STRAIGHT LINES
Exercise 7.1

Contents

1 Problem

Q3. AD and BC are equal perpendiculars to a line segment. Show that CD bisects AB.

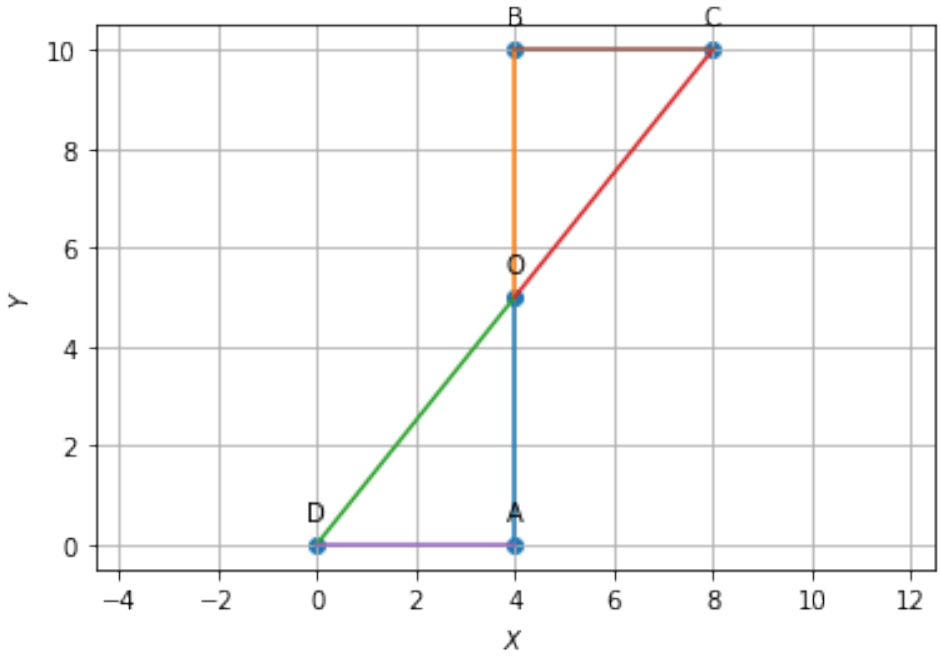

Figure 1:

2 Construction

The input parameters are the lengths a and c.

Symbol	Value	Description
a	4	AD=BC
c	3	OA=OB
θ	$\arctan\left(\frac{c}{a}\right)$	$\angle D = \angle C$
$\mathbf{O}$	$\sqrt{a^2+c^2}\begin{pmatrix} cos\theta \\ sin\theta \end{pmatrix}$	Point $\mathbf{O}$

3 Solution

Given:

$$AD = BC \tag{1}$$
$$\angle CBO = \angle DAO \tag{2}$$

To prove :

$$\angle ODA = \angle OCB \tag{3}$$

Proof
The directional vectors are:

$$\mathbf{m_1} = \mathbf{O} - \mathbf{D} \tag{4}$$
$$\mathbf{m_2} = \mathbf{D} - \mathbf{A} \tag{5}$$

The Normal vectors are:

$$\mathbf{n_1} = \mathbf{O} - \mathbf{C} \tag{6}$$
$$\mathbf{n_2} = \mathbf{C} - \mathbf{B} \tag{7}$$
$$\tag{8}$$

$$\theta_1 = cos^{-1}\left(\frac{\mathbf{m_1}^T\mathbf{m_2}}{\|\mathbf{m_1}\|\|\mathbf{m_2}\|}\right) \tag{9}$$
$$\theta_2 = \cos^{-1}\left(\frac{\mathbf{n_1}^T\mathbf{n_2}}{\|\mathbf{n_1}\|\|\mathbf{n_2}\|}\right) \tag{10}$$

If $\theta_1 = \theta_2$

$$\triangle OBC \cong \triangle OAD \tag{11}$$
$$OA = OB \tag{12}$$