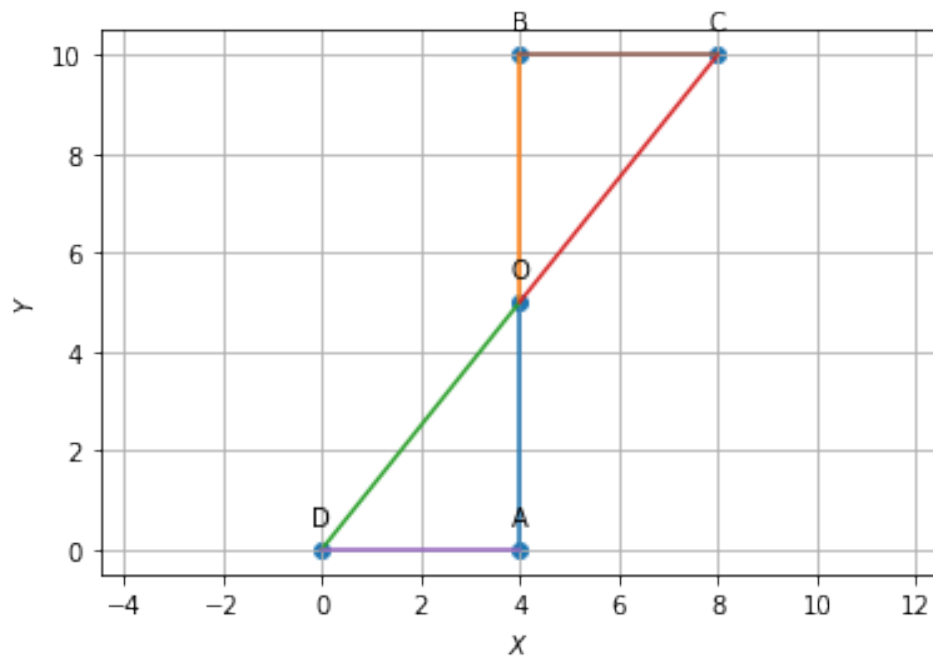


Figure 1:

2 Construction

The input parameters are the lengths a and c .

Symbol	Value	Description
a	4	$AD=BC$
c	3	$OA=OB$
θ	$\arctan\left(\frac{c}{a}\right)$	$\angle D = \angle C$
\mathbf{O}	$\sqrt{a^2 + c^2} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$	Point \mathbf{O}

3 Solution

Given:

$$AD = BC \quad (1)$$

$$\angle CBO = \angle DAO \quad (2)$$

To prove :

$$\angle ODA = \angle OCB \quad (3)$$

Proof

Given Vectors are

$$\mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 10 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 8 \\ 10 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ and } \mathbf{O} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

The directional vectors are:

$$\mathbf{m}_1 = \mathbf{O} - \mathbf{D} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \quad (4)$$

$$\mathbf{m}_2 = \mathbf{D} - \mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad (5)$$

The Normal vectors are:

$$\mathbf{n}_1 = \mathbf{O} - \mathbf{C} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 8 \\ 10 \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \quad (6)$$

$$\mathbf{n}_2 = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 8 \\ 10 \end{pmatrix} - \begin{pmatrix} 4 \\ 10 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (7)$$

$$\theta_1 = \cos^{-1} \left(\frac{\mathbf{m}_1^T \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \right) \quad (8)$$

$$\theta_2 = \cos^{-1} \left(\frac{\mathbf{n}_1^T \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \right) \quad (9)$$

Substitue (4) and (5) in (8)

$$\theta_1 = \cos^{-1} \left(\frac{\begin{pmatrix} 4 & 5 \end{pmatrix} \begin{pmatrix} -4 \\ 0 \end{pmatrix}}{25.61} \right) = \cos^{-1} \left(\frac{-16}{25.61} \right) \quad (10)$$

Substitue (6) and (7) in (9)

$$\theta_2 = \cos^{-1} \left(\frac{\begin{pmatrix} -4 & -5 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix}}{25.61} \right) = \cos^{-1} \left(\frac{-16}{25.61} \right) \quad (11)$$

From the (10) and (11) $\theta_1 = \theta_2$, so

$$\triangle OBC \cong \triangle OAD \quad (12)$$

$$OA = OB \quad (13)$$