CLASS-11 CHAPTER-10 STRAIGHT LINES

Excercise 10.4

Q6. Find the equation of the line parallel to y-axis and drawn through the point of intersection of the lines x - 7y + 5 = 0 and 3x + y = 0.

Solution

The given line equations represented in vector form:

$$(1 \quad -7) \mathbf{x} = -5 \tag{1}$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{2}$$

(3)

The intersection of two lines is given by:

$$\begin{pmatrix} 1 & -7 & 5 \\ 3 & 1 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{4}$$

$$\begin{pmatrix} 1 & -7 & 5 \\ 3 & 1 & 0 \end{pmatrix} \xrightarrow{R_2 = R_2 - 3R_1} \begin{pmatrix} 1 & -7 & 5 \\ 0 & 22 & -15 \end{pmatrix} \mathbf{x} = 0 \tag{5}$$

$$\stackrel{R_2 = \frac{R_2}{22}}{\longleftrightarrow} \begin{pmatrix} 1 & -7 & 5 \\ 0 & 1 & -\frac{15}{22} \end{pmatrix} \mathbf{x} = 0$$
(6)

$$\stackrel{R_1 = R_1 + 7R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & \frac{5}{22} \\ 0 & 1 & \frac{-15}{22} \end{pmatrix} \mathbf{x} = 0 \tag{7}$$

$$\mathbf{x} = \begin{pmatrix} -\frac{5}{22} \\ \frac{15}{22} \end{pmatrix} \tag{8}$$

(9)

To find the equation of the line parallel to the y-axis and the line passing through the P is given as

$$\mathbf{n}^{\top} \left(\mathbf{x} - \mathbf{P} \right) = 0 \tag{10}$$

Since a line is parallel to the y-axis, normal vector is in the direction of the x-axis is

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{11}$$

Let $\mathbf{P} = \begin{pmatrix} -\frac{5}{22} \\ \frac{15}{22} \end{pmatrix}$ and by using (10) and (11)

$$\begin{pmatrix}
1 & 0
\end{pmatrix}
\begin{pmatrix}
\mathbf{x} - \begin{pmatrix}
-\frac{5}{22} \\
\frac{15}{22}
\end{pmatrix}
\end{pmatrix} = 0$$

$$\begin{pmatrix}
1 & 0
\end{pmatrix}
\mathbf{x} = -\frac{5}{22}$$
(13)

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = -\frac{5}{22} \tag{13}$$

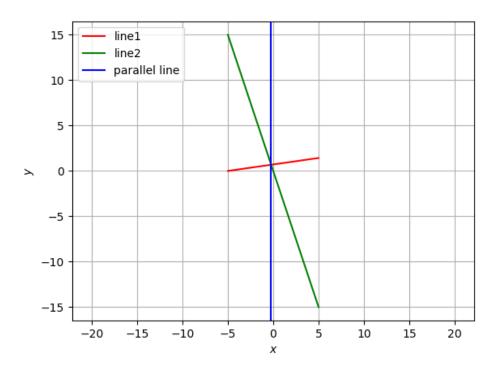


Figure 1: